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A theory of the currency denomination of international trade

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Abstract

The currency denomination of international trade has significant macroeconomic and policy implications. In this paper we solve for the optimal invoicing choice by integrating this microeconomic decision at the level of the firm into a general equilibrium open economy model. Strategic interactions between firms play a critical role. We find that the less competition firms face in foreign markets, as reflected in market share and product differentiation, the more likely they will price in their own currency. We also show that when a set of countries forms a monetary union, the new currency is likely to be used more extensively in trade than the sum of the currencies it replaces.

Keywords: Currency invoicing; Exchange rate pass-through; New open economy macro; Price setting; Monetary union

JEL classification: F41; F30; F33; F10

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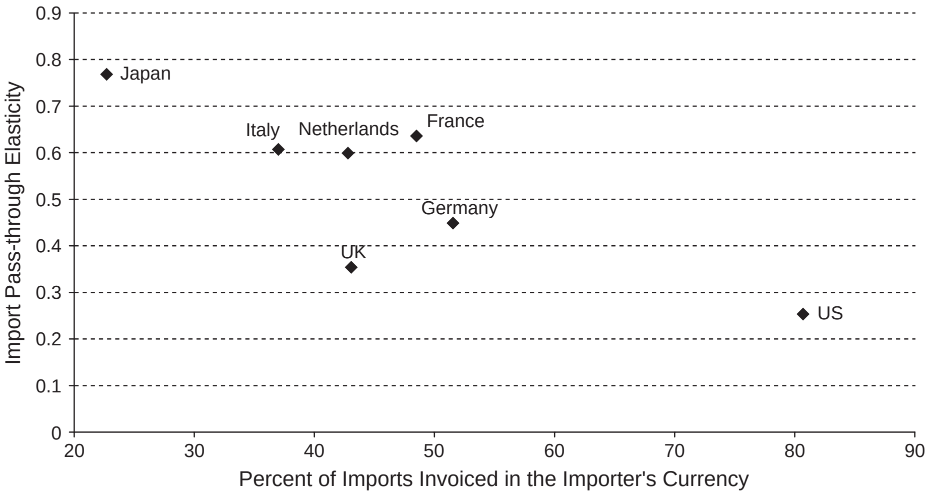

Fig. 1. Pass-through and invoicing.

1. Introduction

The corner stone of new Keynesian macroeconomics is the infrequent adjustment of prices due to small menu costs. At the international level, however, there is an entirely different dimension to this issue. If exporting firms set prices in foreign markets, and infrequently adjust them, in what currency should they set these prices? This question is not grounded in mere theoretical curiosity. It turns out that the invoicing choice, which is a microeconomic one at the level of the firm, has far ranging macroeconomic implications. This has been one of the main messages from the recent "new open economy macroeconomics" literature, which has introduced nominal rigidities in an open economy context. The invoicing choice affects both exchange rate volatility and the impact of the exchange rate on the economy. It has been found to play a critical role for optimal monetary policy and the choice of exchange rate regime.[1] A key channel through which the invoicing choice affects the macro-economy is its impact on the pass-through of exchange rate changes to import prices. If firms set prices in the importer's currency, we should expect zero pass-through. If instead prices are set in the exporter's currency, we

[1] The issue of optimal monetary and exchange rate policy is analyzed in Bacchetta and van Wincoop (2000), Corsetti and Pesenti (2001), Devereux and Engel (2003) and Sutherland (2002). Bacchetta and van Wincoop (1998, 2000) also show that the level of trade and net capital flows are affected by the invoicing choice. Engel (2002a) discusses the impact of the pricing strategy on exchange rate volatility. Engel and Rogers (2001) show that local currency pricing explains a large part of observed deviations from the law of one price. Engel (2002b) and Obstfeld (2002) discuss how the "expenditure switching" effect of exchange rate fluctuations can be affected by the invoicing choice. For general descriptions of the new open economy macro literature, see Lane (2001), Obstfeld and Rogoff (1996), Obstfeld (2001) and Brian Doyle's new open economy macro web page http://www.geocities.com/brian_m_doyle/open.html.

should see full pass-through. Fig. 1 confirms this relationship between invoicing choice and pass-through for a set of seven industrialized countries.[2]

The main objective of this paper is to derive and understand the optimal invoicing decisions in the context of "new open economy macroeconomics" models. While most of the literature has assumed exogenously that firms set prices either in their own currency or in that of the importer, firms are not neutral between these choices. The optimal invoicing choice of firms depends on the uncertainty of their profits under different invoicing strategies. We show that the two crucial factors determining the invoicing choice based on the theory are (i) the market share of an exporting country in a foreign market, and (ii) the extent to which products of domestic firms are substitutes for those of competing foreign firms. The higher the exporting country's market share in an industry, and the more differentiated the products, the more likely firms are to price in the exporter's currency. On the other hand, international competition is strong when the market share of the exporting country is low and its goods are close substitutes with those of foreign competitors. In that case exporting firms are more likely to price in the currencies of their foreign competitors.[3]

While the contribution of this paper is a theoretical one, there is evidence that the factors highlighted by the theory are empirically relevant. Hamada and Horiuchi (1987), analyzing a 1984 survey of Japanese firms, write that "...Japanese firms report that a principal reason for foreign-currency-invoiced export contracts is the hard pressure from international competition." More formal evidence comes from the pass-through literature. Feenstra et al. (1996) show that for the automobile industry a high market share of an exporting country is associated with a relatively high pass-through elasticity for that country's exporters.[4] Yang (1997) finds a positive relationship between U.S. import pass-through elasticities for three and four-digit SIC industries and different proxies of product differentiation.[5] Sectoral invoicing data could provide the most convincing evidence, but such data are scarce. Basevi et al. (1997) and Page (1980) provide some evidence indicating that invoicing in the exporter's currency is more common in more differentiated goods sectors. For aggregate invoicing data Fig. 2 shows a clear positive relationship between the trade-weighted average market share of an exporting country and the fraction of its exports invoiced in the exporter's currency.[6] The United States and Germany have a significantly higher average market share than the other countries and also have the largest fractions invoiced in their own currency. Japan has the lowest fraction

[2] We take the short-term pass-through coefficients from Campa and Goldberg (2002, Table 2) and the invoicing data for the year 1995 from Bekx (1998). In theory the currency in which firms set prices does not need to be the same as the currency of invoicing. Only the currency in which firms set prices is relevant to our analysis. However, Fig. 1 suggests that the two are probably closely related.

[3] Our approach abstracts from transaction and liquidity costs as a determinant of the use of currencies in international trade. See for example Rey (2001) for an interesting contribution on the choice of currency as a medium of exchange.

[4] Along similar lines, Feinberg (1986) finds that import pass-through in Germany is higher in sectors where the import share is larger.

[5] Feenstra et al. (1996) and Yang (1997) also show that these empirical relationships can be understood in the context of partial equilibrium models with flexible prices.

[6] Market share is defined as manufacturing exports to a country divided by total manufacturing sales in that country (gross output plus imports). Since these are aggregate data, the levels are not very meaningful; our interest is in differences across countries.

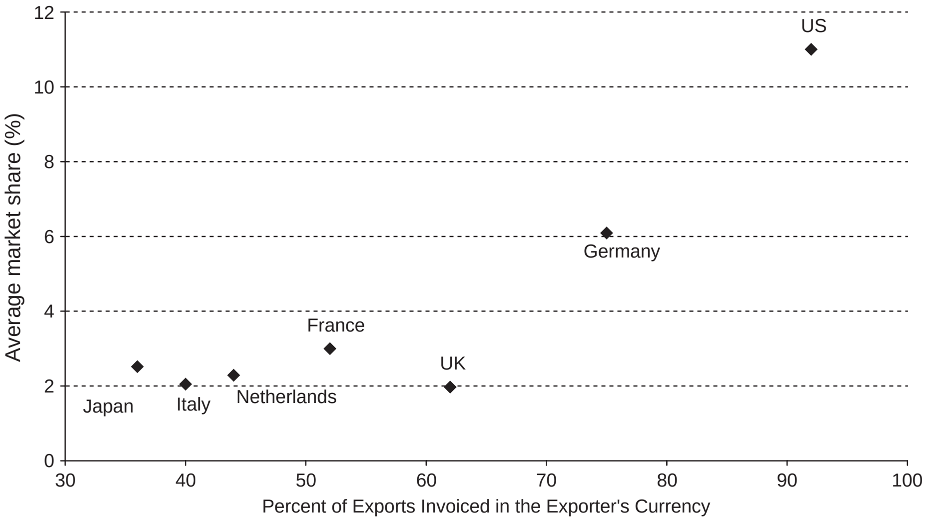

Fig. 2. Invoicing and market share.

of exports invoiced in its own currency. While Japan is the second biggest industrialized country, it has a small market share both because its exports are small relative to its GDP, and because more than half of its exports to industrialized countries go to the United States.[7]

There is an extensive microeconomic literature on currency invoicing, which studies the invoicing decision of a single firm selling in a foreign market and setting the price before the exchange rate is known. The representative papers include Giovannini (1988), Donnenfeld and Zilcha (1991), and Friberg (1998).[8] These papers focus in particular on the role of demand and cost elasticities. A common finding in the literature is that an exporting firm prefers to price in the exporter's (importer's) currency when profits are convex (concave) when pricing in the exporter's currency. But since these models are partial equilibrium, they typically assume that the exchange rate and cost and demand functions are exogenous.

On the other hand, several recent papers take a general equilibrium perspective and look at the optimal currency denomination of trade in the context of the "new open economy macroeconomics". In Bacchetta and van Wincoop (2001), we numerically solve the invoicing decision in a general equilibrium model. The optimal strategy depends on

[7] Japan's goods are also relatively close substitutes with those of competitors. Hooper et al. (1998) find that the overall export price elasticity is higher for Japan than for other industrialized countries, suggesting that Japan's goods are less differentiated than those of others. We have also computed for each country's exports a trade-weighted average elasticity of substitution for 62 commodity-groups, using estimates of elasticities for each of these groups from Hummels (1999). Japan has indeed the highest elasticity.

[8] Johnson and Pick (1997) examine exporters from two countries competing in a third market and show that multiple equilibria can occur. Some papers introduce a distribution sector, so the exporting firm does not sell directly to consumers, but sells to an importing firm. The pricing decision then results from the interactions between the exporter and the importer. See, for example Baron (1976), Bilson (1983) or Viaene and de Vries (1992).

various preference parameters, but the results are difficult to understand as various mechanisms are at work. Devereux et al. (2004) examine the invoicing choice when countries have different levels of monetary volatility. They show that countries with lower monetary volatility prefer to price in their own currency. Corsetti and Pesenti (2002) analyze the interaction between the optimal exchange rate policy and the endogenous pricing decision of firms, when monetary policy is set optimally. They show that the optimal regime is a floating exchange rate where firms prefer to price in their own currency.

In this paper, we link the microeconomic and macroeconomic strands of the literature to understand better the firms' motives to price in a given currency. We integrate the existing partial equilibrium theoretical literature into "new open economy macro" general equilibrium models.

In order to gain intuition about the optimal invoicing strategies we extend the traditional partial equilibrium model in several steps. Each step provides additional insights that would be hard to understand when taken all at once. We first extend the model by allowing firms to take the invoicing decisions of other firms into account. This leads to strategic complementarities. Market share of the exporting country then becomes a critical factor. We consider both a two-country and multi-country version of the partial equilibrium model. The latter provides relevant insights about the implications of European Monetary Union. We then extend the model to a general equilibrium setting, in which the exchange rate is endogenous, by introducing stochastic aggregate demand through monetary shocks. We first keep nominal wages fixed by allowing for nominal rigidities in the labor market. The results then turn out to be essentially the same as in the partial equilibrium model. When allowing for nominal wage flexibility, we first consider a constant real wage. In that case country size plays a role separate from market share. In the last step we allow for real wage volatility. While country size and real wage volatility can theoretically play a role, we argue that empirically they are not very relevant. Finally, we briefly discuss an extension that allows for complete asset markets; the rest of the paper assumes that there is no trade in assets.

It is important to stress that the focus of this paper is entirely a positive one, understanding the key determinants of the currency invoicing choice in a general equilibrium framework. We have no doubt that there are relevant normative implications for monetary policy flowing from the theory, but those will be taken up in future research.[9]

The remainder of the paper is organized as follows. In Section 2 we discuss a partial equilibrium model of invoicing, starting with a framework familiar from the traditional currency invoicing literature. We then extend the model to allow firms to pay attention to the invoicing decisions of competing firms, first in a two-country setup and then in a multi-country setup. We derive results analytically by focusing on small levels of risk. Section 3 builds on the findings of Section 2 by expanding the model to a general equilibrium setup. Section 4 offers conclusions.

[9] Corsetti and Pesenti (2002) make a promising start in this direction.

2. Invoicing choice in partial equilibrium

In this section we first discuss the invoicing decision within a partial equilibrium model that is commonly adopted in the invoicing literature. The results from Section 2.1 are not new, but they form the basis for the extension in Section 2.2 that is the core of the paper. In that section we allow firms to take the invoicing choice of other firms into account, which leads to strategic complementarities.

2.1. A traditional partial equilibrium model

Following the standard approach of the partial equilibrium invoicing literature, firms are assumed to face a demand function $D(p)$, where p is the price faced by the importer, and a cost function $C(q)$ of output. Firms set prices before they know the exchange rate, which is the only source of uncertainty. Each firm has to choose whether to set a price p^{I} in the importer's currency or a price p^{E} in its own currency (the exporter's currency). In the former case $p=p^{\mathrm{I}}$, while in the latter $p=p^{\mathrm{E}}/S$. Profits are then respectively given by: [10]

$$\Pi^{\mathrm{I}} = Sp^{\mathrm{I}}D\left(p^{\mathrm{I}}\right) - C\left(D\left(p^{\mathrm{I}}\right)\right) \tag{1}$$

$$\Pi^{\mathrm{E}} = p^{\mathrm{E}}D\left(p^{\mathrm{E}}/S\right) - C\left(D\left(p^{\mathrm{E}}/S\right)\right) \tag{2}$$

When setting the price in the importer's currency, there is uncertainty about the price denominated in the exporter's currency, Sp^{I}, but there is no demand uncertainty. On the other hand, when setting the price in the exporter's currency, there is only uncertainty about demand, and thus cost, as the price in the importer's currency fluctuates with the exchange rate.[11] Firms need to compare the expected utility of profits under the two price setting options: $EU(\Pi^{\mathrm{E}})-EU(\Pi^{\mathrm{I}})$. A common finding in the literature is that the exporter's (importer's) currency is preferred when Π^{E} is globally convex (concave) with respect to S. This result is entirely independent of the degree of risk-aversion with respect to profits.

Before we discuss the intuition behind this, it is useful to first point out a technical problem when applying this result to any particular set of cost and demand functions. Generally the profit function under exporter's currency pricing has both concave and convex parts, so that this key result of the literature does not apply. Moreover, the result also does not apply in extensions discussed below, whereby profits under importer's currency pricing are a non-linear function of the exchange rate. We avoid these problems by focusing on uncertainty near $S=\bar{S}$, a deterministic exchange rate. We therefore focus on "small" levels of risk, where the variance of S tends to zero. We can then derive all results about optimal invoicing decisions analytically, even for rather complicated general equilibrium structures. Numerical simulations (not reported in this paper) show that higher levels of risk generally lead to the same results as under small amounts of risk.

[10] In partial equilibrium, we do not need to distinguish between real and nominal quantities, since the aggregate price level is implicitly constant.

[11] Thus, in partial equilibrium the currency denomination of trade is similar to fixing the price or the quantity when demand is uncertain. Therefore, the analysis of Klemperer and Meyer (1986) can be applied in this context.

We evaluate the impact of a small amount of risk on the optimal pricing strategy by taking the marginal derivative of $EU(\Pi^{\mathrm{E}})-EU(\Pi^{\mathrm{I}})$ with respect to the variance σ^2 of the nominal exchange rate, evaluated at $\sigma^2=0$. Let U' and U'' be the first and second order derivatives of utility with respect to profits and $\bar{S}=E(S)$. In the Appendix we prove the following Lemma.

Lemma 1. *Let $\Pi^E(S;\, x)$ and $\Pi^I(S;\, x)$ be two profit functions, where x is a vector of parameters that depend on σ^2. Assume that $\partial(\Pi^E-\Pi^I)/\partial x=0$ and $\Pi^E=\Pi^I$ at $\sigma^2=0$. Holding $E(S)=\bar{S}$ constant, for any twice differentiable utility function U(.) we have*

$$\frac{\partial\left[EU\left(\Pi^{\mathrm{E}}\right)-EU\left(\Pi^{\mathrm{I}}\right)\right]}{\partial\sigma^2}=0.5U''\left[\left(\frac{\partial\Pi^{\mathrm{E}}}{\partial S}\right)^2-\left(\frac{\partial\Pi^{\mathrm{I}}}{\partial S}\right)^2\right]+0.5U'\frac{\partial^2\left(\Pi^{\mathrm{E}}-\Pi^{\mathrm{I}}\right)}{\partial S^2} \tag{3}$$

All derivatives are evaluated at $S=\bar{S}$ and $\sigma^2=0$.

In our example x represents the prices that the firm sets. The condition in Lemma 1 is indeed satisfied since the first order derivative of profits with respect to the price is zero. We therefore do not have to be concerned about the effect of σ^2 on optimal prices. Prices can simply be held constant at their deterministic levels, where $p^{\mathrm{I}}=p^{\mathrm{E}}/\bar{S}$. This feature, which also holds for general equilibrium models, simplifies the analysis tremendously.

Under our assumptions, the curvature of profits matters for the optimal pricing decision, but not the curvature of the utility function. For the profit functions (1) and (2) the marginal derivative of profits with respect to the exchange rate is the same, i.e., $\partial\Pi^{\mathrm{E}}/\partial S=\partial\Pi^{\mathrm{I}}/\partial S$, when firms set prices optimally. Intuitively, the effect of the exchange rate on both profit functions is the same if prices can be immediately adjusted to the exchange rate. But since a change in prices has no first order effect on profits, a change in the exchange rate affects both profit functions identically even for preset prices. The first term on the right hand side of Eq. (3) is then zero, so that the rate of risk aversion does not matter. Since $U'>0$, the second term on the right hand side implies that for a marginal increase in the variance of the exchange rate, expected utility is higher under the pricing system with the largest convexity (second order derivative) of profits.

To gain further intuition, we now consider a specific set of constant elasticity demand and cost functions:

$$D(p)=p^{-\mu} \tag{4}$$

$$C(q)=wq^{\eta} \tag{5}$$

where μ is the price elasticity of demand and w the wage rate. The cost function is convex for $\eta>1$. It follows directly from the production function $q=L^{(1/\eta)}$, where L is labor input and the capital stock is held constant in the short run. η is therefore the reciprocal of the labor share and is generally somewhere between 1 and 2.

Applying Lemma 1 to these specific cost and demand functions leads to the following Proposition:

Proposition 1. *Consider a firm exporting to a foreign market, which faces demand and cost functions given by Eqs. (4) and (5). For small levels of risk as defined in Lemma 1, the firm chooses the following pricing strategy:*

- *If $\mu(\eta-1)<1$, the firm prices in the exporter's currency*
- *If $\mu(\eta-1)>1$, the firm prices in the importer's currency.*

The proposition is illustrated in Fig. 3, which plots the two profit functions for marginal deviations of S from $\bar{S}$, holding prices constant at the deterministic level. The derivative of profits with respect to the exchange rate is positive, that is, a depreciation raises profits. As discussed above, the first order derivative is the same whether the firm prices in the importer's or exporter's currency. When $\mu(\eta-1)<1$ (>1), profits are convex (concave) when the firm prices in the exporter's currency and for $S \neq \bar{S}$ are always larger (smaller) than when the firms price in the importer's currency.

One can also interpret the results in the context of price and demand uncertainty, which have an effect both on the variance and expectation of profits. Since the first order derivative of profits with respect to the exchange rate is identical under the two invoicing strategies, the first order effect on the variance is the same. This explains why the rate of risk-aversion does not matter. We therefore only have to consider the impact on expected profits. Under importer's currency pricing the profit function is linear in the exchange rate and expected profits are unaffected. When firms price in the exporter's currency, two factors affect expected profits. First, when $\eta>1$ the cost function is convex, implying that a rise in demand raises costs more than a decline in demand lowers costs. The demand volatility that arises when firms price in the exporter's currency therefore lowers expected profits, making pricing in the importer's currency more attractive. This effect is stronger the larger μ, which raises demand volatility. On the other hand, the expected level of

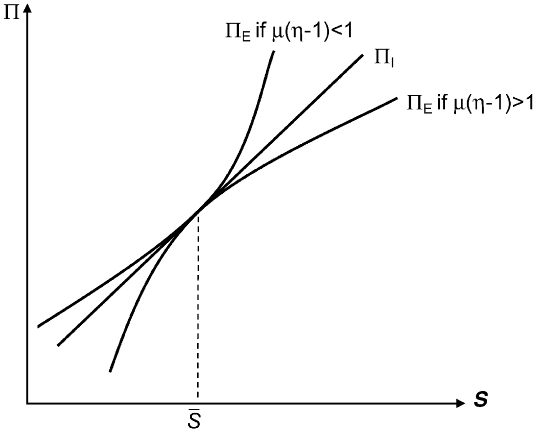

Fig. 3. Profit functions.

demand rises since demand is a convex function of the exchange rate and is proportional to S^{μ}. This raises expected profits when pricing in the exporter's currency. The first effect dominates when $(\eta-1)\mu>1$.

2.2. Introducing strategic complementarities: the role of market share

We now extend the model to highlight the role of strategic complementarities and market share when multiple domestic firms compete in a foreign market. One can think of the model described so far as that of one firm exporting to a foreign market dominated by foreign firms that set the price in their own currency. Results change, however, if we allow the exporting country to have a large market share. In that case an exporting firm is concerned with the invoicing decisions of other exporters that it is competing with. For now, we assume that all exporting firms are from the same country, leaving the case of multiple exporting countries to the next subsection.

We consider a particular industry in which N exporting firms from the Home country sell in the market of the Foreign country, which has N^* domestic firms. The market share $n=N/(N+N^*)$ of the exporting country becomes a critical element of the analysis. Assuming CES preferences with elasticity $\mu>1$ among the different products, the demand for goods from firm j is

$$D(p_j, P^*) = \frac{1}{N+N^*}\left(\frac{p_j}{P^*}\right)^{-\mu} d^*, \tag{6}$$

where p_j is the price set by the firm measured in the importer's currency. The industry price index P^* in the Foreign country is given by:

$$P^* = \left(\sum_{i=1}^{N+N^*} \frac{1}{N+N^*} p_i^{1-\mu}\right)^{\frac{1}{(1-\mu)}} \tag{7}$$

d^* is the real level of Foreign spending on goods in the industry, which is equal to the nominal level of spending divided by the industry price index. We hold d^* constant in the partial equilibrium model, but it will be stochastic in the general equilibrium model discussed in the next section. It is assumed that the total number of firms is large enough so that an individual firm does not affect the industry price index.

A fraction f of Home country firms sets a price p^{E} in their own (exporter's) currency, while a fraction $1-f$ sets a price p^{I} in the importer's currency. We assume that foreign firms set a price p^{H*} in their own currency, since our focus is on the invoicing decisions of exporters.[12] The overall industry price index (Eq. (7)) faced by Foreign country consumers is then

$$P^* = \left((1-n)\left(p^{H*}\right)^{1-\mu} + nf\left(p^{\mathrm{E}}/S\right)^{1-\mu} + n(1-f)\left(p^{\mathrm{I}}\right)^{1-\mu}\right)^{1/(1-\mu)} \tag{8}$$

[12] Since local producers typically price in local currency, we do not analyze their optimal currency pricing. If local firms were to price in foreign currency, this would obviously increase the incentives of foreign firms to price in foreign currency.

The price index depends on the exchange rate to the extent that Home firms price in the exporter's currency, which leads to a price p^{E}/S in the Foreign currency. One can think of the case typically considered in the literature as one where n is infinitesimally small, so that the industry price index is simply $p^{H}*$, which is a constant.

We consider two types of equilibria, Nash equilibria and coordination equilibria. Nash equilibria are the outcome of a Nash game, where each firm makes an optimal invoicing decision conditional on the invoicing decisions of all other firms. In general there are multiple Nash equilibria. In the coordination equilibrium, home country firms coordinate on the invoicing decision that is optimal for each firm. Nash equilibria can be found by applying Lemma 1 for each firm conditional on the invoicing strategy chosen by other firms. The coordination equilibrium can also be found by applying Lemma 1, by assuming that all firms use the same strategy. Applying this to the demand function (Eq. (6)), we obtain Proposition 2.

Proposition 2. *Consider firms exporting to a foreign market, facing cost and demand functions given by Eqs. (5) and (6). Define* $\bar{n}=0.5-0.5/\mu(\eta-1)$. *For small levels of risk as defined in Lemma 1, firms choose the following pricing strategies:*

- *If* $\mu(\eta-1)<1$, *firms price in the exporter's currency*
- *If* $\mu(\eta-1)>1$ *and* $n<\bar{n}$ *firms price in the importer's currency*
- *If* $\mu(\eta-1)>1$ *and* $n>\bar{n}$ *there are three Nash equilibria: (i) all price in exporter's currency, (ii) all price in importer's currency, (iii) a fraction prices in the exporter's currency, while the rest prices in the importer's currency. If firms coordinate they prefer to all price in the exporter's currency if either n or the rate of risk-aversion are large enough.*

The Proposition implies that the market share of the exporting country is crucial for the pricing decision. If the market share is small, below the cutoff $\bar{n}$, the results are unchanged relative to Proposition 1. In particular, firms price in the importer's currency if demand is sufficiently price elastic. If the market share is above the cutoff $\bar{n}$ there are multiple equilibria when $\mu(\eta-1)>1$. One of these equilibria is one in which all firms price in the exporter's currency. This is the preferred equilibrium when firms coordinate on the invoicing strategy if either they are sufficiently risk-averse or their market share is sufficiently large. These results imply that firms are more likely to price in the exporter's currency if their country's market share is large.

The Proposition is further illustrated in Fig. 4. For each of the three cases of Proposition 2, it graphs $(\partial[EU(\Pi^{\mathrm{E}})-EU(\Pi^{\mathrm{I}})]/\partial\sigma^2)$ as a function of f. When $\mu(\eta-1)<1$ the expected utility from profits is highest when pricing in the exporter's currency, independently of the pricing strategy chosen by other firms (line A). When $\mu(\eta-1)>1$ firms prefer to price in the importer's currency when all other exporting firms do so as well (f=0, in lines B and C). But the more other firms price in the exporter's currency, the more attractive it becomes for the marginal firm to do so as well. This is reflected in the upward sloping line.

The positive slope represents strategic complementarities. In order to understand it, consider the invoicing choice of a marginal firm. The relative price of its goods is less sensitive to the exchange rate, leading to reduced demand uncertainty, the more of its

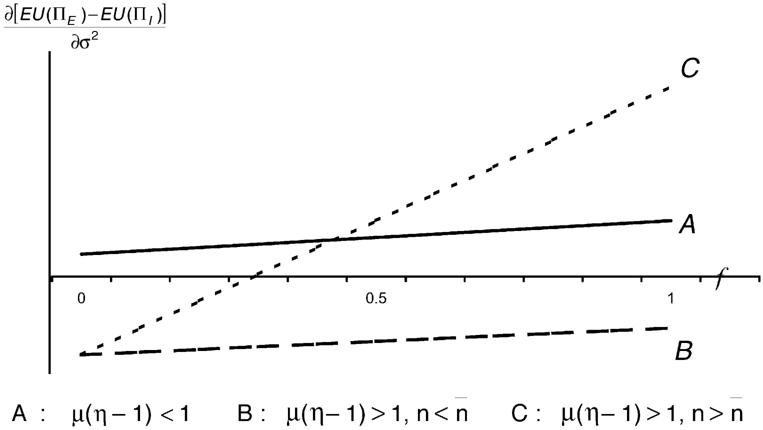

Fig. 4. Profit differences vs. the proportion of firms pricing in their own currency.

competitors choose the same invoicing strategy. If the marginal firm prices in the importer's currency, demand uncertainty increases when more of its competitors price in the exporter's currency. Since demand uncertainty lowers expected profits when the cost function is convex, it is more attractive for a marginal firm to price in the exporter's currency when more of its competitors do the same.

The importance of this strategic complementarity depends on the market share of the exporting country. When the exporting country has small market share, the pricing strategy of competing firms from the exporting country has relatively little impact on the overall industry price index. This is illustrated with line B, where the slope is relatively flat. Firms then still prefer to price in the importer's currency. But when the market share of the exporting country is large, as illustrated with line C, firms prefer to price in the exporter's currency when all other firms do the same. In the extreme case where n=1, so that the exporting country is completely dominant, there is no demand uncertainty at all when all firms price in the exporter's currency. In the case of line C, there is also a third equilibrium in mixed strategies. However, this equilibrium is unstable and we ignore it.

If $n>\bar{n}$ and firms coordinate on the invoicing strategy, they all prefer to price in the exporter's currency if n is large enough or if the rate of risk-aversion is high enough.[13] In order to understand this, we consider the case where n=1 and where all firms price in the same currency. In this case there is no demand uncertainty, since relative prices within the industry are constant. However, there is still price uncertainty when firms price in the importer's currency. This does not affect expected profits, but raises the variance of profits. If firms are risk-averse, they prefer to price in the exporter's currency.[14]

[13] Notice that risk aversion plays a role under coordination, but it does not affect the Nash equilibria.

[14] We have also worked out the model when there is a finite number of firms that are each large enough to affect the industry price index. The algebra then becomes considerably more complicated, but the main result of this section, that pricing in the exporter's currency is more likely the larger the market share of the exporting country, remains unaltered.

2.3. Multiple exporting countries

So far we have assumed that there is only one exporting country. We now consider how results are affected when there are multiple countries exporting to a particular market, while otherwise maintaining the partial equilibrium setup of the previous subsection.

Assume that there are Z countries that all sell to a particular market. A fraction n_i of firms selling to this market is from country i. In principle there could be as many as Z currencies, although it is possible that some countries use the same currency. Let $x(i)$ denote the country in whose currency firms from country i invoice their sales. They can price in the exporter's currency, so that $x(i)$=i, the importer's currency, or the currency of any other country. We now look at the invoicing decision of a marginal firm from a particular exporting country, say country 1. In the Appendix we use a straightforward generalization of Lemma 1 to prove the following Proposition.

Proposition 3. *Consider a set of firms selling in a particular market with a fraction n_i of firms from country i (i = 1,. . .,Z). Each firm faces cost and demand functions given by Eqs.* (5) *and* (6). *Firms from country i price in the currency of country x(i). Let the exchange rate S_x be the units of country 1's currency per unit of country x's currency. A marginal firm from country 1 then prefers to invoice in the currency of country x that minimizes*

$$\mathrm{var}(S_x) + \mu(\eta - 1)\mathrm{var}\left(\sum_{i=1}^{N} n_i S_{x(i)} - S_x\right). \tag{9}$$

It is still the case than when $\mu(\eta-1)$ is sufficiently small, firms prefer to price in their own (exporter's) currency since var(S_1)=0. The larger $\mu(\eta-1)$, the more firms care about demand risk, which is minimized by invoicing in the currency that is most "similar" to the average invoicing currency chosen by competitors.

There can again be multiple Nash equilibria, even more than before due to the multiple currencies. Rather than consider all Nash equilibria in the general setup just described, we illustrate with some simple examples two key results that are listed in the following Proposition.

Proposition 4. *Consider a setup where firms from multiple countries sell to a particular foreign market. Each firm faces cost and demand functions given by Eqs. (5) and (6). Then two general results apply:*

1. *If none of the countries has a large market share, they are more likely than in a two-country setup to invoice in their own currency. Even for a high demand elasticity μ they may choose to invoice in their own currency.*
2. *If a set of countries form a monetary union they are more likely to invoice in their own currency. Imports by the monetary union are also more likely to be invoiced in the union's currency.*

We now discuss two simple examples that illustrate Proposition 4. In a two-country setup at least one country has half of the market share. This no longer needs to be the case with multiple countries selling in a particular market. Consider the extreme case where Z is very large and each country has an equal number of firms, so that $n_i=1/Z$. To further simplify matters, assume that all bilateral exchange rates have the same variance and correlation ρ.[15] It is then easily verified from Proposition 3 that for $\rho<0.5$ or $\rho>0.5$ and $\mu(\eta-1)<1/(2\rho-1)$ there is an equilibrium where all firms price in their own currency. Unless ρ is close to one, firms are happy to price in their own currency even for a high demand elasticity μ. In the two-country model firms from a country with a small market share price only in their own currency when $\mu(\eta-1)<1$. In that case firms from the importing country are necessarily dominant if the market share of the exporter is small. Firms from the exporting country are then inclined to price in the importer's currency to reduce demand risk. In the multi-country example considered here, demand risk is not reduced by pricing in the importer's currency if none of the other firms do so.

The second part of Proposition 4 is relevant in the context of EMU.[16] It suggests that the European Monetary Union (EMU) is likely to lead to more invoicing in euros than in the sum of the currencies it replaced. For illustrative purposes we again use a simple example. Assume that there are Z European countries that export to one non-European country, say Japan. Each European country has an equal number of firms, accounting for a total market share of α. The Japanese firms have a market share of $1-\alpha$ and price in their own currency in their own market. We again assume that all bilateral exchange rates have the same variance and correlation ρ. We restrict ourselves to two possible equilibria: (i) all European firms invoice in their own currency, or (ii) all European firms invoice in yen. Define $x=1/\mu(\eta-1)$. Assume that $\mu(\eta-1)>1$ and

$$(1-x)<2\alpha<\frac{1}{1-\rho}(1-x) \tag{10}$$

Using Proposition 3 it can then be shown that before EMU all European firms invoice in yen, while after EMU there is a Nash equilibrium where all firms invoice in euro.[17] The latter is the preferred Nash equilibrium under coordination.

The lesson to be drawn from this is that if multiple countries adopt the same currency, the market share that matters is that of the entire currency union, not that of individual countries. The concept of a "country" only has meaning here to the extent that currencies differ. EMU creates a big single currency area, with a larger market share than that of any of the individual countries that make up the currency union. For trade between EMU and the rest of the world we are therefore likely to see more invoicing in euros than pre-EMU

[15] To be more precise, let S_{ij} be the units of country i currency per unit of country j currency. The variance of S_{ij} and the correlation $\rho=\mathrm{corr}(S_{ij}, S_{ik})$ are assumed to be the same for all i, j, k.

[16] See, for example, Hartmann (1998), for a discussion of the currency denomination of trade in the EMU context.

[17] If we change the example by assuming that before EMU the variance of bilateral European exchange rates is less than the variance of European currencies relative to the yen, the condition(10) changes to $(1-x)<2\alpha<\frac{1}{1-\rho r}(1-x)$, where r is the standard deviation of bilateral European exchange rates divided by the standard deviation of European currencies relative to the yen. The example remains qualitatively unchanged, although it of course becomes less likely that a country will switch from pricing in yen to pricing in Euros when the variance of bilateral European rates is close to zero before EMU (r close to zero).

invoicing in the currencies that are replaced by the euro. While the example is for European exports, one can easily develop similar examples for European imports.

There is one caveat though. The increased invoicing in euros may not be immediate. In the example above, even after EMU there is still a Nash equilibrium whereby all European firms invoice in yen. Under coordination this is not the preferred invoicing choice, but without coordination history is likely to matter. The model's implication that history matters may explain for example why in Fig. 2 the UK is a bit of an outlier, invoicing more in pounds than can be expected based on market share.

3. Invoicing choice in general equilibrium

When going from a partial to a general equilibrium setup, the exchange rate is no longer exogenous. The source of uncertainty in the model shifts to a more fundamental set of factors. In this paper we only consider shocks to money supplies, which are equivalent to money demand shocks. Money is introduced through a cash-in-advance constraint. The per capita money supplies are M and M^* in the Home and Foreign country. The endogeneity of the exchange rate only matters to the extent that other elements of the cost and demand functions are also affected by the monetary shocks. This is indeed the case as both the aggregate demand for goods and wages are affected by the monetary shocks. In the partial equilibrium model these were both held constant. Another change is that we adopt a representative agent framework. This implies that firms maximize

$$Eu_c \frac{\Pi}{P}$$

where u_c is the marginal utility of consumption and P is the consumer price index.[18] The critical changes relative to partial equilibrium are the endogenous aggregate demand and wages, both of which are correlated with the exchange rate. In this section, we describe the model and provide the main results. The proof of these results and the solution of the model can be found in the Appendix of Bacchetta and van Wincoop (2002).

We consider a three-sector model, with two tradables sectors and one non-tradables sector. We introduce a non-tradables sector for a more realistic sensitivity of the overall consumer price index to the exchange rate. To distinguish between country size and market share in the industry, we consider two tradables sectors, A and B, with firms from both countries operating in both sectors.[19] We assume that the large Home country is dominant in sector A and the small Foreign country is dominant in sector B. In that case we have four configurations of market dominance and country size: (i) the large country operating in sector A where it is dominant, (ii) the large country operating in sector B

[18] The representative agent framework is chosen mainly for convenience and because it is standard in the new open economy macro general equilibrium literature. It is not critical to the results reported in this section. For example, if we instead assumed that "capitalists" own the firms and consume profits (maximize the expected utility of profits as in the partial equilibrium case), while "workers" consume labor income, the results reported below remain unaltered.

[19] This is similar to Tille (2002), and more general than most of the "new open economy macro" literature, which assumes that each country is completely specialized in only one sector.

where it is not dominant, (iii) the small country operating in sector B where it is dominant, and (iv) the small country operating in sector A where it is not dominant.

Mathematically this is done as follows. Let J be an integer. The number of firms in the large country in sectors A and B is $N_{\mathrm{A}}=J^2$ and $N_{\mathrm{B}}=1$, while the number of firms in the small country is $N_{\mathrm{A}}^*=1$ and $N_{\mathrm{B}}^*=J$. In both countries the share of firms in the non-tradables sector is α_N of the total number of firms. The total number of firms is also equal to the total number of consumers, which is N for the large country and N^* for the small country. We then let $J\rightarrow\infty$, so that the small country is infinitesimally small relative to the large country, while the market shares of the large country in sector A and the small country in sector B are infinitesimally close to 1. We refer to that simply as market dominance.

Since the optimal currency pricing strategies depend critically on the profit functions of exporters, we now discuss how the general equilibrium setup changes the demand and cost functions of the Home country. We always refer to the small Foreign country with a * superscript.

3.1. Demand and cost

3.1.1. Demand

The elasticity of substitution of consumption across sectors is assumed to be one and is therefore smaller than the elasticity $\mu>1$ of substitution among the goods within each sector. The overall consumption index is

$$c = c_A^{\alpha_A} c_B^{\alpha_B} c_N^{\alpha_N}$$

where c_i is a CES index with elasticity μ among the output of all firms in sector i. The consumption share α_i of sector i is also equal to the fraction of firms operating in sector i.

We focus on demand by Foreign residents faced by Home exporters. The cash-in-advance constraint implies that total nominal income of the Foreign country is equal to the total money supply, which is N^*M^*.[20] Letting again the superscripts E and I refer to prices set in respectively the exporter's and importer's currencies, the demand by Foreign consumers for a Home firm z in sector i is

$$D_i^{\mathrm{E}*} = \frac{1}{N+N^*}\left(\frac{p_i^{\mathrm{E}}(z)}{SP_i^*}\right)^{-\mu}\frac{N^*M^*}{P_i^*} \tag{12}$$

when the firm prices in the exporter's currency, and

$$D_i^{\mathrm{I}*} = \frac{1}{N+N^*}\left(\frac{p_i^{\mathrm{I}}(z)}{P_i^*}\right)^{-\mu}\frac{N^*M^*}{P_i^*} \tag{13}$$

[20] This is the case both when the buyer's currency and when the seller's currency is used for payment. The currency in which payment takes place may be the same or different from the currency in which prices are set. In the model these two are entirely separable.

when the firm prices in the importer's currency. The sectoral price index is

$$P_i^* = \left((1-n_i)\left(p_i^H*\right)^{1-\mu} + n_i f_i\left(p_i^{\mathrm{E}}/S\right)^{1-\mu} + n_i(1-f_i)\left(p_i^{\mathrm{I}}\right)^{1-\mu}\right)^{1/(-\mu)} \tag{14}$$

where n_i is the fraction of sector i firms that are from the Home country and p_i^H* is the price of domestically sold goods in the Foreign country.

These demand functions and sectoral price indices are the same as in the partial equilibrium model. The only difference is that aggregate real sectoral demand, referred to as $d*$ in the partial equilibrium model, is now $N*M*/P_i^*$ and therefore depends on monetary shocks. The fact that aggregate demand is stochastic is only relevant for invoicing decisions to the extent that it is correlated with the exchange rate. The equilibrium exchange rate can be solved from the Home money market equilibrium condition:

$$NM = \sum_i N_i\left(p_i^H D_i^H + f_i p_i^{\mathrm{E}} D_i^{\mathrm{E}}* + (1-f_i) S p_i^{\mathrm{I}} D_i^{\mathrm{I}}*\right) \tag{15}$$

It can be shown that

$$S = \frac{M}{M*} \tag{16}$$

The exchange rate is therefore simply the ratio of the money supplies.

3.1.2. Cost

The functional form of the cost function also remains unchanged when we move to a general equilibrium setup. To be consistent with the partial equilibrium model we assume that each firm sells exclusively either to the Home market or to the Foreign market. Since the Home market is much larger than the Foreign market, it is assumed that the capital stock of tradables firms that sell to the Home country is correspondingly larger. To be precise, the production function for a tradables firm z in sector i is $L_i(z)^{1/\eta}\ K_i(z)^{1-1/\eta}$, where $L_i(z)$ is labor input and $K_i(z)$ is the capital stock. The latter is assumed to be 1, $N/(N+N*)$ and $N*/(N+N*)$ respectively for a non-tradables firm, a tradables firm that sells to the Home market and a tradables firm that sells to the Foreign market. These capital stocks are proportional to the level of sales in the deterministic equilibrium. The cost function for all firms then remains the same as (Eq. (5)) if we scale both cost and output by the size of the capital stock.

The only change relative to partial equilibrium is that the wage rate is now generally stochastic. When we allow for a flexible wage rate, it is determined by equilibrium in the labor market. With a utility function $u(c,\ l)$ of consumption and leisure, labor supply follows from the first order condition

$$\frac{w}{P} = \frac{u_1}{u_c} \tag{17}$$

where consumption is $c{=}M/P$ and $P{=}P_{\mathrm{A}}^{\alpha_{\mathrm{A}}}P_{\mathrm{B}}^{\alpha_{\mathrm{B}}}P_N^{\alpha_N}$ is the consumer price index. With a time endowment of 1, aggregate labor supply is $L{=}N(1{-}l)$. Aggregate labor demand is:

$$L=\sum_i N_i\Big[\big(K_i^H\big)^{1-\eta}\big(D_i^H\big)^{\eta}+f_i\big(K_i^{\mathrm{E}}\big)^{1-\eta}\big(D_i^{\mathrm{E}}*\big)^{\eta}+(1-f_i)\big(K_i^{\mathrm{I}}\big)^{1-\eta}\big(D_i^{\mathrm{I}}*\big)^{\eta}\Big]$$

The superscript H refers to firms selling to the domestic market. For the non-tradables sector there is only domestic demand. The equilibrium wage rate can be solved by equating aggregate labor supply and demand.

3.2. Results

3.2.1. Rigid nominal wages

In order to stay as close as possible to the partial equilibrium model, we first consider a constant nominal wage and then move to the flexible wage case.[21] We again derive the analytical results based on small levels of risk. Assuming that M and M* have the same variance σ^2, we consider the derivative of $E\frac{u_c}{P}\left(\Pi^{\mathrm{E}}-\Pi^{\mathrm{I}}\right)$ with respect to σ^2 at $\sigma^2{=}0$.[22] Using a generalization of Lemma 1, we can derive the following Proposition when nominal wages are preset.

Proposition 5. *Consider the general equilibrium model with rigid nominal wages. For "small levels of risk", firms choose the following pricing strategies:*

- *If $(\mu-1)(\eta-1)<1$, firms price in the exporter's currency*
- *If $(\mu-1)(\eta-1)>1$ and the exporting country has a negligible market share, firms price in the importer's currency*
- *If $(\mu-1)(\eta-1)>1$ and the exporting country is dominant in the market, there are at least two Nash equilibria: (i) all exporting firms price in exporter's currency, (ii) all price in importer's currency. If firms coordinate, they prefer to price in the exporter's currency when they are risk-averse.*

Proposition 5 is qualitatively identical to Proposition 2 in the partial equilibrium model. Market share is still the critical factor in determining the currency denomination of trade.

[21] A standard approach of introducing nominal wage rigidities is the one discussed in Obstfeld and Rogooff (1996), whereby labor supply is heterogeneous and there are menu costs associated with changing wages. Total labor input is a CES index of heterogeneous labor supplies. Labor is monopolistically supplied and each agent sets the wage rate before uncertainty is resolved. The details do not concern us here and the level at which wages are preset is irrelevant for the results.

[22] In doing so we hold the correlation between the money supplies constant. One can also hold the ratio of the variances of M and M* constant at a level different from one in order to study the effect of monetary risk on optimal currency invoicing. This is the issue addressed in Devereux et al. (2004). In this paper we assume that money supplies have the same variance.

Country size plays no role. The only difference is that the term $\mu(\eta-1)$ in Proposition 2 is now replaced by $(\mu-1)(\eta-1)$. The parameter region where all firms invoice in the exporter's currency has therefore expanded a bit. This is because the demand risk associated with invoicing in the exporter's currency has been reduced. When firms price in the exporter's currency a depreciation raises demand. But a depreciation tends to be associated with a decline in the Foreign money supply M^*, which lowers demand. This offsetting effect reduces demand risk, making pricing in the exporter's currency more attractive.

3.2.2. Flexible wages

When we allow for flexible nominal wages, results can change significantly relative to those in Proposition 5. In Bacchetta and van Wincoop (2002), we consider the case where real wages are constant. This case introduces an asymmetry between the large and the small country. The wage rate is proportional to the consumer price index, which in general depends on exchange rate fluctuations in the small country but not in the large country. We find that this makes it more likely that firms from the small country invoice in the importer's currency. Although this shows that country size could matter theoretically, it does not appear to be an important factor in practice. The second largest country in the world, Japan, invoices much less in its own currency than smaller developed countries.

When wages are flexible and real wages volatile, our main result is in the form of a warning. Allowing for strongly pro-cyclical or anti-cyclical real wages can lead to invoicing results that are starkly at odds with the evidence. This can best be illustrated with a simple example. Assume that preferences take the following form:

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} + \alpha l \tag{18}$$

For the large country the real wage rate becomes proportional to M^{γ}. γ is therefore a measure of the degree of pro-cyclicality of real wages. It can be shown that when the cyclicality parameter γ is larger than μ, and $\eta<2$, all firms in both sectors in both countries price in the importer's currency.[23] An increase in the money supply raises the wage rate, but also leads to a depreciation, which increases demand when firms invoice in the exporter's currency. The positive correlation between wages and demand when firms invoice in the exporter's currency increases expected costs and lowers expected profits.

As has been extensively documented, real wages are neither strongly pro-cyclical nor strongly anti-cyclical. Allowing for strong cyclicality in real wages therefore contradicts most evidence. The fact that for strongly cyclical real wages one can easily get invoicing

[23] More generally, the following equilibria apply to firms from the large country. When $(\eta-1)(\mu-1)<1-\gamma$ all firms price in the exporter's currency. When $(\eta-1)(\mu-1)<\gamma-1$ all firms in all countries price in the importer's currency. When $(\eta-1)(\mu-1)$ is larger than both $1-\gamma$ and $\gamma-1$, all firms in non-dominant sectors price in the importer's currency, while firms in the dominant sectors price in the exporter's currency.

results that contradict the data should therefore not be much of a concern. If anything, it tells us that one has to be careful in choosing parameter values of a new open economy macro model when deriving the optimal invoicing results. It is easy to choose parameter values that lead to misleading results. For example, with preferences such as Eq. (18), the parameter γ plays a dual role, determining both the rate of relative risk-aversion and the cyclicality of real wages.[24]

3.3. Complete asset markets

Throughout the paper we have assumed that no assets are traded internationally. We now briefly discuss the implications of allowing for complete asset markets, so that there is full risk-sharing across the two countries. Assuming that nominal wages are rigid, the only impact of risk-sharing on the profit functions is through its effect on aggregate demand. Risk-sharing does not qualitatively alter the main results of the paper, but it makes invoicing in the importer's currency more likely. The Appendix of Bacchetta and van Wincoop (2002) describes the algebraic details. Here we only discuss the intuition.

In our small–large country model, only the small country is able to share risk in a way that affects its per capita consumption. Although that is a special case, it can be verified that the direction in which risk-sharing affects the results is the same when we allow for two equally sized countries that both benefit from risk-sharing. The Nash equilibria for Foreign country firms remain the same as in Proposition 5. Since per capita consumption of the Home country remains unchanged, the profit functions of Foreign country exporters remain unchanged. The profit functions of Home country exporters change as aggregate demand by Foreign country residents is no longer proportional to M^*. Aggregate demand in general depends positively on both M^* and M as a result of the risk-sharing.

We saw in Proposition 5 that in the case of no risk-sharing, pricing in the exporter's currency became somewhat more likely than in the partial equilibrium model. Demand risk is weakened when firms price in the exporter's currency since the rise in demand as a result of a depreciation tends to be offset by a decline in aggregate Foreign demand as a result of a drop in M^*. With full risk-sharing, Foreign demand depends positively on both Home and Foreign money supplies, so that the off setting effect is smaller (and could even go the other way). Pricing in the exporter's currency therefore becomes less attractive.

4. Conclusions

The recent new open economy macroeconomics literature has shown that the currency in which prices are set has significant implications for trade flows, capital

[24] In Devereux et al. (2004), γ plays the additional role of money demand elasticity. Money demand is modeled through money in the utility function by augmenting the preferences in Eq. (18) with the log of the real money supply. They additionally assume η=1. In that case neither country size, nor market share matter. The level of μ also does not affect the equilibrium invoicing strategies.

flows, nominal and real exchange rates, as well as optimal monetary and exchange rate policies. Since one of the main objectives of the recent literature is to bring microfoundations to macroeconomic analysis, it is natural to consider the optimal pricing strategy of firms within the context of this literature. Our main approach has been to build intuition by starting from a simpler partial equilibrium framework, which has also allowed us to connect the traditional partial equilibrium literature on currency invoicing with the more modern general equilibrium new open economy macro models.

We find that the two main factors determining the invoicing choice are market share and differentiation of goods. The higher the market share of an exporting country, and the more differentiated its goods, the more likely its exporters will price in the exporter's currency. In the introduction we briefly discussed some evidence consistent with these findings. There is clearly a need for further empirical work to confirm that these are critical factors. We find that the conclusions about optimal price setting in partial equilibrium models carry over to a general equilibrium context when wages are rigid. We also find that when allowing for flexible wages in a general equilibrium context one has to be very careful about parameter choices. In particular, parameter choices that lead to strongly cyclical real wages can lead to highly misleading results.

There are two important directions for future research. First, since the focus of this paper has been on positive economics (understanding currency invoicing), we have ignored the normative implications. In previous work (Bacchetta and van Wincoop (2000)) we have addressed the welfare implications of exchange rate regimes holding fixed the invoicing choice of firms. It is clear though that the invoicing choice is affected by monetary and exchange rate policies, which needs to be taken into account. Corsetti and Pesenti (2002) represents the first attempt in this direction. A second direction for research involves the distinction between trade prices and retail prices. Several authors have emphasized the fact that exchange rate changes are passed on to a larger extent to import prices than to consumer prices. In this paper we have made no distinction between the two. In Bacchetta and van Wincoop (2003) we extend the current framework by assuming that exporters do not sell directly to consumers, but sell intermediate goods used as inputs by domestic final goods producers. We show that final goods producers are more likely to price in the local currency than exporters.

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Appendix A

Proof of Lemma 1. Take the two profit functions $\Pi^{\mathrm{E}}(S;x)$ and $\Pi^{\mathrm{I}}(S;x)$ considered in the Lemma. We examine small levels of risk around $\bar{S}$=$E(S)$. Since $\partial(\Pi^{\mathrm{E}}-\Pi^{\mathrm{I}})/\partial x$=0 we only need to consider profits as a function of S, holding x constant at its deterministic level. It is assumed that $\Pi^{\mathrm{E}}(\bar{S})=\Pi^{\mathrm{I}}(\bar{S})$. Let $f(S)=U(\Pi^{\mathrm{E}})-U(\Pi^{\mathrm{I}})$. We have $f(\bar{S})=0$ and:

$$f_S = U'\left(\Pi^{\mathrm{E}}\right)\frac{\partial \Pi^{\mathrm{E}}}{\partial S} - U'\left(\Pi^{\mathrm{I}}\right)\frac{\partial \Pi^{\mathrm{I}}}{\partial S}$$

and

$$f_{SS}(\bar{S}) = U''\left[\left(\frac{\partial \Pi^{\mathrm{E}}}{\partial S}\right)^2 - \left(\frac{\partial \Pi^{\mathrm{I}}}{\partial S}\right)^2\right] + U'\left[\frac{\partial^2 \Pi^{\mathrm{E}}}{\partial S^2} - \frac{\partial^2 \Pi^{\mathrm{I}}}{\partial S^2}\right]$$

where U'' and U' are evaluated at $\bar{S}$.

Then take a second-order Taylor expansion around $\bar{S}$:

$$f(S) = f\left(\bar{S}\right) + f_S\left(\bar{S}\right)\left(S-\bar{S}\right) + \frac{1}{2}f_{SS}\left(\bar{S}\right)\left(S-\bar{S}\right)^2$$

Its expected value is:

$$Ef(S) = \frac{1}{2}f_{SS}\left(\bar{S}\right)\sigma^2$$

Using the equation for $f_{SS}(\bar{S})$ and assuming that $\bar{S}$ is constant gives Lemma 1. Higher order terms in the Taylor expansion do not matter. In order to see this, assume that $S-\bar{S}$ is equal to σy, where y has expectation 0, variance 1 and its distribution does not depend on σ or, even weaker, the derivative of $E\,y^n$ (n>2) with respect to σ (evaluated at σ=0) is finite. The derivative of $E(S-\bar{S})^n=\sigma^n E(y^n)$ with respect to σ^2 is then zero for n>2 when evaluated at σ=0. □

Proof of Proposition 1. From Eqs. (1), (2), (4) and (5), the firm's profit functions are:

$$\Pi^{\mathrm{I}} = S\left(p^{\mathrm{I}}\right)^{1-\mu} - w\left(p^{\mathrm{I}}\right)^{-\eta\mu} \tag{19}$$

$$\Pi^{\mathrm{E}} = S^{\mu}\left(p^{\mathrm{E}}\right)^{1-\mu} - wS^{\eta\mu}\left(p^{\mathrm{E}}\right)^{-\eta\mu} \tag{20}$$

First, notice that for σ^2=0 and $E(S)=\bar{S}=1$, the optimal price set by the firm is the same, i.e., $p^{\mathrm{E}}=p^{\mathrm{I}}\equiv\bar{p}$,where:

$$(\mu-1)\bar{p}^{1-\mu} = \eta\mu w\bar{p}^{-\eta\mu} \tag{21}$$

Then, Lemma 1 applies since $\Pi^E(1; x)=\Pi^I(1; x)$ and $\partial(\Pi^E-\Pi^I)/\partial x=0$, where $x=(p^E, p^I, w)$. Then it can be shown that:

$$\frac{\partial\left[EU\left(\Pi^E\right)-EU\left(\Pi^I\right)\right]}{\partial\sigma^2}=0.5U'\frac{\partial^2\Pi^E}{\partial S^2}=0.5U'(\mu-1)\bar{p}^{1-\mu}[1-\mu(\eta-1)]$$

whose sign depends on the sign of $1-\mu(\eta-1)$. □

Proof of Proposition 2. With strategic complementarities, we first examine the incentives of a marginal firm given the behavior of the other firms. Then we determine the Nash equilibrium such that the marginal firm does not deviate. In the coordination case, all firms take the same action simultaneously. Without loss of generality we set $d^*/(N+N^*)=1$. Using Eq. (6), profits of firm i are:

$$\Pi_i^I=Sp_i^I\left(p_i^I/P^*\right)^{-\mu}-w\left(p_i^I/P^*\right)^{-\eta\mu} \tag{22}$$

$$\Pi_i^E=S^\mu p_i^E\left(p_i^E/P^*\right)^{-\mu}-wS^{\eta\mu}\left(p_i^E/P^*\right)^{-\eta\mu} \tag{23}$$

where P^* is given by Eq. (8). First notice that for $\sigma^2=0$ and $S=\bar{S}=1$, all firms choose the same price: $p^H=p^E=p^I\equiv\bar{p}$. This implies that in the deterministic equilibrium $P^*=\bar{p}$. The optimal price is given by:

$$(\mu-1)\bar{p}=\eta\mu w \tag{24}$$

In computing the derivatives of profits, the main difference with Proposition 1 is that P^* depends on S. From Eq. (8), we have:

$$\frac{\partial P^*}{\partial S}=-\bar{p}nfS^{\mu-2}\left[1-n+nfS^{\mu-1}+n(1-f)\right]^{\frac{\mu}{1-\mu}}$$

which evaluated at $S=\bar{S}=1$ gives $\partial P^*/\partial S=-nf\bar{p}$. Then we can show that:

$$\frac{\partial\left[EU\left(\Pi^E\right)-EU\left(\Pi^I\right)\right]}{\partial\sigma^2}=U'(\mu-1)\frac{\bar{p}}{2}[1-\mu(\eta-1)(1-2fn)]$$

The sign of this expression depends on the sign of $1-\mu(\eta-1)(1-2fn)$. There can be three types of Nash equilibria:

- $f=0$ and $EU(\Pi^E)<EU(\Pi^I)$. In this case, all firms price in the importer's currency and the marginal firm still prefers pricing in the importer's currency.
- $f=1$ and $EU(\Pi^E)>EU(\Pi^I)$. All firms price in their own currency and the marginal firm prefers pricing in its own currency.
- $EU(\Pi^E)=EU(\Pi^I)$ and $0<f<1$. This is a mixed equilibrium, where a proportion f of firms prices in their own currency and the marginal firm is indifferent as to its pricing strategy.

There is an equilibrium where all firms price in the importer's currency ($f=0$) when $\mu(\eta-1)>1$. There is an equilibrium where all firms price in the exporter's currency

(f=1)when either $n>\bar{n}$ or $n<\bar{n}$ and $\mu(\eta-1)<1$. When $n>\bar{n}$ there is also a mixed equilibrium with $0<f<1$.

When firms coordinate, they consider their best pricing strategy given that all the others do the same. Thus, if they price in their own currency, they assume f=1 and thus $\partial P^*/\partial S=-n\bar{p}$; when they price in the importers currency, they assume f=0 and thus $\partial P^*/\partial S$=0. This implies:

$$\frac{\partial\left[EU\left(\Pi^{\mathrm{E}}\right)-EU\left(\Pi^{\mathrm{I}}\right)\right]}{\partial\sigma^2}=0.5U'\bar{p}(1-n)\{n\mu+(1-n)(\mu-1)(1-\mu(\eta-1))\}$$
$$-0.5U''\bar{p}^2n(2-n)$$

For n close to one or $n>0$ and U'' sufficiently large, this expression is positive, so firms prefer to price in the exporter's currency. □

Proof of Proposition 3. Assume there are Z countries and that the price of all firms is equal to one. We consider the currency pricing decision of a marginal firm in country 1 exporting to country 2. Country 1 exporters compare profits when they price in their own currency Π^1 or in any other currency Π^x .Let S_j be the exchange rate of country j with respect to country 1, i.e., the quantity of country 1 currency per one unit of country j currency. Following the same argument as before, we can evaluate profits at the deterministic prices $\bar{p}=\eta\mu w/(\mu-1)$. Profits can be written as:

$$\Pi^x=S_x\bar{p}\left(S_x\bar{p}/\tilde{P}\right)^{-\mu}-w\left(\bar{p}S_x/\tilde{P}\right)^{-\eta\mu}$$

$$\Pi^1=\left(\bar{p}/\tilde{P}\right)^{-\mu}-w\left(\bar{p}/\tilde{P}\right)^{-\eta\mu}$$

where $\tilde{P}$ is the price index in country 2 but expressed in country 1 currency ($\tilde{P}=S_2P_2$):

$$\tilde{P}^{1-\mu}=\bar{p}^{1-\mu}\sum_{i=1}^{Z}n_iS_{x(i)}^{1-\mu}.$$

Since profits now depend on multiple exchange rates, we apply a generalization of Lemma 1 with S replaced by multiple exchange rates. If we are interested in small amounts of risk, as defined in Lemma 1, it is sufficient to look at a second order Taylor approximation of $f(S_x, S_{x(1)}, \ldots, S_{x(Z)})=U(\Pi^x)-U(\Pi^1)$. This yields:

$$E(f)=\frac{1}{2}\frac{\partial^2f}{\partial S_x^2}\mathrm{var}(S_x)+\sum_{i=1}^{Z}\frac{\partial^2f}{\partial S_x\partial S_{x(i)}}\mathrm{cov}\left(S_x,S_{x(i)}\right) \tag{25}$$

Evaluating at the deterministic equilibrium we have:

$$\frac{\partial^2f}{\partial S_x^2}=U'(\mu-1)\bar{p}(\mu-\eta\mu-1)$$

$$\frac{\partial^2f}{\partial S_x\partial S_{x(i)}}=U'(\mu-1)\bar{p}\mu(\eta-1)n_i$$

By substituting the above expressions into Eq. (25) we get:

$$\frac{2E(f)}{(\mu-1)U'\bar{p}} = (\mu-\eta\mu-1)\mathrm{var}(S_x) + 2\sum_{i=1}^{Z}\mu(\eta-1)n_i\mathrm{cov}\left(S_x, S_{x(i)}\right)$$

The firms choose to price in the currency x for which this expression is largest, which is equivalent to choosing the currency x that minimizes the expression (Eq. (9)) in Proposition 3. □

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