Self-Inflicted Unemployment Scarring and Stigma*

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Abstract

Long-term scars of unemployment include higher ex-post displacement and income losses, as well as lower re-employment for longer unemployment spells (stigma). Human capital explanations assume it increases wages and re-employment and decreases displacement risk, but rely on tenure-based and/or employer decided acquisition only. We consider an alternative where investment decisions are made by workers, allowing for displacement and re-employment risks hedging and assuming that the investment technology is independent of the employment status. We calculate analytically the joint optimal investment by the employed and the unemployed. We identify two dynamically stable steady-state values with a lower one for the unemployed generating cyclical dynamics whereby human capital optimally falls during unemployment spells and increases again upon re-employment. It follows that scarring and stigma are endogenously generated as a by-product of decisions made by agents and are therefore *self-inflicted*. We close the analysis by a counterfactual exercise allowing to gauge and confirm the importance of employment risks hedging in total demand for human capital and that of moral hazard issues in the design of UIB programs.

Keywords— Human capital; unemployment duration dependence; unemployment stigma and scarring; work displacement; re-employment probability.

JEL classification— I26, J24, J64, J65

1 Introduction

In addition to contemporaneous income losses,[1] unemployment imposes long-term scars on agents.[2] Indeed, a well-documented stylized fact is that the unemployment history is the best predictor of future spells (e.g. Arulampalam et al., 2001, p. F577). More precisely, unemployment spells in a worker's history lower his probability of being employed at any point in time and also impose wage cuts upon re-employment, whereas displacement rates are higher for re-employed, compared to long-tenured workers (seniority rules, last-in-first-out, LIFO). Moreover, the duration of the unemployment spell has a negative impact on the re-employment probabilities (unemployment stigma).[3] Although persistent, these unemployment scars and stigma do not appear to be permanent.[4]

Human capital (HK) is often invoked as explanation for unemployment scarring and stigma.[5] This approach relies on two hypotheses. First, employers value human capital as reflected in higher wages, higher re-employment and lower displacement probabilities and use the workers' history as a screening mechanism to identify potential depletion of human capital. Second, human capital acquisition is primarily decided by employers, and/or mechanically provided through employment tenure. Unemployed workers face more rapid depreciation and/or are unable to invest and thus see their human capital mechanically deplete over their entire unemployment spell.[6] This human capital depreciation is sanctioned by employers, leading to lower wages and re-employment (scarring)

[1]The U.S. weighted average UI replacement rate in 2010-2011 was 0.41 and varied between 0.30 (AK, LA) and more than 0.49 (AZ, HI, RI). Source: U.S. Department of Labor.

[2]See also Carrington and Fallick (2014); Huckfeldt (2014); Fang and Silos (2012); Krebs (2007); Farber (2005); Rogerson and Schindler (2002) for other evidence and discussion on unemployment scarring.

[3]See Eriksson and Rooth (2014); Kroft et al. (2013) for discussion.

[4]For example, Eliason and Storrie (2006) rely on Swedish data to identify higher income and unemployment sensitivity up to 12 years after displacement. Using Norwegian data, Nilsen and Reiso (2011) also find evidence of lower labor market attachment up to ten years after being unemployed, although that effects fades with the passage of time.

[5]Examples from the Search and Matching (S&M) literature include Shimer (2005); Moscarini and Postel-Vinay (2013) for models with stochastic TFP shocks and Bagger et al. (2014) with stochastic idiosyncratic shocks. Some S&M emphasis on long-term unemployment scars can be found in Stern (1990) (scarring) and Kroft et al. (2014, 2013); Fujita and Moscarini (2013) (stigma).

[6]Hence, evidence provided by Quintini and Venn (2013) shows a decline in mathematics, verbal and cognitive skills among displaced workers with downgraded re-employment.

and higher displacement rates upon re-employment (LIFO), as well as unemployment duration dependence (stigma).

The human capital rationale, although heuristically appealing, hardly relates to traditional models of investment and capital theory. Indeed, investment is either mechanic, or decided by others than the primary holders of human capital. It is also subject to arbitrary assumptions regarding different investment technologies and depreciation rates across employment statuses. Furthermore, standard models of human capital and labor market outcomes are unhelpful as they abstract from the employment risk diversification motives in order to focus exclusively on the higher wage benefits of investing one one's own capital.[7]

This paper bridges this gap by readdressing scarring and stigma through the lens of investment theory, to which we append employment risks diversification. We start by maintaining the first assumption that human capital leads to higher wages and recall rates and lower displacement. The scope is therefore partial equilibrium where – unlike S&M – labor demand is taken as given and not solved endogenously. We replace the second hypothesis of mechanic/external acquisition by reinstating the investment decision with employed and unemployed agents only. Hence capital acquisition is status-dependent, but neither mechanic, nor chosen by others and thus fully consistent with standard investment and capital theory. We also append a realistic model of UI benefits. In particular, an incomplete replacement is applied to the last employment income (and corresponding human capital level), which is locked-in for the entire duration of the unemployment spell. Hence UIB are constant whether or not human capital is adjusted during unemployment. Finally, we allow for (but do not impose) differences in human capital technology across employment statuses, as well as for firm- or sector-specific capital losses incurred upon occurrence of displacement. Abstracting from both in our baseline setup allows us to emphasize scarring dynamics resulting from optimal investment policies, instead of from

[7] See Ben-Porath (1967); Heckman (1976); Kredler (2014) for human capital models with higher wages motives and Rogerson and Schindler (2002); Krebs (2003); Huggett et al. (2011); Cervellati and Sunde (2013) for models that append undiversifiable income risks.

arbitrary parametric restrictions. We later reinstate status-dependent technology and specific capital to gauge their respective contributions.

Solving this dynamic model is particularly challenging for two reasons. First, the two value functions (i.e. for employed and unemployed) are intertwined with one another, as the returns to investing when employed depend on what is selected when unemployed and vice versa. Second, both the displacement and re-employment arrival rates are endogenous functions of the human capital decided by the agent. Unemployment risk exposure is thus (partially) diversifiable, which enriches the motives for investing, but significantly complicates the model's solution. We circumvent this problem through two-step expansion methods developed in Hugonnier, Pelgrin and St-Amour (2013). First, we solve analytically a restricted version where the arrival rates governing displacement and re-employment are exogenous. We then do a first-order expansion on this solution by focusing on the key parameter governing the endogeneity of the arrival rates.

Relying on Displaced Workers Survey (DWS) data, we calibrate and simulate these analytical solutions to address three questions. First, we ask whether this model can generate endogenous unemployment scarring and stigma. At first glance, it is not obvious that it should. Scarring and stigma both impose significant costs to agents, yet are primarily related to displacement and reemployment. Since the exposure to these risks can be hedged through human capital whose acquisition is decided upon by agents, workers should seek to avoid scarring and stigma by investing more when employed (to reduce displacement) and more when unemployed (to accelerate reemployment and limit duration dependence). We nonetheless confirm that the resulting optimal human capital dynamics are consistent with both scarring and stigma. This finding rests on two main results. First, investment by the unemployed is positive, but always lower than for the employed. Moreover, two (i.e one per employment status) steady-state levels of human capital exist, are dynamically stable and always lower for the unemployed. Combining the two entails cyclical optimal dynamics. Upon unemployment, human capital optimally falls towards the lower unemployed steady state and increases towards the higher employed steady state upon re-employment. Since re-employment (resp. displacement) and wages are increasing (resp. decreasing) functions, unemployment spells thus internally induce lower recall rates and lower wages (scarring) and higher displacement upon re-employment (LIFO). Moreover, since human capital falls continuously before reaching the steady state, duration dependence obtains naturally. Importantly, to the extent that scarring and stigma depend on displacement and re-employment events, that the arrival rates of the latter are human capital-dependent and that the investment in the capital is decided by workers, any induced scarring and stigma arise as an optimal dynamic strategy and must therefore be *self-inflicted*.

Second, since our model innovates from standard human capital theory in that dimension, we gauge the importance of displacement and re-employment risks hedging in total demand for human capital. By removing endogenous exposure and adjusting the parameters to maintain the mean displacement/reemployment rates constant, we show that the marginal effects of diversification strongly dominate any higher wage considerations in investment decisions. Third, we also measure the policy effects of UI generosity and of base (i.e. human capital independent) income on total investment. Standard search models associate more generous programs with reduced search efforts and longer unemployment spells (e.g. Chetty, 2008). We offer an alternative moral hazard explanation whereby generous UIB reduces the motives for investing, lowering the steady-state values and increasing unemployment through higher displacement and lower re-employment. Similar effects obtain when base income is lowered.

This paper contributes to discussions of human capital in labor market dynamics. We highlight the importance of employment risks hedging as additional motivation for investing in one's own human capital. This complements the traditional higher wages argument for more investment in training, education, ... Moreover, these employment risks are widely assumed to be the result of systemic macro shocks and cannot be insured against through market instruments. This reasoning justifies both active macro stabilization and UIB policies. We show instead that displacement and re-employment risks *can* be hedged through agents' decisions and that long-term scars can obtain optimally through investment by workers. Finally, we highlight the strong moral hazard risks in making the UIB programs more generous. This results in lowering the incentives for investing, with ensuing higher displacement and lower wages and re-employment.

2 DWS evidence on employment risks and human capital

We resort to Displaced Workers' Survey (DWS) data to provide *prima facie* evidence of the scarring effects of unemployment spells on current employment status and wages. Towards that effect, we construct an unbalanced panel of all bi-annual waves between years 1994 and 2010. In addition to respondents' data on schooling, gender, age, current wage, . . . , DWS provides detailed information on whether the agent has been displaced over the last three years (dw) and if yes, on last job tenure (ljten), last job wage (ljwage), on whether he has worked in the interim (worked) as well as on the number of weeks without work (wkswo). This information is useful to establish scarring and stigma patterns.

Table 1 highlights the scarring effects of past unemployment spells by contrasting the current employment status and hourly wages for previously non-displaced and displaced respondents. To limit the effects of long-term unemployment, we restrict our sample of displaced workers to those having worked since the time of displacement. The results indicate that having been displaced results in a statistically significant 7.8% higher level of unemployment. The wage cut of re-employed displaced workers is also significant, representing on average 8.8% over our 16 years sample.

Table 2 presents descriptive evidence of the hedging capacity of human capital against unemployment risks, as well as of the positive human capital gradient with respect to wages. When capital is proxied by the education level, the data points towards lower unemployment and displacement risks, as well as higher re-employment probabilities for

Table 1: Unemployment scarring

Displaced	Observations	Employed	Unemployed	Current job hr. wage
No	494,760	95.90%	4.10%	6.94$
Yes	44,598	88.05%	11.95%	6.35$
All	539,538	95.24%	4.76%	6.83$

Notes: Displaced Workers Survey. Unbalanced panel sample, bi-annual data, waves 1994–2010. Current status of workers remaining in the labor force and having worked since displacement for displaced workers. Displaced: over the last three years at time of interview.

the better educated. Unsurprisingly, higher levels of education are also associated with higher levels of current wages.

Table 2: Employment risks and hourly wages by education levels

Level	Unemployment	Displacement	Re-employment	Current hr. wage
Less than HS	13.8%	9.7%	49.4%	4.64$
HS	7.5%	5.0%	61.7%	6.25$
Some college	5.4%	3.5%	65.9%	6.75$
College	3.3%	1.9%	69.4%	9.13$
Advanced	2.1%	1.2%	75.2%	12.12$
All	6.3%	4.0%	63.4%	6.63$

Notes: Displaced Workers Survey. Pooled sample, bi-annual data, waves 1994–2010. Displacement: Currently unemployed, conditional upon being employed in previous wave. Re-employment: Currently employed, conditional upon being unemployed in previous wave. Current wages are real hourly wages. Mean of all waves.

The scarring and hedging evidence is corroborated in Table 3 which reports unbalanced panel regression outputs with year random effects. In columns (1) and (2), the re-employment (displacement) is also found to be increasing (decreasing) in the education level. If we measure human capital by job tenure instead (ljten), columns (1) and (2) again confirm that workers with more experience are re-employed at a faster rate and less likely to be displaced. Evidence of duration dependence (stigma) is also apparent whereby the number of weeks without work (wkswo) has a depressing effect on re-employment probabilities and an increasing effect on displacement risk. The latter can also be interpreted as indication of "Last-in, first-out" practices, whereby by previously displaced workers with long unemployment spells are more likely to be displaced again than workers with uninterrupted tenure.

In column (3), we regress the current wages of previously displaced workers that have been re-employed, controlling for past wages, along with other covariates. The GLS estimates point again to a higher wage for the better educated, whereas long tenured workers, as well as workers with long spells of unemployment face significant wage cuts upon re-employment.

Table 3: Regression output

	Dependent variable		
	Re-employment	Displacement	Current wage
	(1)	(2)	(3)
educ	0.0953	−0.1294	0.1918
	(3.93)	(−4.00)	(3.34)
ljten	0.0130	−0.1095	−0.0370
	(3.01)	(−9.39)	(−3.90)
wkswo	−0.0039	0.0042	−0.0154
	(−3.67)	(3.08)	(−5.89)
Estimator	Probit	Probit	GLS
Covariates	yes	yes	yes
Random effects	yes	yes	yes
Obs	9,509	4,176	1,968

Notes: T-statistics in parentheses. Sources: Displaced Workers Survey. Unbalanced panel sample, bi-annual data, waves 1994–2010. Re-employment: Currently employed, conditional upon being unemployed in previous wave. Displacement: Currently unemployed, conditional upon having worked since last lost job. Income scarring: Percentage drop in income over previous income if re-employed following unemployment spell. Main regressors are the education level (educ), the last job tenure (ljten), as well as the number of weeks without work (wkswo). Other covariates include race, gender, age, union, last job wage. Random effects computed at the household id levels.

Overall, we conclude that the scarring costs associated with unemployment are significant and that duration dependence is apparent. Fortunately, whether measured by education or by job tenure, human capital appears to be a significant hedge against these costs. The next section describes a theoretical model incorporating these elements. Consistent with Tables 2, and 3, we assume that labor demand puts value on acquired human capital with higher re-employment, lower displacement probabilities, as well as higher wages. Taking these labor market characteristics as given, we let agents select their investment in human capital and verify whether the resulting dynamics are consistent with scarring and stigma costs identified in Tables 1, and 3.

3 Model

Consider an economy where agents are characterized by two sources of heterogeneity: Human capital $H \in \mathbb{R}_+$ and labor market status $i = e, u$ (i.e. employed, unemployed).[8] The former is defined as the publicly measurable set of skills accumulated by workers over their lifetime. We assume that investment in human capital is decided by agents and takes place both within (e.g. through experience or voluntary training) and outside (e.g. through formal and informal education) employment.[9] The pecuniary (e.g. tuition fees, books, software, ...) and indirect (e.g. opportunity cost of time and effort spent acquiring skills) investment costs are borne by individuals.[10] Human capital provides no direct utility to the agent, but is valued by employers, as reflected in more favorable conditions with respect to wages, firing and hiring for those agents with higher skill levels. Although our perspective is on general human capital, we allow for part of that capital to be immediately depreciated upon a displacement event in order to reflect firm- or industry-specific components that have limited value to outside employers.

[8]We abstract from additional sources of heterogeneity, such as differences in family background, preferences, or ability that are discussed in Heckman (2008); Polachek et al. (2013) in the context of HK models.

[9]See Kräkel (2016); Flinn et al. (2017) for on-the-job training decisions by workers and Jacobson et al. (2005b,a); Heckman and Smith (2004); Heckman et al. (1999) for participation in social training programs.

[10]See Becker (1962, 1993); Acemoglu and Pischke (1999) for the relevance of cost-sharing with workers in general and specific human capital contexts.

Labor market statuses are stochastic and the transition matrix between employment and unemployment spells is agent-specific, in that it depends on the accumulated level of human capital. Employed agents receive an income that is continuously adjusted to reflect changes in human capital. Conversely, unemployed agents receive unemployment benefits that are set at a fraction of the last employment revenue; the benefits are constant for the duration of the unemployment spell. Agents thus select optimal investment paths taking into account its joint benefits in terms of income premia and employment risk diversification.

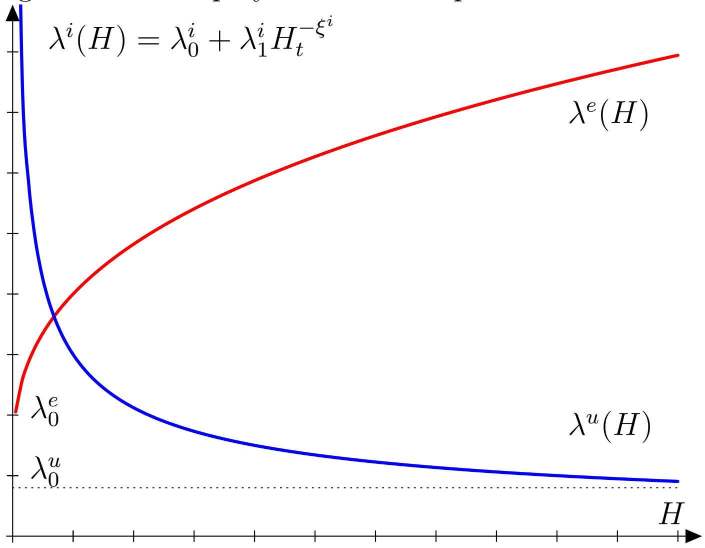
Employment statuses A person's time-t labor market status $i_t \in \{e, u\}$ follows a Poisson stochastic process with each agent being either employed, or unemployed. Importantly, the arrival intensity is assumed to be dependent of the human capital level. More specifically, let T^i, be the random time of job displacement ($i_t = u$) from current employment, or re-employment ($i_t = e$) from current unemployment, with Poisson arrival intensities $\lambda^i : \mathbb{R}_+ \to \mathbb{R}_{++}$ defined as:

$$\begin{aligned} \lambda^i(H_t) &= \lim_{\tau \to 0} \frac{1}{\tau} \Pr\left[t < T^i < t + \tau \mid H_t\right], \quad i \in \{e, u\} \\ &= \lambda_0^i + \lambda_1^i H_t^{-\xi^i}, \quad \lambda_0^i, \lambda_1^i \geq 0; \quad \xi^i > -1. \end{aligned} \tag{1}$$

Hence, imposing $\xi^u > 0$ in (1) entails decreasing and convex work displacement intensities, whereas $\xi^e \in (-1, 0)$ yields increasing concave re-employment intensities.

As shown in Figure 1, an agent can thus reduce his exposure to conditional employment risks by investing in his human capital which decreases his displacement intensity $\lambda^u(H)$, as well as increases his re-employment intensity $\lambda^e(H)$. The parameters λ_1^i capture the endogeneity of the employment risks exposure and play a key role in the solution method discussed below. The parameters ξ^i govern the extent of diminishing returns to investment.

Figure 1: Re-employment and Displacement Intensities



Notes: $\lambda^e(H)$: re-employment intensity. $\lambda^u(H)$: displacement intensity.

Income process The income process $Y_t = Y(H_t, \overline{H}, i_t) \in \mathbb{R}^+$ is status- and human-capital-dependent:

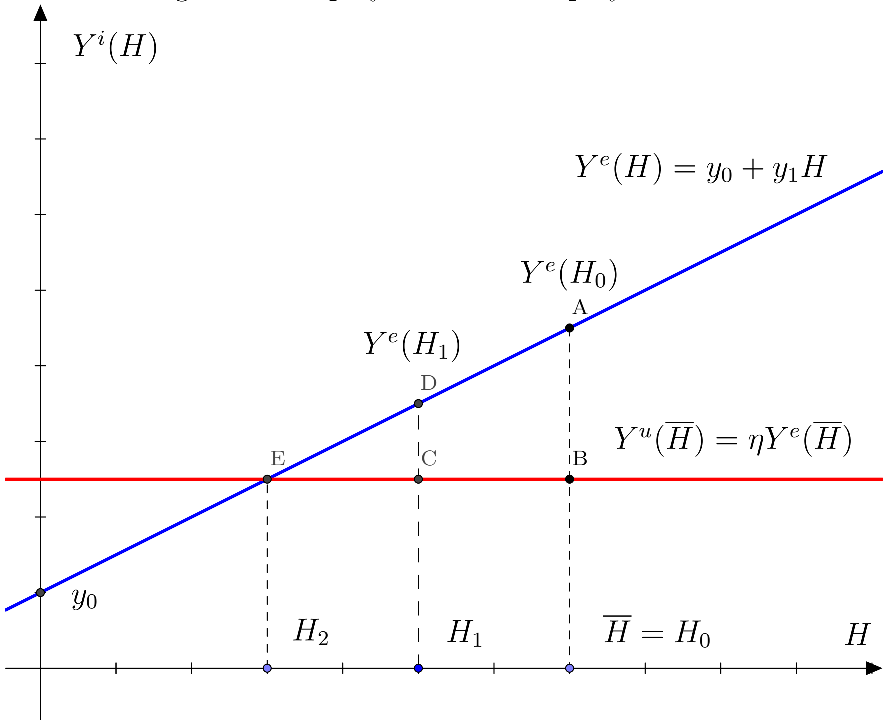
$$Y(H_t, \overline{H}, e) = Y^e(H_t) = y_0 + y_1 H_t, \tag{2a}$$

$$Y(H_t, \overline{H}, u) = Y^u(\overline{H}) = \eta Y^e(\overline{H}), \tag{2b}$$

where $\eta \in (0, 1)$ is the UI replacement rate and where $\overline{H}$ is the last measurable human capital level before the unemployment spell begins (i.e. *lock-in* capital).

Figure 2 also shows that employment income $Y^e(H)$ increases in human capital which can continuously be altered through the agent's investment decisions. Upon job loss at human capital level H_0, unemployment income at point B is a fraction η of the last employment income $Y^u(\overline{H}) = \eta Y^e(H_0)$ and remains fixed throughout the duration of the unemployment spell. For example, if human capital declines to H_1 during unemployment, UI income remains constant, whereas the income upon re-employment income at point D is lower than previously, $Y^e(H_1) < Y^e(H_0)$. Consistent with standard UI policies, investment decisions during the unemployment spell thus affect the displacement and

Figure 2: Employed and Unemployed Income



re-employment probabilities, as well as the re-employment wage, but not the UI benefits (see St-Amour, 2015, for alternative UIB with continuous adjustments).

Note further that the income loss (resp. gain) associated with displacement (resp. re-employment):

$$\begin{aligned} \Delta Y(H, \overline{H}) &= Y^e(H) - Y^u(\overline{H}) \\ &= (1-\eta)y_0 + y_1(H - \eta\overline{H}) \end{aligned} \tag{3}$$

is an increasing function of H and can become negative if human capital depreciates sufficiently during the unemployment spell, i.e. for $H < H_2$ in Figure 2. Indeed, beyond point E, UIB benefits are more generous than what would be earned upon re-employment, thereby lowering incentives to invest in order to augment re-employment probability.

Human capital dynamics The law of motion for the agent's human capitals, $\mathrm{d}H_t = \mathrm{d}H_t(I_t, H_t, i_t)$, is status-dependent and is given by:

$$\mathrm{d}H_t = -\delta^i H_t \mathrm{d}t + P^i I_t^{\alpha} H_t^{1-\alpha} \mathrm{d}t, \quad \alpha, \delta^i \in (0,1) \tag{4}$$

The accumulation process (4) is standard in the HK literature, (e.g. Ben-Porath, 1967; Heckman, 1976; Huggett et al., 2006; Kredler, 2014) and captures continuous, as opposed to period-specific (e.g. young age only) investment I_t. The Cobb-Douglas gross investment function $P^i I_t^{\alpha} H_t^{1-\alpha} \mathrm{d}t$ is monotone increasing and concave in its arguments. Total factor productivity is denoted P^i, whereas depreciation δ^i can be interpreted as technological obsolescence of acquired skills. Differences in productivity and depreciation capture changes in the returns to investment depending on the employment status (e.g faster depreciation and/or lower productivity for the unemployed).

In addition to continuous adjustments through (4), we assume that a share $1 - \phi \in (0, 1)$ of the agent's human capital is lost upon occurrence of unemployment. More precisely, a newly displaced agent's capital H_t is only worth ϕH_t to prospective employers for income and reemployment intensity purposes. This non-stochastic jump in human capital captures firm- or industry-specific capital that is foregone when employment is terminated. Both the effects on displacement/re-employment and on firm-specific capital loss are fully internalized in the agent's investment decisions, as shown next.

Preferences All agents are infinitely-lived, risk-neutral and select dynamic investment in human capital I_t to maximize the expected discounted (at rate ρ) value of net income flow, subject to the dynamics for human capital, the distributional assumptions and income function. More specifically, the value function can be written as:

$$V(H_0, \overline{H}, i_0) = \sup_I \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} \left[Y(H_t, \overline{H}, i_t) - I_t \right] \mathrm{d}t \geq 0, \tag{5}$$

subject to the intensities (1), the income rate (2) and the human capital law of motion (4). Under risk-neutrality, observe that negative net income $Y_t - I_t < 0$ always remains feasible and can be achieved by implicit borrowing (at rate $r = \rho$), as long as the expected net present value $V(H_0, \overline{H}, i_0)$ remains non-negative.[11]

Letting $V^e(H), V^u(H, \overline{H})$ denote the pair of value functions and invoking the Law of Iterated Expectations allows the agent's problem (5) to be written as a joint optimization system:

$$V^e(H_0) = \sup_I \int_0^\infty e^{-\int_0^t (\rho + \lambda^u(H_s))\mathrm{d}s} \left[Y^e(H_t) - I_t + \lambda^u(H_t) V^u(\phi H_t, H_t)\right] \mathrm{d}t, \quad (6a)$$

$$V^u(H_0, \overline{H}) = \sup_I \int_0^\infty e^{-\int_0^t (\rho + \lambda^e(H_s))\mathrm{d}s} \left[Y^u(\overline{H}) - I_t + \lambda^e(H_t) V^e(H_t)\right] \mathrm{d}t, \quad (6b)$$

where the corresponding Hamilton-Jacobi-Bellman (HJB) representation is:

$$\begin{aligned} 0 = \sup_I & - \rho V^e(H) - \lambda^u(H) \left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) - I \\ & + V^e_H(H) \left[-\delta^e H + P^e I^\alpha H^{1-\alpha}\right], \\ 0 = \sup_I & - \rho V^u(H, \overline{H}) - \lambda^e(H) \left[V^u(H, \overline{H}) - V^e(H)\right] + Y^u(\overline{H}) - I \\ & + V^u_H(H, \overline{H}) \left[-\delta^u H + P^u I^\alpha H^{1-\alpha}\right]. \end{aligned}$$

Calculating the first-order conditions and substituting back into the objective function reveals that the joint HJB system simplifies to:

$$\begin{aligned} 0 = & - \rho V^e(H) - \lambda^u(H) \left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) \quad (7a) \\ & - \delta^e H V^e_H(H) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^e V^e_H(H)\right]^{\frac{1}{1-\alpha}}, \\ 0 = & - \rho V^u(H, \overline{H}) - \lambda^e(H) \left[V^u(H, \overline{H}) - V^e(H)\right] + Y^u(H) \quad (7b) \\ & - \delta^u H V^u_H(H, \overline{H}) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^u V^u_H(H, \overline{H})\right]^{\frac{1}{1-\alpha}}. \end{aligned}$$

[11] As will be seen shortly, the optimal strategy never involves borrowing at the parameter set used below, such that non-negative value function is never binding. St-Amour (2015) considers the case where risk-averse agents have no access to borrowing for human capital investment. The main findings obtained through numerical solutions remain qualitatively similar to the ones of this paper.

The bi-variate system of first-order differential equations (7) has no analytical solution due to the endogeneity and nonlinear functional forms used for the intensity functions (1). St-Amour (2015) relies on Chebyshev polynomials to calculate numerical solutions. We resort instead to a two-step approximate closed-form solution method developed in Hugonnier, Pelgrin and St-Amour (2013). First we remove the endogeneity in the employment intensities by imposing $\lambda_1^i = 0$ in (1). This exogenous employment risks case yields a closed-form solution (referred to as order-0 solution) for $V_0^i(H, \overline{H}), I_0^i(H, \overline{H})$. Second, we rewrite the endogenous intensity component as $\lambda_1^i = \epsilon \overline{\lambda}_1^i, i = e, u$ for some constants $\overline{\lambda}_1^i$ and perturbation ϵ and perform a first-order expansion of the value functions around the $\epsilon = 0$ solution:

$$V^e(H, \epsilon) \approx V^e(H, 0) + \epsilon V_\epsilon^e(H, 0),$$

$$V^u(H, \overline{H}, \epsilon) \approx V^u(H, \overline{H}, 0) + \epsilon V_\epsilon^u(H, \overline{H}, 0).$$

Once the approximate solution for the value functions is obtained, any relevant associated variable such as investment and human capital growth is thus recovered through a similar expansion. In particular, any function F involving the value functions can be approximated as:

$$F^e(H, \epsilon) \approx F^e(H, 0) + \epsilon F_\epsilon^e(H, 0),$$

$$F^u(H, \overline{H}, \epsilon) \approx F^u(H, \overline{H}, 0) + \epsilon F_\epsilon^u(H, \overline{H}, 0).$$

4 Optimal investment

We now calculate the optimal investment, starting first with the exogenous displacement and re-employment (order-0), followed by the more general case where both are endogenous.

4.1 Order-0 allocation

Theorem 1 (exogenous employment risks) *Let* $\lambda_1^e = \lambda_1^u = 0$ *and assume that the order-0 transversality and regularity conditions conditions* (14) *in Appendix A hold. Then:*

1. *The indirect utility functions of employed and unemployed agents are given as:*

$$V_0^e(H) = A_0^e + A_h^e H \tag{8a}$$

$$V_0^u(H, \overline{H}) = A_0^u + A_h^u H + A_b^u \overline{H} \tag{8b}$$

2. *The optimal investment functions are given as:*

$$I_0^e(H) = H\left(P^e \alpha A_h^e\right)^{\frac{1}{1-\alpha}} \tag{9a}$$

$$I_0^u(H) = H\left(P^u \alpha A_h^u\right)^{\frac{1}{1-\alpha}} \tag{9b}$$

3. *The optimal human capital growth functions are given as:*

$$g_0^e = -\delta^e + P^{e\frac{1}{1-\alpha}}\left(\alpha A_h^e\right)^{\frac{\alpha}{1-\alpha}} \tag{10a}$$

$$g_0^u = -\delta^u + P^{u\frac{1}{1-\alpha}}\left(\alpha A_h^u\right)^{\frac{\alpha}{1-\alpha}} \tag{10b}$$

where the parameters (A^e, A^u) *are given in Appendix B.*

The expression A_h^i in the indirect utility functions (8) capture the marginal value (i.e. shadow price) of human capital. The last measurable human capital level before the unemployment spell begins $\overline{H}$ is valued under unemployment, but not for employed agents. Since, for the employed, UIB revenues set $\overline{H} = H$ when unemployment begins, the value function simplifies to a function of H only. The optimal investment in (9) shows that the investment-to-capital ratio is constant and increasing in the shadow price. Consequently, the growth rates (10) are constant, so that no steady-state exists at the order zero.

4.2 Order-1 allocation

Theorem 2 (endogenous employment risks) *Assume that the order-0 transversality and regularity conditions conditions* (14) *in Appendix A hold. Then, up to a first-order approximation,*

1. *The indirect utility functions of employed and unemployed agents are given as:*

$$\begin{aligned} V^e(H) =& V_0^e(H) + B_u^e \lambda_1^u H^{-\xi^u} + B_{1u}^e \lambda_1^u H^{1-\xi^u} + B_e^e \lambda_1^e H^{-\xi^e} \\ &+ B_{1e}^e \lambda_1^e H^{1-\xi^e}, \end{aligned} \tag{11a}$$

$$\begin{aligned} V^u(H,\overline{H}) =& V_0^u(H,\overline{H}) + B_u^u \lambda_1^u H^{-\xi^u} + B_{1u}^u \lambda_1^u H^{1-\xi^u} + B_e^u \lambda_1^e H^{-\xi^e} \\ &+ B_{1e}^u \lambda_1^e H^{1-\xi^e} + B_b^u \overline{H} \lambda_1^e H^{-\xi^e}, \end{aligned} \tag{11b}$$

2. *The optimal investment functions are given as:*

$$\begin{aligned} I^e(H) =& I_0^e(H) + C_u^e \lambda_1^u H^{-\xi^u} + C_{1u}^e \lambda_1^u H^{1-\xi^u} + C_e^e \lambda_1^e H^{-\xi^e} \\ &+ C_{1e}^e \lambda_1^e H^{1-\xi^e}, \end{aligned} \tag{12a}$$

$$\begin{aligned} I^u(H,\overline{H}) =& I_0^u(H) + C_u^u \lambda_1^u H^{-\xi^u} + C_{1u}^u \lambda_1^u H^{1-\xi^u} + C_e^u \lambda_1^e H^{-\xi^e} \\ &+ C_{1e}^u \lambda_1^e H^{1-\xi^e} + C_b^u \overline{H} \lambda_1^e H^{-\xi^e}. \end{aligned} \tag{12b}$$

3. *The optimal human capital growth functions are given as:*

$$\begin{aligned} g^e(H) =& g_0^e + D_u^e \lambda_1^u H^{-1-\xi^u} + D_{1u}^e \lambda_1^u H^{-\xi^u} + D_e^e \lambda_1^e H^{-1-\xi^e} \\ &+ D_{1e}^e \lambda_1^e H^{-\xi^e}, \end{aligned} \tag{13a}$$

$$\begin{aligned} g^u(H,\overline{H}) =& g_0^u + D_u^u \lambda_1^u H^{-1-\xi^u} + D_{1u}^u \lambda_1^u H^{-\xi^u} + D_e^u \lambda_1^e H^{-1-\xi^e} \\ &+ D_{1e}^u \lambda_1^e H^{-\xi^e} + D_b^u \overline{H} \lambda_1^e H^{-1-\xi^e}. \end{aligned} \tag{13b}$$

where the order-0 values $V_0^e(H), V_0^u(H,\overline{H}), I_0^e(H), I_0^u(H,\overline{H})$ and $g_0^e(H), g_0^u(H,\overline{H})$ are given in Theorem 1 and where the parameters (B^e, B^u), (C^e, C^u) and (D^e, D^u) are given in Appendix C.

When contrasted with Theorem 1, the order-1 results of Theorem 2 show that the investment shares of human capital $I^i(H,\overline{H})/H$ are no longer constant. It follows that neither are the optimal growth functions $g^i(H,\overline{H})$, such that steady state values $H^i_{SS}(\overline{H})$ may exist, contrary to the exogenous employment risks case. Finally, a role for the lock-in capital $\overline{H}$ is reinstated for optimal investment and growth for the unemployed; employed investment and growth remain unaffected for reasons that will be discussed shortly.

5 Simulation

In order to better understand the dynamics of employment statuses and income induced by those of the human capital, we rely on the order-1 optimal rules in Theorem 2 to simulate the model. We first focus on the restricted case of status-independent technology by imposing $\delta^i = \delta$ and $P^i = P$ in (4) and we abstract from firm-specific capital loss upon displacement by restricting $\phi = 1$ in the HJB (7). These restrictions are imposed so as to emphasize scarring and stigma stemming from optimal investment strategies, instead of from parametric assumptions. In effect, this baseline scenario renders eventual scarring and stigma effects less likely since the agent is able to fully self-insure against scars without facing additional status-dependent penalties, such as less-efficient technology, faster depreciation or firm-specific capital loss upon unemployment. We will reinstate both status-dependent technology and firm-specific capital loss in the comparative statics exercise below.

Our simulation follows the Monte Carlo procedure outlined in Appendix D. The calibration is selected so as to match the theoretical moments calculated from the simulation to their observed counterparts in Tables 1 and 2. More precisely, we use the resulting simulated employment histories $S = \{S_j\}_{j=1}^n$ where $S_j = \{S_{j,t}\}_{t=1}^T$ and $S_{j,t} \in (e,u)$ are the employment statuses for agent $j = 1, 2, \ldots n$, in order to compute the main moments of interest. More precisely, let the displaced index be defined by $D_{j,t} = \mathbb{1}(\cup_{k=1}^{3} S_{j,t-k} = u)$, i.e. having been unemployed at least once over the last three periods. The moments to be matched are the probability of being currently unemployed conditional upon having been displaced $\Pr(u_t \mid D_t = 1)$, or not $\Pr(u_t \mid D_t = 0)$, as well as the income loss conditional upon displacement in the last three periods $\Delta Y_t(e_t \mid D_t = 1)$ which are matched to the DWS data in Table 1. We also rely on the unemployment, i.e. $\Pr(S_t = u)$, the displacement, i.e. $\Pr(S_t = u \mid S_{t-1} = e)$ and the re-employment, i.e. $\Pr(S_t = e \mid S_{t-1} = u)$ rates, which are matched to the DWS population values from Table 2. The calibration is undertaken subject to the four order-0 transversality and regularity conditions conditions (14) in Appendix A. The selected calibration in Table 4.a does match the moments reasonably well in Table 4.b.

Figure 3.a plots the optimal investment in human capital for employed (blue, left-hand scale) and unemployed (red, right-hand scale) agents, in functions of H and for mid-level $\overline{H} = 0.5 * (a + b)$ lock-in capital level, where a, b delimit the range of initial human capital levels. First, investment for unemployed agents is always lower, i.e. $I^u(H, \overline{H}) < I^e(H), \forall H, \overline{H}$. Second, investment is falling in human capital for the employed, but is U-shaped for the unemployed. Indeed, conflicting income and employment risks effects imply that investment can be non-monotone in H.

One the one hand, an increase in H raises the employed agent's revenues $Y^e(H)$ and thus available resources for investing for the employed. Moreover, equation (3) shows that it also raises the value at risk in case of unemployment $\Delta Y(H, \overline{H})$. Both elements concur to increase investment. However, because UI income is fixed at lock in level $\overline{H}$, higher human capital has no effects on available resources for the unemployed $Y^u(\overline{H})$, yet increases the income gain $\Delta Y(H, \overline{H})$ in case of re-employment. Again, these income effects raise incentives for investing in human capital. On the other hand, increasing H also reduces the likelihood of displacement, while increasing the re-employment probability, thereby reducing the incentives for investment. Diminishing returns in adjusting the

Table 4: Calibration and moments matching

(a) Calibrated parameters

Equation s	Parameters		
Intensities (1)	λ_0^e	λ_1^e	ξ^e
	0.185	1.065	-0.1
	λ_0^u	λ_1^u	ξ^u
	0.0225	0.0095	0.3
Income (2)	y_0	y_1	η
	0.05	0.55	0.5
Dynamics (4)	δ^e, δ^u	α	P^e, P^u
	0.175	0.8	0.25
HJB (7) and Apx. D	ρ	ϕ	T
	0.05	1.0	200
	a	b	n
	0.05	2.0	10'000

(b) Observed and simulated moments

	Table 2			Table 1		
	$\Pr(u)$	$\Pr(e\|u)$	$\Pr(u\|e)$	$\Pr(u\|D)$	$\Pr(u\|N)$	$\Delta Y(e\|D)$
Data	0.0635	0.6343	0.0403	0.1195	0.0410	0.0850
Model	0.0662	0.6238	0.0439	0.1977	0.0436	0.0715

Notes: D: Displaced in last three periods, N: Not displaced, u: Unemployed, e: Employed. Corresponding data from Tables 1 and 2.

arrival intensities $\lambda^i(H)$ entail that the marginal effect on employment risk is stronger at low H. Our calibration reveals that the employment risk effect dominates the income effect for the employed, as well as for the unemployed with low human capital. At high H, the income effect is stronger for the unemployed and investment increases in human capital.

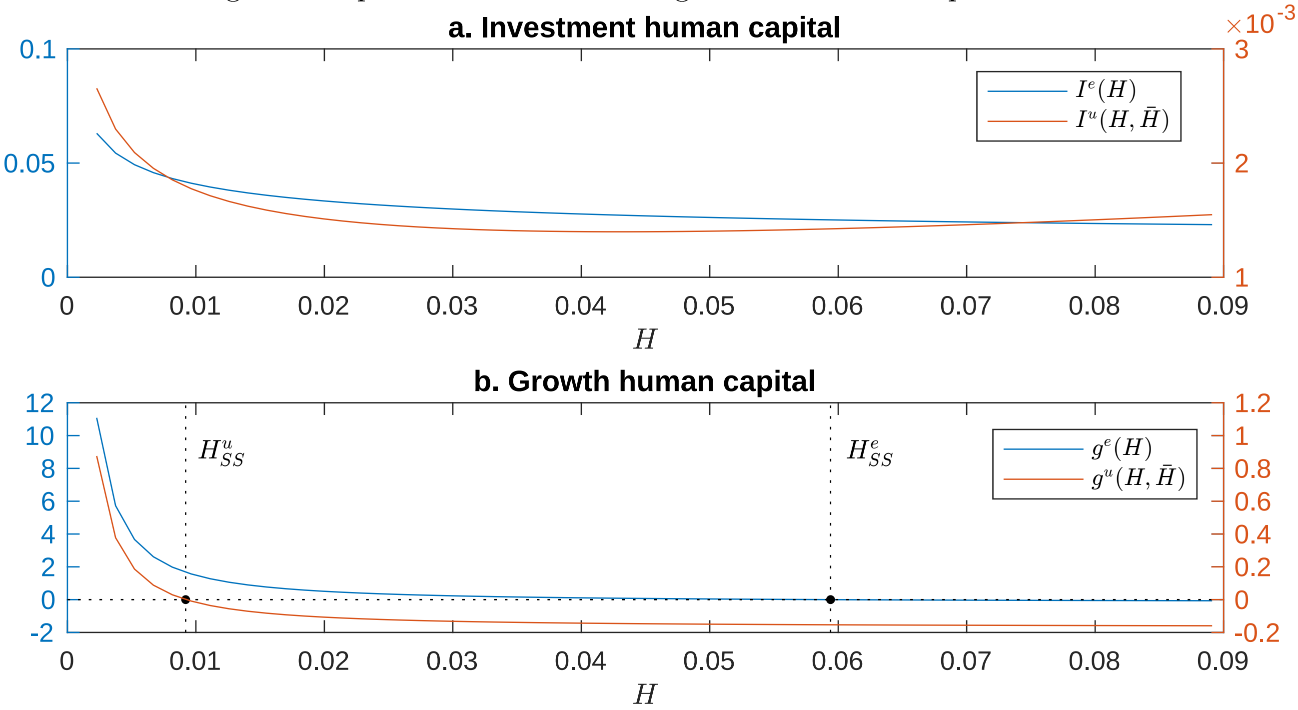
Third, our calibration entails that $C_b^u, D_b^u < 0$, indicating that the investment and growth are both lower for unemployed agents with high lock-in capital, although the net effect is weak due to two opposing forces. On the one hand, a high lock-in capital raises

UI revenues available for investing. On the other hand, the discussion of (3) revealed that the attractiveness of investing, and therefore increasing the likelihood of re-employment is reduced due to more generous UIB income for high $\overline{H}$. Our results indicate that the two effects more or less offset one another.

Figure 3.b shows the optimal human capital dynamics for employed (blue) and unemployed (red) agents, again evaluated at mid-level lock-in capital levels. These results show that steady-state levels exist, are unique given status and $\overline{H}$ and are dynamically stable. In particular, the higher levels of investment for the employed workers translate into higher steady-states compared to the unemployed, with $H^e_{SS} = 0.0599$ compared to $H^u_{SS}(\overline{H}) = 0.0092$. Again, it can be shown that the low effect of lock-in capital on unemployed investment entails that its effects on the steady-state $H^u_{SS}(\overline{H})$ is also very moderate. Consistent with Panel A results, lower investments at high lock-in capital leads to lower steady-state , i.e. $\partial H^u_{SS}(\overline{H})/\partial\overline{H} < 0$. Importantly, dynamic stability implies cyclical dynamics whereby a long-tenured worker who is displaced at H^e_{SS} will optimally choose a depletion of his human capital until either a new lower steady state H^u_{SS} obtains, or he is re-employed, after which human capital will grow again up to H^e_{SS}.

Figure 4 plots a sample of the simulated optimal trajectories for human capital $\{H_{j,t}\}$. Consistent with Figure 3.b, dynamic paths converge rapidly towards the dynamically stable steady-state level associated with employment $H^e_{SS} = 0.0599$ (dotted red line). Each dip in $H_{j,t}$ is caused by a job displacement; once re-employed, the paths converge again towards H^e_{SS}. A prolonged unemployment spell is associated with a constant fall in capital towards the unemployment steady state H^u_{SS}. Since the unemployment probability $\Pr(u) = 6.35\%$ is low, most of the dynamic paths hover around the employed steady-state value H^e_{SS}.

Figure 3: Optimal investment and growth in human capital



Notes: a. Investment for employed ($I^e(H)$, in blue, left-hand scale) and unemployed ($I^u(H, \overline{H})$, in red, right-hand scale) computed from (12) at calibrated parameter values. b. Growth rates for employed ($g^e(H)$, in blue, left-hand scale) and unemployed ($g^u(H, \overline{H})$, in red, right-hand scale) computed from (13). Steady-states for employed (H^e_{SS}) and for unemployed (H^u_{SS}).

6 Self-inflicted unemployment scars and stigma

Figure 5 plots the optimal dynamics of human capital. First, in Panel A, a long-tenured worker with steady-state capital H^e_{SS} and who is displaced moves from a to b on the optimal human capital growth path. From the previous analysis, human capital then optimally depletes for the entire duration of the unemployment spell and moves towards the new lower steady state in c. Once attained, the capital remains at steady-state H^u_{SS} for the duration of the unemployment spell. Upon re-employment, the agent's capital moves to point d after which capital increases again back to the former steady-state H^e_{SS}.

Next, Figure 6 shows how these human capital dynamics translate into unemployment stigma and last-in-first-out. The long-tenured displaced worker moves from a to b on the re-employment intensity function. As human capital optimally falls, so does the recall

Figure 4: Simulated optimal trajectories

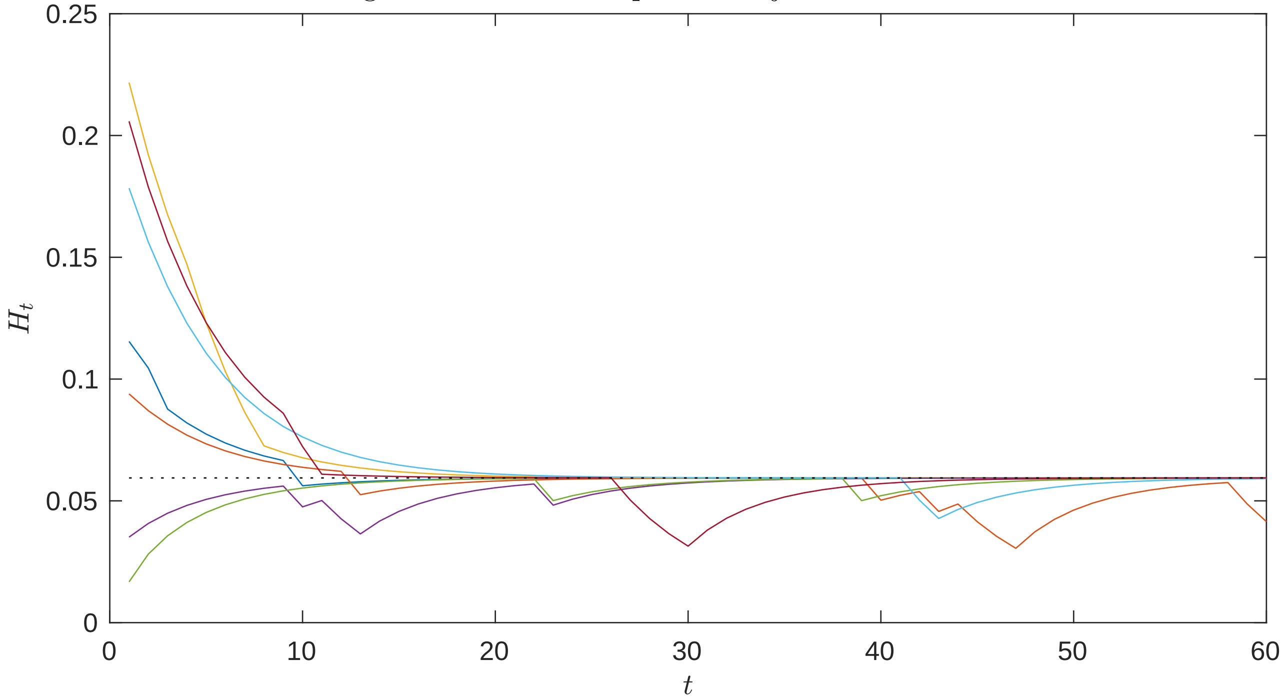
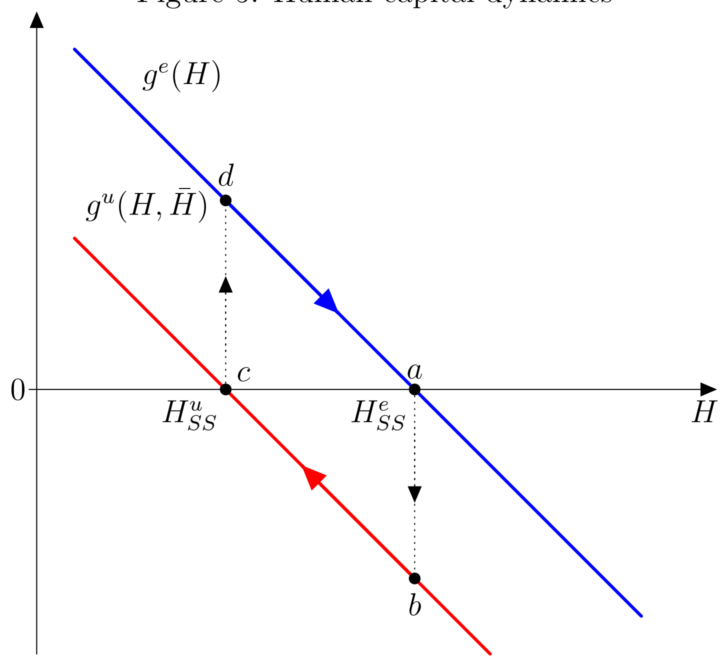


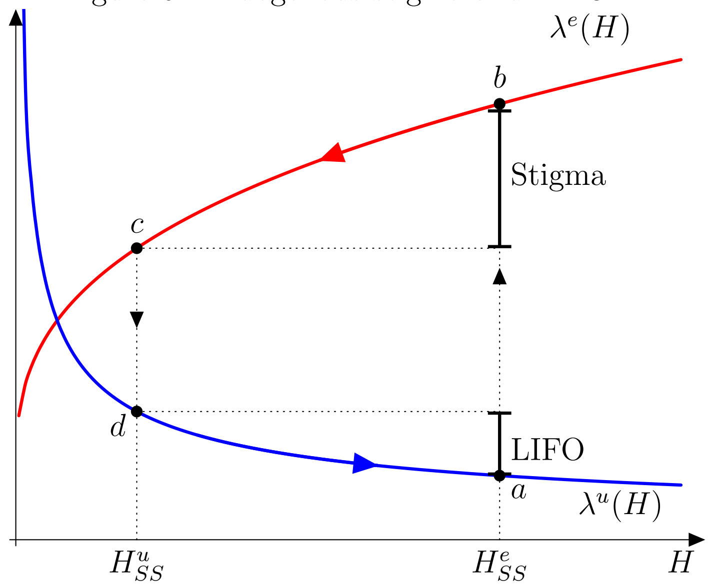
Figure 5: Human capital dynamics



Notes: $g^e(H)$: Optimal human capital growth conditional on employment in (13a). $g^u(H, \overline{H})$: Optimal human capital growth conditional on unemployment in (13b), for capital H and UIB lock-in capital $\overline{H}$.

probability with intensity moving towards c. Duration dependence endogenously obtains as the longer the duration spell, the more important is the associated unemployment stigma, i.e. the fall in $\lambda^e(H)$. Upon re-employment, the agent moves to point d on the $\lambda^u(H)$ intensity and is subject to a higher displacement probability due to the optimal fall in human capital. This LIFO effect persists up to the period where the former steady state H^e_{SS} is attained in point a.

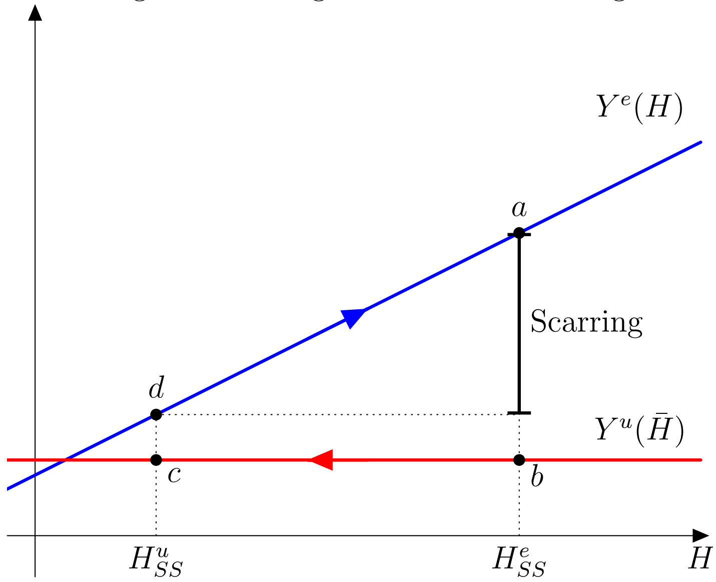
Figure 6: Endogenous stigma and LIFO



Notes: $\lambda^e(H)$: re-employment intensity; $\lambda^u(H)$: displacement intensity, under dynamics described in Figure 5.

The model also generates endogenous income scarring effects of unemployment, as evidenced in Figure 7. A displaced long-tenured worker suffers a drop in income from a to b. As human capital is optimally depleted towards c, the UIB revenues remain unaffected due to the lock-in feature. However, upon re-employment, the agent's labor income is now lower at d, with the longer the unemployment spell, the more important the drop in wages upon re-employment. The model thus endogenously generates wage dynamics that are consistent with income scarring effects of unemployment (e.g. Guvenen et al., 2017; Jacobson et al., 2005b, 1993) .

Figure 7: Endogenous income scarring



Notes: $Y^e(H)$: employment income. $Y^u(\bar{H})$: unemployment income, under dynamics described in Figure 5.

The predicted unemployment scars and stigma can thus be characterized as *self-inflicted*, to the extent that they stem from optimal human capital dynamics decided by agents exclusively. Indeed, we have relied on simple and empirically motivated characterization of labor demand whereby human capital is valued by employers, resulting in higher wages, lower displacement and higher re-employment probabilities. Traditional explanations of scarring and stigma based on screening practices by employers are therefore not required to explain this phenomenon. Importantly, neither are ad-hoc hypotheses, such as (i) more important depreciation rates, (ii) less efficient production technology of human capital, or (iii) mechanic depreciation for the unemployed. Indeed, our baseline calibration assumes identical laws of motion for human capital under employed and unemployed statuses and depletion or growth is decided optimally by employed and unemployed workers. Observe finally that, although long-lasting, the predicted unemployment scarring and stigma are not permanent. Indeed, a sufficiently long employment history pushes human capital up to its former steady-state level H^e_{SS}, such that scars eventually heal, as found in the data (see footnote 4).

7 Counter-factual analysis

We now conduct a counter-factual analysis to gauge the effects of parametric changes on our results. In particular, starting with the optimal allocation $I = I(H, \overline{H}; \theta)$, we modify the deep parameters to $\tilde{\theta}$ and recompute the optimal rules $\tilde{I} = I(H, \overline{H}; \tilde{\theta})$. Three exercises are performed. We first start by assessing the effects of the endogenous exposure to employment risks on the demand for human capital. We next measure the changes in optimal dynamics resulting from policy changes in the UIB, and base income regimes. Finally, we assess the effects of additional unemployment costs in the form of a higher depreciation rate and of firm-specific human capital that is depleted upon displacement. The effects on the baseline results are reported in Table 5.

Table 5: Hedging motives and comparative statics

		Hedging		Policy		Unempl. costs	
Variable	Base	(1)	(2)	(3)	(4)	(5)	(6)
H^e_{SS}	0.0599	−70.0%	−97.3%	−58.1%	−41.5%	133.4%	216.1%
H^u_{SS}	0.0094	15.0%	−96.9%	−58.2%	−41.7%	85.4%	215.6%
I	0.0240	−69.6%	−97.4%	−57.4%	−41.9%	134.9%	217.3%
H	0.0570	−69.5%	−97.4%	−58.4%	−41.8%	133.4%	219.1%
$\Pr(u)$	0.0662	17.9%	18.7%	18.4%	10.7%	−13.2%	−17.1%
$\Pr(e\|u)$	0.6238	0.0%	−16.6%	−4.2%	−2.6%	4.0%	5.7%
$\Pr(u\|e)$	0.0439	20.8%	0.0%	14.7%	8.6%	−11.0%	−14.4%
$\Pr(u\|D)$	0.1977	0.7%	33.0%	10.0%	5.8%	−8.8%	−12.1%
$\Pr(u\|N)$	0.0436	20.8%	0.8%	14.8%	8.9%	−10.3%	−13.3%
$\Delta Y(e\|D)$	0.0715	−78.2%	−95.4%	−44.1%	12.0%	73.5%	63.4%

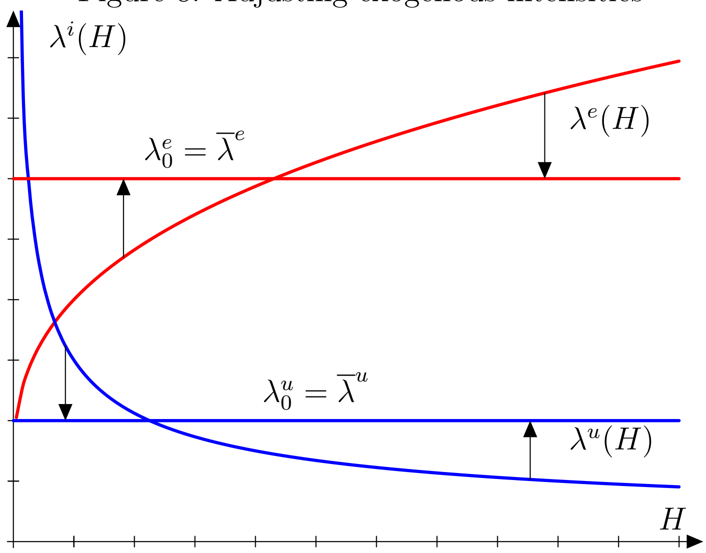
Notes: Percentage changes from base scenario. (1) Exogenous re-employment, $(\lambda^e_0, \lambda^e_1) = (0.9778, 0)$ instead of $(0.185, 1.065)$. (2) Exogenous displacement, $(\lambda^u_0, \lambda^u_1) = (0.0449, 0)$ instead of $(0.0225, 0.0095)$. (3) UIB high, $\eta = 0.80$ instead of 0.50. (4) Base income low, $y_0 = 0.0250$ instead of 0.0500. (5) High unemployment depreciation, $\delta^u = 0.2012$ instead of 0.175. (6) Firm-specific human capital loss $\phi = 0.85$ instead of 1.0.

7.1 Gauging the hedging

Traditional human capital models focus on higher wages as primary motives (e.g. Ben-Porath, 1967; Heckman, 1976; Kredler, 2014) and incorporate at most undiversifiable employment risks (corresponding to our order-0 case $\lambda_1^i = 0$, e.g. as in Rogerson and Schindler, 2002; Krebs, 2003; Huggett et al., 2011; Cervellati and Sunde, 2013). A main contribution of our model is thus to allow for possible hedging of these risks by agents, in addition to the usual income motives for human capital accumulation.

We assess the marginal contributions of self-insurance against employment risks to the investment, human capital, unemployment, displacement and re-employment. This exercise is performed by removing only the re-employment ($\lambda_1^e = 0$) and only the displacement ($\lambda_1^u = 0$) endogeneity in (1), with corresponding solutions given in Theorem 1. Since the intensities are mechanically lowered, we re-adjust the base intensity so as to maintain the mean displacement and re-employment rates in Table 4.b. As seen in Figure 8, this adjustment is however not neutral and tends to benefit low human capital agents by providing them with higher re-employment and lower displacement rates; high human capital agents are disadvantaged for the opposite reasons.

Figure 8: Adjusting exogenous intensities



The first two columns of Table 5 reports how the variables of interest are affected by exogenous employment risks, relative to baseline levels. First, removing the capacity to hedge re-employment risk in column 1 lowers the attractiveness of investing in human capital and results in 69% drops in investment and capital levels. By construction, the re-employment $\Pr(e|u)$ is unaffected, while displacement $\Pr(u|e)$ is increased due to the sharp drop in human capital, resulting in a increase in unemployment. Because displacement cannot be hedged, the scarring effect on unemployment $\Pr(u|D)$ is moderate, whereas the increase in $\Pr(u|N)$ is large relative to baseline scenario. Second, exogenous displacement in column 2 also lowers the incentives to invest in human capital although the effects are much stronger, almost entirely wiping out investment and capital. By construction the displacement risk $\Pr(u|e)$ is unaffected, but re-employment $\Pr(e|u)$ falls sharply, leading to an increase in unemployment rate. Having been displaced has a strong scarring cost in terms of being currently unemployed. In both cases, the fall in H is associated with a narrowing down of the human capital gap between those who have and who haven't been displaced. Consequently, the income scar ΔY is less important relative to baseline.

The fall in investing when employment risks are exogenous obtains from two different reasons. Indeed, from Figure 8, higher re-employment and lower displacement probabilities reduce the incentives for investing for those agents with low human capital. Moreover, agents with high human capital witness a strong drop in the returns to investment when hedging capacities are removed; they respond by decreasing investment. Contrasting the effects of re-employment and displacement endogeneity reveals that the latter has a much more potent effect on capital accumulation.

7.2 UIB and base income policies

In Table 5, column 3, we investigate the effect of more generous unemployment insurance by increasing the UI replacement rate η to 0.80 in (2b). The outcome is a 57% decrease in investment and capital, inducing a deterioration in displacement, re-employment and unemployment. The effects on unemployment and income scars of having been displaced are positive and important. In column 4, we next analyze changes in the base income y_0 in (2a) by imposing a 50% drop in the latter. Again, the reduction in disposable income leads to important cuts in investment. The corresponding drop in mean human capital leads to increases in unemployment, stemming from higher displacement and lower re-employment rates and also induces more scarring effects of displacement. Again, the fall in human capital leads to less important income scarring $\Delta Y(e|D)$.

The reason for these similar depressing effects of UI and base income policies on investment and capital can be deduced from (3) which shows that the income loss associated with unemployment $\Delta Y(H, \overline{H})$ is a decreasing function of η and is increasing in base income y_0. More generous UIB and/or lower base income thus both reduce the income loss associated with unemployment and gains from re-employment, thereby decreasing the incentives for investing. Our results are thus consistent with strong moral hazard responses to UIB generosity, whereby both employed and unemployed agents invest less in their human capital and face higher displacement and lower re-employment probabilities as a result.

7.3 Additional costs of unemployment

Our simulated results have thus far abstracted from additional disadvantages of begin displaced, such as lower returns to investment and loss of firm-specific human capital. However, the results in Theorem 2 make it possible to calculate the effects of such costs. First, in column 5, we augment the depreciation rate of human capital when unemployed by 15% to $\delta^u = 0.2012 > \delta^e = 0.1750$. In column 6, we introduce depletion of firm-specific human capital by imposing a $1 - \phi = 15\%$ loss on the capital stock upon displacement. Both comparative statics in columns 5 and 6 convey the same message. An increase in the unemployment tolls lead to a surge in self-insurance against these costs. The agent reacts to the additional penalties by sharply increasing investment and human capital stock. It follows directly that labor market conditions improve (lower unemployment, higher re-employment, lower risks and scars of displacement). In both cases, the increase in H also widens the human capital gap between those who have and who haven't been displaced, leading to more important income scars ΔY relative to baseline.

Overall our results are consistent with strong moral hazard implications when employment risks can be hedged through human capital investment. Any improvement in the cost of being unemployed (e.g. through more generous UIB) lowers the incentives for investment, with corresponding deteriorations in labor market outcomes; any increase in unemployment costs induces precautionary investment in self-insurance that improves labor market outcomes.

8 Conclusion

In addition to the contemporaneous drop in income due to incomplete UI replacement, unemployment imposes significant long-term scarring and stigma costs on agents. In particular, displacement (re-employment) probabilities are higher (lower), whereas wages upon re-employment are lower following unemployment spells. Moreover, the duration of unemployment spells significantly compounds the magnitude of these costs.

Human capital has long been suspected as potential rationale for these costs. Accelerated depreciation during unemployment associated with screening by employers for imperfectly observed human capital levels has been invoked as the main drivers for scarring and stigma. This explanation has been favored by Search and Matching models. However, capital accumulation is very mechanic in S&M settings: acquisition can occur only through tenure, and/or is decided by the employers only. It thus fails to account for endogenous investment decisions by agents and the possibility to acquire human capital during unemployment spells. Moreover, traditional models of human capital abstract from endogenizing employment risks exposures in the investment decisions made by agents. As with S&M models, displacement and re-employment exposure are completely exogenous and human capital level in HK models is decided by agents whose only motivation is the associated higher income.

This paper has taken the alternative approach or endogenizing human capital decisions by employed and unemployed workers alike and by endogenizing their exposure to displacement and re-employment risks. The combination of both entails that risk exposure is therefore partially diversifiable. Contrary to others, our model can integrate or abstract from status-dependent human capital accumulation technology and from firm- or sector-specific capital depletion upon displacement. For our baseline scenario, these additional tolls of unemployment are shut down. It follows that any acquisition and depletion of human capital and resulting unemployment scarring and stigma are entirely endogenous, rather than mechanic.

The solution of this model is complicated by the fact that the two value functions (employed and unemployed) are intertwined with one another and because the model with human capital arrival rates can be re-written as one with endogenous discounting across the two statuses. We resorted to linear expansion methods to circumvent this problem and obtain analytical approximations of the optimal investing strategies

We first investigated whether and confirmed that this framework is capable of generating unemployment scarring and stigma. The key theoretical element behind this result is that the model generates two status-dependent and dynamically stable steady-states for human capital, with the one for the unemployed always being lower. Changes in employment statuses thus trigger cyclical dynamics characterized by endogenous depletion of acquired human capital when unemployed and accumulation upon re-employment. Since re-employment (displacement), as well as wages intensities are increasing (decreasing) functions of human capital, scarification and stigmatisation endogenously obtains. Because they depend entirely on optimal decisions made by workers instead of by employers, scarring and stigma are therefore self-inflicted.

Such a remarkable result is non-trivial. To the extent that scarring and stigma both impose substantial costs to workers, that they depend on accumulated human capital and that the latter can be adjusted by agents, the optimal strategy could have been to self-insure against these risks by investing more to prevent displacement if employed and in favor of re-employment if unemployed. However, our results show that this is not the case. The cushioning against downward income risks offered by UI programs, as well as imperfect replacement rates entails that moral hazard and low income prevent the unemployed from investing more to avoid long-term costs. Conversely, incorporating incremental tolls of displacement, such as added depreciation and/or depletion of firm-specific capital for the unemployed leads to an increase in self insurance through additional investment and human capital.

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A Order-0 transversality and regularity conditions

The required transversality and regularity conditions for the order-0 solutions are:

$$0 < \rho + \lambda_0^e + \delta^u - \left(\alpha P^{u\frac{1}{\alpha}} A_h^u\right)^{\frac{\alpha}{1-\alpha}}, \tag{14a}$$

$$0 < \rho + \lambda_0^u + \delta^e - \left(\alpha P^{e\frac{1}{\alpha}} A_h^e\right)^{\frac{\alpha}{1-\alpha}}, \tag{14b}$$

$$\phi\lambda_0^e\lambda_0^u < \left(\rho + \lambda_0^e + \delta^u - \left(\alpha P^{u\frac{1}{\alpha}} A_h^u\right)^{\frac{\alpha}{1-\alpha}}\right)\left(\rho + \lambda_0^u + \delta^e - \left(\alpha P^{e\frac{1}{\alpha}} A_h^e\right)^{\frac{\alpha}{1-\alpha}}\right), \tag{14c}$$

B Order-0 parameters

Proof. At the optimum, the order-0 HJB (7) corresponding to $\lambda_1^e, \lambda_1^u = 0$ can be written as:

$$\begin{aligned} 0 = &- \rho V^e(H) - \lambda_0^u \left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) \\ &- \delta^e H V_H^e(H) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^e V_H^e(H)\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{15a}$$

$$\begin{aligned} 0 = &- \rho V^u(H, \overline{H}) - \lambda_0^e \left[V^u(H, \overline{H}) - V^e(H)\right] + Y^u(H) \\ &- \delta^u H V_H^u(H, \overline{H}) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} H \left[P^u V_H^u(H, \overline{H})\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{15b}$$

Consider candidate solution:

$$V_0^e(H) = A_0^e + A_h^e H \tag{16a}$$

$$V_0^u(H, \overline{H}) = A_0^u + A_h^u H + A_b^u \overline{H} \tag{16b}$$

Substituting the candidate solutions (16) in (15) yields:

$$0 = \tilde{A}_0^e + \tilde{A}_h^e H \tag{17a}$$

$$0 = \tilde{A}_0^u + \tilde{A}_h^u H + \tilde{A}_b^u \overline{H} \tag{17b}$$

Assuming the transversality and regularity conditions conditions (14) hold, we can individually set the implicit parameters $\tilde{A}^e, \tilde{A}^u$ to zero in (17) and obtain that the parameters in Theorem 1 are:

$$A_0^u = \frac{y_0\left(\lambda_0^e + \eta\left(\rho + \lambda_0^u\right)\right)}{\rho\left(\lambda_0^e + \rho + \lambda_0^u\right)}; \quad A_b^u = \frac{\eta y_1}{\lambda_0^e + \rho}; \quad A_0^e = \frac{y_0\left(\lambda_0^e + \rho + \eta\lambda_0^u\right)}{\rho\left(\lambda_0^e + \rho + \lambda_0^u\right)}$$

and where A_h^e, A_h^u jointly solve:

$$0 = A_h^e\lambda_0^e - A_h^u\left(\delta^u + \lambda_0^e + \rho\right) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(P^u A_h^u\right)^{\frac{1}{1-\alpha}}$$
$$0 = \lambda_0^u\left(\phi A_h^u + \frac{\eta y_1}{\lambda_0^e + \rho}\right) + (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(P^e A_h^e\right)^{\frac{1}{1-\alpha}} - A_h^e\left(\delta^e + \rho + \lambda_0^u\right) + y_1$$

The optimal investment and growth functions follow directly by substituting (A^e, A^u) in (9) and (10).

■

C Order-1 parameters

Proof. Without loss of generality, rewrite the endogenous component in intensities (1) as $\lambda_1^i = \epsilon\overline{\lambda}_1^i, i = e, u$ for some constants $\overline{\lambda}_1^i$ and perturbation ϵ. The order-1 solution proceed as a first-order Taylor expansion around the order-0 solution corresponding to $\epsilon = 0$. First, the corresponding order-1 HJB can be written as:

$$\begin{aligned} 0 = \sup_I - \rho V^e(H) - \left(\lambda_0^u + \epsilon\overline{\lambda}_1^u H^{-\xi^u}\right)\left[V^e(H) - V^u(\phi H, H)\right] + Y^e(H) - I \\ + V_H^e(H)\left[-\delta^e H + P^e I^\alpha H^{1-\alpha}\right], \end{aligned} \tag{18a}$$

and

$$\begin{aligned} 0 = \sup_I - \rho V^u(H, \overline{H}) - \left(\lambda_0^e + \epsilon\overline{\lambda}_1^e H^{-\xi^e}\right)\left[V^u(H, \overline{H}) - V^e(H)\right] + Y^u(\overline{H}) - I \\ + V_H^u(H, \overline{H})\left[-\delta^u H + P^u I^\alpha H^{1-\alpha}\right]. \end{aligned} \tag{18b}$$

Second, consider candidate solutions given by:

$$
\begin{aligned}
V^e(H) =& V_0^e(H) + \epsilon \Big(B^e H + B_u^e \overline{\lambda}_1^u H^{-\xi^u} + B_{1u}^e \overline{\lambda}_1^u H^{1-\xi^u} + B_e^e \overline{\lambda}_1^e H^{-\xi^e} \\
& + B_{1e}^e \overline{\lambda}_1^e H^{1-\xi^e} \ ),
\end{aligned}
\tag{19a}
$$

and

$$
\begin{aligned}
V^u(H,\overline{H}) =& V_0^u(H,\overline{H}) + \epsilon \Big(B^u H + B_u^u \overline{\lambda}_1^u H^{-\xi^u} + B_{1u}^u \overline{\lambda}_1^u H^{1-\xi^u} + B_e^u \overline{\lambda}_1^e H^{-\xi^e} \\
& + B_{1e}^u \overline{\lambda}_1^e H^{1-\xi^e} + B_b^u \overline{H} \ \overline{\lambda}_1^e H^{-\xi^e} \ ).
\end{aligned}
\tag{19b}
$$

Third, we solve for I^e, I^u using guess (19) in HJB (18) and express optimal investment as a first-order expansion around $\epsilon = 0$. Fourth, we substitute this first-order solution back in the HJB, again do a first-order expansion around $\epsilon = 0$ and individually solve the implicit parameters B as follows:

B^e	0
B_u^e	$\frac{(\eta-1)y_0\phi^{\xi^u}(\lambda_0^e+g_0^u\xi^u+\rho)}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^u}(g_0^e\xi^u+\rho+\lambda_0^u)(\lambda_0^e+g_0^u\xi^u+\rho)-\lambda_0^e\lambda_0^u)}$
B_{1u}^e	$-\frac{\phi^{\xi^u}(\lambda_0^e+g_0^u(\xi^u-1)+\rho)((\lambda_0^e+\rho)(A_h^e-\phi A_h^u)-\eta y_1)}{(\lambda_0^e+\rho)(\phi^{\xi^u}(g_0^e(\xi^u-1)+\rho+\lambda_0^u)(\lambda_0^e+g_0^u(\xi^u-1)+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B_e^e	$-\frac{(\eta-1)y_0\lambda_0^u}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_0^e+\rho)-\lambda_0^e\lambda_0^u)}$
B_{1e}^e	$\frac{\lambda_0^u(\phi A_h^e(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)-\phi A_h^u(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)-\eta y_1((\xi^e-1)g_0^u+\lambda_0^e+\rho))}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)(\phi^{\xi^e}((\xi^e-1)g_0^e+\rho+\lambda_0^u)((\xi^e-1)g_0^u+\lambda_0^e+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B^u	$0,$
B_u^u	$\frac{(\eta-1)y_0\lambda_0^e\phi^{\xi^u}}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^u}(g_0^e\xi^u+\rho+\lambda_0^u)(\lambda_0^e+g_0^u\xi^u+\rho)-\lambda_0^e\lambda_0^u)}$
B_{1u}^u	$-\frac{\lambda_0^e\phi^{\xi^u}((\lambda_0^e+\rho)(A_h^e-\phi A_h^u)-\eta y_1)}{(\lambda_0^e+\rho)(\phi^{\xi^u}(g_0^e(\xi^u-1)+\rho+\lambda_{\mathrm{u0}})(\lambda_0^e+g_0^u(\xi^u-1)+\rho)-\phi\lambda_0^e\lambda_0^u)}$
B_b^u	$-\frac{\eta y_1}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)}$
B_e^u	$-\frac{(\eta-1)y_0\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)}{(\lambda_0^e+\rho+\lambda_0^u)(\phi^{\xi^e}(\xi^e g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_{\mathrm{e0}}+\rho)-\lambda_0^e\lambda_0^u)}$
B_{1e}^u	$\frac{\phi^{\xi^e}((A_h^e-A_h^u)(\lambda_0^e+\rho)((\xi^e-1)g_0^e+\rho+\lambda_0^u)(\xi^e g_0^u+\lambda_0^e+\rho)-\eta y_1\lambda_{\mathrm{e0}}\lambda_0^u\phi^{-\xi^e})}{(\lambda_0^e+\rho)(\xi^e g_0^u+\lambda_0^e+\rho)(\phi^{\xi^e}((\xi^e-1)g_0^e+\rho+\lambda_0^u)((\xi^e-1)g_0^u+\lambda_0^e+\rho)-\phi\lambda_0^e\lambda_0^u)}$

where the (A^i, g_0^i) parameters are given in Appendix B and Theorem 1. Substituting back for $\lambda_1^i = \epsilon\overline{\lambda}_1^i$ yields the optimal solution in Theorem 2.

Investment and growth Given the parameters (B^e, B^u), the parameters (C^e, C^u) for the investment functions are obtained as:

$$C^e = \begin{pmatrix} C^e_u \\ C^e_{1u} \\ C^e_e \\ C^e_{1e} \end{pmatrix} = \kappa^e \begin{pmatrix} -\xi^u B^e_u \\ (1-\xi^u) B^e_{1u} \\ -\xi^e B^e_e \\ (1-\xi^e) B^e_{1e} \end{pmatrix}, \quad C^u = \begin{pmatrix} C^u_u \\ C^u_{1u} \\ C^u_e \\ C^u_{1e} \\ C^u_b \end{pmatrix} = \kappa^u \begin{pmatrix} -\xi^u B^u_u \\ (1-\xi^u) B^u_{1u} \\ -\xi^e B^u_e \\ (1-\xi^e) B^u_{1e} \\ -\xi^e B^e_b \end{pmatrix}$$

where we have set:

$$\kappa^i \equiv \frac{\left[P^i \alpha \left(A^i_h\right)^\alpha\right]^{\frac{1}{1-\alpha}}}{1-\alpha}, \quad i = e, u$$

Given the parameters (C^e, C^u), the parameters (D^e, D^u) for the growth functions are obtained as:

$$D^i = \frac{C^i}{A^i_h}, \quad i = e, u.$$

■

D Simulation

We begin by calibrating the main parameters and by initializing the employment status and human capital for a population of agents $j = 1, 2, \ldots, n$:

- The employment status is drawn from the unconditional population rates: $S_{j,0} \sim \{e, u\}$.
- Both the initial capital $H_{j,0}$ and the initial lock-in capital $\overline{H}_{j,0}$ are independently drawn from a uniform distribution over interval $[a, b]$.

Next, the recursive phase is obtained for $\forall j$ and $\forall t = 0, 1, 2, \ldots, T$ as follows:

1. Set the employment status $i = S_{j,t}$, in order to compute the optimal investment (12) and welfare (11), as well as the displacement/re-employment exposures and income

as:

$$I_{j,t} = I^i(H_{j,t}, \overline{H}_{j,t}), \qquad V_{j,t} = V^i(H_{j,t}, \overline{H}_{j,t}),$$
$$\lambda_{j,t} = \lambda^i(H_{j,t}), \qquad Y_{j,t} = Y^i(H_{j,t}, \overline{H}_{j,t}).$$

2. Use the law of motion (4) to update human capital and the Poisson distribution to update employment status as:

$$H_{j,t+1} = H_{t+1}(I_{j,t}, H_{j,t}), \qquad \overline{H}_{j,t+1} = \mathbb{1}_t^e H_{j,t+1} + \mathbb{1}_t^u \overline{H}_{j,t},$$
$$S_{j,t+1} \sim \text{Poisson}(\lambda_{j,t}).$$