Valuing Life as an Asset, as a Statistic, and at Gunpoint[1]

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November 22, 2017

[1]Financial support by the Swiss Finance Institute is gratefully acknowledged. We have benefited from very useful discussions with, and comments from Georges Dionne, Pierre-Carl Michaud, and Michel Normandin. The usual disclaimer applies.

Abstract

The Human Capital (HK), and Statistical Life Values (VSL) differ sharply in their empirical pricing of a human life and lack a common theoretical background, to justify these differences. This paper makes four contributions to the theory, and measurement of the latter. First, we provide a unified framework to formally define, and relate the Hicksian willingness to pay (WTP) to avoid death risks, the HK, and the VSL. Second, we use this setting to introduce a third life value calculated *at Gunpoint* (GPV), i.e. the maximal WTP to avoid certain, instantaneous death. Third, we associate a flexible human capital model to the common framework to characterize the WTP and the three life valuations in closed-form. Fourth, we structurally estimate these solutions. Our results confirm that the strong curvature of the WTP explains why the VSL (8.43 M$) is much higher than either the HK (438 K$) or the GPV (452 K$) values.

Keywords: Values of Human Life, Human Capital, Value of Statistical Life, Willingness to pay, Equivalent Variation, Morbidity, Mortality, Structural Estimation.

JEL Classification: J17, D91, G11.

The life you save may be your own (Schelling, 1968)

1 Introduction

1.1 Motivation and outline

Motivation Evaluating the price of a human life has long generated a deep interest in economic research.[1] Indeed, life valuations are often relied upon in public health and safety debates, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. Economic life values are also resorted to in occupational, or end-users' wrongful death litigation.

Three main sources of difficulty render the pricing of life particularly challenging. First, a human life is by definition non-divisible. This implies that any marginal valuation, e.g. via small incremental risks to life, must eventually be integrated back into a unitary life value. Second, a human life is non-marketed. The absence of equilibrium prices implies that the economic value of a human life must be inferred from relevant and measurable proxies such as foregone income, or responses to changes in mortality risks. Finally, ethical considerations induce significant discomfort in computing – and eventually comparing – the life values of identified persons.

The two most-widely used life valuation frameworks differ in how these challenges are dealt with. The Human Capital (HK) approach associates the value of an individual's life to the economic value embodied in his human capital. Relying on standard asset pricing, the HK value is the present value of the dividend stream associated with human capital, where the dividend is proxied by the marketed labor income, net of the measurable expenses needed to maintain that capital. The Value of a Statistical Life (VSL), introduced by Drèze (1962) and Schelling (1968), relies on a stated, or inferred, willingness to pay (WTP) to avert (resp. attain) small increases (resp. reductions) in exposure to death risks. Under appropriate assumptions, a collective WTP to save one unidentified (i.e. statistical) life can be recovered through a linear aggregation of the individual WTP's. Focusing on the value of an unidentified, rather than personalized, life thus conveniently

[1] Landefeld and Seskin (1982) make reference to human-capital based evaluations of the value of life dating back to Petty (1691).

avoids addressing the uncomfortable ethical issues associated with computing the value of someone's life.

Notwithstanding cautionary claims to the contrary,[2] both the HK and the VSL are ultimately gauging the value of an identical underlying object and should presumably come up with similar answers to the question of how much a human life is worth. However, despite pricing a common element, these two valuations yield vastly different values in practice.[3] Understanding these differences is complicated by the absence of common theoretical underpinnings that encompass both valuations. Consequently, most HK and VSL evaluations are reduced-form empirical exercises that rely on minimal theoretical foundations and are performed within disjoint settings that further complicate comparisons. Our main objective is to provide this common framework and exploit it to formally identify and empirically measure the two life values.

Contributions This paper makes four different contributions to the theory and the measurement of life values. Our first contribution proposes a unified theoretical background linking both the Human Capital and Statistical Life values. We start from a generic dynamic human capital problem in which an agent facing an uncertain horizon selects investment in his human capital, where the latter augments labor income. Assuming the existence of a solution to this problem satisfying weak preference for life over death, we use standard asset pricing to define the HK value as the discounted dividend stream, i.e. the income, net of investment, *along the optimal dynamic path*. Second, we rely on the associated indirect utility (i.e. the welfare at the optimum) which we combine with the Hicksian Equivalent Variation (EV, Hicks, 1946) to formally define the willingness to pay to avoid any exogenous change in death risk exposure. The theoretical VSL can then be defined formally in two equivalent ways: (i) as the (negative of the) marginal rate of substitution (MRS) between death exposure and wealth, calculated through the indirect

[2] In his opening remarks, Schelling (1968, p. 113) writes

> "This is a treacherous topic and I must choose a nondescriptive title to avoid initial misunderstanding. It's not the worth of a human life that I shall discuss, but of 'life saving', of preventing death. And it's not a particular death, but a statistical death. What it is worth to reduce the probability of death – the statistical frequency of death – within some identifiable group of people, none of whom expects to die except eventually. "

[3] For example, Huggett and Kaplan (2016) identify HK values between 300 K–900 K$, whereas the U.S. Department of Transportation recommends using a VSL-type amount of 9.4 M$ (U.S. Department of Transportation, 2016).

utility and (ii) as the marginal WTP (MWTP) with respect to death risk. This common setup ensures that the HK and the VSL are both evaluated from a single underlying dynamic problem.

Our second contribution also relies on this unified theoretical framework to define a *third* valuation alternative that forthrightly addresses the measurement challenges and can serve as comparison benchmark. The objectives are to gauge the economic value of a human life without recourse to indirect proxies and/or arbitrary aggregation assumptions. Instead, we address the non-divisibility and non-marketability by resorting to the unitary shadow value of life accruing to its main beneficiary, i.e. the willingness to pay that leaves an agent indifferent between living and dying in a highwaymen threat. The Hicksian EV again provides a natural theoretical background to elicit this shadow value. We refer to the corresponding amount as the *Gunpoint Value of Life* (GPV).[4] To paraphrase Schelling (1968)'s seminal title, in our highwaymen valuation, 'the life you save *is* your own'.

Compared to the HK and VSL alternatives, the Gunpoint Value presents several advantages that are discussed in further details in Section 1.2. First, the Gunpoint Value does not uniquely ascribe the economic worth of an agent to the labor income he generates, but instead accounts for all pledgeable disposable resources, including financial wealth. Second, the GPV does not extrapolate measurable responses to small probabilistic changes in the likelihood of death, but instead explicitly values a person's life as *an entity* and does so without external assumptions regarding integrability from marginal to total value of life. Finally, any ethical discomfort in valuing someone else's life is addressed by having that person compute his *own* intrinsic value.

[4] Explicit references to a Gunpoint Value of life can be found in the academic literature (with emphasis added):

> "We argue that living, like other goods, has diminishing marginal utility–the willingness to pay for an additional year of life falls with how many years one has to live. This is in contrast to how the value of a statistical life-year is taught and explained: it is often prefaced with claiming that it is not how much people are willing to pay *to avoid having a gun put to their head* (presumably ones wealth). However, terminal care decisions are often exactly of that nature." (Philipson et al., 2010, p. 2)

or in the media:

> "But how do you put a dollar value on a life, even in a generic sense? It wouldn't work for researchers to survey Americans *at gunpoint* and ask how much they would pay not to die. Instead, an unlikely academic field has grown up to extrapolate life's value from the everyday decisions of average Americans." (Fahrenthold, 2008)

Our third contribution is also theoretical and consists of analytical calculations of the WTP, as well as of the Human Capital, Statistical Life and Gunpoint Values of life. To do so, we rely on a flexible human capital model that is applicable to alternative capital definitions (e.g. skills or health). This model guarantees weak preference for life over death and yields closed-form solutions, allowing us to compute the analytic expressions corresponding to the life values. Since these solutions stem from the same underlying model, they are thus directly comparable. We can therefore assess the contribution to value of fundamentals, such as preferences, risk distributions, or technology, as well as financial and human resources and thus investigate how the WTP, HK, VSL and GPV are theoretically related to one another.

Our fourth and final contribution is empirical and consists of *structural* estimates of the three different life valuations. More precisely, we adopt a revealed-preference perspective to estimate the structural parameters of the model, using PSID data that correspond to the optimal investment, consumption, portfolio and health insurance policies. We can then combine the structural parameters with observed wealth and health status to calculate the analytical expressions for the Human Capital, Statistical and Gunpoint Values of life. Whereas the latter is new and has not been previously estimated, the HK and the VSL can be contrasted with reduced-form estimates in an out-of-sample assessment of our results.

Main findings Our analytical results reveal that the willingness to pay, as well as all three life valuations are increasing in the value of the human capital. The latter reflects technological and distributional parameters, but is independent of preferences. Moreover, standard properties of the indirect utility, that are verified for our model, imply that the willingness to pay is increasing and concave in the increment in death risk. It follows that the slope for small, finite increments (corresponding to the empirical VSL) – understates the marginal WTP (corresponding to the theoretical VSL). Moreover, when the elasticity of inter-temporal substitution (EIS) is larger than one, we show that the WTP is well-defined for any detrimental change and that it is bounded above by a finite limiting value.

The Gunpoint value is equal to net total wealth, i.e. financial wealth, plus the shadow value of human capital, minus capitalized subsistence consumption. It is independent of both preferences and mortality exposure. Since death is instantaneous and certain in a highwayman threat, the agent is willing to pay out all pledgeable resources, net of unpledgeable subsistence requirements. For the same reasons, attitudes towards towards risk, or time, as well as current level of death risk play no role in the GPV. Furthermore, the Gunpoint and Human Capital Values display similarities in calculating the net present value of optimal foregone net consumption (GPV) and of foregone net income (HK). Finally, the Gunpoint is equal to the limiting WTP when the EIS is larger than one. The curvature of the WTP then implies that both the empirical and theoretical VSL overstate the value an agent attributes to his own life.

Our structural estimation shows that the model's parameters are realistic and confirm that the estimated EIS is larger than one, i.e. that the limiting WTP is the GPV. The predicted life valuations are also realistic. Indeed, the estimates for the mean HK value (438 K$) and the mean VSL (8.43 M$) are well in line with those obtained in the reduced-form literature. The average Gunpoint Value (452 K$) is in the same range as the HK valuation. Importantly, our results confirm the strong concavity of the willingness to pay. This curvature – rather than disjoint valuation concepts – is thus the main element explaining the much higher values of life obtained under the VSL, compared to alternative values.

After a review of the relevant literature in Section 1.2, the rest of the paper is organized as follows. We first introduce the formal links between the HK, WTP, VSL and GPV in Section 2. Sections 3 and 4 present the benchmark model and corresponding life values. The empirical strategy is discussed in Section 5, with structural parameters and values of life estimates reviewed in Section 6 and concluding remarks presented in Section 7.

1.2 Related literature

1.2.1 Human Capital values of life

The HK model associates the economic value of a person to the value of his human capital that is entirely depreciated at death. That value is obtained by pricing the expected discounted stream of its associated dividends that are foregone upon death, i.e. the lifetime labor income flows, net of associated investment.[5] Well-known issues related to this approach include the appropriate rate of discounting, the endogeneity of income

[5]See Jena et al. (2009); Huggett and Kaplan (2013, 2016) for applications.

and investment, as well as the treatment of non-labor activities.[6] As for HK models, we do calculate the net present value of the stream of human capital dividends that are lost upon death. Unlike HK models however, that value is computed in closed-form. In particular, we fully account for the endogeneity of the human capital stock and therefore of its associated income and investment expenditures. We therefore encompass the relevant technological and distributional considerations, such as the capital production technology, its deterministic and stochastic depreciation, the income-capital gradient, as well as the duration of the dividends stream.

1.2.2 Value of a Statistical Life

The empirical VSL alternative relies on explicit and implicit evaluations of the Hicksian WTP for a small reduction in fatality risk which is then linearly extrapolated to obtain the value of life. Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.[7] Examples of the latter include responses to prices and fines in the use of life-saving measures such smoke detectors, speed limitations, or seat belt regulations. The Hedonic Wage (HW) variant of the implicit VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/worker characteristics, the wage elasticity with respect to job fatality risk can be estimated and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008).

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. He argues that the assumed exogeneity of the change in fatality risk can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. He also argues that agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Ashenfelter further contends that it is unclear whose

[6]Conley (1976) provides additional discussion of HK approaches while Huggett and Kaplan (2016) address the discounting issues.

[7]A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the implicit VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta*-meta analysis of the stated- and revealed-preferences valuations of life.

preferences are involved in the risk/income tradeoff and how well these arbitrage are understood. For example, if high fatality risk employment attracts workers with low risk aversion and/or high discount rates, then generalizing the wages risk gradient to the entire population could understate the true value of life. Moreover, because wages are an equilibrium object in the HW variant of the VSL, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death. Finally, as was the case for HK measures, HW estimates relate primarily to workers and are hardly adaptable to other non-employed groups, such as young, elders, or the unemployed.

Moreover the passage from statistical towards personalized life valuations is unclear. Pratt and Zeckhauser (1996) argue that concentrating the costs and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e. large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment to avoid increases in risk, thereby reducing the WTP as risk increases.[8] Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our Gunpoint Value. Their conjecture is warranted in our calculations. When faced with certain death, an individual is willing to pay much less than what can be inferred from the VSL.

Hall and Jones (2007) propose a semi-structural measure of life value. They adopt a marginal value perspective by equating the VSL to the marginal cost of saving a human life. The cost of reducing mortality risk can be imputed by specifying and estimating a health production function and by linking health status to death risks. Dividing this cost by the required change in death risk amount yields a VSL-inspired life value, e.g. corresponding to 1.9 M$ for an individual aged 40-44 (Hall and Jones,

[8]Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay $1 million to reduce the associated mortality risk by 10%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.

2007, Tab. 1, p. 60). Like them, we model and estimate the health production and death distribution. However, our valuations framework does not exclusively rely on technological and distributional parameters, but includes preferences and health and wealth statuses.

Our approach offers other advantages in calculating the value of life. First the theoretical and empirical values stem from a common model and are thus directly comparable and interpretable. Second, we rely on a widely-used panel (PSID) accounting for households' consumption, financial and health-related decisions to elicit the WTP and life valuations. Consequently, these values are representative and can be generalized to the entire population. Third, we make no assumption on the shape of the WTP function but rather derive its properties from the indirect utility function measured at the optimal allocation. Indeed, we show that, consistent with economic intuition, the marginal value ascribed to small increases in death intensity is positive, but falling in the latter.

2 A Common Framework for Life Valuation

This section outlines a common framework that will be relied upon to formally define and compare the Human Capital, Statistical, as well as Gunpoint Values of Life. Our main building block is an underlying human capital problem for which the optimal policies and associated indirect utility function can be solved. We combine these solutions with standard asset pricing and Hicksian variational analysis to characterize the three life valuations.

2.1 Underlying Human Capital Problem

Consider an agent's human capital problem defined by a stochastic age at death T^m, an instantaneous death probability $\mathcal{P} \in [0,1]$, a human capital H and associated increasing income function $Y(H)$, a financial wealth W, as well as the relevant distributional assumptions with respect to mortality, human and financial assets. For this program, the agent selects the money value of investment in his human capital I and other controls X (e.g. consumption, asset allocation, ...) so as to maximize utility U:

$$\begin{aligned} V(W,H,\mathcal{P}) &= \sup_{I,X} U, \quad \text{subject to:} \\ \mathrm{d}H &= \mathrm{d}H(H,I), \\ \mathrm{d}W &= \mathrm{d}W(W,Y(H),I,X). \end{aligned} \tag{1}$$

We assume that the agent's preferences and constraints in (1) satisfy standard properties such that the indirect utility $V = V(W,H,\mathcal{P})$ is monotone increasing and concave in W. We further assume weak preference for life over death. In particular, the indirect utility is well-defined, decreasing and convex for all levels of death risk exposure $\mathcal{P}$ and satisfies:

$$V(W,H,\mathcal{P}) \geq V^m > -\infty, \quad \forall\, W,H,\mathcal{P}, \tag{2}$$

where V^m denotes the finite utility at death. Standard examples of the latter include the seminal Yaari (1965); Hakansson (1969) paradigm ($V^m \equiv 0$), or 'warm glow' effects of bequeathed wealth ($V^m = V^m(W_{T^m})$, e.g. Yogo (2016); French and Jones (2011); De Nardi et al. (2009)). Observe that monotonicity, curvature and finite utility assumptions imply the existence of decreasing and convex indifference curves in the wealth and life probability $(1 - \mathcal{P})$ space.

2.2 Human Capital Value of Life

As previously mentioned, the Human Capital Value of life is the market value of the net dividend flow associated with human capital and that is foregone upon death (e.g. Huggett and Kaplan, 2016, 2013). In our setting, this net dividend is the marketed income $Y(H)$, minus the money value of associated investment expenses I, where both are evaluated at the optimum to problem (1):

Definition 1 (HK value of life) *The Human Capital Value of life* $v_{h,t} = v_h(W_t, H_t, \mathcal{P}_0)$ *is the expected discounted present value over stochastic horizon* T^m *of labor revenue flows, net of investment costs:*

$$v_{h,t} = E_t \int_0^{T^m} m_{t,\tau} \left[Y(H_\tau^*) - I_\tau^* \right] \mathrm{d}\tau, \tag{3}$$

where $m_{t,\tau}$ *is a stochastic discount factor induced by the assets' prices and* (H^*, I^*) *are evaluated along the optimal path solving* (1)*.*

As a canonical example with Poisson mortality, assume constant values for the death intensity λ_m, the interest rate r and the non-stochastic growth rate g^n for net income $Y^n = Y(H) - I$. The HK value then simplifies to:

$$v_h = \frac{Y^n}{r + \lambda_m - g^n}. \tag{4}$$

The human capital value of life in this special case is therefore decreasing in both the death risk λ_m and interest rate r and is increasing in both the net income level Y^n, as well as net growth rate g^n.

2.3 Willingness to pay

Next, consider a permanent exogenous change Δ in the instantaneous probability of death from base level $\mathcal{P}_0$. We rely on the indirect utility (1) to define the Hicksian Equivalent Variation as follows:

Definition 2 (WTP) *The maximal willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ to avoid a permanent change Δ in death risk exposure $\mathcal{P}$ is implicitly given as the solution to:*

$$V(W - v, H, \mathcal{P}_0) = V(W, H, \mathcal{P}_0 + \Delta), \tag{5}$$

where $V(W, H, \mathcal{P})$ solves (1)*.*

For unfavorable changes $\Delta > 0$, equation (5) indicates indifference between paying the equivalent variation $v > 0$ to remain at base risk and not paying, but face higher death risk. For favorable changes $\Delta < 0$, the agent is indifferent between receiving compensation $-v > 0$ and foregoing lower death risk exposure.[9]

Observe that if the indirect utility is well-defined over all $\mathcal{P}$, then the willingness to pay is well-defined over all change in death risk Δ. Note also that the monotonicity and curvature assumptions on the indirect utility $V(W, H, \mathcal{P})$ in (1) are sufficient to yield a monotone increasing and concave willingness to pay with respect to increment in death risk Δ. To see this, substitute $v(W, H, \mathcal{P}_0, \Delta)$ in (5), take derivatives and re-arrange to obtain:

$$\frac{\partial v}{\partial \Delta} = \frac{-V_{\mathcal{P}}}{V_W} \geq 0, \tag{6a}$$

$$\frac{\partial^2 v}{\partial \Delta^2} = \frac{V_{\mathcal{P}\mathcal{P}} - V_{WW}\left(\partial v / \partial \Delta\right)^2}{-V_W} \leq 0. \tag{6b}$$

Monotonicity $V_W \geq 0$ and preference for life $V_{\mathcal{P}} \leq 0$ therefore induce a willingness to pay v that is increasing in Δ, whereas the diminishing marginal utility of wealth $V_{WW} \leq 0$, and of survival probability $V_{\mathcal{P}\mathcal{P}} \geq 0$ are sufficient to induce a concave WTP function in mortality risk exposure.

[9] An alternative formulation relies instead on the Hicksian willingness to accept compensation (WTA) to face Δ, implicitly defined as the solution to:

$$V(W + v^a, H, \mathcal{P}_0 + \Delta) = V(W, H, \mathcal{P}_0).$$

This WTA perspective is however not suitable for Gunpoint settings in the absence of bequests. Indeed, whereas paying out the WTP in a highwaymen threat is rational, accepting compensation against certain death when terminal wealth in not bequeathed and life is preferred is not. Since we abstract from bequests in our benchmark model in Section 3, we therefore adopt the WTP perspective in (5).

2.4 Value of Statistical Life

The VSL is a measure of the marginal rate of substitution between the probability of life and wealth, evaluated at base risk (e.g. Aldy and Smyth, 2014; Andersson and Treich, 2011; Bellavance et al., 2009). In the context of the model (1), it is thus the negative of the MRS between $\mathcal{P}$ and W evauated at continuation utility $V(W, H, \mathcal{P})$. The WTP property (6a) establishes that this MRS is also the marginal willingness to pay (MWTP) evaluated at base risk:

Definition 3 (VSL) *The Value of a Statistical Life $v_s = v_s(W, H, \mathcal{P}_0)$ is the negative of the marginal rate of substitution between the probability of death and wealth and also the marginal WTP evaluated at base risk:*

$$v_s = \left.\frac{-V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})}\right|_{\mathcal{P}=\mathcal{P}_0} = \lim_{\Delta \to 0} \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}, \tag{7a}$$

$$= \left.\frac{\partial v(W, H, \mathcal{P}_0, \Delta)}{\partial \Delta}\right|_{\Delta=0}, \tag{7b}$$

where $V(W, H, \mathcal{P})$ solves (1) *and $v(W, H, \mathcal{P}_0, \Delta)$ solves* (5)*.*

Figure 1 illustrates the indifference curve (in blue) in the wealth and life probability space. The VSL in (7a) is the slope of the red tangent evaluated at base death risk $\mathcal{P}_0$ and is equivalent to the total wealth spent to save one life corresponding to the distance [a,d] (e.g. Andersson and Treich, 2011, Fig. 17.1, p. 398).

Moreover, contrasting the theoretical definition of the VSL as a MWTP in (7a) with its empirical counterpart reveals that the latter can also be interpreted as the slope of the willingness to pay to avoid small changes in death risk. To see this, consider a canonical example (e.g. Aldy and Viscusi, 2007), whereby we suppose that each agent $i = 1, 2, \ldots, n$ has WTP of $v^i(W^i, H^i, \mathcal{P}_0, \Delta)$ for a common reduction $\Delta = n^{-1}$ in death risk. Assuming identical preferences, wealth and capital, the empirical value of a statistical life is obtained as:

$$v_s^e = \sum_{i=1}^{n} v^i(W^i, H^i, \mathcal{P}_0, \Delta) = n\, v(W, H, \mathcal{P}_0, \Delta) = \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}, \tag{8}$$

and represents the collective willingness to pay to save one unidentified individual and is equal to a slope of the WTP for Δ small. The theoretical measure of the VSL (7b) is therefore the limiting value of the slope in (8) when the change Δ tends to zero.

Figure 2 illustrates the links between the WTP and the theoretical and empirical measures of the VSL. From properties (6), the willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ (in blue) is an increasing, concave function of the change in death risk Δ. The theoretical VSL v_s in (7b) is the marginal willingness to pay, i.e. the slope of the red tangent evaluated at base death risk ($\Delta = 0$). If the willingness to pay can be computed for all Δ, then the VSL is equivalent to the linear projection corresponding to the total wealth spent to save one person (i.e. reach $\mathcal{P}_0 + \Delta = 1.0$) and is equal to the distance [a,d]. The empirical Value of a Statistical Life v_s^e in (8) is computed for a small change $\Delta_e > 0$ and corresponds to the slope of the green line; equivalently, it is the distance [e,f]. As Figure 2 makes clear, the empirical VSL measure v_s^e will understate its theoretical counterpart v_s when Δ_e is large and when the WTP is concave.

2.5 Gunpoint Value of Life

We next introduce the Gunpoint Value (GPV) as a third valuation of life. To do so, we combine preference for life (2) with the Hicksian Equivalent Variation in (5) to define the GPV as follows:

Definition 4 (GPV) *The Gunpoint Value $v_g = v_g(W, H, \mathcal{P}_0)$ is the maximal WTP to avoid certain, instantaneous death and is implicitly given as the solution to:*

$$V(W - v_g, H, \mathcal{P}_0) = V^m \tag{9}$$

where $V(W, H, \mathcal{P})$ solves (1) *and satisfies* (2).

The Gunpoint Value $v_g(W, H, \mathcal{P}_0)$ in (9) is implicitly defined as the maximal payment that leaves the agent indifferent between paying v_g and remaining at base death risk $\mathcal{P}_0$ and not paying and face instantaneous and certain death and attain utility V^m. The willingness to pay v_g can thus be interpreted as the maximal amount paid in order to survive an *ex-ante* unforecastable and *ex-post* credible highwaymen threat. As will become clear shortly, if the willingness to pay $v(W, H, \lambda_{m0}, \Delta)$ can be computed for any

Δ, then the Gunpoint value corresponds to the limiting WTP when death is certain as represented by the distance [b,c] in Figure 2. A concave WTP entails that a linear extrapolation under either the theoretical, or the empirical VSL will thus over-estimate the value attributed to one's own life.

3 A benchmark human capital model

In order to compute the theoretical life values defined in Section 2 we introduce a parametrized human capital model corresponding to its generic counterpart in (1).

3.1 Economic environment

Consider a depreciable human capital model whose law of motion is given by:

$$\mathrm{d}H_t = \left[I_t^{\alpha} H_t^{1-\alpha} - \delta H_t\right] \mathrm{d}t - \phi H_t \mathrm{d}Q_{st}. \tag{10}$$

The term $\delta \in (0,1)$ is a deterministic depreciation, whereas $\mathrm{d}Q_{st}$ is a Poisson depreciation shock with constant intensity λ_{s0}, whose occurrence further depreciates the capital stock by a factor $\phi \in (0,1)$.

The law of motion (10) applies to alternative interpretations of human capital. If H_t is associated with skills (e.g. Ben-Porath, 1967; Heckman, 1976), then investment I_t comprises education and training choices made by the agent whereas $\mathrm{d}Q_{st}$ can be interpreted as stochastic unemployment, or obsolescence shocks that depreciate the human capital stock. If H_t is instead associated with health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), then investment takes place through medical expenses or healthy leisure whereas the stochastic depreciation occurs through morbidity shocks.

In addition to stochastic depreciation, the agent is exposed to Poisson mortality risk with constant intensity λ_{m0}. Within the context of this continuous-time model, the instantaneous death probability $\mathcal{P}$ introduced earlier can be obtained by noting that:

$$\Pr[\mathrm{Death}(t, t+h)] = \lambda_{m0}\, h + o(h), \tag{11}$$

for a small h. In the subsequent life valuation, we will analyze exogenous changes Δ in death risk $\mathcal{P}$ resulting from permanent changes in the exogenous death risk exposure λ_{m0}.

Second, financial wealth W evolves according to the dynamic budget constraint:

$$dW_t = \left[rW_t + Y_t - c_t - I_t\right] \mathrm{d}t + \pi_t \sigma_S \left[\mathrm{d}Z_t + \theta \mathrm{d}t\right] + x_t \left[\mathrm{d}Q_{st} - \lambda_{s0} \mathrm{d}t\right], \tag{12}$$

$$Y_t = y + \beta H_t, \tag{13}$$

where r it the interest rate and $\theta = \sigma_S^{-1}(\mu - r)$ is the market price of financial risk. In addition to investment I_t, the control variables include c_t as consumption, π_t as the risky portfolio and x_t is the units purchased of actuarially-fair depreciation insurance. The latter pays one unit of the numeraire per unit of contract purchased, upon occurrence of the depreciation shock and can be interpreted as unemployment insurance (if H_t is associated with skills) or as medical, or disability insurance (if H_t is associated with health). The income process in (13) comprises an exogenous component y, whereas the expression βH reflects a positive income gradient for agents with higher human capital.

Finally, the indirect utility of an alive agent is defined as:

$$V(W_t, H_t) = \sup_{(c,\pi,x,I)} U_t,$$

where preferences are:

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_\tau) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_\tau} \right) \mathrm{d}\tau, \tag{14}$$

with

$$f(c_t, U_t) = \frac{\rho U_t}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_t} \right)^{1-\frac{1}{\varepsilon}} - 1 \right). \tag{15}$$

The utility U in (14), combined with the Kreps-Porteus aggregator function $f(c, U)$ in (15) corresponds to the stochastic differential utility proposed by Duffie and Epstein (1992). It is characterized by subjective discount rate $\rho > 0$, minimal subsistence consumption $a > 0$ and disentangles the elasticity of inter-temporal substitution (EIS) $\varepsilon \geq 0$, from the agent's constant relative risk aversion with respect to financial risk $\gamma \geq 0$. As explained in Hugonnier et al. (2013) and confirmed in Theorem 1 below, the homogeneity properties of non-expected utility guarantee that the agent prefers life over death, with minimal consumption requirement $c_t \geq a$ implying positive continuation utility and preference of life over death $V_t \geq V^m \equiv 0$.

3.2 Optimal rules

The baseline human capital model of Section 3.1 can be solved in closed form, yielding the following result.

Theorem 1 *Assume that the following conditions hold:*

$$0 < A(\lambda_{m0}) - \max\left(0, r - \lambda_{m0} + \theta^2/\gamma\right), \tag{16a}$$
$$\beta < (r + \delta + \phi\lambda_{s0})^{\frac{1}{\alpha}}. \tag{16b}$$

Then the indirect utility for the agent's problem is:

$$V_t = \Theta(\lambda_{m0}) N_0(W_t, H_t) \geq 0, \tag{17}$$

and generates the optimal rules:

$$\begin{aligned} c_t &= a + A(\lambda_{m0}) N_0(W_t, H_t) \geq 0, \\ \pi_t &= (\theta/(\gamma\sigma_S)) N_0(W_t, H_t), \\ x_t &= \phi P_0(H_t) \geq 0, \\ I_t &= \left(\alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}\right) P_0(H_t) \geq 0, \end{aligned} \tag{18}$$

where any dependence on death intensity λ_{m0} is explicitly stated. The nonnegative human capital value and net total wealth are given as:

$$P_0(H_t) = BH_t, \tag{19}$$
$$N_0(W_t, H_t) = W_t + \frac{y - a}{r} + P_0(H_t), \tag{20}$$

where $B > 0$ solves $g(B) = 0$, s.t. $g'(B) < 0$ in:

$$g(B) = \beta - (r + \delta + \phi\lambda_{s0})B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}} \tag{21}$$

and where the marginal value of net total wealth and the marginal propensity to consume are:

$$\Theta(\lambda_{m0}) = \tilde{\rho} A(\lambda_{m0})^{\frac{1}{1-\varepsilon}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{-\varepsilon}{1-\varepsilon}} \tag{22}$$
$$A(\lambda_{m0}) = \varepsilon\rho + (1 - \varepsilon)\left(r - \lambda_{m0} + 0.5\,\theta^2/\gamma\right) \geq 0. \tag{23}$$

Both the indirect utility (17) and the optimal rules (18) are increasing functions of the market value of the human capital capital $P_0(H_t)$. The price of human capital B

in (19) can be interpreted as a Tobin's-Q. It is implicitly defined in (21) as an increasing function of the income gradient β and is declining in the rate of interest r and the expected depreciation $\delta + \phi\lambda_{s0}$. The market value of human capital is combined with financial wealth W_t and the NPV of the base income stream, net of minimal consumption $(y - a)/r$, to recover net total wealth $N_0(W_t, H_t)$ in (20). Observe that both human capital value and net total wealth are independent of the death intensity λ_{m0}. This independence results from the well-known equivalence between discounting at rate ρ, with Poisson mortality and finite lives and an infinite horizon plus discounting at augmented rate $\rho + \lambda_{m0}$.

Two features of the optimal rules are particularly relevant for life valuation. A first property is that the exposure to exogenous death risk λ_{m0} affects welfare via $\Theta(\lambda_{m0})$ in (22), through its impact on the marginal propensity to consume (MPC) $A(\lambda_{m0})$ exclusively. Equation (23) establishes that this impact crucially depends on the elasticity of inter-temporal substitution ε. An increase in death risk λ_{m0} induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume.

On the one hand, more discounting of future consumption requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC). This effect is dominant at low elasticity of inter-temporal substitution $\varepsilon \in (0, 1)$. In the latter case, the MPC in (23) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\bar{\lambda}_{m0} = \left(\frac{\varepsilon}{1-\varepsilon}\right)\rho + \left(r + \frac{\theta^2}{2\gamma}\right). \tag{24}$$

It follows that the transversality condition (16a) is violated and both the value function and the optimal rules are not well-defined when the EIS is low and the death risk λ_{m0} is above threshold (24).

On the other hand, heavier discounting makes future consumption less desirable and shifts future towards current consumption (i.e. by increasing the MPC). This *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution $\varepsilon > 1$. In this case, the dual regularity conditions of positive MPC and transversality remain verified everywhere, such that both the indirect utility and optimal rules are well-defined even at high levels of death risk exposure. Note in closing that unit elasticity implies exact cancellation of the two effects and results in a mortality risk-independent MPC that is equal to the subjective discount rate ρ. Consequently, the marginal value of net total wealth Θ is also independent of the exposure to death λ_{m0} when $\varepsilon = 1$.

A second key property is that the welfare in (17) is monotone increasing and linear in both wealth and human capital stock and is unconditionally monotone decreasing and convex in death risk exposure since:

$$\Theta'(\lambda_{m0}) = -\tilde{\rho} A(\lambda_{m0})^{\frac{\varepsilon}{1-\varepsilon}} \leq 0, \tag{25a}$$

$$\Theta''(\lambda_{m0}) = \tilde{\rho}\varepsilon A(\lambda_{m0})^{\frac{2\varepsilon-1}{1-\varepsilon}} \geq 0. \tag{25b}$$

Hence, whereas the sign of the effects of death risk λ_{m0} on the MPC (23) depends on the EIS, preference for life implies that it always reduces the marginal value of total wealth (22).

4 Corresponding Values of Life

We next calculate the model-implied life valuations of Section 2 relying on the solution for the benchmark human capital model of Section 3.[10] We will assume throughout that the optimal rules outlined in Theorem 1 are being followed by the agents and will restrict our attention to detrimental changes in death risk $\Delta \geq 0$.

4.1 Human Capital Value of Life

The HK value of life outlined in Definition 1 is computed as follows.

Proposition 1 (HK value) *The Human Capital Value of life solving* (3) *is:*

$$v_h(H, \lambda_{m0}) = C_0 y + C_1 P_0(H) \tag{26}$$

where the non-negative constants C_0, C_1 *are defined by:*

$$C_0 = \frac{1}{r + \lambda_{m0}},$$
$$C_1 = \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_{m0} - (\alpha B)^{\frac{\alpha}{1-\alpha}}},$$

and where $P_0(H)$ *is given in* (19)*.*

The HK value in (26) is thus an increasing affine function of the economic value of human capital stock $P_0(H)$. A wealth-independent optimal investment in (18) implies that v_h is also independent of W. The first term C_0 is the standard NPV of base income y, lowered for exogenous exposure to death risk λ_{m0} (see equation (4)). The second term C_1 is the net present value along the optimal path of the βH^* component of income, net of spending I^*. Indeed, the optimal rules in (18) reveal that the investment-to-capital ratio is $I/H = (\alpha B)^{1/(1-\alpha)}$ and enters negatively in C_1. A higher Tobin's-Q has two conflicting effects on the HK value. One the one hand, a higher market value $P_0(H) = BH$ entails a larger v_h. On the other hand, a higher price of capital justifies a higher investment ratio I/H and lowers v_h.

[10]Time subscripts will henceforth be abstracted from to alleviate notation.

4.2 Willingness to pay to avoid a finite increase in death risk

Next, we can substitute the indirect utility $V(W, H, \lambda_{m0})$ given by (17) in Definition 2, and solve for $v = v(W, H, \lambda_{m0}, \Delta)$ as follows:

Proposition 2 (willingness to pay) *The willingness to pay to avoid a change from λ_{m0} to $\lambda^*_{m0} = \lambda_{m0} + \Delta$ solving (5) is an increasing and concave function of Δ given by:*

$$v(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] N_0(W, H); \tag{27}$$

1. *when $\varepsilon \in (0, 1)$ and $\Delta < \bar{\Delta} = \bar{\lambda}_{m0} - \lambda_{m0}$, or*

2. *when $\varepsilon > 1$ and $\forall\Delta$,*

where net total wealth $N_0(W, H)$ is given in (20), the marginal value $\Theta(\lambda_{m0})$ is given in (22) and where $\bar{\lambda}_{m0}$ is given in (24).

The WTP in (27) equals zero when the increment $\Delta = 0$ or under unit elasticity of inter-temporal substitution for which case Θ is independent from λ_{m0}. For the other cases, it was shown earlier that the marginal value of total wealth $\Theta(\lambda_{m0}) \geq 0$ in (22) is a decreasing and convex function. Consequently, the weights $\Theta(\lambda^*_{m0})/\Theta(\lambda_{m0}) \in [0, 1]$ for detrimental changes $\Delta \geq 0$ and the willingness to pay is an increasing function of net total wealth $N_0(W, H)$.

Furthermore, equation (6) established that a decreasing and convex effect of mortality risk on welfare entails a monotone increasing and concave willingness to pay to avoid death. These properties of the indirect utility were verified in (25) and the implications for the WTP are again confirmed in (27). They are also consistent with standard economic intuition of diminishing marginal valuation of exposure to death (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2017).

The applicable domain of the willingness to pay depends on the elasticity of inter-temporal substitution. At low EIS (case 1), the previous discussion established that the dual restrictions of positive MPC and transversality (16a) are violated when $\lambda^*_{m0} \geq \bar{\lambda}_{m0}$ in (24). Consequently, neither the marginal value $\Theta(\lambda^*_{m0})$ nor the willingness to pay are well-defined for large changes $\Delta \geq \bar{\Delta}$. When the elasticity is high $\varepsilon > 1$ (case 2), positivity and transversality requirements are met for all detrimental changes $\Delta \geq 0$. Consequently, both the marginal value $\Theta(\lambda_{m0}^*)$ and the willingness to pay are well-defined everywhere, as shown in the following.

Corollary 1 (limiting WTP) *If $\varepsilon > 1$, then the willingness to pay in is bounded above by:*

$$\lim_{\Delta \to \infty} v(W, H, \lambda_{m0}, \Delta) = N_0(W, H). \tag{28}$$

Hence the maximal willingness to pay is finite and is equal to net total wealth when the elasticity of inter-temporal substitution is larger than one.

4.3 Value of a Statistical Life

Using Definition 3, and welfare (17), we can calculate the theoretical expression for the VSL implied by the benchmark model as follows.

Proposition 3 (Value of Statistical Life) *The Value of a Statistical Life is:*

$$v_s(W, H, \lambda_{m0}) = \frac{-\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} N_0(W, H), \tag{29}$$

where total wealth $N_0(W, H)$ is given in (20) *and the marginal value $\Theta(\lambda_{m0})$ is given in* (22).

As explained earlier, the marginal value of total wealth $\Theta(\lambda_{m0})$ is unconditionally decreasing in death risk. It follows that the VSL is an increasing function of net total wealth $N_0(W, H)$. Again unit elasticity of inter-temporal substitution entails that $\Theta'(\lambda_{m0}) = 0$, such that the VSL is zero.

Remark 1 (discrete changes per period) The theoretical calculations of the VSL in equation (29) are valid for permanent, infinitesimal changes in the death intensity. In the spirit of the empirical VSL literature, the value of a statistical life can also be computed as the willingness to pay to avoid an exogenous increase Δ in the probability of death over a given time interval (e.g. a change $\Delta = 0.1\%$ per one year period), divided by Δ (see v_s^e in equation (8)). This calculation can also be obtained in closed-form, and involves two steps. First, the new value of the endowed intensity $\lambda_{m0}^*(\Delta, T)$ is computed, corresponding to a change in death risk Δ occurring over a duration of T :

Lemma 1 *A higher likelihood of death of* Δ *per time interval of* $s \in [0, T]$ *corresponds to a permanent increase in the endowed intensity to* $\lambda^*_{m0}(\Delta, T) > \lambda_{m0}$ *given by:*

$$\lambda^*_{m0}(\Delta, T) = \frac{-1}{T} \log \left[e^{-\lambda_{m0} T} - \Delta \right]. \tag{30}$$

Second, we can substitute $\Theta(\lambda^*_{m0}(\Delta, T))$ in the WTP (27), and divide by Δ to obtain the corresponding empirical Value of a Statistical Life.

4.4 Gunpoint Value of Life

Combining Definition 4 and welfare function (17) reveals the following result.

Proposition 4 (Gunpoint value of life) *The willingness to pay to avoid certain death solving* (9) *is given by:*

$$v_g(W, H) = N_0(W, H), \tag{31}$$

where $N_0(W, H)$ *is the net total wealth in* (20)*.*

In the absence of a bequest motive, the agent who is forced to evaluate life at gunpoint is thus willing to pledge all available resources, i.e. his entire financial wealth W, plus the capitalized value of his fixed income endowment y/r. Since human capital is non-transferable and entirely depreciated at death, the agent is also willing to give up the shadow value of his capital $P_0(H) = HB$, an increasing function of the human capital stock and of its Tobin's-Q. However, the previous discussion emphasized that the minimal consumption level a is required at all periods for subsistence. Its costs therefore cannot be pledged in a highwaymen threat, and must be subtracted from the Gunpoint value.

It can also be shown that net total wealth $N_0(W, H)$ is equal to the expected discounted present value of excess consumption along the optimal path.[11] In order to survive, the agent is thus willing to pledge the total value of his optimal consumption stream (net of minimal subsistence). This result can be traced to the homegeneity

[11] In particular, Hugonnier et al. (2013, Prop. 2) show that

$$E_t \int_t^\infty m_{t,\tau}(c^*_\tau - a) \mathrm{d}\tau = N_0(W, H).$$

property under which the foregone utility is measured in the same units as the foregone excess consumption. This interpretation also foreshadows the similarities between the HK (foregone net income stream) and the GPV values of life (foregone net consumption stream).

The links between the WTP in (27) and GPV in (31) crucially depend on the EIS. On the one hand, for $\varepsilon > 1$, Proposition 2 established that the willingness to pay can be computed for all mortality risk increments Δ, while Corollary 1 showed that the WTP is bounded above by net total wealth. The GPV in (31) therefore corresponds to that maximal WTP. On the other hand, at low $\varepsilon \in (0, 1)$, the willingness to pay is not defined for high $\Delta \geq \bar{\Delta}$ and its limiting value cannot be computed. Regardless, the Gunpoint Value again corresponds to net total wealth. The reason stems from the way the GPV is characterized in Definition 4, i.e. the agent pays v_g to avoid receiving the utility $V^m \equiv 0$ that is associated with certain and immediate death. Because the utility at death is a finite given primitive, the Gunpoint Value – unlike the WTP – is always computable for all EIS levels. For the same reason, the Gunpoint Value of life v_g in (31) is also independent from the agent's preferences $(\rho, \varepsilon, \gamma)$, and from the death intensity (λ_{m0}). Because the outcome of death is certain when life is evaluated at gunpoint, the attitudes towards time and risk, as well as the level of exposure to death risk become irrelevant. Since death is instantaneous, attitudes towards inter-temporal substitution are irrelevant as well.

Remark 2 (aging) Our closed-form expressions for the willingness to pay and the three life valuations has abstracted from aging processes. The latter can be incorporated for a wide pattern of age-dependencies, although at some non-negligible computation cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in $\lambda_{m0t}, \lambda_{s0t}, \phi_t, \delta_t,$ or β_t results in age-dependent MPC and Tobin's-Q that solve the system of ODE's:

$$\dot{A}_t = A_t^2 - \left(\varepsilon\rho + (1-\varepsilon)\left(r - \lambda_{m0t} + \theta^2/(2\gamma)\right)\right) A_t,$$

$$\dot{B}_t = (r + \delta_t + \phi_t \lambda_{s0t}) B_t + (1 - 1/\alpha)(\alpha B_t)^{\frac{1}{1-\alpha}} - \beta_t,$$

subject to the boundary condition:

$$\lim_{t\to\infty}(r - \lambda_{m0t} + \theta^2/(2\gamma) - A_t) < 0,$$
$$\lim_{t\to\infty}((\alpha B_t)^{\frac{\alpha}{a-\alpha}} - r - \delta_t - \phi_t \lambda_{s0t}) < 0.$$

Allowing for aging and solving for A_t, B_t implies that the expressions C_{0t}, C_{1t}, the marginal value $\Theta(\lambda_{m0t})$, as well as the human and total wealth $P_{0t}(H), N_{0t}(W, H)$ are also age-dependent and all the relevant calculations can be modified accordingly to compute the WTP and the life values at any age level t.

5 Structural estimation

In order to structurally estimate the willingness to pay and the life valuations, we follow a long tradition associating the agent's human capital to his health (e.g. see the Hicks' lecture by Becker, 2007, for a review). We then estimate the technological, preferences and stochastic parameters for the benchmark model outlined in Section 3 by combining the observed health and wealth statuses with the observed decisions corresponding to (18). Once the structural parameters have been estimated, they can be relied upon to compute the closed-form expressions for the life valuations in Section 4.

5.1 Econometric model

For identification purposes, the econometric model assumes that agents follow the first-order optimal rules to the benchmark model and that they are heterogeneous with respect to their health and wealth statuses, whereas they are homogeneous with respect to the distributional, technological, revenue and preference parameters. To structurally estimate the latter, we use the closed-form expressions given in Theorem 1 to which we append the income equation (13). Specifically, denote by $\mathbf{Y}_j = [c_j, \pi_j, x_j, I_j, Y_j]'$ the 5×1 vector of observed decisions and income for agent $j = 1, 2, \ldots, n$, let $\mathbf{X}_j = [1, W_j, H_j]'$ capture his current wealth and health statuses. Also let $\mathbf{B}(\boldsymbol{\theta})$ denote the 5×3 matrix of closed-form expressions for the optimal rules implicit in equation (18), that are functions of the structural parameters $\boldsymbol{\theta}$. The econometric model relies on Maximum Likelihood to structurally estimates $\boldsymbol{\theta}$ in:

$$\mathbf{Y}_j = \mathbf{B}(\boldsymbol{\theta})\mathbf{X}_j + \mathbf{u}_j \tag{32}$$

where the $\mathbf{u}_j$'s are (potentially correlated) Gaussian error terms. In order to ensure theoretical consistency and augment identification, we estimate the structural parameters in (32) imposing the regularity conditions (16). Note that the key EIS parameter ε is unconstrained and therefore allowed to take positive values below or above one. In light of the strong nonlinearities not all the deep parameters can be identified and a subset of parameters denoted $\boldsymbol{\theta}^c$ are calibrated.

Remark 3 (semi-structural estimation) The structural econometric model in (32) exploits all cross-equations and regularity restrictions by estimating the fully-constrained model. A somewhat simpler semi-structural approach exploits instead the triangular identification by recursively solving for the deep parameters through a sequence of reduced-form estimations. That approach is presented in Appendix D and can be relied upon to generate starting values for the structural model, or when empirical identification issues are observed.

5.2 Data

We use a sample of $n = 8{,}378$ individuals taken from the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix E. We proxy the health variables through the polytomous self-reported health status (Poor, Fair, Good, Very Good and Excellent) that is linearly converted to numeric values from 1 to 4. The financial wealth comprises risky and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation and utility expenses reported in the PSID. Finally, health spending and health insurance expenditures are taken to ve the out-of-pocket spending and premia paid by agents. All nominal values are scaled by 10^{-6} for the estimation.

Tables 1, and 2 present descriptive statistics for the main variables of interest, per health status and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear relation between health and wealth can be deduced. The level of consumption in panel b is increasing in financial wealth, consistent with expectations. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption.

In panel c, stock holdings are very low for all but the fourth and fifth quintiles, illustrating the well-known non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth and devoid of clear health gradients.

Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth and also sharply decreasing in health status.

6 Results

6.1 Structural parameters

Table 3 reports the calibrated (with subscripts ^{c}) and estimated (standard errors in parentheses) model parameters. Overall, the latter are precisely estimated and are consistent with other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.6843$). Although deterministic depreciation is relatively low ($\delta = 1.25\%$), additional depletion brought upon by sickness is important ($\phi = 1.36\%$). Second, exposure to mortality risk is realistic ($\lambda_{m0} = 0.0283$), corresponding to a remaining expected lifetime of $\ell = \lambda_{m0}^{-1} = 35.3$ years, given mean respondent age of 45.68 years in Table 1.[12]

Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ($\beta = 0.0092$), as well as an estimated value for base income that is close to poverty thresholds ($y \times 10^6 = 12.2$ K\$).[13] The financial parameters (μ, σ_S, r) are calibrated from the observed moments of the S&P500 and 30-days T-Bills historical returns. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 2.8953$). The minimal consumption level is realistic and larger than base income ($a \times 10^6 = 14.0$ K\$). As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.2416$) and is consistent with a *Live Fast and Die Young* effect whereby a higher risk of death increases the marginal propensity to consume. Importantly, a high EIS confirms that the willingness to pay is well defined everywhere, that its limiting value exists and that the maximal WTP corresponds to the GPV.

6.2 Estimated valuations

Human Capital Value of Life Using the estimated parameters in Table 3, we can compute the HK value of life $v_h(H)$ given in (26) and reported in Table 4.a. Overall, the human capital values are increasing in H, common across W and are realistic. Indeed, the

[12] The remaining life expectancy at age 45 in the US in 2013 was 36.1 years (all), 34.1 (males) and 37.9 (females) (Arias et al., 2017).

[13] For example, the 2016 poverty threshold for single-agent households under age 65 was 12.5 K$ (U.S. Census Bureau, 2017).

estimated HK values range from 252 K\$ (Poor health) to 536 K\$ (Excellent health), with a mean (median) value of 438 K\$ (465 K\$). These figures compare advantageously with other HK estimates and provide a first out-of-sample confirmation that the structural estimates are realistic.[14]

Value of Statistical Life Table 4.b reports the Values of Statistical Life $v_s(W, H, \lambda_{m0})$ in (29) by observed health and wealth statuses. First, the VSL mean (median) value is 8.43 M\$ (8.79 M\$), with valuations ranging between 2.16 M\$ and 13.27 M\$. These values are well within the ranges usually found in the empirical VSL literature.[15] The concordance of these estimates with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded.

Second, the VSL is increasing in both wealth and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Andersson and Treich, 2011; Robinson and Hammitt, 2016; Murphy and Topel, 2006). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks and are willing to pay less to attain further improvements (or prevent deteriorations). Our benchmark model abstracts from endogenous mortality (see the robustness discussion below) whereas better health increases net total wealth $N_0(W, H)$, such that our estimates unambiguously indicate that the former effect is dominant and that improved health raises the VSL.

Gunpoint Value Table 4.c reports the Gunpoint values $v_g(W, H)$ in (31). The mean (median) GPV is 453 K\$ (472 K\$) and its values are increasing in both health and wealth

[14] Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K\$ at age 20, peaking at less than 900 K\$ at age 45 and falling steadily towards zero afterwards.

[15] A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 M\$ (2000 base year, corresponding to 8.6 M\$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 M\$ and 10 M\$. Robinson and Hammitt (2016) report values ranging between 4.2 and 13.7 M\$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 M\$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 M\$ (2006\$), corresponding to 8.8 M\$ in 2016 (U.S. Environmental Protection Agency, 2017).

and range between 116 K\$ and 712 K\$. The Gunpoint Value is thus of similar magnitude to the Human Capital Value of life and both are much lower than the VSL.

Accounting for the large VSL A first potential explanation for the large VSL is related to aggregation bias. More precisely, our large theoretical estimate could overstate the true *collective* willingness to pay to save someone by failing to account for idiosyncratic differences between agents' willingness to pay.

To address this issue, we can compute the aggregate willingness to pay by summing over the individual WTP's (27) to avoid a change equal to one over the size of the population. More precisely, we calculate:

$$\begin{aligned} v_s^e &= \sum_{i=1}^{n} v^i(W^i, H^i, \mathcal{P}_0, \Delta) = \left[1 - \frac{\Theta(\lambda_{m0}^*)}{\Theta(\lambda_{m0})}\right] \sum_{i=1}^{n} N_0(W^i, H^i) \\ &= \left[1 - \frac{\Theta(\lambda_{m0}^*)}{\Theta(\lambda_{m0})}\right] n N_0(\bar{W}, \bar{H}) = \left[1 - \frac{\Theta(\lambda_{m0}^*)}{\Theta(\lambda_{m0})}\right] \frac{N_0(\bar{W}, \bar{H})}{\Delta}. \end{aligned} \tag{33}$$

Setting $\Delta = 1/8,378$ and $\lambda_{m0}^* = \lambda_{m0} + \Delta$ in (33), we recover an aggregate VSL of 7.54 M\$, which, as expected, is lower, but still close to the mean theoretical value of $v_s(W, H, \lambda_{m0})$ $= 8.43$ M\$. We can also use Lemma 1 to fix an arbitrary duration T and use (30) to identify λ_{m0}^*. Setting $T = 1$ and $\Delta = 1/n$ yields $\lambda_{m0}^* = 0.0284$, which can be substituted in (33) to recover an empirical value of $v_s^e = 7.75$ M\$. This VSL is again lower, but nonetheless close to the theoretical value. We conclude that accounting for aggregation bias in computing the empirical VSL does not alleviate the large discrepancies with HK and GPV valuations.

In order to better understand these differences, it is useful to plot the estimated WTP $v(W, H, \lambda_{m0}, \Delta)$ as a function of the change in death intensity Δ in Figure 3.[16] First, the estimated WTP in blue displays a pronounced curvature, consistent with our theoretical results. Second, we saw from (28) and (31) that that the limit of the willingness to pay when death becomes certain is the net total wealth $N_0(W, H)$ and that this limiting value is also the gunpoint value $v_g(W, H)$ plotted in red. Third, as explained in Proposition 3, the VSL $v_s(W, H, \lambda_{m0})$ is the value of the slope of the yellow tangent of $v(W, H, \lambda_{m0}, \Delta)$ evaluated at $\Delta = 0$. The strongly diminishing MWTP in Figure 3 is informative as to why the VSL is much larger than the Human Capital and Gunpoint values. Indeed,

[16] These valuations are calculated from (27) at the estimated parameters and relying on the mean wealth and health status in Table 1.a ($W = 38,685\$ \times 10^{-6}, H = 2.58$).

the agent is willing to pay 356 K\$ to avoid an increase of $\Delta = 0.15$ which would lower expected remaining lifetime from 35.3 to 5.6 years. This value is already close to the HK and GPV values of 438 K\$ and 454 K\$ who are both much lower than the VSL of 8.4 M\$. Put differently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment.

6.3 Robustness

In order to verify robustness of the results, we consider a more general model of human capital. Hugonnier et al. (2013) study a demand for health framework that is similar to our benchmark, with two key differences. First, the model allows for self-insurance against morbidity and mortality risks by introducing health-dependent intensities:

$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}} \in [\lambda_{s0}, \eta],$$
$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m}.$$

Hence, better health lowers exposure to sickness and death risks and our benchmark model of Section 3 is an exogenous restricted case that imposes $\lambda_{s1}, \lambda_{m1} = 0$. Second, preferences are modified to allow for source-dependent aversion against financial, morbidity and mortality risks. In particular, our preferences in (14) are replaced by:

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_{\tau-}) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} F_k(U_{\tau-}, H_{\tau-}, \Delta_k U_\tau) \right) \mathrm{d}\tau,$$

with the Kreps-Porteus aggregator (15) unchanged, and with penalties for exposure against Poisson sickness and death risks:

$$F_k = U_{t-}\, \lambda_k(H_{t-}) \left[\frac{\Delta_k U_t}{U_{t-}} + u(1; \gamma_k) - u\left(1 + \frac{\Delta_k U_t}{U_{t-}}; \gamma_k\right) \right], \quad \text{where}$$
$$\Delta_k U_t = E_{t-}[U_t - U_{t-} | \mathrm{d}Q_{kt} \neq 0], \quad \text{and} \quad u(x; \gamma_k) = \frac{x^{1-\gamma_k}}{1 - \gamma_k}.$$

Our benchmark specification is thus a restricted case that imposes risk-neutral attitudes towards morbidity ($\gamma_s = 0$) and mortality ($\gamma_m = 0$) risks.

In a separate technical appendix (available upon request), we show that the approximate closed-form expressions for the WTP, HK, VSL and GPV valuations can be obtained. These expressions encompass some adjustments for the endogeneity of health risks exposure and source-dependent risk aversion, yet remain otherwise qualitatively similar. We structurally estimate the Hugonnier et al. (2013) model and compute the life values. These values remain in the same range as our benchmark estimates, with mean HK of 493 K$, VSL of 8.14 M$ and GPV of 460 K$ and again confirm the strong concavity of the WTP. We conclude that our main findings are qualitatively and empirically robust to more general specifications.

7 Conclusion

Computing the money value of a human being has generated a profound and continued interest, with early records dating back to the late XVIIth century. The two main valuation frameworks have centered on the marginal rate of substitution between the probability of living and wealth (VSL) and on a person's human capital value that is destroyed upon death (HK). Despite pricing a common element, the two life valuations yield vastly divergent measures, with the VSL being 10-20 times higher than the HK. Both the very different settings in which the two values are calculated as well as the absence of common theoretical underpinnings complicate any comparison exercise between the HK and VSL.

Our main contribution has been to show that is nonetheless possible to address both issues by relying on a unique human capital problem to analytically compute and structurally estimate the theoretical VSL and HK values. We have also introduced a third valuation reflecting the maximum amount an agent would be willing to pay to save himself from instantaneous and certain death (GPV) as a useful benchmark. These three closed-form for the life values were estimated jointly using a common econometric model and data set. This approach and thus provided direct comparability as well as a unique opportunity to identify the role of the preferences, distributional, and technological parameters on life valuations. Our main findings are twofold. First, we confirmed the relevance of reduced-form estimates with a GPV value of 453 K\$, close to the HK value of 438 K\$, both of which are much lower than the VSL of 8.43 MM\$. Second, we confirmed the standard economic intuition that the willingness to pay to avert death risk is increasing, but strongly concave and finite in mortality exposure. Allowing for a more general model with endogenous sickness and death intensities as well as source-dependent risk aversion only reaffirmed our findings.

Two potential explanations justify the wide disparities between the VSL and other measures. First, as famously pointed out by Schelling (1968), the VSL should be interpreted as the *aggregate* willingness to pay for infinitesimal changes in the mortality risk affecting an entire population. Conversely, the Human Capital and the Gunpoint values measure a market- and individual-based willingness to pay to avoid a large change in death risk (i.e. life versus certain death) that affects a single individual. There is therefore no *ex-ante* reason why the Statistical Life and other values should be equal. However, we saw that calculating an aggregate WTP, while correctly accounting for aggregation bias yields the same values as for the theoretically correct measure of marginal rate of substitution. Equivalently, the aggregate versus individual WTP explanation for the large VSL does not appear to warrant the large gaps with the other life values.

Second, we formally show and empirically verify that the differences are related to the strong curvature and finiteness of the WTP. The theoretical VSL is a linear projection from the marginal willingness to pay, whereas the empirical VSL is a linear approximation to that MWTP. When the WTP is strongly concave and computable everywhere, both theoretical and empirical VSL will strongly overestimate the limiting willingness to pay that corresponds to the Gunpoint Value. The empirical similarities between the HK and GPV values relate to the close parallels in the measured object. The HK measures the net present value of the foregone dividend stream associated with human capital (i.e. income, minus investment costs). The GPV measures the NPV of the foregone utility stream associated with living. The homogeneity properties entail that the latter is also the NPV of the foregone consumption above minimal subsistence requirements.

We saw that the model is fully amenable to a wide variety of aging processes, although at some computation cost. Aging was abstracted from at the estimation stage, but we can conjecture that it would reduce all three life valuations. Indeed, biological limitations to expectancy, increasing depreciation and exposure to sickness as well as death risks would induce optimal dis-savings in both financial assets and human capital and thereby lower net total wealth (Hugonnier et al., 2017). One caveat of our approach is the absence of bequest motives. This omission is related to the technical difficulty in solving our benchmark when bequeathed wealth is optimally chosen. Although it remains unclear how our results would be affected, we can however conjecture that a likely effect would be to reduce the GPV even further. Indeed, the *warm glow* effect of bequest would attenuate the cost of dying and consequently also the WTP to avert death (Philipson et al., 2010). Moreover, bequeathed wealth is illiquid, to the extent that it is set aside for surviving heirs and not to ensure one's own survival. Without affecting human capital, the amount of disposable financial resources that can be pledged in a money-or-death threat would therefore be reduced and consequently so would the VSL and the GPV.

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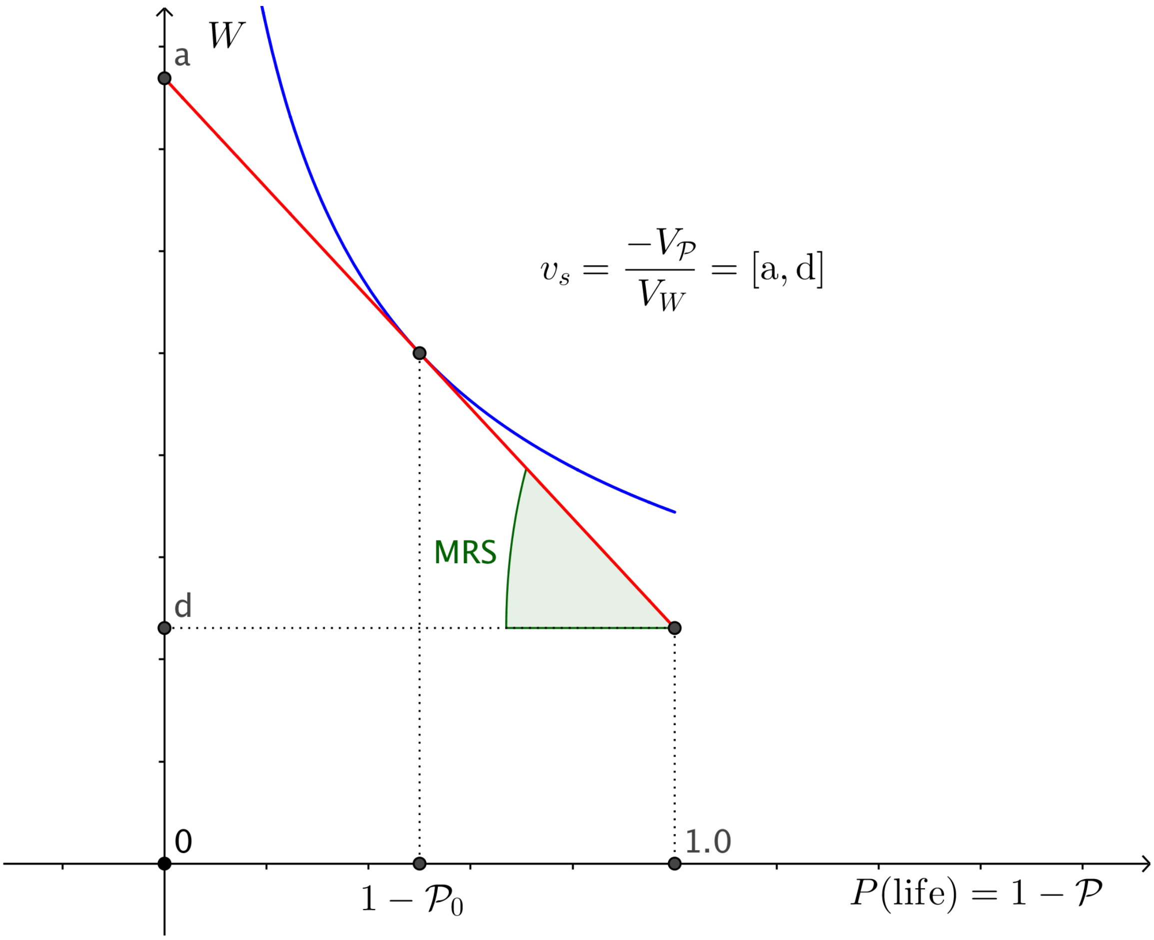
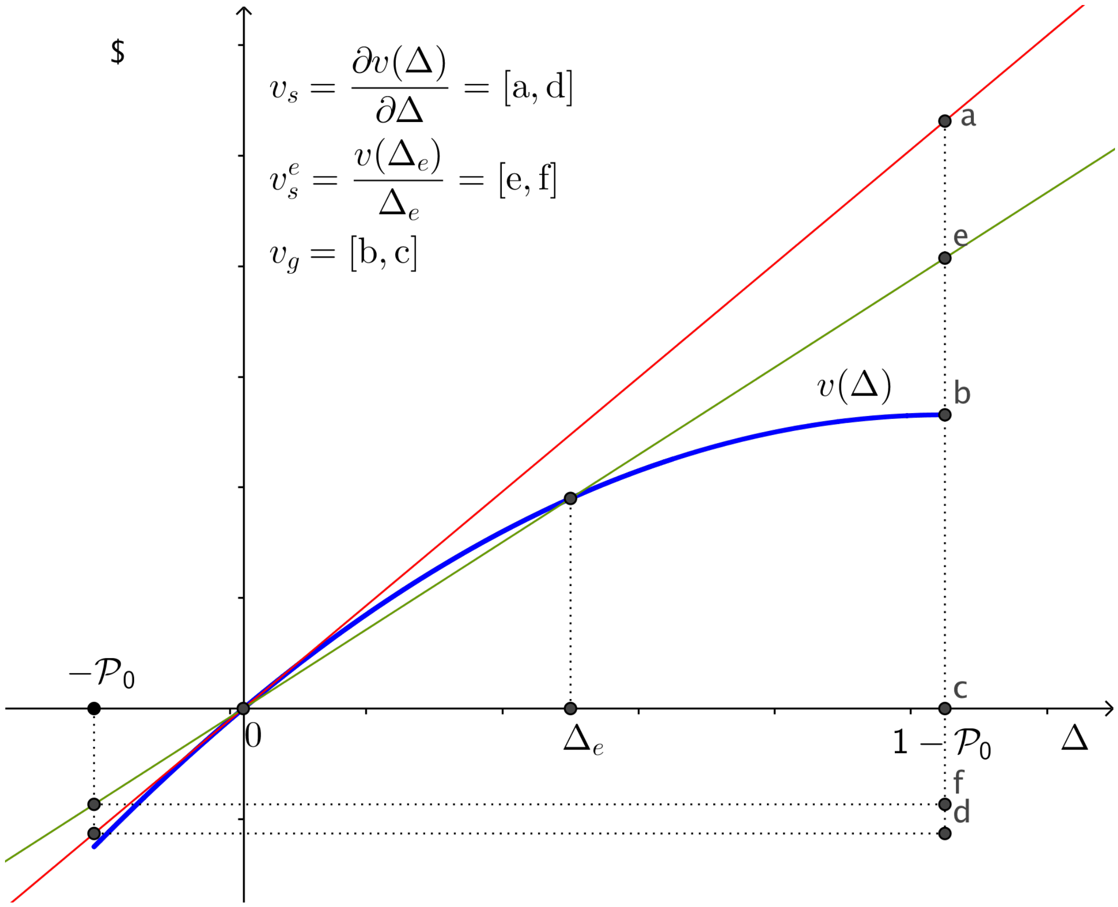
A Figures

Figure 1: Indifference curves, MRS and Value of Statistical Life



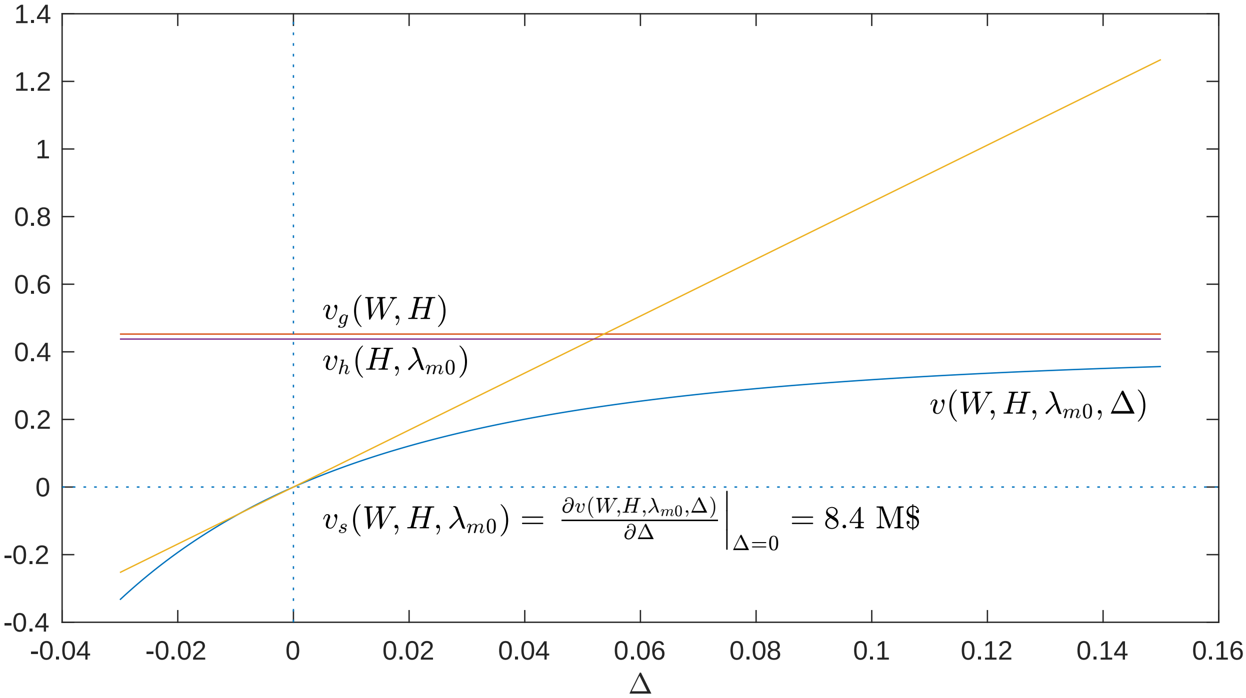
Notes: Reproduced and adapted from Andersson and Treich (2011, Fig. 17.1, p. 398). Indifference curves for indirect utility (1) in blue. v_s: Theoretical Value of Statistical Life in (7a) is the negative of the MRS, i.e. the slope of red tangent equal to distance [a,d].

Figure 2: Willingness to pay and life valuations



Notes: $\mathcal{P}$: instantaneous probability of death. v: Willingness to pay to avoid change Δ in death risk (in blue), evaluated at (W, H) and for base risk $\mathcal{P}_0$. v_s: Theoretical Value of Statistical Life in (7b) is slope of red tangent equal to distance [a,d]. v_s^e: Empirical Value of Statistical Life in (8) is slope of green line and equal to distance [e,f]. v_g: Gunpoint Value of Life in (9) is equal to distance [b,c].

Figure 3: Estimated WTP, HK, VSL and GPV Values of life (in M$)



Notes: At estimated parameter values, for mean wealth and health levels. $v(W,H,\lambda_{m0},\Delta)$ in blue is the willingness to pay to avoid an increase of Δ in exogenous death intensity λ_{m0}; $v_h(H,\lambda_{m0})$ in purple is the Human Capital value of life; $v_g(W,H)$ in red is the Gunpoint value of life; $v_s(W,H,\lambda_{m0})$ is the Value of statistical life and the slope of the yellow tangent evaluated at $\Delta = 0$. In MM$.

B Proofs

B.1 Theorem 1

The benchmark human capital model of Section 3 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death, and depreciation intensities are constant at $\lambda_{m0}, \lambda_{s0}$ (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e. $\gamma_s = \gamma_m = 0$). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1,Theorem 1) yields the the optimal solution in (18).

■

B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital $P_0(H)$ from

$$\begin{aligned} P_0(H) &= E_t \int_0^\infty m_{t,\tau} \left[\beta H_\tau^* - I_\tau^*\right] \mathrm{d}\tau, \\ &= BH. \end{aligned}$$

Straightforward calculations adapt this result to a stochastic horizon T^m, and include the fixed income component y in income (13).

■

B.3 Proposition 2

Combining the Hicksian EV (5) with the indirect utility (17) and the net total wealth in (20) reveals that the WTP v solves:

$$\begin{aligned} \Theta(\lambda_{m0}^*) N_0(W, H) &= \Theta(\lambda_{m0}) N_0(W - v, H) \\ &= \Theta(\lambda_{m0}) \left[N_0(W, H) - v\right] \end{aligned}$$

where we have set $\lambda_{m0}^* = \lambda_{m0} + \Delta$, and using the linearity of the welfare function with respect to wealth. The WTP $v = v(W, H, \lambda_{m0}, \Delta)$ is solved directly as in (27).

Next, by the properties of the marginal value of net total wealth, $\Theta(\lambda_{m0}^*)$ in (25) is monotone decreasing and convex in Δ. It follows directly that the WTP

$$v(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(\lambda_{m0}^*)}{\Theta(\lambda_{m0})}\right] N_0(W, H)$$

is monotone increasing and concave in Δ. When $\varepsilon \in (0, 1)$, the marginal value $\Theta(\lambda_{m0}^*)$, cannot be calculated for $\lambda_{m0} \geq \bar{\lambda}_{m0}$ by the non-negativity and transversality condition (16a). For $\varepsilon > 1$, the marginal value $\Theta(\lambda_{m0}^*) = 0$, as $\Delta \to \infty$. Substituting in (27) reveals that the WTP converges to $N_0(W, H)$ as stated in (28).

■

B.4 Proposition 3

By the VSL definition (7a), and the properties of the Poisson death process (11):

$$v_s = \left.\frac{-V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})}\right|_{\mathcal{P}=\mathcal{P}_0} = \frac{-V_{\lambda_{m0}}(W, H, \lambda_{m0})}{V_W(W, H, \lambda_{m0})}$$

From the properties of the welfare function (17), we have that $V_{\lambda_{m0}} = \Theta'(\lambda_{m0}) N_0(W, H)$, whereas $V_W = \Theta(\lambda_{m0})$. Substituting yields the VSL in (29). ■

B.5 Lemma 1

A higher likelihood of death of Δ over a time interval of $s \in [0, T]$ corresponds to an increase in the endowed intensity to $\lambda_{m0}^*(\Delta) > \lambda_{m0}$:

$$\Delta = \Pr\left[T_m \leq T \mid \lambda_{m0}^*\right] - \Pr\left[T_m \leq T \mid \lambda_{m0}\right],$$

Observing that:

$$\Pr\left[T_m \leq T \mid \lambda\right] = 1 - E\left[e^{-\int_0^T \lambda \mathrm{d}s}\right] = 1 - e^{-T\lambda},$$

and substituting solves for λ_{m0}^* reveals that the latter as stated in (30). ■

B.6 Proposition 4

Combining the Hicksian EV (9) with the indirect utility (17) and the net total wealth in (20) reveals that the WTP v solves:

$$
\begin{aligned}
V^m \equiv 0 &= \Theta(\lambda_{m0}) N_0(W - v_g, H) \\
&= \Theta(\lambda_{m0}) \left[N_0(W, H) - v_g \right]
\end{aligned}
$$

Solving for v_g reveals that it is as stated in (31). Because net total wealth is independent of ε, so is the Gunpoint Value. ■

C Tables

C.1 Data

Table 1: PSID data statistics

	Mean	Std. dev.	Min	Max
Health (H)	2.58	0.80	1	4
Wealth (W)	38 685	122 024	0	1 430 000
Consumption (c)	9 835	11 799	1.047	335 781
Risky holdings (π)	20 636	81 741	0	1 367 500
Insurance (x)	247	718	0	17 754
Health investment (I)	721	2 586	0	107 438
Income (Y)	21 838	37 063	0	1 597 869
Age (years)	45.68	16.46	16	100

Notes: Statistics in 2013 $ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good) and 4.0 (Excellent).

Table 2: PSID data statistics (cont'd)

Health	H_j	Wealth quintiles 1	2	3	4	5
		a. Wealth W_j (\$)				
Poor	1.00	0	139	2 063	11 831	152 151
Fair	1.75	0	145	1 741	12 027	123 083
Good	2.50	0	168	1 802	11 908	120 467
Very good	3.25	0	199	1 823	12 197	118 738
Excellent	4.00	0	192	1 823	12 099	122 135
		b. Consumption c_j (\$)				
Poor	1.00	3 281	4 906	6 558	10 052	7 752
Fair	1.75	4 095	6 888	8 795	11 196	13 368
Good	2.50	5 086	6 526	9 745	11 269	13 336
Very good	3.25	5 989	7 517	10 181	11 131	13 626
Excellent	4.00	5 276	6 897	10 002	12 099	14 628
		c. Stocks π_j (\$)				
Poor	1.00	0	0	0	725	46 497
Fair	1.75	0	5	279	2 309	76 721
Good	2.50	0	1	268	4 320	55 379
Very good	3.25	0	5	192	4 756	68 768
Excellent	4.00	0	0	334	5 801	90 147
		d. Insurance x_j (\$)				
Poor	1.00	165	191	503	723	856
Fair	1.75	181	196	497	775	1 095
Good	2.50	206	219	401	564	852
Very good	3.25	190	284	313	522	797
Excellent	4.00	203	254	366	429	807
		e. Investment I_j (\$)				
Poor	1.00	549	552	2 341	2 936	6 003
Fair	1.75	400	468	968	621	1 250
Good	2.50	243	238	383	500	962
Very good	3.25	276	226	275	435	596
Excellent	4.00	151	192	230	307	451

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth and per health status

C.2 Benchmark model

Table 3: Estimated and calibrated structural parameter values, benchmark model

Parameter	Value	Parameter	Value
	a. Law of motion health (10)		
α	0.6843	δ	0.0125
	(0.3720)		(0.0060)
ϕ	0.0136^c		
	b. Sickness and death intensities		
λ_{s0}	0.0347	λ_{m0}	0.0283
	(0.0108)		(0.0089)
	c. Wealth and income (12), (13)		
y	0.0120	β	0.0092
	(0.0049)		(0.0044)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
	d. Preferences (14), (15)		
γ	2.8953	ε	1.2416
	(1.4497)		(0.3724)
a^c	0.0140	ρ^c	0.0500

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (32), estimated by ML, subject to the regularity conditions (16).

Table 4: Estimated Values of Life, benchmark model (in $)

Health level	Wealth quintile				
	1	2	3	4	5
	a. Human Capital $v_h(W, H, \lambda_{m0})$ in (26)				
Poor			251 968		
Fair			323 127		
Good			394 287		
Very Good			465 446		
Excellent			536 606		
All					
- mean			437 756		
- median			465 446		
	b. Value of Statistical Life $v_s(W, H, \lambda_{m0})$ in (29)				
Poor	2 163 175	2 165 769	2 201 602	2 383 566	4 997 543
Fair	4 370 666	4 373 362	4 403 105	4 594 717	6 663 544
Good	6 578 156	6 581 286	6 611 719	6 799 989	8 822 301
Very Good	8 785 647	8 789 345	8 819 610	9 012 869	10 997 579
Excellent	10 993 137	10 996 714	11 027 098	11 218 530	13 268 351
All					
- mean			8 429 649		
- median			8 787 965		
	c. Gunpoint Value $v_g(W, H)$ in (31)				
Poor	116 121	116 260	118 183	127 951	268 271
Fair	234 620	234 765	236 362	246 647	357 703
Good	353 120	353 288	354 921	365 028	473 587
Very Good	471 619	471 818	473 442	483 817	590 357
Excellent	590 119	590 311	591 942	602 218	712 254
All					
- mean			452 509		
- median			471 744		

Notes: Averages of individual values in the PSID sample, computed at estimated parameter values, multiplied by 1 M$ to correct for scaling used in estimation.

D Semi-structural estimation

Data requirements This approach requires a subset of the data used for the structural model, i.e. W, H, Y, I, π, c, as well as a sickness indicator $S = \mathbb{1}_{sick}$, and use of the Life Tables. The parameters $\rho, r, \theta, \sigma_S$ are calibrated as in the full procedure.

Recursive identification The key parameters and functions are identified recursively as follows:

1. Intensities for the Poisson shocks:

   (a) Sickness:

   $$\hat{\lambda}_{s0} = -\log(1 - E(S))$$

   (b) Death:

   $$\hat{\lambda}_{m0} = 1/\ell(\bar{t})$$

   where ℓ is the remaining life expectancy evaluated at at mean age.

2. Law of movement health:

   $$\frac{\Delta H}{H} = -\delta + \left(\frac{I}{H}\right)^{\alpha} - \phi S + u_H$$

   estimated by NLLS, identifies $(\hat{\delta}, \hat{\alpha}, \hat{\phi})$.

3. Income:

   $$Y = y + \beta H + u_Y$$

   estimated by OLS, identifies $(\hat{y}, \hat{\beta})$.

4. Tobin's-Q:

   $$B = \frac{\hat{\beta} + \left(\frac{1-\hat{\alpha}}{\hat{\alpha}}\right) E\left(\frac{I}{H}\right)}{r + \hat{\delta} + \hat{\lambda}_{s0}\hat{\phi}}.$$

identifies $\hat{B}$.

5. MPC and minimal consumption:

$$c = a + A\left(\frac{\hat{y} - a}{r}\right) + AW + A\hat{B}H + u_c$$

by OLS, identifies $(\hat{a}, \hat{A})$.

6. Human capital and net total wealth:

$$\begin{aligned} P_0(H) &= \hat{B}H \\ N_0(W, H) &= W + \frac{\hat{y} - \hat{a}}{r} + P_0(H) \end{aligned}$$

identifies $\hat{P}_0(H), \hat{N}_0(W, H)$. Substituted in (31) to compute the Gunpoint value.

7. Risk aversion:

$$\pi = (\theta/(\gamma\sigma_S))\hat{N}_0(W, H) + u_\pi$$

by OLS or NLLS, identifies $\hat{\gamma}$.

8. Elasticity of inter-temporal substitution:

$$\varepsilon = \frac{\hat{A} - (r + \hat{\lambda}_{m0} + 0.5\theta^2/\hat{\gamma})}{\rho - (r + \hat{\lambda}_{m0} + 0.5\theta^2/\hat{\gamma})}, \quad \tilde{\rho} = \rho^{\frac{-\hat{\varepsilon}}{1-\hat{\varepsilon}}}$$

identifies $(\hat{\varepsilon}, \tilde{\rho})$.

9. Human capital marginal values:

$$C_0 = \frac{1}{r + \hat{\lambda}_{m0}}, \quad C_1 = \frac{r - (\hat{\alpha}\hat{B})^{\frac{\hat{\alpha}}{1-\hat{\alpha}}}}{r + \hat{\lambda}_{m0} - (\alpha\hat{B})^{\frac{\hat{\alpha}}{1-\hat{\alpha}}}},$$

identifies $(\hat{C}_0, \hat{C}_1)$, which can be combined with $\hat{P}_0(H)$ in computing the HK value (26).

10. Marginal value of total wealth:

$$\begin{aligned} \Theta(\lambda_{m0}) &= \tilde{\rho}\left[\hat{\varepsilon}\rho + (1 - \hat{\varepsilon})(r - \lambda_{m0} + 0.5\theta^2/\hat{\gamma})\right]^{\frac{1}{1-\hat{\varepsilon}}} \\ \Theta'(\lambda_{m0}) &= -\tilde{\rho}\left[\hat{\varepsilon}\rho + (1 - \hat{\varepsilon})(r - \lambda_{m0} + 0.5\theta^2/\hat{\gamma})\right]^{\frac{\hat{\varepsilon}}{1-\hat{\varepsilon}}} \end{aligned}$$

identifies the functions $\hat{\Theta}(\lambda_{m0}), \hat{\Theta}'(\lambda_{m0})$ for any λ_{m0}. Can be combined with $\hat{\lambda}_{m0}$ and $\hat{N}_0(W, H)$ to compute the WTP $v(W, H, \hat{\lambda}_{m0}, \Delta)$ in (27), or to compute the VSL $v_s(W, H, \hat{\lambda}_{m0})$ in (29).

As generated regressors are used for identification, the corresponding standard errors can be calculated through Bootstrap or numerical derivatives.

E Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size) and scaled by 10^{-6} for the estimation. The agents' wealth and health which are constructed as follows:

Health H_j Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

Wealth W_j Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance and are constructed as follows:

Portfolio π_j Money value of financial wealth held in risky assets.

Consumption c_j Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

Health expenditures I_j Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

Health insurance x_j Spending on health insurance premium.