Valuing Life as an Asset, as a Statistic, and at Gunpoint[1]

Julien Hugonnier[1,4,5], Florian Pelgrin[2,6] and Pascal St-Amour[3,4,6]

[1]École Polytechnique Fédérale de Lausanne

[2]EDHEC Business School

[3]HEC Lausanne, University of Lausanne

[4]Swiss Finance Institute

[5]CEPR

[6]CIRANO

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Abstract

The Human Capital (HK) and the Statistical (VSL) values differ sharply in their empirical pricing of a human life. Rationalizing these differences is complicated by the absence of common theoretical and empirical grounds. We rely on a flexible human capital model to provide a unified theoretical framework for life valuations. The optimal investment is used to characterize the HK, whereas the indirect utility yields the willingness to pay (WTP) to avoid increases in mortality risk. The marginal WTP solves for the VSL and the limiting WTP provides an alternative valuation calculated at Gunpoint (GPV). A structural estimation of the analytical solutions with PSID data confirms that the HK (421 K$) and GPV (447 K$) are close to one another and that the strong curvature of the WTP explains a much larger VSL (8.35 M$).

Keywords: Value of Human Life, Human Capital, Value of Statistical Life, Hicksian Willingness to Pay, Equivalent Variation, Mortality, Structural Estimation.

JEL Classification: J17, D15, G11.

1 Introduction

1.1 Motivation and outline

Computing the value of a human life has long generated a deep interest in economic research.[1] Indeed, life valuations are relied upon in public health and safety debates, such as for cost/benefit analyses of life-saving measures in transportation, environmental, or medical settings. They are also important in long-run debates on quality versus quantity of life to determine whether to spend more resources on innovations that foster consumption growth or on those that prolong life expectancy.[2] Moreover, economic life values are resorted to in wrongful death litigation, as well as in terminal care cost/benefit analysis.[3]

The two most widely-used life valuation frameworks are the Human Capital (HK) and the Statistical Life (VSL) values.[4] The HK life value relies on asset pricing to compute the present value of a given person's cash flows corresponding to his labor income, net of the measurable investment expenses. The Value of a Statistical Life, introduced by Drèze (1962) and Schelling (1968), corresponds to a collective marginal rate of substitution between longevity and wealth. Whereas HK values are mainly used for identified life pricing, such as in litigation, the VSL's domain of application relates to public health and safety decisions benefiting unidentified persons,[5] as well as in societal debates on long-term consumption vs longevity choices.

In practice, both HK and VSL valuations of a human life yield strikingly different measures with VSL estimates being much larger than HK values. For example, Huggett and Kaplan (2016) identify HK values between 300 K–900 K$, whereas the U.S. Department of Transportation recommends using a VSL-type amount of 9.4 M$ (U.S. Department of Transportation, 2016). Although it is well recognized that HK and VSL life values

[1] Landefeld and Seskin (1982) make reference to human-capital based evaluations of the value of life dating back to Petty (1691).

[2] See Jones (2016); Jones and Klenow (2016); Hall and Jones (2007); Murphy and Topel (2006); Becker et al. (2005) for discussions.

[3] See Viscusi (2000, 2007) for legal uses of life values and Philipson et al. (2010); Round (2012) for end-of-life discussions.

[4] See Symmons (1938); Mishan (1971) for early descriptions and discussions of HK and VSL life values.

[5] See Ashenfelter (2006) for a critique of the theoretical foundations of the VSL and Viscusi (2000, 2007) for critical discussion of why the VSL is inappropriate relative to the HK in calculating compensating damages in wrongful death litigation.

need not be equal,[6] rationalizing differences of such magnitude is complicated by the absence of common theoretical and empirical grounds linking both valuations. Indeed, most HK and VSL evaluations are reduced-form empirical exercises that neither share joint theoretical underpinnings, nor common database, nor encompassing identification strategy.

This void leaves open a number of important issues that we address in this paper. First, what are the necessary conditions (e.g. preference for life over death, transversality, appropriate rate of discounting, sign and cross-equations restrictions) for joint theoretical consistency of HK and VSL? Second, how do valuations of small (such as the willingness to pay, WTP) or marginal changes in risks to life extend to unitary ones (such as the HK and VSL)? Third, what is the role of technological, preference and distributional parameters, as well as of the agents' statuses (e.g. financial and human wealth) in life valuations? Fourth, how are the HK, WTP and VSL related to each other and to alternative life valuation metrics, such as the limiting WTP? Finally, to what extent can these valuations be empirically revealed by observed financial and human capital decisions of agents?

To answer these questions, we propose a unified dynamic framework yielding model-based WTP, HK and VSL life values. The analytical solutions to this framework satisfy rigorous theoretical restrictions and allow us to clarify the links between the various life valuations. Unlike other approaches based on implicit characterizations, these explicit closed-form life values make it possible to pinpoint precisely the role of modeling and parametric assumptions, as well as of state variables such as wealth or health. Remarkably, the structural estimation of the WTP, HK and VSL using a common household data-set confirms the relevance of reduced-form estimates found in the literature. Importantly, they reassert the central role played by diminishing marginal willingness to pay for longevity in explaining the large discrepancies between the VSL and other measures.

More precisely, we solve a flexible life cycle problem in which an agent facing an uncertain horizon selects consumption, portfolio, insurance, as well as investment in his human capital (e.g. skills or health). The optimal investment and associated net income stream are then capitalized using the stochastic discount factor induced by the agent's opportunity set to characterize the HK value. Moreover, the indirect utility calculated at the optimum can be combined with variational analysis (Hicks, 1946) to define the

[6]See Conley (1976); Shepard and Zeckhauser (1984); Pratt and Zeckhauser (1996); Viscusi (2000, 2007) for discussions.

willingness to pay to prevent increases in mortality risk exposure. The marginal WTP defines the VSL, whereas the limiting WTP yields an alternative life value. In particular, the Gunpoint Value of Life (GPV) is the Hicksian willingness to pay that leaves an agent indifferent between living and immediate death in a highwaymen threat.[7] As argued by Philipson et al. (2010), end-of-life care decisions are more akin to valuing life at gunpoint and are inappropriate settings for VSL-type valuations. The Gunpoint value can also complement HK values in wrongful death litigation cases by establishing how much a person would value his *own* life, instead of having the market perform this valuation.

Our human capital model departs from standard approaches in three key dimensions. First, we rely on recursive utility (Epstein and Zin, 1989, 1991; Duffie and Epstein, 1992) under which non-negative consumption requirements are associated with non-negative utility. This feature guarantees unconditional preference for life (positive utility) over death (zero utility) regardless of parametric values (Hugonnier et al., 2013). Moreover, this preference specification disentangles risk aversion from the elasticity of inter-temporal substitution (EIS). In addition to reconciling theoretical with observed consumption and financial decisions, this separation allows for non-indifference over the timing of the resolution of death uncertainty and consequently much more flexible tradeoffs between quantity (i.e. longevity) and quality (i.e. consumption) of life (e.g. Córdoba and Ripoll, 2017). Second, our model features endogenous human capital accumulation that is subject to stochastic depreciation shocks (e.g. unemployment or illness), where the latter can be interpreted either as skills (e.g. Ben-Porath, 1967; Heckman, 1976) or as health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990).

We use the explicit solutions to this human capital model to derive the analytic expressions for the willingness to pay and the three other life values. These closed-form life valuations pinpoint the contributions of fundamentals, such as preferences, risk distributions, or technology, thereby allowing us to investigate how the WTP, HK, VSL and GPV are theoretically related to one another. We also structurally estimate the model's distributional, technological and preferences parameters by associating human capital to health and by resorting to PSID data that correspond to the optimal consumption, portfolio, as well as health spending and insurance policies. A Revealed Preference perspective then allows us to combine the estimated deep parameters with observed

[7] See Jones-Lee (1974); Cook and Graham (1977); Eeckhoudt and Hammitt (2004) for related definitions.

wealth and health data to estimate the analytical expressions for the willingness to pay, Human Capital, Statistical and Gunpoint Values of life. Our encompassing approach thus ensures that the WTP and the three different life values are computed through a single-step estimation, using a common data set, and imposing compliance with theoretical restrictions.

Our main theoretical and empirical findings can be summarized as follows. First, the properties of the indirect utility guarantee that the willingness to pay to avoid detrimental changes in exposure to death is unconditionally positive, increasing, concave and bounded in the mortality risk increment. Curvature stems directly from the non-linearities in risk probabilities that is made possible by recursive preferences, but abstracted from in VNM utility (e.g. Córdoba and Ripoll, 2017, Section 2.2). It follows that the theoretical VSL (i.e. the marginal WTP) is under-estimated by the empirical VSL used in the literature (i.e. the infra-marginal WTP).[8] Moreover curvature implies that both theoretical and empirical VSL's are larger than the GPV (the WTP's upper bound), where the latter is the shadow value of the agent's disposable resources, net of subsistence requirements. In particular, we show that the ratio of the VSL to this limiting WTP is inversely proportional to the marginal propensity to consume (MPC) and, since the MPC out of wealth is typically much lower than one, the predicted VSL-GPV gap is positive and significant.

Second, the HK and GPV share a number of similarities. Indeed, both measures directly compute the value of a whole life, relying on the expected net present values of human capital dividends (HK) and of consumption above subsistence (GPV), and both are independent of preferences towards risk and time. For the HK and GPV, preference independence is explained by the separation between human capital investment and other decisions under complete markets (e.g. Bodie et al., 1992; Hugonnier et al., 2013; Acemoglu and Autor, 2018). For the GPV, preference independence further reflects the nature of mortality in a Gunpoint valuation: because death is both certain and

[8]Under appropriate assumptions, a collective willingness to pay to save one unidentified (i.e. statistical) life can be recovered through a linear aggregation of the individual WTP's. As a canonical example (e.g. Aldy and Viscusi, 2007), suppose n agents are individually willing to pay $v(\Delta)$ to attain (avert) a small beneficial (detrimental) change $\Delta = 1/n$ in death risk exposure and satisfying $v(0) = 0$. The empirical VSL is the collective WTP: $v_s^e(\Delta) = nv(\Delta) = v(\Delta)/\Delta$, corresponding to the slope of the WTP and approximating the MWTP $v'(\Delta)$. See Remarks 3 and 4 below for discussion.

instantaneous if the agent refuses to pay, attitudes towards risks and time substitution are irrelevant.

Third, our empirical results identify strongly diminishing marginal WTP and accord with the reduced-form HK and VSL estimates reported in the literature, with structural average values of 421 K$ (HK) and 8.35 K$ (VSL). Moreover, the average Human Capital value of life is close to the average Gunpoint benchmark (447 K$), as expected from the theoretical parallels between the two. The small GPV/VSL ratio is consistent with a realistic MPC estimate of 5.36%. The confirmation of large empirical VSL-HK differences, and the finding of HK-GPV similarities in an encompassing framework indicates that the linear extrapolation from a marginal value to a unitary life value when the WTP is strongly concave is the main reason behind the significantly larger VSL estimates.

Finally, our model can be generalized even further by allowing for differential aversion to financial, mortality and morbiditiy risks, as well as for self-insurance against death and sickness risk exposures along the lines of Hugonnier et al. (2013). A structural estimation of the life valuations for this more flexible model confirms our main empirical findings. We conclude that our results are robust to our choice of theoretical framework.

1.2 Related literature

Other researchers have offered encompassing approaches to life valuations. Jones-Lee (1974) proposes a static VNM framework, without human capital considerations which focuses on the utility of wealth when alive and at death to analyze the WTP's properties. Marginal WTP for small changes in death risk yield the VSL whereas a Gunpoint-equivalent life value is studied through the willingness to accept compensation for certain death. Conley (1976); Shepard and Zeckhauser (1984); Rosen (1988) analyze Human Capital and Statistical Life values in a life cycle model with perfect and imperfect capital markets. These studies emphasize the role of the EIS and conclude that the VSL is much larger than the HK under reasonable assumptions. Our main contribution to these analyses are that we calculate and structurally estimate closed-form solutions to a much richer parametrized encompassing framework. In particular, we provide WTP, HK, VSL and GPV solutions under non-expected utility settings with endogenous stochastic human capital accumulation. These formulas are estimated under the full set of theoretical restrictions with a common data base.

In addition to a voluminous body of empirical research on the VSL (and more limited one on the HK), the papers in the literature most related to our analysis are the following. First, Córdoba and Ripoll (2017) concur with us on the relevance of recursive preferences for life valuation. In particular, they emphasize the importance of disentangling attitudes towards risk, from those towards time. This separation allows for non-indifference with respect to the timing of the resolution of survival uncertainty, and guarantees preference for life over death, even at high risk aversion levels. They also contend that more realistic curvature of the willingness to pay for survival can only be attained by allowing non-linear effects of death probabilities on utility that are abstracted from under VNM preferences. Both their discussion and their calibration emphasize a preference for late, rather than early, resolution of death uncertainty, as well as a diminishing marginal willingness to pay for additional longevity (Córdoba and Ripoll, 2017, Sec. 2.2, and Tab. 1). Our structural estimation of the generalized model of Hugonnier et al. (2013) in Section 4.4 confirms this conjecture with an inverse elasticity of inter-temporal substitution that is larger than mortality risk aversion, consistent with a preference for late resolution of death uncertainty and with a concave WTP.

Despite these similarities, the parametrized model of Córdoba and Ripoll (2017) is however different from ours. Their closest analog in Section 3.2 is set in discrete (rather than continuous) time, and lets the agent select consumption only. It fully abstracts from our analysis of endogenous human capital accumulation, stochastic capital depreciation, risky portfolio and insurance choices and their main solutions for life values are characterized for hand-to-mouth consumers only. Moreover they emphasize mortality risk aversion as key determinant of life values in an homethetic recursive preferences specification. In our setting, the agent is risk-neutral with respect to mortality risk,[9] so that the elasticity of inter-temporal substitution is the main driver of mortality preferences, and we allow for non-homotheticity in a recursive utility setting by introducing minimal consumption requirements. Finally, whereas they obtain closed-form solutions for the VSL, they do not explicitly compute the WTP,[10] and fully abstract from both HK and GPV.

[9] Our benchmark model in Section 2 allows for financial risk aversion, but abstracts from aversion towards mortality and depreciation shocks. Both are re-introduced in robustness tests in Section 4.4 that confirm our main findings.

[10] More precisely, the WTP in their setup is simply the VSL times the change in death probability (see the equation before eq. (16)). Instead, we compute the WTP from Hicksian variational analysis and rely on its marginal and limiting properties to characterize the VSL and Gunpoint values.

Bommier et al. (2019) also analyze the implications for life valuation of life cycle models of consumption and portfolio choices with recursive preferences. However, important differences remain between the two approaches. Indeed, Bommier et al. (2019) neither allow for human capital and insurance decisions, nor do they analytically solve their model, and therefore do not formally characterize how structural parameters and state variables affect a broad set of life valuations. More specifically, whereas we rely on the explicit solutions for the optimal human capital dynamics and the indirect utility to analyze the HK, WTP, VSL and GPV, Bommier et al. (2019) use the (unsolved) marginal utility of consumption and of death risk to discuss the implications for the VSL only. Moreover, Bommier et al. (2019) calibrate their model to fit the empirical VSL estimates, and then assess the resulting life cycle paths of consumption, financial market participation and portfolio. Conversely, we structurally estimate the model by relying on a wide set of cross-equations theoretical restrictions in a multivariate econometric setting to fit the observed financial and human capital decisions, and *then* proceed to gauge the empirical implications for the life valuations.

Our parametrized setting directly borrows from Hugonnier et al. (2013). We consider a restricted case of this model where we abstract from source-dependent risk aversion as well as from endogenous exposure to mortality and morbidity risks. This simplifying choice is made for tractatability reasons only; for completeness, we also estimate the unrestricted model and compute the corresponding life valuations. The results (discussed in Section 4.4) confirm all our main findings. Moreover, whereas Hugonnier et al. (2013) do consider a VSL-inspired life valuation, their main emphasis is on separation between financial and health-related choices. The WTP, Gunpoint and HK values are therefore completely abstracted from.

Hall and Jones (2007) propose a semi-structural measure of life value akin to the Value of a Statistical Life. They adopt a marginal value perspective by equating the VSL to the marginal cost of saving a human life. In their setting, the cost of reducing mortality risk can be imputed by estimating a health production function and by linking health status to death risks. Dividing this marginal cost by the change in death risk yields a VSL-inspired life value. Unlike Hall and Jones (2007) we do not measure the health production function through its effects on mortality, but estimate the technology through the measurable effects of investment on future health status.[11] Moreover, our fully structural approach does not indirectly evaluate the marginal value of life via its marginal cost, but rather directly through the individual willingness to pay to avoid changes in death risks.

Finally, we share similarities with Murphy and Topel (2006) who resort to a life cycle model with direct utilitarian services of health to study life valuations. In particular, both continuous-time approaches study permanent changes in Poisson death intensity, under perfect markets assumption, and both identify the VSL as a marginal rate of substitution between longevity and wealth. Moreover, both emphasize the key role of the elasticity of inter-temporal substitution in generating diminishing marginal values. However, contrary to Murphy and Topel (2006), our human capital (i.e. health) is endogenously determined in a stochastic environment, whereas we abstract from nonmarket time (i.e. leisure). The associated VSL, as well as other life measures, are all increasing in health, rather than health-independent. Importantly, whereas Murphy and Topel (2006) posit an arbitrary process for consumption (see eq. (19), p. 885) and restrict their analysis to hand-to-mouth in their calibration, we solve for optimal consumption, portfolio, insurance, and health expenditures. This allows us to analyze and structurally estimate all life valuations – including the HK, WTP and GPV that are abstracted from in Murphy and Topel (2006) – through the prism of the indirect utility function.

The rest of the paper is organized as follows. We first present and solve our human capital model in Section 2. The associated optimal rules and welfare are used to characterize the implied life valuations in Section 3. Section 4 reviews the empirical strategy, with structural parameters and life value estimates. Concluding remarks are presented in Section 5.

2 Human Capital Model

This section characterizes and solves a flexible human capital model involving both stochastic death and depreciation shocks. The solution to this model will then be relied upon in the next section to compute the life valuations.

[11]Indeed, mortality is treated exogenously in our baseline model. The more general setup with endogenous death risk exposure in Section 4.4 yields similar empirical results.

2.1 Economic environment

Planning horizon and human capital We consider a continuous-time human capital problem where the agent's planning horizon is limited by a stochastic age at death T_m satisfying:

$$\lim_{h\to 0}\frac{1}{h}\Pr\left[T_m\in(t,t+h]\mid T_m>t\right]=\lambda_m, \tag{1}$$

such that the probability of death by age t is monotone increasing in the arrival rate $\lambda_m>0$:

$$\Pr(T_m\leq t)=1-e^{-\lambda_m t}. \tag{2}$$

Subsequent analysis will focus on changes in mortality risk exposure stemming from permanent changes in death intensity λ_m.[12]

The agent invests at rate I_t in his human capital H_t whose law of motion is given by:

$$\mathrm{d}H_t=\left(I_t^{\alpha}H_t^{1-\alpha}-\delta H_t\right)\mathrm{d}t-\phi H_t\mathrm{d}Q_{st}. \tag{3}$$

In this equation, the Cobb-Douglas parameter $\alpha\in(0,1)$ captures diminishing returns to investment, $\delta>0$ measures the gradual deterministic depreciation of human capital absent investments, and dQ_{st} is the increment of a Poisson process with constant intensity λ_s whose jumps depreciate the capital stock by a factor $\phi\in(0,1)$.

The law of motion (3) nests alternative interpretations of human capital. If H_t is associated with skills (e.g. Ben-Porath, 1967; Heckman, 1976), then investment I_t comprises education and training choices made by the agent whereas $\mathrm{d}Q_{st}$ can be interpreted as stochastic unemployment, or technological obsolescence shocks that depreciate the human capital stock. If H_t is instead associated with the agent's health (e.g. Grossman, 1972; Ehrlich and Chuma, 1990), then investment takes place through medical expenses or healthy lifestyle decisions whereas the stochastic depreciation occurs through morbidity shocks.

[12] See also Murphy and Topel (2006) for a similar perspective.

Budget constraint The agent's income rate is given by:

$$Y_t = Y(H_t) = y + \beta H_t, \tag{4}$$

and includes both an exogenous base income y and a positive income gradient β for human capital. Individuals can trade in a risk-less asset with rate r, as well as in two risky assets to smooth out shocks to consumption: stocks and insurance against human capital depreciation. Financial wealth W_t evolves according to the dynamic budget constraint:

$$dW_t = (rW_t + Y_t - c_t - I_t)\,\mathrm{d}t + \pi_t\sigma_S\,(\mathrm{d}Z_t + \theta\mathrm{d}t) + x_t\,(\mathrm{d}Q_{st} - \lambda_s\mathrm{d}t)\,, \tag{5}$$

where $\sigma_S > 0$ is the volatility of the stock, $\theta = (\mu - r)/\sigma_S$ is the market price of financial risk and Z_t is a Brownian motion. In addition to investment I_t, the agent selects consumption c_t, the risky portfolio π_t and the number of units x_t of actuarially-fair depreciation insurance. The latter pays one unit of the numeraire upon the occurrence of a depreciation shock, and can be interpreted as unemployment insurance (if H_t is associated with skills), or as medical, or disability insurance (if H_t is associated with health status).

Preferences Following Hugonnier et al. (2013) we define the indirect utility of an alive agent as:

$$V(W_t, H_t) = \sup_{(c,\pi,x,I)} U_t, \tag{6a}$$

where preferences are given by

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_\tau) - \frac{\gamma|\sigma_\tau(U)|^2}{2U_\tau} \right) \mathrm{d}\tau, \tag{6b}$$

where

$$\sigma_t(U) = \frac{1}{dt} d\langle Z, U\rangle$$

denotes the diffusion of the continuation utility process, and $f(c,u)$ is the Kreps-Porteus aggregator function defined by:

$$f(c,u) = \frac{\rho\, u}{1-1/\varepsilon}\left(\left(\frac{c-a}{u}\right)^{1-\frac{1}{\varepsilon}} - 1\right). \tag{6c}$$

The preference specification in (6) belongs to the stochastic differential utility class proposed by Duffie and Epstein (1992) and is the continuous-time analog of the discrete-time recursive preferences of Epstein and Zin (1989, 1991). It is characterized by a subjective discount rate $\rho > 0$, a minimal subsistence consumption level $a > 0$, risk-neutrality with respect to both depreciation shocks and death,[13] and disentangles the agent's elasticity of inter-temporal substitution (EIS) $\varepsilon \geq 0$, from his constant relative risk aversion with respect to financial risk $\gamma \geq 0$. As explained in Hugonnier et al. (2013) and confirmed in Theorem 1 below, the homogeneity properties of our specification implies that any feasible consumption process $c_t - a \geq 0$ is associated with a positive continuation utility and therefore guarantees weak preference for living: $V_t \geq V^m \equiv 0$, where V^m is the utility at death.

Remark 1 The model assumes that the sole motivation for investing in H_t relates to its positive effects on marketed income in (4).[14] However, the valuation of human capital can also be made with respect to its non-marketed utilitarian services. Indeed, the model can be adapted for non-workers by first defining $\tilde{c}_t = c_t - \beta H_t$, then eliminating βH_t in the income equation (4) and finally replacing for $c_t = \tilde{c}_t + \beta H_t$ in the budget constraint (5) and preference equations (6). The agent then selects $\tilde{c}_t$ and the other controls taking into account the utilitarian benefits of human capital. As shown in Hugonnier et al. (2013, Remark 3), the theoretical results are unaffected under this alternative interpretation. This property is especially useful when applying the model to agents who, for reasons of age, illness, or choice are unable or unwilling to work, e.g. in end-of-life analysis (e.g. Philipson et al., 2010; Hugonnier et al., 2017).

[13] See Section 4.4 below for an extension in which the agent is differently averse towards financial (γ), death (γ_m) and depreciation (γ_s) risks.

[14] The generalized model in Section 4.4 allows for additional beneficial effects of human capital as self-insurance against morbidity and mortality risk exposure.

2.2 Optimal rules

The agent's dynamic problem (6), subject to (3) and (5) can either be solved directly through the Hamilton-Jacobi-Bellman (HJB) or in two separate steps.[15] The latter approach involves:

1. An *hypothetical* infinitely-lived agent first solves the optimal investment by maximizing the discounted value of the H-dependent part of net income:

   $$P(H_t) = \sup_{I \geq 0} E_t \int_t^{\infty} \frac{m_\tau}{m_t} (\beta H_\tau - I_\tau) \, \mathrm{d}\tau.$$

   where

   $$m_t = \exp\left(-rt - \theta Z_t - \frac{1}{2}\theta^2 t\right). \tag{7}$$

   is the stochastic discount factor induced by the prices of financial assets. The human wealth $P(H)$ is then combined with the agent's financial wealth and the present value of his base income stream net of minimal consumption expenditures to obtain the agent's net total wealth as:

   $$\begin{aligned} N(W_t, H_t) &= W_t + \frac{y-a}{r} + P(H_t), \\ &= W_t + E_t \int_t^{\infty} \frac{m_\tau}{m_t} \left(Y(H_\tau^*) - I_\tau^* - a\right) \mathrm{d}\tau. \end{aligned} \tag{8}$$

   An important consequence of this characterization is that, due to complete financial markets, the agent's optimal human capital investment can be determined independently of his preferences.

2. The finitely-lived agent then selects the remaining policies $\bar{c}_t = c_t - a$, π_t and $\bar{x}_t = x_t - \phi P(H_t)$ by maximizing utility (6), subject to the law of motion for net total wealth:

   $$\mathrm{d}N_t = (rN_t - \bar{c}_t)\mathrm{d}t + \pi_t \sigma_S (\mathrm{d}Z_t + \theta \mathrm{d}t) + \bar{x}_t \left(\mathrm{d}Q_{st} - \lambda_s \mathrm{d}t\right).$$

[15] See Bodie et al. (1992); Hugonnier et al. (2013); Acemoglu and Autor (2018) for discussion and applications of separability of investment and financial decisions in human capital problems.

The remaining optimal consumption, portfolio and insurance policies, as well as indirect utility function are calculated as functions of $P(H_t)$ and $N(W_t, H_t)$ and encompass explicit adjustments for finite lives where appropriate.

In the context of our parametric model both steps can be carried out, leading to the following result.

Theorem 1 *Assume that the parameters of the model are such that*

$$\beta < (r + \delta + \phi\lambda_s)^{\frac{1}{\alpha}}, \tag{9a}$$

and

$$A(\lambda_m) = \varepsilon\rho + (1-\varepsilon)\left(r - \lambda_m + \frac{1}{2}\theta^2/\gamma\right) > \max\left(0, r - \lambda_m + \frac{\theta^2}{\gamma}\right). \tag{9b}$$

Then,

1. *the human wealth and net total wealth are given as:*

$$P(H_t) = BH_t \geq 0, \tag{10}$$

$$N(W_t, H_t) = W_t + \frac{y-a}{r} + P(H_t) \geq 0, \tag{11}$$

where $B > 0$ is the unique solution to

$$g(B) = \beta - (r + \delta + \phi\lambda_s)B - (1 - 1/\alpha)(\alpha B)^{\frac{1}{1-\alpha}} = 0 \tag{12}$$

such that $g(B) = 0$, and $g'(B) < 0$;

2. *the indirect utility for the agent's problem is:*

$$V_t = V(W_t, H_t, \lambda_m) = \Theta(\lambda_m)N(W_t, H_t) \geq 0, \tag{13a}$$

$$\Theta(\lambda_m) = \tilde{\rho}A(\lambda_m)^{\frac{1}{1-\varepsilon}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{-\varepsilon}{1-\varepsilon}} \tag{13b}$$

and generates the optimal rules:

$$
\begin{aligned}
c_t^* &= c(W_t, H_t, \lambda_m) = a + A(\lambda_m)N(W_t, H_t) \geq 0, \\
\pi_t^* &= \pi(W_t, H_t) = (\theta/(\gamma\sigma_S))N(W_t, H_t), \\
x_t^* &= x(H_t) = \phi P(H_t) \geq 0, \\
I_t^* &= I(H_t) = \left(\alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}\right) P(H_t) \geq 0,
\end{aligned}
\tag{14}
$$

where any dependence on death intensity λ_m is explicitly stated.

Condition (9a) is a transversality restriction for a finite shadow value of human capital, whereas condition (9b) is required to ensure positive marginal propensity to consume out of net wealth $A > 0$, as well as for minimal consumption requirements $c_t > a$. Restrictions (9) jointly ensure that the continuation utility V_t is finite and that the solutions are well-defined. The constant B in (10) can naturally be interpreted as the marginal value (i.e. Tobin's Q) associated with human capital. It is implicitly defined in (12) as an increasing function of the income gradient β and a decreasing function of the rate of interest r and the expected depreciation rate $\delta + \phi\lambda_s$.

Three features of the optimal rules are particularly relevant for life valuation. First, the two-step solution method ensures that both human wealth (10) and the net total wealth (11) are independent of the death intensity λ_m. Second and related, the exposure to exogenous death risk λ_m affects welfare only through $\Theta(\lambda_m)$ in (13b), via its impact on the marginal propensity to consume $A(\lambda_m)$. Equation (9b) establishes that $A'(\lambda_m) = \varepsilon - 1 \lesseqgtr 0$, i.e. this MPC effect is entirely determined by the elasticity of inter-temporal substitution. An increase in death risk λ_m induces heavier discounting of future utility flows, leading to two opposite outcomes on the marginal propensity to consume. On the one hand, more discounting requires shifting current towards future consumption to maintain utility (i.e. by lowering the MPC). This effect is dominant at low elasticity of inter-temporal substitution $\varepsilon \in (0, 1)$. On the other hand, heavier discounting makes future consumption less desirable prompting the agent to shift future towards current consumption (i.e. by increasing the MPC). This *Live Fast and Die Young* effect is dominant at high elasticity of inter-temporal substitution $\varepsilon > 1$.[16] Observe that, separate

[16] Bommier et al. (2019) refer to higher MPC at shorter longevity as a *carpe diem* effect and numerically attribute its relevance to risk aversion. However, the EIS ε, is the sole determinant for the effect of λ_m on $A(\lambda_m)$ in the closed-form (9b). The mortality gradient on the MPC is independent from the financial

ε and γ parameters entail that a high EIS can coincide with high risk aversion, a flexibility that cannot be attained under VNM preferences that impose that the two are inversely related.

Third, the welfare in (13) is increasing in both wealth and human capital stock and is decreasing and convex in the death intensity λ_m at all EIS levels since:

$$\Theta'(\lambda_m) = -\tilde{\rho} A(\lambda_m)^{\frac{\varepsilon}{1-\varepsilon}} \leq 0, \tag{15a}$$

$$\Theta''(\lambda_m) = \tilde{\rho}\varepsilon A(\lambda_m)^{\frac{2\varepsilon-1}{1-\varepsilon}} \geq 0. \tag{15b}$$

Hence, whereas the sign of the effects of death risk λ_m on the MPC (9b) depends on the EIS, preference for life implies that higher mortality exposure unconditionally reduces the marginal value of net total wealth $\Theta(\lambda_m)$ in (13b) and therefore lowers welfare V_t in (13a). Importantly, as shown below in Section 3.2, a decreasing and convex effect of death risk on welfare entails that the willingness to pay to avoid increases in mortality is increasing and concave in death risks.

3 Willingness to Pay and Values of Life

We next calculate the life valuations implied by the solutions in Theorem 1 for the human capital model of Section 2. First, the optimal path for human capital induced by investment at the optimum $\{H(I_t^*)\}_t$ is relied upon to solve for the HK value of life. Second, the associated indirect utility V_t is combined with Hicksian variational analysis for the willingness to pay to avoid increases in death risk. The marginal WTP yields the VSL whereas the limiting WTP yields the Gunpoint value. We assume throughout this section that the parameters of the model satisfy the regularity conditions (9) and abstract from time subscripts whenever possible to alleviate notation.

3.1 Human Capital Value of Life

The Human Capital Value of life is the market value of the net cash flow associated with human capital and that is foregone upon death (e.g. Conley, 1976; Huggett and Kaplan,

risk aversion γ, whereas risk-neutrality with respect to death risk explains why aversion towards the latter plays no role in $A(\lambda_m)$. For the source-dependent extension discussed in Section 4.4, that effect involves both parameters and is augmented to $A'(\lambda_m) = (\varepsilon - 1)/(1 - \gamma_m)$ where $\gamma_m \in [0, 1)$ is mortality risk aversion. See Hugonnier et al. (2013, eq. (24)) for details.

2013, 2016). In our setting, this net cash flow is the marketed income, minus the money value of investment expenses, where both are evaluated at the optimum:

Definition 1 (HK value of life) *The Human Capital Value of life is*

$$v_{h,t} = E_t \int_t^{T_m} \frac{m_\tau}{m_t} \left(Y(H_\tau^*) - I_\tau^*\right) \mathrm{d}\tau, \tag{16}$$

where m_t is the stochastic discount factor induced by the prices of financial assets, I^* denotes the agent's optimal human capital investment, and H^* denotes the corresponding path of his human capital process.

We can substitute investment I^* from (14) in the law of motion (3) to recover the optimal path for human capital H^* and corresponding income flow $Y(H^*)$. Recall also that the agent's investment opportunity set induces a unique stochastic discount factor m_t given by (7). Combining both in (16) leads to the following result.

Proposition 1 (HK) *The Human Capital Value of life is:*

$$v_h(H, \lambda_m) = C_0 \, \frac{y}{r} + C_1 P(H) \tag{17}$$

where the constants $(C_0, C_1) \in [0,1]^2$ are defined by:

$$C_0 = \frac{r}{r + \lambda_m}, \tag{18a}$$

$$C_1 = \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_m - (\alpha B)^{\frac{\alpha}{1-\alpha}}}, \tag{18b}$$

and where human wealth $P(H)$ is given in (10)*.*

Combining Definition (16) with the income equation (4) establishes that the HK value is the present value of the net income flow $(y + \beta H - I^*)$. Unlike in step-1 of the solution method, this present value is computed over a finite horizon and must be therefore be corrected for mortality exposure λ_m. The first term in (17) is the present value y/r of the agent's base income $y = Y(0)$ calculated over an infinite horizon and corrected for the exposure to death risk by multiplying with the constant $C_0 \in [0, 1]$ in (18a). The second term is the net present value $P(H)$ of the human capital cash flow $\beta H_t - I^*$ over an infinite horizon and this value is corrected for finite life by multiplying with the constant $C_1 \in [0,1]$ in (18b).

3.2 Willingness to pay to avoid a change in death risk

Next, consider an *admissible* change Δ in the intensity of death from base level λ_m in (1), i.e. one for which the indirect utility remains well defined when evaluated at the modified death exposure. The analysis of the WTP to avoid imminent death risk in a Gunpoint setting (discussed in Section 3.4) naturally designates the Hicksian Equivalent Variation (EV), rather than Compensating Variation (CV) as the relevant measure of willingness to pay (resp. to accept compensation) to avoid (resp. to forego) detrimental (resp. beneficial) changes in mortality.[17] We use standard variational analysis to define the corresponding Hicksian EV as follows:

Definition 2 (Hicksian Equivalent Variation) *Let $\mathcal{A}$ be the admissible set of permanent changes $\Delta \geq -\lambda_m$ in death intensity such that the conditions* (9) *of Theorem 1 hold when λ_m is evaluated at $\lambda_m^* = \lambda_m + \Delta$. Then the Equivalent Variation to avoid $\Delta \in \mathcal{A}$ is implicitly given as the solution $v = v(W, H, \lambda_m, \Delta)$ to:*

$$V\left(W - v, H; \lambda_m\right) = V\left(W, H; \lambda_m^*\right). \tag{19}$$

where $V(W, H; \lambda_m)$ is an indirect utility function.

For unfavorable changes $\Delta > 0$, the EV (19) indicates a willingness to pay $v > 0$ to remain at base risk instead of facing higher mortality. For favorable changes $\Delta < 0$, the EV is a willingness to accept (WTA) compensation equal to $-v > 0$ to forego lower risk.

The properties of the willingness to pay $v(W, H, \lambda_m, \Delta)$ with respect to the increment in death risk follow directly from those of the indirect utility $V(W, H; \lambda_m)$. In particular,

[17] Whereas paying out the WTP under a gunpoint threat is rational, accepting compensation against certain death when terminal wealth in not bequeathed and life is preferred to death cannot be. Since we abstract from bequests in our benchmark model in Section 2, we therefore adopt the EV, rather than CV perspective and focus on the WTP to avert unfavorable risks in subsequent analysis. For completeness, the extension to CV measures is nonetheless presented in Remark 2 below.

we can substitute the EV v in (19), take derivatives and re-arrange to obtain:

$$v_\Delta = -\frac{V_{\lambda_m}}{V_W}, \tag{20a}$$

$$v_{\Delta\Delta} = \frac{V_{\lambda_m\lambda_m} - V_{WW}v_\Delta^2}{-V_W}, \tag{20b}$$

where a subscript denotes a partial derivative. Monotonicity $V_W \geq 0$ and preference for life over death $V_{\lambda_m} \leq 0$ therefore induce a willingness to pay v that is increasing in Δ, whereas the diminishing marginal utility of wealth $V_{WW} \leq 0$ and of survival probability $V_{\lambda_m\lambda_m} \geq 0$ are sufficient to induce a concave WTP function in mortality risk exposure.

Relying on the indirect utility given in (13) for the human capital problem in Section 2 allows us to solve for the Hicksian variation as follows:

Proposition 2 (Hicksian EV) *The Equivalent Variation is:*

$$v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right] N(W, H). \tag{21}$$

It is increasing and concave in Δ *with*

$$\inf_{\Delta\in\mathcal{A}} v(W, H, \lambda_m, \Delta), = \left[1 - \frac{\Theta(0)}{\Theta(\lambda_m)}\right] N(W, H) \tag{22a}$$

$$\sup_{\Delta\in\mathcal{A}} v(W, H, \lambda_m, \Delta) = N(W, H). \tag{22b}$$

where net total wealth $N(W, H)$ *is given in* (11) *and its marginal value* $\Theta(\lambda_m)$ *is given in* (13b)*.*

The WTP in (21) equals zero if either $\Delta = 0$ or if the agent's elasticity of inter-temporal substitution $\varepsilon = 1$ because in this case the marginal utility of total wealth Θ is independent from λ_m. Moreover, the properties in (15) established that the indirect utility $V(W, H; \lambda_m)$ in (13a) is decreasing and convex in the death intensity λ_m. Consequently, the weights $\Theta(\lambda_m^*)/\Theta(\lambda_m) \in [0, 1]$ for detrimental changes $\Delta \geq 0$ and the willingness to pay is an unconditionally increasing function of net total wealth $N(W, H)$. Combining (15) with (20) confirms a monotone increasing and concave willingness to pay to avoid increases in death risk exposure in (21), consistent with standard economic intuition of diminishing marginal valuation of additional longevity (e.g. Philipson et al., 2010; Córdoba and Ripoll, 2017).

The lower bound on the WTP in (22a) is obtained by setting $\Delta = -\lambda_m$ yielding the WTA a compensation in order to forego zero death risk exposure.[18] From equations (9b) and (13b) this bound exists and is finite. Equation (22b) establishes that the willingness to pay is bounded above by net total wealth $N(W,H)$. When the elasticity of intertemporal substitution is larger than one, this upper bound corresponds to the asymptotic WTP. When the EIS is below one, the upper bound corresponds to a maximal admissible WTP satisfying the transversality constraint (9b) (see Appendix B.3).[19]

Remark 2 A similar reasoning defines the Hicksian Compensating Variation as follows:

$$V\left(W - v^c, H; \lambda_m^*\right) = V\left(W, H; \lambda_m\right)$$

which can be solved as

$$\begin{aligned} v^c(W,H,\lambda_m,\Delta) &= \left[1 - \frac{\Theta(\lambda_m)}{\Theta(\lambda_m^*)}\right] N(W,H), \\ &= \frac{-\Theta(\lambda_m)}{\Theta(\lambda_m^*)} v(W,H,\lambda_m,\Delta). \end{aligned} \tag{23}$$

Since $\Theta'(\lambda_m) < 0$, it follows that $0 < v^c < -v$ for $\Delta < 0$ and $0 < v < -v^c$ for admissible $\Delta > 0$, i.e. the WTP to attain a beneficial or avert a detrimental change in death risk is always less that the corresponding WTA to forego a favorable or accept an unfavorable change in mortality, consistent with standard Hicksian variational analysis (e.g. Smith and Keeney, 2005; Hammitt, 2008).

3.3 Value of a Statistical Life

The VSL is the marginal rate of substitution between life and wealth, evaluated at base risk (e.g. Eeckhoudt and Hammitt, 2004; Murphy and Topel, 2006; Bellavance et al.,

[18]See also Eeckhoudt and Hammitt (2004) for a WTP to fully eliminate mortality risk.

[19]Bommier et al. (2018, p. 13) contend that $\varepsilon \in (0,1)$ implies an upper bound on the death intensity, which could be problematic for elders with low life expectancy. However, this problem is overstated. Indeed, admissibility at low EIS implies that $\lambda_m \leq \bar{\lambda}_m(\varepsilon) = \rho\varepsilon/(1-\varepsilon) + r + 0.5\theta^2/\gamma$. Using the calibrated (ρ, r, θ) and estimated (γ) values in Table 3, as well as the calibrated EIS value by Bommier et al. (2019, p. 20) reveals that the associated minimal longevity is $[\bar{\lambda}_m(\varepsilon = 0.5)]^{-1} = 8.8$ years, well within reasonable ranges for cross-sectional and longitudinal household data. Moreover, our model can be adapted to either age-increasing (see Remark 5) and to health-decreasing (see Section 4.4 death intensities, both allowing for realistic life cycles in expected longevity. Finally, our estimate of the EIS in Table 3 is $\varepsilon = 1.24 > 1$ for which the admissible death intensities are unbounded above.

2009; Andersson and Treich, 2011; Aldy and Smyth, 2014). Adapted to our setting, the VSL is defined as:

Definition 3 (VSL) *The Value of a Statistical Life $v_s = v_s(W, H; \lambda_m)$ is the negative of the marginal rate of substitution between the probability of death and wealth computed from the indirect utility $V(W, H; \lambda_m)$ evaluated at base risk:*

$$v_s = -\frac{V_{\lambda_m}(W, H; \lambda_m)}{V_W(W, H; \lambda_m)} \tag{24}$$

where $V(W, H; \lambda_m)$ is an indirect utility function.

Remark 3 We can rely on the WTP property (20a) to rewrite the VSL in (24) as a marginal willingness to pay:

$$v_s(W, H; \lambda_m) = \frac{\partial v(W, H; \lambda_m, \Delta)}{\partial \Delta} = \lim_{\Delta \to 0} \frac{v(W, H; \lambda_m, \Delta)}{\Delta}. \tag{25}$$

Contrasting the theoretical definition of the VSL as a MWTP in (25) with its empirical counterpart reveals the links between the two measures. Indeed, the empirical VSL commonly relied upon in the literature (e.g. see footnote 8) can be expressed as:

$$v_s^e(W, H; \lambda_m, \Delta) = \frac{v(W, H; \lambda_m, \Delta)}{\Delta}, \tag{26}$$

for small increment $\Delta = 1/n$, where n is the size of the population affected by the change. The theoretical measure of the VSL in (25) is the limiting value of its empirical counterpart in (26) when the change Δ tends to zero. The importance of the bias between the empirical and theoretical VSL's $(v_s^e - v_s)$ will consequently depend on the curvature of the willingness to pay v, as well as on the size and sign of the change Δ, an issue to which we will return shortly.

Using Definition 3 and welfare (13), we can calculate the theoretical expression for the VSL for the parametrized model as follows.

Proposition 3 (VSL) *The Value of a Statistical Life is:*

$$v_s(W, H, \lambda_m) = \frac{1}{A(\lambda_m)} N(W, H), \tag{27}$$

where the marginal propensity to consume $A(\lambda_m)$ *is given in* (9b) *and net total wealth* $N(W, H)$ *is given in* (11).

The Value of a Statistical life is unconditionally positive, increasing in net worth, and decreasing in the MPC. Hence, the WTP to avoid admissible detrimental changes (21), the WTP to attain admissible beneficial ones (23), and the VSL (27) are all unconditionally increasing in wealth and the shadow value of human capital BH.[20] Observe that since the MPC is typically low (e.g. see Carroll, 2001, for a review), the VSL is expected to be significantly larger than net disposable resources $N(W, H)$.

Remark 4 (empirical VSL as a collective WTP) We can rely on our theoretical measure for the individual WTP to compute the collective willingness to pay to save a human life. Given a finite population of agents indexed $j \in \{1, 2, \ldots, n\}$ and a set of social weights $\boldsymbol{\eta} \in \mathbb{R}^n_+$, we can assume homogeneous parameters across agents and exploit the linearity of the WTP function (21) in wealth and human capital to derive the collective WTP as:

$$\sum_{j=1}^{n} \eta_j v_j(W_j, H_j, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda^*_m)}{\Theta(\lambda_m)}\right] \sum_{j=1}^{n} \eta_j N(W_j, H_j).$$

Imposing identical unit weights $\eta_j = 1, \forall j$ yields:

$$\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda^*_m)}{\Theta(\lambda_m)}\right] nN(\bar{W}, \bar{H}) = nv(\bar{W}, \bar{H}, \lambda_m, \Delta).$$

Evaluating the latter at $\Delta = n^{-1}$ yields the empirical VSL (26) measure commonly used in the literature:

$$\sum_{j=1}^{n} v_j(W_j, H_j, \lambda_m, \Delta) = \frac{v(\bar{W}, \bar{H}, \lambda_m, \Delta)}{\Delta} = v^e_s(\bar{W}, \bar{H}, \lambda_m, \Delta),$$

[20]This contradicts the conjecture by Bommier et al. (2018, 2019) that homogeneous recursive preferences with no bequests and low EIS systematically yield negative VSL and that poor individuals with $\varepsilon \in (0, 1)$ would be willing to pay more than rich ones for additional longevity. The apparent contradiction may likely be traced to Bommier et al. (2018, p. 15) making their claim from the MRS based on (unsolved) excess consumption $VSL = -U_{\lambda_m}/U_{c-a}$, rather than from the closed-form solution for $v_s = -V_{\lambda_m}/V_W$. Since (U_t, c_t) are both unsolved endogenous objects, the correct measure is to compute the MRS via the indirect utility as in (24).

i.e. under unit weights, the empirical VSL v_s^e is the collective WTP, corresponding to n times the individual WTP evaluated at mean wealth and human capital.

3.4 Gunpoint Value of Life

We next resort to the Gunpoint Value (GPV) as a third approach to the valuation of life. To do so, we adapt the Hicksian EV in Definition 2 to define the GPV as follows:

Definition 4 (GPV) *The Gunpoint Value v_g is the WTP to avoid certain, instantaneous death and is implicitly given as the solution to:*

$$V(W - v_g, H; \lambda_m) = V^m \tag{28}$$

where $V(W, H; \lambda_m)$ is an indirect utility V^m is the finite utility at certain death.

The willingness to pay v_g can be interpreted as the maximal amount paid to survive an *ex-ante* unforecastable and *ex-post* credible highwaymen threat. It also corresponds to the Ransom value for irreplaceable goods introduced by Cook and Graham (1977) and analyzed by Eeckhoudt and Hammitt (2004) in the context of one's own life. The main difference is that our formulation of the GPV is defined through the indirect utility $V(W, H; \lambda_m)$ in the context of a specific dynamic problem, instead of a generic utility $U(c)$.

Several points are worth mentioning in comparing the GPV with the HK and VSL. First, unlike the HK, the Gunpoint Value does not uniquely ascribe the economic worth of an agent to the capitalized net labor income that agent generates. Second, unlike the VSL, the GPV does not linearly extrapolate measurable responses to small probabilistic changes in the likelihood of death, but instead explicitly values a person's life as an entity and does so without external assumptions regarding integrability from marginal to total value of life. Finally, the GPV is theoretically computable at any admissible death intensity and applicable in life-or-death situations. As such, it is well suited in end-of-life terminal care decisions where neither the HK, nor the VSL are appropriate (Philipson et al., 2010).

Combining Definition 4 with the indirect utility (13), and noting that $V^m \equiv 0$ for preferences (6) reveals the following result for the GPV:

Proposition 4 (GPV) *The Gunpoint Value of life is:*

$$v_g(W, H) = N(W, H), \tag{29}$$

where $N(W, H)$ is the net total wealth in (11).

In the absence of bequest motives, the agent who is forced to evaluate life at gunpoint would be willing to pay the hypothetical (i.e. step-1) value of pledgeable resources. The discussion of net total wealth in (8) establishes that this amount corresponds to his entire financial wealth W, plus the capitalized value of his net income along the optimal path $Y(H^*) - I^*$. However, the previous discussion emphasized that the minimal consumption level a is required at all periods for subsistence. Its cost therefore cannot be pledged in a highwaymen threat and must be subtracted from the Gunpoint value. Indeed, it can be shown (Hugonnier et al., 2013, Prop. 2) that net total wealth $N(W, H)$ is equal to:

$$\begin{aligned} N(W_t, H_t) &= E_t \int_t^\infty \frac{m_\tau}{m_t} (c_\tau^* - a) \, \mathrm{d}\tau. \\ &= W_t + E_t \int_t^\infty \frac{m_\tau}{m_t} (Y(H_\tau^*) - a - I_\tau^*) \, \mathrm{d}\tau. \end{aligned} \tag{30}$$

To survive, the agent is thus willing to pledge the net present value of his optimal consumption stream (net of unpledgeable minimal subsistence), at which point he becomes indifferent between living and dying. This result can be traced to recursive preferences under which the foregone utility is measured in the same units as the foregone excess consumption. Interestingly, since net total wealth is independent from the agent's preferences $(\rho, \varepsilon, \gamma)$ and from the death intensity (λ_m), so is the GPV. Because death is certain and instantaneous when life is evaluated at gunpoint, the attitudes towards time and risk, as well as the level of exposure to death risk become irrelevant.

Combining (30) with Proposition 1 shows that the difference between the Gunpoint and HK values of life can be expressed as:

$$\begin{aligned} v_g(W_t, H_t) - v_h(H, \lambda_m) &= W_t - \frac{a}{r} + E_t \int_{T_m}^\infty \frac{m_\tau}{m_t} (Y(H_\tau^*) - I_\tau^*) \, \mathrm{d}\tau \\ &= W_t - \frac{a}{r} + (1 - C_0) \frac{y}{r} + (1 - C_1) P(H_t) \end{aligned}$$

The first two terms reflect the financial wealth and (capitalized) minimal consumption that affect net total wealth and therefore optimal consumption and welfare, but have no effects on optimal investment and, therefore, on the optimal path for net income $Y(H^*) - I^*$. The third and last terms show the mortality risk adjustments $(C_0, C_1) \in [0,1]^2$ on the net cash flow that are present in the HK value but not in the GPV. Unless minimal consumption requirements a/r are very large, the Gunpoint Value is therefore expected to be larger than the Human Capital Value.

The links between the willingness to pay in (21) and the GPV in (29) are intuitive and follow directly from the properties of the WTP. Indeed, the Gunpoint Value corresponds to the admissible upper bound (22b) on the willingness to pay to avoid a change in death risk exposure:

$$v_g(W, H) = \sup_{\Delta \in \mathcal{A}} v(W, H, \lambda_m, \Delta). \tag{31}$$

This upper bound exists and is finite by admissibility, i.e. compliance with transversality restrictions. Moreover, comparing (27) and (29) establishes that:

$$v_g(W, H) = A(\lambda_m) v_s(W, H, \lambda_m). \tag{32}$$

Estimates of the marginal propensity to consume $A(\lambda_m)$ are typically low, ranging between 2-9% for housing wealth and around 6% for financial wealth (e.g. Carroll et al., 2011, p. 58). Consequently, the predicted gap between the GPV and VSL is positive and large.

To gain further insight and without loss of generality, it is useful to set $t = 1$ in the probability of death (2) and evaluate for:

$$\mathcal{P} \equiv \Pr(T_m \leq 1) = 1 - e^{-\lambda_m},$$

a monotone increasing function of λ_m. The willingness to pay $v(\Delta_{\mathcal{P}}) = v(W, H; \mathcal{P}, \Delta_{\mathcal{P}})$ can then be analyzed over changes $\Delta_{\mathcal{P}} \in [-\mathcal{P}, 1 - \mathcal{P}]$ from base risk $\mathcal{P}$ and is plotted Figure 1. This graph emphasizes the central role of the WTP and illustrates why the theoretical VSL is expected to be larger than its empirical counterpart, and both are expected to be much larger than the GPV.

From properties (20), the WTP (solid blue line) is an increasing, concave function of the change in death risk $\Delta_{\mathcal{P}}$. The theoretical VSL v_s in (25) is the marginal willingness to pay, i.e. the slope of the dashed red tangent evaluated at base death risk ($\Delta_{\mathcal{P}} = 0$). It is equivalent to the linear projection corresponding to the total wealth spent to save one person (i.e. when $\mathcal{P} + \Delta_{\mathcal{P}} = 1.0$) and is equal to the distance [a,f]. The empirical Value of a Statistical Life v_s^e in (26) is computed for a small change $\Delta_{\mathcal{P}}^e > 0$ and is the slope of the dashed-dotted green line; equivalently, it is the linear projection represented by the distance [b,e]. The empirical VSL measure v_s^e will thus understate its theoretical counterpart v_s when $\Delta_{\mathcal{P}}^e \gg 0$ and when the WTP is concave. Moreover, equation (31) establishes that the Gunpoint value corresponds to the admissible upper bound on the WTP, i.e. the limiting WTP when death is certain as represented by the distance [c,d] in Figure 1. A concave WTP entails that a linear extrapolation under either the theoretical, or the empirical VSL will thus overstate the Gunpoint value attributed to one's own life, as confirmed from our discussion of (32).

Remark 5 (Aging) Our closed-form expressions for the willingness to pay and the three life valuations have thus far abstracted from aging processes. The latter can be incorporated although at some non-negligible computational cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in $\lambda_{mt}, \lambda_{st}, \phi_t, \delta_t$, or β_t results in age-dependent MPC and Tobin's-Q that solve the system of ordinary differential equations:

$$\dot{A}_t = A_t^2 - \left(\varepsilon\rho + (1-\varepsilon)\left(r - \lambda_{mt} + \theta^2/(2\gamma)\right)\right) A_t,$$
$$\dot{B}_t = (r + \delta_t + \phi_t \lambda_{st}) B_t + (1 - 1/\alpha)(\alpha B_t)^{\frac{1}{1-\alpha}} - \beta_t,$$

subject to appropriate boundary conditions. Allowing for aging and solving these differential equations for A_t, B_t implies that the solutions for C_{0t}, C_{1t}, the marginal value $\Theta_t(\lambda_{mt})$, as well as the human and total wealth $P_t(H), N_t(W, H)$ are also age-dependent. All the previous results remain applicable with these time-varying expressions.

4 Structural estimation

To estimate the willingness to pay and the three life valuations, we follow a long tradition associating the agent's human capital to his health (e.g. see the Hicks' lecture by Becker, 2007, for a review). We estimate the technological, preferences and parameters for the model outlined in Section 2 by contrasting the theoretical decisions in Theorem 1 to their observed counterparts. The estimated structural parameters can then be combined with observed wealth and health statuses to compute the closed-form expressions for the life valuations in Section 3.

4.1 Econometric model

For identification purposes, the econometric model assumes that agents follow the optimal rules in Theorem 1 and that they differ with respect to their health and wealth statuses but share common preference, technological and distributional parameters $\mathbf{\Theta} \in \mathbb{R}^k_+$. In particular, we use the optimal rules (14) to which we append the income equation (4). Specifically, denote by

$$\mathbf{Y}_j = [c_j, \pi_j, x_j, I_j, Y_j]'$$

the 5×1 vector of observed decisions and income for agents $j = 1, 2, \ldots, n$, and let

$$\mathbf{X}_j = [1, W_j, H_j]'$$

capture current wealth and health statuses. Also let $\mathbf{B}(\mathbf{\Theta})$ denote the 5×3 matrix of closed-form expressions for the optimal rules implicit in equation (14), that are functions of the structural parameters $\mathbf{\Theta}$. The econometric model relies on Maximum Likelihood to structurally estimates the latter in:

$$\mathbf{Y}_j = \mathbf{B}(\mathbf{\Theta})\mathbf{X}_j + \mathbf{u}_j \tag{33}$$

where the $\mathbf{u}_j$'s are (potentially correlated) Gaussian error terms.

In order to ensure theoretical consistency and augment identification, we estimate the structural parameters in (33) imposing the regularity conditions (9). Despite these measures, not all parameters can be estimated and a subset are fixed. In particular, we rely on standard values in the Asset Pricing literature to calibrate the returns process (μ, r, σ_S) and discount rate (ρ). The subsistence consumption (a) is set using poverty thresholds and consumption literature. Finally, the share of health capital lost in sickness (ϕ) is calibrated relying on earlier estimates in the literature, and via a thorough search procedure.

4.2 Data

We use a sample of n = 8,378 individuals taken from the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID). The data construction is detailed in Appendix D. We proxy the health variables through the polytomous self-reported health statuses (Poor, Fair, Good, Very Good and Excellent) that are linearly converted to numeric values ranging from 1 to 4. The financial wealth comprises risky and riskless assets. Using the method in Skinner (1987), we infer the unreported total consumption by extrapolating the food, transportation, and utility expenses reported in the PSID. Finally, health spending and health insurance expenditures are taken to be the out-of-pocket spending and the premia paid by agents. All nominal values are scaled by 10^{-6} for the estimation.

Tables 1, and 2 present descriptive statistics for the main variables of interest, per health status and per wealth quintiles. Table 2.a shows that financial wealth remains very low for the first three quintiles (see also Hubbard et al., 1994, 1995; Skinner, 2007, for similar evidence). Moreover no clear relation between health and wealth can be inferred. The level of consumption in panel b is increasing in financial wealth, consistent with expectations. However, the effects of health remain ambiguous, except for the least healthy who witness a significant drop in consumption. In panel c, stock holdings are very low for all but the fourth and fifth quintiles, illustrating the well-known non-participation puzzle (e.g. Friend and Blume, 1975; Mankiw and Zeldes, 1991). Again, a clear positive wealth gradient is observed, whereas health effects are weakly positive. The health insurance expenses in panel d are modest relative to consumption. They are increasing in wealth and devoid of clear health gradients. Finally, health spending in panel e is of the same order of magnitude as insurance. It is strongly increasing in wealth and also sharply decreasing in health status.

4.3 Estimation results

4.3.1 Structural parameters

Table 3 reports the calibrated (with subscripts ^{c}) and estimated (standard errors in parentheses) model parameters. Overall, the latter are precisely estimated and are consistent with other estimates for this type of model (e.g. Hugonnier et al., 2013, 2017).

First, the health law of motion parameters in panel a are indicative of significant diminishing returns in adjusting health status ($\alpha = 0.6843$). Although deterministic depreciation is relatively low ($\delta = 1.25\%$), morbidity is consequential with realistic additional depletion of $\phi = 1.36\%$,[21] and average waiting time between occurrence of $\lambda_s^{-1} = 28.8$ years. Second, exposure to mortality risk is realistic ($\lambda_m = 0.0283$), corresponding to a remaining expected lifetime of $\lambda_m^{-1} = 35.3$ years, given mean respondent age of 45.26 years in Table 1.[22] Third, the income parameters in panel c are indicative of a significant positive effect of health on labor income ($\beta = 0.0092$), as well as an estimated value for base income that is close to poverty thresholds ($y \times 10^6 = 12.2$ K\$).[23] The financial parameters (μ, σ_S, r) are calibrated from the observed moments of the S&P500 and 30-days T-Bills historical returns. Finally, the preference parameters in panel d indicate realistic aversion to financial risk ($\gamma = 2.8953$). The minimal consumption level is somewhat larger than base income ($a \times 10^6 = 14.0$ K\$), and is close to other calibrated values in the literature.[24] As for other cross-sectional estimates using survey data (Gruber, 2013; Hugonnier et al., 2017), the elasticity of inter-temporal substitution is larger than one ($\varepsilon = 1.2416$) and is consistent with a *Live Fast and Die Young* effect whereby a higher risk of death increases the marginal propensity to consume.[25]

4.3.2 Estimated valuations

Human Capital Value of Life Using the estimated parameters in Table 3, we can compute the HK value of life $v_h(H)$ given in (17) and reported in Table 4.a. Consistent

[21] Hugonnier et al. (2013) estimate $\phi = 1.11\%$ using pooled PSID data from 1999 to 2007.

[22] The remaining life expectancy at age 45 in the US in 2013 was 36.1 years (all), 34.1 (males) and 37.9 (females) (Arias et al., 2017).

[23] For example, the 2016 poverty threshold for single-agent households under age 65 was 12.5 K\$ (U.S. Census Bureau, 2017).

[24] For example, Murphy and Topel (2006, Tab. 2, p. 886) calibrate minimal composite consumption share z_0/z between 5-20%. Using mean consumption level in Table 1 yields $a/c = 14.23\%$.

[25] The EIS estimate is also close to the calibrated value of Córdoba and Ripoll (2017). Indeed, their calibration for $\sigma = 1/\varepsilon = 0.80$ is almost identical to our estimated $1/\hat{\varepsilon} = 1/1.2146 = 0.81$.

with predictions, the human capital values are independent from W and increasing in H, ranging from 250 K\$ (Poor health) to 527 K\$ (Excellent health), with a mean value of 421 K\$. These figures are realistic and compare advantageously with other HK estimates in the literature and provide a first out-of-sample confirmation that the structural estimates are reasonable.[26]

Value of Statistical Life Table 4.b reports the Values of Statistical Life in (27) by observed health and wealth statuses. The VSL mean value is 8.35 M\$, with valuations ranging between 2.17 M\$ and 15.01 M\$. These values are well within the ranges usually found in the empirical VSL literature.[27] The concordance of these estimates with previous findings provides additional out-of-sample evidence that our structural estimates are well grounded. Importantly, our joint theoretical and estimation approaches confirm the large VSL–HK discrepancies identified in the empirical literature.

It is also possible to assess a measure of the marginal vs infra-marginal WTP gap by calculating the empirical VSL measure in (26). Setting $\Delta = 1/n = 1/8,378$ and $\lambda^*_{m0} = \lambda_m + \Delta$, we recover an aggregate VSL of 8.34 M\$, which, as expected, is lower, but close to the mean theoretical value of $v_s(W, H, \lambda_m) = 8.3515$ M\$. This result confirms that the theoretical and empirical values are close to one another, i.e. the individual MWTP is well approximated by the collective WTP corresponding to the empirical VSL when $\Delta = 1/n$ is small (i.e. the sample size is large).

The VSL is increasing in both wealth and especially health. Positive wealth gradients have been identified elsewhere (Bellavance et al., 2009; Andersson and Treich, 2011; Adler et al., 2014) whereby diminishing marginal value of wealth and higher financial values at stake both imply that richer agents are willing to pay more to improve survival probabilities. The literature has been more ambivalent with respect to the health effect (e.g. Murphy and Topel, 2006; Andersson and Treich, 2011; Robinson and Hammitt, 2016). On the one hand better health increases the value of life that is at stake, on

[26]Huggett and Kaplan (2016, benchmark case, Fig. 7.a, p. 38) find HK values starting at about 300 K\$ at age 20, peaking at less than 900 K\$ at age 45 and falling steadily towards zero afterwards.

[27]A meta-analysis by Bellavance et al. (2009, Tab. 6, p. 452) finds mean values of 6.2 M\$ (2000 base year, corresponding to 8.6 M\$, 2016 value). Survey evidence by Doucouliagos et al. (2014) ranges between 6 M\$ and 10 M\$. Robinson and Hammitt (2016) report values ranging between 4.2 and 13.7 M\$. Finally, guidance values published by the U.S. Department of Transportation were 9.6 M\$ in 2016 (U.S. Department of Transportation, 2016), whereas the Environmental Protection Agency relies on central estimates of 7.4 M\$ (2006\$), corresponding to 8.8 M\$ in 2016 (U.S. Environmental Protection Agency, 2017).

the other hand, healthier agents face lower death risks and are thus less willing to pay to attain further improvements (or prevent deteriorations). Since our benchmark model abstracts from endogenous mortality (see the robustness discussion in Section 4.4 for generalization) and better health increases net total wealth $N(W, H)$, our estimates unambiguously indicate that the former effect is dominant and that improved health raises the VSL.

Gunpoint Value Table 4.c reports the Gunpoint values in (29). The mean GPV is 447 K$ and the estimates are increasing in both health and wealth and range between 116 K$ and 804 K$. The Gunpoint Value is thus of similar magnitude to the HK Value of life and both are much lower than the VSL. Indeed, this finding was already foreseeable from equation (32) indicating that the VSL/GPV ratio is inversely proportional to the marginal propensity to consume. Since our estimates reveal that $A(\lambda_m) = 5.36\%$ – a value again well in line with other estimates (e.g. Carroll et al., 2011) – we identify a VSL that is 18.66 times larger than the GPV.

Willingness to pay We emphasized that both the empirical and theoretical VSL will overstate the GPV corresponding to the upper bound on the concave willingness to pay. To help visualize this gap, Figure 2 is the estimated counterpart to Figure 1 and plots the willingness to pay $v(W, H, \lambda_m, \Delta)$ as a function of Δ calculated from (21) at the estimated parameters and relying on the mean wealth and health status.

The strongly concave estimated WTP in Figure 2 is informative as to why the VSL is much larger than the Human Capital and Gunpoint values. Indeed, the agent is willing to pay 37 K$ to avoid an increase of $\Delta = 0.0047$ which shortens his horizon from 35.3 to 30.3 years and would pay 406 K$ to avoid an increase of $\Delta = 0.17$ which lowers expected remaining lifetime from 35.3 to only 5 years. This last value is already close to the HK and GPV values of 421 K$ and 447 K$, which are both much lower than the VSL of 8.35 M$. Equivalently, the linear extrapolation of marginal values that is relied upon in the VSL calculation overstates the willingness to protect one's own life when the WTP is very concave in the death risk increment, as foreshadowed in our discussion of (27) and (32).

4.4 Robustness

To verify the robustness of our results, we now consider a more general model of human capital (Hugonnier et al., 2013) that is similar to our benchmark, with two key differences. First, the model allows for self-insurance against morbidity and mortality risks,[28] obtained by introducing health-dependent intensities:

$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m},$$
$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}},$$

where $H_{t-} = \lim_{s \uparrow t} H_s$ is health prior to the morbidity shock realization. Hence, better health lowers exposure to sickness and death risks and our benchmark model of Section 2 is an exogenous restricted case that imposes $\lambda_{s1}, \lambda_{m1} = 0$. Second, preferences are modified to allow for source-dependent aversion against financial, morbidity and mortality risks. In particular, the preferences in (6b) are replaced by:

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_{\tau-}) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_{\tau-}} - \sum_{k=m}^{s} U_{\tau-}\, \lambda_k(H_{\tau-}) F_k \left(H_{\tau-}, \frac{\Delta_k U_\tau}{U_{\tau-}} \right) \right) \mathrm{d}\tau,$$

with the Kreps-Porteus aggregator defined in (6c) and the penalties for exposure to sickness and death risks defined by

$$F_k = \frac{\Delta_k U_t}{U_{t-}} + \frac{1}{1-\gamma_k} \left[1 - \left(1 + \frac{\Delta_k U_t}{U_{t-}} \right)^{1-\gamma_k} \right],$$

where

$$\Delta_k U_t = E_{t-}[U_t - U_{t-} | \mathrm{d}Q_{kt} \neq 0].$$

Our benchmark specification of Section 2 is thus a restricted case that imposes risk-neutral attitudes towards morbidity ($\gamma_s = 0$) and mortality ($\gamma_m = 0$) risks.

While this model cannot be solved in closed form we show in a separate appendix that the expansion techniques of Hugonnier et al. (2013) can be used to derive an

[28] See also Dalgaard and Strulik (2014, 2017); Galama and van Kippersluis (2019) for life-cycle models with endogenous longevity and (Bennardo and Piccolo, 2014; Galama and van Kippersluis, 2019) for endogenous exposure to health depreciation.

approximate solution that is accurate to the first order in $(\lambda_{s1}, \lambda_{m1})$ and which in turn leads to approximate closed-form expressions for the WTP, HK, VSL and GPV values. These expressions encompass explicit adjustments for the endogeneity of health risks exposure and source-dependent risk aversion, yet remain otherwise qualitatively similar. We structurally estimate the Hugonnier et al. (2013) model and compute the life values. These values remain in the same range as our benchmark estimates, with mean HK of 493 K$, VSL of 8.14 M$ and GPV of 460 K$ and again confirm the strong concavity of the WTP. We conclude that our main findings are qualitatively and empirically robust to more general specifications.[29]

5 Conclusion

Computing the money value of a human life has long generated a profound and continued interest, with early records dating back to the late XVIIth century. The two most widely-used valuation frameworks have centered on the marginal rate of substitution between the probability of living and wealth (VSL) and on a person's human capital value that is destroyed upon death (HK). The two life valuations yield strikingly divergent measures, with the VSL being 10-20 times higher than the HK.

The absence of common theoretical underpinnings and the very different empirical settings in which the two values are calculated complicate any comparison exercise between the HK and VSL. We address this issue via a common human capital problem to analytically compute and structurally estimate the theoretical VSL and HK values. We also introduce a third life value that reflects the maximum amount an agent would be willing to pay to save himself from instantaneous and certain death (GPV) to serve as useful benchmark. The willingness to pay to avoid changes in death risk, as well as the three closed-form for the life values are estimated jointly using a common structural econometric model and PSID data set. This encompassing approach thus provides direct comparability as well as a unique opportunity to identify the role of the preferences, distributional and technological parameters on life valuations.

[29] In addition to robust life valuations, the preference parameter estimates for the generalized model are consistent with a preference for late, as opposed to early resolution of death uncertainty favored by Córdoba and Ripoll (2017). Indeed, our estimated inverse EIS, $1/\hat{\varepsilon} = 1/1.6699 = 0.5988$, is larger than our estimated mortality risk aversion, $\hat{\gamma}_m = 0.2862$.

Our main findings can be summarized as follows. We confirm the large discrepancies with an HK value of 421 K$ and a VSL of 8.35 M$ and show that the Gunpoint value of 447 K$ is similar to the HK. Our results accord with the standard economic intuition that the willingness to pay to avert death risk is increasing, but strongly concave and finite in mortality exposure. Allowing for a more general model with endogenous sickness and death intensities as well as source-dependent risk aversion only reaffirmed our findings. The large HK-VSL gaps are therefore robust to the use of integrated theoretical and empirical measurements.

We show that the HK-VSL differences are related to the strong curvature and finiteness of the WTP. In particular, the theoretical VSL is a linear projection from the marginal willingness to pay, whereas the empirical VSL is a local approximation to that MWTP. When the WTP is strongly concave, both theoretical and empirical VSL will strongly overestimate the limiting willingness to pay that corresponds to the Gunpoint Value. The empirical similarities between the HK and GPV values reflect the close theoretical parallels in the measured object. The HK computes the net present value of the foregone dividend stream associated with human capital (i.e. income, minus investment costs). The GPV measures the NPV of the foregone utility stream associated with living. The homogeneity properties entail that the latter is also the NPV of the foregone consumption above minimal subsistence requirements.

The Human Capital, Willingness to pay, Value of Statistical Life and Gunpoint value of life remain specialized tools that are complementary to one another and are applicable in specific contexts. Our encompassing approach provides single-step HK, WTP, VSL and GPV measurement in fully integrated theoretical and empirical environments. It precisely identifies the roles of human capital, wealth, preferences, technology and distribution parameters in life valuations. Finally, our framework is very general, and can be easily extended along other dimensions (aging, attitudes, self-insurance).

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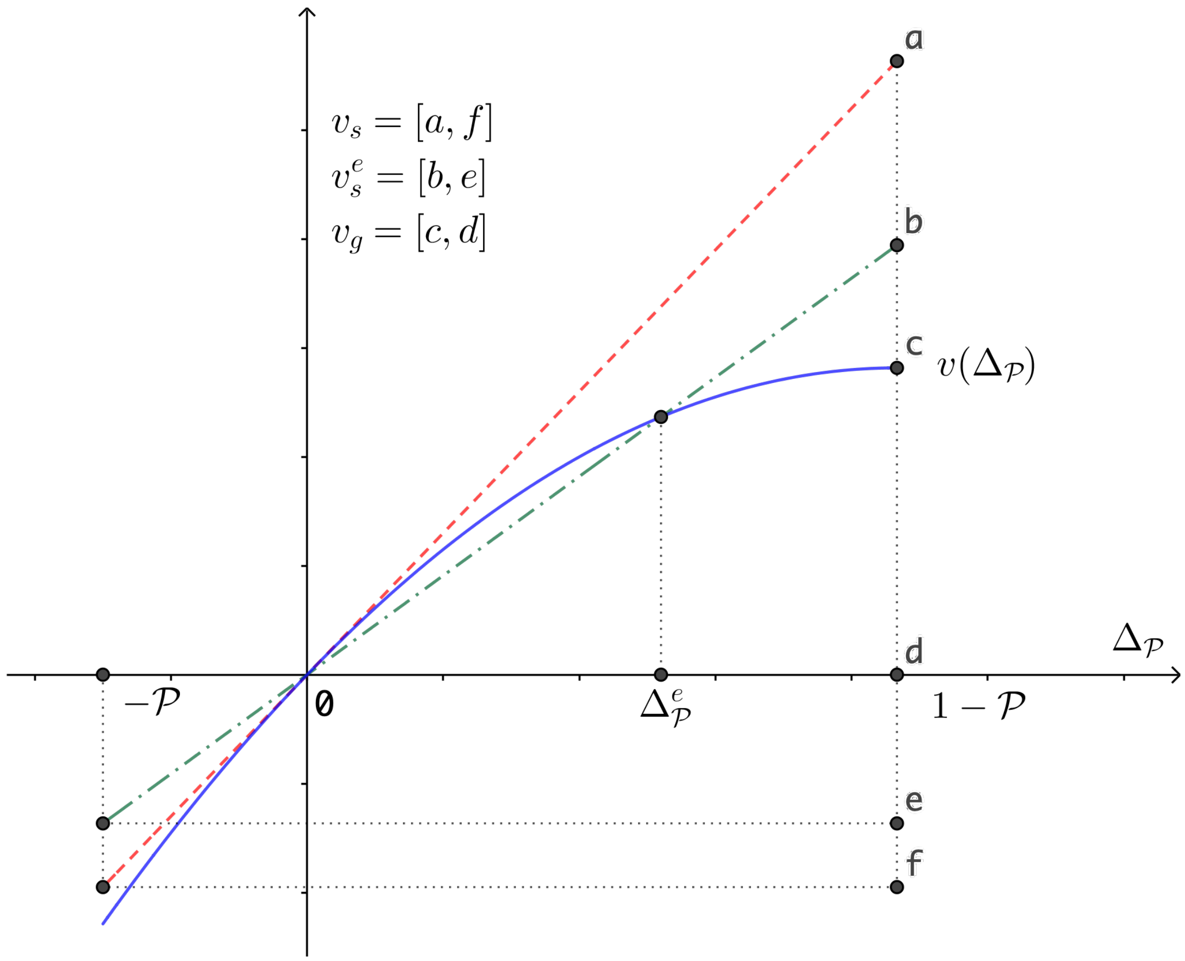
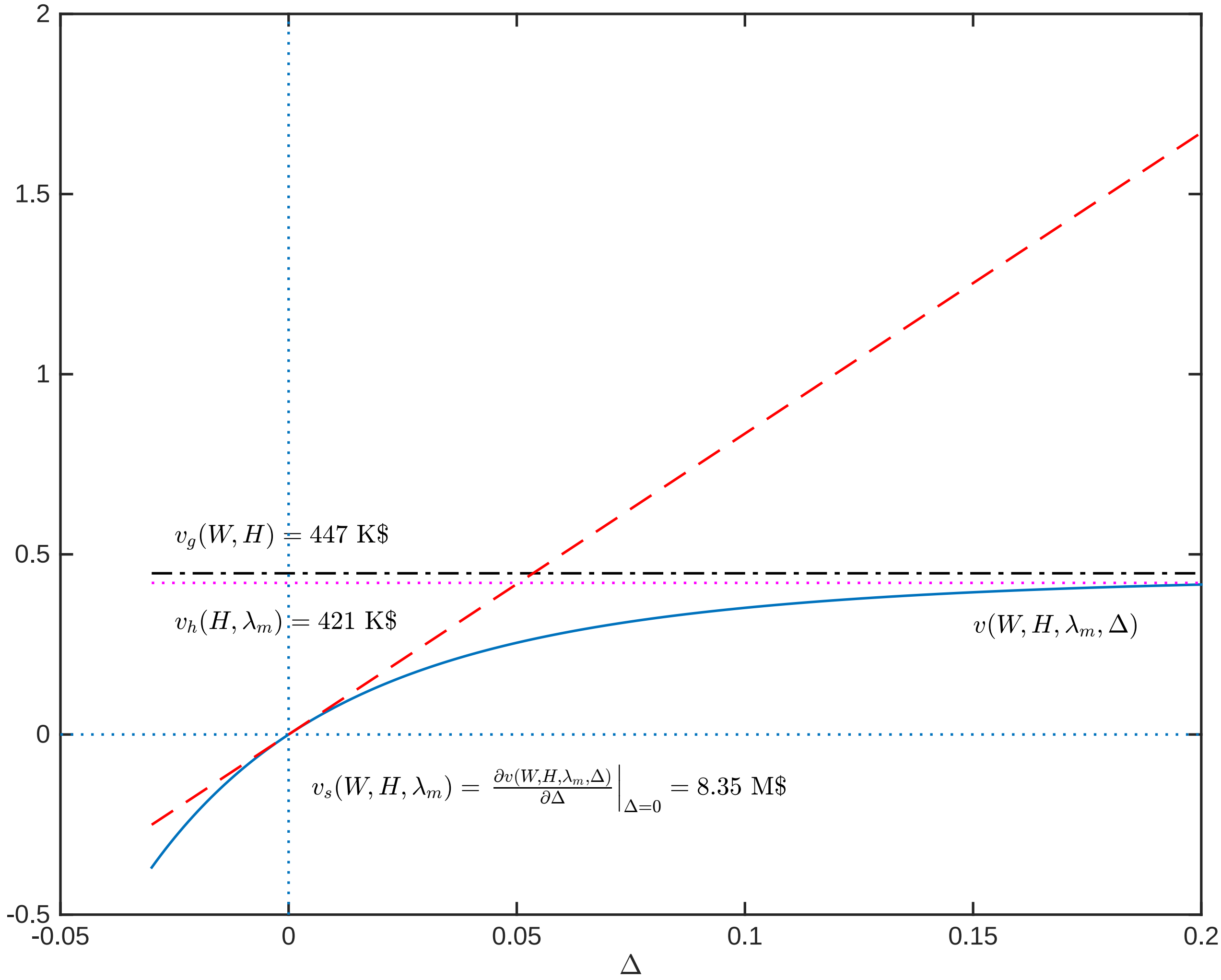
A Figures

Figure 1: Willingness to pay and life valuations



Notes: $\Delta_{\mathcal{P}} \in [-\mathcal{P}, 1-\mathcal{P}]$ is change in the probability of death from base exposure $\mathcal{P} = 1 - e^{-\lambda_m}$. $v(\Delta_{\mathcal{P}}) = v(W, H; \mathcal{P}, \Delta_{\mathcal{P}})$ is the willingness to pay to avoid $\Delta_{\mathcal{P}}$ is solid blue line. $v_s = v'(0)$ is the theoretical Value of Statistical Life in (25) is slope of tangent, i.e. dashed red line and equal to distance [a,f]. $v_s^e = v(\Delta_{\mathcal{P}}^e)/\Delta_{\mathcal{P}}^e$ is the empirical Value of Statistical Life in (26) is slope of dashed-dotted green line and equal to distance [b,e]. $v_g = \sup_{\Delta_{\mathcal{P}}}(v)$ is the Gunpoint Value of Life in (28) is equal to distance [c,d].

Figure 2: Estimated WTP, HK, VSL and GPV Values of life (in M$)



Notes: At estimated parameter values, for mean wealth and health levels. $v(W, H, \lambda_m, \Delta)$ (blue solid line) is the willingness to pay to avoid an increase of Δ in exogenous death intensity λ_m; $v_h(H, \lambda_m)$ (magenta dashed) is the Human Capital value of life; $v_g(W, H)$ (black dashed-dotted) is the Gunpoint value of life; $v_s(W, H, \lambda_m)$ is the Value of statistical life and the slope of the dashed red tangent evaluated at $\Delta = 0$.

B Proofs

B.1 Theorem 1

The benchmark human capital model of Section 2 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death and depreciation intensities are constant at λ_m, λ_s (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e. $\gamma_s = \gamma_m = 0$). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1,Theorem 1) yields the the optimal solution in (14).

■

B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital $P(H)$ from

$$\begin{aligned} P(H) &= E_t \int_t^\infty \frac{m_\tau}{m_t} \left[\beta H_\tau^* - I_\tau^*\right] \mathrm{d}\tau, \\ &= BH. \end{aligned}$$

Straightforward calculations adapt this result to a stochastic horizon T^m and include the fixed income component y in income (4).

■

B.3 Proposition 2

Combining the Hicksian EV (19) with the indirect utility (13a) and using the linearity of the net total wealth in (11) reveals that the WTP v solves:

$$\begin{aligned} \Theta(\lambda_m^*) N(W, H) &= \Theta(\lambda_m) N(W - v, H) \\ &= \Theta(\lambda_m) \left[N(W, H) - v\right] \end{aligned}$$

where we have set $\lambda_m^* = \lambda_m + \Delta$. The WTP $v = v(W, H, \lambda_m, \Delta)$ is solved directly as in (21).

Next, by the properties of the marginal value of net total wealth, $\Theta(\lambda_m^*)$ in (15) is monotone decreasing and convex in Δ. It follows directly from (20) that the WTP

$$v(W, H, \lambda_m, \Delta) = \left[1 - \frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right] N(W, H)$$

is monotone increasing and concave in Δ.

The lower bound follows directly from evaluating finite and admissible $A(\lambda_m^*), \Theta(\lambda_m^*)$ at $\lambda_m^* = 0$ in (21). To compute the upper bound, two cases must be considered:

1. For $0 < \varepsilon < 1$, the MPC in (9b) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\lambda_m^* = \lambda_m + \Delta < \bar{\lambda}_m = \left(\frac{\varepsilon}{1-\varepsilon}\right)\rho + \left(r + \frac{\theta^2}{2\gamma}\right).$$

   Admissibility $\mathcal{A}$ therefore requires $\Delta < \bar{\Delta} = \bar{\lambda}_m - \lambda_m$ for the transversality condition (9b) to be verified. The supremum of the WTP is then $v(W, H, \lambda_m, \bar{\Delta}) = N(W, H)$.

2. For $\varepsilon > 1$, the MPC is monotone increasing and transversality is always verified. Consequently, the WTP is well-defined over the domain $\Delta \geq -\lambda_m$. It follows that:

$$\lim_{\Delta \to \infty} \Theta(\lambda_m + \Delta) = 0$$
$$\lim_{\Delta \to \infty} v(W, H, \lambda_m, \Delta) = N(W, H)$$

   i.e. the willingness to pay asymptotically converges to net total wealth as stated in (22b).

■

B.4 Proposition 3

By the VSL definition (24) and the properties of the Poisson death process (10):

$$v_s = \frac{-V_{\lambda_m}(W, H, \lambda_m)}{V_W(W, H, \lambda_m)}$$

From the properties of the welfare function (13a), we have that $V_{\lambda_m} = \Theta'(\lambda_m)N(W, H)$, whereas $V_W = \Theta(\lambda_m)$. Substituting for Θ in (13b) yields the VSL in (27). ■

B.5 Proposition 4

Combining the Hicksian EV (28) with the indirect utility (13a) and the net total wealth in (11) reveals that the WTP v solves:

$$\begin{aligned} V^m \equiv 0 &= \Theta(\lambda_m)N(W - v_g, H) \\ &= \Theta(\lambda_m)\left[N(W, H) - v_g\right] \end{aligned}$$

Solving for v_g reveals that it is as stated in (29). Because net total wealth is independent of the preference parameters $(\varepsilon, \gamma, \rho)$, so is the Gunpoint Value. ■

C Tables

C.1 Data

Table 1: PSID data statistics

	Model	Mean	Std. dev.	Min	Max
Health	H	2,85	0,80	1	4
Wealth	W	38 685	122 024	0	1 430 000
Consumption	c	9 835	11 799	1,05	335 781
Risky holdings	π	20 636	81 741	0	1 367 500
Insurance	x	247	718	0	17 754
Health investment	I	721	2 586	0	107 438
Income	Y	21 838	37 063	0	1 597 869
Age	t	45	16	16	100

Notes: Statistics in 2013 $ for PSID data used in estimation (8 378 observations). Scaling for self-reported health is 1.0 (Poor), 1.75 (Fair), 2.50 (Good), 3.25 (Very good) and 4.0 (Excellent).

Table 2: PSID data statistics (cont'd)

Health	Wealth quintiles 1	2	3	4	5
			a. Wealth W_j (\$)		
Poor	0	70	1 139	10 357	136 209
Fair	0	71	1 109	10 861	188 044
Good	0	86	1 214	11 207	160 925
Very Good	0	90	1 282	11 654	178 580
Excellent	0	88	1 315	11 974	214 106
			b. Consumption c_j (\$)		
Poor	3 943	3 859	6 216	10 473	18 226
Fair	4 724	5 702	9 256	13 491	15 610
Good	6 459	5 742	9 205	12 457	17 109
Very Good	5 684	5 582	9 442	11 812	15 702
Excellent	6 177	5 616	10 117	11 575	17 465
			c. Stocks π_j (\$)		
Poor	0	0	83	1 402	39 752
Fair	0	1	107	2 811	100 461
Good	0	4	143	3 299	82 499
Very Good	0	3	110	3 673	101 223
Excellent	0	3	116	3 627	125 934
			d. Insurance x_j (\$)		
Poor	50	142	123	304	230
Fair	83	134	162	320	537
Good	132	104	268	335	512
Very Good	106	64	209	316	483
Excellent	108	87	240	314	455
			e. Investment I_j (\$)		
Poor	783	792	852	2 021	4 447
Fair	538	762	777	1 711	2 969
Good	347	482	623	1 219	1 352
Very Good	250	318	422	639	1 070
Excellent	360	327	488	532	861

Notes: Statistics in 2013 $ for PSID data used in estimation. Means per quintiles of wealth and per health status

C.2 Benchmark model

Table 3: Estimated and calibrated structural parameter values, benchmark model

Parameter	Value	Parameter	Value
	a. Law of motion health (3)		
α	0.6843	δ	0.0125
	(0.3720)		(0.0060)
ϕ	0.0136^c		
	b. Sickness (3) and death (1) intensities		
λ_s	0.0347	λ_m	0.0283
	(0.0108)		(0.0089)
	c. Income (4) and wealth (5)		
y	0.0120	β	0.0092
	(0.0049)		(0.0044)
μ	0.108^c	r	0.048^c
σ_S	0.20^c		
	d. Preferences (6)		
γ	2.8953	ε	1.2416
	(1.4497)		(0.3724)
a^c	0.0140	ρ^c	0.0500

Notes: Estimated structural parameters (standard errors in parentheses); c: calibrated parameters. Econometric model (33), estimated by ML, subject to the regularity conditions (9).

Table 4: Estimated Values of Life (in $)

Health level	Wealth quintile 1	2	3	4	5
			a. Human Capital $v_h(W, H, \lambda_m)$ in (17)		
Poor			249 532		
Fair			318 865		
Good			388 198		
Very Good			457 531		
Excellent			526 864		
All					
- mean			420 729		
- median			457 731		
			b. Value of Statistical Life $v_s(W, H, \lambda_m)$ in (27)		
Poor	2 167 573	2 168 877	2 188 829	2 360 907	4 710 118
Fair	4 379 551	4 380 874	4 400 253	4 582 287	7 889 684
Good	6 591 529	6 593 136	6 614 190	6 800 733	9 595 444
Very Good	8 803 507	8 805 188	8 827 429	9 021 052	12 136 981
Excellent	11 015 485	11 017 133	11 040 023	11 238 999	15 012 108
All					
- mean			8 351 519		
- median			8 803 507		
			c. Gunpoint Value $v_g(W, H)$ in (29)		
Poor	116 121	116 191	117 259	126 478	252 329
Fair	234 620	234 691	235 729	245 481	422 664
Good	353 120	353 206	354 334	364 327	514 045
Very Good	471 619	471 709	472 901	483 274	650 199
Excellent	590 119	590 207	591 433	602 093	804 225
All					
- mean			447 405		
- median			471 619		

Notes: Averages of individual values in the PSID sample, computed at estimated parameter values, multiplied by 1 M$ to correct for scaling used in estimation.

D Data

The data construction follows the procedure in Hugonnier et al. (2013). We rely on a sample of 8,378 U.S. individuals obtained by using the 2013 wave of the Institute for Social Research's Panel Study of Income Dynamics (PSID, http://psidonline.isr.umich.edu/). All nominal variables in per-capita values (i.e., household values divided by household size) and scaled by 10^{-6} for the estimation. The agents' wealth and health are constructed as follows:

Health H_j Values of 1.0 (Poor health), 1.75 (Fair), 2.5 (Good), 3.25 (Very good) and 4.0 (Excellent) are ascribed to the self-reported health variable of the household head.

Wealth W_j Financial wealth is defined as risky (i.e. stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless (i.e. checking accounts plus bonds plus remaining IRA's and pension assets) assets.

The dependent variables are the observed portfolios, consumption, health expenditure and health insurance and are constructed as follows:

Portfolio π_j Money value of financial wealth held in risky assets.

Consumption c_j Inferred from the food, utility and transportation expenditures that are recorded in PSID, using the Skinner (1987) method with the updated shares of Guo (2010).

Health expenditures I_j Out-of-pocket spending on hospital, nursing home, doctor, outpatient surgery, dental expenditures, prescriptions in-home medical care.

Health insurance x_j Spending on health insurance premium.