Valuing Life as an Asset, as a Statistic, and at Gunpoint

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Abstract

The Human Capital value (HK) and the Statistical Life Values (VSL) differ sharply in their empirical pricing of a human life. Rationalizing these differences is complicated by the absence of common theoretical and empirical grounds. We provide a unified theoretical framework to relate these two life valuation approaches and use this setting to introduce an alternative valuation calculated at Gunpoint (GPV) as the willingness to pay to avoid certain, instantaneous death. We then provide closed form expression for the different life valuations in the context a flexible, dynamic human capital model that we structurally estimate using PSID data.

Keywords: Value of Human Life, Human Capital, Value of Statistical Life, Hicksian Willingness to Pay, Equivalent Variation, Mortality, Structural Estimation.

JEL Classification: J17, D15, G11.
1 Introduction

Computing the value of a human life has long generated a deep interest in economic research.\(^1\) Indeed, life valuations are relied upon in public health and safety debates, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. They are also important in long-run debates on quality versus quantity of life to determine whether to spend more resources on innovations that foster consumption growth or on those that prolong life expectancy.\(^2\) Finally, economic life values are resorted to in wrongful death litigation, as well as in terminal care cost/benefit analysis.\(^3\)

The two most widely-used life valuation frameworks are the Human Capital (HK) and the Statistical Life (VSL) values.\(^4\) The HK value of a given life relies on asset pricing to compute the present value of the net cash flows associated with that person’s human capital, where the dividend is proxied by the marketed labor income, net of the measurable investment expenses. The Value of a Statistical Life, introduced by Drèze (1962) and Schelling (1968), corresponds to a stated, or inferred, marginal willingness to pay (MWTP) to avert (resp. attain) small increases (resp. reductions) in exposure to death risks.\(^5\) Whereas HK values are mainly used for identified life pricing, such as in litigation, the VSL’s domain of application relates to public health and safety decisions benefiting unidentified persons,\(^6\) as well as in societal debates on long-term consumption vs longevity choices. In practice, both HK and VSL valuations of a human life yield strikingly different measures with VSL estimates 10-20 times larger than HK values.\(^7\)

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\(^1\) Landefeld and Seskin (1982) make reference to human-capital based evaluations of the value of life dating back to Petty (1691).
\(^2\) See Jones (2016); Jones and Klenow (2016); Hall and Jones (2007); Murphy and Topel (2006); Becker et al. (2005) for discussions.
\(^3\) See Viscusi (2000, 2007) for legal uses of life values and Philipson et al. (2010); Round (2012) for end-of-life discussions.
\(^4\) See Mishan (1971) for early description and discussion of HK and VSL life values.
\(^5\) Under appropriate assumptions, a collective WTP to save one unidentified (i.e. statistical) life among the group can be recovered through a linear aggregation of the individual WTP’s. As a canonical example (e.g. Aldy and Viscusi, 2007), suppose \(n\) agents are individually willing to pay \(v(\Delta)\) to attain (avert) a small beneficial (detrimental) change \(\Delta = 1/n\) in death risk exposure and satisfying \(v(0) = 0\). The empirical VSL is the collective WTP: \(v_e(\Delta) = nv(\Delta) = v(\Delta)/\Delta\), i.e. the slope of the WTP approximating the MWTP.
\(^6\) See Viscusi (2000, 2007) for critical discussion of why the VSL is inappropriate relative to the HK in calculating compensating damages in wrongful death litigation.
\(^7\) Huggett and Kaplan (2016) identify HK values between 300 K–900 K$, whereas the U.S. Department of Transportation recommends using a VSL-type amount of 9.4 M$ (U.S. Department of Transportation, 2016).
Although it is well recognized that identified life values (like the HK) and unidentified ones (like the VSL) need not be equal (e.g. Conley, 1976; Shepard and Zeckhauser, 1984; Pratt and Zeckhauser, 1996; Viscusi, 2000, 2007), rationalizing these differences is complicated by the absence of common theoretical and empirical grounds linking both valuations. Indeed, most HK and VSL evaluations are reduced-form empirical exercises that neither share joint theoretical underpinnings, nor common database.

This void between the two approaches leaves open a number of important issues that we address in this paper. First, what are the necessary conditions (e.g. preference for life over death, transversality, appropriate rate of discounting, sign and cross-equations restrictions) for joint theoretical consistency of HK and VSL? Second, how do valuations of marginal risks to life (such as the WTP) extend to unitary ones (such as the HK and VSL)? Third, what is the role of technological, preference and distributional parameters, as well as agents’ statuses (e.g. financial and human wealth) in life valuations? Fourth, how are the HK, WTP and VSL related to each other and to alternative life valuation metrics. Finally, to what extent are these valuations empirically revealed by observed financial and human capital decisions of agents?

To answer these questions, we propose a unified framework yielding model-based WTP, HK and VSL life values that can be computed in closed-form and estimated from a common sample of data. To illustrate, we start from a generic life cycle problem in which an agent facing an uncertain horizon selects arbitrary controls (consumption, portfolio, . . . ), as well as investment in his human capital (e.g. skills or health), where the latter benefits labor income. The optimal investment and associated net income stream are then capitalized using the stochastic discount factor induced by the agent’s opportunity set to characterize the HK value. Moreover, the indirect utility calculated at the optimum can be combined with variational analysis (Hicks, 1946) to define the willingness to pay to prevent increases (or attain decreases) in mortality risk exposure. The marginal WTP defines the VSL, whereas the limiting WTP yields an alternative life value that can be used as convenient benchmark. In particular, we define Gunpoint Value of Life (GPV) as the Hicksian willingness to pay that leaves an agent indifferent between living and immediate death. As argued by Philipson et al. (2010), end-of-life care decisions are more akin to valuing life at gunpoint and are inappropriate settings for VSL-type valuations. The Gunpoint value can also complement HK values in wrongful
death litigation cases by establishing how much a person would value his own life, instead of having the market perform this valuation.

Next, we propose a rich parametrization of the encompassing human capital model to provide analytical calculations of the WTP, as well as of the HK, VSL and GPV values of life. Our framework departs from standard approaches in four key dimensions. First, we rely on source-dependent risk aversion extension of recursive utility (Epstein and Zin, 1989, 1991; Duffie and Epstein, 1992) that measures continuation utility units of consumption. Therefore, non-negative consumption requirements are associated with non-negative utility and thereby guarantee unconditional preference for life (positive utility) over death (zero utility) regardless of parametric values (Hugonnier et al., 2013). Moreover, the preferences we use allow to disentangle aversion toward different dimensions of risk from the elasticity of inter-temporal substitution (EIS). This feature not only reconciles theoretical with observed consumption and financial decisions, but allows for non-indifference over the resolution of death uncertainty’s timing and consequently much more flexible tradeoffs between quantity (i.e. longevity) and quality (i.e. consumption) of life. In particular, such a specification allows for high financial risk aversion (to justify low risky asset holdings) to coexist with both preference for life over death, as well as with high EIS, i.e. easy substitution between quality and quantity of life.

Our model features endogenous human capital accumulation, where the latter can be interpreted either as skills (e.g. Ben-Porath, 1967; Heckman, 1976) or as health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990) and that is subject to stochastic depreciation shocks (e.g. unemployment or illness). We rely on separation properties between human capital and other decisions (Bodie et al., 1992; Hugonnier et al., 2013; Acemoglu and Autor, 2018) to derive closed-form solutions for optimal consumption, portfolio, human capital investment, and insurance against depreciation shocks. Using these explicit solutions we then derive analytic expressions for the willingness to pay and the three other life values. With these expressions at hand we can pinpoint the contributions of fundamentals, such as preferences, risk distributions, or technology, as well as financial and human resources and thus investigate how the WTP, HK, VSL and GPV are theoretically related to one another. We also show how our setup can be further generalized to allow for source-dependent risk aversion against financial, independent aging processes, as well as for self-insurance against mortality and depreciation shocks.
We structurally estimate our parametrized model to provide estimates for the willingness to pay and the three alternative values of life induced by our unified framework. Towards that purpose, we associate human capital to health and adopt a revealed-preference perspective to estimate the model’s distributional, technological and preferences parameters. This is achieved by resorting to PSID data that correspond to the optimal consumption, portfolio, as well as health spending and insurance policies. The estimated deep parameters combined with observed wealth and health data allow us to calculate the analytical expressions for the willingness to pay, Human Capital, Statistical and Gunpoint Values of life. Our encompassing approach thus ensures that the WTP and the three different life values are computed through a single-step common estimation.

Our main findings are threefold. First, the properties of the indirect utility guarantee that the willingness to pay to avoid detrimental changes in exposure to death is increasing, concave and bounded in the mortality risk increment. Curvature stems directly from the non-linearities in risk probabilities that is made possible by recursive preferences, but abstracted from in VNM utility (Córdoba and Ripoll, 2017). It follows that the theoretical VSL (i.e. the marginal WTP) is under-estimated by the empirical VSL (the infra-marginal WTP over finite changes in death risk), and that both theoretical and empirical VSL’s are larger than the GPV (the WTP’s upper bound) corresponding to net total wealth. Indeed, we show that the ratio of the VSL to this limiting WTP is inversely proportional to the marginal propensity to consume (MPC). Since the MPC is typically much lower than one, the predicted VSL-GPV gap is positive and significant.

Second, unlike the VSL, the HK and GPV directly compute the value of a whole life, rather than linearly projecting the willingness to pay for marginal changes in death risk to recover a unitary value. Moreover, both reflect expected net present values of human capital dividends (HK) and of consumption above subsistence (GPV), and both are independent of preferences towards risk and time. For the HK and GPV, preference independence is explained by the separation between investment and other decisions under complete markets. For the GPV, preference independence is further explained by the nature of mortality in a Gunpoint valuation; because death is instantaneous and certain under a highwaymen threat, attitudes towards time substitution and towards risks are irrelevant.
Third, our empirical results confirm strongly diminishing MWTP and accord with the reduced-form HK and VSL estimates reported in the literature, with structural average values of 421 K$ (HK) and 8.35 K$ (VSL). The large HK-VSL discrepancy is therefore not alleviated under joint theoretical and empirical evaluations. Moreover, the average Human Capital value of life is close to the average Gunpoint benchmark (447 K$), as expected from the theoretical parallels between the two. The large VSL/GPV ratio of 18.66 is consistent with a realistic MPC estimate of 5.36%. The confirmation of large empirical VSL-HK differences, and the finding of HK-GPV similarities in an encompassing framework indicates that the linear extrapolation from a marginal value to a unitary life value when the WTP is strongly concave is the main reason behind the much larger VSL estimates. These findings add further weight to cautionary warnings in the literature that the VSL and HK are best perceived as complementary, rather than substitutes to one another.

After a review of the literature in Section 2, the rest of the paper is organized as follows. We introduce the HK, WTP, VSL and GPV in Section 3. Sections 4 and 5 present the parametrized model and corresponding life values. The empirical strategy is discussed in Section 6, with structural parameters and values of life estimates reviewed in Section 7. Concluding remarks are presented in Section 8.

2 Relevant literature

2.1 Most related

The literature most related to our analysis includes recent papers by Córdoba and Ripoll (2017); Hugonnier et al. (2013); Hall and Jones (2007); Murphy and Topel (2006). Córdoba and Ripoll (2017) concur with us on the relevance of recursive preferences for life valuation. In particular, they emphasize the importance of disentangling attitudes towards risk, from those towards time to allow for non-indifference with respect to the timing of the resolution of survival uncertainty, and to guarantee preference for life over death, even at high risk aversion levels. They also contend that more realistic curvature of the willingness to pay for survival can only be attained by allowing non-linear effects of death probabilities on utility. Their calibration emphasizes preference for late, rather

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8See Bommier et al. (2018) however for a discussion of limitations.
than early, resolution of death uncertainty, which is also confirmed by our structural estimation results.

The parametrized model of Córdoba and Ripoll (2017) is however quite different from ours. Their closest analog (Perpetual Youth, Section 3.2) is set in discrete (rather than continuous) time, and lets the agent select consumption only. It fully abstracts from our analysis of endogenous human capital accumulation, stochastic capital depreciation, risky portfolio and insurance choices and their main solutions for life values are characterized for hand-to-mouth consumers only. Moreover they emphasize mortality risk aversion as key determinant of life values in an homothetic recursive preferences specification. In our setting, the agent is risk-neutral with respect to mortality risk, so that the elasticity of inter-temporal substitution is the main driver of mortality preferences, and we allow for non-homotheticity in a recursive utility setting by introducing minimal consumption requirements. Finally, whereas they obtain closed-form solutions for the VSL, they do not explicitly compute the WTP, and fully abstract from both HK and GPV values of life.

Our parametrized setting directly borrows from Hugonnier et al. (2013). We consider a restricted case of this model where we abstract from source-dependent risk aversion as well as from endogenous exposure to mortality and morbidity risks. This simplifying choice is made for tractability reasons only. For completeness, we also estimate the unrestricted model and compute the corresponding life valuations. The results (in a separate technical appendix) confirm all our main findings. Moreover, whereas Hugonnier et al. (2013) do consider a VSL-inspired life valuation, their main emphasis is on separation between financial and health-related choices. The WTP, Gunpoint and HK values are therefore completely abstracted from.

Hall and Jones (2007) propose a semi-structural measure of life value akin to the VSL. They adopt a marginal value perspective by equating the latter to the marginal cost of saving a human life. The cost of reducing mortality risk can be imputed by estimating a health production function and by linking health status to death risks.

We do allow for financial risk aversion in our model, but abstract from aversion towards mortality and depreciation shocks. In a separate technical appendix, we show that our main results are robust to allowing source-dependent aversion to mortality and depreciation risks instead of neutrality.

More precisely, the WTP in their setup is simply the VSL times the change in death probability (see the equation before eq. (16)). Instead, we compute the WTP from Hicksian variational analysis and rely on its properties to characterize the VSL and Gunpoint values.
Dividing this marginal cost by the change in death risk yields a VSL-inspired life value, e.g. corresponding to 1.9 M$ for an individual aged 40-44 (Hall and Jones, 2007, Tab. 1, p. 60). Unlike Hall and Jones (2007) we do not measure the health production function through its effects on mortality, but estimate the technology through the measurable effects of investment on future health status. Moreover, our fully structural approach does not indirectly evaluate the marginal value of life via its marginal cost, but rather directly through the individual willingness to pay to avoid changes in death risks.

Finally, we share similarities with Murphy and Topel (2006) who also resort to a life cycle model with direct utilitarian services of health to study life valuations. In particular, both continuous-time approaches study permanent changes in Poisson death intensity, under perfect markets assumption, and both identify the VSL as a marginal rate of substitution between longevity and wealth. Both emphasize the key role of the elasticity of inter-temporal substitution in generating diminishing marginal values. However, contrary to Murphy and Topel (2006), whereas we abstract from nonmarket time (i.e. leisure), our human capital (i.e. health) is endogenously determined in a stochastic environment. Its non-linear effect entails that our VSL, as well as other life measures, are increasing in health, rather than health-independent. Importantly, whereas they posit an arbitrary process for consumption (see eq. (19), p. 885) and restrict their analysis to hand-to-mouth in their calibration, we solve for optimal consumption, portfolio, insurance, and health expenditures. This allows us to analyze and structurally estimate all life valuations – including the HK, WTP and GPV that are abstracted from in Murphy and Topel (2006) – through the prism of the indirect utility function.

2.2 Human Capital values of life

The HK model associates the economic value of a person to the value of his human capital that is entirely depreciated at death. The latter is obtained by pricing the expected discounted stream of cash-flows until the agent’s time of death, i.e. the lifetime labor income flows, net of associated investment. Well-known issues related to this approach

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11 Indeed, mortality is treated exogenously in our baseline model. The more general setup with endogenous death risk exposure in Section 7.3 yields similar empirical results.
12 See Jena et al. (2009); Huggett and Kaplan (2013, 2016) for different applications.
include the appropriate rate of discounting, the endogeneity of income and investment, as well as the treatment of non-labor activities.\textsuperscript{13}

As for HK models, we do calculate the net present value of the stream of human capital dividends that are lost upon death. Unlike HK models however, that value is computed in closed-form and relying on the stochastic discount factor induced by the assets under consideration in the model. In particular, we fully account for the endogeneity of the human capital stock and of its associated income and investment expenditures. We therefore encompass the relevant technological and distributional considerations, such as the capital production technology, its deterministic and stochastic depreciation, the income-capital gradient, as well as the duration of the dividends stream. Finally, the parametrized model is fully adaptable to non-labor valuation since the flow of marketed income related to human capital can also be equivalently recast as non-marketed utilitarian services (see Remark 1 below).

\subsection*{2.3 Value of a Statistical Life}

The empirical VSL alternative relies on explicit and implicit evaluations of the Hicksian WTP for a small reduction in fatality risk which is then linearly extrapolated to obtain the value of life. Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.\textsuperscript{14} Examples of the latter include responses to prices and fines in the use of life-saving measures such as smoke detectors, speed limitations, or seat belt regulations. The Hedonic Wage (HW) variant of the implicit VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/worker characteristics, the wage elasticity with respect to job fatality risk can be estimated and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008; Shogren and Stamland, 2002).

Ashenfelter (2006) provides a critical assessment of the VSL’s theoretical and empirical underpinnings. He argues that the assumed exogeneity of the change in fatality risk can

\textsuperscript{13}Conley (1976) provides additional discussion of HK approaches while Huggett and Kaplan (2016) address the discounting issues.

\textsuperscript{14}A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the implicit VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a \textit{meta-meta} analysis of the stated- and revealed-preferences valuations of life.
be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. He also argues that agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Ashenfelter further contends that it is unclear whose preferences are involved in the risk/income tradeoff and how well these arbitrage are understood. For example, if high fatality risk employment attracts workers with low risk aversion and/or high discount rates, then generalizing the wages risk gradient to the entire population could understate the true value of life. An argument related to Ashenfelter’s preferences indeterminacy can be made for the HW variant of the VSL. Because wages are an equilibrium outcome, they encompass both labor demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death.

Our approach addresses many of the issues raised by Ashenfelter (2006). First, we fully allow for endogenous adjustments in the optimal allocations resulting from changes in death risk exposure when we compute the willingness to pay and the VSL. Second, agency issues are absent as the agent bears the entire costs and benefits of changes in mortality. Third, whose preferences are at stake is not an issue as the latter are jointly estimated with the WTP and life valuations by resorting to a widely-used panel of households (PSID). Consequently, these values can safely be considered as representative of the general population. Fourth, labor demand considerations are absent as our partial equilibrium approach takes the return on investment as mortality-risk independent in characterizing the agent’s optimal human capital allocations. More fundamentally, we neither rely on the wage/fatality nexus, nor on any other proxy and we make no assumption on the shape of the WTP function but rather derive its properties from the indirect utility function induced by the optimal allocation.

Our results also confirm early conjectures on the pitfalls associated with personalizing unidentified VSL life valuations. Indeed, Pratt and Zeckhauser (1996) argue that concentrating the costs and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the dead anyway effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e. large groups
facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or high payment effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment, thereby reducing the marginal WTP as mortality exposure increases.\textsuperscript{15} Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our highwaymen threat. Their conjecture is warranted in our calculations. We show that the willingness to pay is finite and bounded above by the Gunpoint Value. Diminishing MWTP entails that the latter is much lower than what can be inferred from the VSL.

\textbf{2.4 Gunpoint value of life}

Early references to a Gunpoint value include Jones-Lee (1974) who analyzes the Hicksian Compensating Variation (CV) for changes in the probability of dying in a static setting. The extreme case where the latter tends to one corresponds to a willingness to accept compensation for imminent death. Jones-Lee (1974) shows that this WTA exists and is finite when the least upper bound on the utility at death (e.g. from bequeathed wealth) is large relative to reference expected utility. Our analysis abstracts from bequests and normalizes utility at death to zero, so that the Hicksian Equivalent Variation (EV), i.e. the WTP to avoid death is the appropriate Gunpoint measure and we show formally that it corresponds to the least upper bound on the WTP.

Other early references include Cook and Graham (1977) who study the demand for insurance against the loss of irreplaceable goods, defined as one where personal valuation considerations dominate market ones, i.e as having no readily identifiable market-provided replacement in the case of loss (e.g. a family pet, health, a spouse’s, or a child’s life). The willingness to pay to avoid this loss is defined as the Ransom value. If the ransom is a normal good (i.e. is increasing in wealth), Cook and Graham (1977) show that the state-dependent marginal utility of wealth, conditional on loss, is less than that of wealth minus ransom, conditional on no loss. The agent consequently optimally under-insures at actuarially fair contracts. Under sufficiently large wealth effects on ransom, the agent does

\textsuperscript{15}Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay $1 million to reduce the associated mortality risk by 10\%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.
not insure against the loss of the irreplaceable good, but against the associated wealth loss. For example, he then selects a life insurance against a spouse’s death corresponding to foregone income (plus eventual burial expenses) that has clear analogs to the HK value. Finally, they show that the MRS between wealth and death (corresponding to the VSL) is necessarily larger than the Ransom value.

Eeckhoudt and Hammitt (2004) rely on this framework to focus on the impact of risk aversion on four measures of life value: the VSL, the WTP to fully eliminate death risk (i.e. \( P > 0 \rightarrow P^* = 0 \)), or to partially lower it (i.e. \( P > 0 \rightarrow P^* < P_0 \)) and the WTP to eliminate the certainty of death (i.e. \( P = 1 \rightarrow P^* = 0 \)). The latter corresponds to Cook and Graham (1977)’s Ransom value where the irreplaceable good is one’s own life. In the special case where both the utility and marginal utility of wealth at death are zero (e.g. in the absence of bequest value), they confirm that the Ransom value is the agent’s wealth and is independent of attitudes toward risk.

The Ransom value of Cook and Graham (1977); Eeckhoudt and Hammitt (2004) is clearly related to the Gunpoint value as both depend on Hicksian WTP to avoid certain death in gauging a person’s own value. The main difference is that we do not rely on a generic utility, but instead we base our analysis on the indirect utility associated to a dynamic human capital problem, with source dependent risk aversion to characterize the WTP, VSL and GPV. This approach allows us to encompass the HK value as well, to link the different measures and to fully identify the role of preferences, distributional and technological parameters on life valuation.

Implicit references to a GPV are also found in the context of end-of-life care. For example, Philipson, Becker, Goldman and Murphy (2010) contend that “[the VSL] is often prefaced with claiming that it is not how much people are willing to pay to avoid having a gun put to their head (presumably one’s wealth). However, terminal care decisions are often exactly of that nature” (Philipson et al., 2010, p. 2, emphasis added). We confirm their conjecture that financial wealth is entirely pledged in a highwaymen threat, however we show that so is the agent’s human wealth. Since our application associates the latter to health, we thus provide explicit adjustment for an agent’s health status in his life valuations in the spirit of the Quality-Adjusted Life Years (QALY, e.g. Round, 2012). One could also argue that HK measures are inappropriate in terminal care situations where agents are unable to work. The GPV we propose handles such case...
by equivalently associating the value of health capital to the utilitarian services it can provide (see Remark 1).

Finally, Murphy and Topel (2006) also implicitly refer to a Gunpoint value in their parametrized analysis of the value of a life year (i.e. utility and net savings at given age). Indeed, commenting on a key variable in their VSLY analysis, they write that “[t]he ratio $z_0/z$ asks how much of current composite consumption individuals would sacrifice before they would rather be dead” (p. 885). However, a closer analysis reveals that this ratio, which they calibrate between 5-20% of composite consumption, rather corresponds to a minimal consumption ratio in their non-homothetic VNM preferences. Indeed, these values accord well to our calibration yielding a minimal consumption ratio representing 14.23% of consumption. Whereas we show that the Gunpoint value i.e. the total wealth that leaves the agent indifferent between life and death corresponds to the expected discounted value of the lifetime consumption stream, and is therefore much larger than minimal consumption.

3 A Framework for Life Valuation

We follow a long literature in focusing on Hicksian variation obtained from the indirect utility function induced by a life cycle model to characterize the willingness to pay, VSL and HK (e.g. Conley, 1976; Shepard and Zeckhauser, 1984; Rosen, 1988; Murphy and Topel, 2006). We depart from that literature by endogenizing human capital investment, by explicitly resorting to asset pricing of the net dividends flow to compute the HK value at the optimum, and by relying on the WTP to contrast the Statistical and Gunpoint Values of Life. The corresponding analytical expressions for a parametrized model (presented in Section 4) will be computed in Section 5.

3.1 Generic Human Capital Problem

Consider a continuous-time human capital problem where the agent’s planning horizon is limited by a stochastic age at death $T_m$ satisfying:

$$\mathcal{P}(t) = \Pr(T_m \leq t) = 1 - e^{-\lambda_m t}. \quad (1)$$
for some constant $\lambda_m > 0$ that models the arrival rate of death. In the subsequent analysis we focus on changes in the death exposure $P = P(t)$ at a given horizon $t > 0$ that we normalize to one.$^{16}$

Denote the agent’s financial wealth by $W$ and assume that his income rate is given by some increasing function $Y(H)$ of his human capital by $H$ (e.g. skills, training, or health). The agent has access to a complete financial market and selects the money value $I$ of investment in his human capital and as well as a set $X$ of other controls (e.g. consumption, asset allocation, . . . ) to maximize a utility index $U$ subject to the law of motion of his financial wealth and human capital. The indirect utility function associated with this problem is defined by

$$V(W, H; P) = \sup_{(I, X)} U,$$

s.t. $$dH = dH(H, I),$$
$$dW = dW(W, Y(H), I, X).$$

and we assume that the agent’s preferences and constraints are such that this indirect utility function is increasing in $H$, increasing and concave in $W$, and decreasing and convex in $P$. This guarantees that:

$$V(W, H; P) \geq V^m \equiv V(W, H, 1) > -\infty, \quad \forall W, H; P,$$

i.e. the agent exhibits weak preference for life over death,$^{17}$ and that his indirect utility admits decreasing and convex indifference curves in the wealth and life probability space as illustrated in Figure 1.

### 3.2 Human Capital Value of Life

The Human Capital Value of life is the market value of the net dividend flow associated with human capital and that is foregone upon death (e.g. Huggett and Kaplan, 2016).

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$^{16}$Since the death probability $P(t)$ is a strictly increasing function of the intensity parameter $\lambda_m$ we have that an increase in $P(1)$ is equivalent to an increase of $\lambda_m$ and thus induces a upward shift in the mortality exposure of the agent at every horizon. (See Murphy and Topel, 2006, for a similar perspective on changes in death intensity).

$^{17}$See Yaari (1965); Hakansson (1969) or Yogo (2016); French and Jones (2011); De Nardi et al. (2009) for different modellings of the utility at death $V_m$. 
In our setting, this net dividend is the marketed income, minus the money value of investment expenses, where both are evaluated at the optimum to problem (2):

**Definition 1 (HK value of life)** The Human Capital Value of life is

\[
v_{h,t} = E_t \int_t^{T_m} \frac{m_{t \tau}}{m_t} (Y(H^*_\tau) - I^*_\tau) \, d\tau,
\]

where \( m_t \) is the stochastic discount factor induced by the prices of financial assets, \( I^* \) denotes the agent’s optimal human capital investment, and \( H^* \) denotes the corresponding path of his human capital process.

As a canonical example, assume that \( m_t = e^{-rt} \) for some constant interest rate \( r \) and that the net cash flow of the agent at the optimum is given by \( Y(H^*) - I^* = Y(0) + Y_n(H^*) \), where \( Y_n(H^*) \) grows at some constant rate \( g_n < r + \lambda_m \) along the optimal path. The HK value in (4) then simplifies to:

\[
v_h = \frac{Y(0)}{r + \lambda_m} + \frac{Y_n(H^*)}{r + \lambda_m - g_n}.
\]

The human capital value of life in this special case is therefore decreasing in both the death risk \( \lambda_m \) and interest rate \( r \) and is increasing in both \( Y(0), Y_n(H) \), as well as the growth rate \( g_n \).

### 3.3 Willingness to pay

Next, consider a change in the probability of death from base level \( \mathcal{P} \) in (1). We rely on the indirect utility (2) to define the Hicksian Equivalent and Compensating Variations as follows:

**Definition 2 (Hicksian Variation)** Consider a permanent change \( \Delta \in [-\mathcal{P}, 1-\mathcal{P}] \) in base death risk exposure \( \mathcal{P} \).

1. The Equivalent Variation \( v^e = v^e(W, H; \mathcal{P}) \) to avoid \( \Delta \) is implicitly given as the solution to:

\[
V(W - v^e, H; \mathcal{P}) = V(W, H; \mathcal{P} + \Delta),
\]
2. The Compensating Variation \( v^c = v^c(W, H; \mathcal{P}) \) to attain \( \Delta \) is implicitly given as the solution to:

\[
V(W - v^c, H; \mathcal{P} + \Delta) = V(W, H; \mathcal{P}), \tag{6b}
\]

where \( V(W, H; \mathcal{P}) \) solves (2) and satisfies (3).

Definition 2 characterizes both the willingnesses to pay (WTP) and to accept (WTA) from Equivalent and Compensating variational perspectives. For unfavorable changes \( \Delta > 0 \), the EV (6a) indicates a willingness to pay \( v^e > 0 \) to remain at base risk instead of facing higher mortality, whereas the CV (6b) defines a willingness to accept compensation \(-v^c > 0\) for higher death risk. For favorable changes \( \Delta < 0 \), the EV is a WTA equal to \(-v^e > 0\) to forego lower risk, whereas the CV \( v^c > 0 \) is the WTP to attain the reduction in mortality. The analysis of the WTP to avoid imminent death risk in a Gunpoint setting naturally designates the Hicksian EV measure (6a) as the relevant willingness to pay. Indeed, whereas paying out the WTP in a highwaymen threat is rational, accepting compensation against certain death when terminal wealth in not bequeathed and life is preferred to death cannot be. Since we abstract from bequests in our benchmark model in Section 4, we therefore adopt the EV perspective in (6a) and we will henceforth set \( v = v^e \) as the WTP to avert unfavorable risks in subsequent analysis (see Remark 2 below for further discussion of EV-CV links).

The properties of the willingness to pay with respect to the increment in death risk \( v \) follow directly from those of the indirect utility \( V(W, H; \mathcal{P}) \) in (2). In particular, consider the case where preference for life in (3) induces a decreasing and convex indirect utility with respect to the death probability \( \mathcal{P} \). We can substitute \( v \) in (6a), take derivatives and re-arrange to obtain:

\[
v_\Delta = -\frac{V_P}{V_W} \geq 0, \tag{7a}
\]

\[
v_{\Delta \Delta} = \frac{V_{PP} - V_{WW}v_\Delta^2}{-V_W} \leq 0, \tag{7b}
\]

where a subscript denotes a partial derivative. Monotonicity \( V_W \geq 0 \) and preference for life over death \( V_P \leq 0 \) therefore induce a willingness to pay \( v \) that is increasing in \( \Delta \),
whereas the diminishing marginal utility of wealth $V_{WW} \leq 0$ and of survival probability $V_{PP} \geq 0$ are sufficient to induce a concave WTP function in mortality risk exposure.

### 3.4 Value of Statistical Life

The VSL is a measure of the marginal rate of substitution between the probability of life and wealth, evaluated at base risk (e.g. Aldy and Smyth, 2014; Andersson and Treich, 2011; Bellavance et al., 2009; Murphy and Topel, 2006; Eeckhoudt and Hammitt, 2004). In the context of our framework and relying on the WTP property (7a), the VSL can be defined as follows:

**Definition 3 (VSL)** *The Value of a Statistical Life $v_s = v_s(W, H; P)$ is the negative of the marginal rate of substitution between the probability of death and wealth computed from the indirect utility $V(W, H; P)$ evaluated at base risk:*

$$v_s = -\frac{V_P(W, H; P)}{V_W(W, H; P)}$$

where $V(W, H; P)$ solves (2) and satisfies (3).

Figure 1 illustrates the indifference curve (in blue) in the wealth and life probability space. The VSL in (8a) is the slope of the dashed red tangent evaluated at base death risk $P$ and is equivalent to the total wealth spent to save one life corresponding to the distance $[a, d]$ (e.g. Andersson and Treich, 2011, Fig. 17.1, p. 398). Equivalently, the marginal rate of substitution between life and wealth in (8a) is implicitly associated to the relative price of a (non-marketable) life.

We can rely on the WTP property (7a) to rewrite the VSL in (8a) as a marginal willingness to pay:

$$v_s(W, H; P) = \frac{\partial v(W, H; P, \Delta)}{\partial \Delta} = \lim_{\Delta \to 0} \frac{v(W, H; P, \Delta)}{\Delta}.$$  

Contrasting the theoretical definition of the VSL as a MWTP in (8b) with its empirical counterpart reveals the links between the two measures. Indeed, the empirical VSL commonly relied upon in the literature (e.g. see footnote 5) can be expressed as:

$$v_s^e(W, H; P, \Delta) = \frac{v(W, H; P, \Delta)}{\Delta},$$

16
for small increment $\Delta = 1/n$, where $n$ is the size of the population affected by the change. The theoretical measure of the VSL in (8b) is the limiting value of its empirical counterpart in (9) when the change $\Delta$ tends to zero. The importance of the bias between the empirical and theoretical VSL’s $(v_s^e - v_s)$ will consequently depend on the curvature of the willingness to pay $v$, as well as on the size and sign of the change $\Delta$, an issue to which we will return shortly.

### 3.5 Gunpoint Value of Life

We next introduce the Gunpoint Value (GPV) as a third approach to the valuation of life. To do so, we combine preference for life (3) with the Hicksian Equivalent Variation in (6a) to define the GPV as follows:

**Definition 4 (GPV)** *The Gunpoint Value $v_g$ is the WTP to avoid certain, instantaneous death and is implicitly given as the solution to:*

$$V(W - v_g, H; \mathcal{P}) = V^m$$  \hspace{1cm} (10)

where $V(W, H; \mathcal{P})$ solves (2) and $V^m$ is the utility at death.

The willingness to pay $v_g$ can be interpreted as the maximal amount paid to survive an *ex-ante* unforecastable and *ex-post* credible highwaymen threat. It also corresponds to the Ransom value for irreplaceable goods introduced by Cook and Graham (1977) and analyzed by Eeckhoudt and Hammitt (2004) in the context of one’s own life. The main difference is that the GPV is defined through the indirect utility $V(W, H; \mathcal{P})$ in the context of a specific dynamic problem, instead of a generic utility $U(W; \mathcal{P})$. Note that the weak preference for life (3) ensures that indifference between life and death $V = V^m < -\infty$ exists and therefore that a Gunpoint value may be computed.

Several points are worth mentioning in comparing the GPV with the HK and VSL. First, unlike the HK, the Gunpoint Value does not uniquely ascribe the economic worth of an agent to the capitalized net labor income that agent generates. Second, unlike the VSL, the GPV does not extrapolate measurable responses to small probabilistic changes in the likelihood of death, but instead explicitly values a person’s life as an entity and does so without external assumptions regarding integrability from marginal to total value.
of life. Finally, the GPV is theoretically computable at any death intensity and applicable in life-or-death situations. As such, it is well suited in end-of-life terminal care decisions where neither the HK, nor the VSL are appropriate (Philipson et al., 2010).

3.6 Connections between the WTP, VSL and the GPV

Figure 2 illustrates the central role of the willingness to pay in linking the GPV to the theoretical and empirical VSL. From properties (7), the WTP $v = v(W, H; P, \Delta)$ (solid blue line) is an increasing, concave function of the change in death risk $\Delta \in [-P, 1-P]$ when the indirect utility $V(W, H; P)$ is decreasing and convex in $P$.

The theoretical VSL $v_s$ in (8b) is the marginal willingness to pay, i.e. the slope of the dashed red tangent evaluated at base death risk ($\Delta = 0$). It is equivalent to the linear projection corresponding to the total wealth spent to save one person (i.e. when $P + \Delta = 1.0$) and is equal to the distance $[a,f]$. The empirical Value of a Statistical Life $v_e$ in (9) is computed for a small change $\Delta^e > 0$ and is the slope of the dashed-dotted green line; equivalently, it is the linear projection represented by the distance $[b,e]$. The empirical VSL measure $v_e$ will thus understate its theoretical counterpart $v_s$ when $\Delta^e \gg 0$ and when the WTP is concave. Moreover, as will become clear shortly, the Gunpoint value corresponds to an admissible upper bound on the WTP, i.e. the limiting WTP when death is certain as represented by the distance $[c,d]$ in Figure 2. A concave WTP entails that a linear extrapolation under either the theoretical, or the empirical VSL will thus overstate the Gunpoint value attributed to one’s own life.

4 A Parametrized Human Capital Model

We now parametrize the generic human capital model in Section 3.1 to compute the willingness to pay and theoretical life values defined in Sections 3.2–3.5.

4.1 Economic environment

Consider a stochastic, depreciable human capital process $H_t$ whose law of motion is given by:

$$dH_t = \left(I_t H_t^{1-\alpha} - \delta H_t\right) dt - \phi H_t dQ_{st}. \quad (11)$$
In this equation, the Cobb-Douglas parameter $\alpha \in (0, 1)$ captures diminishing returns to investment, $\delta > 0$ captures the gradual deterministic depreciation of human capital absent investments, and $dQ_{st}$ is the increment of a Poisson process with constant intensity $\lambda_s$ whose jumps depreciate the capital stock by a factor $\phi \in (0, 1)$.

The law of motion (11) applies to alternative interpretations of human capital. If $H_t$ is associated with skills (e.g. Ben-Porath, 1967; Heckman, 1976), then investment $I_t$ comprises education and training choices made by the agent whereas $dQ_{st}$ can be interpreted as stochastic unemployment, or technological obsolescence shocks that depreciate the human capital stock. If $H_t$ is instead associated with health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), then investment takes place through medical expenses or healthy lifestyle decisions whereas the stochastic depreciation occurs through morbidity shocks.

The agent’s income is given by:

$$Y_t = Y(H_t) = y + \beta H_t,$$

and includes both an exogenous base income $y$ and a positive income gradient $\beta$ for higher human capital. Individuals can trade in two risky assets to smooth out shocks to consumption: stocks and insurance against human capital depreciation. Financial wealth $W_t$ evolves according to the dynamic budget constraint:

$$dW_t = (rW_t + Y_t - c_t - I_t) \, dt + \pi_t \sigma_S \, (dZ_t + \theta \, dt) + x_t \, (dQ_{st} - \lambda_s \, dt),$$

where $r$ is the interest rate, $\sigma_S > 0$ is the volatility of the traded risky asset, and $\theta = (\mu - r)/\sigma_S$ is the market price of financial risk. In addition to investment $I_t$, the control variables include consumption $c_t$, the risky portfolio $\pi_t$ and the number of units $x_t$ of actuarially-fair depreciation insurance. The latter pays one unit of the numeraire upon the occurrence of a depreciation shock, and can be interpreted as unemployment insurance (if $H_t$ is associated with skills) or as medical, or disability insurance (if $H_t$ is associated with health).
Following Hugonnier et al. (2013) we define the indirect utility of an alive agent as:

$$V(W_t, H_t) = \sup_{(c, \pi, x, t)} U_t,$$

where preferences are given by

$$U_t = E_t \int_t^{T_m} \left( f(c_{\tau}, U_{\tau}) - \frac{\gamma|\sigma_{\tau}(U)|^2}{2U_{\tau}} \right) d\tau,$$

with the Kreps-Porteus aggregator function

$$f(c, u) = \frac{\rho u}{1 - \frac{1}{\epsilon}} \left( \left( \frac{c - a}{u} \right)^{1 - \frac{1}{\epsilon}} - 1 \right).$$

The preference specification in (14) is a source dependent risk aversion extension of the stochastic differential utility proposed by Duffie and Epstein (1992) as the continuous-time analog of the discrete-time recursive preferences of Epstein and Zin (1989, 1991). It is characterized by a subjective discount rate $\rho > 0$, a minimal subsistence consumption level $a > 0$, risk-neutrality with respect to both depreciation shocks and death, and disentangles the agent’s elasticity of inter-temporal substitution ($EIS$) $\varepsilon \geq 0$, from his constant relative risk aversion with respect to financial risk $\gamma \geq 0$. As explained in Hugonnier et al. (2013) and confirmed in Theorem 1 below, the homogeneity properties of our specification implies that any feasible consumption process is associated with a positive continuation utility and therefore guarantees weak preference of life over death.

**Remark 1** The model assumes that the sole motivation for investing in $H_t$ relates to its positive effects on marketed income in (12). However, the valuation of human capital can also be made with respect to its non-marketed services. Indeed, the model can be adapted for non-workers by first defining $\tilde{c}_t = c_t - \beta H_t$, then eliminating $\beta H_t$ in the income equation (12) and finally replacing for $c_t = \tilde{c}_t + \beta H_t$ in the budget constraint (13) and preference equations (14). The agent then selects $\tilde{c}_t$ and the other controls taking into account the beneficial utilitarian flow of human capital. As shown in Hugonnier

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18See Section 7.3 below for an extension in which the agent is differently risk averse towards these two sources of risk.

19The generalized model in Section 7.3 allows for additional beneficial effects of human capital on morbidity and mortality risk exposure.
et al. (2013, Remark 3), the theoretical results are unaffected under this alternative interpretation.

4.2 Optimal rules

The agent’s dynamic problem (14), subject to (11) and (13) can either be solved directly through the Hamilton-Jacobi-Bellman (HJB) or in two separate stages, following the method outlined in Hugonnier et al. (2013). The two-step approach involves:

1. An hypothetical infinitely-lived agent first solves the optimal investment by maximizing the discounted value of the $H$-dependent part of net income:

$$P(H_t) = \sup_{I \geq 0} E_t \int_t^\infty \frac{m_\tau}{m_t} (\beta H_\tau - I_\tau) \, d\tau.$$ (15)

where

$$m_t = \exp \left( -rt - \theta Z_t - \frac{1}{2} \theta^2 t \right).$$

is the stochastic discount factor induced by the prices of financial assets. The human wealth $P(H)$ is then combined with the agent’s financial wealth and the present value of his base income stream net of minimal consumption expenditures to obtain the agent’s net total wealth as:

$$N(W_t, H_t) = W_t + \frac{y-a}{r} + P(H_t),$$

$$= W_t + E_t \int_t^\infty \frac{m_\tau}{m_t} (Y(H_\tau^*) - I_\tau^* - a) \, d\tau.$$ (16)

An important consequence of this characterization is that, due to complete financial markets, the agent’s optimal human capital investment can be determined independently of his preferences.

2. The finitely-lived agent then selects the remaining policies $\bar{c}_t = c_t - a$, $\pi_t$ as well as $\bar{x}_t = x_t - \phi P(H_t)$ by maximizing utility (14), subject to the law of motion for net

\footnote{See also Bodie et al. (1992); Acemoglu and Autor (2018) for discussion of separability in human capital problems.}
total wealth:

\[ dN_t = (rN_t - \bar{\epsilon}_t)dt + \pi_t\sigma_S(dZ_t + \theta dt) + \bar{x}_t(dQ_{st} - \lambda_s dt). \]

The remaining optimal consumption, portfolio and insurance policies, as well as indirect utility function are calculated as functions of \( P(H_t) \) and \( N(W_t, H_t) \) and encompass explicit adjustments for finite lives where appropriate.

In the context of our parametric model both of the above optimization problems can be solved in closed form. Putting together these explicit solutions leads to the following result.

**Theorem 1** Assume that the parameters of the model are such that

\[ \beta < (r + \delta + \phi \lambda_s)^{\frac{1}{2}}, \tag{17a} \]

and

\[ A(\lambda_m) = \varepsilon r + (1 - \varepsilon) \left( r - \lambda_m + \frac{1}{2} \frac{\theta^2}{\gamma} \right) > \max \left( 0, r - \lambda_m + \frac{\theta^2}{\gamma} \right). \tag{17b} \]

Then,

1. the human wealth and net total wealth are given as:

   \[ P(H_t) = BH_t, \tag{18} \]
   \[ N(W_t, H_t) = W_t + \frac{y - a}{r} + P(H_t), \tag{19} \]

   where \( B > 0 \) is the unique solution to

   \[ g(B) = \beta - (r + \delta + \phi \lambda_s)B - (1 - 1/\alpha)(\alpha B)^{-\frac{1}{\alpha}} = 0 \tag{20} \]

   such that \( g(B) = 0 \); and
2. The indirect utility for the agent’s problem is:

\[ V_t = \Theta(\lambda_m)N(W_t, H_t) \geq 0, \]  
\[ \Theta(\lambda_m) = \hat{\rho}A(\lambda_m)^{\frac{1}{1-\varepsilon}} \geq 0, \quad \hat{\rho} = \rho^{\frac{\varepsilon}{1-\varepsilon}} \]  

and generates the optimal rules:

\[ c^*_t = a + A(\lambda_m)N(W_t, H_t) \geq 0, \]  
\[ \pi^*_t = \left(\frac{\theta}{(\gamma \sigma S)}\right)N(W_t, H_t), \]  
\[ x^*_t = \phi P(H_t) \geq 0, \]  
\[ I^*_t = \left(\alpha^{\frac{1}{1-\varepsilon}}B^{\frac{\varepsilon}{1-\varepsilon}}\right)P(H_t) \geq 0, \]  

where any dependence on death intensity \( \lambda_m \) is explicitly stated.

The regularity conditions (17) are required to ensure positive marginal propensity to consume out of net wealth \( A > 0 \), for minimal consumption requirements \( c_t > a \) and for appropriate transversality restriction for both the value function and the shadow value of human capital. The constant \( B \) in (18) can naturally be interpreted as the marginal value (i.e. Tobin’s \( Q \)) associated with human capital. It is implicitly defined in (20) as an increasing function of the income gradient \( \beta \) and a decreasing function of the rate of interest \( r \) and the expected depreciation rate \( \delta + \phi \lambda_s \).

Three features of the optimal rules are particularly relevant for life valuation. First, the two-step solution method ensures that both human wealth (18) and the net total wealth (19) are independent of the death intensity \( \lambda_m \). Second and related, the exposure to exogenous death risk \( \lambda_m \) affects welfare via \( \Theta(\lambda_m) \) in (21b), through its impact on the marginal propensity to consume (MPC) \( A(\lambda_m) \) exclusively. Equation (17b) establishes that this MPC effect crucially depends on the elasticity of inter-temporal substitution \( \varepsilon \).

An increase in death risk \( \lambda_m \) induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume. On the one hand, more discounting of future consumption requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC). This effect is dominant at low elasticity of inter-temporal substitution \( \varepsilon \in (0, 1) \) for which the MPC in (17b) is monotone decreasing. On the other hand, heavier discounting makes future consumption less desirable and shifts
future towards current consumption (i.e. by increasing the MPC). This *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution \( \varepsilon > 1 \). Unlike VNM, separate \( \gamma, \varepsilon \) entail that a high EIS can coincide with high risk aversion.

Third, the welfare in (21) is increasing in both wealth and human capital stock and is decreasing and convex in the death intensity \( \lambda_m \) at all EIS levels since:

\[
\begin{align*}
\Theta'(\lambda_m) &= -\tilde{\rho}A(\lambda_m)^{1-\varepsilon} \leq 0, \\
\Theta''(\lambda_m) &= \tilde{\rho}\varepsilon A(\lambda_m)^{2\varepsilon-1} \geq 0.
\end{align*}
\]

Hence, whereas the sign of the effects of death risk \( \lambda_m \) on the MPC (17b) depends on the EIS, preference for life implies that higher mortality exposure always reduces the marginal value of net total wealth (21b) and therefore lowers welfare in (21a). Importantly, as shown in (7), a decreasing and convex effect of death risk on welfare entails that the willingness to pay is increasing and concave with respect to changes in the latter.\(^{21}\)

## 5 Willingness to Pay and Values of Life

We next calculate the model-implied life valuations of Section 3 relying on the solution for the parametrized human capital model of Section 4. We assume throughout this section that the parameters of the model satisfy the regularity conditions of Theorem 1 and abstract from time subscripts whenever possible to alleviate notation.

### 5.1 Human Capital Value of Life

Recall that the agent’s investment opportunity set induces a unique stochastic discount factor given by (15). Using this process to compute the HK value of life leads to the following result.

**Proposition 1 (HK value)** The Human Capital Value of life solving (4) is:

\[
v_h(H, \lambda_m) = C_0 \frac{y}{r} + C_1 P(H)
\]

\(^{21}\)Using the fact that the death intensity \( \lambda_m \) and the base mortality exposure \( P \) are related by \( \lambda_m = -\log(1-P) \) it is easily shown that the indirect utility function is decreasing and convex in \( P \), as required to obtain the properties assumed in (7).
where the constants \((C_0, C_1) \in [0, 1]^2\) are defined by:

\[
C_0 = \frac{r}{r + \lambda_m}, \tag{25a}
\]

\[
C_1 = \frac{r - (\alpha B)^{\frac{1}{1-\alpha}}}{r + \lambda_m - (\alpha B)^{\frac{1}{1-\alpha}}}, \tag{25b}
\]

and where human wealth \(P(H)\) is given in (18).

Combining Definition (4) and the income equation (12) establishes that the HK value is the present value of the net income flow \(y + \beta H - I^*\). Unlike in step-1 of the solution method, this present value is computed over a finite horizon and must be therefore be corrected for mortality exposure \(\lambda_m\) in the constants \(C_0\) and \(C_1\). Similar to the canonical HK value in (5), the first term in (24) is the present value \(y/r\) of the agent’s base income \(y = Y(0)\) calculated over an infinite horizon and corrected for the exposure to death risk by multiplying with the constant \(C_0 \in [0, 1]\) in (25a). The second term is the net present value \(P(H)\) of the human capital cash flow \(\beta H_t - I^*\) over an infinite horizon and this value is corrected for finite life by multiplying with the constant \(C_1 \in [0, 1]\) in (25b).

5.2 Willingness to pay to avoid a change in death risk

We can next substitute the indirect utility \(V(W, H, \lambda_m)\) given by (21a) in Definition 2 and solve the resulting equation for the willingness to pay to avoid a change \(\Delta\) in the death intensity.\(^{22}\) For that purpose, only the admissible changes for which the indirect utility remains well defined when evaluated at the modified death intensity may be considered. Specifically, we denote by \(\mathcal{A}\) the set of \(\Delta \geq -\lambda_m\) such that the conditions of Theorem 1 hold when the mortality intensity \(\lambda_m\) is modified to \(\lambda_m^* = \lambda_m + \Delta\). With this notation we have the following result:

**Proposition 2 (willingness to pay)** The Equivalent Variation willingness to pay to avoid an admissible change \(\Delta \in \mathcal{A}\) is explicitly given by

\[
v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right] N(W, H). \tag{26}
\]

\(^{22}\)Studying the willingness to pay to avoid a change in the death intensity \(\lambda_m\) rather than to avoid a change in the base death exposure \(P\) allows to derive simpler formulas but is without loss of generality. Indeed, since \(P = 1 - e^{-\lambda_m}\) we have that a change of size \(\Delta\) in the death intensity corresponds to a change of size \(e^{-\lambda_m}(1 - e^{-\Delta})\) in the base death exposure \(P\).
It is increasing and concave in $\Delta$ with

\[
\inf_{\Delta \in A} v(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(0)}{\Theta(\lambda_m)} \right] N(W, H) \quad (27a)
\]

\[
\sup_{\Delta \in A} v(W, H, \lambda_m, \Delta) = N(W, H). \quad (27b)
\]

where net total wealth $N(W, H)$ is given in (19) and its marginal value $\Theta(\lambda_m)$ is given in (21b).

The WTP in (26) equals zero if either $\Delta = 0$ or if the agent’s elasticity of inter-temporal substitution $\varepsilon = 1$ because in this case the marginal utility of total wealth $\Theta$ is independent from $\lambda_m$. For the other cases, it was shown earlier that $\Theta(\lambda_m) \geq 0$ in (21b) is a decreasing and convex function. Consequently, the weights $\Theta(\lambda^*_m)/\Theta(\lambda_m) \in [0, 1]$ for detrimental changes $\Delta \geq 0$ and the willingness to pay is an increasing function of net total wealth $N(W, H)$.

Equation (27a) establishes that the lower bound on the WTP is obtained by setting $\Delta = -\lambda_m$ yielding the WTA a compensation in order to forego zero death risk exposure. From equations (17b) and (21b) this bound exists and is finite. Equation (27b) establishes that the willingness to pay is bounded above by net total wealth $N(W, H)$. When the elasticity of inter-temporal substitution is larger than one, this upper bound corresponds to the asymptotic WTP. When the EIS is below one, the upper bound corresponds to a maximal admissible WTP satisfying the transversality constraint (17b) (see Appendix B.3).

**Remark 2** A similar reasoning allows to solve the Hicksian Compensating Variation in (6b) as:

\[
v^c(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda_m)}{\Theta(\lambda^*_m)} \right] N(W, H), = \frac{-\Theta(\lambda_m)}{\Theta(\lambda^*_m)} v(W, H, \lambda_m, \Delta).
\]

Since $\Theta'(\lambda_m) < 0$, it follows that $0 < v^c < -v$ for $\Delta < 0$ and $0 < v < -v^c$ for admissible $\Delta > 0$, i.e. the WTP to attain a beneficial or avert a detrimental change in death risk is always less that the corresponding WTA to forego a favorable or accept an unfavorable change in mortality, consistent with standard Hicksian variational analysis (e.g. Hammitt, 2008; Smith and Keeney, 2005).

23See also Eeckhoudt and Hammitt (2004) for a WTP to fully eliminate mortality risk.
5.3 Value of a Statistical Life

Using Definition 3 and welfare (21), we can calculate the theoretical expression for the VSL for the parametrized model as follows.

**Proposition 3 (Value of Statistical Life)** The Value of a Statistical Life is:

\[ v_s(W, H, \lambda_m) = \frac{1}{A(\lambda_m)} N(W, H), \]  

(28)

where the marginal propensity to consume \( A(\lambda_m) \) is given in (17b) and net total wealth \( N(W, H) \) is given in (19).

The Value of a Statistical life reflects the marginal rate of substitution between wealth and life. It is unconditionally decreasing in the MPC and increasing in net worth. Observe that since the MPC is typically low (e.g. see Carroll, 2001, for a review), the VSL is expected to be significantly larger than net disposable resources \( N(W, H) \), an issue to which we will return shortly.

**Remark 3 (empirical VSL as a collective WTP)** We can rely on our theoretical measure for the individual WTP to compute the collective willingness to pay to save a human life. Given a finite population of agents indexed \( j \in \{1, 2, \ldots, n\} \) and a set of social weights \( \eta \in \mathbb{R}_+^n \), we can assume homogeneous parameters \( \theta = \theta, \forall j \) and exploit the linearity of the WTP function (26) in wealth and human capital to derive the collective WTP as:

\[
\sum_{j=1}^{n} \eta_j v_j(W_j, H_j, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda_m)}{\Theta(\lambda_m^*)} \right] \sum_{j=1}^{n} \eta_j N(W_j, H_j).
\]

(29)

Two special cases of identical weights \( \eta \) in (29) are worth mentioning:

1. Proportional weights \( \eta_j = 1/n \) yield:

\[
\bar{v}(W, H, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)} \right] N(\bar{W}, \bar{H}) = v(\bar{W}, \bar{H}, \lambda_m, \Delta).
\]

where \( \bar{H} \) and \( \bar{W} \) denote the weighted averages of agents’ human capital and financial wealth. The collective WTP – corresponding to the mean willingness to pay \( \bar{v} \) – is the WTP (26) evaluated at the mean wealth and human capital (\( \bar{W}, \bar{H} \)).
2. Unit weights \( \eta_j = 1, \forall j \) yield:

\[
\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_m, \Delta) = \left[ 1 - \frac{\Theta(\lambda^*_m)}{\Theta(\lambda_m)} \right] nN(\bar{W}, \bar{H}) = nv(\bar{W}, \bar{H}, \lambda_m, \Delta).
\]

Evaluating the latter at \( \Delta = n^{-1} \) yields the empirical VSL measure commonly used in the literature:

\[
\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_m, \Delta) = \frac{v(\bar{W}, \bar{H}, \lambda_m, \Delta)}{\Delta} = v^e_s(\bar{W}, \bar{H}, \lambda_m, \Delta).
\] (30)

Hence, the empirical VSL \( v^e_s \) in (9), or (30) is indeed a collective WTP, under homogeneity and unit social weights assumptions and corresponds to an infra-marginal WTP, i.e. a slope between two points on the willingness to pay, where the latter is evaluated at mean wealth and human capital \((\bar{W}, \bar{H})\). As discussed earlier, a concave WTP implies that \( v^e_s \leq v_s \) for \( \Delta \geq 0 \), i.e. the empirical measure under-estimates the theoretical VSL corresponding to the MWTP (see Figure 2).

5.4 Gunpoint Value of Life

Combining Definition 4 and (21) reveals the following result for the GPV.

**Proposition 4 (Gunpoint value of life)** The willingness to pay to avoid instantaneous and certain death solving (10) is given by:

\[
v_g(W, H) = N(W, H),
\] (31)

where \( N(W, H) \) is the net total wealth in (19).

In the absence of bequest motives, the agent who is forced to evaluate life at gunpoint would be willing to pay the hypothetical (i.e. step-1) value of pledgeable resources. The discussion of net total wealth in (16) establishes that this amount corresponds to his entire financial wealth \( W \), plus the capitalized value of his net income along the optimal path \( Y(H^*) - I^* \). However, the previous discussion emphasized that the minimal consumption level \( a \) is required at all periods for subsistence. Its cost therefore cannot be pledged in a highwaymen threat and must be subtracted from the Gunpoint value.
Interestingly, since net total wealth is independent of the agent’s risk aversion, elasticity of inter-temporal substitution, and death risk exposure, so is the GPV. The reason stems from the way the GPV is characterized in Definition 4 as the unitary value of a life, rather than by integrating marginal changes in death risk exposure. The agent therefore pays $v_g$ to avoid receiving the utility $V^m$ that is associated with certain and immediate death. Because the utility at death is a finite primitive and is normalized at zero, the Gunpoint Value is computable for all EIS levels. For the same reason, the Gunpoint Value of life $v_g$ in (31) is also independent from the agent’s preferences ($\rho, \varepsilon, \gamma$) and from the death intensity ($\lambda_m$). Because death is certain and instantaneous when life is evaluated at gunpoint, the attitudes towards time and risk, as well as the level of exposure to death risk become irrelevant.

It can also be shown (Hugonnier et al., 2013, Prop. 2) that net total wealth $N(W, H)$ is equal to the present value over an infinite horizon of excess consumption along the optimal path or, equivalently, to the sum of financial wealth and the present value over an infinite horizon of income net of human capital and subsistence consumption expenditures along the optimal path:

\[
N(W_t, H_t) = E_t \int_t^{\infty} \frac{m_\tau}{m_t} (c_\tau^* - a) \, d\tau \\
= W_t + E_t \int_t^{\infty} \frac{m_\tau}{m_t} (Y(H_\tau^*) - a - I_\tau^*) \, d\tau
\] (32)

To survive, the agent is thus willing to pledge the net present value of his optimal consumption stream (net of unpledgeable minimal subsistence), at which point he becomes indifferent between living and dying. This result can be traced to the fact that as a result of specification of preferences the foregone utility is measured in the same units as the foregone excess consumption.

Combining (32) with the result of Theorem 1 shows that the difference between the Gunpoint and HK values of life can be expressed as:

\[
v_g(W_t, H_t) - v_h(H, \lambda_m) = W_t - \frac{a}{r} + E_t \int_{T_m}^{\infty} \frac{m_\tau}{m_t} (Y(H_\tau^*) - I_\tau^*) \, d\tau \\
= W_t - \frac{a}{r} + (1 - C_0) \frac{y}{r} + (1 - C_1) P(H_t)
\]
The first two terms reflect the financial wealth and (capitalized) minimal consumption that affect net total wealth and therefore optimal consumption and welfare, but have no effects on optimal investment and, therefore, on the optimal path for net income \(Y(H^*) - I^*\). The third and last terms show the mortality risk adjustments \((C_0, C_1) \in [0, 1]^2\) on the net cash flow that are present in the HK value but not in the GPV. Unless minimal consumption requirements \(a/r\) are very large, the Gunpoint Value is therefore expected to be larger than the Human Capital Value.

The links between the willingness to pay in (26) and the GPV in (31) are intuitive and follow directly from the properties of the WTP. Indeed, the Gunpoint Value corresponds to the admissible upper bound (27b) on the willingness to pay to avoid a change in death risk exposure:

\[
v_g(W, H) = \sup_{\Delta \in A} v(W, H, \lambda_m, \Delta).
\]

This upper bound exists and is finite by admissibility, i.e. compliance with transversality restrictions. A concave willingness to pay thus implies that the VSL will necessarily over-value the GPV (see Figure 2). Indeed, comparing (28) and (31) establishes that:

\[
v_g(W, H) = A(\lambda_m)v_s(W, H, \lambda_m).
\]  

Estimates of the marginal propensity to consume \(A(\lambda_m)\) are typically low, ranging from 2-9% for housing wealth and 6% for financial wealth (e.g. Carroll et al., 2011, p. 58). Consequently, the predicted gap between the GPV and VSL is positive and large.

Finally, our closed-form results also have implications for the optimal long-run allocation between safety- and consumption-enhancing innovations. Indeed, contrasting excess consumption \(\bar{c}_t = c_t - a\) with the Statistical and Gunpoint Values of life reveals that the growth rates are linked as follows:

\[
E_t \left[ \frac{d\bar{c}_t}{\bar{c}_t} \right] = E_t \left[ \frac{dN_t}{N_t} \right] = E_t \left[ \frac{dv_{st}}{v_{st}} \right] = E_t \left[ \frac{dv_{gt}}{v_{gt}} \right].
\]

Along the optimal path, consumption above subsistence \(\bar{c}_t\) grows at the same rate as the Statistical and Gunpoint Values of life, i.e. at the rate of growth of net total wealth.
This could warrant allocating equal resources to life-saving and consumption-enhancing innovations.\textsuperscript{24}

**Remark 4 (Aging)** Our closed-form expressions for the willingness to pay and the three life valuations have thus far abstracted from aging processes. The latter can be incorporated although at some non-negligible computation cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in $\lambda_{mt}$, $\lambda_{st}$, $\phi_t$, $\delta_t$, or $\beta_t$ results in age-dependent MPC and Tobin’s-$Q$ that solve the system of ordinary differential equations:

\[
\begin{align*}
\dot{A}_t &= A_t^2 - \left(\varepsilon_\rho + (1 - \varepsilon) \left( r - \lambda_{mt} + \theta^2/(2\gamma) \right) \right) A_t, \\
\dot{B}_t &= (r + \delta_t + \phi_t \lambda_{st}) B_t + (1 - 1/\alpha)(\alpha B_t)^{1-\alpha} - \beta_t,
\end{align*}
\]

subject to appropriate boundary conditions. Allowing for aging and solving for the ODE’s $A_t, B_t$ implies that the solutions for $C_0t, C_1t$, the marginal value $\Theta_t(\lambda_{mt})$, as well as the human and total wealth $P_t(H), N_t(W, H)$ are also age-dependent. All the previous results remain applicable with these time-varying expressions.

6 Structural estimation

To structurally estimate the willingness to pay and the three life valuations, we follow a long tradition associating the agent’s human capital to his health (e.g. see the Hicks’ lecture by Becker, 2007, for a review). We estimate the technological, preferences and parameters for the model outlined in Section 4 by contrasting the theoretical decisions to their observed counterparts. Once the structural parameters have been estimated, they can be relied upon to compute the closed-form expressions for the life valuations in Section 5.

\textsuperscript{24}In particular, Jones (2016, p. 567) writes that:

“When the value of life rises faster than consumption, economic growth leads to a disproportionate concern for safety. This concern may be so strong that it is desirable that consumption growth be restrained. […] It would clearly be desirable to have precise estimates of the value of life and on how this has changed over time; in particular, does it indeed rises faster than consumption?”
6.1 Econometric model

For identification purposes, the econometric model assumes that agents follow the optimal rules and that they differ with respect to their health and wealth statuses but share common preference, technological and distributional parameters $\theta \in \mathbb{R}^k$. In particular, we use the closed-form expressions in Theorem 1 to which we append the income equation (12). Specifically, denote by

$$Y_j = [c_j, \pi_j, x_j, I_j, Y_j]'$$

the $5 \times 1$ vector of observed decisions and income for agent $j = 1, 2, \ldots, n$, let

$$X_j = [1, W_j, H_j]'$$

capture his current wealth and health statuses. Also let $B(\theta)$ denote the $5 \times 3$ matrix of closed-form expressions for the optimal rules implicit in equation (22), that are functions of the structural parameters $\theta$. The econometric model relies on Maximum Likelihood to structurally estimates the latter in:

$$Y_j = B(\theta)X_j + u_j$$  \hspace{1cm} (34)

where the $u_j$’s are (potentially correlated) Gaussian error terms.

In order to ensure theoretical consistency (i.e. $\theta \in \mathcal{A}$) and augment identification, we estimate the structural parameters in (34) imposing the regularity conditions (17). Despite these measures, not all parameters can be estimated and a subset are calibrated. In particular, we rely on standard values in the Asset Pricing literature to calibrate the returns process ($\mu, \tau, \sigma_S$) and discount rate ($\rho$). The subsistence consumption ($a$) is set using poverty thresholds and consumption literature. Finally, the share of health capital lost in sickness ($\phi$) is calibrated relying on the Stochastic Health Production literature, and via a thorough search procedure.

\footnote{Observe that this identifying hypothesis is consistent with the aggregation assumptions required to elicit the empirical VSL as a collective WTP (see Remark 3).}
6.2 Data

We use a sample of $n = 8,378$ individuals taken from the 2013 wave of the Institute for Social Research’s Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix D. We proxy the health variables through the polytomous self-reported health statuses (Poor, Fair, Good, Very Good and Excellent) that are linearly converted to numeric values ranging from 1 to 4. The financial wealth comprises risky and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation, and utility expenses reported in the PSID. Finally, health spending and health insurance expenditures are taken to be the out-of-pocket spending and the premia paid by agents. All nominal values are scaled by $10^{-6}$ for the estimation.

Tables 1, and 2 present descriptive statistics for the main variables of interest, per health status and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear relation between health and wealth can be inferred. The level of consumption in panel b is increasing in financial wealth, consistent with expectations. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption.

In panel c, stock holdings are very low for all but the fourth and fifth quintiles, illustrating the well-known non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth and devoid of clear health gradients. Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth and also sharply decreasing in health status.
7 Results

7.1 Structural parameters

Table 3 reports the calibrated (with subscripts c) and estimated (standard errors in parentheses) model parameters. Overall, the latter are precisely estimated and are consistent with other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.6843$). Although deterministic depreciation is relatively low ($\delta = 1.25\%$), morbidity is consequential with realistic additional depletion of $\phi = 1.36\%$ and average waiting time between occurrence of $\lambda_s^{-1} = 28.8$ years. Second, exposure to mortality risk is realistic ($\lambda_m = 0.0283$), corresponding to a remaining expected lifetime of $\lambda_m^{-1} = 35.3$ years, given mean respondent age of 45.26 years in Table 1. Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ($\beta = 0.0092$), as well as an estimated value for base income that is close to poverty thresholds ($y \times 10^6 = 12.2$ K$). The financial parameters ($\mu, \sigma, r$) are calibrated from the observed moments of the S&P500 and 30-days T-Bills historical returns. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 2.8953$). The minimal consumption level is somewhat larger than base income ($a \times 10^6 = 14.0$ K$), and is close to other calibrated values in the literature. As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.2416$) and is consistent with a Live Fast and Die Young effect whereby a higher risk of death increases the marginal propensity to consume. 

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26Hugonnier et al. (2013) estimate $\phi$ to be 1.11% using pooled PSID data from 1999 to 2007.
27The remaining life expectancy at age 45 in the US in 2013 was 36.1 years (all), 34.1 (males) and 37.9 (females) (Arias et al., 2017).
28For example, the 2016 poverty threshold for single-agent households under age 65 was 12.5 K$ (U.S. Census Bureau, 2017).
29For example, Murphy and Topel (2006, Tab. 2, p. 886) calibrate minimal composite consumption share $z_0/\bar{z}$ between 5-20%. Using mean consumption level in Table 1 yields $a/c = 14.23\%$.
30The EIS estimate is also close to the calibrated value of Córdoba and Ripoll (2017). Indeed, their calibration for $\sigma = 1/\varepsilon = 0.80$ is almost identical to our estimated $1/\hat{\varepsilon} = 1/1.2146 = 0.81$. 

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7.2 Estimated valuations

Human Capital Value of Life  Using the estimated parameters in Table 3, we can compute the HK value of life $v_h(H)$ given in (24) and reported in Table 4.a. Consistent with predictions, the human capital values are independent from $W$ and increasing in $H$, ranging from 250 K$ (Poor health) to 527 K$ (Excellent health), with a mean value of 421 K$. These figures are realistic. For example, setting $Y(0) = y$, using (scaled) mean income 21,838$ minus expenses of 721 $ in Table 1 to compute $Y_n(H)$, a mortality exposure $\lambda_m = 0.0283$ and a constant net income growth rate $g_n = 1\%$ yields a canonical HK value in (5) equal to 475 K$. Our structural estimates also compare advantageously with HK estimates in the literature and provide a first out-of-sample confirmation that the structural estimates are reasonable.\(^{31}\)

Value of Statistical Life  Table 4.b reports the Values of Statistical Life in (28) by observed health and wealth statuses. The VSL mean value is 8.35 M$, with valuations ranging between 2.17 M$ and 15.01 M$. These values are well within the ranges usually found in the empirical VSL literature.\(^{32}\) The concordance of these estimates with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded. Importantly, our joint theoretical and estimation approaches confirm the large VSL–HK discrepancies identified in the empirical literature.

It is also possible to assess a measure of the marginal vs infra-marginal WTP gap by calculating the empirical VSL measure in (30). Setting $\Delta = 1/n = 1/8,378$ and $\lambda_{m0} = \lambda_m + \Delta$, we recover an aggregate VSL of 8.34 M$, which, as expected, is lower, but very close to the mean theoretical value of $v_s(W, H, \lambda_m) = 8.3515$ M$. This result confirms that the theoretical and empirical values are close to one another, i.e. the individual MWTP is well approximated by the collective WTP corresponding to the empirical VSL when $\Delta = 1/n$ is small (i.e. the sample size is large).

\(^{31}\)Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K$ at age 20, peaking at less than 900 K$ at age 45 and falling steadily towards zero afterwards.

\(^{32}\)A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 M$ (2000 base year, corresponding to 8.6 M$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 M$ and 10 M$. Robinson and Hammitt (2016) report values ranging between 4.2 and 13.7 M$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 M$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 M$ (2006$), corresponding to 8.8 M$ in 2016 (U.S. Environmental Protection Agency, 2017).
The VSL is increasing in both wealth and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Andersson and Treich, 2011; Robinson and Hammitt, 2016; Murphy and Topel, 2006). On the one hand better health increases the value of life that is at stake, on the other hand, healthier agents face lower death risks and are thus less willing to pay to attain further improvements (or prevent deteriorations). Since our benchmark model abstracts from endogenous mortality (see the robustness discussion in Section 7.3 for generalization) and better health increases net total wealth $N(W, H)$, our estimates unambiguously indicate that the former effect is dominant and that improved health raises the VSL.

**Gunpoint Value** Table 4.c reports the Gunpoint values in (31). The mean GPV is 447 K$ and the estimates are increasing in both health and wealth and range between 116 K$ and 804 K$. The Gunpoint Value is thus of similar magnitude to the HK Value of life and both are much lower than the VSL. Indeed, this finding was already foreseeable from equation (33) indicating that the VSL/GPV ratio is inversely proportional to the marginal propensity to consume. Since our estimates reveal that $A(\lambda_m) = 5.36\%$ – a value again well in line with other estimates (Carroll et al., 2011) – we identify a VSL that is 18.66 times larger than the GPV.

**Willingness to pay** We emphasized that both the empirical and theoretical VSL will overstate the GPV corresponding to the upper bound on the concave willingness to pay. To help visualize this gap, Figure 3 is the estimated counterpart to Figure 2 and plots the willingness to pay $v(W, H, \lambda_m, \Delta)$ as a function of $\Delta$ calculated from (26) at the estimated parameters and relying on the mean wealth and health status.

The strongly concave estimated WTP in Figure 3 is informative as to why the VSL is much larger than the Human Capital and Gunpoint values. Indeed, the agent is willing to pay 37 K$ to avoid an increase of $\Delta = 0.0047$ which shortens his horizon from 35.3 to 30.3 years and would pay 406 K$ to avoid an increase of $\Delta = 0.17$ which lowers expected remaining lifetime from 35.3 to only 5 years. This last value is already close to the HK
and GPV values of 421 K$ and 447 K$, which are both much lower than the VSL of 8.35 M$. Equivalently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one’s own life when the WTP is very concave in the death risk increment, as foreshadowed in our discussion of (28) and (33).

7.3 Robustness

To verify robustness of our results, we consider a more general model of human capital. Hugonnier et al. (2013) study a demand for health framework that is similar to our benchmark, with two key differences. First, the model allows for self-insurance against morbidity and mortality risks by introducing health-dependent intensities:

\[
\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1}H_{t-}^{-\xi_m}, \\
\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1}H_{t-}^{-\xi_s}},
\]

where \(H_{t-} = \lim_{s \uparrow t} H_s\) is health prior to the morbidity shock realization. Hence, better health lowers exposure to sickness and death risks and our benchmark model of Section 4 is an exogenous restricted case that imposes \(\lambda_{s1}, \lambda_{m1} = 0\). Second, preferences are modified to allow for source-dependent aversion against financial, morbidity and mortality risks. In particular, the preferences in (14b) are replaced by:

\[
U_t = E_t \int_t^{T_m} \left( f(c, U_{\tau-}) - \frac{\gamma|\sigma_c(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} U_{\tau-} \lambda_k(H_{\tau-})F_k \left( H_{\tau-}, \frac{\Delta_k U_{\tau-}}{U_{\tau-}} \right) \right) \, d\tau,
\]

with the Kreps-Porteus aggregator defined in (14c) and the penalties for exposure to sickness and death risks defined by

\[
F_k = \frac{\Delta_k U_t}{U_{t-}} + \frac{1}{1 - \gamma_k} \left[ 1 - \left( 1 + \frac{\Delta_k U_t}{U_{t-}} \right)^{1-\gamma_k} \right],
\]

where

\[
\Delta_k U_t = E_t [-U_t - U_{t-}] \, dQ_{kt} \neq 0.
\]
Our benchmark specification of Section 4 is thus a restricted case that imposes risk-neutral attitudes towards morbidity ($\gamma_s = 0$) and mortality ($\gamma_m = 0$) risks.

While this model cannot be solved in closed form we show in a separate appendix that the expansion techniques of Hugonnier et al. (2013) can be used to derive an approximate solution that is accurate to the first order in ($\lambda_s$, $\lambda_m$) and which in turn leads to approximate closed-form expressions for the WTP, HK, VSL and GPV values. These expressions encompass explicit adjustments for the endogeneity of health risks exposure and source-dependent risk aversion, yet remain otherwise qualitatively similar. We structurally estimate the Hugonnier et al. (2013) model and compute the life values. These values remain in the same range as our benchmark estimates, with mean HK of 493 K$, VSL of 8.14 M$ and GPV of 460 K$ and again confirm the strong concavity of the WTP. We conclude that our main findings are qualitatively and empirically robust to more general specifications.\(^{33}\)

## 8 Conclusion

Computing the money value of a human life has long generated a profound and continued interest, with early records dating back to the late XVII\(^{th}\) century. The two most widely-used valuation frameworks have centered on the marginal rate of substitution between the probability of living and wealth (VSL) and on a person’s human capital value that is destroyed upon death (HK). The two life valuations yield strikingly divergent measures, with the VSL being 10-20 times higher than the HK.

The absence of common theoretical underpinnings and the very different empirical settings in which the two values are calculated complicate any comparison exercise between the HK and VSL. We address this issue via a unique and generic human capital problem to analytically compute and structurally estimate the theoretical VSL and HK values. We also introduce a third life value that reflects the maximum amount an agent would be willing to pay to save himself from instantaneous and certain death (GPV) to serve as useful benchmark. The willingness to pay to avoid changes in death risk, as well as the three closed-form for the life values are estimated jointly using a common structural

\(^{33}\)In addition to robust life valuations, the preference parameter estimates for the generalized model are consistent with a preference for late, as opposed to early resolution of death uncertainty favored by Córdoba and Ripoll (2017). Indeed, our estimated inverse EIS, $1/\hat{\xi} = 1/1.6699 = 0.5988$, is larger than our estimated mortality risk aversion, $\hat{\gamma}_m = 0.2862$. 

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econometric model and PSID data set. This encompassing approach thus provides direct comparability as well as a unique opportunity to identify the role of the preferences, distributional and technological parameters on life valuations.

Our main findings can be summarized as follows. We confirm the large discrepancies with an HK value of 421 K$ and a VSL of 8.35 M$ and show that the Gunpoint value of 447 K$ is similar to the HK. Our results accord with the standard economic intuition that the willingness to pay to avert death risk is increasing, but strongly concave and finite in mortality exposure. Allowing for a more general model with endogenous sickness and death intensities as well as source-dependent risk aversion only reaffirmed our findings. The large HK-VSL gaps are therefore robust to the use of integrated theoretical and empirical measurements.

We show that the HK-VSL differences are related to the strong curvature and finiteness of the WTP. In particular, the theoretical VSL is a linear projection from the marginal willingness to pay, whereas the empirical VSL is a local approximation to that MWTP. When the WTP is strongly concave, both theoretical and empirical VSL will strongly overestimate the limiting willingness to pay that corresponds to the Gunpoint Value. The empirical similarities between the HK and GPV values reflect the close theoretical parallels in the measured object. The HK computes the net present value of the foregone dividend stream associated with human capital (i.e. income, minus investment costs). The GPV measures the NPV of the foregone utility stream associated with living. The homogeneity properties entail that the latter is also the NPV of the foregone consumption above minimal subsistence requirements.

The Human Capital, Willingness to pay, Value of Statistical Life and Gunpoint value of life remain specialized tools that are complementary to one another and are applicable in specific contexts. Our encompassing approach provides single-step HK, WTP, VSL and GPV measurement in fully integrated theoretical and empirical environments. It precisely identifies the roles of human capital, wealth, preferences, technology and distribution parameters in life valuations. Importantly, our framework is very general, and can be easily extended along other dimensions (aging, attitudes, self-insurance).
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A Figures

**Figure 1:** Indifference curves, MRS and Value of Statistical Life

\[ v_s = \frac{-V_P}{V_W} = [a, d] \]

*Notes:* Reproduced and adapted from Andersson and Treich (2011, Fig. 17.1, p. 398). Indifference curves for indirect utility (2) is solid blue line. \( v_s \): Theoretical Value of Statistical Life in (8a) is the MRS between life and wealth, i.e. the slope of tangent, i.e. dashed red line and equal to distance \([a,d]\).
Notes: $\Delta$ is change in the probability of death from base exposure $P$. $v(\Delta)$: Willingness to pay to avoid $\Delta$ is solid blue line. $v_s = v'(0)$: Theoretical Value of Statistical Life in (8b) is slope of tangent, i.e. dashed red line and equal to distance $[a,f]$. $v^e_s = v(\Delta)/\Delta$: Empirical Value of Statistical Life in (9) is slope of dashed-dotted green line and equal to distance $[b,e]$. $v_g = \sup_{\Delta} (v)$: Gunpoint Value of Life in (10) is equal to distance $[c,d]$. 
**Figure 3:** Estimated WTP, HK, VSL and GPV Values of life (in M$)

\[
v_s(W,H;\lambda_m) = \left. \frac{\partial v(W,H;\lambda_m,\Delta)}{\partial \Delta} \right|_{\Delta=0} = 8.35 \text{ M}$
\[
v_h(H;\lambda_m) = 421 \text{ K}$
\[
v_g(W,H) = 447 \text{ K}$

**Notes:** At estimated parameter values, for mean wealth and health levels. \(v(W,H,\lambda_m,\Delta)\) (blue solid line) is the willingness to pay to avoid an increase of \(\Delta\) in exogenous death intensity \(\lambda_m\); \(v_h(H,\lambda_m)\) (magenta dashed) is the Human Capital value of life; \(v_g(W,H)\) (black dashed-dotted) is the Gunpoint value of life; \(v_s(W,H,\lambda_m)\) is the Value of statistical life and the slope of the dashed red tangent evaluated at \(\Delta = 0\).
B Proofs

B.1 Theorem 1

The benchmark human capital model of Section 4 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death and depreciation intensities are constant at $\lambda_m, \lambda_s$ (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e. $\gamma_s = \gamma_m = 0$). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1, Theorem 1) yields the the optimal solution in (22).

B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital $P(H)$ from

$$P(H) = E_t \int_t^\infty \frac{m^s}{m^t} [\beta H^*_t - I^*_t] d\tau,$$

$$= BH.$$

Straightforward calculations adapt this result to a stochastic horizon $T^m$ and include the fixed income component $y$ in income (12).

B.3 Proposition 2

Combining the Hicksian EV (6a) with the indirect utility (21a) and using the linearity of the net total wealth in (19) reveals that the WTP $v$ solves:

$$\Theta(\lambda^*_m) N(W, H) = \Theta(\lambda_m) N(W - v, H)$$

$$= \Theta(\lambda_m) [N(W, H) - v]$$

where we have set $\lambda^*_m = \lambda_m + \Delta$. The WTP $v = v(W, H, \lambda_m, \Delta)$ is solved directly as in (26).
Next, by the properties of the marginal value of net total wealth, $\Theta(\lambda^*_m)$ in (23) is monotone decreasing and convex in $\Delta$. It follows directly from (7) that the WTP

$$v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda^*_m)}{\Theta(\lambda^*_m)}\right] N(W, H)$$

is monotone increasing and concave in $\Delta$.

The lower bound follows directly from evaluating finite and admissible $A(\lambda^*_m), \Theta(\lambda^*_m)$ at $\lambda^*_m = 0$ in (26). To compute the upper bound, two cases must be considered:

1. For $0 < \varepsilon < 1$, the MPC in (17b) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\lambda^*_m = \lambda_m + \Delta < \bar{\lambda}_m = \left(\frac{\varepsilon}{1 - \varepsilon}\right) \rho + \left(r + \frac{\theta^2}{2\gamma}\right).$$

Admissibility $A$ therefore requires $\Delta < \bar{\Delta} = \bar{\lambda}_m - \lambda_m$ for the transversality condition (17b) to be verified. The supremum of the WTP is then $v(W, H, \lambda_m, \bar{\Delta}) = N(W, H)$.

2. For $\varepsilon > 1$, the MPC is monotone increasing and transversality is always verified. Consequently, the WTP is well-defined over the domain $\Delta \geq -\lambda_m$. It follows that:

$$\lim_{\Delta \to \infty} \Theta(\lambda_m + \Delta) = 0$$

$$\lim_{\Delta \to \infty} v(W, H, \lambda_m, \Delta) = N(W, H)$$

i.e. the willingness to pay asymptotically converges to net total wealth as stated in (27b).

B.4 Proposition 3

By the VSL definition (8a) and the properties of the Poisson death process (18):

$$v_s = \frac{-V_{\lambda m}(W, H, \lambda_m)}{V_W(W, H, \lambda_m)}$$
From the properties of the welfare function (21a), we have that $V_{\lambda_m} = \Theta'(\lambda_m)N(W, H)$, whereas $V_W = \Theta(\lambda_m)$. Substituting for $\Theta$ in (21b) yields the VSL in (28).

\[ V_{\lambda_m} = \Theta'(\lambda_m)N(W, H), \]

**B.5 Proposition 4**

Combining the Hicksian EV (10) with the indirect utility (21a) and the net total wealth in (19) reveals that the WTP $v$ solves:

\[ V_{\lambda_m} \equiv 0 = \Theta(\lambda_m)N(W - v_g, H) \]
\[ = \Theta(\lambda_m) [N(W, H) - v_g] \]

Solving for $v_g$ reveals that it is as stated in (31). Because net total wealth is independent of the preference parameters $(\varepsilon, \gamma, \rho)$, so is the Gunpoint Value.
### Tables

#### C.1 Data

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health</td>
<td>(H)</td>
<td>2.85</td>
<td>0.80</td>
<td>1</td>
</tr>
<tr>
<td>Wealth</td>
<td>(W)</td>
<td>38 685</td>
<td>122 024</td>
<td>0</td>
</tr>
<tr>
<td>Consumption</td>
<td>(c)</td>
<td>9 835</td>
<td>11 799</td>
<td>1.05</td>
</tr>
<tr>
<td>Risky holdings</td>
<td>(\pi)</td>
<td>20 636</td>
<td>81 741</td>
<td>0</td>
</tr>
<tr>
<td>Insurance</td>
<td>(x)</td>
<td>247</td>
<td>718</td>
<td>0</td>
</tr>
<tr>
<td>Health investment</td>
<td>(I)</td>
<td>721</td>
<td>2 586</td>
<td>0</td>
</tr>
<tr>
<td>Income</td>
<td>(Y)</td>
<td>21 838</td>
<td>37 063</td>
<td>0</td>
</tr>
<tr>
<td>Age</td>
<td>(t)</td>
<td>45</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

*Notes:* Statistics in 2013 $ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good) and 4.0 (Excellent).
Table 2: PSID data statistics (cont’d)

<table>
<thead>
<tr>
<th>Health</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wealth quintiles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Wealth $W_j$ ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>0</td>
<td>70</td>
<td>1 139</td>
<td>10 357</td>
<td>136 209</td>
</tr>
<tr>
<td>Fair</td>
<td>0</td>
<td>71</td>
<td>1 109</td>
<td>10 861</td>
<td>188 044</td>
</tr>
<tr>
<td>Good</td>
<td>0</td>
<td>86</td>
<td>1 214</td>
<td>11 207</td>
<td>160 925</td>
</tr>
<tr>
<td>Very Good</td>
<td>0</td>
<td>90</td>
<td>1 282</td>
<td>11 654</td>
<td>178 580</td>
</tr>
<tr>
<td>Excellent</td>
<td>0</td>
<td>88</td>
<td>1 315</td>
<td>11 974</td>
<td>214 106</td>
</tr>
<tr>
<td>b. Consumption $c_j$ ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>3 943</td>
<td>3 859</td>
<td>6 216</td>
<td>10 473</td>
<td>18 226</td>
</tr>
<tr>
<td>Fair</td>
<td>4 724</td>
<td>5 702</td>
<td>9 256</td>
<td>13 491</td>
<td>15 610</td>
</tr>
<tr>
<td>Good</td>
<td>6 459</td>
<td>5 742</td>
<td>9 205</td>
<td>12 457</td>
<td>17 109</td>
</tr>
<tr>
<td>Very Good</td>
<td>5 684</td>
<td>5 582</td>
<td>9 442</td>
<td>11 812</td>
<td>15 702</td>
</tr>
<tr>
<td>Excellent</td>
<td>6 177</td>
<td>5 616</td>
<td>10 117</td>
<td>11 575</td>
<td>17 465</td>
</tr>
<tr>
<td>c. Stocks $\pi_j$ ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>0</td>
<td>0</td>
<td>83</td>
<td>1 402</td>
<td>39 752</td>
</tr>
<tr>
<td>Fair</td>
<td>0</td>
<td>1</td>
<td>107</td>
<td>2 811</td>
<td>100 461</td>
</tr>
<tr>
<td>Good</td>
<td>0</td>
<td>4</td>
<td>143</td>
<td>3 299</td>
<td>82 499</td>
</tr>
<tr>
<td>Very Good</td>
<td>0</td>
<td>3</td>
<td>110</td>
<td>3 673</td>
<td>101 223</td>
</tr>
<tr>
<td>Excellent</td>
<td>0</td>
<td>3</td>
<td>116</td>
<td>3 627</td>
<td>125 934</td>
</tr>
<tr>
<td>d. Insurance $x_j$ ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>50</td>
<td>142</td>
<td>123</td>
<td>304</td>
<td>230</td>
</tr>
<tr>
<td>Fair</td>
<td>83</td>
<td>134</td>
<td>162</td>
<td>320</td>
<td>537</td>
</tr>
<tr>
<td>Good</td>
<td>132</td>
<td>104</td>
<td>268</td>
<td>335</td>
<td>512</td>
</tr>
<tr>
<td>Very Good</td>
<td>106</td>
<td>64</td>
<td>209</td>
<td>316</td>
<td>483</td>
</tr>
<tr>
<td>Excellent</td>
<td>108</td>
<td>87</td>
<td>240</td>
<td>314</td>
<td>455</td>
</tr>
<tr>
<td>e. Investment $I_j$ ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>783</td>
<td>792</td>
<td>852</td>
<td>2 021</td>
<td>4 447</td>
</tr>
<tr>
<td>Fair</td>
<td>538</td>
<td>762</td>
<td>777</td>
<td>1 711</td>
<td>2 969</td>
</tr>
<tr>
<td>Good</td>
<td>347</td>
<td>482</td>
<td>623</td>
<td>1 219</td>
<td>1 352</td>
</tr>
<tr>
<td>Very Good</td>
<td>250</td>
<td>318</td>
<td>422</td>
<td>639</td>
<td>1 070</td>
</tr>
<tr>
<td>Excellent</td>
<td>360</td>
<td>327</td>
<td>488</td>
<td>532</td>
<td>861</td>
</tr>
</tbody>
</table>

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth and per health status.
C.2 Benchmark model

Table 3: Estimated and calibrated structural parameter values, benchmark model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Law of motion health (11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.6843</td>
<td>$\delta$</td>
<td>0.0125</td>
</tr>
<tr>
<td>(0.3720)</td>
<td></td>
<td></td>
<td>(0.0060)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0136$^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Sickness and death intensities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>0.0347</td>
<td>$\lambda_m$</td>
<td>0.0283</td>
</tr>
<tr>
<td>(0.0108)</td>
<td></td>
<td></td>
<td>(0.0089)</td>
</tr>
<tr>
<td>c. Income (12) and wealth (13)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$</td>
<td>0.0120</td>
<td>$\beta$</td>
<td>0.0092</td>
</tr>
<tr>
<td>(0.0049)</td>
<td></td>
<td></td>
<td>(0.0044)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.108$^c$</td>
<td>$r$</td>
<td>0.048$^c$</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.20$^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Preferences (14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.8953</td>
<td>$\varepsilon$</td>
<td>1.2416</td>
</tr>
<tr>
<td>(1.4497)</td>
<td></td>
<td></td>
<td>(0.3724)</td>
</tr>
<tr>
<td>$a^c$</td>
<td>0.0140</td>
<td>$\rho^c$</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

Notes: Estimated structural parameters (standard errors in parentheses); $c$: calibrated parameters. Econometric model (34), estimated by ML, subject to the regularity conditions (17).
Table 4: Estimated Values of Life (in $)

<table>
<thead>
<tr>
<th>Health level</th>
<th>Wealth quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>a. Human Capital $v_h(W, H, \lambda_m)$ in (24)</td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>249 532</td>
</tr>
<tr>
<td>Fair</td>
<td>318 865</td>
</tr>
<tr>
<td>Good</td>
<td>388 198</td>
</tr>
<tr>
<td>Very Good</td>
<td>457 531</td>
</tr>
<tr>
<td>Excellent</td>
<td>526 864</td>
</tr>
<tr>
<td>All</td>
<td>- mean 420 729</td>
</tr>
<tr>
<td>All</td>
<td>- median 457 731</td>
</tr>
<tr>
<td>b. Value of Statistical Life $v_s(W, H, \lambda_m)$ in (28)</td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>2 167 573 2 168 877 2 188 829 2 360 907 4 710 118</td>
</tr>
<tr>
<td>Fair</td>
<td>4 379 551 4 380 874 4 400 253 4 582 287 7 889 684</td>
</tr>
<tr>
<td>Good</td>
<td>6 591 529 6 593 136 6 614 190 6 800 733 9 595 444</td>
</tr>
<tr>
<td>Very Good</td>
<td>8 803 507 8 805 188 8 827 429 9 021 052 12 136 981</td>
</tr>
<tr>
<td>Excellent</td>
<td>11 015 485 11 017 133 11 040 023 11 238 999 15 012 108</td>
</tr>
<tr>
<td>All</td>
<td>- mean 8 351 519</td>
</tr>
<tr>
<td>All</td>
<td>- median 8 803 507</td>
</tr>
<tr>
<td>c. Gunpoint Value $v_g(W, H)$ in (31)</td>
<td></td>
</tr>
<tr>
<td>Poor</td>
<td>116 121 116 191 117 259 126 478 252 329</td>
</tr>
<tr>
<td>Fair</td>
<td>234 620 234 691 235 729 245 481 422 664</td>
</tr>
<tr>
<td>Good</td>
<td>353 120 353 206 354 334 364 327 514 045</td>
</tr>
<tr>
<td>Very Good</td>
<td>471 619 471 709 472 901 483 274 650 199</td>
</tr>
<tr>
<td>Excellent</td>
<td>590 119 590 207 591 433 602 093 804 225</td>
</tr>
<tr>
<td>All</td>
<td>- mean 447 405</td>
</tr>
<tr>
<td>All</td>
<td>- median 471 619</td>
</tr>
</tbody>
</table>

Notes: Averages of individual values in the PSID sample, computed at estimated parameter values, multiplied by 1 M$ to correct for scaling used in estimation.
D Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research’s Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size) and scaled by $10^{-6}$ for the estimation. The agents’ wealth and health are constructed as follows:

**Health** $H_j$ Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

**Wealth** $W_j$ Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA’s or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA’s and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance and are constructed as follows:

**Portfolio** $\pi_j$ Money value of financial wealth held in risky assets.

**Consumption** $c_j$ Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

**Health expenditures** $I_j$ Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

**Health insurance** $x_j$ Spending on health insurance premium.