CHAPTER THIRTEEN

Modeling Exchange Rates with Incomplete Information

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13.1 Introduction

Most models of exchange rate determination make a set of heroic assumptions about the information with which investors operate in the foreign exchange (FX) market. In particular, investors are assumed to (i) have identical information; (ii) perfectly know the model; and (iii) use all available information at all times. These assumptions are typical in macroeconomics and are technically convenient. However, recent research has shown that these abstractions about the information structure have crucial implications and that relaxing them can shed light on a wide range of important exchange rate puzzles. In this chapter, we review a number of models that we have developed in previous work to relax these restrictive assumptions on information. We also review some related literature.

It is not difficult to argue that the ‘‘benchmark’’ information structure commonly used in models of exchange rate determination bears little resemblance to reality. The assumption of common information held by all investors is inconsistent with various observations. First, there is an enormous volume of trade in the FX market (larger than in any other financial market), reflecting differences among investors. Second, investors have different expectations about

Handbook of Exchange Rates, First Edition. Edited by Jessica James, Ian W. Marsh, and Lucio Sarno.

future macro variables such as GDP and prices as well as future exchange rates themselves. Third, the close link between exchange rates and order flow, first documented by Evans and Lyons (2002), suggests that the exchange rate primarily aggregates private as opposed to public information.

That investors perfectly know the model is also a radical simplification of reality. There exists a considerable amount of uncertainty about the model and about structural parameters. This implies a learning process by investors, which affects their behavior. It also makes policy, especially monetary policy, more difficult. A substantial literature has documented parameter instability in macroeconomic data, while the implications of model uncertainty for monetary policy have also been investigated. There is also widespread evidence of parameter instability in financial data (see Pastor and Veronesi (2009) for a survey), including exchange rates (Rossi, 2006).

Finally, the assumption that everyone uses all available information at all times ignores the cost of continuous information processing. There are two ways in which information processing is limited. First, as we will discuss later on, most financial institutions and individual investors do not actively manage the FX exposure of external claims. They do not continuously adjust their FX holdings based on all available information as it is costly to do so. Second, even when they do change their portfolios, decisions are usually made on the basis of only a limited set of information. The best known example of this behavior is the carry trade, which may be conditioned only on interest rate differentials.

Through some simple examples, we illustrate that relaxing these restrictive assumptions about the information structure allows us to shed light on some of the biggest puzzles related to exchange rates, such as the disconnect between exchange rates and macro fundamentals and the forward discount puzzle. Our strategy is to start from a standard exchange rate model, the monetary model, and introduce various types of incomplete information. We consider only small deviations from the benchmark case, so that investors still use what they know about the model's structure to form their expectations.

The remainder of this chapter is organized as follows. In Section 13.2, we start by discussing a standard "benchmark" monetary model of exchange rate determination that makes the usual set of restrictive assumptions about the information structure. The subsequent three sections relax some of these assumptions, one at a time. In Section 13.3, we allow for information heterogeneity across investors. In Section 13.4, we introduce model uncertainty in the form of time-varying structural parameters that are unknown. Finally, in Section 13.5, we discuss what happens when investors do not continuously process all available information. Section 13.6 provides concluding remarks.

13.2 Basic Monetary Model

The simplest dynamic model of exchange rate determination is the monetary model. We examine the impact of incomplete information within a two-country version of this standard framework. The model is described by the following four equations:

$$m_t = p_t + \phi y_t - \alpha i_t \tag{13.1}$$

$$m_t^* = p_t^* + \phi y_t^* - \alpha i_t^* \tag{13.2}$$

$$p_t = p_t^* + s_t \tag{13.3}$$

$$E_t(s_{t+1} - s_t) = i_t - i_t^* + \psi_t \tag{13.4}$$

Equation (13.1) is a standard money market equilibrium equation, with m_t being the log-money supply, p_t the log-price level, y_t the log-output level, and i_t the interest rate. Equation (13.2) is the analogous equation for the Foreign country.[1] Equation (13.3) is a purchasing power parity equation and Equation (13.4) is an interest rate parity equation. ψ_t is the deviation from uncovered interest rate parity (UIP).

Substituting Equations (13.1)–(13.3) into Equation (13.4) we obtain a first-order difference equation with a familiar solution

$$s_t = (1-\lambda)\left[f_t + E_t \sum_{j=1}^{\infty} \lambda^j f_{t+j}\right] - \lambda\left[\psi_t + E_t \sum_{j=1}^{\infty} \lambda^j \psi_{t+j}\right] \tag{13.5}$$

where $f_t = m_t - m_t^* - \phi(y_t - y_t^*)$ and $\lambda = \alpha/(1+\alpha)$.

With full information, expectations can be computed from the known process for the fundamental f_t and the UIP deviation ψ_t. For example, when they follow an autoregressive (AR) process with AR coefficients of, respectively, ρ_f and ρ_ψ, we have

$$s_t = \frac{1-\lambda}{1-\lambda\rho_f} f_t - \frac{\lambda}{1-\lambda\rho_\psi}\psi_t \tag{13.6}$$

In this case the exchange rate is directly linked to the observed macro fundamentals f_t and ψ_t.

The implicit assumption behind Equation (13.6) is that investors have no information about future fundamental shocks. However, the solution is very similar when agents receive public signals about future fundamentals, such as public news variables that are featured in the literature on the impact of news shocks.[2] For example, let $v_t = f_{t+1} + \varepsilon_t$ be a piece of public information about

[1] This traditional money market equilibrium can easily be replaced by an interest rate rule, which is more typical in DSGE models. Equation 13.1 can be written as $i_t = \kappa_0 + \kappa_1(p_t - \overline{p}) + \kappa_2(y_t - \overline{y}) + \kappa_3(m_t - \bar{m})$. Often, other variables appear in interest rate rules, such as the current or expected inflation rate, but this does not fundamentally change the specification. It just involves replacing one fundamental variable in the interest rate rule, such as $m_t - \bar{m}$, with another fundamental variable, such as $\pi_t - \bar{\pi}$, where π_t is the inflation rate.

[2] See for examples Beaudry and Portier (2006), Devereux and Engel (2006), Jaimovich and Rebelo (2008), and Lorenzoni (2010).

f_{t+1}, where the variance of ε_{t+1} is σ_v^2. Together with the signal $f_{t+1} = \rho_f f_t + \varepsilon_{t+1}^f$, signal extraction implies $E_{t+1} f_{t+1} = a_1 f_t + a_2 v_t$, where $a_1 = \rho_f / (\sigma_f^2 d)$, $a_2 = 1/(\sigma_v^2 d)$ and $d = (1/\sigma_v^2) + (1/\sigma_f^2)$. Since $E_t f_{t+j} = \rho_f^{j-1} E_t f_{t+1}$ for $j > 1$, we then have

$$s_t = (1-\lambda)\frac{1+\lambda(a_1-\rho_f)}{1-\lambda\rho_f} f_t + (1-\lambda)\frac{\lambda a_2}{1-\lambda\rho_f} v_t - \frac{\lambda}{1-\lambda\rho_\psi}\psi_t \qquad (13.7)$$

The exchange rate again depends on a set of publicly observed variables, with v_t now added to the list.

This model contains all restrictive assumptions about the information structure alluded to in the introduction. All agents have the same public information. They all know the model. The parameters of the model are constant and known. Finally, all agents continuously adjust their portfolio based on all available information. This latter assumption is generally made in rational expectation dynamic portfolio choice models. In these models, the expected excess return on Foreign bonds (the UIP deviation) is then equal to a risk premium.

The model has many implications that are at odds with the data. First, it implies that the exchange rate is exclusively determined by public information. This stands in sharp contrast to the widespread evidence of a disconnect between exchange rates and observed macro variables. The best illustration of this disconnect is the well-known Meese–Rogoff puzzle. Meese and Rogoff (1983) tried to explain exchange rate movements with observed macroeconomic fundamentals and found that a fundamental-based model cannot outperform a random walk.[3] Their findings imply that the limited explanatory power of observed macro fundamentals is dominated by small sample estimation errors of reduced form parameters. This generates an even weaker fit than not using any macro fundamentals at all, as in the random walk model.

Notice that the puzzle here is not why the exchange rate is a random walk. Engel and West (2005) have shown that the benchmark model above can generate near-random-walk behavior when the discount rate λ is close to 1 and the fundamental is an I(1) variable. The puzzle, rather, is the very limited explanatory power of observed macro fundamentals. Even when the discount rate is close to 1 and the exchange rate is close to a random walk, in standard models, changes in the exchange rate are fully determined by changes in observed macro fundamentals.

The model also implies a stable relationship between exchange rates and fundamentals. As we discuss later on, there is plenty of evidence that this relationship is highly unstable. It is for this very reason that Meese and

[3]More precisely, Meese and Rogoff (1983) estimate a linear exchange rate model based on standard fundamentals such as money supply, output, and interest rates. They use the estimated model to do a one-period-ahead forecast, but use the actual future fundamental (which implies this is not a true forecast). They do this for several periods using rolling regressions and compute the RMSE. They do the same exercise by predicting the exchange rate with a random walk. The RMSE for the random walk model is generally lower than that for the model based on fundamentals.

Rogoff (1983) conducted rolling regressions to reestimate model parameters each month.

Finally, the model suffers from the well-known forward discount puzzle for standard justifications of the UIP deviation ψ_t. This is most clear when we set ψ_t equal to zero. Equation (13.4) then implies that high interest rate currencies tend to depreciate, while in reality, the evidence consistently shows that they tend to appreciate. The puzzle can potentially be explained when ψ_t is a time-varying risk premium, as in standard models where agents continuously adjust their portfolio. But so far the quest for such a model matching the data has remained unsuccessful.[4]

We now turn to generalizations of the simple information structure above and discuss how they can generate a better fit to the data.

13.3 Information Heterogeneity

The first deviation from the benchmark we consider is information heterogeneity as analyzed in Bacchetta and van Wincoop (2006). There is symmetric information dispersion in the sense that agents have private signals, but no agent has superior information. There are two types of information heterogeneity. First, agents have private information about the future level of the fundamental. Second, agents have private trading needs that are only known to themselves and are unrelated to expectations about the future fundamental. Examples of this are private liquidity needs, hedging needs, or private investment opportunities. This leads to a source of demand or supply of Foreign bonds that is unrelated to expected returns and is unobservable in the aggregate.

The main implication of having private information about future fundamentals is that the exchange rate becomes a source of information. Since the exchange rate reflects demand or supply from heterogeneous agents, it aggregates information about future fundamentals. However, the exchange rate is still a noisy signal, as in the noisy rational expectation literature, because of the unrelated private trading needs.

These two types of information heterogeneity lead to three changes to the model Eqs. (13.1)–(13.4). First, the UIP deviation ψ_t is equal to a risk premium. The "nonspeculative" liquidity or hedging needs are unrelated to expected returns and represent a separate source of risk. This risk premium is unobserved as it depends on the aggregate net supply of Foreign bonds associated with liquidity or hedge trade. While agents know their own liquidity or hedge trade, they cannot observe it at the aggregate.

The second change is that the expectation $E_t s_{t+1}$ now needs to be replaced by the average expectation $\bar{E}_t s_{t+1}$ across all agents. We assume that there is

[4]See surveys by Lewis (1995) and Engel (1996). Burnside et al. (2011) find that there is very little connection between excess returns on currency strategies and a wide range of possible risk factors. Verdelhan (2010) has had some success based on a model with habit formation, but his explanation relies on the close link between consumption and real exchange rates that is not observed in the data.

a continuum of agents in the interval [0,1]. Finally, agents receive a private signal about future fundamentals. For simplicity, we assume that agents receive a private signal about the fundamental next period. Agent i receives the signal $v_t^i = f_{t+1} + \varepsilon_t^{v,i}$, where the signal error $\varepsilon_t^{v,i}$ has a $N(0, \sigma_v^2)$ distribution.[5]

In addition, we make the simplifying assumptions that ψ_t is i.i.d. with variance σ_ψ^2 and that f_t follows a random walk $f_{t+1} = f_t + \varepsilon_{t+1}^f$. The variance of ε_{t+1}^f is σ_f^2. Substituting Equations (13.1)–(13.3) into Equation (13.4), we have

$$s_t = \lambda \bar{E}_t s_{t+1} + (1 - \lambda) f_t - \lambda \psi_t \tag{13.8}$$

The model is solved in three steps. First, conjecture a solution

$$s_t = (1 - \lambda_f) f_t + \lambda_f f_{t+1} - \lambda_\psi \psi_t \tag{13.9}$$

Second, for each investor, compute the expectation of f_{t+1}. This is done by solving a standard signal extraction problem using three sources of information: the random walk process $f_{t+1} = f_t + \varepsilon_{t+1}^f$, which is public information, the private signal, and the exchange rate equation. The exchange rate signal is $(s_t - (1 - \lambda_f) f_t)/\lambda_f = f_{t+1} - \lambda_\psi \psi_t / \lambda_f$. This gives

$$E_t^i f_{t+1} = \frac{\beta^f f_t + \beta^v v_t^i + \beta^s (s_t - (1 - \lambda_f) f_t)/\lambda_f}{D} \tag{13.10}$$

where $\beta^f = 1/\sigma_f^2$, $\beta^v = 1/\sigma_v^2$, $\beta^s = \lambda_f^2/(\lambda_\psi^2 \sigma_\psi^2)$, and $D = \beta^f + \beta^v + \beta^s$. Finally, we use this result to compute the expectation of s_{t+1}. Using Equation (13.9) and aggregating over agents, $\bar{E}_t s_{t+1}$ becomes a linear expression in f_t, f_{t+1}, and s_t. Substituting the result into Equation (13.8), we can solve for the unknown parameters λ_f and λ_ψ.

This last step gives two equations in the unknowns λ_f and λ_ψ, with $\lambda_f > 0$ and $\lambda_\psi > \lambda$. We can compare this solution to that of the public information model in which there is no information heterogeneity. In that case, the solution is Equation (13.6) with $\rho_f = 1$ and $\rho_\psi = 0$, so that $\lambda_f = 0$ and $\lambda_\psi = \lambda$.

Information heterogeneity therefore affects the exchange rate solution in two ways. First, the exchange rate now depends on the unobserved future fundamental f_{t+1} as agents trade on the basis of their private signals about this future fundamental. Second, the impact of the unobserved fundamental ψ_t is now amplified as λ_ψ, bigger than in the common knowledge model. This results from rational confusion over what is driving the exchange rate. An increase in the risk premium ψ_t on Foreign bonds leads to an appreciation of the domestic

[5]When private signals provide information about fundamentals further in the future, it gives rise to higher order expectations, as shown in Bacchetta and van Wincoop (2006) (see also Bacchetta and van Wincoop, 2008).

currency. But there is a magnification effect under information heterogeneity.[6] Agents do not know whether the appreciation is the result of an increase in the risk premium or is simply due to more favorable private signals that others have about the future fundamental. As they give some weight to the second possibility, their expectation of f_{t+1} drops, leading to a further appreciation.

These results imply a stronger disconnect between the exchange rate and observed fundamentals than under public information. They also imply that, depending on publicly observed information, the exchange rate contains information about future macro fundamentals. This is consistent with evidence reported by Engel and West (2005) and Froot and Ramadorai (2005). These results become even stronger when agents have private information about fundamentals further into the future. The rational confusion then becomes persistent. Even when ψ_t is entirely transitory, a shock to ψ_t will affect the exchange rate for T periods when agents have information about fundamental T periods into the future.

This model can also explain the close relationship between order flow and exchange rates. Evans and Lyons (2002), who first documented this relationship, define order flow as the "net of buyer-initiated and seller-initiated orders." The initiator of a transaction is the trader who acts on the basis of private information. The close link between order flow and exchange rates therefore suggests that most information is private. In the modern FX market, where almost all trade is electronic, private information is mostly channeled through market orders.

In Bacchetta and van Wincoop (2006) we break the demand for Foreign bonds into a component that only depends on private information and a component that depends on public information and the exchange rate. The first component of demand is submitted through market orders (order flow), while the second component is submitted through limit orders.[7] We then show that the exchange rate is driven by (i) public information and (ii) order flow. We show that the model can generate a very close link between the exchange rate and order flow as seen in the data.

13.4 Model Uncertainty

The second deviation from the benchmark case consists in considering the impact of model uncertainty, while going back to the assumption of common information across all agents. Model uncertainty was first introduced into exchange rate models in the late 1980s in order to explain the persistent

[6]Rational confusion can also occur without heterogeneity, as in Takagi (1991) who assumes that investors cannot distinguish between two fundamental shocks. However, there is no magnification effect in this case, as investors do not use the exchange rate as a source of information on others' signals.

[7]This simple allocation between market and limit orders does not affect the model's equilibrium. The solution would become much more complex if private information influenced limit orders.

expectational errors of market participants about future exchange rates and to explain the high exchange rate volatility. In the second half of the 1970s and 1980s, the dollar consistently depreciated more than investors expected, while in the early 1980s it appreciated more than investors expected. Contributions by Lewis (1989) and Kaminsky (1993) showed that such persistent expectational errors can, in fact, be perfectly rational when there is uncertainty about model parameters. Lewis (1989) considers the standard monetary model, but assumes the existence of a one-time change in the constant term of the money demand equation. By observing the data, agents gradually learn about the new value of the constant term. Kaminsky (1993) assumes that money growth is equal to a drift term plus a random innovation. The drift term can switch between two values based on a Markov process. In both cases, agents learn about the unknown parameters through Bayesian updating.

To illustrate the mechanism for such consistent expectational errors, assume that the fundamental f_t in our simple monetary model follows the process

$$\Delta f_t = \delta + \beta \Delta f_{t-1} + v_t \tag{13.11}$$

Investors do not know δ. They form Bayesian expectations by observing Δf_t, starting with a prior belief δ_0. A large value of Δf_t can be the result of either a high value of δ or a large draw of the transitory shock v_t.

Now assume that δ increases, leading to a large value of Δf_t. Investors will then increase their expectation of δ, but not as much as the actual change in δ as they give weight to the possibility that there is only a transitory increase in Δf_t associated with v_t. This means that actual future values of Δf are larger than investors expect. The exchange rate therefore depreciates more than investors expect. This will continue as long as the expectation of δ by investors is below the true value. Since the learning process is gradual, this can indeed last a long time, leading to persistent expectational errors. Nonetheless, agents are perfectly rational.

Tabellini (1988) emphasized that such a framework can lead to increased exchange rate volatility relative to the case in which parameters are known. The logic behind this is as follows. An increase in v_t leads to an exchange rate depreciation. However, when δ is unknown, agents will increase their expectation of δ, which raises the expectation of future levels of Δf, which in turn leads to an even larger depreciation.

Bacchetta and van Wincoop (2011) emphasize a different implication of model uncertainty. They show that it can lead to a highly unstable reduced form relationship between the exchange rate and macro fundamentals even if the true structural parameters are constant.[8] This is driven by uncertainty about the level of parameters that generates confusion about the interpretation of the data. We now develop this point by introducing structural parameter uncertainty in the model.

[8]This instability, however, is not sufficient to explain the Meese–Rogoff result. For a discussion, see Bacchetta et al. (2010).

Let us add money demand shocks ν_t and ν_t^* to the money demand equations (Eqs. (13.1) and (13.2)) and define $b_t = \nu_t - \nu_t^*$. Assume that these aggregate money demand shocks are unobserved, so that b_t is an unobserved macro fundamental. From Equations (13.1)–(13.3) of the monetary model we have

$$i_t - i_t^* = \frac{1}{\alpha}s_t - \frac{1}{\alpha}(m_t - m_t^*) - \frac{1}{\alpha}(\phi(y_t - y_t^*) + b_t) \tag{13.12}$$

Assume that agents do not know the value of the parameter ϕ. They also do not know the value of b_t. However, through interest rates, money supplies, and exchange rate, they do learn the value of

$$\phi(y_t - y_t^*) + b_t \tag{13.13}$$

For illustrative purposes, we make a couple of simplifying assumptions. First, we assume that $m_t - m_t^*$ and $y_t - y_t^*$ follow random walk processes. Second, we assume that b_t is i.i.d. with variance σ_b^2. Finally, we assume that starting in period 1 the parameter ϕ is drawn from a distribution with mean $\bar{\phi}$ and standard deviation σ_ϕ^2. Agents can learn over time about the value of the parameter from the observation of $\phi(y_t - y_t^*) + b_t$.

Substituting the expression for the interest differential Eq. (13.12) into Eq. (13.4) and solving s_t by integrating forward gives

$$s_t = (m_t - m_t^*) - ((1-\lambda)\phi + \lambda E_t\phi)(y_t - y_t^*) + (1-\lambda)b_t \tag{13.14}$$

This implies that the impact of the fundamental $y_t - y_t^*$ on the exchange rate is

$$\frac{\partial s_t}{\partial(y_t - y_t^*)} = -((1-\lambda)\phi + \lambda E_t\phi) \tag{13.15}$$

We can compare this to the case where ϕ is a known constant. From Equation (13.6), setting $\rho_f = 1$, the derivative is $-\phi$. As mentioned before, the discount rate λ is close to 1. This implies that the impact of the fundamental $y_t - y_t^*$ on the exchange rate depends almost exclusively on the expectation of ϕ rather than ϕ itself.

The expectation of ϕ may bear very little relationship to the actual ϕ. To see this, we use Kalman filter formulas to update expectations of ϕ. Let p_t be the perceived variance of ϕ at time t. We start in period 1 with $E_1\phi = \bar{\phi}$ and $p_1 = \sigma_\phi^2$. Subsequently, the expectation and variance evolve according to

$$p_t = p_{t-1}\alpha_t \tag{13.16}$$

$$\alpha_t = \frac{\sigma_b^2}{(y_t - y_t^*)^2 p_{t-1} + \sigma_b^2} \tag{13.17}$$

$$E_t\phi = \alpha_t E_{t-1}\phi + (1-\alpha_t)\phi - \frac{p_{t-1}}{(y_t - y_t^*)^2 p_{t-1} + \sigma_b^2}(y_t - y_t^*)b_t \tag{13.18}$$

α_t captures the speed of learning. In a more general example with multiple unknown parameters and persistence of b_t, Bacchetta and van Wincoop (2011) show that learning can be very slow. It may take more than a century for the variance to be reduced by half.

The key equation is Equation (13.18), which shows how the expectation of ϕ evolves over time. If the last term on the right-hand side is equal to zero, the expectation is a weighted average of the expectation last period (with weight α_t that is close to 1) and the true parameter ϕ. But it is the last term that is key here. It depends on the product of $y_t - y_t^*$ and b_t. The expectation of the unknown parameter therefore depends on the product of an observed and an unobserved fundamental.

How is this possible? The reason is another type of rational confusion, which we refer to as a scapegoat effect (Bacchetta and van Wincoop, 2004). Consider an increase in the unobserved fundamental b_t. Using information about interest rates and the exchange rate, agents only know the aggregate of $-\phi(y_t - y_t^*) + b_t$. When b_t is positive and $(y_t - y_t^*)$ is positive, agents do not know whether $-\phi(y_t - y_t^*) + b_t$ is large because b_t is large or the unknown parameter ϕ is low. They give at least some weight to the latter possibility, therefore reducing the expectation of ϕ, as we can see formally from Equation (13.18). Relative output becomes the scapegoat for what is really a shock to another unobserved fundamental.

The scapegoat effect implies that the relationship between the exchange rate and observed macro fundamentals can become highly unstable, and in a way that is unrelated to time variation in structural parameters themselves. In Bacchetta and van Wincoop (2011), we show that the expectation of the structural parameters can move far away from the actual unknown structural parameters, both over short and long horizons. This results in a very unstable reduced form relationship between the exchange rate and macro fundamentals.

This finding is consistent with survey evidence in the literature. Cheung and Chinn (2001) conducted a survey of US FX traders and found that the weight that traders attached to different macro indicators varies considerably over time. More recently, Fratzscher et al. (2011) use 9 years of survey data for 12 currencies to show that the weight that FX traders attach to different macro fundamentals as determinants of exchange rates varies significantly over time. They also show that these time-varying survey weights lead to time variation in the reduced form relationship between exchange rates and macro fundamentals. Finally, they provide evidence of scapegoat effects by showing that the survey weights depend on the interaction of fundamentals and noise as in Equation (13.18), using order flow data to measure the noise.[9]

[9]There is also some econometric evidence of parameter instability in reduced form exchange rate equations. See Rossi (2006) and Sarno and Valente (2009).

13.5 Infrequent Decision Making

As discussed in Section 13.2, in most applications, the UIP deviation in Equation (13.4) is a risk premium. Equating the expected excess return on Foreign bonds to a risk premium follows from any portfolio Euler equation that represents a trade-off between Home and Foreign bonds. It implicitly assumes that agents make new portfolio decisions each period on the basis of all available information. This assumption, although entirely standard in the literature, is nonetheless, a very strong and nonrealistic one. It implicitly assumes that all traders actively manage their FX exposure. Although there now exists an industry, developed in the late 1980s, that actively manages FX exposure (hedge funds, currency overlay managers, leveraged funds), it manages only a tiny fraction of cross-border financial holdings.[10] Banks themselves actively manage FX positions mostly intraday. Mutual funds are not allowed by law to actively reallocate between Home and Foreign assets. A Europe fund is a Europe fund and cannot suddenly start investing in US bonds. Similarly, a global bond fund cannot suddenly start shorting one country's bonds when expected returns make this attractive. Moreover, Lyons (2001) reports that financial institutions rarely devote their own proprietary capital to currency strategies. Finally, individual investors are well known to make very infrequent portfolio decisions, especially regarding pension fund allocations.

In the models that we have discussed so far, we have assumed that Equation (13.4) holds and that agents reallocate their portfolio between Home and Foreign bonds each period on the basis of all available information. We now turn to the model in Bacchetta and van Wincoop (2010) in which agents make infrequent portfolio decisions. Infrequent decisions imply that information is only gradually incorporated into the exchange rate. As initially argued by Froot and Thaler (1990) and Lyons (2001), the slow incorporation of information leads to excess return predictability and could explain the forward premium puzzle. The key aspect is not the frequency of trading, but the frequency of portfolio decision making. There is a cost to active portfolio management that makes it optimal for agents to take only infrequent portfolio decisions. To capture this feature, the model assumes overlapping investors who make a portfolio decision only in their first period. In subsequent periods, investors may trade to rebalance their portfolio, but they do not make any decisions on a new portfolio, as this is costly.

The model replaces Equations (13.1)–(13.3), which connect the interest differential to the exchange rate and some macro fundamentals, with a simple AR(1) process for the interest differential. This represents a gradually changing interest rate target. In practice, we set the Home interest rate equal to a constant r and let the Foreign interest rate vary over time based on an AR process.[11]

[10]See Bacchetta and van Wincoop (2010) and Sager and Taylor (2006) for a discussion.

[11]The constant Home interest rate is the result of an exogenous constant real interest rate of r and a zero-inflation monetary policy in the Home country.

The heart of the model is associated with Equation (13.4), which now changes as agents make infrequent portfolio decisions. Assume that there are overlapping generations (OLG) of agents who live T periods and who make one portfolio decision for the next T periods when born. The portfolio decision involves the allocation between Home and Foreign nominal bonds. Investors now care about the excess return on Foreign bonds over the next T periods as they make one portfolio decision for T periods. Let $q_{t+k} = s_{t+k} - s_{t+k-1} + i^*_{t+k-1} - i_{t+k-1}$ be the excess return on Foreign bonds from $t + k - 1$ to $t + k$. The excess return from t to $t + T$ is then $q_{t,t+T} = q_{t+1} + \cdots + q_{t+T} = s_{t+T} - s_t - fd_t - \cdots - fd_{t+T-1}$, where $fd_t = i_t - i^*_t$ is the forward discount.

Agents only consume in the last period of life. Assuming a constant rate of relative risk aversion γ, the fraction allocated to the Foreign bond is

$$b_t = \bar{b} + \frac{E_t(q_{t,t+T})}{\gamma\sigma^2} \tag{13.19}$$

where $\bar{b}$ is a constant and σ^2 depends on the risk associated with future excess returns and is constant as well in equilibrium.[12]

Agents are born with wealth of 1, which accumulates over time due to returns on their portfolio. For an investor born at time $t - k$, wealth at time t is $W_{t-k,k} = \prod_{j=1}^{k} R^p_{t+k-j}$, where R^p_{t+k-j} is the portfolio return from $t + k - j - 1$ to $t + k - j$, which is equal to $1 + r + b_t q_{t+k-j}$. Bond market equilibrium is represented by

$$\sum_{k=1}^{T} b_{t-k+1} W_{t-k+1,t} + X_t = BS_t \tag{13.20}$$

Here X_t represents exogenous purchases of Foreign bonds by noise or liquidity traders, which is calibrated to match observed exchange rate volatility and the well-known near-random-walk behavior of the exchange rate. The supply of bonds is on the right-hand side. The Foreign bond supply is fixed at B in Foreign currency, which translates to BS_t in the Home currency.

The model is solved by substituting the expressions for the optimal portfolios and wealth and then log-linearizing. This leads to a complicated difference equation in the exchange rate that is solved numerically. The only stochastic driver is the forward discount, which follows an AR process.

The model can account for the forward discount puzzle. The basic logic is very simple. Consider an increase in the Foreign interest rate. This leads to an increased demand for Foreign bonds, causing an appreciation of the Foreign currency. However, as agents adjust their portfolios gradually (simplified in the model through the OLG structure), there is a continued shift toward Foreign bonds that leads to a steady appreciation of the Foreign currency. This accounts for the well-established stylized fact that high interest rate currencies tend to

[12] The precise expression is $\sigma^2 = (1 - (1/\gamma))\text{var}_t(q_{t,t+T}) + (1/\gamma)\sum_{k=1}^{T} \text{var}_t(q_{t+k})$.

appreciate (the forward discount or Fama puzzle). It is also consistent with the evidence presented in Eichenbaum and Evans (1995) that after an interest rate increase, a currency continues to appreciate for 8–12 quarters before it starts to depreciate.

Four comments are worth making about this result. First, there is the question of who sells the Foreign bonds when agents continue to shift their portfolio to Foreign bonds. The answer is that the "inactive" agents at any point in time, which account for a fraction $(T - 1)/T$ of all agents, automatically take the other side through portfolio rebalancing. As the Foreign currency appreciates, these inactive agents sell Foreign bonds in order to rebalance their portfolios. Notice that this does not involve a new portfolio decision. They simply sell to keep the portfolio share allocated to Foreign bonds constant.

Second, there is the question of whether making infrequent portfolio decisions is optimal. Of course, if there is no cost to portfolio decision making, all agents would actively manage their portfolios at all times. However, the industry that actively manages FX positions charges steep fees for their services. The fees depend on the risk of the fund. At 20% risk (standard deviation of return), the typical fee is a 1% management fee plus 20% of profits, which in practice amounts to about 4%. Bacchetta and van Wincoop (2010) found that at such fees, it is indeed optimal for agents to not actively manage their portfolios. While active portfolio management leads to higher expected portfolio returns, it also involves considerable risk as future exchange rates are hard to predict. As a result, the welfare gains from active management are not sufficient to offset the fees charged.

Third, an important question is how these results change when we allow for many currencies. Diversification of the portfolio across many currencies can reduce the overall risk exposure, which can make active FX portfolio management optimal. Bacchetta and van Wincoop (2010) considered an extension calibrated to six countries (five currencies). As the risk is now diminished, it indeed becomes optimal for investors to actively manage their portfolio. However, as some agents start to actively manage their portfolio and therefore actively exploit expected excess return opportunities, in equilibrium, these expected excess returns become smaller. This in turn makes it less attractive to actively manage portfolios. There is then an equilibrium that is such that the gain from active portfolio management is exactly equal its cost and only a small fraction of agents actively manage their portfolios, as seen in the data. At the same time, the calibration shows that the excess return predictability in equilibrium corresponds closely to that seen in the data.

Finally, there might be another source of incomplete information processing in addition to infrequent decisions. When investors change their portfolio, they may do this on the basis of a limited set of information. Investors may simply observe the interest differential, as with carry trade, and invest in the high interest rate currency. Alternatively, investors may simply assume that the exchange rate follows a random walk. Bacchetta and van Wincoop (2010) introduce these assumptions in the context of infrequent trading and show that the model generates an even more negative coefficient in the Fama regression. In

Bacchetta and van Wincoop (2007), we focus on the random walk hypothesis in forming exchange rate expectations. We show that with active trading, such an assumption leads to strongly counterfactual positive Fama coefficients. However, with infrequent trading, the model can match the data.

13.6 Conclusion

In this chapter, we have reviewed the implications of various forms of incomplete information in an otherwise standard model of exchange rate determination. Deviations from the complete information paradigm allow us to explain various exchange rate puzzles, such as the disconnect between exchange rates and fundamentals and the forward premium puzzle.

The focus of this chapter is mainly influenced by our previous research and does not represent an exhaustive review of the existing literature. While we have examined incomplete information in versions of the standard monetary model, some papers have examined this issue in alternative models. For example, Roberts (1995) assumes imperfect information on the persistence of a shock in a dynamic Mundell–Fleming model. However, a reduced form approach is more difficult to interpret as learning is not based on optimal inference. Martinez-García (2010) introduced imperfect information in a DSGE model. He showed that consumption reacts less to shocks. This can explain that relative consumption is less volatile than exchange rates, that is, the well-known Backus–Smith puzzle.

We have also restricted our discussion to rational expectations frameworks. An entirely different direction is to consider deviations from rational expectations, where expectations are typically based on rules that ignore all or part of the information from the model. In particular, models of adaptive learning have been applied to exchange rates in many papers (Chakraborty and Evans, 2008; Lewis and Markiewicz, 2009). Often in these analyses, there is no structural model uncertainty and recursive learning schemes converge to rational expectations equilibria. In contrast, Gourinchas and Tornell (2004) consider a model where agents have incorrect beliefs about the process of the interest rate and never learn. Other models introduce more exogenous expectational rules, such as Mark and Wu (1998) and the well-known model by Frankel and Froot (1988) of chartists and fundamentalists (De Grauwe and Grimaldi, 2005). Goldberg and Frydman (1996) assume imperfect knowledge of the underlying model, so that agents use the relevant variables but ignore the model's structure and thus the precise weights of each variable. These types of models have been used to account for a wide range of exchange rate features, such as the exchange rate disconnect, high exchange rate volatility, persistent expectational errors, and the forward discount puzzle.

Acknowledgments

We would like to thank an anonymous referee for helpful comments. Bacchetta gratefully acknowledges financial support from the National Centre of

Competence in Research “Financial Valuation and Risk Management”(NCCR FINRISK) and from the Swiss Finance Institute.

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