Valuing Life as an Asset, as a Statistic, and at Gunpoint[1]

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Abstract

The Human Capital (HK), and Statistical Life Values (VSL) differ sharply in their empirical pricing of a human life and lack a common theoretical background, to justify these differences. We first contribute to the theory, and measurement of life value by providing a unified framework to formally define, and relate the Hicksian willingness to pay (WTP) to avoid changes in death risks, the HK, and the VSL. Second, we use this setting to introduce a benchmark life value calculated at Gunpoint (GPV), i.e. the maximal WTP to avoid certain, instantaneous death. Third, we associate a flexible human capital model to the common framework to characterize the WTP and the three life valuations in closed-form. Fourth, our structural estimates of these solutions yield mean life values of 8.35 M$ (VSL), 421 K$ (HK) and 447 K$ (GPV). We confirm that the strong curvature of the WTP, rather than segmented frameworks, explains why the VSL is much higher than other values.

Keywords: Value of Human Life, Human Capital, Value of Statistical Life, Willingness to pay, Equivalent Variation, Mortality, Structural Estimation.

JEL Classification: J17, D91, G11.

The life you save may be your own

This is a treacherous topic... (Schelling, 1968)

1 Introduction

1.1 Motivation and outline

Motivation and objectives Evaluating the price of a human life has long generated a deep interest in economic research.[1] Indeed, life valuations are often relied upon in public health and safety debates, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. Economic life values are also resorted to in occupational, or end-users' wrongful death litigation.

Three main sources of difficulty render the pricing of life particularly challenging. First, a human life is by definition non-divisible. This implies that any marginal valuation, e.g. via small incremental risks to life, must eventually be integrated back into a unitary life value. Second, a human life is non-marketed. The absence of equilibrium prices implies that the economic value of a human life must somehow be inferred from relevant and measurable proxies such as foregone income, or responses to changes in mortality risks. Finally, ethical considerations induce significant discomfort in computing – and eventually comparing – the life values of identified persons.

The two most-widely used life valuation frameworks differ in how these challenges are dealt with. The Human Capital (HK) approach does not balk at valuing the life of an individual and associates the latter to the economic value embodied in his human capital and that is fully depreciated upon death. Relying on standard asset pricing, the HK value is the present value of the dividend stream associated with human capital, where the dividend is proxied by the marketed labor income, net of the measurable expenses required to maintain that capital. The Value of a Statistical Life (VSL), introduced by Drèze (1962) and Schelling (1968), relies on a stated, or inferred, willingness to pay (WTP) to avert (resp. attain) small increases (resp. reductions) in exposure to death risks. Under appropriate assumptions, a collective WTP to save one unidentified (i.e.

[1] Landefeld and Seskin (1982) make reference to human-capital based evaluations of the value of life dating back to Petty (1691).

statistical) life can be recovered through a linear aggregation of the individual WTP's.[2] Focusing on the value of an unidentified, rather than personalized, life thus conveniently avoids addressing the uncomfortable issues associated with computing and comparing the values of identified lives.

Both the HK and the VSL are pricing the same underlying object and should presumably come up with similar answers to the question of how much a human life is worth. However, these two valuations yield vastly different prices in practice with VSL estimates 10-20 times larger than HK values.[3] Rationalizing these differences is complicated by the absence of common theoretical underpinnings that encompass both valuations. Consequently, most HK and VSL evaluations are reduced-form empirical exercises that never exploit joint theoretical restrictions and that are performed within disjoint data settings that further complicate comparisons.

This void between the two approaches leaves open a number of questions that we address in this paper. In particular, can a common theoretical and empirical framework help in rationalizing any differences between the HK and the VSL? Can this framework also yield a reasonable value of life metric against which the two alternatives can be gauged? More fundamentally, what lessons can we learn from an encompassing approach about the interpretation and applicability of the alternative measures in pricing the economic value of a human life?

Contributions To answer these fundamental questions, we first propose a unified theoretical background linking both the Human Capital and Statistical Life values. We start from a generic dynamic human capital problem in which an agent facing an uncertain horizon selects investment in his skills or his health, where human capital augments labor income. Assuming the existence of a solution to this problem satisfying weak preference for life over death, we use standard asset pricing to define the HK value as the discounted dividend stream, i.e. the income, net of investment, *along the optimal dynamic path*. We next rely on the associated indirect utility (i.e. the welfare at the optimum) which we combine with the Hicksian Equivalent Variation (EV, Hicks, 1946) to define the

[2] As a canonical example (e.g. Aldy and Viscusi, 2007), suppose n agents are individually willing to pay $v(\Delta)$ to attain (avert) a small beneficial (detrimental) change $\Delta = 1/n$ in death risk exposure. The empirical VSL is the collective WTP: $v_s^e(\Delta) = nv(\Delta) = v(\Delta)/\Delta$, i.e. the slope of the WTP.

[3] Huggett and Kaplan (2016) identify HK values between 300 K–900 K\$, whereas the U.S. Department of Transportation recommends using a VSL-type amount of 9.4 M\$ (U.S. Department of Transportation, 2016).

willingness to pay to avoid any detrimental change in death risk exposure. The theoretical VSL can then be derived formally in two equivalent ways: (i) as the (negative of the) marginal rate of substitution (MRS) between death exposure and wealth, calculated through the indirect utility and (ii) as the marginal WTP (MWTP) with respect to death risk, calculated through the EV. This encompassing setup ensures that the HK and the VSL are both evaluated from a common underlying dynamic problem.

Second, we make use of this unified theoretical framework to define a *third* valuation alternative that forthrightly addresses the measurement challenges and can serve as comparison benchmark. The objectives are to gauge the economic value of a human life without recourse to indirect proxies and/or arbitrary aggregation assumptions. Instead, we address the non-divisibility and non-marketability by resorting to the unitary shadow value of life accruing to its main beneficiary, i.e. the willingness to pay that leaves an agent indifferent between living and dying in a highwaymen threat. The Hicksian EV again provides a natural theoretical background to elicit this shadow value which we refer to as the Gunpoint Value of Life (GPV).[4] To paraphrase Schelling (1968)'s seminal title, 'the life you save *is* your own' in our highwaymen valuation. Consequently, any discomfort in valuing someone else's life can be circumvented by having that person compute his *own* intrinsic value.

Third, we rely on a parametrized version of the encompassing human capital model to provide analytical calculations of the WTP, as well as of the Human Capital, Statistical Life and Gunpoint Values of life. This model is flexible enough to be applicable to either a skills (Ben-Porath, 1967; Heckman, 1976) or a health (Grossman, 1972; Ehrlich and Chuma, 1990) interpretation. It guarantees weak preference for life over death and yields closed-form solutions, allowing us to compute the analytic expressions corresponding to

[4] Explicit references (with emphasis added) to a Gunpoint Value of life include:

> "We argue that living, like other goods, has diminishing marginal utility–the willingness to pay for an additional year of life falls with how many years one has to live. This is in contrast to how the value of a statistical life-year is taught and explained: it is often prefaced with claiming that it is not how much people are willing to pay *to avoid having a gun put to their head* (presumably ones wealth). However, terminal care decisions are often exactly of that nature." (Philipson et al., 2010, p. 2)

and in the media:

> "But how do you put a dollar value on a life, even in a generic sense? It wouldn't work for researchers to survey Americans *at gunpoint* and ask how much they would pay not to die. Instead, an unlikely academic field has grown up to extrapolate life's value from the everyday decisions of average Americans." (Fahrenthold, 2008)

the willingness to pay, as well as the three life values. We can therefore assess the contributions of fundamentals, such as preferences, risk distributions, or technology, as well as financial and human resources and thus investigate how the WTP, HK, VSL and GPV are theoretically related to one another.

Finally, we proceed with a structural estimation of our parametrized model and report closed-form estimates of the willingness to pay and the three different life valuations. More precisely, we adopt a revealed-preference perspective to estimate the structural parameters of the human capital model, using PSID data that correspond to the optimal investment, consumption, portfolio and health insurance policies. We can then combine the structural parameters with observed wealth and health status to calculate the analytical expressions for the WTP, Human Capital, Statistical and Gunpoint Values of life. The HK and the VSL can be contrasted with reduced-form estimates in an out-of-sample assessment of our results, compared to each other, as well as with the estimated Gunpoint benchmark.

Main findings First, we innovate from the literature by showing that the Hicksian willingness to pay, Statistical, Human Capital and Gunpoint values of life *can* be jointly characterized and structurally estimated from a common dynamic human capital problem. Standard properties of this problem – that are verified in our application and estimation – guarantee that the willingness to pay to avoid detrimental (or to accept compensation to forego beneficial) changes in death risk is increasing, concave and bounded in the latter. It follows that the theoretical VSL (i.e. the marginal WTP) is under-estimated by the empirical VSL (the average WTP over discrete changes in death risk, see footnote 2), and that both theoretical and empirical VSL's over-estimate the GPV (the WTP's upper bound). Importantly, we show that the ratio of the VSL to this limiting WTP is inversely proportional to the marginal propensity to consume (MPC). Since the MPC is typically much lower than one, the predicted VSL-GPV gap is significant.

Secondly, unlike the VSL, both the HK and GPV directly compute the value of a whole life, rather than linearly projecting marginal threats to recover a unitary value. The Human Capital and Gunpoint values display further similarities in that both reflect expected net present values of human capital dividends (HK) and of consumption above subsistence (GPV), and both are independent of preferences towards risk and time. For

HK, this independence reflects the market-based asset pricing of the dividends flow, for the GPV, it reflects the instantaneous and certain nature of death in a Gunpoint valuation.

Third, our empirical results confirm that the willingness to pay is strongly concave and bounded above by the GPV. They are also consistent with the reduced-form HK and VSL estimates reported in the literature, with structural average values of 421 K$ (HK) and 8.35 K$ (VSL). Importantly this large discrepancy between the HK and the VSL *cannot* be explained by a disjoint theoretical and empirical evaluation. Moreover, the average Human Capital value of life is close to the average Gunpoint benchmark (447 K$), consistent with the theoretical parallels between the two. The large VSL/GPV ratio of 18.66 is consistent with a realistic MPC estimate of 5.36%.

The corroboration of large empirical VSL-HK differences, and the finding of HK-GPV similarities in an encompassing framework indicates that other reasons must be assessed to understand why the VSL yields much larger estimates. Towards that purpose, it is useful to revert to the original warnings by Schelling (1968) that the VSL should *not* be taken as a human life value, but rather as a collective willingness to avoid small changes in death risk.[5] We argue that the problem neither lies from that interpretation, nor from the approximations taken in the empirical literature to elicit it. Indeed, we formally show that the empirical VSL can be derived in closed form as a collective WTP and that the bias with the theoretical MWTP is small for large samples. Rather, the problem stems from the linear extrapolation from that marginal value to a unitary life value that is directly measured by the HK and GPV. The strong concavity of the WTP necessarily entails a significant approximation error from a linear projection to a holistic life value.

These findings also confirm early conjectures on the pitfalls associated with personalizing unidentified life valuations. Commenting on the VSL, Pratt and Zeckhauser (1996) argue that concentrating the costs and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e.

[5] In his opening remarks, Schelling (1968, p. 113) writes

> "This is a treacherous topic and I must choose a nondescriptive title to avoid initial misunderstanding. It's not the worth of a human life that I shall discuss, but of 'life saving', of preventing death. And it's not a particular death, but a statistical death. What it is worth to reduce the probability of death – the statistical frequency of death – within some identifiable group of people, none of whom expects to die except eventually. "

large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment to avoid increases in risk, thereby reducing the WTP as risk increases.[6] Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our highwaymen threat. Their conjecture is warranted in our calculations. We show that the willingness to pay is finite and bounded above by the Gunpoint Value. Diminishing MWTP entails that the latter is much lower than what can be inferred from the VSL.

A main take-away message of this paper is therefore that the three life pricing are relevant tools that should remain specialized in their respective applications. If society ends up paying for policies that result in small changes in death risk exposure affecting large populations, then the VSL is clearly appropriate; its extrapolation to a human life value is not. In the latter case, the Human Capital and Gunpoint Values are better suited and applications such as wrongful death litigation or terminal care decisions should revert to the HK or GPV frameworks.

After a review of the relevant literature in Section 1.2, the rest of the paper is organized as follows. We first introduce the formal links between the HK, WTP, VSL and GPV in Section 2. Sections 3 and 4 present the benchmark model and corresponding life values. The empirical strategy is discussed in Section 5, with structural parameters and values of life estimates reviewed in Section 6. We discuss and interpret these results in Section 7, with concluding remarks presented in Section 8.

1.2 Related literature

1.2.1 Human Capital values of life

The HK model associates the economic value of a person to the value of his human capital that is entirely depreciated at death. The latter is obtained by pricing the expected

[6] Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay $1 million to reduce the associated mortality risk by 10%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.

discounted stream of its associated dividends that are foregone upon death, i.e. the lifetime labor income flows, net of associated investment.[7] Well-known issues related to this approach include the appropriate rate of discounting, the endogeneity of income and investment, as well as the treatment of non-labor activities.[8] As for HK models, we do calculate the net present value of the stream of human capital dividends that are lost upon death. Unlike HK models however, that value is computed in closed-form. In particular, we fully account for the endogeneity of the human capital stock and therefore of its associated income and investment expenditures. We therefore encompass the relevant technological and distributional considerations, such as the capital production technology, its deterministic and stochastic depreciation, the income-capital gradient, as well as the duration of the dividends stream.

1.2.2 Value of a Statistical Life

The empirical VSL alternative relies on explicit and implicit evaluations of the Hicksian WTP for a small reduction in fatality risk which is then linearly extrapolated to obtain the value of life. Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.[9] Examples of the latter include responses to prices and fines in the use of life-saving measures such smoke detectors, speed limitations, or seat belt regulations. The Hedonic Wage (HW) variant of the implicit VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/worker characteristics, the wage elasticity with respect to job fatality risk can be estimated and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008). Hall and Jones (2007) propose a semi-structural measure of life value. They adopt a marginal value perspective by equating the VSL to the marginal cost of saving a human life. The cost of reducing mortality risk can be imputed by specifying and estimating a health production function and by linking health

[7]See Jena et al. (2009); Huggett and Kaplan (2013, 2016) for applications.

[8]Conley (1976) provides additional discussion of HK approaches while Huggett and Kaplan (2016) address the discounting issues.

[9]A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the implicit VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta*-meta analysis of the stated- and revealed-preferences valuations of life.

status to death risks. Dividing this cost by the required change in death risk amount yields a VSL-inspired life value, e.g. corresponding to 1.9 M$ for an individual aged 40-44 (Hall and Jones, 2007, Tab. 1, p. 60).

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. He argues that the assumed exogeneity of the change in fatality risk can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. He also argues that agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Ashenfelter further contends that it is unclear whose preferences are involved in the risk/income tradeoff and how well these arbitrage are understood. For example, if high fatality risk employment attracts workers with low risk aversion and/or high discount rates, then generalizing the wages risk gradient to the entire population could understate the true value of life. Moreover, because wages are an equilibrium object in the HW variant of the VSL, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death. Finally, as was the case for HK measures, HW estimates relate primarily to workers and are hardly adaptable to other non-employed groups, such as young, elders, or the unemployed.

Our approach offers other advantages in calculating the value of life. First the theoretical and empirical values stem from a common model and are thus directly comparable and interpretable. Second, we model and estimate the human capital production. However, our valuations framework does not exclusively rely on technological and distributional parameters, but includes preferences and human capital and wealth statuses. Third, we rely on a widely-used panel (PSID) accounting for households' consumption, financial and health-related decisions to elicit the WTP and life valuations. Consequently, these values are representative and can be generalized to the population. Fourth, we make no assumption on the shape of the WTP function but rather derive its properties from the indirect utility function measured at the optimal allocation. Indeed, we show that, consistent with economic intuition, the marginal value ascribed to small increases in death intensity is positive, but falling in the latter.

2 A Common Framework for Life Valuation

This section outlines a common framework that will be relied upon to formally define and compare the Human Capital, Statistical, as well as Gunpoint Values of Life. Our main building block is an underlying human capital problem for which the optimal policies and associated indirect utility function can be solved. We combine these solutions with standard asset pricing and Hicksian variational analysis to characterize the three life valuations.

2.1 Underlying Human Capital Problem

Consider an agent's human capital problem defined by a stochastic age at death T^m, an instantaneous death probability $\mathcal{P} \in [0, 1]$, a human capital H (e.g. skills or health) and associated increasing income function $Y(H)$, a financial wealth W, as well as the relevant distributional assumptions with respect to mortality, human and financial assets. For this program, the agent selects the money value of investment in his human capital I and other controls X (e.g. consumption, or asset allocation, ...) so as to maximize utility U:

$$\begin{aligned} V(W, H, \mathcal{P}) = \sup_{I,X} \; U, &\quad \text{subject to:} \\ \mathrm{d}H = \mathrm{d}H(H, I), & \\ \mathrm{d}W = \mathrm{d}W(W, Y(H), I, X). & \end{aligned} \tag{1}$$

We assume that the agent's preferences and constraints in (1) satisfy standard properties such that the indirect utility $V = V(W, H, \mathcal{P})$ is monotone increasing and concave in W. We further assume weak preference for life over death. In particular, the indirect utility is well-defined, decreasing and convex for all levels of death risk exposure $\mathcal{P}$ and satisfies:

$$V(W, H, \mathcal{P}) \geq V^m > -\infty, \quad \forall W, H, \mathcal{P}, \tag{2}$$

where V^m denotes the finite utility at death. Standard examples of the latter include the seminal Yaari (1965); Hakansson (1969) paradigm ($V^m \equiv 0$), or 'warm glow' effects of bequeathed wealth ($V^m = V^m(W_{T^m})$, e.g. Yogo (2016); French and Jones (2011); De Nardi et al. (2009)). Observe that monotonicity, curvature and finite utility assumptions imply the existence of decreasing and convex indifference curves in the wealth and life probability $(1-\mathcal{P})$ space (see Figure 1).

2.2 Human Capital Value of Life

The Human Capital Value of life is the market value of the net dividend flow associated with human capital and that is foregone upon death (e.g. Huggett and Kaplan, 2016, 2013). In our setting, this net dividend is the marketed income $Y(H)$, minus the money value of associated investment expenses I, where both are evaluated at the optimum to problem (1):

Definition 1 (HK value of life) *The Human Capital Value of life* $v_{h,t} = v_h(W_t, H_t, \mathcal{P}_0)$ *is the expected discounted present value over stochastic horizon* T^m *of labor revenue flows, net of investment costs:*

$$v_{h,t} = E_t \int_0^{T^m} m_{t,\tau} \left[Y(H_\tau^*) - I_\tau^* \right] \mathrm{d}\tau, \tag{3}$$

where $m_{t,\tau}$ *is a stochastic discount factor induced by the assets' prices and* (H^*, I^*) *are evaluated along the optimal path solving* (1).

As a canonical example with Poisson mortality, assume constant values for the death intensity λ_m, the interest rate r and the non-stochastic growth rate g^n for net income $Y^n = Y(H) - I$. The HK value then simplifies to:

$$v_h = \frac{Y^n}{r + \lambda_m - g^n}. \tag{4}$$

The human capital value of life in this special case is therefore decreasing in both the death risk λ_m and interest rate r and is increasing in both the net income level Y^n, as well as its growth rate g^n.

2.3 Willingness to pay

Next, consider a permanent exogenous change Δ in the instantaneous probability of death from base level $\mathcal{P}_0$. We rely on the indirect utility (1) to define the Hicksian Equivalent Variation as follows:

Definition 2 (WTP) *The willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ to avoid a permanent change Δ in death risk exposure $\mathcal{P}$ is implicitly given as the solution to:*

$$V\left(W - v, H, \mathcal{P}_0\right) = V\left(W, H, \mathcal{P}_0 + \Delta\right), \tag{5}$$

where $V(W, H, \mathcal{P})$ solves (1)*.*

For unfavorable changes $\Delta > 0$, equation (5) indicates indifference between paying the equivalent variation $v > 0$ to remain at base risk and not paying, but face higher death risk. For favorable changes $\Delta < 0$, the agent is indifferent between receiving compensation $-v > 0$ and foregoing lower death risk exposure.[10]

Observe that if the indirect utility is well-defined over all $\mathcal{P}$, then the willingness to pay is well-defined over all changes in death risk Δ. Note also that the monotonicity and curvature assumptions on the indirect utility $V(W, H, \mathcal{P})$ in (1) are sufficient to yield a monotone increasing and concave willingness to pay with respect to increment in death risk Δ. To see this, substitute $v(W, H, \mathcal{P}_0, \Delta)$ in (5), take derivatives and re-arrange to obtain:

$$\frac{\partial v}{\partial \Delta} = \frac{-V_{\mathcal{P}}}{V_W} \geq 0, \tag{6a}$$

$$\frac{\partial^2 v}{\partial \Delta^2} = \frac{V_{\mathcal{P}\mathcal{P}} - V_{WW}\left(\partial v / \partial \Delta\right)^2}{-V_W} \leq 0. \tag{6b}$$

Monotonicity $V_W \geq 0$ and preference for life $V_{\mathcal{P}} \leq 0$ therefore induce a willingness to pay v that is increasing in Δ, whereas the diminishing marginal utility of wealth $V_{WW} \leq 0$, and of survival probability $V_{\mathcal{P}\mathcal{P}} \geq 0$ are sufficient to induce a concave WTP function in mortality risk exposure.

[10] An alternative formulation relies instead on the Hicksian willingness to accept compensation (WTA) to face Δ, implicitly defined as the solution to:

$$V\left(W + v^a, H, \mathcal{P}_0 + \Delta\right) = V\left(W, H, \mathcal{P}_0\right).$$

This WTA perspective is however not suitable for Gunpoint settings in the absence of bequests. Indeed, whereas paying out the WTP in a highwaymen threat is rational, accepting compensation against certain death when terminal wealth in not bequeathed and life is preferred is not. Since we abstract from bequests in our benchmark model in Section 3, we therefore adopt the WTP perspective in (5).

2.4 Value of Statistical Life

The VSL is a measure of the marginal rate of substitution between the probability of life and wealth, evaluated at base risk (e.g. Aldy and Smyth, 2014; Andersson and Treich, 2011; Bellavance et al., 2009). In the context of the model (1), it is thus the negative of the MRS between $\mathcal{P}$ and W evaluated at continuation utility $V(W, H, \mathcal{P})$. The WTP property (6a) establishes that this MRS is also the marginal willingness to pay (MWTP) evaluated at base risk.

Definition 3 (VSL) *The Value of a Statistical Life $v_s = v_s(W, H, \mathcal{P}_0)$ is the negative of the marginal rate of substitution between the probability of death and wealth and also the marginal WTP evaluated at base risk:*

$$v_s = \left. \frac{-V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})} \right|_{\mathcal{P}=\mathcal{P}_0} \tag{7a}$$

$$= \lim_{\Delta \to 0} \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta} = \left. \frac{\partial v(W, H, \mathcal{P}_0, \Delta)}{\partial \Delta} \right|_{\Delta=0}, \tag{7b}$$

where $V(W, H, \mathcal{P})$ solves (1) *and $v(W, H, \mathcal{P}_0, \Delta)$ solves* (5).

Figure 1 illustrates the indifference curve (in blue) in the wealth and life probability space. The VSL in (7a) is the slope of the red tangent evaluated at base death risk $\mathcal{P}_0$ and is equivalent to the total wealth spent to save one life corresponding to the distance [a,d] (e.g. Andersson and Treich, 2011, Fig. 17.1, p. 398).

Moreover, contrasting the theoretical definition of the VSL as a MWTP in (7b) with its empirical counterpart reveals that the latter can also be interpreted as the slope of the willingness to pay to avoid small changes in death risk. Indeed the empirical VSL commonly relied upon in the literature (e.g. footnote 2) can be expressed as:

$$v_s^e(W, H, \mathcal{P}_0, \Delta) = \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}, \tag{8}$$

for small change $\Delta = 1/n$, where n is the size of the population affected by the change. The theoretical measure of the VSL v_s in (7b) is therefore the limiting value of its empirical counterpart v_s^e when the change Δ tends to zero. We formally clarify the conditions under which the empirical measure v_s^e can also be interpreted as a collective WTP in Remark 1 below.

2.5 Gunpoint Value of Life

We next introduce the Gunpoint Value (GPV) as a third valuation of life. To do so, we combine preference for life (2) with the Hicksian Equivalent Variation in (5) to define the GPV as follows:

Definition 4 (GPV) *The Gunpoint Value* $v_g = v_g(W, H, \mathcal{P}_0)$ *is the WTP to avoid certain, instantaneous death and is implicitly given as the solution to:*

$$V(W - v_g, H, \mathcal{P}_0) = V^m \tag{9}$$

where $V(W, H, \mathcal{P})$ *solves* (1) *and satisfies* (2), *and where* V^m *is the utility at death.*

The Gunpoint Value $v_g(W, H, \mathcal{P}_0)$ in (9) is implicitly defined as the payment that leaves the agent indifferent between paying v_g and remaining at base death risk $\mathcal{P}_0$ and not paying and face instantaneous and certain death and attain utility V^m. The willingness to pay v_g can thus be interpreted as the maximal amount paid in order to survive an *ex-ante* unforecastable and *ex-post* credible highwaymen threat.

Compared to the HK and VSL alternatives, the Gunpoint Value presents several advantages. First, unlike the HK, the Gunpoint Value does not uniquely ascribe the economic worth of an agent to the labor income he generates, but instead accounts for all pledgeable disposable resources, including financial wealth. Second, unlike the VSL, the GPV does not extrapolate measurable responses to small probabilistic changes in the likelihood of death, but instead explicitly values a person's life as *an entity* and does so without external assumptions regarding integrability from marginal to total value of life. Finally, instead of calculating an external valuation of someone's life, the GPV circumvents ethical discomforts by letting someone compute his own intrinsic value through his individual willingness to pay to save *himself*.

2.6 Clarifying the links between the WTP, VSL, and the GPV

Figure 2 establishes the central role of the willingness to pay in linking the theoretical and empirical Statistical Values of Life, as well as the benchmark Gunpoint values of life. From properties (6), the WTP $v = v(W, H, \mathcal{P}_0, \Delta)$ (in blue) is an increasing, concave function of the change in death risk Δ. The theoretical VSL v_s in (7b) is the marginal willingness to pay, i.e. the slope of the red tangent evaluated at base death risk ($\Delta = 0$). It is equivalent to the linear projection corresponding to the total wealth spent to save one person (i.e. when $\mathcal{P}_0 + \Delta = 1.0$) and is equal to the distance [a,f]. The empirical Value of a Statistical Life v_s^e in (8) is computed for a small change $\Delta^e > 0$ and corresponds to the slope of the green line; equivalently, it is the linear projection represented by the distance [b,e]. The empirical VSL measure v_s^e will thus understate its theoretical counterpart v_s when Δ^e is large and when the WTP is concave. Moreover, as will become clear shortly, the Gunpoint value corresponds to the upper bound on the WTP, i.e. the limiting WTP when death is certain as represented by the distance [c,d] in Figure 2. A concave WTP entails that a linear extrapolation under either the theoretical, or the empirical VSL will thus overstate the value attributed to one's own life.

3 A benchmark human capital model

We now proceed with a parametrized version of our benchmark human capital model in Section 2.1 in order to compute the willingness to pay and theoretical life values defined in Sections 2.2–2.5.

3.1 Economic environment

Consider a depreciable human capital H_t whose law of motion is given by:

$$\mathrm{d}H_t = \left[I_t^{\alpha} H_t^{1-\alpha} - \delta H_t\right] \mathrm{d}t - \phi H_t \mathrm{d}Q_{st}. \tag{10}$$

The term $\delta \in (0,1)$ is a deterministic depreciation, whereas $\mathrm{d}Q_{st}$ is a Poisson depreciation shock with constant intensity λ_{s0}, whose occurrence further depreciates the capital stock by a factor $\phi \in (0,1)$.

The law of motion (10) applies to alternative interpretations of human capital. If H_t is associated with skills (e.g. Ben-Porath, 1967; Heckman, 1976), then investment I_t comprises education and training choices made by the agent whereas $\mathrm{d}Q_{st}$ can be interpreted as stochastic unemployment, or technological obsolescence shocks that depreciate the human capital stock. If H_t is instead associated with health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), then investment takes place through medical expenses or healthy leisure decisions whereas the stochastic depreciation occurs through morbidity shocks.

In addition to stochastic depreciation, the agent is exposed to Poisson mortality risk with constant intensity λ_{m0}. Within the context of this continuous-time model, the instantaneous death probability $\mathcal{P}$ introduced earlier can be obtained by noting that:

$$\Pr[\text{Death}(t, t+h)] = \lambda_{m0}\, h + o(h), \tag{11}$$

for a small h. In the subsequent life valuation, we will associate exogenous changes Δ from base death risk $\mathcal{P}_0$ resulting from permanent changes in the exogenous death risk exposure λ_{m0}.

Financial wealth W_t evolves according to the dynamic budget constraint:

$$dW_t = [rW_t + Y_t - c_t - I_t]\,\mathrm{d}t + \pi_t \sigma_S\,[\mathrm{d}Z_t + \theta \mathrm{d}t] + x_t\,[\mathrm{d}Q_{st} - \lambda_{s0}\mathrm{d}t]\,, \tag{12}$$

$$Y_t = y + \beta H_t, \tag{13}$$

where r it the interest rate and $\theta = \sigma_S^{-1}(\mu - r)$ is the market price of financial risk. In addition to investment I_t, the control variables include c_t as consumption, π_t as the risky portfolio and x_t is the units purchased of actuarially-fair depreciation insurance. The latter pays one unit of the numeraire per unit of contract purchased, upon occurrence of the depreciation shock and can be interpreted as unemployment insurance (if H_t is associated with skills) or as medical, or disability insurance (if H_t is associated with health). The income process in (13) comprises an exogenous component y, whereas the expression βH reflects a positive income gradient for agents with higher human capital.

Finally, the indirect utility of an alive agent is defined as:

$$V(W_t, H_t) = \sup_{(c,\pi,x,I)} U_t,$$

where preferences are:

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_\tau) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_\tau} \right) \mathrm{d}\tau, \tag{14}$$

with

$$f(c_t, U_t) = \frac{\rho U_t}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_t} \right)^{1-\frac{1}{\varepsilon}} - 1 \right). \tag{15}$$

The utility U in (14), combined with the Kreps-Porteus aggregator function $f(c, U)$ in (15) corresponds to the stochastic differential utility proposed by Duffie and Epstein (1992), i.e. the continuous-time analog to the discrete-time Epstein and Zin (1989, 1991) preferences. It is characterized by subjective discount rate $\rho > 0$, minimal subsistence consumption $a > 0$ and disentangles the elasticity of inter-temporal substitution (EIS) $\varepsilon \geq 0$, from the agent's constant relative risk aversion with respect to financial risk $\gamma \geq 0$. As explained in Hugonnier et al. (2013) and confirmed in Theorem 1 below, the homogeneity properties of non-expected utility guarantee that the agent prefers life over death, with minimal consumption requirement $c_t \geq a$ implying positive continuation utility and preference of life over death $V_t \geq V^m \equiv 0$.

3.2 Optimal rules

The baseline human capital model of Section 3.1 can be solved in closed form, yielding the following result.

Theorem 1 *Assume that the following conditions hold:*

$$0 < A(\lambda_{m0}) - \max\left(0, r - \lambda_{m0} + \theta^2/\gamma\right), \tag{16a}$$

$$\beta < (r + \delta + \phi\lambda_{s0})^{\frac{1}{\alpha}}. \tag{16b}$$

Then the indirect utility for the agent's problem is:

$$V_t = \Theta(\lambda_{m0}) N_0(W_t, H_t) \geq 0, \tag{17}$$

and generates the optimal rules:

$$\begin{aligned} c_t =& a + A(\lambda_{m0}) N_0(W_t, H_t) \geq 0, \\ \pi_t =& (\theta/(\gamma\sigma_S)) N_0(W_t, H_t), \\ x_t =& \phi P_0(H_t) \geq 0, \\ I_t =& \left(\alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}\right) P_0(H_t) \geq 0, \end{aligned} \tag{18}$$

where any dependence on death intensity λ_{m0} is explicitly stated. The human wealth and net total wealth are given as:

$$P_0(H_t) = BH_t \geq 0, \tag{19}$$

$$N_0(W_t, H_t) = W_t + \frac{y-a}{r} + P_0(H_t) \geq 0, \tag{20}$$

where $B > 0$ solves $g(B) = 0$, s.t. $g'(B) < 0$ in:

$$g(B) = \beta - (r + \delta + \phi\lambda_{s0})B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}} \tag{21}$$

and where the marginal value of net total wealth and the marginal propensity to consume are:

$$\Theta(\lambda_{m0}) = \tilde{\rho} A(\lambda_{m0})^{\frac{1}{1-\varepsilon}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{-\varepsilon}{1-\varepsilon}} \tag{22}$$

$$A(\lambda_{m0}) = \varepsilon\rho + (1-\varepsilon)\left(r - \lambda_{m0} + 0.5\,\theta^2/\gamma\right) \geq 0. \tag{23}$$

Both the indirect utility (17) and the optimal rules (18) are increasing functions of the agent's human wealth, i.e. the market value of his human capital $P_0(H_t)$. The price of human capital B in (19) can be interpreted as a Tobin's-Q. It is implicitly defined in (21) as an increasing function of the income gradient β and is declining in the rate of interest r and the expected depreciation $\delta + \phi\lambda_{s0}$. The human wealth is combined with financial wealth W_t and the NPV of the base income stream, net of minimal consumption $(y-a)/r$, to recover net total wealth $N_0(W_t, H_t)$ in (20). Observe that both human wealth and net total wealth are independent of the death intensity λ_{m0}. This independence results from the well-known equivalence between discounting at rate ρ, with Poisson mortality and finite lives and an infinite horizon plus discounting at augmented rate $\rho + \lambda_{m0}$.[11]

Two features of the optimal rules are particularly relevant for life valuation. A first property is that the exposure to exogenous death risk λ_{m0} affects welfare via $\Theta(\lambda_{m0})$ in (22), through its impact on the marginal propensity to consume (MPC) $A(\lambda_{m0})$ exclusively. Equation (23) establishes that this impact crucially depends on the elasticity of inter-temporal substitution ε. An increase in death risk λ_{m0} induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume. On the one hand, more discounting of future consumption requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC). This effect is dominant at low elasticity of inter-temporal substitution $\varepsilon \in (0, 1)$ and the MPC in (23) is monotone decreasing. On the other hand, heavier discounting makes future consumption less desirable and shifts future towards current consumption (i.e. by increasing the MPC). This *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution $\varepsilon > 1$. Note in closing that unit elasticity implies exact cancellation of the two effects and results in a mortality risk-independent MPC that is equal to the subjective discount rate ρ. Consequently, the marginal value of net total wealth Θ is also independent of the exposure to death λ_{m0} when $\varepsilon = 1$.

A second key property is that the welfare in (17) is monotone increasing and linear in both wealth and human capital stock and is unconditionally monotone decreasing and convex in death risk exposure since:

$$\Theta'(\lambda_{m0}) = -\tilde{\rho} A(\lambda_{m0})^{\frac{\varepsilon}{1-\varepsilon}} \leq 0, \tag{24a}$$

$$\Theta''(\lambda_{m0}) = \tilde{\rho} \varepsilon A(\lambda_{m0})^{\frac{2\varepsilon - 1}{1-\varepsilon}} \geq 0. \tag{24b}$$

Hence, whereas the sign of the effects of death risk λ_{m0} on the MPC (23) depends on the EIS, preference for life implies that it always reduces the marginal value of net total wealth (22).

[11]In particular, a finite-horizon problem with positive death exposure and discounting ρ can be equivalently recast as an infinite horizon program with augmented discounting. Under completeness, the agent can sell a claim to his income process to recover human wealth and net total wealth. The optimal rules are obtained as functions of the latter, with heavier discounting capturing finite lives $\rho + \lambda_{m0}$ (see Hugonnier et al., 2013, for details).

4 Willingness to Pay and Values of Life

We next calculate the model-implied life valuations of Section 2 relying on the solution for the benchmark human capital model of Section 3. We will assume throughout that the optimal rules outlined in Theorem 1 are being followed by the agents and will abstract from time subscripts to alleviate notation.

4.1 Human Capital Value of Life

The HK value of life outlined in Definition 1 is computed as follows.

Proposition 1 (HK value) *The Human Capital Value of life solving* (3) *is:*

$$v_h(H, \lambda_{m0}) = C_0 y + C_1 P_0(H) \tag{25}$$

where the non-negative constants C_0, C_1 *are defined by:*

$$\begin{aligned} C_0 &= \frac{1}{r + \lambda_{m0}}, \\ C_1 &= \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_{m0} - (\alpha B)^{\frac{\alpha}{1-\alpha}}}, \end{aligned} \tag{26}$$

and where $P_0(H)$ *is given in* (19).

The HK value in (25) is an increasing affine function of the economic value of human capital stock $P_0(H)$. A wealth-independent optimal investment in (18) implies that v_h is also independent of W. The first term C_0 is the standard NPV of base income y, lowered for exogenous exposure to death risk λ_{m0} – see equation (4). The second term C_1 is the net present value along the optimal path of the βH^* component of income, net of spending I^*. Indeed, the optimal rules in (18) reveal that the investment-to-capital ratio is $I/H = (\alpha B)^{1/(1-\alpha)}$ and enters negatively in C_1. A higher Tobin's-Q has two conflicting effects on the HK value. One the one hand, a higher market value $P_0(H) = BH$ entails a larger v_h. On the other hand, a higher price of human capital justifies a higher investment ratio I/H and lowers v_h.

4.2 Willingness to pay to avoid a finite increase in death risk

Next, we can substitute the indirect utility $V(W, H, \lambda_{m0})$ given by (17) in Definition 2, and solve for $v = v(W, H, \lambda_{m0}, \Delta)$ as follows:

Proposition 2 (willingness to pay) *The willingness to pay to avoid a change from* λ_{m0} *to* $\lambda^*_{m0} = \lambda_{m0} + \Delta$ *solving* (5) *is given by:*

$$v(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] N_0(W, H), \tag{27}$$

an increasing and concave function of Δ *that is bounded by:*

$$\inf_{\Delta} v(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(0)}{\Theta(\lambda_{m0})}\right] N_0(W, H) \tag{28}$$

$$\sup_{\Delta} v(W, H, \lambda_{m0}, \Delta) = N_0(W, H). \tag{29}$$

where net total wealth $N_0(W, H)$ *is given in* (20) *and the marginal value* $\Theta(\lambda_{m0})$ *is given in* (22).

The WTP in (27) equals zero whenever the increment $\Delta = 0$, as well as under unit elasticity of inter-temporal substitution $\varepsilon = 1$. In this case, the marginal value of total wealth Θ is independent from λ_{m0}. For the other cases, it was shown earlier that $\Theta(\lambda_{m0}) \geq 0$ in (22) is a decreasing and convex function. Consequently, the weights $\Theta(\lambda^*_{m0})/\Theta(\lambda_{m0}) \in [0, 1]$ for detrimental changes $\Delta \geq 0$ and the willingness to pay is an increasing function of net total wealth $N_0(W, H)$.

Furthermore, equation (6) established that a decreasing and convex effect of mortality risk on welfare entails a monotone increasing and concave willingness to pay to avoid death. These properties of the indirect utility were verified in (24) and the implications for the WTP are again confirmed in (27). They are also consistent with standard economic intuition of diminishing marginal valuation of exposure to death (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2017).

Equation (28) establishes that the lower bound on the WTP is obtained by setting $\Delta = -\lambda_{m0}$. From equations (22) and (23) this bound exists and is finite. Equation (29) establishes that the willingness to pay is bounded above by net total wealth $N_0(W, H)$. When the elasticity of inter-temporal substitution is larger than one, this upper bound corresponds to the asymptotic WTP. When the EIS is below one, the upper bound corresponds to a maximal admissible WTP satisfying the transversality constraint (16a). This case is discussed in further details in Appendix B.3.

4.3 Value of a Statistical Life

Using Definition 3, and welfare (17), we can calculate the theoretical expression for the VSL implied by the benchmark model as follows.

Proposition 3 (Value of Statistical Life) *The Value of a Statistical Life is:*

$$\begin{aligned} v_s(W, H, \lambda_{m0}) &= \frac{-\Theta'(\lambda_{m0})}{\Theta(\lambda_{m0})} N_0(W, H), \\ &= \frac{1}{A(\lambda_{m0})} N_0(W, H), \end{aligned} \tag{30}$$

where total wealth $N_0(W, H)$ *is given in* (20)*, the marginal value* $\Theta(\lambda_{m0})$ *is given in* (22) *and the MPC* $A(\lambda_{m0})$ *is given in* (23)*.*

The Value of a Statistical life, corresponding to the marginal willingness to pay to avoid increases in death risk, is unconditionally decreasing in the MPC and increasing in net worth. Observe that since the MPC is typically low (see Carroll, 2001, for a review), the VSL is thus much larger than net disposable resources $N_0(W, H)$, an issue to which we will return shortly.

Remark 1 (empirical VSL as a collective WTP) The empirical VSL literature emphasizes that it measures a *collective*, rather than individual, willingness to pay to save a human life that can be captured by the average willingness to pay over small Δ in (8). This claim can be verified by relying on a social welfare function calculated via our indirect utility. Given a population of size n, as well as any set of weights $\boldsymbol{\eta} \in \mathbb{R}^n_+$ and corresponding social welfare function $\sum_{j=1}^{n} \eta_j V_j(W_j, H_j, \lambda_{m0})$, the Hicksian WTP can be written as:

$$\sum_{j=1}^{n} \eta_j V_j(W_j - v_j, H_j, \lambda_{m0}) = \sum_{j=1}^{n} \eta_j V_j(W_j, H_j, \lambda_{m0} + \Delta).$$

Resorting to the indirect utility function (17), we can assume homogeneous parameters for preferences $(\rho, \gamma, \varepsilon, a)$, human capital technology (α, δ, ϕ) and risk exposition $(\lambda_{s0}, \lambda_{m0})$

and exploit the linearity of welfare in wealth and human capital to derive the (weighted) collective WTP as:

$$\sum_{j=1}^{n} \eta_j v_j(W_j, H_j, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] \sum_{j=1}^{n} \eta_j N_0(W_j, H_j).$$

Two special cases of identical weights $\boldsymbol{\eta}$ are worth mentioning:

1. Proportional weights $\eta_j = 1/n, \forall j$ yield:

$$\begin{aligned}\bar{v}(W, H, \lambda_{m0}, \Delta) &= \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] N_0(\bar{W}, \bar{H}),\\ &= v(\bar{W}, \bar{H}, \lambda_{m0}, \Delta),\end{aligned}$$

   i.e. the mean willingness to pay is the WTP (27) evaluated at the mean wealth and human capital.

2. Unit weights $\eta_j = 1, \forall j$ yield the (unweighted) collective WTP:

$$\begin{aligned}\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_{m0}, \Delta) &= \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] \sum_{j=1}^{n} N_0(W_j, H_j),\\ &= \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] n N_0(\bar{W}, \bar{H}),\\ &= n v(\bar{W}, \bar{H}, \lambda_{m0}, \Delta).\end{aligned}$$

   Evaluating the latter at $\Delta = n^{-1}$ yields the empirical VSL measure commonly used in the literature:

$$\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_{m0}, \Delta) = \frac{v(\bar{W}, \bar{H}, \lambda_{m0}, \Delta)}{\Delta} = v_s^e(\bar{W}, \bar{H}, \lambda_{m0}, \Delta). \tag{31}$$

Hence, the empirical VSL v_s^e in (8), or (31) is a collective WTP, under homogeneity and unit social welfare weights assumptions. Moreover, the collective WTP corresponds to a slope between two points on the willingness to pay, evaluated at mean wealth and human capital. As discussed earlier, a concave WTP implies that the $v_s^e \leq v_s$, i.e. the empirical measure under-estimates the theoretical VSL corresponding to the MWTP and given in (30) (see Figure 2).

Remark 2 (discrete changes per period) The theoretical calculations of the VSL in equation (30) are valid for permanent, infinitesimal changes in the death intensity. In the spirit of the empirical VSL literature, the value of a statistical life can also be computed as the willingness to pay to avoid an exogenous increase Δ in the probability of death over a given time interval (e.g. a change $\Delta = 0.1\%$ per one year period), divided by Δ. This calculation can also be obtained in closed-form, and involves two steps. First, the new value of the endowed intensity $\lambda^*_{m0}(\Delta, T)$ is computed, corresponding to a change in death risk Δ occurring over a duration of T :

Lemma 1 *A higher likelihood of death of Δ per time interval of $s \in [0, T]$ corresponds to a permanent increase in the endowed intensity to $\lambda^*_{m0}(\Delta, T) > \lambda_{m0}$ given by:*

$$\lambda^*_{m0}(\Delta, T) = \frac{-1}{T} \log \left[e^{-\lambda_{m0} T} - \Delta \right]. \tag{32}$$

Second, we can substitute $\Theta(\lambda^*_{m0}(\Delta, T))$ in the WTP (27), and divide by Δ to obtain the corresponding empirical Value of a Statistical Life.

4.4 Gunpoint Value of Life

Combining Definition 4 and welfare function (17) reveals the following result.

Proposition 4 (Gunpoint value of life) *The willingness to pay to avoid certain death solving* (9) *is given by:*

$$v_g(W, H) = N_0(W, H), \tag{33}$$

where $N_0(W, H)$ is the net total wealth in (20)*.*

In the absence of a bequest motive, the agent who is forced to evaluate life at gunpoint is thus willing to pledge all available resources, i.e. his entire financial wealth W, plus the capitalized value of his fixed income endowment y/r. Since human capital is non-transferable and entirely depreciated at death, the agent is also willing to give up his human wealth $P_0(H) = HB$, an increasing function of the human capital stock and of its Tobin's-Q. However, the previous discussion emphasized that the minimal consumption level a is required at all periods for subsistence. Its costs therefore cannot be pledged in a highwaymen threat, and must be subtracted from the Gunpoint value.

It can also be shown that net total wealth $N_0(W, H)$ is equal to the expected discounted present value of excess consumption along the optimal path.[12] In order to survive, the agent is thus willing to pledge the total value of his optimal consumption stream (net of minimal subsistence). This result can be traced to the homegeneity property under which the foregone utility is measured in the same units as the foregone excess consumption. This interpretation also foreshadows the similarities between the HK (foregone net income stream) and the GPV values of life (foregone net consumption stream).

The links between the WTP in (27) and GPV in (33) are intuitive and follow directly from (29) showing that the WTP is bounded above by net total wealth. The Gunpoint Value therefore corresponds to that limiting WTP. A concave willingness to pay thus implies that the VSL will over-value the GPV (see Figure 2). Indeed, comparing (30) and (33) establishes that:

$$v_g(W, H) = A(\lambda_{m0})v_s(W, H, \lambda_{m0}). \tag{34}$$

Estimates of the marginal propensity to consume $A(\lambda_{m0})$ are typically low, ranging from 2-9% for housing wealth and 6% for financial wealth (e.g. Carroll et al., 2011, p. 58). Consequently, the predicted gap between the GPV and VSL is large.

Interestingly, since net total wealth is independent of risk aversion and elasticity of inter-temporal substitution, so is the GPV. The reason stems from the way the GPV is characterized in Definition 4, i.e. as the unitary value of a life, rather than by integrating marginal changes in death risk exposure. The agent therefore pays v_g to avoid receiving the utility $V^m \equiv 0$ that is associated with certain and immediate death. Because the utility at death is a finite primitive, the Gunpoint Value is always computable for all EIS levels. For the same reason, the Gunpoint Value of life v_g in (33) is also independent from the agent's preferences $(\rho, \varepsilon, \gamma)$, and from the death intensity (λ_{m0}). Because the

[12] In particular, Hugonnier et al. (2013, Prop. 2) show that

$$E_t \int_t^{\infty} m_{t,\tau}(c_\tau^* - a)\mathrm{d}\tau = N_0(W, H).$$

outcome of death is certain when life is evaluated at gunpoint, the attitudes towards time and risk, as well as the level of exposure to death risk become irrelevant. Since death at gunpoint is instantaneous, attitudes towards inter-temporal substitution are irrelevant as well.

Remark 3 (aging) Our closed-form expressions for the willingness to pay and the three life valuations have abstracted from aging processes. The latter can be incorporated for a wide pattern of age-dependencies, although at some non-negligible computation cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in $\lambda_{m0t}, \lambda_{s0t}, \phi_t, \delta_t,$ or β_t results in age-dependent MPC and Tobin's-Q that solve the system of ODE's:

$$\begin{aligned}
\dot{A}_t &= A_t^2 - \left(\varepsilon\rho + (1-\varepsilon)\left(r - \lambda_{m0t} + \theta^2/(2\gamma)\right)\right) A_t, \\
\dot{B}_t &= (r + \delta_t + \phi_t \lambda_{s0t}) B_t + (1 - 1/\alpha)(\alpha B_t)^{\frac{1}{1-\alpha}} - \beta_t,
\end{aligned}$$

subject to the boundary condition:

$$\begin{aligned}
&\lim_{t\to\infty} (r - \lambda_{m0t} + \theta^2/(2\gamma) - A_t) < 0, \\
&\lim_{t\to\infty} ((\alpha B_t)^{\frac{\alpha}{a-\alpha}} - r - \delta_t - \phi_t \lambda_{s0t}) < 0.
\end{aligned}$$

Allowing for aging and solving for A_t, B_t implies that the expressions C_{0t}, C_{1t}, the marginal value $\Theta_t(\lambda_{m0t})$, as well as the human and total wealth $P_{0t}(H), N_{0t}(W, H)$ are also age-dependent and all the relevant calculations can be modified accordingly to compute the WTP and the life values at any age level t.

5 Structural estimation

In order to structurally estimate the willingness to pay and the life valuations, we follow a long tradition associating the agent's human capital to his health (e.g. see the Hicks' lecture by Becker, 2007, for a review). We then estimate the technological, preferences and stochastic parameters for the benchmark model outlined in Section 3 by combining the observed health and wealth statuses with the observed decisions corresponding to (18).

Once the structural parameters have been estimated, they can be relied upon to compute the closed-form expressions for the life valuations in Section 4.

5.1 Econometric model

For identification purposes, the econometric model assumes that agents follow the first-order optimal rules to the benchmark model and that they differ with respect to their health and wealth statuses, whereas they share common preference, technological, and distributional parameters. Observe that this identifying hypothesis is consistent with the aggregation assumptions required to elicit the empirical VSL as a collective WTP (see Remark 1). To structurally estimate the model's parameters, we use the closed-form expressions given in Theorem 1 to which we append the income equation (13). Specifically, denote by $\mathbf{Y}_j = [c_j, \pi_j, x_j, I_j, Y_j]'$ the 5×1 vector of observed decisions and income for agent $j = 1, 2, \ldots, n$, let $\mathbf{X}_j = [1, W_j, H_j]'$ capture his current wealth and health statuses. Also let $\mathbf{B}(\boldsymbol{\theta})$ denote the 5×3 matrix of closed-form expressions for the optimal rules implicit in equation (18), that are functions of the structural parameters $\boldsymbol{\theta}$. The econometric model relies on Maximum Likelihood to structurally estimates the latter in:

$$\mathbf{Y}_j = \mathbf{B}(\boldsymbol{\theta})\mathbf{X}_j + \mathbf{u}_j \tag{35}$$

where the $\mathbf{u}_j$'s are (potentially correlated) Gaussian error terms. In order to ensure theoretical consistency and augment identification, we estimate the structural parameters in (35) imposing the regularity conditions (16). Note that the key EIS parameter ε is unconstrained and therefore allowed to take positive values below or above one. In light of the strong nonlinearities not all the deep parameters can be identified and a subset of parameters denoted $\boldsymbol{\theta}^c$ are calibrated.

Remark 4 (semi-structural estimation) The structural econometric model in (35) exploits all cross-equations and regularity restrictions by estimating the fully-constrained model. A somewhat simpler semi-structural approach exploits instead the triangular identification by recursively solving for the deep parameters through a reduced-form estimation of (35). That approach is presented in Appendix D and can be relied upon to generate starting values for the structural model, or when empirical identification issues are observed.

5.2 Data

We use a sample of $n = 8{,}378$ individuals taken from the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix E. We proxy the health variables through the polytomous self-reported health status (Poor, Fair, Good, Very Good and Excellent) that is linearly converted to numeric values from 1 to 4. The financial wealth comprises risky and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation and utility expenses reported in the PSID. Finally, health spending and health insurance expenditures are taken to be the out-of-pocket spending, as well as premia paid by agents. All nominal values are scaled by 10^{-6} for the estimation.

Tables 1, and 2 present descriptive statistics for the main variables of interest, per health status and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear relation between health and wealth can be deduced. The level of consumption in panel b is increasing in financial wealth, consistent with expectations. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption.

In panel c, stock holdings are very low for all but the fourth and fifth quintiles, illustrating the well-known non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth and devoid of clear health gradients. Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth and also sharply decreasing in health status.

6 Results

6.1 Structural parameters

Table 3 reports the calibrated (with subscripts ^{c}) and estimated (standard errors in parentheses) model parameters. Overall, the latter are precisely estimated and are consistent with other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.6843$). Although deterministic depreciation is relatively low ($\delta = 1.25\%$), additional depletion brought upon by sickness is important ($\phi = 1.36\%$). Second, exposure to mortality risk is realistic ($\lambda_{m0} = 0.0283$), corresponding to a remaining expected lifetime of $\ell = \lambda_{m0}^{-1} = 35.3$ years, given mean respondent age of 45.26 years in Table 1.[13]

Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ($\beta = 0.0092$), as well as an estimated value for base income that is close to poverty thresholds ($y \times 10^6 = 12.2$ K\$).[14] The financial parameters (μ, σ_S, r) are calibrated from the observed moments of the S&P500 and 30-days T-Bills historical returns. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 2.8953$). The minimal consumption level is realistic and larger than base income ($a \times 10^6 = 14.0$ K\$). As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.2416$) and is consistent with a *Live Fast and Die Young* effect whereby a higher risk of death increases the marginal propensity to consume. Importantly, a high EIS confirms that the willingness to pay is well defined everywhere (see Appendix B.3), and that the limiting value on the WTP corresponds to the GPV.

6.2 Estimated valuations

Human Capital Value of Life Using the estimated parameters in Table 3, we can compute the HK value of life $v_h(H)$ given in (25) and reported in Table 4.a. Overall, the human capital values are common across W and increasing in H, ranging from 250 K$

[13]The remaining life expectancy at age 45 in the US in 2013 was 36.1 years (all), 34.1 (males) and 37.9 (females) (Arias et al., 2017).

[14]For example, the 2016 poverty threshold for single-agent households under age 65 was 12.5 K$ (U.S. Census Bureau, 2017).

(Poor health) to 527 K$ (Excellent health), with a mean value of 421 K$. These figures are realistic. For example, using mean income 21,838$ minus expenses of 721 $ in Table 1, mortality exposure $\lambda_{m0} = 0.0283$ and a constant net income growth rate $g^n = 2\%$ yields a canonical HK value in (4) equal to 375 K$. Our structural estimates also compare advantageously with other HK estimates and provide a first out-of-sample confirmation that the structural estimates are reasonable.[15]

Value of Statistical Life Table 4.b reports the Values of Statistical Life $v_s(W, H, \lambda_{m0})$ in (30) by observed health and wealth statuses. First, the VSL mean value is 8.35 M$, with valuations ranging between 2.17 M$ and 15.01 M$. These values are well within the ranges usually found in the empirical VSL literature.[16] The concordance of these estimates with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded. Importantly they confirm that the large VSL-HK gap is *not* an artefact of disjoint estimation devoid of common theoretical linkages.

Second, the VSL is increasing in both wealth and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Andersson and Treich, 2011; Robinson and Hammitt, 2016; Murphy and Topel, 2006). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks and are willing to pay less to attain further improvements (or prevent deteriorations). Since our benchmark model abstracts from endogenous mortality (see the robustness discussion in Section 6.3 for generalization) and better health increases net total wealth $N_0(W, H)$, our estimates unambiguously indicate that the former effect is dominant and that improved health raises the VSL.

[15] Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K$ at age 20, peaking at less than 900 K$ at age 45 and falling steadily towards zero afterwards.

[16] A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 M$ (2000 base year, corresponding to 8.6 M$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 M$ and 10 M$. Robinson and Hammitt (2016) report values ranging between 4.2 and 13.7 M$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 M$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 M$ (2006$), corresponding to 8.8 M$ in 2016 (U.S. Environmental Protection Agency, 2017).

Gunpoint Value Table 4.c reports the Gunpoint values $v_g(W,H)$ in (33). The mean GPV is 447 K\$ and its values are increasing in both health and wealth and range between 116 K\$ and 804 K\$. The Gunpoint Value is thus of similar magnitude to the Human Capital Value of life and both are much lower than the VSL. Indeed, this finding was already foreseeable from equation (34) indicating that the VSL/GPV ratio is inversely proportional to the marginal propensity to consume. Since our estimates reveal that $A(\lambda_{m0}) = 5.36\%$ – a value again well in line with other estimates (Carroll et al., 2011) – we identify a VSL that is 18.66 times larger than the GPV.

6.3 Robustness

In order to verify robustness of the results, we consider a more general model of human capital. Hugonnier et al. (2013) study a demand for health framework that is similar to our benchmark, with two key differences. First, the model allows for self-insurance against morbidity and mortality risks by introducing health-dependent intensities:

$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}} \in [\lambda_{s0}, \eta],$$
$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m}.$$

Hence, better health lowers exposure to sickness and death risks and our benchmark model of Section 3 is an exogenous restricted case that imposes $\lambda_{s1}, \lambda_{m1} = 0$. Second, preferences are modified to allow for source-dependent aversion against financial, morbidity and mortality risks. In particular, our preferences in (14) are replaced by:

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_{\tau-}) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} F_k(U_{\tau-}, H_{\tau-}, \Delta_k U_\tau) \right) \mathrm{d}\tau,$$

with the Kreps-Porteus aggregator (15) unchanged, and with penalties for exposure against Poisson sickness and death risks:

$$F_k = U_{t-}\, \lambda_k(H_{t-}) \left[\frac{\Delta_k U_t}{U_{t-}} + u(1;\gamma_k) - u\left(1 + \frac{\Delta_k U_t}{U_{t-}}; \gamma_k\right) \right], \quad \text{where}$$
$$\Delta_k U_t = E_{t-}[U_t - U_{t-} | \mathrm{d}Q_{kt} \neq 0], \quad \text{and} \quad u(x;\gamma_k) = \frac{x^{1-\gamma_k}}{1-\gamma_k}.$$

Our benchmark specification is thus a restricted case that imposes risk-neutral attitudes towards morbidity ($\gamma_s = 0$) and mortality ($\gamma_m = 0$) risks.

In a separate technical appendix (available upon request), we show that the approximate closed-form expressions for the WTP, HK, VSL and GPV valuations can be obtained. These expressions encompass some adjustments for the endogeneity of health risks exposure and source-dependent risk aversion, yet remain otherwise qualitatively similar. We structurally estimate the Hugonnier et al. (2013) model and compute the life values. These values remain in the same range as our benchmark estimates, with mean HK of 493 K\$, VSL of 8.14 M\$ and GPV of 460 K\$ and again confirm the strong concavity of the WTP. We conclude that our main findings are qualitatively and empirically robust to more general specifications.

7 Discussion: Accounting for the large VSL

7.1 Disjoint theoretical and empirical frameworks?

Our empirical results yields three main messages. First, contrasting reduced-form estimates obtained from separate settings with fully structural ones from a common model and data set produces very similar estimates for the HK and VSL life values. Equivalently, segmented reduced-form approaches do not exhibit readily identifiable biases when contrasted with a fully-encompassing theoretical and empirical approach. A second and related message is that the large discrepancies between the two main life valuations are therefore not a result of separate theoretical and empirical frameworks; these differences remain when we structurally estimate the closed-form expressions for the VSL and HK from a common setup. Third, the Human Capital Value is much closer to a natural benchmark given by the Gunpoint Value than the VSL is. Explanations other than segmented theoretical and empirical frameworks must therefore be analyzed to understand why the VSL is more than 18 times larger than the other two life values.

7.2 Collective WTP vs individual MWTP?

Much has been made about a fundamental characterization of the VSL as a *collective* willingness to pay to save unidentified lives. When contrasted with an *individual* marginal

WTP to save oneself, one could argue that there are no reasons to expect that the two should be equal (e.g. Pratt and Zeckhauser, 1996). We helped dispel this ambiguity by formally showing that the collective WTP can be calculated in closed-form and that this expression indeed corresponds to the average WTP value favored by the empirical VSL under specific aggregation assumptions (see Remark 1). However, we also showed that the latter nonetheless remains an approximation to the theoretical VSL that will *under*-state the individual marginal rate of substitution under diminishing MWTP. The HK-VLS gaps are therefore larger when relying on the true MRS measure rather than on its empirical proxy.

We can verify this claim by computing the collective WTP corresponding to the empirical VSL v_s^e given in (31). Setting $\Delta = 1/n = 1/8,378$ and $\lambda_{m0}^* = \lambda_{m0} + \Delta$, we recover an aggregate VSL of 8.3400 M\$, which, as expected, is lower, but very close to the mean theoretical value of $v_s(W, H, \lambda_{m0}) = 8.3515$ M\$. We can also use Lemma 1 to fix an arbitrary duration $T = 1$ for change Δ and compute $\lambda_{m0}^*(1/n, T)$ in (32), as well as $\Delta_T = \lambda_{m0}^* - \lambda_{m0}$. Substituting the latter in (31) recovers an almost identical empirical value of $v_s^e = 8.3396$ M\$.

Both results thus confirm that the theoretical and empirical values are close to one another, i.e. the individual MWTP is well approximated by the collective WTP corresponding to the empirical VSL when $\Delta = 1/n$ is small (i.e. the sample size is large). Equivalently, we cannot rely on any alleged opposition between a collective willingness to pay and an individual marginal WTP to rationalize the large VSL values.

7.3 Diminishing MWTP?

We also showed that both the empirical and theoretical VSL will overstate the GPV corresponding to the upper bound on the willingness to pay when the WTP is concave. To help visualize this gap, Figure 3 is the estimated counterpart to Figure 2 and plots the willingness to pay $v(W, H, \lambda_{m0}, \Delta)$ in function of Δ calculated from (27) at the estimated parameters and relying on the mean wealth and health status. First, the estimated WTP in blue displays a pronounced curvature, consistent with our theoretical results. Second, equations (29) and (33) established that the upper bound of the willingness to pay is the net total wealth $N_0(W, H)$ and that this limiting value is also the Gunpoint value $v_g(W, H)$ plotted in red. Third, equation (7) identified the VSL $v_s(W, H, \lambda_{m0})$ as the

MWTP, i.e. the value of the slope of the yellow tangent of $v(W, H, \lambda_{m0}, \Delta)$ evaluated at $\Delta = 0$.

The strongly diminishing MWTP in Figure 3 is informative as to why the VSL is much larger than the Human Capital and Gunpoint values. Indeed, the agent is willing to pay 37 K$ to avoid an increase of $\Delta = 0.0047$ which shortens his horizon from 35.3 to 30.3 years and would pay 406 K$ to avoid $\Delta = 0.17$ which lowers expected remaining lifetime from 35.3 to only 5 years. This last value is already close to the HK and GPV values of 421 K$ and 447 K$ who are both much lower than the VSL of 8.35 M$. Equivalently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment, as foreshadowed in our discussion of (34) showing that the VSL is much larger than total pledgeable resources.

7.4 Back to basics?

To summarize, our results confirm that the important discrepancies between the Human Capital and Statistical Life Values are not a consequence of treating the two separately in theoretical and empirical implementations. Moreover, opposing aggregate WTP vs individual MWTP's neither alleviates, nor rationalizes the large gaps. Furthermore, the HK valuation is much closer to the natural benchmark given by the Gunpoint Value, i.e. the upper bound on the willingness to pay. All elements point to the strongly diminishing marginal willingness to pay to avoid increases in death risk exposure as the sole remaining explanation.

Seen from that perspective only reaffirms the appropriateness of Schelling (1968)'s warning that the VSL should *not* be interpreted as the value of a given human being. Rather it is a local measure of a rate of substitution between wealth and life. Indeed, the empirical VSL adequately gauges a collective willingness to pay to avoid (attain) a detrimental (beneficial) change in death risk exposure. Moreover, it does adequately measure an individual marginal WTP when the changes are small. Consequently, relying on the VSL appears fully warranted to compute a collective value on small indiscriminate reductions on mortality for which society will ultimately end up paying the costs.

For the same reason that the MRS is a seldom a good proxy for a unitary value for nonlinear indifference curves, the problem lies rather in the linear extrapolation from that local to a holistic measure of life value. When the WTP is concave, the VSL will produce large approximation errors when gauged against more appropriate benchmarks for identified life valuations, such as the HK or the GPV. This extrapolation, to paraphrase Schelling's wording, is *treacherous* and best left to methods that abstract from integrating marginal values and value the whole life instead. For purposes such as wrongful death litigation, curative vs terminal care decisions, the HK and GPV appear the better alternative.

8 Conclusion

Computing the money value of a human being has generated a profound and continued interest, with early records dating back to the late XVIIth century. The two widely-used valuation frameworks have centered on the marginal rate of substitution between the probability of living and wealth (VSL) and on a person's human capital value that is destroyed upon death (HK). Despite pricing a common element, the two life valuations yield vastly divergent measures, with the VSL being 10-20 times higher than the HK. Both the absence of common theoretical underpinnings as well as the very different empirical settings in which the two values are calculated complicate any comparison exercise between the HK and VSL.

We have shown that is nonetheless possible to address both issues by relying on a unique human capital problem to analytically compute and structurally estimate the theoretical VSL and HK values. We have also introduced a third valuation reflecting the maximum amount an agent would be willing to pay to save himself from instantaneous and certain death (GPV) as a useful benchmark. The willingness to pay to avoid changes in death risk, as well as the three closed-form for the life values were estimated jointly using a common structural econometric model and data set. This approach thus provided direct comparability as well as a unique opportunity to identify the role of the preferences, distributional, and technological parameters on life valuations.

Our main findings are twofold. First, we confirmed the relevance of reduced-form estimates with a GPV value of 447 K\$, close to the HK value of 421 K\$, both of which are much lower than the VSL of 8.35 MM\$. Second, we confirmed the standard economic intuition that the willingness to pay to avert death risk is increasing, but strongly concave and finite in mortality exposure. Allowing for a more general model with endogenous sickness and death intensities as well as source-dependent risk aversion only reaffirmed our findings. The large HK-VSL gaps are therefore *not* an artefact of segmented theoretical and empirical concepts.

Two potential explanations justify the wide disparities between the VSL and other measures. First, as famously pointed out by Schelling (1968), the VSL should be interpreted as the *aggregate* willingness to pay for infinitesimal changes in the mortality risk affecting an entire population. Conversely, the Human Capital and the Gunpoint values measure a market- and individual-based willingness to pay to avoid a large change in death risk (i.e. life versus certain death) that affects a single individual. There is therefore no *ex-ante* reason why the Statistical Life and other values should be equal.

However, our theoretical results helped clarify this ambiguity by highlighting the close linkages between the VSL and the other valuations via the willingness to pay. We also accounted for aggregation in formally showing that the empirical VSL is indeed a collective WTP, but that it will *under*-estimate the marginal rate of substitution captured by the theoretical VSL. Moreover, our empirical estimates showed that the extent of this bias is small when population size is large. Equivalently, the aggregate WTP versus individual MWTP explanation for the large VSL does not rationalize the large gaps with the other life values.

Second, we formally showed and empirically verified that the differences are related to the strong curvature and finiteness of the WTP. The theoretical VSL is a linear projection from the marginal willingness to pay, whereas the empirical VSL is a local approximation to that MWTP. When the WTP is strongly concave, both theoretical and empirical VSL will strongly overestimate the limiting willingness to pay that corresponds to the Gunpoint Value. The empirical similarities between the HK and GPV values relate to the close parallels in the measured object. The HK computes the net present value of the foregone dividend stream associated with human capital (i.e. income, minus investment costs). The GPV measures the NPV of the foregone utility stream associated with living. The homogeneity properties entail that the latter is also the NPV of the foregone consumption above minimal subsistence requirements.

We concluded by reiterating Schelling (1968)'s warning that the VSL is not a human value, but should remain employed in instances for which is was specifically designed.

Whereas it is fully adequate for gauging an aggregate willingness to collectively pay for unidentified small reductions in mortality affecting and paid for by large populations, it produces significant errors when linearly integrated to value a given human life. Methods such as the HK or the GPV abstract from integrating marginal values and calculate unit life values instead. These approaches appear better suited when identified life values are required (e.g. in wrongful death litigation, curative vs terminal care decisions).

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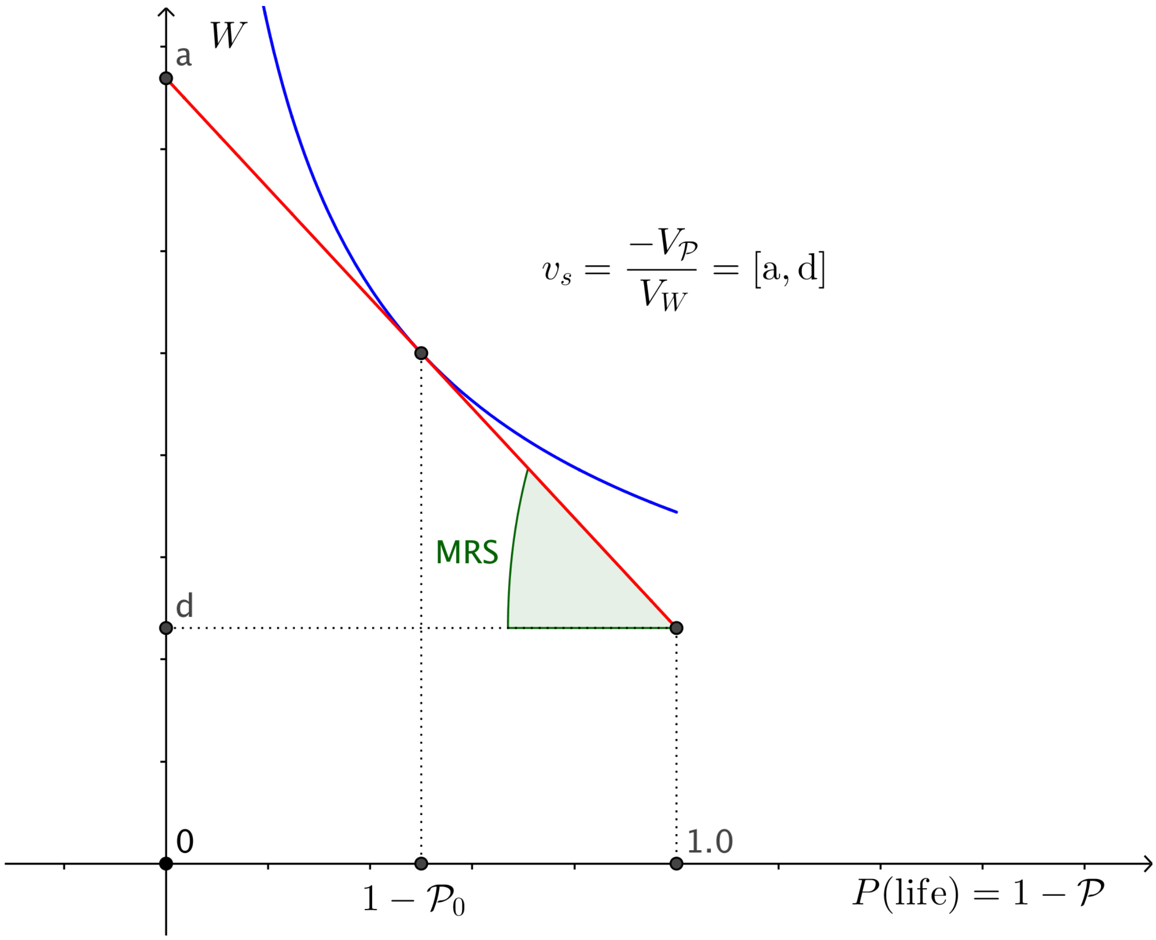
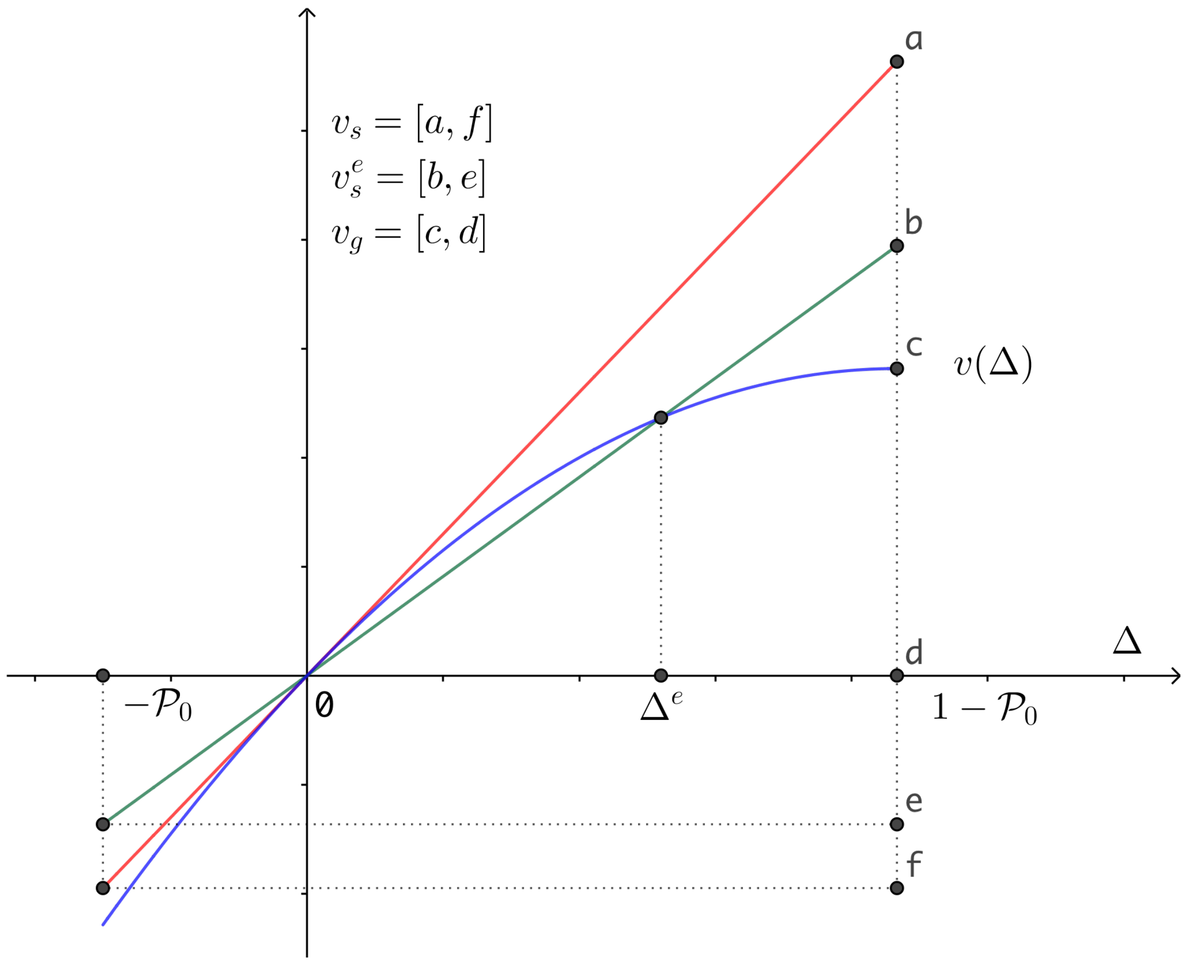
A Figures

Figure 1: Indifference curves, MRS and Value of Statistical Life



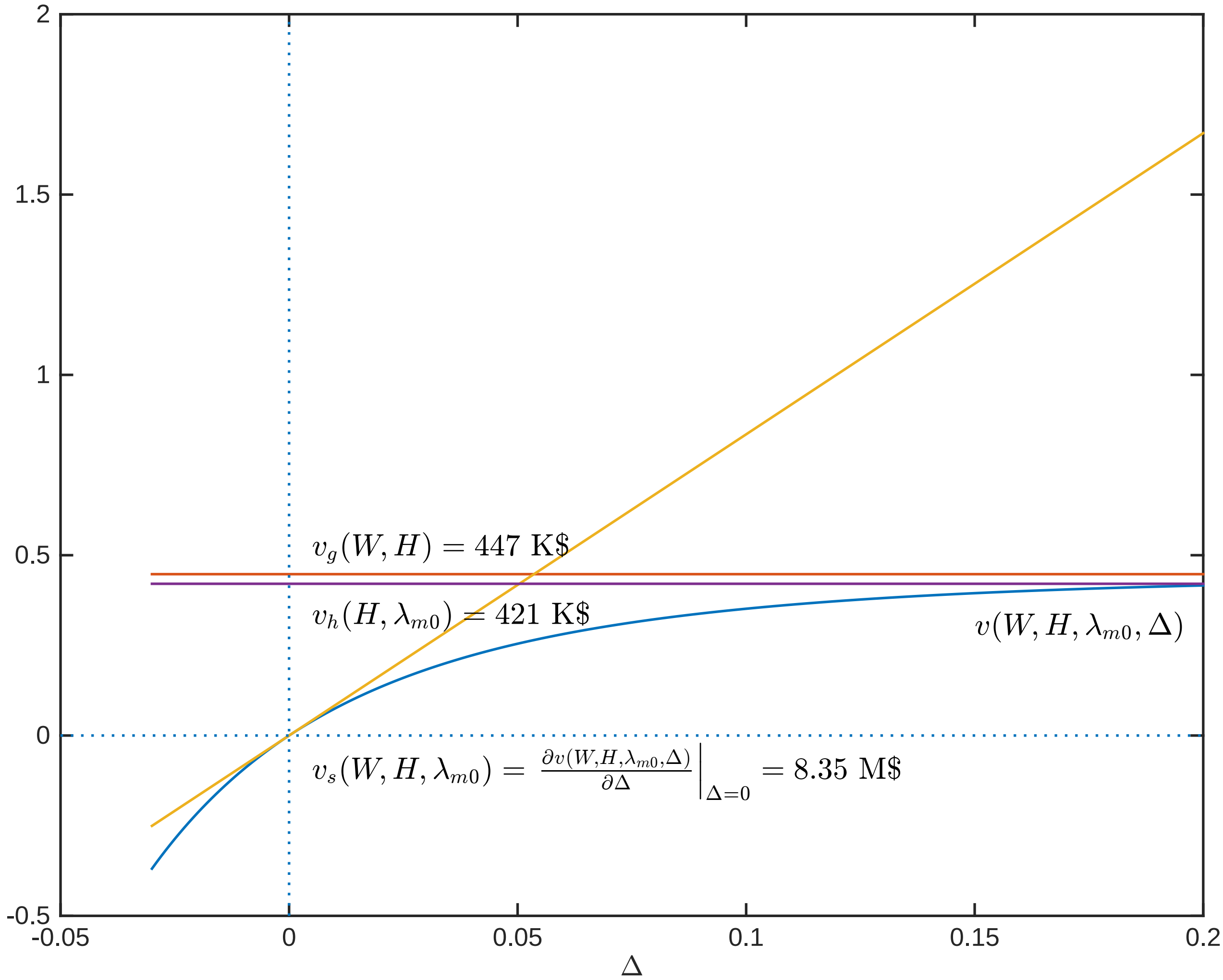
Notes: Reproduced and adapted from Andersson and Treich (2011, Fig. 17.1, p. 398). Indifference curves for indirect utility (1) in blue. v_s: Theoretical Value of Statistical Life in (7a) is the negative of the MRS, i.e. the slope of red tangent equal to distance [a,d].

Figure 2: Willingness to pay and life valuations



Notes: Δ is change in the instantaneous probability of death from base exposure $\mathcal{P}_0$. $v(\Delta)$: Willingness to pay to avoid Δ (in blue). $v_s = v'(0)$: Theoretical Value of Statistical Life in (7b) is slope of red tangent equal to distance [a,f]. $v_s^e = v(\Delta^e)/\Delta^e$: Empirical Value of Statistical Life in (31) is slope of green line and equal to distance [b,e]. $v_g = \sup_\Delta(v)$: Gunpoint Value of Life in (9) is equal to distance [c,d].

Figure 3: Estimated WTP, HK, VSL and GPV Values of life (in M$)



Notes: At estimated parameter values, for mean wealth and health levels. $v(W, H, \lambda_{m0}, \Delta)$ in blue is the willingness to pay to avoid an increase of Δ in exogenous death intensity λ_{m0}; $v_h(H, \lambda_{m0})$ in purple is the Human Capital value of life; $v_g(W, H)$ in red is the Gunpoint value of life; $v_s(W, H, \lambda_{m0})$ is the Value of statistical life and the slope of the yellow tangent evaluated at $\Delta = 0$.

B Proofs

B.1 Theorem 1

The benchmark human capital model of Section 3 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death, and depreciation intensities are constant at $\lambda_{m0}, \lambda_{s0}$ (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e. $\gamma_s = \gamma_m = 0$). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1,Theorem 1) yields the the optimal solution in (18).

■

B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital $P_0(H)$ from

$$
\begin{aligned}
P_0(H) &= E_t \int_0^\infty m_{t,\tau} \left[\beta H^*_\tau - I^*_\tau\right] \mathrm{d}\tau, \\
&= BH.
\end{aligned}
$$

Straightforward calculations adapt this result to a stochastic horizon T^m, and include the fixed income component y in income (13).

■

B.3 Proposition 2

Combining the Hicksian EV (5) with the indirect utility (17) and the net total wealth in (20) reveals that the WTP v solves:

$$
\begin{aligned}
\Theta(\lambda^*_{m0}) N_0(W, H) &= \Theta(\lambda_{m0}) N_0(W - v, H) \\
&= \Theta(\lambda_{m0}) \left[N_0(W, H) - v\right]
\end{aligned}
$$

where we have set $\lambda^*_{m0} = \lambda_{m0} + \Delta$, and using the linearity of the welfare function with respect to wealth. The WTP $v = v(W, H, \lambda_{m0}, \Delta)$ is solved directly as in (27).

Next, by the properties of the marginal value of net total wealth, $\Theta(\lambda^*_{m0})$ in (24) is monotone decreasing and convex in Δ. It follows directly that the WTP

$$v(W, H, \lambda_{m0}, \Delta) = \left[1 - \frac{\Theta(\lambda^*_{m0})}{\Theta(\lambda_{m0})}\right] N_0(W, H)$$

is monotone increasing and concave in Δ.

At low EIS, the MPC in (23) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\bar{\lambda}_{m0} = \left(\frac{\varepsilon}{1-\varepsilon}\right)\rho + \left(r + \frac{\theta^2}{2\gamma}\right).$$

It follows that the transversality condition (16a) is violated and the marginal value $\Theta(\lambda^*_{m0})$, cannot be calculated for $\lambda_{m0} \geq \bar{\lambda}_{m0}$. The supremum of the WTP is then $v(W, H, \lambda_{m0}, \bar{\Delta}) = N_0(W, H)$. For $\varepsilon > 1$, the MPC is monotone increasing, and transversality is always verified. Consequently, the WTP is well-defined over the domain $\Delta \geq -\lambda_{m0}$. It follows that:

$$\lim_{\Delta \to \infty} \Theta(\lambda_{m0} + \Delta) = 0$$
$$\lim_{\Delta \to \infty} V(W, H, \lambda_{m0}, \Delta) = N_0(W, H)$$

i.e. the willingness to pay asymptotically converges to net total wealth as stated in (29).

■

B.4 Proposition 3

By the VSL definition (7a), and the properties of the Poisson death process (11):

$$v_s = \left.\frac{-V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})}\right|_{\mathcal{P}=\mathcal{P}_0} = \frac{-V_{\lambda_{m0}}(W, H, \lambda_{m0})}{V_W(W, H, \lambda_{m0})}$$

From the properties of the welfare function (17), we have that $V_{\lambda_{m0}} = \Theta'(\lambda_{m0}) N_0(W, H)$, whereas $V_W = \Theta(\lambda_{m0})$. Substituting yields the VSL in (30). ■

B.5 Lemma 1

A higher likelihood of death of Δ over a time interval of $s \in [0, T]$ corresponds to an increase in the endowed intensity to $\lambda^*_{m0}(\Delta) > \lambda_{m0}$:

$$\Delta = \Pr\left[T_m \leq T \mid \lambda^*_{m0}\right] - \Pr\left[T_m \leq T \mid \lambda_{m0}\right],$$

Observing that:

$$\Pr\left[T_m \leq T \mid \lambda\right] = 1 - E\left[e^{-\int_0^T \lambda \mathrm{d}s}\right] = 1 - e^{-T\lambda},$$

and substituting solves for λ^*_{m0} reveals that the latter as stated in (32). ■

B.6 Proposition 4

Combining the Hicksian EV (9) with the indirect utility (17) and the net total wealth in (20) reveals that the WTP v solves:

$$\begin{aligned} V^m \equiv 0 &= \Theta(\lambda_{m0}) N_0(W - v_g, H) \\ &= \Theta(\lambda_{m0}) \left[N_0(W, H) - v_g\right] \end{aligned}$$

Solving for v_g reveals that it is as stated in (33). Because net total wealth is independent of ε, so is the Gunpoint Value. ■

C Tables

C.1 Data

Table 1: PSID data statistics

	Model	Mean	Std. dev.	Min	Max
Health	H	2,85	0,80	1	4
Wealth	W	38 685	122 024	0	1 430 000
Consumption	c	9 835	11 799	1,05	335 781
Risky holdings	π	20 636	81 741	0	1 367 500
Insurance	x	247	718	0	17 754
Health investment	I	721	2 586	0	107 438
Income	Y	21 838	37 063	0	1 597 869
Age	t	45	16	16	100

Notes: Statistics in 2013 $ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good) and 4.0 (Excellent).

Table 2: PSID data statistics (cont'd)

	Wealth quintiles				
Health	1	2	3	4	5
	a. Wealth W_j ($)				
Poor	0	70	1 139	10 357	136 209
Fair	0	71	1 109	10 861	188 044
Good	0	86	1 214	11 207	160 925
Very Good	0	90	1 282	11 654	178 580
Excellent	0	88	1 315	11 974	214 106
	b. Consumption c_j ($)				
Poor	3 943	3 859	6 216	10 473	18 226
Fair	4 724	5 702	9 256	13 491	15 610
Good	6 459	5 742	9 205	12 457	17 109
Very Good	5 684	5 582	9 442	11 812	15 702
Excellent	6 177	5 616	10 117	11 575	17 465
	c. Stocks π_j ($)				
Poor	0	0	83	1 402	39 752
Fair	0	1	107	2 811	100 461
Good	0	4	143	3 299	82 499
Very Good	0	3	110	3 673	101 223
Excellent	0	3	116	3 627	125 934
	d. Insurance x_j ($)				
Poor	50	142	123	304	230
Fair	83	134	162	320	537
Good	132	104	268	335	512
Very Good	106	64	209	316	483
Excellent	108	87	240	314	455
	e. Investment I_j ($)				
Poor	783	792	852	2 021	4 447
Fair	538	762	777	1 711	2 969
Good	347	482	623	1 219	1 352
Very Good	250	318	422	639	1 070
Excellent	360	327	488	532	861

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth and per health status

C.2 Benchmark model

Table 3: Estimated and calibrated structural parameter values, benchmark model

Parameter	Value	Parameter	Value
a. Law of motion health (10)			
α	0.6843	δ	0.0125
	(0.3720)		(0.0060)
ϕ	0.0136^c		
b. Sickness and death intensities			
λ_{s0}	0.0347	λ_{m0}	0.0283
	(0.0108)		(0.0089)
c. Wealth and income (12), (13)			
y	0.0120	β	0.0092
	(0.0049)		(0.0044)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
d. Preferences (14), (15)			
γ	2.8953	ε	1.2416
	(1.4497)		(0.3724)
a^c	0.0140	ρ^c	0.0500

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (35), estimated by ML, subject to the regularity conditions (16).

Table 4: Estimated Values of Life (in $)

Health level	Wealth quintile				
	1	2	3	4	5
			a. Human Capital $v_h(W, H, \lambda_{m0})$ in (25)		
Poor			249 532		
Fair			318 865		
Good			388 198		
Very Good			457 531		
Excellent			526 864		
All					
- mean			420 729		
- median			457 731		
			b. Value of Statistical Life $v_s(W, H, \lambda_{m0})$ in (30)		
Poor	2 167 573	2 168 877	2 188 829	2 360 907	4 710 118
Fair	4 379 551	4 380 874	4 400 253	4 582 287	7 889 684
Good	6 591 529	6 593 136	6 614 190	6 800 733	9 595 444
Very Good	8 803 507	8 805 188	8 827 429	9 021 052	12 136 981
Excellent	11 015 485	11 017 133	11 040 023	11 238 999	15 012 108
All					
- mean			8 351 519		
- median			8 803 507		
			c. Gunpoint Value $v_g(W, H)$ in (33)		
Poor	116 121	116 191	117 259	126 478	252 329
Fair	234 620	234 691	235 729	245 481	422 664
Good	353 120	353 206	354 334	364 327	514 045
Very Good	471 619	471 709	472 901	483 274	650 199
Excellent	590 119	590 207	591 433	602 093	804 225
All					
- mean			447 405		
- median			471 619		

Notes: Averages of individual values in the PSID sample, computed at estimated parameter values, multiplied by 1 M$ to correct for scaling used in estimation.

D Semi-structural estimation

Reduced-form estimation The reduced-form (RF) analog to the econometric model (35) can be written as:

$$\mathbf{Y}_j = \mathbf{B}\mathbf{X}_j + \mathbf{e}_j$$

with $\mathbf{Y}_j = [c_j, \pi_j, x_j, I_j, Y_j]'$ and $\mathbf{X}_j = [1, W_j, H_j]'$ and identifies the 5×3 parameters $\mathbf{B}$ via a multivariate estimation, with positive sign constraints imposed.

Recursive identification We retain the calibration procedure for $\rho, \sigma_S, r, \theta$ and identify the key parameters from the estimated RF parameters $\mathbf{B}$ through the following sequence:

1. Base revenue and health sensitivity of income:

$$y = B_0^Y, \quad \beta = B_H^Y$$

2. Tobin's-Q:

$$B = \frac{B_H^c}{B_W^c} = \frac{B_H^\pi}{B_W^\pi}$$

3. Health production and morbidity risk exposure:

$$0 = \left(\frac{\log(\alpha) + \log(B)}{1-\alpha}\right) - \log(B_H^I)$$

non-linearly solves for α, whereas

$$\phi = \frac{B_H^x}{B}$$
$$\tilde{\delta} = (\delta + \lambda_{s0}\phi) = \frac{\beta}{B} - r + (1-\alpha)(\alpha B)^{\frac{\alpha}{1-\alpha}}$$

4. Death intensity, given Life Tables (Arias et al., 2017),

$$\lambda_{m0} = 1/\ell(\bar{t})$$

where ℓ is the remaining life expectancy evaluated at at mean age.

5. Marginal propensity to consume:

$$A = B_W^c$$

6. Minimal subsistence consumption:

$$a = \left(B_0^c - \frac{Ay}{r} \right) \left(1 - \frac{A}{r} \right)^{-1}$$

7. Risk aversion:

$$\gamma = \frac{\theta}{B_W^\pi \sigma_S} = \frac{\theta}{B_H^\pi \sigma_S B} = \frac{\theta}{B_0^\pi \sigma_S} \left(\frac{y-a}{r} \right)$$

8. Elasticity of inter-temporal substitution:

$$\varepsilon = \frac{A - (r + \lambda_{m0} + 0.5\theta^2/\gamma)}{\rho - (r + \lambda_{m0} + 0.5\theta^2/\gamma)}$$

This procedure identifies $(y, \beta, B, \alpha, \phi, \tilde{\delta}, \lambda_{m0}, A, a, \gamma, \varepsilon)$, from which the functions $P_0(H)$ in (19), $N_0(W, H)$ in (20), the marginal value of net total wealth $\Theta(\lambda_{m0}), \Theta(\lambda_{m0}^*)$, and $\Theta'(\lambda_{m0})$ in (22), as well as C_0, C_1 in (26) can be recovered to compute the willingness to pay v and life values v_h, v_s, v_g. Note that this semi-structural identification does not fully exploit all the cross-equations restrictions that are explicitly incorporated in our full-structural approach.

E Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size) and scaled by 10^{-6} for the estimation. The agents' wealth and health which are constructed as follows:

Health H_j Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

Wealth W_j Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance and are constructed as follows:

Portfolio π_j Money value of financial wealth held in risky assets.

Consumption c_j Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

Health expenditures I_j Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

Health insurance x_j Spending on health insurance premium.