Valuing Life as an Asset, as a Statistic, and at Gunpoint[1]

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Abstract

The Human Capital (HK), and Statistical Life Values (VSL) lack a common theoretical background, and differ sharply in their empirical pricing of a human life. This paper makes four contributions to the theory, and measurement of the latter. First, we provide a unified framework to formally define, and relate the HK, and the VSL. Second, we use this setting to introduce a third life value calculated *at Gunpoint* (GPV), i.e. the maximal Hicksian willingness to pay to avoid certain, instantaneous death. Third, we associate a flexible human capital model to the common framework to characterize the three life valuations in closed-form. Fourth, we structurally estimate the three life values. Our results confirm the relevance of reduced-form HK, and VSL estimates, identify the role of technological, distributional, and preferences parameters, and clarify the formal links between the alternative valuations.

Keywords: Values of Human Life, Human Capital, Value of Statistical Life, Willingness to pay, Equivalent Variation, Endogenous Morbidity, Endogenous Mortality, Structural Estimation.

JEL Classification: J17, D91, G11.

1 Introduction

The life you save may be your own (Schelling, 1968)

1.1 Motivation and outline

Motivation Evaluating the price of a human life has long generated a deep interest in economic research.[1] Indeed, life valuations are often used in public health and safety debates, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. Economic life values are also resorted to in wrongful death litigation, for example in cases involving occupational or end-users' exposure to fatality.

Three main sources of difficulty make the life pricing exercise particularly challenging. First, a human life is by definition non-divisible, i.e. a life must be valued as an entity, and not partially. This implies that any marginal valuation, e.g. via small incremental risks to life, must eventually be integrated back into a unitary life value. Second, a human life is non-marketed, i.e. the life of someone else life cannot be acquired through markets. The absence of equilibrium prices implies that the economic value of a human life must somehow be inferred from relevant, and measurable proxies. Finally, ethical considerations induce significant discomfort in computing – and eventually comparing – the life values of different persons.

The two most-widely used life valuation frameworks differ in how these three challenges are dealt with. First, the Human Capital (HK) approach does not balk at personalizing valuations, and associates the value of an individual's life to the economic value embodied in his human capital. Relying on standard asset pricing, the HK valuation is the expected present value of the dividend stream associated with human capital, where the dividend is proxied by the marketed labor income, net of measurable expenses to maintain that capital. Second, the Value of a Statistical Life (VSL), introduced by Drèze (1962); Schelling (1968), relies on a stated, or inferred willingness to pay (WTP) to avert (resp. attain) small increases (resp. reductions) in exposure to death risks as its main building block. Under specific preferences assumptions, a collective WTP to save one unidentified (i.e. statistical) life can be recovered through a linear aggregation of the

[1] Landefeld and Seskin (1982) reference human-capital based evaluations of the value of life dating back to Petty (1691).

individual WTP's. Focusing on the value of an unidentified, rather than personalized, agent's life thus conveniently avoids addressing the uncomfortable ethical issues associated with the latter.

Notwithstanding cautionary claims to the contrary,[2] both the HK, and the VSL are ultimately gauging the value of an identical underlying object, and should presumably come up with similar answers to the single question of how much is worth a human life. However, despite pricing a common element, the two life valuations yield vastly different values in practice.[3] Understanding these differences is complicated by the absence of common theoretical underpinnings that encompass both valuations. Consequently, most HK, and VSL evaluations are reduced-form empirical exercises that rely on minimal theoretical foundations, and are performed within disparate settings that further complicate comparisons.

Contributions This paper makes four different contributions to the theory, and the measurement of life values. Our first contribution proposes a common theoretical background linking both the Human Capital, and Statistical Life values. We start from a generic dynamic human capital problem in which an agent facing an uncertain horizon selects investment in his human capital, where the latter augments labor income. Assuming the existence of a solution to this problem satisfying weak preference for life over death, we use standard asset pricing to define the HK value as the discounted dividend stream, i.e. the income, net of investment, along the *optimal dynamic path*. Second, we rely on the associated indirect utility (i.e. the welfare at the optimum) which we combine with the Hicksian Equivalent Variation (EV, Hicks, 1946) to formally define the maximal willingness to pay to avoid any exogenous change in death risk exposure. The VSL can then be defined formally in two equivalent ways: (i) as the (negative of the) marginal rate of substitution (MRS) between death exposure and wealth, calculated through the

[2]In his opening remarks, Schelling (1968, p. 113) writes

> "This is a treacherous topic and I must choose a nondescriptive title to avoid initial misunderstanding. It's not the worth of a human life that I shall discuss, but of 'life saving', of preventing death. And it's not a particular death, but a statistical death. What it is worth to reduce the probability of death – the statistical frequency of death – within some identifiable group of people, none of whom expects to die except eventually. "

[3]For example, Huggett and Kaplan (2016) identify HK values between 300 K–900 K\$, whereas the U.S. Department of Transportation recommends using a VSL-type amount of 9.4 MM\$ (U.S. Department of Transportation, 2016).

indirect utility, and (ii) as the marginal WTP with respect to death risk. This common setup ensures that the HK, and the VSL are both evaluated from a single, micro-founded perspective corresponding to a shared underlying dynamic problem.

Our second contribution also relies on this common theoretical framework to define a *third* valuation alternative that forthrightly addresses the three aforementioned measurement challenges. The objectives are to gauge the economic value of a human life without recourse to indirect proxies and/or arbitrary aggregation assumptions. Instead, we address the non-divisibility, non-marketability, as well as ethical issues by resorting to the unitary shadow value of life accruing to its main beneficiary, i.e. the willingness to pay that leaves an agent indifferent between living and dying. The Hicksian EV again provides a natural theoretical background to elicit this shadow value, and can be thought of as asking a question most of us would rather avoid having to answer:

> "What is the maximum amount you would be willing to pay in order to survive in a credible 'your money or your life' highwayman threat or, equivalently, how much would you value your own life?"

We refer to the corresponding amount as the *Gunpoint Value of Life* (GPV). To paraphrase Schelling (1968)'s seminal title, 'the life you save *is* your own' in our highwaymen valuation.

Compared to the HK, and VSL alternatives, the Gunpoint Value presents several advantages that are discussed in further details in Section 1.2. First, unlike the HK models, the Gunpoint Value does not uniquely ascribe the economic worth of an agent to the labor income he generates, but instead accounts for all pledgeable disposable resources, including financial wealth. Moreover, the GPV also incorporates human capital services other than income determination, such as self-insurance against income shocks. Second, unlike the VSL, the GPV does not extrapolate measurable responses to small probabilistic changes in the likelihood of death, but instead explicitly values a person's life as *an entity*, and does so without external assumptions regarding integrability from marginal to total value of life. Finally, the GPV directly confronts the ethical pitfalls in calculating an external life value. Unlike the VSL which computes a collective willingness to pay to save *someone*, we let someone compute his own intrinsic value through his individual willingness to pay to save *himself*.

Our third contribution is also theoretical, and consists of analytical calculations of the WTP, as well as of the Human Capital, Statistical Life, and Gunpoint Values of life. To do so, we rely on a flexible human capital model that we developed elsewhere (Hugonnier et al., 2013). This model follows a long tradition in associating an agent's human capital to his health, and endogenizes investment, and exposure to sickness, and death risks, while guaranteeing weak preference for life over death. It is thus general enough to accommodate other well-known health demand models as testable special cases. Importantly, closed-form solutions are available, allowing us to compute the analytic expressions corresponding to the formally-defined life values. In particular, we can price the net dividend to human capital along the optimal path to calculate the HK value. We can use the indirect utility to compute the Hicksian willingness to pay, from which we calculate the VSL. Finally, we are able to use the WTP to calculate the maximal amount an agent would pay to avoid certain, instantaneous death to recover the GPV. These three closed-form life valuations, as well as the WTP stem from the same underlying model, and are thus directly comparable. Consequently, we can precisely assess the contribution to value of fundamentals, such as preferences, risks distributions, or technology, as well as financial, and human resources. This analysis allows us to pinpoint how the HK, VSL, and GPV are related to one another.

Our fourth, and final contribution is empirical, and consists of *structural* estimates of the three different life valuations. More precisely, we adopt a revealed-preference perspective to estimate the structural parameters of the benchmark model, using PSID data that correspond to the optimal investment, consumption, portfolio, and health insurance policies. We can then combine the structural parameters with observed wealth, and health status to calculate the analytical expressions for the Human Capital, Statistical, and Gunpoint Values of life. Whereas the latter is new, and has not been previously estimated, the HK, and the VSL can be contrasted with reduced-form estimates in an out-of-sample assessment of our results.

Main findings The generic human capital model yields the optimal investment from which the net dividends stream along the optimal path can be recovered to calculate the HK value of life. It also produces the indirect utility through which the WTP, VSL, and GPV are calculated. Standard properties of the indirect utility implies that the willingness to pay is increasing, and concave in the increment in death risk. It follows that the marginal WTP (corresponding to the theoretical VSL) overstates the limiting WTP (corresponding to the GPV), and that the slope for small increments (corresponding to the empirical VSL) understates the theoretical Value of a Statistical Life.

Applying our benchmark human capital model first reveals that the market value of the net dividend stream corresponding to the HK value equals the capitalized fixed income, plus the shadow value of the human capital. The latter is the capital stock, times its morbidity-adjusted Tobin's-Q. The market-based HK value is independent of individual characteristics such as preferences, and wealth, and is pricing income, technological, and distributional features exclusively. Second, the willingness to pay is a weighted average of total disposable resources, and of a term encompassing preferences, as well as endogenous mortality exposure. The WTP is lower for high elasticity of inter-temporal substitution through a *Live Fast and Die Young* effect whereby higher death risk exposure is substituted more easily by higher consumption. In the limit, the WTP converges to net total wealth only, i.e. financial wealth, minus capitalized subsistence consumption, plus the Human Capital value of life. Third, the MWTP corresponding to the theoretical VSL is also increasing in net total wealth and is lower at high elasticity. Fourth, the Gunpoint value is the limiting WTP, and is equal to net total wealth. Since death is instantaneous and certain in a highwayman threat, the agent thus pays out all pledgeable resources, net of unpledgeable subsistence requirements, and attitudes towards towards risk, or time play no role in the GPV. Finally, the theoretical predictions are qualitatively robust for the restricted case corresponding to Grossman (1972); Ehrlich and Chuma (1990).

Our structural estimation shows that the deep parameters are realistic, and that preferences are elastic with respect to inter-temporal substitution. Morever, the restricted case is formally rejected when tested against our general benchmark. The estimates for the HK value (738 K$), and the VSL (8.14 MM$) are well in line with those obtained in the reduced-form literature. The Gunpoint Value (460 K$) is in the same range as the HK, with a lower value explained by the financial wealth deficit with respect to capitalized minimal consumption. Importantly, our results confirm the strong concavity of the willingness to pay, i.e. that the MWTP is decreasing in the increment in death risk exposure. This curvature – rather than disjoint valuation concepts – is thus the main element explaining the much higher values of life obtained under the VSL, compared to the HK, and Gunpoint values.

From a different perspective, Pratt and Zeckhauser (1996) argue that concentrating the costs, and benefits of death risk changes (an extreme case of which is the GPV) leads to two opposing effects on valuation, and therefore on the VSL. The willingness to pay increases as the longevity gains are personalized, possibly being infinite in the case on one's own life value (*Dead anyways* effect). However, concentrating the costs lowers the WTP through decreasing marginal utility of wealth, and of life (*High payments* effect). They conjecture that the latter effect dominates the former in the case of large changes in death risk. Our theoretical, and empirical results unambiguously confirm this conjecture, and explain why the Gunpoint, and the closely-related HK value are both much lower than the Statistical Value of Life.

The rest of the paper is organized as follows. We first present the related literature in Section 1.2, while Section 2 formally defines, and discusses the links between the WTP, GPV, and VSL. The benchmark human capital model is described in Section 3, and the application relying on this model is outlined in Section 4. The empirical strategy is discussed in Section 5, with deep parameters, and values of life estimates being presented in Section 6.

1.2 Related literature

1.2.1 Human Capital values of life

The HK models evaluate the human capital embodied in the expected discounted net value of the lifetime labor income flows, net of associated investment, and that are foregone upon death.[4] Well-known issues related to this approach include the treatment of non-labor activities, the appropriate rate of discounting, and the endogeneity of survival probabilities.[5]

As for HK models, we do calculate the net present value of income streams that are lost upon death. Unlike HK models however, that value is computed in closed-form, i.e. accounting for potential endogeneities linked to the income stream and/or the

[4]See Jena et al. (2009) for partial- and general-equilibrium HK life values.

[5]Conley (1976) provides additional discussion of HK approaches while Huggett and Kaplan (2016) address the discounting issues.

rate of discounting. Furthermore, whereas our modeling strategy does allow for labor income flows, this hypothesis is not restrictive for two reasons. First, by assuming that labor income is health-dependent, the reduced capacity to work for unhealthy individuals is explicitly taken into account. Since health is an adjustable variable, any endogeneity of labor income is thus implicitly accounted for. Second, Hugonnier et al. (2013) show that the base model with health-dependent labor income and health-independent preferences can be rewritten as an equivalent one with health-independent income, and direct preference for health. Put differently, our model choice is equivalent to one with only exogenous income (which could be zero), and where agents directly value better health in the instantaneous utility function. Finally, discounting is also internally determined, and explicitly incorporates the endogeneity of exposure to death risk. Agents thus fully internalize their adjustable longevity in discounting future choices, and resources.

1.2.2 Value of a Statistical Life

The empirical VSL alternative relies instead on explicit and implicit evaluations of the Hicksian WTP for a small reduction in fatality risk which is then linearly extrapolated to obtain the value of life.[6] Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.[7] Examples of the latter include responses to prices and fines in the use of life-saving measures such smoke detectors, speed limitations or seat belt regulations. Implicit VSL research also exploits the fatality risk and wages nexus in labor markets to identify the death-income tradeoff. In particular, the Hedonic Wage (HW) variant of VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/workers characteristics, the wage elasticity with respect to job fatality risk can be estimated, and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008).

[6] See equation (7) below for a canonical example of the empirical VSL.

[7] A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the revealed-preference VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta*-meta analysis of the stated- and revealed-preferences valuations of life.

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. First, the assumed exogeneity of the change in fatality risk can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. Second, agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Third, and related, whose preferences are involved in the risk/income tradeoff and how well these arbitrage are understood often remains an open question. For example, high fatality risk employment may attract workers with low risk aversion and/or high time discount rates; generalizing the wages risk gradient to the entire population could understate true valuation of life. Moreover, because wages are an equilibrium object in the HW variant of the VSL, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death. Finally, as was the case for HK measures, HW estimates relate primarily to workers, and are hardly adaptable to other non-employed groups, such as young, elders, or the unemployed.

Our approach offers several advantages in calculating the value of life. First, by emphasizing the destruction of the human capital in the willingness to pay to avoid certain death, we bridge a gap between HK and VSL literature. Unlike HK however, we do not uniquely associate the service flows of human capital to labor revenues, but explicitly calculate other self-insurance services provided by health. Second, and related, any endogeneity of morbidity and mortality risk exposition is fully accounted for in the model. Indeed, we explicitly ascribe an increase in fatality risk to the exogenous component in death intensity; the optimal WTP fully accounts for possible adjustments to such exogenous increases in death risk via the endogenous elements to mortality risk.

Third, by focusing on a unified theoretical setting with an individual human capital problem, the question of whose risks preferences are involved is not an issue in our setup. Indeed, we rely on a widely-used panel (PSID) accounting for households' consumption, financial, and health-related decisions to elicit the WTP, and life valuations. Resorting to such a representative panel ensures that these values can therefore be generalized to the entire population. Unlike the HW variant of the VSL, our GPV approach neither relies exclusively on equilibrium objects such as wages, nor does it apply uniquely to workers to elicit the WTP. This also means that the GPV reflects the value of life to a representative subset of those who are primarily concerned, i.e. the holders of the life capital. Finally, unlike the empirical VSL, our theoretical setup makes no assumption on the shape of the WTP, but rather establishes its properties through the optimization process. Indeed, we show that, consistent with economic intuition, the marginal value ascribed to small increases in death intensity is positive, but falling in the latter. The direct implication is that the linear extrapolation that is implicit in the VSL sharply overestimates the value of one's own life.

A related, albeit semi-structural measure of life value is provided by Hall and Jones (2007). As for the VSL, they adopt a marginal value perspective by equating the latter to the marginal cost of saving a human life. By specifying, and separately estimating a technology for health production, and inversely relating mortality risks exposure to health status, they impute the cost of reducing mortality by a given amount. Dividing this cost by that amount yields a VSL-inspired life value, e.g. corresponding to 1.9 MM$ for an individual aged 40-44 (Hall and Jones, 2007, Tab. 1, p. 60). As for them, we model, and estimate the health production, and death distributional processes. However, our valuations framework does not exclusively rely on technological and distributional parameters, but includes preferences, and status in explicitly computing the marginal and total values of life along the optimal path.

2 A Common Framework for Life Valuation

This section outlines a common framework that will be relied upon to formally define, and link the Human Capital, Statistical, and Gunpoint Values of Life. Our main building block is an underlying human capital problem for which the optimal policies, and associated indirect utility function can be solved. We combine these solutions with standard asset pricing, and Hicksian Variation to characterize the three life valuations.

2.1 Underlying Human Capital Problem

Consider an agent's human capital problem defined by a stochastic age at death T_m, an instantaneous death probability $\mathcal{P} \in [0, 1]$, a human capital H, and associated increasing income function $Y(H)$, a financial wealth W, as well as the relevant distributional assumptions with respect to mortality, human, and financial assets. For this program, the agent selects the money value of investment in his human capital I, and other controls X so as to maximize utility U:

$$\begin{aligned} V(W,H,\mathcal{P}) &= \sup_{I,X} U, \quad \text{subject to:} \\ \mathrm{d}H &= \mathrm{d}H(H,I), \\ \mathrm{d}W &= \mathrm{d}W(W,Y(H),I,X). \end{aligned} \tag{1}$$

We assume that the agent's preferences, and constraints in (1) satisfy standard properties such that the indirect utility $V = V(W,H,\mathcal{P})$, i.e. the agent's continuation utility at the optimal policies, is monotone increasing and concave in the (W,H) space. We further assume weak preference for life over death. In particular, the indirect utility is assumed to be decreasing and convex in death probability $\mathcal{P}$, and satisfies:

$$V(W,H,\mathcal{P}) \geq V^m \equiv V(W,H,1) > -\infty, \quad \forall W,H,\mathcal{P}, \tag{2}$$

where V^m denotes the finite utility at death. Standard examples of the latter include the seminal Yaari (1965); Hakansson (1969) paradigm ($V^m \equiv 0$), or 'warm glow' effects of bequeathed wealth ($V^m = V(W_{T_m},H,\mathcal{P})$, e.g. Yogo (2016); French and Jones (2011); De Nardi et al. (2009)). Observe that monotonicity, curvature, and finite utility assumptions imply the existence of decreasing and convex indifference curves in the wealth, and life probability $(1-\mathcal{P})$ space.

2.2 Human Capital Value of Life

First, as is well known, the Human Capital Value of life is the market value of the net dividend flow associated with human capital, and that is foregone upon death (e.g. Huggett and Kaplan, 2016, 2013). In our setting, this net dividend is the marketed income $Y(H)$, minus the money value of associated investment expenses I, where both are evaluated at the optimum to problem (1):

Definition 1 (HK value of life) *The Human Capital Value of life $v_{h,t} = v_h(W_t,H_t,\mathcal{P}_0)$ is the expected discounted present value over stochastic horizon T_m of labor revenue flows,* *net of investment costs:*

$$v_{h,t} = E_t \int_0^{T_m} m_{t,\tau} \left[Y(H^*_\tau) - I^*_\tau \right] \mathrm{d}\tau, \tag{3}$$

where $m_{t,\tau}$ is a stochastic discount factor induced by the assets' prices, and (H^*, I^*) are evaluated along the optimal path solving (1)*.*

As a canonical example, assume deterministic horizon T_m, constant discount rate r, and deterministic growth rate $g^n < r$ for net income $Y^n = Y(H) - I$, for which the HK value simplifies to:

$$v_h = \frac{Y^n}{r - g^n} \left(1 - e^{-T_m(r-g^n)}\right).$$

The human capital value of life in this special case is therefore decreasing in interest rate r, and increasing in net income level Y^n, net growth rate g^n, and horizon length T_m.

2.3 Willingness to pay

Next, consider a permanent exogenous change Δ in the instantaneous probability of death from base level $\mathcal{P}_0$ to $\mathcal{P}_0^* = \mathcal{P}_0 + \Delta$, and rely on the indirect utility (1) to define the Hicksian Equivalent Variation as follows:

Definition 2 (WTP) *The maximal willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ to avoid a permanent change Δ in death risk exposure $\mathcal{P}$ is implicitly given as the solution to:*

$$V\left(W - v, H, \mathcal{P}_0\right) = V\left(W, H, \mathcal{P}_0 + \Delta\right), \tag{4}$$

where $V(W, H, \mathcal{P})$ solves (1)*.*

For unfavorable changes $\Delta > 0$, equation (4) indicates indifference between paying the equivalent variation $v > 0$ to remain at base risk $\mathcal{P}_0$, and not paying, but face higher death risk $\mathcal{P}_0^* > \mathcal{P}_0$; for favorable changes $\Delta < 0$, the agent is indifferent between accepting $-v > 0$ and foregoing lower death risk exposure.[8]

[8] An alternative formulation relies instead on the Hicksian willingness to accept compensation (WTA) to face Δ, implicitly defined as $v^a = v^a(W, H, \mathcal{P}_0, \Delta)$ in:

$$V\left(W + v^a, H, \mathcal{P}_0 + \Delta\right) = V\left(W, H, \mathcal{P}_0\right).$$

Note also that the monotonicity, and curvature assumptions on the indirect utility $V(W,H,\mathcal{P})$ in (1) are sufficient to yield a monotone increasing, and concave willingness to pay with respect to increment in death risk Δ. To see this, substitute $v(W,H,\mathcal{P}_0,\Delta)$ in (4), take derivatives, and re-arrange to obtain:

$$\frac{\partial v}{\partial \Delta} = \frac{-V_{\mathcal{P}}}{V_W} \geq 0, \tag{5a}$$

$$\frac{\partial^2 v}{\partial \Delta^2} = \frac{V_{\mathcal{P}\mathcal{P}} - V_{WW}\left(\partial v/\partial \Delta\right)^2}{-V_W} \leq 0, \tag{5b}$$

i.e. the marginal willingness to pay (MWTP) is equal to the (negative of the) marginal rate of substitution between death risk, and wealth, and is decreasing in mortality risk exposure.

2.4 Value of Statistical Life

Third, as is well known, the VSL is a measure of the marginal rate of substitution (MRS) between the probability of life and wealth, evaluated at base risk (e.g. Aldy and Smyth, 2014; Andersson and Treich, 2011; Bellavance et al., 2009). In the context of the continuation utility $V(W,H,\mathcal{P})$, is is thus the negative of the MRS between $\mathcal{P}$, and W. Equivalently, the discussion of the WTP properties in equation (5a) establishes that this MRS is also the marginal willingness to pay evaluated at base risk, such that:

Definition 3 (VSL) *The Value of a Statistical Life $v_s = v_s(W,H,\mathcal{P}_0)$ is the negative of the marginal rate of substitution between the probability of death, and wealth, and also the marginal WTP evaluated at base risk:*

$$v_s = \left.\frac{-V_{\mathcal{P}}(W,H,\mathcal{P})}{V_W(W,H,\mathcal{P})}\right|_{\mathcal{P}=\mathcal{P}_0}, \tag{6a}$$

$$= \frac{\partial v(W,H,\mathcal{P}_0,\Delta)}{\partial \Delta} = \lim_{\Delta \to 0} \frac{v(W,H,\mathcal{P}_0,\Delta)}{\Delta}, \tag{6b}$$

where $V(W,H,\mathcal{P})$ solves (1)*, and $v(W,H,\mathcal{P}_0,\Delta)$ solves* (4)*.*

This WTA perspective is however not suitable for Gunpoint settings in the absence of bequests. Indeed, whereas paying out the WTP in a highwaymen threat is rational, accepting compensation against instantaneous, and certain death when terminal wealth in not bequeathed, and life is preferred is not. Since we abstract from bequests in our benchmark model in Section 3, we therefore adopt the WTP perspective in (4).

Figure 1 plots the indifference curves (in blue) in the wealth, and life probability space. The VSL in (6a) is the slope of the red tangent evaluated at base death risk $\mathcal{P}_0$, and is equivalent to the total wealth spent to save one life corresponding to the distance [a,d] (e.g. Andersson and Treich, 2011, Fig. 17.1, p. 398).

Moreover, contrasting the theoretical definition of the VSL as a MWTP in (6b) with its empirical counterpart reveals that the latter can also be interpreted as a slope of the willingness to pay to avoid small changes in death risk. To see this, consider a canonical example (e.g. Aldy and Viscusi, 2007), whereby we suppose that each agent $i = 1, 2, \ldots, N$ has WTP of $v^i(W, H, \mathcal{P}_0, \Delta)$ for a $\Delta = N^{-1}$ reduction in death risk. Assuming identical preferences, and statuses, the empirical value of a statistical life is obtained as:

$$v_s^e = \sum_{i=1}^{N} v^i(W, H, \mathcal{P}_0, \Delta) = N v(W, H, \mathcal{P}_0, \Delta) = \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}, \tag{7}$$

i.e. the collective willingness to pay to save one unidentified individual is equal to a slope of the WTP for Δ small. The theoretical measure of the VSL (6b) is therefore the limiting value of the slope in (7) when the change Δ tends to zero. Figure 2 illustrates the difference between the theoretical, and empirical VSL valuations. From properties (5), the willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ (in blue) is an increasing, concave function of the change in death risk Δ. The theoretical VSL v_s in (6b) is the marginal willingness to pay ($\Delta \to 0$), i.e. the slope of the red tangent evaluated at base death risk. Equivalently, it is the linear projection corresponding to the total wealth spent to save one person (i.e. reach $\mathcal{P}_0^* = 1.0$) and it is equal to the distance [a,d]. The empirical Value of a Statistical Life v_s^e in (7) is computed for a small change $\Delta > 0$, and corresponds to the slope of the green line; equivalently, it is the distance [e,f]. As Figure 2 makes clear, the empirical VSL measure v_s^e will understate its theoretical counterpart v_s when Δ is large, and when the WTP is concave, i.e under diminishing marginal values of wealth, and of additional expected life. Our analytical characterization of the VSL will consequently rely on its theoretical definition in equations (6).

2.5 Gunpoint Value of Life

We next introduce the Gunpoint Value (GPV) as a third valuation of life. To do so, we combine preference for life (2) with the Hicksian Equivalent Variation in (4) to define the GPV as follows.

Definition 4 (GPV) *The Gunpoint Value* $v_g = v_g(W, H, \mathcal{P}_0)$ *is the maximal WTP to avoid a change* $\Delta = 1 - \mathcal{P}_0$ *and is implicitly given as the solution to:*

$$V(W - v_g, H, \mathcal{P}_0) = V^m \tag{8}$$

where $V(W, H, \mathcal{P})$*, and* V^m *solve* (1)*, and satisfies* (2)*.*

The Gunpoint Value $v_g(W, H, \mathcal{P}_0)$ in equation (8) is implicitly defined as the maximal payment that leaves the agent indifferent between paying v_g and remaining at base death risk $\mathcal{P}_0$, and not paying and face instantaneous and certain death $\mathcal{P}_0^* = \mathcal{P}_0 + \Delta = 1$. The willingness to pay v_g can thus be interpreted as the maximal amount paid at gunpoint in order to survive an *ex-ante* unforecastable, and *ex-post* credible highwaymen threat. Figure 2 illustrates the links between the WTP, VSL, and the GPV. The Gunpoint Value is the limiting WTP when death is certain, and is equal to the distance [b,c]. A linear extrapolation under either the theoretical, or the empirical VSL will thus over-estimate the value attributed to one's own life when the WTP is increasing and concave.

To summarize, the Human Capital v_h, as well as the willingness to pay v, Statistical v_s, and Gunpoint v_g Values of life can be computed jointly from the optimal policies X^*, I^* , and indirect utility $V(W, H, \mathcal{P})$ associated with a generic underlying human capital problem. These values will in general be functions of financial W, and human capital H, as well as depend on base exposure to death risk P_0. Finally, the differences between the empirical, and theoretical Statistical, and Gunpoint Values crucially depend on the properties of the WTP function.

3 A Benchmark Human Capital Model

Our objective is to calculate the valuations (v_h, v, v_s, v_g) that are formally defined in Section 2 as functions on the optimal policies I^*, X^*, and associated indirect utility $V(W, H, \mathcal{P})$. Towards that aim, we resort to a dynamic model of human capital that we developed elsewhere. The life cycle framework of Hugonnier et al. (2013) associates an agent's human capital H to his health, and focuses on endogenously-determined health expenditures in a setting where the time horizon is finite and stochastic. Modeling health as human capital follows a long tradition (e.g. see the Hicks Lecture by Becker, 2007, for a review), and is motivated by several analogies. First, while productive skills can be augmented via education, training, and experience, health can be accumulated through medical spending, and leisure choices made by the agent. Second, both skills and health are durable capital, and subject to depreciation, either via obsolescence, or biological decline. Third, skills and health are both non-marketed, and non-transferable, and are fully depreciated at death. Fourth, skills affect income both in levels, and in self-insurance against adverse labor market shocks, while health determines the capacity to work, and provides self-insurance against sickness shocks.

3.1 Overview

For completeness, the main building blocks of the Hugonnier et al. (2013) are briefly summarized here. First, health $H \geq 0$ is a durable and depreciable good whose law of motion is given by:

$$\mathrm{d}H_t = \left[I_t^{\alpha} H_{t-}^{1-\alpha} - \delta H_{t-} \right] \mathrm{d}t - \phi H_{t-} \mathrm{d}Q_{st} \tag{9}$$

where $\delta \in (0, 1)$ is a deterministic depreciation, and $\mathrm{d}Q_s$ is a stochastic morbidity shock whose occurrence further depreciates the health stock by a factor $\phi \in (0, 1)$, and where $H_{t-} = \lim_{s \uparrow t} H_s$ is health prior to occurrence of the sickness shock. Health investment $I \geq 0$ is subject to diminishing returns, and is motivated in part by self-insurance services. Indeed, healthier agents face lower Poisson morbidity intensity $\lambda_s(H)$, and can also lower their Poisson mortality intensity $\lambda_m(H)$ given by:

$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}} \in [\lambda_{s0}, \eta], \tag{10}$$

$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m} \tag{11}$$

The parameters $\lambda_{k1} \geq 0, k = s, m$, determine the extent to which exposure to morbidity and mortality can be adjusted through better health, with $\xi_k \geq 0$ representing diminishing returns, whereas the parameters $\lambda_{k0} \geq 0$ capture endowed exogenous exposure. Within the context of this continuous-time model, the instantaneous death probability $\mathcal{P}$ introduced earlier can be obtained by noting that:

$$\Pr[\text{Death}(t, t+h)] = \lambda_m(H)h + o(h),$$

for a small h. In the subsequent life valuation, we will henceforth analyze exogenous changes Δ in death risk $\mathcal{P}$ resulting from permanent changes in the exogenous death risk exposure λ_{m0} in the death intensity (11).

Second, financial wealth W evolves according to the budget constraint:

$$dW_t = [rW_{t-} + Y_t - c_t - I_t]\,\mathrm{d}t + \pi_t \sigma_S [\mathrm{d}Z_t + \theta \mathrm{d}t] + x_t [\mathrm{d}Q_{st} - \lambda_s(H_{t-})\mathrm{d}t], \tag{12}$$

$$Y_t = y + \beta H_t \tag{13}$$

where, in addition to I, the control variables include c as consumption, π as the risky portfolio, and x as demand for actuarially-fair health insurance, and where r it the interest rate, and $\theta = \sigma_S^{-1}(\mu - r)$ is the market price of financial risk. The income process in (13) comprises an exogenous component y, whereas the expression βH provides further motivation for investing in one's health, and can equivalently be interpreted as improved work capacity for healthier agents, or as utilitarian services implicitly procured by better health (see Hugonnier et al., 2013, Remark 3).

Finally, the agents' objectives are:

$$V(W_t, H_t) = \sup_{(c,\pi,x,I)} U_t,$$

where preferences are:

$$U_t = 1_{\{T_m > t\}} E_t \int_t^{T_m} \left(f(c_\tau, U_{\tau-}) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} F_k(U_{\tau-}, H_{\tau-}, \Delta_k U_\tau) \right) \mathrm{d}\tau, \tag{14}$$

with

$$f(c_t, U_{t-}) = \frac{\rho\, U_{t-}}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_{t-}} \right)^{1-\frac{1}{\varepsilon}} - 1 \right), \tag{15}$$

$$F_k = U_{t-}\, \lambda_k(H_{t-}) \left[\frac{\Delta_k U_t}{U_{t-}} + u(1; \gamma_k) - u\left(1 + \frac{\Delta_k U_t}{U_{t-}}; \gamma_k \right) \right], \tag{16}$$

where $\Delta_k U_t = E_{t-}[U_t - U_{t-} | \mathrm{d}Q_{kt} \neq 0]$ is the expected utility jump associated with either morbidity ($k = s$), or mortality ($k = m$). The utility U in (14), combined with the Kreps-Porteus aggregator function $f(c, U)$ in (15), and risk exposure penalties in (16) is generalized from Duffie and Epstein (1992). It is characterized by subjective discount rate $\rho > 0$, minimal subsistence consumption $a \geq 0$, as well as by non-expected utility which disentangles the elasticity of inter-temporal substitution (EIS) $\varepsilon \geq 0$, from source-dependent aversion with respect to the three sources of risk. Indeed, as is the case for the traditional penalty for financial risk exposure $\sigma_t(U) = \mathrm{d}\langle U, Z\rangle_t/\mathrm{d}t$ (with aversion γ), the agent is separately penalized for being exposed to the Poisson morbidity risk F_s (with aversion γ_s), and for exposure to the Poisson mortality risk F_m (with aversion $\gamma_m \in (0, 1)$) where $u(\cdot; \gamma_k)$ is the CRRA function with curvature γ_k for $k = s, m$.

3.2 Special cases

The model of Hugonnier et al. (2013) generalizes other demand-for-health frameworks found in the literature. In particular, the following models are nested as special cases:

1. **Exogenous morbidity** by restricting $\lambda_{s1} = 0$ in (10) after readjusting base intensity λ_{s0} upwards to maintain mean exposure;

2. **No morbidity** by restricting $\phi = 0$ in (9), and $\lambda_{s0} = \lambda_{s1} = 0$ in (10), as well as by shutting down the demand for health insurance $x = 0$ in (12), and the aversion to morbidity risk $\gamma_s = 0$, in penalty (16);

3. **Exogenous expected horizon** by restricting $\lambda_{m1} = 0$ in (11) after readjusting λ_{m0} upwards base intensity to maintain mean exposure;

4. **Von Neumann-Morgenstern (VNM) utility** by restricting $\varepsilon = 1/\gamma$ in aggregator (15);

5. **Source-independent risk aversion** constant relative risk aversion (CRRA) obtains by imposing $a = 0$ (no minimal consumption), with source-independent risk aversion is by restricting $\gamma_s = \gamma_m = 0$ (no morbidity risk), or $\gamma_s = \gamma$ (if morbidity risk) in (15), and (16).

As an example, Ehrlich and Chuma (1990), who characterize the solutions to the seminal Grossman (1972) model, can be obtained by jointly imposing restrictions 2–5. The restricted model then becomes:

$$\begin{aligned} V(W_t, H_t) &= \sup_{(c,\pi,I)} U_t, \\ U_t &= 1_{\{T_m > t\}} E_t \int_t^{T_m} e^{-\rho\tau} u\left(c_\tau; \gamma\right) \mathrm{d}\tau, \end{aligned} \tag{17}$$

where $u(\cdot;\gamma)$ is the CRRA function with curvature γ, and subject to:

$$\begin{aligned} \mathrm{d}H_t &= \left[I_t^\alpha H_t^{1-\alpha} - \delta H_t\right] \mathrm{d}t, \\ \lambda_m(H_t) &= \lambda_{m0}, \\ dW_t &= \left[rW_t + Y_t - c_t - I_t\right] \mathrm{d}t + \pi_t \sigma_S \left[\mathrm{d}Z_t + \theta \mathrm{d}t\right], \\ Y_t &= y + \beta H_t. \end{aligned} \tag{18}$$

Compared to our benchmark Hugonnier et al. (2013), the Grossman (1972); Ehrlich and Chuma (1990) model in (17), and (18) thus abstracts entirely from morbidity (and associated health insurance, and risk aversion), while it supposes an exogenously set horizon, and risk neutrality with respect to death risk. In addition, VNM preferences restrict financial risk aversion to be the inverse of the elasticity of inter-temporal substitution. The theoretical, and empirical implications of these restrictions for life valuation will be reviewed in Sections 4.5, 6.1, and 6.3.

3.3 Key features for life valuation

Four features of the Hugonnier et al. (2013) framework are especially relevant for life valuation purposes. First, this model follows standard approaches dating back to Yaari (1965); Hakansson (1969) in normalizing utility at death $V^m \equiv 0$. Second, unlike VNM models with CRRA preferences, non-expected utility guarantees that the indirect utility is measured in the same units as consumption, regardless of parameter values. It follows that positive excess consumption $c - a \geq 0$ – a necessary condition for subsistence – is associated with positive welfare, and therefore (weak) preference for life over death $V \geq 0 = V^m$, as required in (2).[9] Third, under a Poisson mortality assumption, it is possible to rewrite the original problem with finite and stochastic horizon T_m and constant discounting ρ as an infinite horizon program with health-dependent discounting $\rho + \lambda_m(H)$ (see Hugonnier et al., 2013, for details).

Fourth, as summarized in Appendix B.1, the agent's problem (14) can be solved for the choice variables (c, π, x, I), and therefore for the indirect utility V from which the expression for life valuations introduced in Section 2 can be computed. The approximate solution proceeds in two steps, first a closed-form expression (referred to as order-0) is recovered for the exogenous intensity case corresponding to $\lambda_s(H) = \lambda_{s0}$ in (10), and $\lambda_m(H) = \lambda_{m0}$ in (11). Then a first-order expansion around $\lambda_{s1}, \lambda_{m1} = 0$ is performed to obtain the endogenous intensities adjustments, and thus recover the approximate solutions (referred to as order-1).

Under regularity, and transversality conditions (37) outlined in Appendix B.1, these approximate first-order solutions reveal two important characteristics for valuing a human life. First, the indirect utility associated with the agent's problem in the Hugonnier et al. (2013) model is given as:

$$V(W, H, \lambda_{m0}) = \Theta(\lambda_{m0}) \left[N_1(W, H) - \lambda_{m1} H^{-\xi_m} l_m(\lambda_{m0}) N_0(W, H)\right], \tag{19}$$

where any dependence on λ_{m0} is explicitly stated. The expression $N_0(W, H)$ captures the agent's order-0 (i.e. without endogenous intensities) net total wealth along the optimal path:

$$N_0(W, H) = W + \frac{y - a}{r} + \underbrace{HB}_{P_0(H)}. \tag{20}$$

This net worth is the sum of financial wealth W, plus the present value of the fixed-income stream, net of minimal consumption $(y - a)/r$, plus $P_0(H) = HB$ which is the order-0 shadow value of the human capital along the optimal path. The price $B \geq 0$ is

[9]See Rosen (1988); Hall and Jones (2007) for a discussion of the violation of weak preference for life (2) in VNM-CRRA settings. See Hugonnier et al. (2013) for why this condition is verified under non-expected utility.

the health's Tobin-Q solving $g(B) = 0$, subject to $g'(B) < 0$ in:

$$g(B) = \beta - (r + \delta + \phi\lambda_{s0})B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}}, \tag{21}$$

and is increasing in health sensitivity of income β, and decreasing in depreciation parameters $(\delta, \phi, \lambda_{s0})$. The order-1 net total wealth corrects net worth for endogenous morbidity and is given as:

$$\begin{aligned} N_1(W, H) &= N_0(W, H) - \lambda_{s1} l_s H^{-\xi_s} P_0(H) \\ &= W + \frac{y - a}{r} + \underbrace{HB\left[1 - \lambda_{s1} l_s H^{-\xi_s}\right]}_{P_1(H)} \end{aligned} \tag{22}$$

and is lower than $N_0(W, H)$ at all wealth and health levels. Indeed, the order-1 human capital $P_1(H)$ in (22) adjusts the shadow value of the health capital for optimal sickness risk exposure $\lambda_{s1} l_s H^{-\xi_s}$, where:

$$\begin{aligned} l_s &= \frac{\phi(\eta - \lambda_{s0})}{r - F(1 - \xi_s)} \geq 0, \\ F(x) &= x(\alpha B)^{\frac{\alpha}{1-\alpha}} - x\delta - \lambda_{s0}\left[1 - (1 - \phi)^x\right]. \end{aligned} \tag{23}$$

Because the health shock intensity (10) is mechanically higher for $\lambda_{s1} > 0$, the health stock is more subject to stochastic depreciation; its value must be discounted accordingly, and is lower than under exogenous exposure to sickness $\lambda_{s1} = 0$. The analytical solutions (21), and (23) indicate that $P_1(H)$ – and therefore net total wealth – is a function of the parameters in the health production technology (9), morbidity risk (10) and income (13) only, and is independent of attitudes with respect to risk, and time, as well as of horizon length. The horizon independence stems from the equivalence with an infinite horizon problem discussed earlier.

Second, the exposure to exogenous death risk λ_{m0} affects welfare through its impact on the marginal propensity to consume (MPC) exclusively. To see this, note that the marginal value of $N_1(W, H)$ in (19) is given as:

$$\Theta(\lambda_{m0}) = \rho \left(\frac{A(\lambda_{m0})}{\rho}\right)^{\frac{1}{1-\varepsilon}} \geq 0, \tag{24}$$

where $A(\lambda_{m0})$ is the marginal propensity to consume given by:

$$A(\lambda_{m0}) = \varepsilon\rho + (1-\varepsilon)\left(r - \frac{\lambda_{m0}}{1-\gamma_m} + \frac{\theta^2}{2\gamma}\right) \geq 0, \tag{25}$$

which also conditions the endogenous mortality adjustment term:

$$l_m(\lambda_{m0}) = \frac{1}{(1-\gamma_m)[A(\lambda_{m0}) - F(-\xi_m)]} \geq 0. \tag{26}$$

The effect of mortality exposure on the MPC in (25) crucially depends on the elasticity of inter-temporal substitution ε. An increase in death risk λ_{m0} induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume. First, more discounting makes future consumption less desirable and shifts future towards current consumption (i.e. by increasing the MPC); this *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution $\varepsilon > 1$. Second, higher discounting of future consumption requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC); that effect is dominant at low elasticity of inter-temporal substitution $\varepsilon \in (0,1)$. Observe in the latter case that an upper bound on death intensity is required to maintain non-negativity of the MPC in (25):

$$\lambda_{m0} \leq \bar{\lambda}_{m0} = (1-\gamma_m)\left[\left(\frac{\varepsilon}{1-\varepsilon}\right)\rho + \left(r + \frac{\theta^2}{2\gamma}\right)\right], \quad \text{when } \varepsilon \in (0,1). \tag{27}$$

Finally, unit elasticity implies exact cancellation of the two effects, and results in a mortality risk-independent MPC that is equal to the subjective discount rate ρ.

We note in closing that whereas the sign of the effects of death risk λ_{m0} on the MPC (25) depends on the EIS, preference for life implies that it always reduces the marginal value of total wealth, i.e. $\Theta'(\lambda_{m0}) \leq 0, \forall \varepsilon$ in (24). By non-negativity of (A, Θ), it follows that $\Theta(\lambda_{m0}) = 0$ when $\lambda_{m0} \to \infty$, i.e. the agent's marginal value of net total wealth converges to zero as mortality becomes more certain.

4 Application to Values of Life

We next calculate the associated closed-form expressions for life valuation relying on the solution for the benchmark human capital model of Hugonnier et al. (2013), starting with the HK value of life in Section 4.1, followed by the WTP in Section 4.2, the VSL in Section 4.3, and the GPV in Section 4.4. In these sections, we will assume throughout that the optimal rules verifying the regularity conditions outlined in Appendix B.1 are being followed by the agents. We also resort to a similar first-order approximation to compute the valuations. We then close this section by comparing the valuation results for the baseline model with those in other life cycle models of demand for health in Section 4.5.

4.1 Human Capital Value of Life

The HK value of life outlined in Definition 1 is computed as follows.

Proposition 1 (HK value) *Up to a first order approximation, the Human Capital Value of life* (3) *is:*

$$v_h(H) = \frac{y}{r} + P_1(H) \tag{28}$$

where $P_1(H)$ *is the human capital value in* (22).

From equation (3) in Definition 1, the first component y/r in the HK value is the NPV of the fixed component of income (13) over an infinite horizon. The second component $P_1(H)$ is the expected discounted value of the health-dependent part of income βH, net of expenses I, along the optimal path. As equations (19), and (22) make clear, the Human Capital Value of life is thus embedded in the net total wealth $N_1(W, H)$, and therefore directly linked with welfare.

For reasons discussed earlier, the determinants of $P_1(H)$ are independent of preferences, and horizon length, and reflect only the technology, income, and sickness risk. In particular, the presence of morbidity risk exposure $l_s\lambda_{s1}H^{-\xi_s}$ along the optimal path augments the expected depreciation of the human capital stock. It consequently lowers the economic value of H, and therefore the HK value of life. Moreover, the discussion of (21) showed that the Tobin's-Q (and therefore $P_1(H)$) is higher under a higher health gradient in labor income or utility services, or lower depreciation rates, and sickness exposure. Because all these elements are more plausible for younger agents, and since health declines over the life cycle (e.g. Pelgrin and St-Amour, 2016) the model would thus be consistent with a higher Human Capital Value of life for young adults, compared to elders, independently of horizon length.

4.2 Willingness to pay to avoid a finite increase in death risk

Next, we can substitute the indirect utility $V(W, H, \lambda_{m0})$ given by (19) in the implicit equivalent variation in Definition 2, and solve for $v = v(W, H, \lambda_{m0}, \Delta)$ as follows:

Proposition 2 (willingness to pay) *Up to a first order approximation, the maximal willingness to pay solving* (4) *to avoid a change from* λ_{m0} *to* $\lambda^*_{m0} = \lambda_{m0} + \Delta$ *is given by:*

$$
\begin{aligned}
v(W, H, \lambda_{m0}, \Delta) = &\left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] N_1(W, H) \\
&+ \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})} \lambda_{m1} H^{-\xi_m} \left[l_m(\lambda^*_{m0}) - l_m(\lambda_{m0})\right] N_0(W, H),
\end{aligned}
\tag{29}
$$

where total wealth $N_0(W, H)$*, and* $N_1(W, H)$ *are given in* (20)*, and* (22)*, and where the* $\Theta(\cdot)$*, and* $l_m(\cdot)$ *functions are given in* (24) *and* (26)*.*

The WTP in (29) equals zero when the increment $\Delta = 0$, and is otherwise a weighted average of two components: the first-order net total wealth $N_1(W, H)$, and the change in the endogenous mortality adjustment that is induced by a change in the endowed intensity $\lambda_{m1} H^{-\xi_m} \left[l_m(\lambda^*_{m0}) - l_m(\lambda_{m0})\right] N_0(W, H)$. Indeed, it was shown earlier that the marginal value of total wealth $\Theta(\lambda_{m0}) \geq 0$ in (24) is a decreasing function, regardless of the EIS. Consequently, the weights $\Theta(\lambda^*_{m0})/\Theta(\lambda_{m0}) \in [0, 1]$ for detrimental changes $\Delta \geq 0$.

Moreover, we saw earlier that a change in λ_{m0} affects welfare $V(W, H, \lambda_{m0})$ through the $\Theta(\lambda_{m0})$, and the $l_m(\lambda_{m0})$ channels, both of which transit through the MPC, $A(\lambda_{m0})$. This is naturally reflected in the WTP (29) via its effects on $\Theta(\lambda^*_{m0})$, and $l^m(\lambda^*_{m0})$. Interestingly, we saw that unit elasticity cancels out the two conflicting effects on the MPC $A(\lambda_{m0})$, and therefore implies that $\Theta(\lambda_{m0}) = \Theta(\lambda^*_{m0})$, and $l_m(\lambda_{m0}) = l_m(\lambda^*_{m0})$. Consequently, $v(W, H, \lambda_{m0}, \Delta) = 0, \forall \Delta$ and for all wealth, and health levels, i.e. the agent with unit elasticity is indifferent to an increase in the risk of death, regardless of the magnitude of the change, and is therefore unwilling to pay to avoid $\Delta > 0$.

Third, since $l_m(\lambda_{m0})$ in (26) is declining in the MPC, it follows that $l_m(\lambda^*_{m0}) - l_m(\lambda_{m0}) < 0$ when $\varepsilon > 1$, i.e. agents with elastic preferences more easily substitute additional consumption when faced with a shorter horizon, and are therefore willing to pay less to avert $\Delta > 0$. Finally, the weights $\Theta(\lambda^*_{m0})/\Theta(\lambda_{m0})$ are falling as exogenous death risk λ^*_{m0} increases. This implies that the elements capturing preferences towards time or towards the various sources of risk (embedded in $l_m(\lambda_{m0})$), as well as the possibility to adjust death risk (i.e. $\lambda_{m1}H^{-\xi_m}$) gradually lose any relevance as the change in the endowed death intensity becomes large. This is formalized in the following result.

Corollary 1 (limiting WTP) *Up to a first order approximation,*

$$\lim_{\Delta \to \bar{\Delta}} v(W, H, \lambda_{m0}, \Delta) = N_1(W, H), \quad \textit{if} \quad \begin{cases} \varepsilon > 1, \ \textit{and } \bar{\Delta} = +\infty, \ \textit{or} \\ \varepsilon \in (0, 1), \ \textit{and } \bar{\Delta} = \bar{\lambda}_{m0} - \lambda_{m0}. \end{cases}$$

where net total wealth $N_1(W, H)$ is given in (22)*, and where the maximal admissible death intensity $\bar{\lambda}_{m0}$ is computed using* (27)*,*

Hence, when preferences are sufficiently elastic with respect to inter-temporal substitution, the willingness to pay asymptotically converges to the morbidity-adjusted net total wealth $N_1(W, H)$ as death becomes certain. It also converges to $N_1(W, H)$ when preferences are inelastic, and the exogenous death risk intensity attains its maximal admissible value $\bar{\lambda}_{m0}$. For the other cases of finite Δ, the shape of the willingness to pay $v(W, H, \lambda_{m0}, \cdot)$ in function of the death risk increment Δ crucially depends on the EIS ε, as well as on the other parameters, and the health level, and is difficult to establish ex-ante.[10] We will instead perform an empirical evaluation below (see Figure 3).

4.3 Value of a Statistical Life

Using Definition 3, and welfare (19), we can calculate the theoretical expression for the VSL consistent with the benchmark model as follows.

[10] Rosen (1988) stresses the importance of inter-temporal substitution in valuing longevity. See also Córdoba and Ripoll (2013) for the importance of the EIS in value of life calculations, as well as Huggett and Kaplan (2016) for EIS effects on human capital valuation.

Proposition 3 (Value of Statistical Life) *Up to a first order approximation, the Value of a Statistical Life solving* (6) *is:*

$$v_s(W, H, \lambda_{m0}) = \frac{-\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} N_1(W, H) + \lambda_{m1} H^{-\xi_m} l'_m(\lambda_{m0}) N_0(W, H), \tag{30}$$

where total wealth $N_0(W, H)$*, and* $N_1(W, H)$ *are given in* (20)*, and* (22)*, and where the* $\Theta(\lambda_{m0})$*, and* $l_m(\lambda_{m0})$ *functions are given in* (24) *and* (26)*.*

First, as explained earlier, the marginal value of total wealth $\Theta(\lambda_{m0})$ is a decreasing function for all levels of EIS. It follows that the VSL is an increasing function of first-order net total wealth $N_1(W, H)$. However, the marginal effects of λ_{m0} on the endogenous death risk factor l_m depends on the elasticity of inter-temporal substitution. The VSL is consequently lower ($l'_m(\lambda_{m0}) < 0$) when the agent's preferences are sufficiently elastic with respect to time (i.e. $\varepsilon > 1$) and higher otherwise. Finally, unit elasticity again entails that $\Theta'(\lambda_{m0}) = l'_m(\lambda_{m0}) = 0$ and therefore that $v_s(W, H, \lambda_{m0}) = 0$.

Remark 1 (discrete changes per period) The theoretical calculations of the VSL in equation (30) are valid for permanent, infinitesimal changes in the death intensity. In the spirit of the empirical VSL literature, the value of a statistical life can also be computed as the maximal willingness to pay to avoid an exogenous increase Δ in the probability of death over a given time interval (e.g. a change $\Delta = 0.1\%$ per one year period), divided by Δ (see v_s^e in equation (7)). This calculation involves two steps. First, the new value of the endowed intensity $\lambda^*_{m0}(H, \Delta, T)$ is computed, corresponding to a change in death risk Δ occurring over a duration of T (see Lemma 1 in Appendix C.5). Second, one substitutes $\lambda^*_{m0}(H, \Delta, T)$ in $\Theta(\lambda^*_{m0})$, and $l_m(\lambda^*_{m0})$ in the WTP (29), and divides by Δ to obtain the corresponding Value of a Statistical Life.

4.4 Gunpoint Value of Life

Again relying on the welfare function (19), and resorting to a first-order approximation to the GPV in (8) in Definition 4 reveals the following result.

Proposition 4 (Gunpoint value of life) *Up to a first order approximation, the maximum willingness to pay solving* (8) *to avoid certain death is given by:*

$$\begin{aligned} v_g(W,H) &= N_1(W,H) \\ &= v_h(H) + W - \frac{a}{r}. \end{aligned} \tag{31}$$

where $N_1(W,H)$ *is the net total wealth in* (22)*, and* $v_h(H)$ *is the HK value in* (28)*.*

We saw earlier that $N_1(W,H) = W + (y-a)/r + P_1(H)$ captures order-1 net total wealth. In the absence of a bequest motive, and under perfect markets, the agent who is forced to evaluate life at gunpoint is thus willing to pledge all available resources, i.e. his entire financial wealth W, plus the capitalized value of his fixed income endowment y/r, plus the morbidity-adjusted value of his human capital $P_1(H)$. However, the previous discussion emphasized that the minimal consumption level a is required at all periods for subsistence. It therefore cannot be pledged in a highwaymen threat, and must be subtracted from the Gunpoint value. It follows that the GPV can be higher or lower than the HK value depending on the level of financial wealth relative to capitalized subsistence consumption.

The links between the Gunpoint, and the Human Capital values of life are intuitive. Since human capital is non-transferable, and is entirely depreciated at death, the agent is also willing to give up the shadow value of his health capital $P_1(H) = HB[1 - \lambda_{s1} l_s H^{-\xi_s}]$. For reasons discussed earlier, this shadow value is likely to be lower for older individuals, consistent with a lower GPV for elders. Furthermore, for reasons previously discussed, the shadow value of health must be adjusted downwards for the endogeneity of the agent's exposure to health shocks. Finally, the shadow price of health $p_1(H) = P_1(H)/H$ is monotone increasing and concave, such that healthier agents face lower sickness risks, and thus value more highly their health capital that is fully depreciated at death.

Interestingly, the shadow value of the health stock $P_1(H)$, and therefore the Gunpoint Value of life v_g in (31) are both independent of the attitudes towards death risk γ_m and of the endogenous components in the mortality intensity (λ_{m1}, ξ_m). Consequently, neither aversion to death risk nor health's ability to ward off death determine the Gunpoint Value of life. This result stems from the way the GPV is evaluated. Because the outcome of death is certain when life is evaluated at gunpoint, both the attitudes towards death *risk* and the ability to marginally alter exposure to that risk become irrelevant. Indeed, this element could already be inferred by combining Figure 2, and Corollary 1 which showed that the willingness to pay $v(W, H, \lambda_{m0}, \Delta)$ is converging to $N_1(W, H)$ that is independent of preferences, and horizon, as the exogenous death risk λ^*_{m0} becomes large, and death becomes certain.

Furthermore, unlike the WTP, the Gunpoint value is independent of the exogenous death intensity λ_{m0}. This again stems from the way $v_g(W, H)$ is computed, i.e. as a willingness to pay to avert certain, and instantaneous death regardless of *how* death occurs. Put differently, both base exposure, and the specific mechanism – be it through increases in λ_{m0} or not – are irrelevant, only the outcome is. Moreover, the Gunpoint value is independent of the EIS ε. Because death is instantaneous in a highwaymen threat, attitudes towards inter-temporal substitution are irrelevant as well.

Finally, it can also be shown that net total wealth $N_1(W, H)$ is equal to the expected discounted present value of excess consumption along the optimal path.[11] In order to survive, the agent is thus willing to pledge the total value of his optimal consumption stream (net of minimal subsistence). This result can be traced to the homegeneity between the value and excess consumption functions in the non-expected utility setting we consider (see Section 3.3). The foregone utility is measured in the same units as the foregone excess consumption when gauging the Gunpoint value of life.

4.5 Comparison with other models of human capital

We mentioned in Section 3.2 that the Hugonnier et al. (2013) generalizes other well-known life cycle models of health demand. In particular, the Ehrlich and Chuma (1990) version of the Grossman (1972) model in (17)-(18) abstracts from morbidity altogether, as well as from endogenous mortality, while assuming VNM preferences, without minimal consumption requirements. Appendix B.2 reports the indirect utility, and optimal rules for this restricted model. Relying on these restrictions, the corresponding Human Capital Value, willingness to pay, as well as Statistical, and Gunpoint Values then simplify to:

[11] In particular, Hugonnier et al. (2013, Prop. 2) establishes that:

$$E_t \int_t^\infty m_{t,\tau}(c^*_\tau - a)\mathrm{d}\tau = N_1(W, H).$$

Corollary 2 (Valuation for restricted case) *The HK, WTP, VSL, and GPV corresponding to the Grossman (1972); Ehrlich and Chuma (1990) model in* (17), *and* (18) *are given by:*

$$\tilde{v}_h(H) = \frac{y}{r} + \tilde{P}_0(H), \tag{32}$$

$$\tilde{v}(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\tilde{\Theta}(\lambda^*_{m0})}{\tilde{\Theta}(\lambda_{m0})}\right] \tilde{N}_0(W, H), \tag{33}$$

$$\tilde{v}_s(W, H, \lambda_{m0}) = \frac{-\tilde{\Theta}'(\lambda_{m0})}{\tilde{\Theta}(\lambda_{m0})} \tilde{N}_0(W, H), \tag{34}$$

$$\tilde{v}_g(W, H) = \tilde{N}_0(W, H), \tag{35}$$

where the modified expressions for marginal value $\tilde{\Theta}(\lambda_{m0})$, *and order-0 human* $\tilde{P}_0(H)$, *and net total wealth* $\tilde{N}_0(W, H)$ *are given in Appendix B.2.*

These restricted results qualitatively confirm those for the more general model. In the absence of endogenous mortality and morbidity, as well as minimal consumption adjustments, the relevant net wealth measure is the order-0, $\tilde{N}_0(W, H)$. If the other parameters remain constant, the absence of morbidity risk exposure ($\lambda_{s0} = 0$) raises the Tobin's-Q of health, $\tilde{B}$, which, combined with $\lambda_{s1} = 0$ results in higher human capital $\tilde{P}_0(H) > P_1(H)$. Consequently, the HK value (32) is increased. Moreover, net total wealth is also higher $\tilde{N}_0(W, H) > N_1(W, H)$. The WTP $\tilde{v}$ in (33) remains an increasing share of order-0 net total wealth, and converges to the latter as exposure to death risk increases. The Value of a Statistical Life $\tilde{v}_s$ in (34) is again increasing in net total wealth, whereas the Gunpoint Value $\tilde{v}_g$ in (35) confirms that all available net worth is spent to survive a highwaymen threat. Importantly, equation (39) in Appendix B.2 establishes that the marginal value $\tilde{\Theta}(\lambda_{m0})$ is a decreasing, and convex function at all parameter values. Consistent with Figure 2, the WTP $\tilde{v}$ in (33) is therefore an increasing, and concave function in the death risk increment Δ. It follows that the linear projection bias of the VSL discussed earlier is unconditionally present for the restricted model.

We conclude that whereas the main theoretical findings are qualitatively robust to the choice of human capital model (within the generalized class of the health models we consider), abstracting from sickness risk as well as from minimal consumption requirements increases the human capital, and net worth that can be used in life valuation.

5 Structural estimation

5.1 Econometric model

The econometric model that we rely upon assumes that agents are heterogeneous with respect to their health, and wealth statuses, and are homogeneous with respect to the distributional, revenue, and preference parameters. To structurally estimate these deep parameters, we use the quadri-variate closed-form expressions for the optimal rules in Theorem 1, to which we append the exogenous income equation (13). Specifically, let $\mathbf{Y}_j = [c_j - a, \pi_j, x_j, I_j]'$ denote the vector of optimal excess consumption, portfolio, health insurance, and health spending for agents $j = 1, 2, \ldots, n$. The estimated econometric model rules are:

$$\begin{bmatrix} \mathbf{Y}_j \\ Y_j \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^*(W_j, H_j) \\ y + \beta H_j \end{bmatrix} + \mathbf{u}_j, \tag{36}$$

where the optimal rules $\mathbf{Y}^*(W_j, H_j)$ are given in equation (38) in Appendix B.1, and where the $\mathbf{u}_j$'s are (potentially contemporaneously correlated) Gaussian error terms.

To ensure theoretical consistency, we estimate the structural parameters in (36) imposing the full set of regularity conditions in (37) in Appendix B.1. In light of the strong nonlinearities not all the deep parameters can be identified, and a subset are calibrated. We resort to a two-stage, iterative Maximum Likelihood procedure. In stage one, we fix the curvature parameters in the Poisson intensity functions ξ_k, for $k = m, s$, then estimate the remaining deep parameters. In the first stage two, we condition on the latter to re-estimate the ξ_k. In the second stage, we iterate on this procedure until a fixed point is reached.

5.2 Data

We use a sample of 8 378 individuals taken from the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix E. We proxy the health variables through the polytomous self-reported health status that is linearly converted to numeric values from 1 to 4. The financial wealth comprises risky, and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation, and utility expenses reported in the PSID. Finally, health expenditures and insurance are respectively the out-of-pocket spending, and premia paid by agents. All nominal values are scaled by 10^{-6} for the estimation.

Tables 1, and 2 present descriptive statistics, as well as mean values (in \$) for the main variables of interest, per health status, and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear effects of the health status on wealth levels can be deduced. The level of consumption in panel b is increasing in financial wealth. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption.

In panel c, stock holdings are very low for all but the fourth, and fifth quintiles, illustrating the non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth, and devoid of clear health gradients. Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth, and also sharply decreasing in health status.

6 Results

Sections 6.1, and 6.2 report the estimated parameters, and life valuations for the benchmark model. Section 6.3 revisits these results for the restricted model. Finally, Section 6.4 provides concluding remarks.

6.1 Structural parameters

Table 3 reports the calibrated (with subscripts ^{c}), and estimated (standard errors in parentheses) deep parameters. Overall, the latter are precisely estimated, and are close to other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.70$). Although depreciation is relatively low ($\delta = 1.09\%$), additional depletion brought upon by sickness is consequential ($\phi = 1.36\%$). Second, the sickness and death intensities parameters in panel c are consistent with endogeneous morbidity, and mortality ($\lambda_{k1}, \xi_k \neq 0$ for $k = s, m$). Moreover, high convexity parameters ($\xi_k > 1$) indicate strongly diminishing returns in adjusting exposure to death and sickness risks. Consistent with intuition, self-insurance against morbidity risk is more potent than against the risk of dying ($\lambda_{s1}, \xi_s > \lambda_{m1}, \xi_m$). Finally, a large calibrated value for η entails that sickness risks increase very steeply as health falls.

Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ($\beta = 0.0095$), as well as a realistic calibrated value for base income ($y \times 10^6 = 12.2$ K\$).[12] The returns process parameters (μ, r, σ_S) are calibrated at standard values. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 3.52$), and to mortality risk ($\gamma_m = 0.29$), where the latter is less than one as required, as well as a high calibrated value for aversion to morbidity risk ($\gamma_s = 7.4$). As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.67$), and is consistent with a *Live Fast and Die Young* effect whereby a higher risk of death increases the marginal propensity to consume. Observe that the inverse of the EIS is nonetheless larger than the mortality risk aversion ($1/\varepsilon = 0.60 > 0.29 = \gamma_m$), an issue to which we will return shortly. The minimal consumption level is realistic, and larger than base income ($a \times 10^6 = 14.4$ K\$). Finally, we note in closing that all the restrictions in Section 3.2 that are associated with the Grossman (1972); Ehrlich and Chuma (1990) model (17), and (18) are individually rejected, confirming the relevance of the more general Hugonnier et al. (2013) benchmark.

6.2 Estimated valuations

Human Capital Value of Life Using the estimated parameters in Table 3, we can compute the HK value of life $v_h(H)$ given in (28), and reported in Table 4. Overall, the human capital values are realistic, with a mean value of 738 K\$. Indeed, the estimated HK values range from 393 K\$ (Poor health) to 913 K\$ (Excellent health) and compare

[12] For example, the 2016 poverty threshold for single-agent households under age 65 was 12.5 K\$ (U.S. Census Bureau, 2017).

advantageously with other HK estimates,[13] and provide a first out-of-sample confirmation that the structural estimates are realistic. The HK values are independent of wealth, and are increasing in the health level. From the health capital $P_1(H) = HB[1 - \lambda_{s1} l_s H^{-\xi_s}]$, healthy agents have more human capital HB at stake, as well as a lower exposure to sickness risks $\lambda_{s1} H^{-\xi_s}$. Both elements concur to yield a higher economic value of their human capital stock.

Value of Statistical Life Again relying on the estimated structural parameters, Table 5 reports the Values of Statistical Life $v_s(W, H, \lambda_{m0})$ in (30) by observed health, and wealth statuses. First, the VSL values average 8.14 MM$, and are ranging between 1.48 MM$, and 12.97 MM$, well within the ranges usually found in the empirical VSL literature.[14] The concordance of these values with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded. The human capital $P_1(H)$ captures a low value to the VSL, representing between 4.5% and 9.4% of the levels, indicating that financial wealth is the main determinant of $v_s(W, H, \lambda_0)$.

Second, the VSL is increasing in both wealth, and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Andersson and Treich, 2011; Robinson and Hammitt, 2016; Murphy and Topel, 2006). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks, and are willing to pay less to attain further improvements (or prevent deteriorations). Our estimates unambiguously indicate that the former effect is dominant and that better health raises the VSL.

[13] Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K$ at age 20, peaking at less than 900 K$ at age 45, and falling steadily towards zero afterwards.

[14] A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 MM$ (2000 base year, corresponding to 8.6 MM$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 MM$, and 10 MM$. Robinson and Hammitt (2016) report values ranging between 4.2, and 13.7 MM$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 MM$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 MM$ (2006$), corresponding to 8.8 MM$ in 2016 (U.S. Environmental Protection Agency, 2017).

Gunpoint Value Using the point estimates of the deep parameters, Table 6 reports the Gunpoint values $v_g(W, H)$ in (31). The GPV is increasing in both health and wealth, and ranges between 88 K\$, and 730 K\$, with an average of 460 K\$. Contrasting these valuations with the low observed financial wealth in Table 2.a reveals that the bulk of the Gunpoint value captures human capital, with $P_1(H)$ capturing between 77% (high wealth), and 157% (low wealth) of net total wealth $N_1(W, H)$. As discussed earlier, the low observed wealth W, and high minimal consumption $a > 0$ explain why the Gunpoint is lower than the HK value.

Whereas it is of similar magnitude to the Human Capital Value of life, the Gunpoint Value is much lower than the VSL. To understand these differences, it is useful to chart the estimated counterpart of the willingness to pay first introduced in Figure 2. In particular, Figure 3 plots the estimated $v(W, H, \lambda_{m0}, \Delta)$ in function of the death intensity $\lambda^*_{m0} = \lambda_{m0} + \Delta$.[15] First, the estimated WTP in blue is an increasing, and concave function that equals zero at $\lambda^*_{m0} = \lambda_{m0} = 0.0244$, is negative for $\Delta < 0$, and positive for positive increments. The pronounced curvature of the WTP is consistent with standard economic intuition of diminishing marginal valuation of exposure to death (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2016). Concavity of the WTP is also expected when the reciprocal of the EIS is larger than mortality risk aversion (as was found in Table 3.c) in other life valuation literature using Non-Expected Utility (see Córdoba and Ripoll, 2016, for discussion).

Second, we saw from Corollary 1 that for $\varepsilon > 1$, the limit of the willingness to pay when death becomes certain – i.e. when λ^*_{m0} tends to infinity – is the morbidity-adjusted net total wealth $N_1(W, H)$. From Proposition 4, this limiting value is also the gunpoint value $v_g(W, H)$ plotted in red. Third, as explained in Proposition 3, the VSL $v_s(W, H, \lambda_{m0})$ is the value of the slope of the yellow tangent of $v(W, H, \lambda^*_{m0})$ evaluated at $\Delta = 0$ or, equivalently, the value of the yellow tangent evaluated as $\lambda^*_{m0} = 1 + \lambda_{m0}$. The pronounced curvature of the WTP in Figure 3 is informative as to why the VSL is much larger than the Gunpoint value. Put differently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment.

[15] These valuations are calculated from (29) at the estimated parameters, and relying on the mean wealth and health status in Table 1.a ($W = 38,685\$ \times 10^{-6}, H = 2.58$).

6.3 Comparisons with restricted model of human capital

Our discussion of the estimated parameters in Table 3 revealed that the theoretical restrictions associated with the Grossman (1972); Ehrlich and Chuma (1990) model (17), and (18) were rejected, thereby validating our benchmark model over the restricted one. Despite this statistical evidence, we re-estimate the restricted model to verify empirical robustness. In particular, the econometric model (36), is modified where optimal rules are now given by (41), and the model is re-estimated subject to the modified regularity conditions (40) outlined in Appendix B.2.

The estimated parameters for the restricted model are reported in Table 7, whereas the associated valuations $\tilde{v}_h, \tilde{v}_s$, and $\tilde{v}_g$ are reported in Tables 8, 9, and 10. Overall, the unrestricted deep parameters remain similar, with the exception of a higher depreciation rate δ which over-compensates the absence of morbidity risk.[16] Larger depreciation results in a lower Tobin's-Q, and explains a lower HK value (634 K\$ vs 737 K\$). Conversely, the VSL (16.7 MM\$ vs 8.1 MM\$), and GPV (661 K\$ vs 460 K\$) results are higher, confirming our discussion of Corollary 2. Abstracting from sickness risks, combined with the absence of minimal consumption by imposing $a = 0$ results in higher net total wealth $\tilde{N}_0(W, H) > N_1(W, H)$. Consequently, we find higher life values for the VSL, and GPV. We conclude that while the theoretical valuations are qualitatively similar, the absence of adjustments for sickness exposure, and for minimal consumption requirements results in quantitative adjustments for the restricted model that do not overturn our main conclusions. Formal testing unambiguously rejects the associated restrictions.

6.4 Discussion, and Caveats

Our structural estimation maintains the general conclusion that the Value of a Statistical Life is much larger than other valuations. As famously pointed out by Schelling (1968), and widely recognized by the literature, the VSL gauges the aggregate willingness to pay for infinitesimal changes in the risk of dying indiscriminately affecting entire populations. Conversely, the Gunpoint value measures the willingness to pay to avoid a large change in death risk (i.e. life versus certain death), and affecting a single individual. There

[16] In particular, the depreciation rate for the restricted model $\delta = 0.0272$ is more than twice the deterministic plus expected stochastic depreciation for the benchmark: $\delta + \phi\lambda_{s0} = 0.0113$.

is therefore no *ex-ante* reason why the Statistical Life, and Gunpoint values should be equal.

Indeed, Pratt and Zeckhauser (1996) argue that concentrating the costs, and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e. large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment to avoid increases in risk, thereby reducing the WTP as risk increases.[17] Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our Gunpoint Value. Their conjecture is warranted in our calculations. When faced with certain death, an individual is willing to pay much less than what can be inferred from the VSL. Indeed, while total financial wealth, plus the value of the human capital, net of subsistence costs, are paid out in the Gunpoint value, these resources are limited, and much less than what society might collectively be willing to pay to save one unidentified life.

Given that three different answers are provided to the same question, which of the Human Capital, Statistical Life, or Gunpoint Value should be relied upon to measure the value of a human life? To the extent that they measure fundamentally different objects, we may contend that *all* should be used. Put differently, the HK, VSL, and GPV are complements, rather than substitutes, and their relevance should depend on the underlying motivation for computing a life value. All in all, the VSL is more appropriate in issues involving collective choices that involve small changes in death probabilities affecting large subsets of the population, and for which society is the ultimate payer of the associated costs. Well-known examples identified in the literature include general safety measures with respect to transportation, or public health. The GPV is inappropriate for such cases in that it gauges what someone would be willing to pay to survive, not what a society is collectively willing to pay to indiscriminately save someone in the group. The

[17] Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay $1 million to reduce the associated mortality risk by 10%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.

HK and the GPV are closely linked and should thus be relied upon in situations where the risk of dying concerns a single individual, such as the continued life support decisions, wrongful death litigation, or life insurance where the GPV would gauge the value of life ascribed by its main beneficiary, with full adjustments for his health, financial, and net total wealth statuses. The HK value can be applied in instances where labor income is the main source of human wealth.

Two caveats of our approach are worth mentioning. A first limitation is the absence of bequest motives. This omission is related to the technical difficulty in solving the Hugonnier et al. (2013) model when bequeathed wealth is optimally chosen. Although it remains unclear how our results would be affected, we can however conjecture that a likely effect would be to reduce the GPV even further. Indeed, *warm glow* effects of bequest would attenuate the cost of dying, and consequently also the WTP to avert death. Moreover, bequeathed wealth is illiquid, to the extent that it is set aside for surviving heirs, and not to ensure one's own survival. Without affecting human capital, the amount of disposable financial resources that can be pledged in a money-or-death threat would therefore be reduced, and consequently so would the GPV.

A second limitation is the absence of aging in our valuation. A fair treatment would involve time varying parameters, which are made possible in the original paper of Hugonnier et al. (2013, Appendix B), yet are technically more challenging. Although a complete derivation is again beyond the scope of this paper, we can conjecture that aging should also reduce the GPV. Indeed, biological limits to life expectancy would generate optimal dis-saving of financial wealth, lowering even further the Gunpoint value for elders. Moreover, as mentioned earlier, the marginal Q of health, B, is a declining function of the health depreciation parameters (which would likely increase in age) and an increasing function of health gradient of income (which should fall in age). A lower value of human capital for elders would then lead to less resources being paid out to survive a credible death threat.

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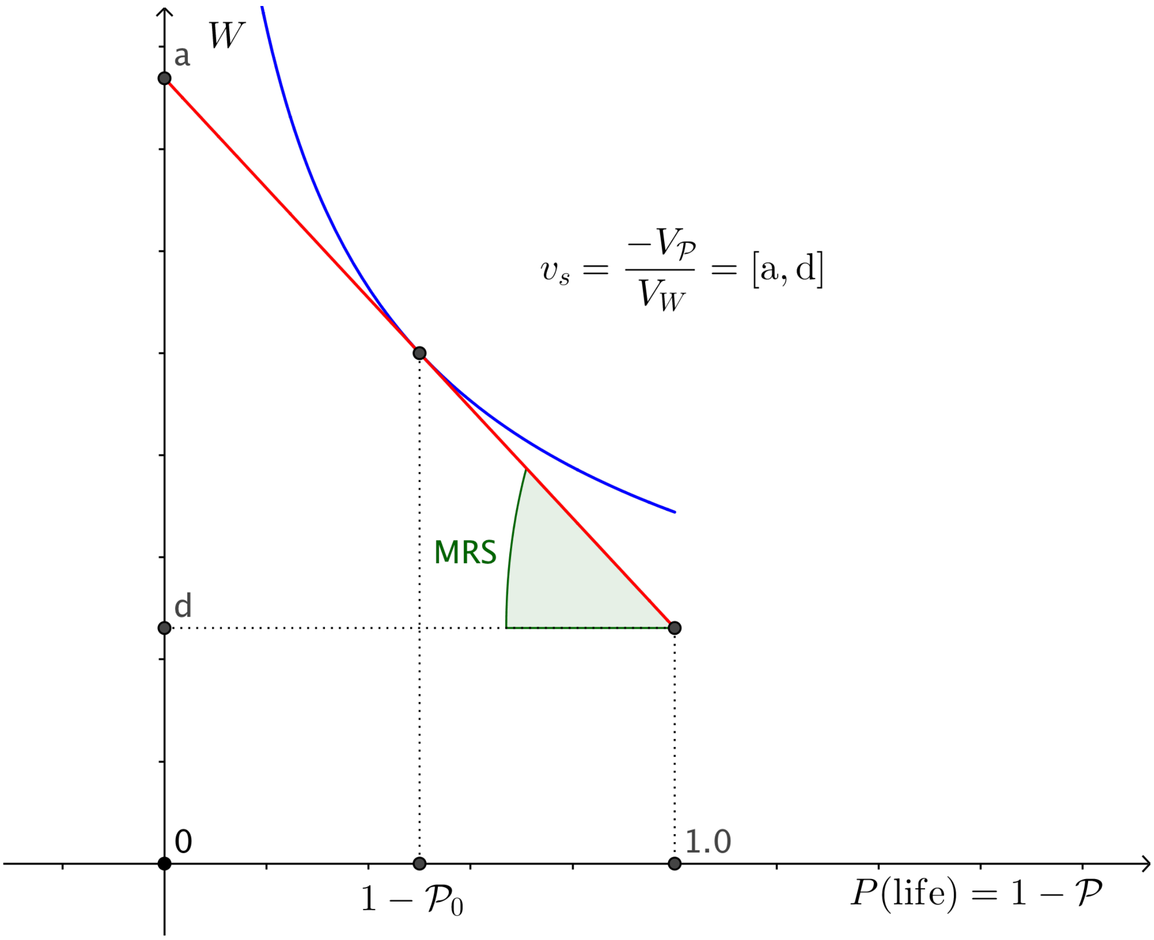
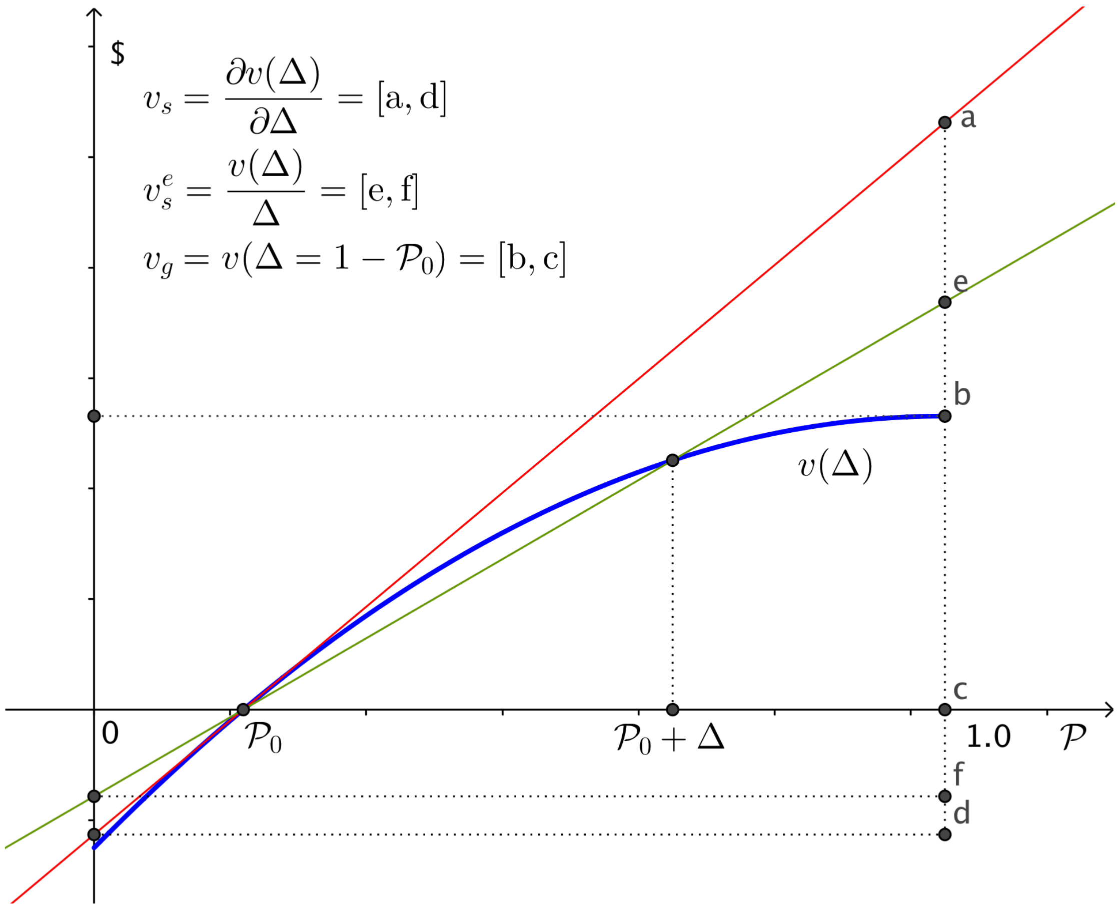
A Figures

Figure 1: Indifference curves, MRS, and Value of Statistical Life



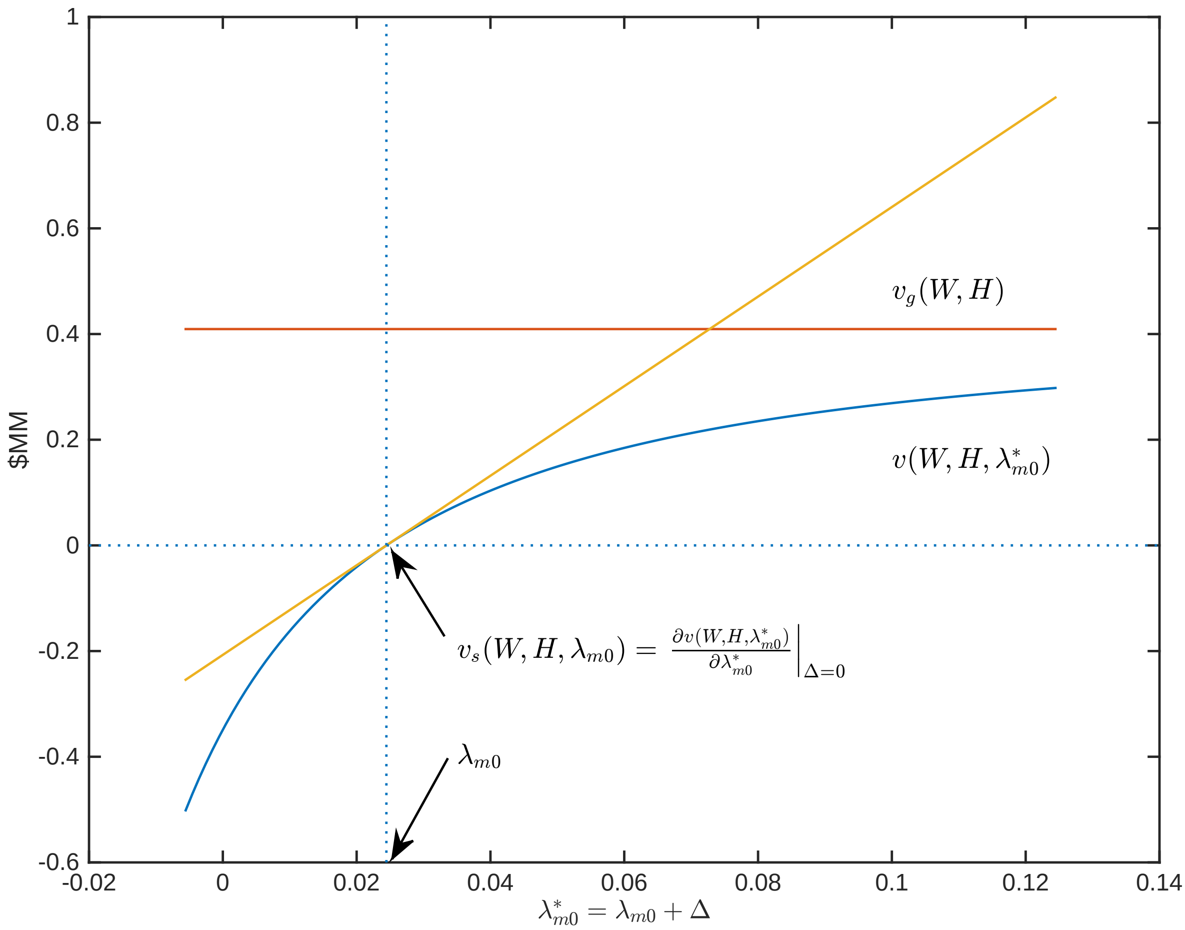
Notes: Reproduced and adapted from Andersson and Treich (2011, Fig. 17.1, p. 398). Indifference curves for indirect utility (1) in blue. v_s: Theoretical Value of Statistical Life in (6a) is the negative of the MRS, i.e. the slope of red tangent equal to distance [a,d].

Figure 2: Willingness to pay, Empirical, and Theoretical VSL, and Gunpoint Value of Life



Notes: $\mathcal{P}$: instantaneous probability of death. v: Willingness to pay to avoid change Δ in death risk (in blue), evaluated at (W, H), and for base risk $\mathcal{P}_0$. v_s: Theoretical Value of Statistical Life in (6b) is slope of red tangent equal to distance [a,d]. v_s^e: Empirical Value of Statistical Life in (7) is slope of green line, and equal to distance [e,f]. v_g: Gunpoint Value of Life in (8) is equal to distance [b,c].

Figure 3: Estimated willingness to pay, Statistical Life and Gunpoint Values



Notes: At estimated parameter values, for mean wealth and health levels. $v(W, H, \lambda^*_{m0})$ in blue is the maximum willingness to pay to avoid an increase of Δ in exogenous death intensity λ_{m0}; $v_g(W, H)$ in red is the Gunpoint value of life; $v_s(W, H, \lambda_{m0})$ is the Value of statistical life, and the slope of the yellow tangent evaluated at λ_{m0}. In MM$.

B Solutions for human capital models

B.1 Solution for the benchmark Hugonnier et al. (2013) model

The main results of Hugonnier et al. (2013, Prop. 1, 2, and Thm. 1, 2) show that the optimal policy can be characterized as follows.

Theorem 1 (Indirect utility and optimal policy) *Assume that the following regularity conditions are verified:*

$$\begin{aligned}
&\beta < (r+\delta+\phi\lambda_{s0})^{\frac{1}{\alpha}},\\
&0 < A(\lambda_{m0}) - \max\left(0, r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right),\\
&0 < \min\left(\frac{\lambda_{m0}}{1-\gamma_m}, r\right) - F(1-\xi_s),\\
&0 < A(\lambda_{m0}) - \max\left(0, r - \frac{\lambda_{m0}}{1-\gamma_m} + \theta^2/\gamma\right) - F(-\xi_m).
\end{aligned} \tag{37}$$

Then, up to a first-order approximation, the nonnegative indirect utility of an alive agent is given by (19) *and generates the optimal policy functions:*

$$\begin{aligned}
c_t =& a + A(\lambda_{m0})\Big[N_1(W_{t-}, H_{t-}) - (1-\varepsilon)\lambda_{m1}H_{t-}^{-\xi_m} l_m(\lambda_{m0}) N_0(W_{t-}, H_{t-})\Big]\\
\pi_t =& (\theta/(\gamma\sigma_S))N_0(W_{t-}, H_{t-}) - \lambda_{s1}(\theta/(\gamma\sigma_S)) l_s H_{t-}^{-\xi_s} P_0(H_{t-})\\
x_t =& \phi P_0(H_{t-}) - \lambda_{m1}\chi(\xi_m)(1-1/\gamma_s) l_m(\lambda_{m0}) H_{t-}^{-\xi_m} N_0(W_{t-}, H_{t-})\\
&- \lambda_{s1}\chi(\xi_s - 1) l_s H_{t-}^{-\xi_s} P_0(H_{t-}),\\
I_t =& K P_0(H_{t-}) + \lambda_{m1}(\xi_m K/(1-\alpha)) l_m(\lambda_{m0}) H_{t-}^{-\xi_m} N_0(W_{t-}, H_{t-})\\
&+ \lambda_{s1}((\xi_s - 1)K/(1-\alpha)) l_s H_{t-}^{-\xi_s} P_0(H_{t-}),
\end{aligned} \tag{38}$$

where $K = \alpha^{1/(1-\alpha)} B^{\alpha/(1-\alpha)}$, $\chi(x) = 1 - (1-\phi)^{-x}$, *and where any dependence on the endowed mortality rate* λ_{m0} *is explicitly stated, and where the nonnegative order-0 value of human capital and nonnegative orders-0 and 1 of net total wealth are defined as by* (20), *and* (22).

B.2 Solution for the restricted Grossman (1972); Ehrlich and Chuma (1990) model

The Hugonnier et al. (2013) framework nests the Grossman (1972); Ehrlich and Chuma (1990) model. In particular, the exogenous morbidity, and mortality restriction $\lambda_{k1} = 0, k = s, m$ corresponds to the order-0 case analyzed in Hugonnier et al. (2013, Prop. 1, Thm. 1). Moreover, imposing $\lambda_{s0} = \phi = 0$ in the $g(\cdot)$ equation yields $\tilde{B}$. Imposing no subsistence $a = 0$, VNM preferences $\varepsilon = 1/\gamma$, and source-independent risk aversion $\gamma_k = 0, k = s, m$ restrictions in $\Theta(\lambda_{m0}), A(\lambda_{m0})$ yields:

$$\tilde{N}_0(W, H) = W + \frac{y}{r} + \tilde{B}H,$$

where $\tilde{B}$ solves $g(\tilde{B}) = 0$ s.t. $g'(\tilde{B}) < 0$ in

$$g(\tilde{B}) = \beta - (r + \delta)\tilde{B} - (1 - 1/\alpha)(\alpha\tilde{B})^{\frac{1}{1-\alpha}},$$

and where

$$\tilde{\Theta}(\lambda_{m0}) = \rho \left(\frac{\tilde{A}(\lambda_{m0})}{\rho} \right)^{\frac{\gamma}{\gamma-1}}, \quad \tilde{A}(\lambda_{m0}) = \frac{\rho}{\gamma} + \left(\frac{\gamma - 1}{\gamma} \right) \left(r - \lambda_{m0} + \frac{\theta^2}{2\gamma} \right). \tag{39}$$

The associated theoretical restrictions are:

$$\begin{aligned} &\beta < (r + \delta)^{\frac{1}{\alpha}}, \\ &0 < \tilde{A}(\lambda_{m0}) - \max\left(0, r - \lambda_{m0} + \theta^2/\gamma\right), \end{aligned} \tag{40}$$

Given these elements, the welfare corresponding to the restricted case is:

$$\tilde{V}(W, H, \lambda_{m0}) = \tilde{\Theta}(\lambda_{m0})\tilde{N}_0(W, H),$$

and generates the optimal policy functions:

$$\begin{aligned}
\tilde{c}_t =& a + \tilde{A}(\lambda_{m0})\tilde{N}_0(W_{t-}, H_{t-}) \\
\tilde{\pi}_t =& (\theta/(\gamma\sigma_S))\tilde{N}_0(W_{t-}, H_{t-}) \\
x_t \equiv& 0 \\
\tilde{I}_t =& \tilde{K}\tilde{P}_0(H_{t-}),
\end{aligned} \tag{41}$$

where $\tilde{K} = \alpha^{1/(1-\alpha)}\tilde{B}^{\alpha/(1-\alpha)}$, and where any dependence on the endowed mortality rate λ_{m0} is explicitly stated.

C Proofs

C.1 Proof of Proposition 1

From definition (3), the NPV of the fixed component y in income (13) is y/r. The results in Hugonnier et al. (2013, Prop. 2) show that the NPV of the health-dependent part of income net of expenses along the optimal path is:

$$E_t \int_0^{T_m} m_{t,\tau} \left[\beta H^*_\tau - I^*_\tau\right] \mathrm{d}\tau = P_1(H),$$

as required.

C.2 Proof of Proposition 2

Following Hugonnier et al. (2013), we can define $L_k(H) = H^{-\xi_k} l_k$, for $k = s, m$, and set $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$ for some strictly positive constants $\bar{\lambda}_{k1}$, and for $k = m, s$, such that the value function in (19) can be written as:

$$\begin{aligned} V(W, H, \lambda_{m0}, \epsilon) &= \Theta(\lambda_{m0}) \left[N_0(W, H) - \epsilon\bar{\lambda}_{s1} L_s(H) P_0(H)\right] \\ &\qquad - \Theta(\lambda_{m0})\epsilon\bar{\lambda}_{m1} L_m(H, \lambda_{m0}) N_0(W, H), \\ &= \Theta(\lambda_{m0}) \left[N_1(W, H, \epsilon) - \epsilon\bar{\lambda}_{m1} L_m(H, \lambda_{m0}) N_0(W, H)\right], \end{aligned} \tag{42}$$

where the first-order total wealth $N_1(W, H, \epsilon)$ is implicitly defined. The indirect utility (42) is obtained by Hugonnier et al. (2013) through a first-order Taylor expansion of the agent's problem around small deviations $\epsilon \approx 0$. By a similar reasoning, the first-order approximation to the Hicksian compensating value $v(\epsilon) = v(W, H, \lambda^*_{m0}, \epsilon) \geq 0$ in (4) to prevent any increase in the endowed death intensity $\lambda^*_{m0} > \lambda_{m0}$ is given as:

$$\begin{aligned} 0 &= V(W - v(\epsilon), H, \lambda_{m0}, \epsilon) - V(W, H, \lambda^*_{m0}, \epsilon) \\ &= \nabla V(W, H, \lambda^*_{m0}, \epsilon) \\ &\approx \nabla V(W, H, \lambda^*_{m0}, 0) + \epsilon \nabla V_\epsilon(W, H, \lambda^*_{m0}, 0). \end{aligned}$$

Straightforward calculations using the indirect utility (42) reveal that:

$$\begin{aligned}\nabla V(W,H,\lambda_{m0}^*,0) &= V(W-v(0),H,\lambda_{m0},0) - V(W,H,\lambda_{m0}^*,0)\\ &= -\Theta v(0) + (\Theta-\Theta^*)N_0(W,H)\end{aligned}$$

where $\Theta = \Theta(\lambda_{m0})$, and $\Theta^* = \Theta(\lambda_{m0}^*)$ are given in (**??**). Setting $\nabla V(W,H,\lambda_{m0}^*,0) = 0$ uniquely solves for $v(0)$ as:

$$v(0) = \left(1 - \frac{\Theta^*}{\Theta}\right) N_0(W,H). \tag{43}$$

Similarly, we obtain:

$$\begin{aligned}\nabla V_\epsilon(W,H,\lambda_{m0}^*,0) =& - V_W(W-v(0),H,\lambda_{m0},0)v'(0)\\ &+ V_\epsilon(W-v(0),H,\lambda_{m0},0) - V_\epsilon(W,H,\lambda_{m0}^*,0),\\ =& - \Theta v'(0) + \bar{\lambda}_{m1}\Theta L_m(H)v(0)\\ &- \bar{\lambda}_{m1}\left[\Theta L_m(H) - \Theta^* L_m^*(H)\right] N_0(W,H)\\ &- \bar{\lambda}_{s1}\left[\Theta - \Theta^*\right] L_s(H)P_0(H),\end{aligned}$$

where $L_m(H) = L_m(H,\lambda_{m0})$, and $L_m^*(H) = L_m(H,\lambda_{m0}^*)$ are given in (**??**). Again setting $\nabla V_\epsilon(W,H,\lambda_{m0}^*,0) = 0$ uniquely solves for $v'(0)$ as:

$$v'(0) = -\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H) - L_m^*(H)\right] N_0(W,H) - \bar{\lambda}_{s1}\left[1-\frac{\Theta^*}{\Theta}\right] L_s(H)P_0(H) \tag{44}$$

The corresponding Hicksian value $v(\epsilon)$ obtains by substituting the solutions (43) and (44) in the first-order expansion of the compensating value:

$$\begin{aligned}v(\epsilon) \approx& v(0) + \epsilon v'(0)\\ =& \left[1-\frac{\Theta^*}{\Theta}\right]\left[N_0(W,H) - \epsilon\bar{\lambda}_{s1}L_s(H)P_0(H)\right]\\ &- \epsilon\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H)-L_m^*(H)\right] N_0(W,H)\\ =& \left[1-\frac{\Theta^*}{\Theta}\right] N_1(W,H,\epsilon) - \epsilon\bar{\lambda}_{m1}\frac{\Theta^*}{\Theta}\left[L_m(H)-L_m^*(H)\right] N_0(W,H).\end{aligned}$$

Substituting back $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$, and using total wealth (22) yields (29). ■

C.3 Proof of Corollary 1

When the agent's preferences are sufficiently elastic with respect to time (i.e. $\varepsilon > 1$), the marginal propensity to consume $A(\lambda_{m0})$ in (25) is a linear increasing function, such that $l_m^* < l_m$ is decreasing. Since $\Theta(\lambda_{m0})$ was found to be increasing and convex for all ε it follows that

$$\lim_{\lambda_{m0}^* \to +\infty} v(W, H, \lambda_{m0}^*) = N_1(W, H), \quad \text{if } \varepsilon > 1,$$

Conversely, when the elasticity is low, i.e. $\varepsilon \in (0, 1)$, the marginal propensity to consume $A(\lambda_{m0})$ is a linear decreasing function. To maintain non-negativity of the MPC, the maximal admissible increase in the death intensity must be bounded above by $\bar{\lambda}_{m0}$ in (27). Since $A(\bar{\lambda}_{m0}) = 0$, and $\varepsilon \in (0, 1)$, it follows that $\Theta(\bar{\lambda}_{m0}) = 0$ in (24), while $l_m(\bar{\lambda}_{m0})$ is finite in (26), and

$$v(W, H, \bar{\lambda}_{m0}) = N_1(W, H), \quad \text{if } \varepsilon \in (0, 1)$$

as stated. ■

C.4 Proof of Proposition 3

Using a similar reasoning and standard principles, the VSL can be calculated as the negative of the MRS between death intensity λ_{m0}, and wealth:

$$\begin{aligned} v_s(W, H, \lambda_{m0}, \epsilon) &= \frac{-V_{\lambda_{m0}}(W, H, \lambda_{m0}, \epsilon)}{V_W(W, H, \lambda_{m0}, \epsilon)} \\ &\approx v_s(W, H, \lambda_{m0}, 0) + \epsilon \frac{\partial v_s(W, H, \lambda_{m0}, \epsilon)}{\partial \epsilon}\bigg|_{\epsilon=0}, \end{aligned}$$

where

$$\begin{aligned} v_s(W, H, \lambda_{m0}, 0) &= \frac{N_0(W, H)\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} \\ \frac{\partial v_s(W, H, \lambda_{m0}, \epsilon)}{\partial \epsilon}\bigg|_{\epsilon=0} &= \frac{-\bar{\lambda}_{s1} BHL_s(H)\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} - \bar{\lambda}_{m1} N_0(W, H) \frac{\partial L_m(H, \lambda_{m0})}{\partial \lambda_{m0}}. \end{aligned}$$

Re-arranging terms, using the definition of $N_1(W, H)$ in (22), and substituting for $\lambda_{k1} = \epsilon\bar{\lambda}_{k1}$ yields the VSL in (30). Note that the alternative calculation through the marginal willingness to pay

$$v_s(W, H, \lambda_{m0}, \epsilon) = \left.\frac{\partial v(W, H, \lambda^*_{m0}, \epsilon)}{\partial \lambda^*_{m0}}\right|_{\Delta=0}$$

yields the same value of statistical life (30). ■

C.5 Discrete changes per period in death intensity (Remark 1)

Lemma 1 *A higher likelihood of death of Δ per time interval of $s \in [0, T]$ corresponds to a permanent increase in the endowed intensity to $\lambda^*_{m0}(H, \Delta, T) > \lambda_{m0}$ given by:*

$$\lambda^*_{m0}(H, \Delta, T) = \frac{-1}{T} \log \left[e^{-\lambda_{m0} T} - \frac{\Delta}{1 - \lambda_{m1} k(H, T)} \right],$$

where,

$$\begin{aligned} k(H, T) &= H^{-\xi_m} \left(\frac{e^{\psi T} - 1}{\psi} \right) \geq 0, \\ \psi &= \xi_m \left[\delta - (\alpha B)^{\frac{\alpha}{1-\alpha}} \right] + \lambda_{s0} \left[(1 - \phi)^{-\xi_m} - 1 \right] \geq 0. \end{aligned}$$

Proof A higher likelihood of death of Δ over a time interval of $s \in [0, T]$ corresponds to an increase in the endowed intensity to $\lambda^*_{m0}(\Delta, H) > \lambda_{m0}$:

$$\begin{aligned} \Pr \left[T_m \leq T \mid \lambda^*_{m0} \right] &= \Pr \left[T_m \leq T \mid \lambda_{m0} \right] + \Delta, \\ &= 1 - E \left[e^{-\int_0^T \lambda^*_m(\Delta, H_s) \mathrm{d}s} \right], \end{aligned}$$

where we have set $\lambda^*_m(\Delta, H) = \lambda^*_{m0}(\Delta, H) + \lambda_{m1} H^{-\xi_m}$ in (11). Solving for λ^*_{m0} through a first-order expansion around benchmark $\lambda_{k1} = 0, k = m, s$ reveals that the latter is as:

$$\lambda^*_{m0}(\Delta, H) = \frac{-1}{T} \log \left[e^{-\lambda_{m0} T} - \frac{\Delta}{1 - \lambda_{m1} k(H)} \right],$$

where,

$$\begin{aligned} k(H) &= E \int_0^T H_s^{-\xi_m} \mathrm{d}s = H^{-\xi_m} \left(\frac{e^{\psi T} - 1}{\psi} \right) \geq 0, \\ \psi &= \xi_m \left[\delta - (\alpha B)^{\frac{\alpha}{1-\alpha}} \right] + \lambda_{s0} \left[(1 - \phi)^{-\xi_m} - 1 \right] \geq 0. \end{aligned}$$

as stated. ■

C.6 Proof of Proposition 4

Again by a similar reasoning, the first-order approximation to gunpoint value of life $v_g(\epsilon) = v_g(W, H, \epsilon)$ in (8) is implicitly given as:

$$
\begin{aligned}
0 &= V(W - v_g(\epsilon), H, \lambda_{m0}, \epsilon) \\
&\approx V(W - v_g(0), H, \lambda_{m0}, 0) + \epsilon V_\epsilon(W - v_g(0), H, \lambda_{m0}, 0).
\end{aligned}
$$

Straightforward calculation indicate that:

$$
V(W - v_g(0), H, \lambda_{m0}, 0) = \Theta\left[N_0(W, H) - v_g(0)\right]
$$

whereas,

$$
V_\epsilon(W - v_g(0), H, \lambda_{m0}, 0) = \Theta\left[-v_g'(0) - \bar{\lambda}_{s1} L_s(H) P_0(H) - \bar{\lambda}_{m1} L_m(H)\left(N_0(W, H) - v_g(0)\right)\right].
$$

Again equating each terms to zero uniquely solves for $v_g(0), v_g'(0)$ and reveals that:

$$
\begin{aligned}
v_g(\epsilon) &\approx v_g(0) + \epsilon v_g'(0) \\
&= N_0(W, H) - \epsilon \bar{\lambda}_{s1} L_s(H) P_0(H).
\end{aligned}
$$

Substituting back $\lambda_{s1} = \epsilon \bar{\lambda}_{s1}$, and using total wealth (22) yields (31). ■

D Tables

D.1 Data

Table 1: PSID data statistics

	Mean	Std. dev.	Min	Max
Health (H)	2.58	0.80	1	4
Wealth (W)	38 685	122 024	0	1 430 000
Consumption (c)	9 835	11 799	1.047	335 781
Risky holdings (π)	20 636	81 741	0	1 367 500
Insurance (x)	247	718	0	17 754
Health investment (I)	721	2 586	0	107 438
Income (Y)	21 838	37 063	0	1 597 869
Age (years)	45.68	16.46	16	100

Notes: Statistics in 2013 \$ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good), and 4.0 (Excellent).

Table 2: PSID data statistics (cont'd)

Health	H_j	Wealth quintiles 1	2	3	4	5
		a. Wealth W_j ($)				
Poor	1.00	0	139	2 063	11 831	152 151
Fair	1.75	0	145	1 741	12 027	123 083
Good	2.50	0	168	1 802	11 908	120 467
Very good	3.25	0	199	1 823	12 197	118 738
Excellent	4.00	0	192	1 823	12 099	122 135
		b. Consumption c_j ($)				
Poor	1.00	3 281	4 906	6 558	10 052	7 752
Fair	1.75	4 095	6 888	8 795	11 196	13 368
Good	2.50	5 086	6 526	9 745	11 269	13 336
Very good	3.25	5 989	7 517	10 181	11 131	13 626
Excellent	4.00	5 276	6 897	10 002	12 099	14 628
		c. Stocks π_j ($)				
Poor	1.00	0	0	0	725	46 497
Fair	1.75	0	5	279	2 309	76 721
Good	2.50	0	1	268	4 320	55 379
Very good	3.25	0	5	192	4 756	68 768
Excellent	4.00	0	0	334	5 801	90 147
		d. Insurance x_j ($)				
Poor	1.00	165	191	503	723	856
Fair	1.75	181	196	497	775	1 095
Good	2.50	206	219	401	564	852
Very good	3.25	190	284	313	522	797
Excellent	4.00	203	254	366	429	807
		e. Investment I_j ($)				
Poor	1.00	549	552	2 341	2 936	6 003
Fair	1.75	400	468	968	621	1 250
Good	2.50	243	238	383	500	962
Very good	3.25	276	226	275	435	596
Excellent	4.00	151	192	230	307	451

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth, and per health status

D.2 Benchmark model (Hugonnier et al., 2013)

Table 3: Estimated and calibrated structural parameter values, benchmark model

Parameter	Value	Parameter	Value
a. Law of motion health (9)			
α	0.7045	δ	0.0109
	(0.1799)		(0.0048)
ϕ	0.0136^c		
b. Sickness and death intensities (10), (11)			
λ_{s0}	0.0316	λ_{s1}	0.0088
	(0.0152)		(0.0042)
ξ_s	2.9802	η	50^c
	(1.0262)		
λ_{m0}	0.0244	λ_{m1}	0.0045
	(0.0087)		(0.0022)
ξ_m	1.0686		
	(0.4497)		
c. Wealth, and income (12), (13)			
y	0.0122^c	β	0.0095
			(0.0045)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
d. Preferences (15), (16)			
γ	3.5242	ε	1.6699
	(1.3316)		(0.5911)
a	0.0146	γ_m	0.2862
	(0.0037)		(0.1212)
γ_s	7.4^c	ρ	0.05^c

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (36), estimated by iterative 2-stages ML, subject to the regularity conditions (37).

Table 4: Estimated Human Capital Value of Life, benchmark model (in $)

Health level	
Poor	392 736
Fair	534 280
Good	662 459
Very Good	787 946
Excellent	912 547
All	
- mean	737 887
- median	787 946

Notes: At estimated parameter values, using $v_h(H)$ in (28), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

Table 5: Estimated Value of Statistical Life, benchmark model (in $)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	1 477 135	1 479 511	1 512 348	1 679 099	4 074 525
Fair	4 009 020	4 011 551	4 039 474	4 219 360	6 161 584
Good	6 309 322	6 312 286	6 341 112	6 519 437	8 434 923
Very Good	8 563 359	8 566 878	8 595 675	8 779 565	10 668 049
Excellent	10 802 205	10 805 618	10 834 609	11 017 266	12 973 124
All					
- mean			8 142 566		
- median			8 565 013		

Notes: At estimated parameter values, using $v_s(W, H, \lambda_{m0})$ in (30), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

Table 6: Estimated Gunpoint Value of Life, benchmark model (in $)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	87 935	88 074	89 998	99 766	240 086
Fair	229 479	229 624	231 220	241 506	352 562
Good	357 658	357 826	359 460	369 566	478 125
Very Good	483 146	483 344	484 969	495 343	601 884
Excellent	607 747	607 939	609 570	619 846	729 881
All					
- mean			460 087		
- median			483 243		

Notes: At estimated parameter values, using $v_g(W, H)$ in (31), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

D.3 Restricted model (Grossman, 1972; Ehrlich and Chuma, 1990)

Table 7: Estimated and calibrated structural parameter values, restricted model

Parameter	Value	Parameter	Value
	a. Law of motion health (18)		
α	0.6940	δ	0.0272
	(0.1739)		(0.0123)
ϕ	0		
	b. Sickness and death intensities (18)		
λ_{s0}	0	λ_{s1}	0
ξ_s	0	η	0
λ_{m0}	0.0336	λ_{m1}	0
	(0.0087)		
ξ_m	0		
	c. Income, and wealth (13), (18)		
y	0.0122^c	β	0.0095
			(0.0048)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
	d. Preferences (17)		
γ	2.7832	ε	$1/\gamma$
	(1.2209)		
a	0	γ_m	0
γ_s	0	ρ	0.05^c

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (36), where optimal rules are given by (41), estimated by iterative 2-stages ML, subject to the regularity conditions (40).

Table 8: Estimated Human Capital Value of Life, restricted model

Health level	
Poor	382 814
Fair	479 162
Good	575 509
Very Good	671 856
Excellent	768 204
All	
- mean	634 365
- median	671 856

Notes: At estimated parameter values, using $\tilde{v}(H)$ in (32), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

Table 9: Estimated Value of Statistical Life, restricted model (in $)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	9 667 389	9 670 905	9 719 481	9 966 156	13 509 723
Fair	12 100 495	12 104 150	12 144 470	12 404 224	15 208 774
Good	14 533 601	14 537 843	14 579 100	14 834 323	17 575 815
Very Good	16 966 707	16 971 721	17 012 749	17 274 735	19 965 254
Excellent	19 399 813	19 404 662	19 445 851	19 705 361	22 484 145
All					
- mean			16 701 792		
- median			16 970 897		

Notes: At estimated parameter values, using $\tilde{v}_s(W, H)$ in (34), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

Table 10: Estimated Gunpoint Value of Life, restricted model (in $)

Health level	Wealth quintile				
	1	2	3	4	5
Poor	382 814	382 954	384 877	394 645	534 965
Fair	479 162	479 306	480 903	491 189	602 245
Good	575 509	575 677	577 311	587 417	695 976
Very Good	671 856	672 055	673 680	684 054	790 594
Excellent	768 204	768 396	770 027	780 303	890 339
All					
- mean			661 366		
- median			672 022		

Notes: At estimated parameter values, using $\tilde{v}_g(W, H)$ in (35), multiplied by 1 MM$ to correct for scaling used in estimation. All reported entries are averages of individual values and shares in the PSID sample.

E Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size), and scaled by 10^{-6} for the estimation. The agents' wealth and health which are constructed as follows:

Health H_j Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

Wealth W_j Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance, and are constructed as follows:

Portfolio π_j Money value of financial wealth held in risky assets.

Consumption c_j Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

Health expenditures I_j Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

Health insurance x_j Spending on health insurance premium.