

Corporate Cash and Employment*

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Abstract

In the aftermath of the U.S. financial crisis, both a sharp drop in employment and a surge in corporate cash have been observed. In this paper, based on U.S. data, we argue that the negative relationship between the corporate cash ratio and employment is systematic, both over time and across firms. We develop a dynamic general equilibrium model where heterogenous firms need cash and external liquid funds in their production process. We analyze the dynamic impact of aggregate shocks and the cross-firm impact of idiosyncratic shocks. We show that external liquidity shocks generate a negative comovement between the cash ratio and employment, as documented in the data.

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1 Introduction

In the aftermath of the U.S. financial crisis, both a sharp decline in employment and an accumulation of cash held by firms have been observed. While both variables are part of firms' decisions, they are typically not considered jointly in the literature. To what extent are these two features related? Holding liquid assets facilitates the firm's ability to pay for the wage bill. But employment and cash decisions also react to changes in firms' environment, e.g., changes in credit conditions. Therefore, examining these two variables jointly sheds light on the role of financial shocks on employment, especially during the crisis. The contribution of this paper is twofold. First, it provides stylized facts on the relationship between the corporate cash position and employment. Second, it delivers an explanation to the empirical evidence by building a tractable dynamic general equilibrium framework, including both cash and employment decisions. This framework sheds a new light on the impact of financial shocks by focusing on firms' external liquidity.

We first document a robust negative comovement between the corporate cash ratio and employment on U.S. data, which is not specific to the recent financial crisis. Using Flow-of-Funds data over the period 1980-2015, the correlation between HP-filtered employment and the share of liquid assets in total assets is -0.43 . Moreover, using firm-level data from Compustat, the cross-firm correlation between employment and the cash ratio is on average -0.22 over the same period.¹ Section 2 provides a detailed description of this empirical analysis.

To understand the optimal cash and employment decisions, we consider an infinite-horizon general equilibrium model with heterogeneous firms that need internal and external liquid funds in their production process. Liquidity is closely related to labor because firms have liquidity needs in order to finance the wage bill, which is part of working capital. We adopt a structure similar to Christiano and Eichenbaum (1995), who divide periods into two subperiods. In the first subperiod, firms use credit to install capital, while they resort to liquid funds to pay workers in the second subperiod. In contrast to the literature introducing working capital in macroeconomic models (see Christiano et al., 2010, for a survey), we assume that firms do not have full access to external liquidity and thus cannot borrow all their short-term needs. This generates a demand for cash related to the wage bill. In line with this "cash-in-advance" assumption, the data suggest a positive relationship between firms' cash holding and their wage bill.

Liquidity that is external to the firm may take several forms, such as credit lines, trade credits, trade receivables to customers, or late wage payments. Liquidity shocks are changes in the availability of external liquidity and affect the demand

¹Melcangi (2016) documents a similar stylized fact for the UK.

for cash. In addition to liquidity shocks, we assume that firms may be hit by technology shocks and by changes in their ability to obtain long-term credit (i.e., standard credit shocks). Shocks can be at the aggregate or at the idiosyncratic level.

The model is designed to be tractable so that several results can be derived analytically. We show that liquidity shocks can explain the negative comovement between employment and the corporate cash ratio. A reduction in external liquidity generates two effects. On the one hand, lower liquidity reduces the financial opportunities of firms and depresses labor demand. On the other hand, the reduction in external liquidity makes the production process more intensive in cash to ensure that wages are fully financed. Firms' assets are then tilted towards cash. Combining these two effects implies that the cash ratio increases while employment declines. This mechanism differs from the simpler alternative explanation that firms hoard cash in bad times, when employment is low. While this behavior can also contribute to a negative comovement between cash and employment, it does not fully explain all patterns of the empirical evidence, as we report in Section 2. Therefore, the mechanism we propose in this paper should be seen as complementary to other channels.

Using cash ratio data enables us to identify liquidity shocks. We find that the aggregate liquidity shocks that are generated by a calibrated version of our model are empirically plausible. Notably, they are highly correlated with the use of short-term loans or of commercial paper. They are also consistent with the tightening of liquidity conditions in the aftermath of the Lehman crisis reported in the literature.² Therefore, our model sheds a different light on financial shocks, by focusing on liquidity shocks. Besides, our quantitative analysis shows that liquidity shocks can explain a large share of output volatility, and an even larger share of employment volatility.

The introduction of firms heterogeneity in the model allows us to investigate to what extent idiosyncratic liquidity shocks can explain the negative cross-firm correlation between the cash ratio and labor observed in the data. The model is parameterized using moments distribution from firm-level data. Despite its simplicity, it performs relatively well quantitatively to reproduce the negative cross-firm correlation since it gives a correlation of -0.13 , while it is -0.22 in the data.

²Gilchrist and Zakrajšek (2012) argue that banks cut the corporate lines of credits during the crisis. Ivashina and Scharfstein (2010) show that firms initially drew heavily on their credit lines, but that subsequently credit conditions tightened. Campello et al. (2011) show that some firms had their credit lines canceled and that other firms had to renegotiate their credit lines with a higher cost. More generally, credit line agreements may contain restrictive covenants that may limit the ability of borrowers to draw on their lines. See also Chari et al. (2008) or Kahle and Stulz (2013).

The optimal choice of corporate liquidity is rarely introduced in macroeconomic models, even in models with financial frictions. When it is, the focus is on investment, not labor. Liquid assets are usually held by households, typically in the form of money, to finance their consumption.³ However, firms also have liquidity needs. Papers incorporating firms' liquidity are typically in the spirit of Holmstrom and Tirole (2011) and Woodford (1990); they include Aghion et al. (2010), Kiyotaki and Moore (2012), Bacchetta and Benhima (2015), or Cui and Radde (2015). However, these papers do not specifically analyze employment fluctuations.

While the link between liquidity and employment has not received much attention so far, our analysis is related to several strands of the literature. First, there is a growing literature that incorporates firms' financial frictions in a macroeconomic context. For instance, Covas and den Haan (2011) and Jermann and Quadrini (2012) analyze corporate external finance decisions over the business cycle, such as debt and equity. However, these papers do not introduce cash. For example, in their theoretical model, Jermann and Quadrini (2012) have working capital that is fully financed by an intra-period loan. Other papers focus more closely on the relationship between financial factors and the labor market. This literature stresses the role of financial frictions influencing labor demand.⁴ Most of these papers provide a more detailed analysis of the labor market than we do, but they do not consider cash holdings. Our analysis focuses on the impact of liquidity conditions on labor demand.

Our paper is also related to a vast theoretical literature in corporate finance on firms' cash holdings and corporate saving. Our approach shares features with several recent papers that provide analyses at the firm level or in environments with heterogeneous firms. Some papers are particularly close to our approach as they focus on the role of financing conditions on cash decisions.⁵ Our paper differs from this literature by focusing on employment, which plays a key role in the working capital management. Another difference is that we make a clear distinction between liquid and less liquid assets. The recent dynamic models in the corporate finance literature consider cash a negative debt or as a residual between cash flow and investment.⁶ One exception is Melcangi (2016), who studies the effect of variations

³There are obviously some exceptions. For example, Stockman (1981) considers a cash-in-advance constraint both for consumption and capital.

⁴See for instance Wasmer and Weil (2004), Benmelech et al. (2011), Monacelli et al. (2011), Boeri et al. (2012), Chodorow-Reich (2014), Karabarbounis and Neiman (2012), Pagano and Pica (2012) or Petrosky-Nadeau and Wasmer (2013), Bentolila et al. (2015), Mehrotra and Sergeyev (2015).

⁵See for example, Bolton et al. (2013), Eisfeldt and Muir (2016), Falato et al. (2013), Gao (2013), and Hugonnier et al. (2015). Some papers consider other determinants of firms' cash holdings (Armenter and Hnatkovska, 2011; Boileau and Moyen, 2012).

⁶This contrasts with an older corporate finance literature, see Holmstrom and Tirole (2011).

in the availability of intra-period loans on the employment and cash accumulation behavior of firms. Our paper also differs from most of the literature is that we adopt a general-equilibrium, business-cycle approach. The general-equilibrium analysis is important in the context of employment as this is an input that is not generated by the firm (in contrast to capital). As a result, market-clearing wage fluctuations can potentially offset partial equilibrium effects. This is particularly relevant in the context of liquidity management as the wage bill affects firms' liquidity needs. The business-cycle approach enables us to assess the relative importance of financial shocks.

Finally, our approach is consistent with the findings of the empirical literature on the determinants of corporate cash.⁷ This literature stresses in particular the precautionary motive to save cash and shows that this motive increases with cash flow uncertainty or with more uncertain access to capital markets (see for instance Almeida et al., 2004). Some papers have also analyzed the use of short-term credits, like credit lines, and their interaction with corporate cash holdings. They tend to show that cash is a substitute to credit lines, as suggested by our analysis. For instance, Campello et al. (2011) find a negative correlation between cash and credit lines.⁸

The rest of the paper is organized as follows. Section 2 investigates the negative comovement between the corporate cash ratio and employment and the relationship between wages and cash. Section 3 presents the model and shows the basic mechanism that can lead to this negative relationship, using a baseline version of the model that allows for an analytical solution. In Section 4, we calibrate the model to analyze quantitatively the dynamic impact of aggregate shocks. We first consider the baseline model and then a more realistic extended model. We also consider several extensions to that benchmark and the impact of liquidity uncertainty shocks. In Section 5, we examine the effect of idiosyncratic shocks on cross-firm correlations and Section 6 concludes. Several results are derived in the Appendices.

2 Stylized Facts

In this section, we document a negative comovement in the U.S. between the corporate cash ratio and employment. We also investigate the connections between cash holdings and the wage bill.

⁷See, for example, Bates et al. (2009) and Almeida et al. (2014) for surveys.

⁸Similarly, Sufi (2009) and Lins et al. (2010) show that internal cash is used more in bad times while firms are more likely to use credit lines in good times. Acharya et al. (2013) build a model to show that firms would rather use credit lines instead of cash reserve when they face a low aggregate risk.

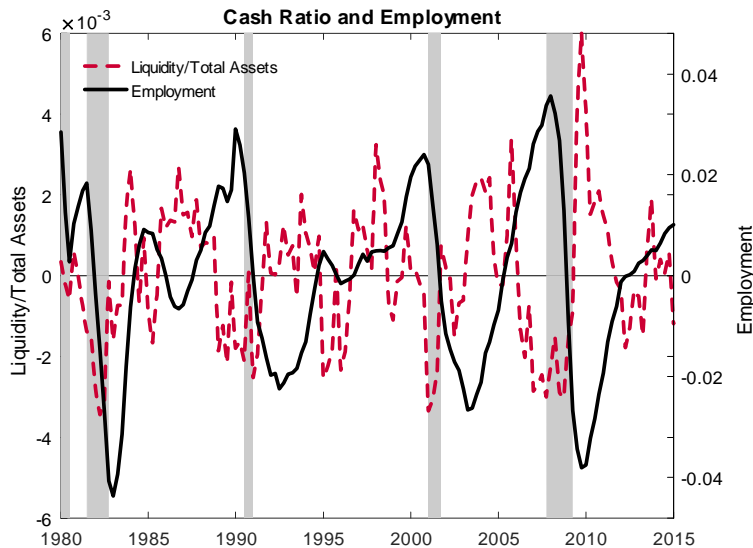
2.1 Corporate Cash Ratio and Employment

The negative correlation between cash ratio and employment can be found both in aggregate terms and at the firm level.

2.1.1 Aggregate Data

We first illustrate the aggregate relationship between the two variables over the business cycle. We use quarterly data in the non-farm non-financial corporate sector. The cash ratio, defined as the share of corporate liquidity in total assets, is built from the U.S. Flow of Funds. Cash is measured as the sum of private foreign deposits, checkable deposits and currency, total time and savings deposits and money market mutual fund shares. Corporate employment is drawn from the Bureau of Labor Statistics. Figure 1 displays the HP-filtered component of employment (transformed in log) and the cash ratio over the sample 1980q1-2015q3.

Figure 1: **Corporate Liquidity and Employment (aggregate data).**



Note: Employment is expressed in logarithm and both variables are HP-filtered.

The figure shows a negative comovement between the two variables that is particularly striking during the Great Recession since the corporate liquidity ratio experienced a large boom from 2009 while employment has been strongly depressed. Since then, employment has recovered and the liquidity ratio has been less volatile. Over the whole sample, the contemporaneous correlation between employment and the cash ratio is -0.43 and is significant at 1%. Table 1 of the online Appendix presents several robustness tests confirming this result, even though there are several cases where the correlation is less negative. Not surprisingly, the negative correlation is smaller by excluding the Great Recession (-0.19 , significant at 10%).

Interestingly, abstracting from the money market mutual fund shares in the definition of liquidity also leads to a smaller correlation (-0.16 , significant at 5%). This result suggests that liquid financial instruments can be part of the explanation regarding the negative correlation.⁹

2.1.2 Firm-level Data

The stylized fact documented above is driven by macroeconomic shocks common to all firms. In order to capture the heterogeneity among firms, we assess the correlation between the corporate cash ratio and employment using yearly disaggregated firm-level data from Compustat. The sample contains U.S. non-financial firms from 1980 to 2014. We focus on firms that are active at least 10 years over the period. We exclude financial and utilities firms, firms which are not incorporated in the U.S. market and those engaged in major mergers.¹⁰ This is justified by the fact that part of the stock of cash holding is affected by acquisition.¹¹ We use the number of employees per firm (Compustat data item #29) as our measure of employment. The corporate cash ratio is defined as the ratio between cash and short term investment (Compustat data item #1) and the book value of assets (Compustat data item #6). Figure 2 draws the scatter plot with the corporate cash ratio (horizontal axis) and the log of employment (vertical axis).

The unconditional cross-section correlation between employment and cash ratio is -0.22 on average and it is significant at 1%.¹² To go further in the analysis, we estimate by OLS the following regression equation

$$\log(EMP_{it}) = \beta_1 + \beta_2 \left(\frac{CHE}{AT} \right)_{it} + \beta_3 \mathbf{X}_{it} + \zeta \mathbf{y}_t + \xi \mathbf{z}_i + \varepsilon_{it}, \quad (1)$$

where $\log(EMP_{it})$ is the log of the number of employees for firm i at time t , $\left(\frac{CHE}{AT} \right)_{it}$ is the cash ratio, \mathbf{X}_{it} is a vector of firm-specific control variables. We control for unobservable heterogeneity at the firm level by introducing firm fixed effects (\mathbf{z}_i). The regression also includes year fixed effects through \mathbf{y}_t to account for macroeconomic fluctuations. Finally, control variables, included in \mathbf{X}_{it} , are the log of total

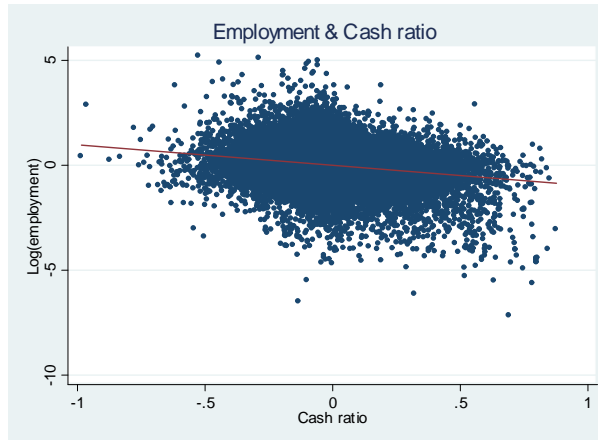
⁹In order to avoid any spurious correlation, we also compute the correlation when cash is divided by the one-quarter lagged value of total assets instead of its current value. The correlation is still negative (-0.35) and significant. Figure 2 in the online appendix displays the liquidity ratio and employment over a longer period (1962q1-2015q3) and we find that the correlation is lower than in the benchmark sample (-0.27). Also, Figure 3 in the online appendix shows that the correlation between the cash *level* and employment (both in log and HP filtered) is insignificant.

¹⁰Using Compustat data items, we remove firms when $6000 < SIC < 6999$, $4900 < SIC < 4949$, $curcd \neq USD$ and $sale_fn = AB$.

¹¹The sample is reduced to 18 052 firms. Data description and descriptive statistics are provided in the online appendix.

¹²Table 3 in the online appendix presents a series of robustness checks of this correlation.

Figure 2: **Unconditional correlation between the cash ratio and employment (firm-level).**



Note: For both variables, we remove the firm-specific linear trend.

assets ($\log(AT_{it})$) measuring the size of the firm; cash flow ($CFLOW_{it}$), measuring firms’ internal funds; the leverage ratio (LEV_{it}) capturing the relative demand for credit; and the log of capital expenditures ($\log(CAPX_{it})$) capturing the investment policy of the firm. All variables are firm-specific linearly detrended.¹³

In Table 1, Columns (1)-(4) show a robust negative correlation between employment and the cash ratio. The estimates for β_2 is -0.82 and it is significant at 1% when we control for all the firm-specific variables. The negative firm-level correlation is robust to the inclusion of years fixed effects, which indicates that it is not driven exclusively by business cycle effects like the cost of cash. Similarly, by controlling for cash flows, we take into account the fact that labor layoffs mechanically generate cash flows. This also controls for “real option value” of cash, to the extent that current cash flows contain information on the future state of the firms.¹⁴

Table 2 provides robustness checks and additional regressions. Column (1) shows that the correlation is still negative when we control for the size of the firm by using sales rather than total assets. The result is also unaffected when the 10% largest firms are dropped from the sample, as recommended by Covas and den Haan (2011) (Column (2)). In this paper, we argue that the negative idiosyncratic correlation between cash holding and employment is driven by the availability of externality liquidity: facing a reduction in external liquid funds, a firms reduces employment and raises the amount of cash in its portfolio to finance the wage bill, which is part

¹³Data are described in details in the online appendix.

¹⁴The correlation is also robust to the use of lagged variables and variables in differences (see Tables 5-6 in the online appendix).

Table 1. **Benchmark estimation:** Employment and Cash Ratio

Dependent Variable: $\log(\text{EMP}_{it})$				
	(1)	(2)	(3)	(4)
$(\frac{\text{CHE}}{\text{AT}})_{it}$	-0.975** (0.039)	-0.880** (0.030)	-0.866** (0.029)	-0.818** (0.024)
$\log(\text{AT})_{it}$		0.573** (0.008)	0.572** (0.008)	0.523** (0.007)
CFLOW_{it}			-0.000 (0.000)	-0.000** (0.000)
LEV_{it}				-0.001 (0.002)
$\log(\text{CAPX})_{it}$				0.009** (0.003)
R-squared	0.06	0.44	0.44	0.49
Firm fixed effects	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes
Observations	140 705	140 705	139 967	133 217

Notes: Robust standard errors are in brackets. A */** next to the coefficient indicates significance at the 10/5 percent level

Table 2. **Robustness:** Employment and Cash Ratio

Dependent Variable:	$\log(\text{EMP}_{it})$			$\log(\text{INVT}_{it})$
	(1)	(2)	(3)	(4)
$(\frac{\text{CHE}}{\text{AT}})_{it}$	-0.349** (0.023)	-0.828** (0.026)		-1.255** (0.034)
$(\frac{\text{ST DEBT}}{\text{TOTAL DEBT}})_{it}$			0.122** (0.020)	
$\log(\text{AT})_{it}$		0.514** (0.008)	0.513** (0.008)	0.746** (0.011)
$\log(\text{SALE})_{it}$	0.407** (0.009)			
CFLOW_{it}	-0.000** (0.000)	-0.000 (0.001)	-0.000** (0.000)	-0.000* (0.000)
LEV_{it}	-0.009** (0.002)	0.000 (0.002)	-0.001 (0.002)	-0.014* (0.005)
$\log(\text{CAPX})_{it}$	0.141** (0.003)	0.084** (0.003)	0.100** (0.003)	0.035** (0.004)
R-squared	0.46	0.45	0.45	0.44
Firm fixed effects	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes
Observations	131 327	99 371	130 544	116 532

Notes: Robust standard errors are in brackets. A */** next to the coefficient indicates significance at the 10/5 percent level

of working capital. Results in Columns (3) and (4) are in line with this intuition. Column (3) reveals a positive correlation between employment and external liquid funds proxied by the share of short-term debt to total debt.¹⁵ Column (4) uses an alternative measure of working capital, namely inventories, as a dependent variable and shows that its correlation with the cash ratio is still negative and significant.

2.2 Financing Wages with Cash

An important assumption of our model is that cash holding decisions are determined by wage bill financing. We use firm-level data to investigate the plausibility of this assumption by analyzing the relationship between cash *level* and wages. From our database, we observe that cash represents 36% of firms' staff expenses (median value).¹⁶ In our model, we assume that corporate cash is used to finance end-of-period wages. We assess this link by regressing the future amount of staff expenses (XLR_{it+1}), which proxies the expected future wage, on the current cash level (both expressed in log). Table 3, Columns (1)-(4) displays the results of the estimation.

In Column (1), we estimate the conditional correlation between future wages and cash level, including fixed effects and year-sector-fixed effects and without control. The year-sector-fixed effects are included because it is very likely that the correlation depends on the type of sector we consider. In Columns (2) and (3), we control for the current amount of staff expenses and the size of the firm. The FE estimator suffers from the Nickell bias in the presence of a lagged dependent variable, but our relatively large time dimension reduces this bias. In Column (4) we use the typical Arellano–Bond (GMM) estimation to take this endogeneity issue more specifically into account. The Hansen test for overidentification suggests that we do not reject the exogeneity of the instruments, even though our results might be sensitive to the choice of instruments set. In all specifications, there is a positive and significant relationship between the cash level and future staff expenses. This result suggests that firms hold more cash prior to a rise in staff expenses, which is in line with our model's assumption.

We investigate further this relationship by looking at data at the industry level.

¹⁵The short-term debt is defined as the total debt (Compustat data item #34), minus the long-term debt (Compustat data item #142), which represents debt obligations due in more than one year. Notice that the cash ratio is significantly and negatively correlated with the share of short-term debt (-0.13). See online appendix for details.

¹⁶The series staff expense (Compustat data item #42) includes salaries, wages, pension costs, profit sharing and incentive compensation, payroll taxes and other employee benefits. The scarce availability of this variable reduces the sample to 2224 firms. The online appendix shows that the distribution of firms' size and cash ratio is slightly affected (see Table 11 in the online appendix). In addition, the correlation between the cash ratio and employment is -0.18 and still significant at 1%.

Table 3. Wages and Cash

Dependent Variable: $\log(\text{XLR})_{it+1}$				
	(1)	(2)	(3)	(4)
$\log(\text{CHE})_{it}$	0.165** (0.010)	0.015** (0.004)	0.041** (0.004)	0.021** (0.00)
$\log(\text{XLR})_{it}$		0.530** (0.023)	0.627** (0.03)	0.728** (0.01)
$\log(\text{AT})_{it}$		0.239** (0.012)		
$\log(\text{SALE})_{it}$			0.110** (0.01)	0.180** (0.00)
R-squared	0.30	0.97	0.97	
Sector-year fixed effects	yes	yes	yes	no
Firm fixed effects	yes	yes	yes	no
Estimation	OLS	OLS	OLS	SYS-GMM
Hansen test (p-value)	–	–	–	0.70
Arellano-Bond test (p-value), AR(2)	–	–	–	0.22
Observations	18 644	18 642	18 100	18 133

Notes: Robust standard errors are in brackets. A */** next to the coefficient indicates significance at the 10/5 percent level. In Column (4), the estimation is by two-step system GMM. All explanatory variables dated in t-2 and longer are used as instruments.

One might expect that labor-intensive industries experience a stronger correlation between cash level and wage. The NBER-CES Manufacturing Industry Database provides a measure of labor share, defined as the ratio between payroll and value added, by year and by industry. We merge this database with the Compustat database in order to get the (median) amount of cash, wages and total assets. The sample covers 1980-2009 and consists in 103 industries. As previously, we analyze the relationship between current cash holding and future staff expenses (both expressed in log), but we now add an interaction term between cash level and the labor share. This interaction is significantly positive and therefore we find that the correlation between cash and future wages is stronger for industries that rely more on labor.¹⁷ These two pieces of evidence go in favor of our assumption that cash is a key financing source of wages for firms.

3 A Dynamic Model of Corporate Cash Holdings

The single-good economy is inhabited by infinitely-lived heterogeneous entrepreneurs, identical households, deposit institutions and a government. Entrepreneurs

¹⁷Estimation results are provided in Table 14 of the online appendix.

produce, hire labor, invest, borrow, and hold cash. Households work, consume, lend to entrepreneurs and hold short-term deposits. There are two types of debt: short-term debt and long-term debt. Liquidity is modelled by dividing each period into two subperiods, which we refer to as beginning-of-period and end-of-period. We define short-term debt as the debt that can be issued between end-of-period t and beginning-of-period $t + 1$.¹⁸ Long-term debt is debt that can be issued for the full period. It is illiquid in the sense that the long-term debt market does not open at end-of-period. The firm faces two different credit constraints attached to each type of debt. These two types of debt matter because firms face liquidity issues during the production process. Firms have a liquidity need at end-of-period as they have to pay for the wage bill.¹⁹ This liquidity need can be covered either by short-term debt, which we refer to as “external liquidity”, or by cash holdings. Therefore, the need for cash is affected by changes in the availability of external liquidity. We first describe the problem of entrepreneurs and then turn to their optimal behavior, focusing on optimal labor demand and cash. We then close the model and characterize analytically the properties of the model in a baseline case.

3.1 Entrepreneurs

There is a continuum of entrepreneurs of length 1. Entrepreneur $i \in [0, 1]$ maximizes

$$E_t \sum_{s=0}^{\infty} \beta^s u(c_{it+s}), \quad (2)$$

where c_{it+s} is his consumption in period $t + s$ and $u(\cdot)$ is a strictly increasing and concave function. Entrepreneur i produces Y_{it} out of capital K_{it} and labor l_{it} through the production function

$$Y_{it} = F(K_{it}, A_{it}l_{it}), \quad (3)$$

where $F(\cdot)$ is a standard constant-return-to-scale production function and A_{it} is total factor productivity (TFP). Capital depreciates at rate δ . In the baseline version of the model, we abstract from adjustment costs. TFP is composed of an aggregate component and an idiosyncratic one

$$A_{it} = A_t + \epsilon_{it}^A, \quad (4)$$

¹⁸This debt can be considered as intra-temporal since there are no consumption decisions during this interval.

¹⁹For convenience we only consider labor as end-of-period input. In a related context, Gao (2013) considers raw material instead of labor.

where A_t follows an AR(1) process and ϵ_{it}^A follows a Markov process, with $E(A_t) = A$ and $\int_0^1 \epsilon_{it}^A di = 0$.

Entrepreneurs enter beginning-of-period t with initial wealth Ω_{it} and can borrow in illiquid debt D_{it} to pay for their consumption, their capital, and cash M_{it} . Debt D_{it} is illiquid in the sense that it can only be issued at beginning-of-period. We follow Jermann and Quadrini (2012) by assuming that firms benefit from a subsidy on debt, so the gross interest rate on debt is $r_t = \tau R_t$, with $0 < \tau < 1$, where R_t is the before-tax interest rate.²⁰ Cash bears no interest. The firms' beginning-of-period budget constraint is

$$\Omega_{it} + D_{it} = c_{it} + K_{it} + M_{it}. \quad (5)$$

The cash ratio m_{it} is defined as the proportion of cash to total assets, i.e., $m_{it} \equiv M_{it}/(K_{it} + M_{it})$. As D_{it} is never negative in equilibrium, it is never part of gross assets.²¹ Initial wealth is made of output, the remaining capital stock, and unused cash minus the gross interest rate payment on debt and the cost associated with external liquidity used in the previous subperiod

$$\Omega_{it} = Y_{it-1} + (1 - \delta)K_{it-1} + \widetilde{M}_{it-1} - r_{t-1}D_{it-1} - r_{t-1}^L L_{it-1}, \quad (6)$$

where \widetilde{M}_{it-1} is unused cash, L_{it-1} is external liquidity obtained in the previous end-of-period and $r_t^L \geq 1$ is the cost associated with it.

Liquidity shocks (defined below) affect the magnitude of external liquidity L_{it} available to firms. At end-of-period t , firms need to pay for wages, before production takes place, which is at the next beginning-of-period. Hence, they have to pay out of their cash or any liquid funds they obtain in that end-of-period. They face the following liquidity constraint

$$M_{it} + L_{it} \geq w_t l_{it}, \quad (7)$$

where w_t is the wage rate. Unused cash is simply defined as $\widetilde{M}_{it} = M_{it} - L_{it} - w_t l_{it}$. It will be equal to zero in equilibrium in all of our analysis. We assume that liquidity is constrained by lenders. Due to standard moral hazard arguments, a fraction $0 \leq \kappa_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used

²⁰This tax advantage of debt is also found in Hennessy and Whited (2005). It reflects the firms' preference for debt over equity (pecking order). In our model, this pecking order is represented by the fact that firms will have a tendency to consume (which corresponds to distributing dividends) and as a consequence they will be leveraged up to the maximum level.

²¹ D_{it} is non-negative because all firms are always constrained due to the debt subsidy and because we abstract from equity issuance. If some firms were unconstrained, they could choose a negative D_{it} , and thus hold both bonds and cash.

as collateral for debt repayments, i.e.,

$$r_t^L L_{it} \leq \kappa_{it}(1 - \delta)K_{it}. \quad (8)$$

We will assume that $r_t > r_t^L$, so that $r_t^L L_{it} = \kappa_{it}(1 - \delta)K_{it}$. Shocks to κ_{it} are therefore liquidity shocks, i.e., shocks that affect the amount of external liquidity.²²

The liquidity shock κ_{it} is assumed to be composed of an aggregate component and an idiosyncratic one

$$\kappa_{it} = \kappa_t + \epsilon_{it}^\kappa, \quad (9)$$

where κ_t follows an AR(1) process and ϵ_{it}^κ follows a Markov process, with $E(\kappa_t) = \kappa$ and $\int_0^1 \epsilon_{it}^\kappa di = 0$. In our baseline analysis, we simply assume that κ_{it} is known at beginning-of-period t , but we relax this assumption in the more general numerical analysis.

Finally, we assume that the entrepreneur faces a standard credit constraint at beginning-of-period t .²³ A fraction $0 \leq \phi_{it} \leq 1$ of the capital stock at the beginning-of-period has to be used as collateral for debt repayments

$$r_t D_{it} \leq \phi_{it}(1 - \delta)K_{it}. \quad (10)$$

In principle, the two constraints (8) and (10) could be related. However, we specify them independently as we will build κ_{it} directly from the data.

The parameter ϕ_{it} is composed of an aggregate component and a firm-specific one

$$\phi_{it} = \phi_t + \epsilon_i^\phi, \quad (11)$$

where ϕ_t follows an AR(1) process with $E(\phi_t) = \phi$ and $\int_0^1 \epsilon_i^\phi di = 0$.

In this paper, we make the distinction between a standard credit shock, ϕ_{it} , and a liquidity shock, κ_{it} . The former can be viewed as a standard disturbance on the banking sector since it affects the long-term credit. The latter corresponds to an exogenous change in the availability of external liquid funds, as for instance a variation in the supply of credit lines or a restriction in the ability to extend trade credit. We argue that distinguishing credit from liquidity shocks matters because the latter generates a negative correlation between employment and the cash ratio and it explains a notable share of output fluctuations.

²²External liquidity could also vary with the proportion of wages that have to be paid at end-of-period.

²³The presence of credit constraints at the beginning-of-period is not crucial to the main mechanisms we analyze, but it allows to study the impact of credit market shocks. Moreover, it is a convenient assumption with heterogeneous firms, as it puts a limit to the size of the most productive firms.

3.2 Optimal Cash Holding and Employment

Before closing the model, it is interesting to examine the relationship between cash and employment in partial equilibrium, that is, for given interest rates and wages. Entrepreneurs maximize (2) subject to (5), (7) and (10). The optimization of the entrepreneur is described in details in Appendix A. We assume that shocks are anticipated so the random variables A_{it} , κ_{it} and ϕ_{it} are known at beginning-of-period t . While this assumption is natural for ϕ_{it} since firms borrow long-term at beginning-of-period, it is less so for κ_{it} , as firms borrow short-term at end-of-period. However, this assumption is useful to derive analytical results. It will be relaxed in the quantitative analysis in Section 4.3, where κ_{it} will be learned at the end-of-period. As cash does not yield any interest, one can verify that (7) is always binding so that $\widetilde{M}_{it} = 0$.

It is convenient to express production as a function of the capital-labor ratio $k_{it} = K_{it}/l_{it}$. We have $F(K_{it}, A_{it}l_{it}) = A_{it}l_{it}f(k_{it}/A_{it})$ where $f(k) = F(k, 1)$. The optimality conditions with respect to l_{it} and K_{it} imply that the capital-labor ratio is described by (see Appendix A)

$$k_{it} = A_{it}\tilde{k}(\tilde{w}_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L), \quad (12)$$

where $\tilde{w}_{it} = w_t/A_{it}$. As shown in Appendix A, $\tilde{k}(\cdot)$ is increasing in \tilde{w}_{it} , ϕ_{it} and κ_{it} . Indeed, a lower wage makes production less intensive in capital as opposed to labor. Besides, as capital is the collateral, lower ϕ_{it} and κ_{it} reduce the collateral value of capital and thus have a negative effect on the capital-labor ratio. The effect of a reduction in TFP, A_{it} , is more ambiguous as it reduces both the marginal productivity of labor and capital, leading to a reduction in both inputs. In the Cobb-Douglas case where $F(K_{it}, A_{it}l_{it}) = K_{it}^\alpha (A_{it}l_{it})^{1-\alpha}$, however, we can show that overall, a lower productivity increases the capital-labor ratio when $\delta > 0$. In that case, a reduction in A_{it} affects the marginal productivity of labor relatively more than the return on capital, because it does not affect the remaining stock of capital.

The cash ratio, which is a key variable in our analysis because it reflects the cash-intensity of production, can be derived from the above results. Using (7), (12), and $r_t^L L_{it} = \kappa_{it}(1 - \delta)K_{it}$, we find

$$\frac{M_{it}}{K_{it}} = \frac{1}{k_{it}} [w_t - \kappa_{it}(1 - \delta)k_{it}/r_t^L] = \frac{w_t}{k_{it}} - \kappa_{it} \frac{(1 - \delta)}{r_t^L}. \quad (13)$$

The demand for cash per unit of capital is equal to the demand for cash per unit of labor, divided by the capital-labor ratio. The demand for cash per unit of labor

is itself simply equal to the liquidity need per unit of labor (w_t), minus external liquidity per unit of labor ($\kappa_{it}(1 - \delta)k_{it}/r_t^L$). A decrease in κ_{it} has two effects: a direct negative effect as it diminishes the access to external finance and an indirect negative collateral effect as the capital-labor ratio decreases. These two effects both increase the cash ratio. A decrease in ϕ_{it} also increases the cash ratio, but only through the negative collateral effect. In contrast, a decrease in A_{it} increases the capital-labor ratio and as a result it decreases the cash ratio. Equation (13) then implies that the cash ratio, which depends solely on M_{it}/K_{it} , comoves negatively with κ_{it} and positively with A_{it} .

To analyze labor demand, we will focus on cases where entrepreneurs are credit-constrained and have log utility. Appendix A shows that the credit constraint is binding whenever the wage paid by firms, w_t , is lower than the marginal return of labor, which boils down to a function of A_{it} , κ_{it} , ϕ_{it} and the interest rates: $w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$. Moreover, with log utility Appendix A shows that optimal consumption is $c_{it} = (1 - \beta)\Omega_{it}$. In that case, it is useful to rewrite the constraint (5) using (7), (10), and $L_{it} = \kappa_{it}(1 - \delta)K_{it}$. This gives

$$\beta\Omega_{it} + \frac{\phi_{it}(1 - \delta)K_{it}}{r_t} + \frac{\kappa_{it}(1 - \delta)K_{it}}{r_t^L} = K_{it} + w_t l_{it}. \quad (14)$$

Equation (14) gives the budget constraint aggregated over the two subperiods. Total financing of firms, on the left-hand side, pays for inputs, on the right-hand side. Both the long-term and short-term financing conditions, represented respectively by ϕ_{it} and κ_{it} , affect the capacity of firms to finance labor l_{it} and capital k_{it} . Using (14), the optimal behavior of entrepreneurs, for given interest rates and wages, is described in the following proposition.

Proposition 1 (Individual policy functions) *Suppose that $u(c_{it}) = \ln(c_{it})$. If $r_t > r_t^L > 1$, then there exists a function w^* such that, if $w_t < w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$, then the liquidity constraint (7) and the credit constraints (8) and (10) are binding, k_{it} is given by (12) and the policy functions for K_{it} , M_{it} , l_{it} , D_{it} , and Ω_{it+1} satisfy:*

$$l_{it} = Z_{it}\Omega_{it} \quad (15)$$

$$K_{it} = k_{it}Z_{it}\Omega_{it} \quad (16)$$

$$M_{it} = [w_t - \kappa_{it}(1 - \delta)k_{it}/r_t^L]Z_{it}\Omega_{it} \quad (17)$$

$$D_{it} = \phi_{it}(1 - \delta)k_{it}Z_{it}\Omega_{it}/r_t \quad (18)$$

$$\Omega_{it+1} = [(1 - \kappa_{it} - \phi_{it})(1 - \delta)k_{it} + A_{it}f(k_{it})]Z_{it}\Omega_{it} \quad (19)$$

where

$$Z_{it} = \frac{\beta}{[k_{it} + w_t] - (\kappa_{it}/r_t^L + \phi_{it}/r_t)(1 - \delta)k_{it}}. \quad (20)$$

Proof. See Appendix A. ■

We call Z_{it} the financial multiplier. It measures the impact of a change in wealth on labor demand. Notice that a decline in the financing conditions ϕ_{it} or κ_{it} implies a smaller Z_{it} , everything else equal. A worsening of financing conditions has thus a negative effect on inputs, including labor. However, it also decreases the capital-labor ratio as the collateral value of capital declines, which has a positive effect on labor. Under standard assumptions, the direct negative effect dominates, as shown in the following corollary.

Corollary 1 *Under the Cobb-Douglas production function, ceteris paribus, firms with lower financing conditions κ_{it} or ϕ_{it} have lower employment l_{it} and a higher cash ratio m_{it} . Moreover, a lower productivity A_{it} affects negatively employment l_{it} but has a negative effect on the cash ratio m_{it} .*

Proof. See Appendix A. ■

Corollary 1 illustrates the main mechanism in the model. An expected decrease in κ_{it} implies a smaller amount of available liquid funds at end-of-period t . As a response, firms naturally increase the proportion of cash in their portfolio, as seen in (13). At the same time, they reduce their labor demand and their production, as outside funding decreases. The same occurs with a decline in ϕ_{it} , but the increase in cash ratio is milder. This increase takes place as firms reduce their capital stock relative to labor and hence relative to their liquidity needs, because of the indirect collateral effect. On the opposite, with a decline in productivity A_{it} , firms increase their capital-labor ratio, which has a negative effect on their cash ratio, as their liquidity needs decline in proportion to capital. At the same time, labor declines. Note that these results hold under partial equilibrium, but give an important hint on the general equilibrium behavior of the economy.

3.3 Closing the Model

The model is closed by introducing identical households, deposit institutions and a government. Households supply labor, long-term debt and make short-term deposits at deposit institutions while deposit institutions supply short-term debt to firms. The government supplies money to firms and deposit institutions. Money is supplied hyperelastically so that its price is always equal to one. The wage rate w_t and the interest rates r_t , R_t and r_t^L are then determined endogenously.

Identical households have utility U_t with the discount factor β

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s [v(c_{t+s}^h, l_{t+s-1})], \quad (21)$$

where c_t^h and l_{t-1} are respectively the household's consumption and the labor supplied to firms at the beginning of period (to produce Y_{t-1}).

At the end of $t-1$, households receive wages $w_{t-1}l_{t-1}$ and deposit them at a deposit institution.²⁴ Deposit institutions lend L_{t-1} short term to entrepreneurs and hold the rest at the central bank, in quantity $M_{t-1} - \widetilde{M}_{t-1}$. The intermediation cost for a deposit institution is linear in total short-term lending, with a unit cost of $\psi > 0$. We assume that deposit institutions behave competitively so the return on households deposits is 1 and the cost of short-term lending is $r_{t-1}^L = 1 + \psi > 1$.

These deposits are then used by households at beginning-of-period t to consume and invest in long-term bonds D_t^h , along with the return on last period bonds $R_{t-1}D_{t-1}^h$.²⁵ Households also receive transfers T_t from the government. This yields the following household budget constraint

$$w_{t-1}l_{t-1} + R_{t-1}D_{t-1}^h + T_t = c_t^h + D_t^h. \quad (22)$$

We use GHH preferences which take the form

$$v(c^h, l) = \frac{\left(c^h - \bar{w} \frac{l^{1+1/\eta}}{1+1/\eta}\right)^{1-\sigma}}{1-\sigma}, \quad (23)$$

where $\eta > 0$ is the Frisch elasticity of labor supply, \bar{w} is a positive constant, and $1/\sigma > 0$ is the elasticity of intertemporal substitution. Households' optimization then implies that, in equilibrium (see Appendix B for details), bonds are priced via the Euler equation

$$E_t \left[\left(c_t^h - \bar{w} \frac{l_{t-1}^{1+1/\eta}}{1+1/\eta} \right)^{-\sigma} - \beta R_t \left(c_{t+1}^h - \bar{w} \frac{l_t^{1+1/\eta}}{1+1/\eta} \right)^{-\sigma} \right] = 0, \quad (24)$$

where $E_t(\cdot)$ is the expectation as of beginning-of-period t .

Additionally, households have a labor supply $l^s(w_t)$ that depends positively on the wage rate. In our specification, we have (see Appendix B)

$$l^s(w_t) = (w_t/\bar{w})^\eta. \quad (25)$$

²⁴We assume it is too costly for households to provide short-term loans to entrepreneurs.

²⁵Households do not hold money at beginning-of-period because it is strictly dominated by long-term bonds as a saving instrument and there is no specific liquidity service of money.

The wage rate is then determined endogenously so that $l^s(w_t) = \int_0^1 l_{it} di$, where l_{it} is labor demand by firm i in period t . According to Proposition 1, $l_{it} = l(w_t, r_t, r_t^L, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it})$, so the equilibrium wage is defined by

$$l^s(w_t) = \int_0^1 l(w_t, r_t, r_t^L, A_{it}, \kappa_{it}, \phi_{it}, \Omega_{it}) di, \quad (26)$$

We assume, without loss of generality, that only firms issue long-term and short-term bonds, so that $D^h = D$. By contrast, only the government can issue money M_t to meet the demands of firms and depositors. The government has a budget constraint at beginning-of-period t

$$T_t + R_{t-1}(1 - \tau)D_{t-1} = M_t - M_{t-1}. \quad (27)$$

Finally, the consolidated household's and government's budget constraints, using $D^h = D$ and $\tau R = r$, yield

$$w_{t-1}l_{t-1} + r_{t-1}D_{t-1}^h + M_{t-1} = c_t^h + D_t^h + M_t. \quad (28)$$

3.4 Equilibrium in the Baseline Case

We first consider a tractable baseline case. This is the case when the interest rate is constant, which occurs when $\sigma = 0$ so that households have an infinite elasticity of intertemporal substitution. In this case, $R_t = 1/\beta$ so $r_t = \tau/\beta$. We therefore let the wage adjust, while the interest rate is constant. We will relax this assumption when we turn to the fully calibrated version of the model. We still assume that the random variables A_{it} , κ_{it} and ϕ_{it} are known at beginning-of-period t . We consider separately aggregate and idiosyncratic shocks.

3.4.1 Aggregate Shocks

In the absence of idiosyncratic shocks, the only potential source of heterogeneity between firms is their wealth. Since labor demand is linear in wealth, we can then write $l_t = \int_0^1 l(w_t, r_t, r_t^L, A_t, \kappa_t, \phi_t, \Omega_{it}) di = l(w_t, r_t, r_t^L, A_t, \kappa_t, \phi_t, \Omega_t)$ where $\Omega_t = \int_0^1 \Omega_{it} di$. This holds for K_t , M_t and D_t as well. We consider a constrained equilibrium defined as follows:

Definition 1 (Constrained equilibrium under aggregate shocks only) *For a given aggregate wealth Ω_t and a given realization of A_t , κ_t and ϕ_t , a constrained period- t equilibrium is a level of employment l_t , of capital K_t , of cash M_t , of debt D_t , of financial multiplier Z_t and of future wealth Ω_{t+1} satisfying Equations (15) to (20),*

where $r_t = \tau/\beta$, $r_t^L = 1 + \psi$, the wage w_t clears the labor market so that $l^s(w_t) = l_t$ where $l^s(w_t)$ satisfies the labor supply equation (25) and k_t is the corresponding capital-labor ratio given by Equation (12). Finally, the equilibrium wage w_t must be strictly lower than $w^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$.

Since aggregate labor demand depends on A_t , κ_t , ϕ_t , r_t , r_t^L and Ω_t , the equilibrium wage also depends on those variables: $w_t = w(A_t, \kappa_t, \phi_t, r_t, r_t^L, \Omega_t)$. For an individual firm, we saw that the credit constraint is binding whenever $w_t < w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$. At the aggregate level, we can show that there exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$ so that $w_t < w^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$ is equivalent to $\Omega_t < \Omega^*$. When the wage is low, firms want to use all their resources to produce. However, because firms' resources are limited by the credit constraints, the aggregate labor demand is low when the aggregate wealth is low, which maintains the equilibrium wage at a low level and firms are constrained in equilibrium. It is shown in Appendix A that for the steady state to be constrained we need $\tau < 1$. Individual agents and the aggregate economy will fluctuate around a constrained steady state. Therefore, for small enough shocks, the economy will be constrained, as assumed in our analysis. Intuitively, on the one hand, a wage that is lower than the marginal productivity of labor makes the credit constraint binding, as stated in Proposition 1. On the other hand, the credit constraint makes the equilibrium wage dependent on aggregate wealth. When $\tau < 1$, the net interest rate $r_t - 1$ is below the propensity to consume out of wealth $1/\beta - 1$, so firms never accumulate sufficient wealth to be able to provide an equilibrium wage equal to marginal productivity.²⁶

3.4.2 Idiosyncratic Shocks

In the absence of aggregate shocks, the constrained equilibrium is defined as follows:

Definition 2 (Constrained equilibrium under idiosyncratic shocks only) *For a given period- t distribution of wealth, productivity and liquidity $\{\Omega_{it}, A_{it}, \kappa_{it}\}_{i \in [0,1]}$, a constrained period- t equilibrium is given by the firm-specific levels of employment l_{it} , of capital K_{it} , of cash M_{it} , of debt D_{it} , of Z_{it} and of future wealth Ω_{it+1} satisfying Equations (15) to (20), where $r_t = \tau/\beta$, $r_t^L = 1 + \psi$, the wage w_t clears the labor market such that $l^s(w_t) = \int_0^1 l_{it} di$ is satisfied with $l^s(w_t)$ following the labor supply equation (25) and k_{it} is the corresponding capital-labor ratio given by Equation (12). Finally, the equilibrium wage must satisfy $w_t < w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$ for all $i \in [0, 1]$.*

²⁶We also need $\psi < \tau/\beta - 1$, to guarantee that $r > r^L$ in the steady state and hence that the short-term credit constraint is binding as well. Note that $r = r^L$ would still yield a binding short-term credit constraint as long as the long-term credit constraint is binding.

In our simulation exercise in Section 5, we will check ex post that we do have $w_t < w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$ for all i .

4 Macroeconomic Effects of Liquidity Shocks

In this section, we focus on the effects of aggregate shocks, assuming that all entrepreneurs are identical. The assessment of idiosyncratic shocks is addressed in Section 5. First, we parametrize the model and we analyze the dynamic impact of productivity and financial shocks in the baseline model. We then relax several assumptions in the model and we assess quantitatively the importance of these shocks over the business cycle. We derive the series of shocks that are consistent with the theoretical model and examine their properties. We then analyze the robustness to various extensions, namely a more general production function, rigid wages, a separable utility function, or adjustment costs. Finally, we examine the impact of liquidity uncertainty.

4.1 Parametrization

Table 4 details the model’s parametrization.

In the baseline model, we set $\sigma = 0$ such that households have an infinite elasticity of intertemporal substitution. In that case, both interest rates are constant over time. We relax this assumption in Section 4.3 by introducing time-varying long-term interest rate with $\sigma = 1$. As standard in the literature, we set the share of capital in production, α , to 0.36, the Frisch parameter, η , is set to unity and we assume an annual capital depreciation rate of 10% ($\delta = 0.025$). The firms’ discount factor is set to $\beta = 0.975$ which – combined with a subsidy on net interest debt payments, τ , to 40% – generates an effective steady-state annual gross interest rate of 6.30%.²⁷ ψ is set to target an annual steady state short-term interest rate of 1.2% ($r^L = 1.003$) so that the cost of using liquidity, r_t^L , is lower than the gross interest rate.

The liquidity parameter κ and the credit parameter ϕ are calibrated in order to match two empirical targets, using aggregate data. Precisely, the model has to replicate the mean of the cash ratio and the debt to output ratio over the sample, i.e., 3.4% and 53%, respectively.²⁸ It follows that steady-state level of financial

²⁷The value of β is somewhat low, but it ensures that firms are always constrained. If we set $\beta = 0.985$ as in Jermann and Quadrini (2012) firms are still constrained in 90 percent of the cases. Notice that the calibrated value of τ is in the same order than the average corporate tax rate in the US in 2015 (see OECD.stat database).

²⁸As in Section 2, the cash ratio is defined as the share of liquidity to total assets from the non-financial corporate business sector. The debt to output ratio is measured by the ratio between

Table 4. Parametrization of the Model

		Value
σ^{-1}	Elasticity of intertemporal substitution	1.00
η	Frisch parameter	1.00
α	Elasticity of output wrt capital	0.30
δ	Capital depreciation rate	0.025
β	Firms discount factor	0.975
τ	Tax advantage	0.40
r	Gross long-term interest rate	1.016
r^L	Liquidity cost	1.003
ϕ	Collateral share for debt	0.09
κ	Collateral share for liquidity	0.075
κ_i	Firm-specific collateral share for liquidity	[0.01; 0.091]
A	Productivity shock	1.00
A_i	Firm-specific productivity shock	[0.94; 1.07]

shocks are set to $\kappa = 0.075$ and $\phi = 0.09$. Finally, we normalize A to unity.

4.2 Dynamic Impact of Aggregate Shocks in the Baseline Model

We start by illustrating the link between employment and the cash ratio in the baseline model described in Section 3.4. We examine the impulse response functions (IRFs) to a reduction in liquidity (κ_t), credit (ϕ_t), and technology (A_t) from their steady-state level. Each shock is normalized to generate a one percent reduction of output from its steady state.²⁹ For the sake of comparison, the autoregressive parameters for the technology, credit and liquidity shocks are all set arbitrarily to 0.8.³⁰ The IRFs are computed by determining the equilibrium wage, w_t , that clears

credit market instruments (liabilities) from the non-financial corporate business sector and the gross value added in the business sector. Data sources are available in the online appendix.

²⁹In practice, the TFP shock has to be smaller ($\sigma_A = 0.05$) than financial shocks ($\sigma_\phi = \sigma_\kappa = 0.9$). This normalization makes the impulse responses comparable. In Section 4.3, we proceed to a formal analysis of shocks' contribution to the business cycle.

³⁰In Section 4.3.2, we identify model-based shocks series using the data and we show that the credit shock is more persistent than the liquidity shock. For the analysis of IRFs, we have chosen to make the two shocks equivalently persistent in order to highlight the transmission mechanisms

the labor market and using in turn the policy functions (15) to (19).³¹ Figure 3 displays the IRFs of a set of variables in percentage deviation from the steady state. The marker-solid, solid and dashed lines correspond to a response to κ_t , ϕ_t and A_t , respectively.

The upper panel in Figure 3 displays the responses of the cash ratio and employment. While the response of labor is identical and negative for the three shocks, the cash ratio reacts strongly to the liquidity shock. Both credit and TFP shocks have barely any impact on the cash ratio. These results coincide with the main mechanism discussed through Corollary 1. Considering a negative liquidity shock, i.e., a decline in κ_t , firms have smaller external liquid funds to pay for wage bills. The cash ratio m_t rises through two channels. First, there is a direct effect as firms need to compensate for the reduced access to external liquidity by relying more on internal liquidity. Second, there is an indirect collateral effect, since the collateral value of capital is reduced relative to labor, which reduces the value of assets relative to liquidity needs. Altogether, these two channels drive the cash ratio in the same upward direction. In the case of a negative credit shock, only the collateral motive plays a role on the cash ratio which slightly increases. The reason of this modest increase is that the credit shock does not *directly* affect the structure of the portfolio between internal and external liquidity. Additionally, a reduction in financial opportunities (i.e., shortage in external liquidity and credit) lowers labor demand through the financial multiplier. Therefore, employment l_t declines. When it comes to a negative technology shock, the comovement between employment and the cash ratio is different. As explained above, a decline in productivity A_t rises the capital-labor ratio which increases in turn the scale of assets as compared to liquidity needs and generates a subtle reduction in the cash ratio. The other effect, more standard, is to decrease employment through a tighter financial multiplier.

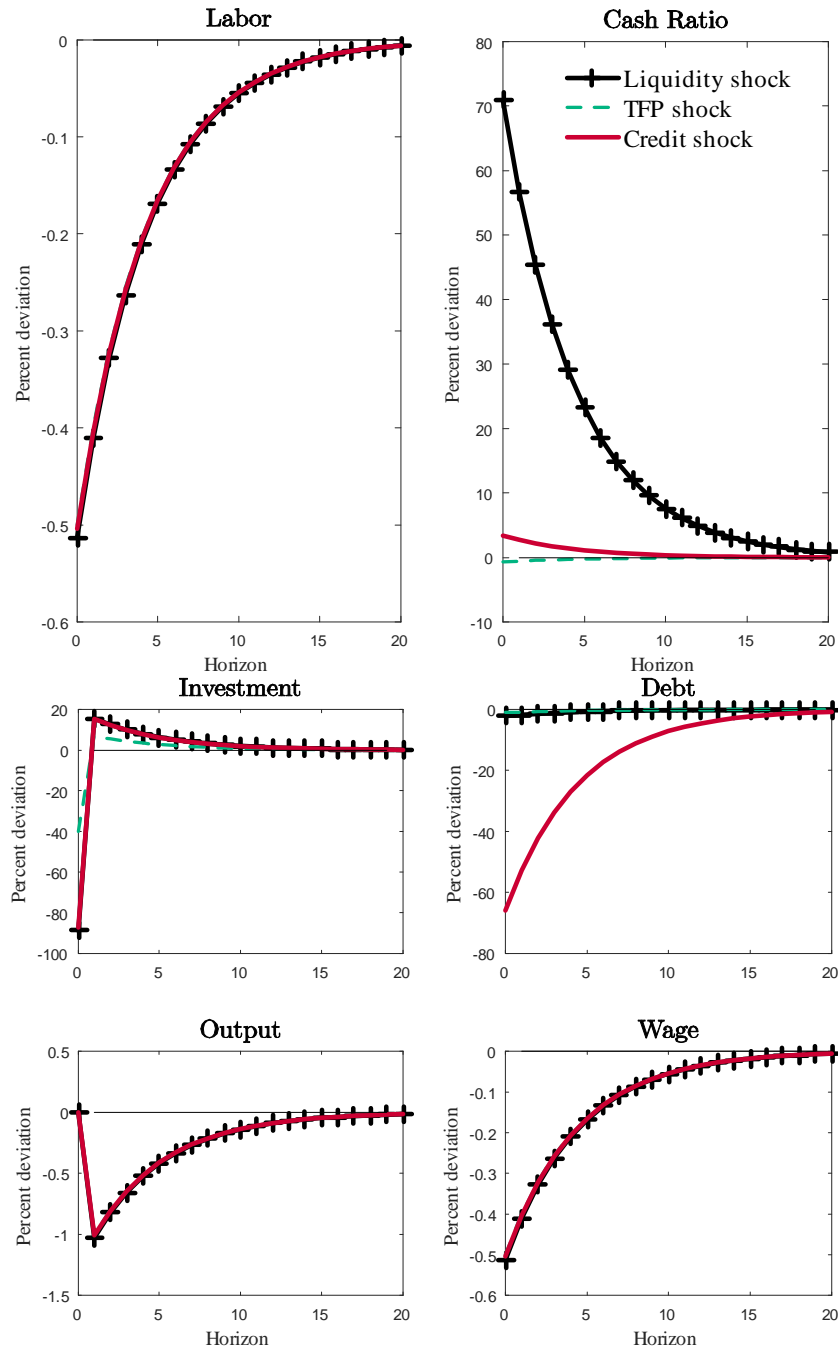
The lower panel in Figure 3 shows the remaining IRFs. The three recessionary shocks generate a decline in wages and therefore a reduction in liquidity needs. Debt responds mostly to the credit shock although it evolves in the same pattern as output in all experiments, which is in line with Covas and den Haan (2012) and Jermann and Quadrini (2012) who stress that debt is procyclical. Similarly, investment is initially depressed in response to the three shocks due to the reduction in external financing, but later rebounds to reconstitute the capital stock.

This analysis stresses the transmission channels through which liquidity shocks generate a negative relationship between employment and the cash ratio in the baseline model. For convenience we assumed that liquidity shocks are known at the beginning-of-period. This implies that these shocks have the same impact on inputs

of the shocks irrespective of their intrinsic properties.

³¹We check that we do have $w_t < w_t^*$ every period.

Figure 3: Impulse Response Functions to liquidity, TFP and credit shocks in the baseline model.



Note: The solid lines with markers correspond to the IRFs to an external liquidity shock (κ). The dashed lines correspond to the IRFs to a TFP shock (A). The solid lines correspond to the IRFs to a credit shock (ϕ). The size of each shock is normalized to generate a 1% deviation of output from its steady-state value.

as credit shocks. From equation (14), it can be seen that κ_t and ϕ_t affect in the same way the consolidated budget constraint of the beginning and end-of-period. The inclusion of the two subperiods is a convenient manner to introduce cash in our model. However, since firms borrow short-term at the end-of-period, it seems more natural to assume that liquidity shocks are not observed at the beginning of the period. To understand the specific impact of liquidity shocks on the economy, beyond its link with cash, we therefore assume that κ_t is known only at the end-of-period, that is when liquidity needs arise. We also enrich the model with standard preferences in order to assess quantitatively the role of liquidity shocks in the cycle.

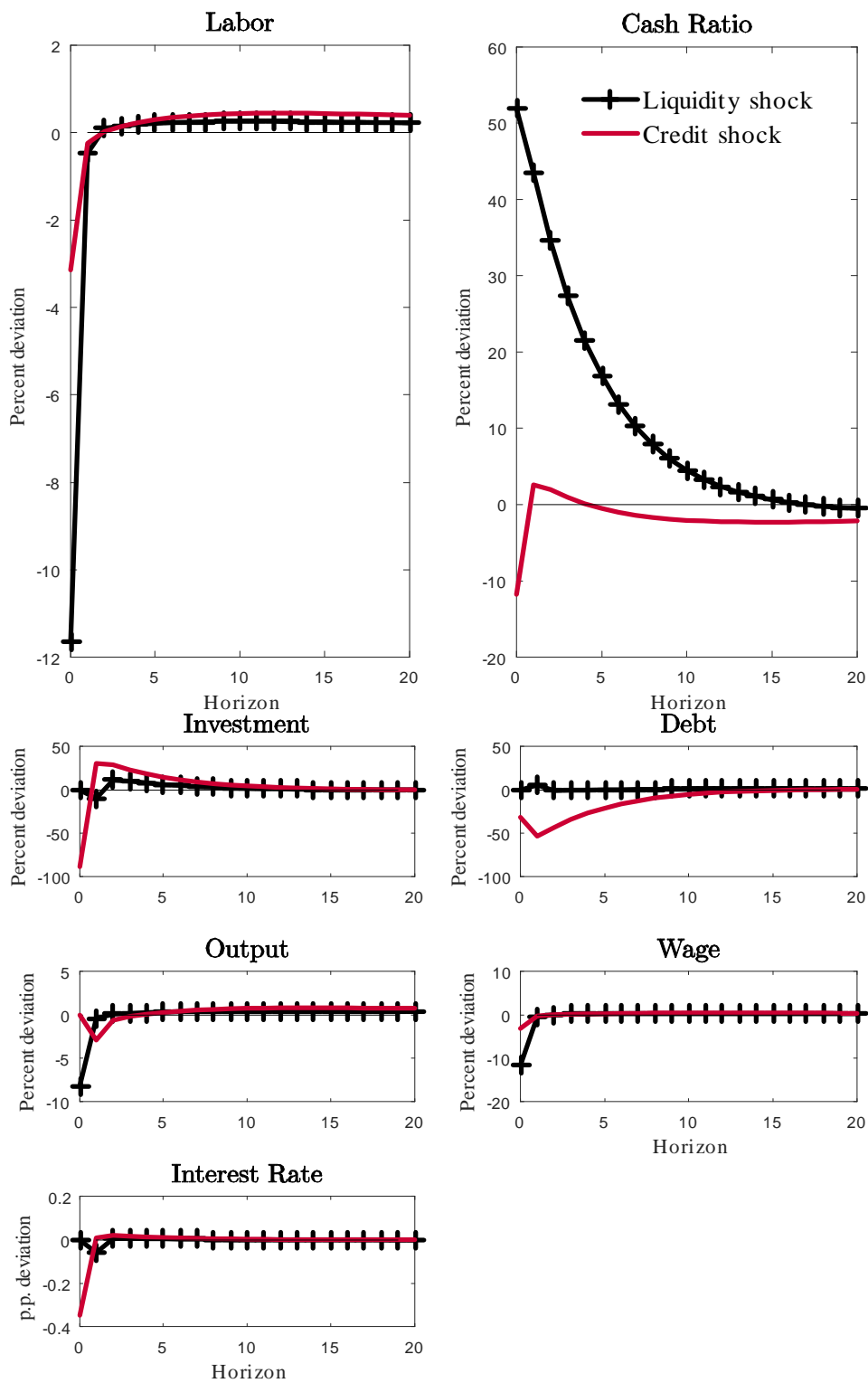
4.3 A More General Quantitative Analysis

In the baseline model described in Section 3.4, the utility of households is linear in consumption ($\sigma = 0$) implying that the long- and short-run interest rates are constant over time. This assumption has the advantage to make the model tractable. We now relax this assumption by allowing R_t and r_t to adjust endogenously. To do so, σ is set to 1, which corresponds to a unitary elasticity of intertemporal substitution for households (log-utility). As a consequence, the model is fully solved numerically. As explained above, we also assume that κ_t is revealed only at the end of period t , when short-term debt is decided. This assumption allows us to explicitly differentiate the two types of debt since credit can be seen as inter-temporal debt while liquidity can be seen as intra-temporal debt. In this section, we first illustrate the extended model's properties by looking at the IRFs of the two financial shocks. We then construct shocks series from the extended model and we investigate whether it fits with some standard features of the data. Finally, we focus on additional model's extensions to check the robustness of our results.

4.3.1 Impulse Response Functions in the Extended Model

Figure 4 illustrates the effect of negative financial shocks on the economy using the same identical persistence parameters and shock values as for the baseline model. The IRFs now show different responses of labor and investment. The response of labor is much stronger for the liquidity shock than for the credit shock, because the former shock is known only at the end-of-period, when investment decisions have already been made and labor is the only remaining variable of adjustment. When hit by a credit shock, firms adjust both investment and labor, so investment responds negatively on impact, while the response of labor is milder. By allocating the resource cut to both labor and investment, the firm can limit the impact of a credit shock on output as compared to an equally-sized liquidity shock.

Figure 4: Impulse Response Functions to liquidity, TFP and credit shocks in the extended model.



Note: The solid lines with markers correspond to the IRFs to an external liquidity shock (κ). The solid lines correspond to the IRFs to a credit shock (ϕ). The size of each shock is identical to the one calibrated in Figure 3.

Now that the interest rate is endogenous, both financial shocks lead to a decrease in the interest rate, as they are both recessionary. This has two notable consequences. First, after a negative credit shock, labor demand drops more than in the baseline case, as the drop in interest rate increases the value of collateral, and makes firms more prone to cut on labor than on investment. This affects wages negatively and limits the liquidity needs, making the cash ratio respond negatively. The liquidity shock is therefore the only shock that drives a negative correlation between labor and the cash ratio. Second, debt slightly increases after a liquidity shock, as the credit constraint is alleviated by the decrease in interest rate.

4.3.2 Model-based Shocks

The theoretical framework is used to construct the three series we are interested in, namely, TFP (A_t), liquidity (κ_t) and credit (ϕ_t). Let \hat{x}_t denote the log-deviation of the variable x_t from its deterministic trend, corresponding to HP-filtered empirical data (detailed below). We assume that the liquidity, short-term and long-term credit constraints are always binding, and check ex-post that it is indeed the case for the estimated shocks series.

For technology, we consider the Cobb-Douglas production function in loglinear terms

$$\hat{A}_t = \left[\frac{1}{1-\alpha} \right] \hat{Y}_t - \hat{l}_t - \left[\frac{\alpha}{1-\alpha} \right] \hat{K}_t. \quad (29)$$

For the credit series, we use the loglinearized version of the credit constraint, given by Equation (10)

$$\hat{\phi}_t = \hat{r}_t + \hat{D}_t - \hat{K}_t. \quad (30)$$

Finally, the liquidity series is constructed using the liquidity and short-term credit constraints (see Equations (7) and (8))

$$\hat{\kappa}_t = \left[\frac{wl/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} (\hat{w}_t + \hat{l}_t) - \left[\frac{M/Y}{(1-\delta)K/Y} \right] \frac{1}{\kappa} \hat{M}_t - \hat{K}_t. \quad (31)$$

All the parameters are taken from the parametrization and the model's steady-state. We use empirical data of output (\hat{Y}_t), measured as the gross value added in the business sector from NIPA. The wage bill ($\hat{w}_t + \hat{l}_t$) is measured as the hourly compensation index multiplied by hours worked in the nonfarm business sector from BLS. Debt series (\hat{D}_t) is measured by credit market instruments (liabilities) from the non-financial corporate business sector from Flow of Funds. The long-term interest rate, \hat{r}_t , is measured by the 10-year treasury constant maturity rate. Capital (\hat{K}_t) is measured using total capital expenditures and consumption of fixed capital of non-financial corporate business sector from Flow of Funds, as in Jermann and

Quadrini (2012). Output, \hat{Y}_t , is measured as the Gross Value Added of the business sector from NIPA. Cash is defined as the sum of private foreign deposits, checkable deposits and currency, total time and savings deposits and money market mutual fund shares from Flow of Funds. Employment is from BLS. All the nominal series are deflated by the price index for gross value added in the business sector from NIPA.³²

Figure 5 plots the series of TFP, credit and liquidity, constructed from Equations (29)-(31). Over the sample 1980q1 to 2015q3, the liquidity series features less persistence than the credit series and those two are more volatile than productivity.³³ Regarding the recent period, the economy experienced a reduction in $\hat{\kappa}_t$, below its trends, which can be viewed as a shortage in external liquidity supply. It peaks at the end of 2008, in the midst of the banking liquidity crisis. It is now a well-known fact that firms drew down on their lines of credits during that period, while banks started restricting new commitments.³⁴ As a result, firms' effective access to liquidity only started to fall at the beginning of 2009, which is consistent with our liquidity measure. This negative liquidity shock was accompanied by a reduction in $\hat{\phi}_t$, interpreted as a negative credit shock. Our model predicts that the Great Recession was mostly driven by financial shocks, i.e., liquidity and credit shocks, rather than a technology shock. This latter result is in line with Jermann and Quadrini (2012) who construct a generic financial shock. Notice also that liquidity drops briefly in 2001, during the dot-com bubble. The declines of 1998 and 2006 in our liquidity measure do not, however, seem related to any recession and probably reflect noise in our measure. Indeed, this is a coarse measure based on a stylized model, which does not account for all potential drivers of cash.

We feed the model with the productivity and financial shocks identified above in order to recover the historical path of key macroeconomic variables and to assess the importance of financial shocks to the business cycle.³⁵ Figure 6 displays the empirical series of output, employment, wages and short-term loans and their counterpart model-based series.³⁶ To complement the analysis, Table 5 provides the

³²Details on data sources are provided in the online appendix. All data are at a quarterly frequency.

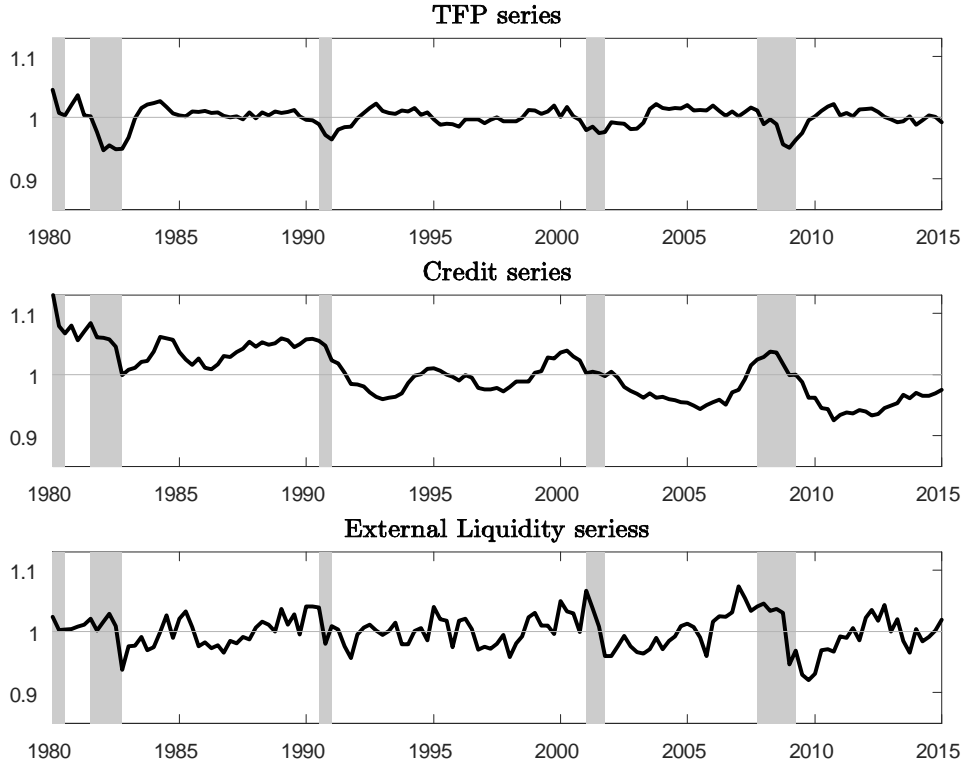
³³The autoregressive parameters of \hat{A}_t , $\hat{\phi}_t$ and $\hat{\kappa}_t$ are respectively 0.78, 0.92 and 0.55 and their variances are 0.03, 0.08 and 0.06.

³⁴See Ivashina and Scharfstein (2010), Cornett et al. (2011) and Gilchrist and Zakrajšek (2012).

³⁵We estimate a SVAR(1) model on $\hat{\kappa}_t$, $\hat{\phi}_t$ and \hat{A}_t using the Choleski decomposition to orthogonalize the three shocks. The SVAR model is then incorporated into the theoretical model, feeding the orthogonalized residuals into the model. The orthogonalization of shocks allows us to include shocks one by one in order to understand how they affect the historical path of the variables.

³⁶The Survey of Terms of Business Lending provides measures of banking loans to business firms for different maturities. We can therefore extract the short-term loans in the data which are defined as the total amount of loans provided by domestic banks of a maturity of less than one year. Data are available from 1997q2 to 2015q3 and are HP-filtered.

Figure 5: Model-based Shocks to Technology, Credit and Liquidity.



Note: Employment is expressed in logarithm and both variables are HP-filtered.

correlation between the simulated and the observed series when all structural shocks are included into the model (as in Figure 6) and when only one shock remains.

Table 5: Correlation between Empirical and Model-based Series

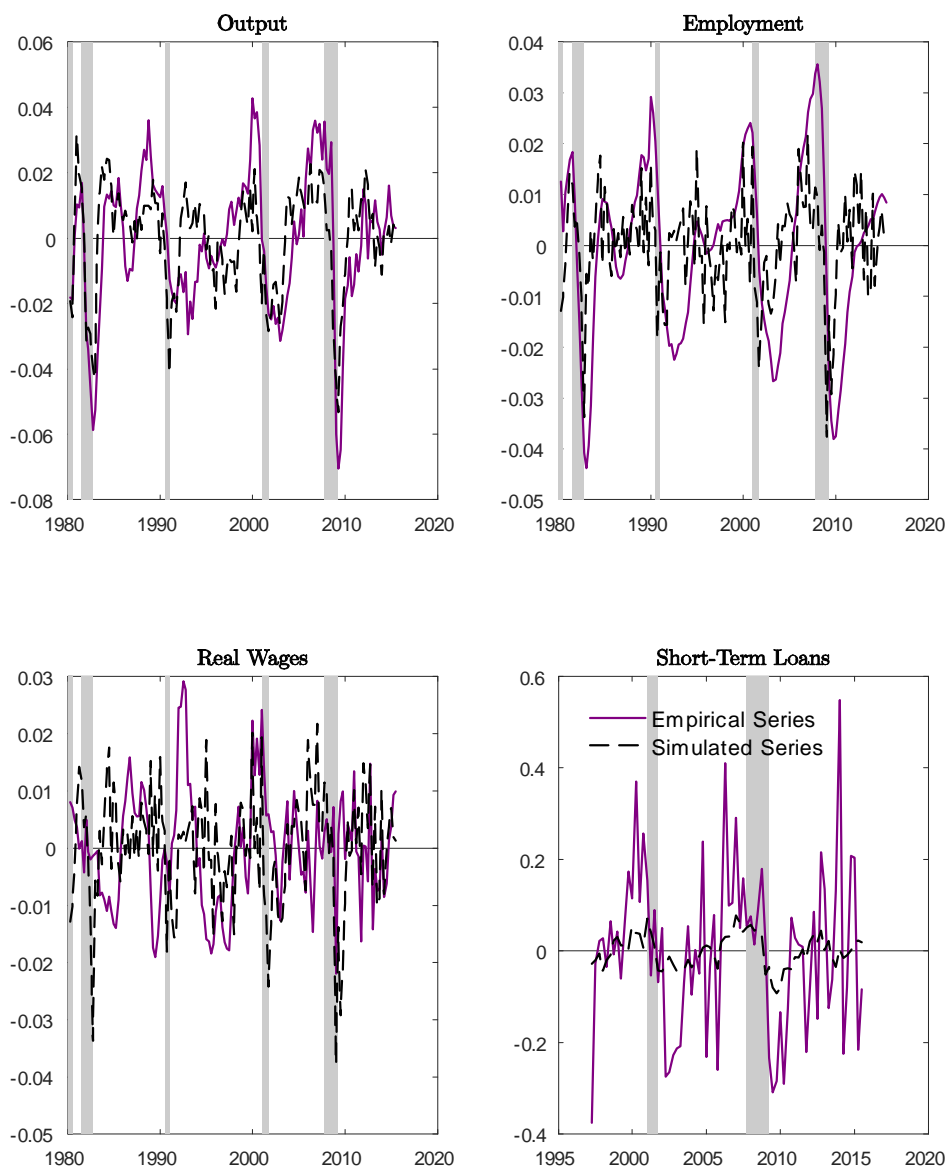
	All shocks	$\hat{\kappa}_t$ only	$\hat{\phi}_t$ only	\hat{A}_t only
Output	0.65**	0.30**	0.24**	0.54**
Employment	0.42**	0.21**	0.20**	0.37**
Real Wages	0.14*	0.16**	-0.20*	0.13
Short-term Loans	0.60**	0.48**	0.31**	0.42**
Cash Ratio	0.70**	0.80**	0.14*	0.17*

Notes: A */** next to the coefficient indicates significance at the 10/5 percent level

As shown in Figure 6, simulated output fits quite well the empirical series, with a correlation of 0.65. In particular, the model is able to generate a recession of a strong magnitude in 2008. Table 5 shows that the TFP shock is a key element to explain the historical path of output since the correlation between the two series is 0.54 if the TFP shock only is included.³⁷ Figure 6 shows that the model is not able to

³⁷The correlation is 0.36 if both financial shocks are included (not shown).

Figure 6: Simulated and Empirical Macroeconomic Variables.



Note: Output and wages are expressed in real terms, deflated by the price index for gross value added in the business sector. All series are expressed in log and HP-filtered.

capture the “jobless recovery” observed in the data after the 90s’ crisis. Apart from that, simulated employment fits well its empirical counterpart with a correlation of 0.42. The model fails to generate realistic movements of the real wage since the correlation is significant only at 10%. Despite its simplicity, our model is able to generate a series of external liquid funds that fits well the historical evolution of our measure short-term loans. The correlation of 0.60 is striking given that short-term loans are not used in the construction of our shocks. Unsurprisingly, the liquidity shock is more appropriate than the credit shock to generate this correlation. This result is even more notable when we look at the cash ratio since the correlation between the two series is 0.80 when the liquidity shock only is included while the correlation is 0.14 (0.17, resp.) when the credit (TFP, resp.) shock only is included.

To complete this analysis, we can compute the correlation between two simulated series, namely employment and cash ratio. It amounts to -0.47 which matches surprisingly well the data (-0.43). This result confirms that the extended model captures the main stylized fact documented in Section 2.

4.3.3 Variance Decomposition

Now that we have shown that the extended model does a satisfactory job in replicating empirical data, we investigate by how much each shock contributes to a set of variables’ volatility. Table 6 shows the variance decomposition of key variables in the theoretical model.

Table 6: Shocks’ Contribution to Volatility

	$\hat{\kappa}_t$	$\hat{\phi}_t$	\hat{A}_t
Output	0.27	0.13	0.60
Employment	0.60	0.09	0.31
Cash ratio	0.40	0.12	0.48
Short-term Loans	0.73	0.05	0.22

Financial shocks explain 40% of output volatility while the contribution reaches 69% for employment. This result coincides with Jermann and Quadrini (2012) who emphasize that financial shocks are key drivers of macroeconomic fluctuations in a large-scale estimated model. We enrich this finding by arguing that liquidity shocks not only explain a negative correlation between employment and cash ratio, but are also an important source of fluctuations. Unsurprisingly, they explain most the variability of the short-term loans in our model (73%). Moreover, it is worth noticing that 27% of output volatility, 60% of employment volatility and 40% of cash ratio volatility are explained by liquidity shocks. This can be explained by the high

estimated volatility of liquidity shocks and to the model, which features a relatively higher impact of liquidity shocks on the cash ratio, output and employment, as illustrated in Figure 4.

4.4 Sensitivity Analysis

In this section, we assess the robustness of our results to several extensions. We modify the extended model presented above in several directions by assuming (1) real wage rigidities, (2) separable preferences for households, (3) a CES production function and, (4) capital adjustment costs. Table 7 compares key theoretical moments with their empirical counterparts. We discuss each extension one by one.³⁸

Table 7: Second-order Moments Comparison

	Data	Benchmark (1)	RWR (2)	Sep. Pref. (3)	CES (1)	K Adj. Cost (4)
σ_Y	0.02	0.02	0.02	0.02	0.02	0.02
σ_ℓ/σ_Y	0.95	0.62	0.74	0.62	0.77	0.72
σ_I/σ_Y	4.09	4.83	4.64	5.12	4.36	2.37
σ_w/σ_Y	0.60	0.62	0.46	0.62	0.77	0.71
σ_L/σ_Y	9.93	1.99	1.81	2.01	2.06	1.68
$corr(\hat{\ell}_t, \hat{Y}_t)$	0.79	0.57	0.46	0.57	0.50	0.72
$corr(I_t, \hat{Y}_t)$	0.84	0.88	0.90	0.91	0.63	0.89
$corr(\hat{\ell}_t, \widehat{CR}_t)$	-0.43	-0.47	-0.32	-0.49	-0.79	-0.14

4.4.1 Real Wage Rigidities

Liquidity needs, which are central to our analysis, depend on real wage dynamics. In a perfectly competitive environment, wages have a mitigating effect, as they decrease following a negative shock. By introducing real wage rigidities, we affect this liquidity need channel. We follow Blanchard and Galí (2007) by assuming a partial adjustment of the wage: $\hat{w}_t = \zeta \hat{w}_{t-1} + (1 - \zeta) \widehat{mrs}_t$ where \widehat{mrs}_t is the marginal rate of substitution between consumption and leisure. We set the degree of wage rigidity ζ to 0.5 as suggested by Blanchard and Galí (2010). As expected, wages are less volatile which implies that liquidity needs are more sluggish. Lower

³⁸Figure 4 in the online appendix plots the variance decomposition of output, employment and cash ratio for each model's extension. We show that our results are robust. In particular, liquidity shocks about 30% of output fluctuation irrespective of the setup we consider.

variation in wages is combined with more volatile employment which matches the data better. The correlation between employment and cash ratio is lower than in the extended model (-0.32) because real wages decrease by less in response to liquidity shocks, generating eventually a rise in labor demand.

4.4.2 Separable Utility Function

Another way to evaluate the importance of the model’s working capital channel is to modify households’ preferences. In the extended model, we assume GHH utility function: the reduction in consumption following a recessionary shock is not accompanied with a rise in labor supply, i.e., the wealth effect on labor supply is absent. Therefore, labor supply is only driven by the substitution effect between consumption and leisure implying that consumption and labor go in the same direction. Alternatively, we could assume that households have a separable utility function such that the utility function (23) is replaced by $v(c^h, l) = \log(c^h) - \bar{w} \frac{l^{1+1/\eta}}{1+1/\eta}$. We find that the reduction in employment following a negative shock is partially compensated by the negative wealth effect that boosts labor supply. However, the simulated moments are not sensitive to this specification as shown in Table 7.

4.4.3 CES Production Function

We allow the production function to follow a more general CES specification with $F(K_t, A_t l_t) = \left[\alpha_k K_t^{\frac{\varpi-1}{\varpi}} + \alpha_l (A_t l_t)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}}$, where ϖ is the elasticity of substitution between capital and labor and α_k and α_l are distribution parameters. Notice that the CES production function nests the benchmark Cobb-Douglas case when $\varpi = 1$ and $\alpha_k + \alpha_l = 1$. We set $\varpi = 0.8$.³⁹ Parameters α_k and α_l are “re-parameterized” following the strategy suggested by Cantore and Levine (2012) such that $\alpha_k = \alpha(Y/K)^{\frac{\varpi-1}{\varpi}} = 0.61$ and $\alpha_l = (1 - \alpha)(Y/l)^{\frac{\varpi-1}{\varpi}} = 0.64$. Table 7 shows that the volatility of all variables is hardly changed by the modification of the production function. The most notable difference is a higher correlation between employment and cash ratio (-0.79). The reason for a more negative correlation is the increased contribution of liquidity shocks. With lower substitutability between labor and capital, liquidity shocks play a larger role in explaining employment fluctuations.

³⁹There is no consensus in the literature regarding the parametrization of elasticity ϖ . León-Ledesma et al. (2010) emphasize the identification issues resulting from the estimation of this parameter. Klump et al. (2012) provide a survey where it can be shown that the estimated values vary between 0.5 and 0.9 in the literature.

4.4.4 Capital Adjustment Costs

As standard in the literature, we finally add capital adjustment costs. The law of motion for capital becomes $K_t = (1 - \delta)K_{t-1} + I_t - \frac{\Phi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}$, where $\Phi \geq 0$ governs the size of the adjustment costs. We calibrate this parameter to $\Phi = 15$. Table 7 shows that investment is slightly less volatile than in the benchmark model while the correlation between employment and cash ratio is lower than in the data (-0.14). The correlation is less negative since liquidity shocks have a lower contribution to fluctuations: with capital adjustment, employment reacts more to productivity and credit shocks.

4.5 Liquidity Uncertainty

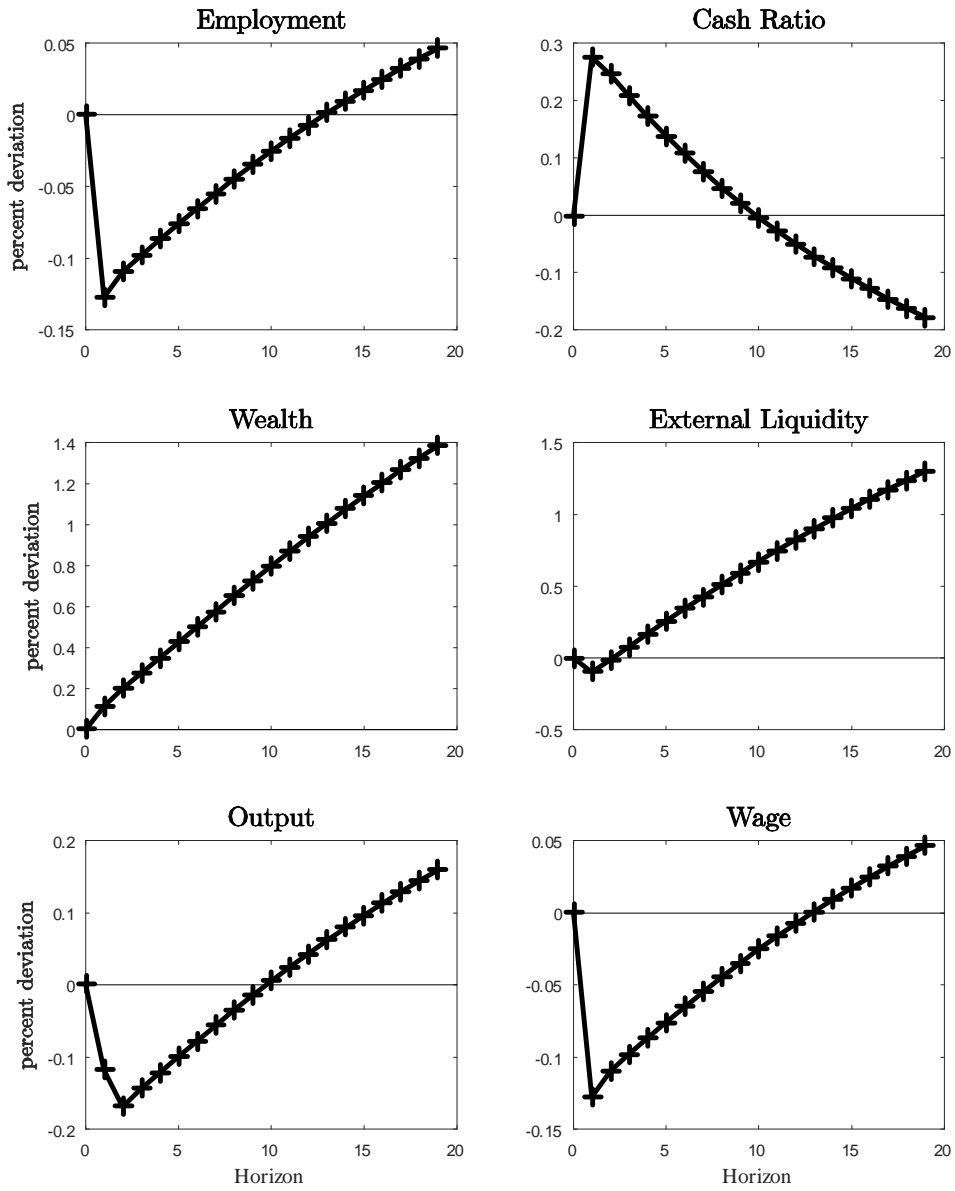
Finally, we examine the impact of liquidity uncertainty by allowing for time-varying volatility of liquidity shocks (e.g., in the spirit of Justiniano and Primiceri, 2008). We assume that κ_t follows the process $\log(\kappa_t) = (1 - \rho_\kappa) \log(\kappa) + \rho_\kappa \log(\kappa_{t-1}) + \sigma_{\kappa t} \varepsilon_{\kappa t}$ where ρ_κ measures the persistence of the liquidity shock and $\sigma_{\kappa t}$ measures the liquidity uncertainty such that $\log(\sigma_{\kappa t}) = (1 - \rho_\sigma) \log(\sigma_\kappa) + \rho_\sigma \log(\sigma_{\kappa t-1}) + \sigma_{\sigma_\kappa} \varepsilon_{\sigma_\kappa}$.⁴⁰ Figure 7 displays the IRFs of key variables to one percent rise in liquidity uncertainty.

Facing more uncertainty regarding the availability of external liquid funds, firms have a precautionary saving behavior by increasing the amount of cash they own in their portfolio. They also accumulate more collateral (capital) which, combined with the amount of cash they hold, increases wealth. Ultimately, higher collateral generates an increase in external funds, i.e., long and short-term debt. Additionally, uncertain perspectives on future liquidity conditions reduce the demand for labor, generating a drop in wages and a recession.⁴¹ Eventually, the greater financial opportunities and the wealth expansion stimulates employment and output while firms need less cash in their portfolio. This result confirms that liquidity prospect by firms are key to understand the negative comovement between labor and the cash ratio.

⁴⁰Following Fernandez-Villaverde et al. (2011), we make a third-order approximation of the model which is simulated 2096 periods when all shocks hit the economy. The first 2000 periods are dropped to delete the effects of the initial conditions on the simulation and the ergodic mean is computed from the 96 remaining periods. We then compute the effect of the uncertainty shock, by hitting $\varepsilon_{\sigma_\kappa}$, in deviation from the ergodic mean. Following Gilchrist et al. (2014), we set $\rho_\sigma = 0.9$, the results being unaffected by this parametrization.

⁴¹This result contrasts with the literature which looks at uncertainty for TFP shocks. In the standard RBC model, in response to more uncertain future economic conditions, households save more and supply more labor which, under a perfectly competitive labor market, leads to a rise in output (see Basu and Bundick, 2015).

Figure 7: Impulse Response Functions to Uncertainty Shocks on Liquidity.



Note: The shock is a 10 percent deviation from the ergodic mean of the conditional standard deviation of liquidity shocks.

5 Cross-firms Correlations

We now assess whether the baseline model is able to explain the cross-firm evidence of a negative correlation between cash and employment. To examine this issue, we reintroduce heterogeneous firms that are hit by idiosyncratic productivity shocks ϵ_{it}^A and liquidity shocks ϵ_{it}^κ . Instead we assume for simplicity that the aggregate economy does not fluctuate by setting $A_t = A$, $\kappa_t = \kappa$. We also assume that credit constraints do not vary across firms and time and set $\phi_{it} = \phi$. Because the capital intensity and financial multiplier differ across firms, the model does not allow for linear aggregation, despite the fact that the policy functions are linear at the individual level. Therefore, we use a numerical method to solve the model.

5.1 Calibration

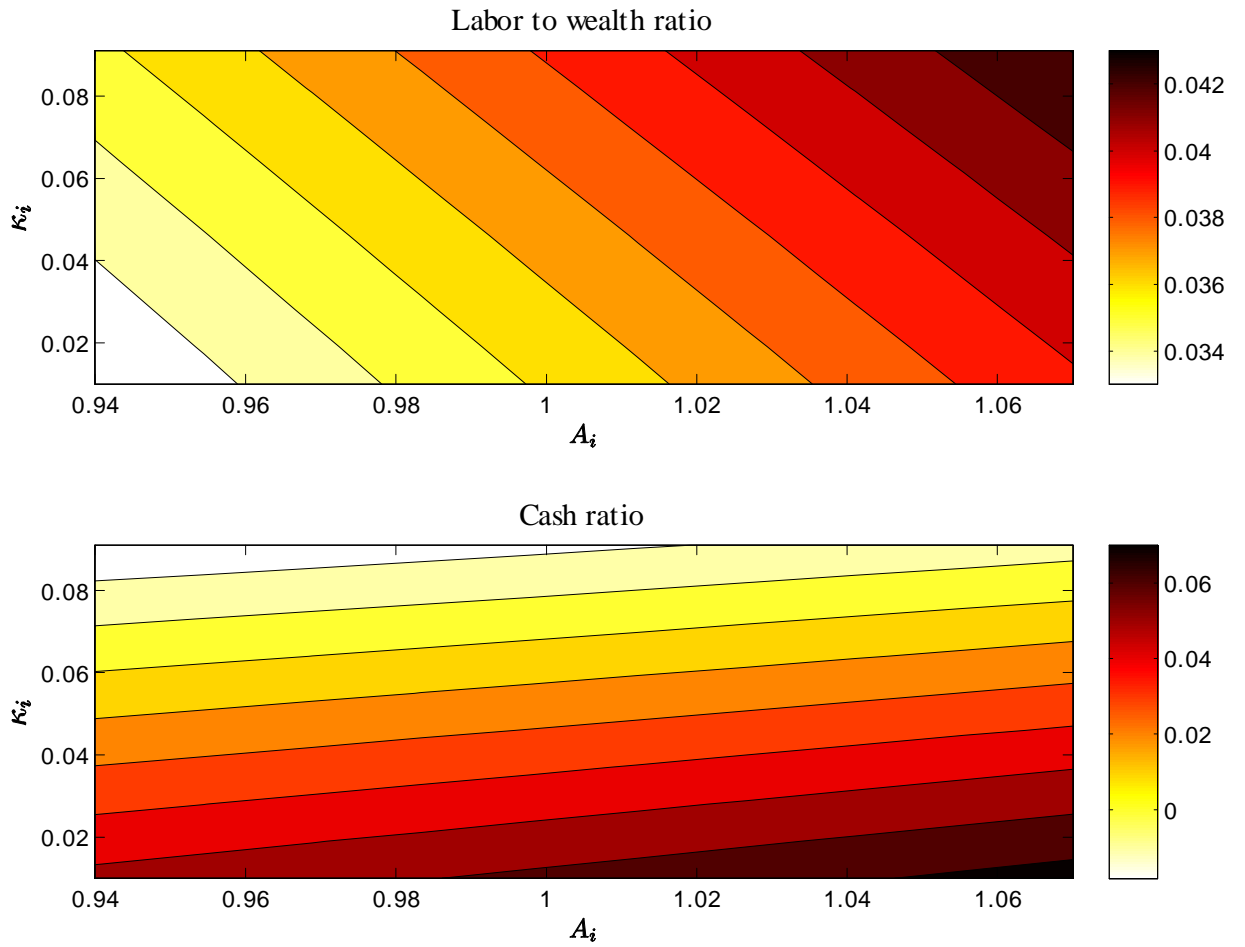
Beside the parametrization described previously, we aim at calibrating a range for $\kappa_{it} = \kappa + \epsilon_{it}^\kappa$ and $A_{it} = A + \epsilon_{it}^A$. We assume that these shocks can take 10 equidistant possible realizations. The two shocks are assumed to follow an independent first-order Markov process with transition probability of $\frac{0.25}{9}$. More precisely, each firm has a probability of 75% to stay in the same state for κ (A) and a probability of 25% to switch to one of the 9 other states, with an identical probability for each of these states. We calibrate the range for κ_{it} and A_{it} (namely, we set the minimum and maximum values) to match some distribution moments observed at the firm level. Table 4 provides the interquartile values to match, computed from the Compustat database described in Section 2. The range of the idiosyncratic liquidity and productivity shocks κ_{it} and A_{it} are set to reproduce the interquartile ratio for our two variables of interest, namely the cash ratio and employment. This implies $\kappa_{it} \in [0.01; 0.091]$ and $A_{it} \in [0.94; 1.07]$. All the other parameters are parametrized as described in Section 4.1. The numerical method to obtain the steady-state wage and distribution of firms is described in Appendix D.

5.2 Results

The upper panel of Table 8 displays firm-level moments computed from the stationary distribution. Interestingly, our stylized model provides a negative cross-firm correlation between the cash ratio and employment, equals to -0.13 under our benchmark calibration. This number is somewhat smaller than the unconditional correlation found in the data (-0.22).

To understand this result, Figure 8 shows the impact of an idiosyncratic innovation of κ_{it} and A_{it} on the value of the labor normalized by wealth (ℓ_{it}/Ω_{it}) and the cash ratio (m_{it}), both weighted by the distribution probability.

Figure 8: Value of the labor to wealth ratio (l_i/ω_i) and the cash ratio (m_i).



Note: All values of l_i/Ω_i and m_i are weighted by the distribution probability.

Table 8. Simulated Moments

Benchmark Calibration		Data	Model
$\frac{m_{75\%}}{m_{25\%}}$	Interquartile ratio of m	10.04	8.79
$\frac{\ell_{75\%}}{\ell_{25\%}}$	Interquartile ratio of ℓ	1.47	1.45
$corr(m, \ell)$	Correlation(cash ratio; labor)	-0.22	-0.13

Average value of labor and cash ratio by class of firms		ℓ	m
Ω_i	bottom 50%	0.67	0.03
	top 50%	1.05	0.03
κ_i	bottom 50%	0.77	0.05
	top 50%	0.81	0.01
A_i	bottom 50%	0.74	0.02
	top 50%	0.84	0.03

Note: In the upper panel, the empirical correlation between m and ℓ is computed after removing the firm-specific linear trend from data. In the lower panel, all the values of labor and the cash ratio are weighted by the distribution probability.

This figure shows that, as κ_{it} decreases, the cash ratio is higher and labor is lower for a given Ω_{it} . Differently, firms facing a negative productivity shock adjust both labor and the cash ratio downward. Consequently, even though the two shocks predict an opposite correlation between employment and the cash ratio, our calibrated liquidity shock is strong enough to generate a reasonable negative correlation. When the amount of liquid funds is reduced, firms are able to finance less labor with the same amount of cash. To accommodate for this shock, they both accumulate more cash in order to pay for the wage bill and diminish their level of labor to limit the wage bill.

However, while the normalized labor (l_{it}/Ω_{it}) is independent of Ω_{it} according to Proposition 1, the level of labor l_{it} is driven by the size of the firm Ω_{it} , which depends on the history of shocks. As a consequence, the correlation between the cash ratio and labor is determined not only by A_{it} and κ_{it} as suggested by Figure 8, but also by Ω_{it} . The lower panel of Table 8 complements the previous figure by showing the weighted value of these variables by class of firms. While firms with a level of wealth below median have on average a substantially lower level of employment than firms with a level of wealth above median, their cash ratio is

about the same on average. On the one hand, idiosyncratic innovations on liquidity (κ) and technology (A) affect the cash ratio and labor, as shown in Figure 8. On the other hand, they also affect firms' wealth and therefore employment for a given level of cash. This heterogeneity of wealth generates noise that further dampens the correlation.

We can also show that the credit constraint affects the correlation between the cash ratio and employment through a multiplier effect. To do so, we compare our benchmark calibration with an alternative one where ϕ is calibrated at a lower value ($\phi = 0.005$ rather than 0.09). This value aims at replicating the first quartile of the debt-to-sales ratio distribution from our Compustat database. This strategy implies that firms are more financially-constrained in the alternative calibration than in the baseline. We find that the constraint on long-term credit has little impact on our results since the correlation goes from -0.13 in the benchmark to -0.12 . Turning to the data, we find more clear-cut results since firms with the lower debt-to-sales ratio, that are more likely to be credit constrained, have a more negative correlation between the cash ratio and employment. Precisely, the 25 percent smaller firms in terms of debt ratio have a correlation of -0.23 compared to -0.19 for the top 25 percent.⁴²

6 Conclusion

This paper has documented a negative comovement between the corporate cash ratio and employment. Even though such a relationship may appear surprising at first sight, we show that it can be explained by liquidity shocks. These shocks make production less attractive or more difficult to finance, while they also generate a need for liquidity necessary to pay wage bills, which can be satisfied by holding more cash. Moreover, we argue that our analysis is useful in understanding the motives for firms' cash holdings and in shedding light on the dominant shocks during the financial crisis.

Besides explaining an interesting stylized fact, the simple model developed in this paper could be extended to analyze the role of corporate liquidity in a macroeconomic environment. Several extensions could be of interest. First, instead of focusing on the business cycle frequency, the model could be used to examine longer term developments. The model would actually be consistent with the documented gradual increase in cash holdings if we assume changes in the production process that imply more end-of-period payments (e.g., with more extensive use of just-in-time technologies as reported in Gao, 2013, or with an increase in production

⁴²See Section 2.3.5 in the online appendix for details.

outsourcing). Second, for a better analysis of the financial crisis, it would be of interest to introduce demand shocks. Finally, the role of policy intervention would be a natural extension. The last two extensions would be related to the existing DSGE literature incorporating working capital to study monetary policy.

Appendix

A The Entrepreneur's Problem

Entrepreneurs maximize (2) subject to (5), (6), (7), (8), (10), and $\tilde{M}_t \geq 0$. They also take into account the production function $Y_{it} = F(K_{it}, A_{it}l_{it})$. The production function has constant returns to scale so we can write $Y_{it} = A_{it}l_{it}f(k_{it}/A_{it})$, with $f(k) = F(k, 1)$ and with k the capital-labor ratio K/l . All three shocks A_{it} , ϕ_{it} and κ_{it} are known at the beginning-of-period. We denote by $E_t(\cdot)$ the expectation conditional on the beginning-of-period information. The Lagrangian problem is

$$\begin{aligned} \mathcal{L}_{it} = & E_t \sum_{s=t}^{\infty} \beta^{s-t} \{ u(c_{is}) \\ & + \gamma_{is} \left[\tilde{M}_{is-1} + Y_{is-1} + (1 - \delta)K_{is-1} - r_{s-1}D_{is-1} - r_{ts-1}^L L_{is-1} + D_{is} - c_{is} - K_{is} - M_{is} \right] \\ & + \eta_{is} \left[M_{is} + L_{is} - w_t l_{is} - \tilde{M}_{is} \right] \\ & + \lambda_{is} \left[\phi_{is}(1 - \delta)K_{is} - r_s D_{is} \right] \\ & + \nu_{is} \left[\kappa_{is}(1 - \delta)K_{is} - r_s^L L_{is} \right] \\ & + \mu_{is} \tilde{M}_{is} \} \end{aligned}$$

The entrepreneur's program yields the following first-order conditions with respect to l_{it} , c_{it} , K_{it} , D_{it} , M_{it} , \tilde{M}_{it} and L_{it} :

$$w_t \eta_{it} = A_{it} F_{lit} \beta E_t \gamma_{it+1} \quad (32)$$

$$u'(c_{it}) = \gamma_{it} \quad (33)$$

$$\gamma_{it} = \beta F_{Kit} E_t \gamma_{it+1} + (1 - \delta)(\phi_{it} \lambda_{it} + \kappa_{it} \nu_{it}) \quad (34)$$

$$\gamma_{it} = \beta r_t E_t \gamma_{it+1} + r_t \lambda_{it} \quad (35)$$

$$\gamma_{it} = \eta_{it} \quad (36)$$

$$\eta_{it} = \beta E_t \gamma_{it+1} + \mu_{it} \quad (37)$$

$$\eta_{it} = \beta r_t^L E_t \gamma_{it+1} + r_t^L \nu_{it} \quad (38)$$

Studying these FOCs indicates which constraints are binding. Since $\gamma_{it} = u'(c_{it}) > 0$, then $\eta_{it} > 0$ according to (36), which implies that both budget constraints are

binding. Moreover, using (32), (35) and (36), we obtain:

$$\beta \left(\frac{A_{it}F_{lit}}{w_t} - r_t \right) E_t \gamma_{it+1} = r_t \lambda_{it}$$

This implies that whenever $w_t r_t < A_{it} F_{lit}$, the long-term credit constraint is binding ($\lambda_{it} > 0$). Besides, using (35), (36) and (38), we find:

$$\beta(r_t - r_t^L) E_t \gamma_{it+1} + r_t \lambda_{it} = r_t^L \nu_{it} \quad (39)$$

Therefore, if $r_t > r_t^L$, then the short-term credit constraint is binding ($\nu_{it} > 0$). Finally, using (37) and (38), we find:

$$\beta(r_t^L - 1) E_t \gamma_{it+1} + r_t^L \nu_{it} = \mu_{it}$$

Therefore, if $r_t^L > 1$, then the entrepreneurs hold no excess money ($\mu_{it} > 0$).

Assume now that $r_t > r_t^L > 1$ and make the guess that $\lambda_{it} > 0$ (we will determine later under which conditions the long-term credit constraint is indeed binding). Then all the constraints are binding and we can write $\tilde{M}_{it} = 0$, $D_{it} = \phi(1 - \delta)K_{it}/r_t$ and $M_{it} = w_t l_{it} - \kappa_{it}(1 - \delta)K_{it}/r_t^L$. We can then rewrite the objective as

$$\begin{aligned} \mathcal{L}_{it} = & E_t \sum_{s=t}^{\infty} \beta^{s-t} \{u(c_{is}) \\ & + \gamma_{is} [Y_{is-1} + (1 - \delta)K_{is-1}(1 - \kappa_{is-1} - \phi_{is-1}) \\ & - c_{it} - K_{is}[1 - (1 - \delta)(\phi_{is}/r_{is} + \kappa_{is}/r_{is}^L)] - w_{is}l_{is}\} \end{aligned} \quad (40)$$

The optimality conditions with respect to c_{it} , l_{it} and K_{it} are:

$$\gamma_{it} = u'(c_{it}) \quad (41)$$

$$w_t \gamma_{it} = A_{it} F_{lit} \beta E_t \gamma_{it+1} \quad (42)$$

$$[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] \gamma_{it} = \beta E_t \gamma_{it+1} [F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})] \quad (43)$$

Combining (42) with (43), we obtain:

$$\frac{w_t}{A_{it}} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] F_{lit}}{F_{Kit} + (1 - \delta)(1 - \phi_{it} - \kappa_{it})}$$

F has constant returns to scale so we can write: $F(K, Al) = Alf(K/Al)$. Therefore, $F_K(K, Al) = f'(K/Al)$ and $F_l(K, Al) = f(K/Al) - Kf'(K/Al)/Al$. As a

consequence, $w_t/A_{it} = \tilde{w}(\tilde{k}_{it}, \phi_{it}, \kappa_{it})$, with $\tilde{k}_{it} = K_{it}/A_{it}l_{it}$ and

$$\tilde{w}(\tilde{k}_{it}, \phi_{it}, \kappa_{it}, r_t, r_t^L) = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)][f(\tilde{k}_{it}) - \tilde{k}_{it}f'(\tilde{k}_{it})]}{f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})} \quad (44)$$

Since F is increasing in l , $F_{lit} = f(\tilde{k}_{it}) - \tilde{k}_{it}f'(\tilde{k}_{it}) > 0$. Besides, since F is concave in K , we have $f'' < 0$. We can show that this implies that \tilde{w} is strictly increasing in \tilde{k} . If there exists a solution $\tilde{k}(\tilde{w}_{it}, \phi_{it}, \kappa_{it}, r_t, r_t^L)$ to that equation, then this solution is unique.⁴³ Finally, k_{it} is then given by $k_{it} = A_{it}\tilde{k}(\tilde{w}_{it}, \phi_{it}, \kappa_{it}, r_t, r_t^L)$.

Note that the long-term credit constraint is binding whenever $\tilde{w}_{it}r_t < F_{lit}$. Combining this inequality with (44), we find that this is equivalent to:

$$\begin{aligned} f'(\tilde{k}_{it}) + (1 - \delta)(1 - \kappa_{it} - \phi_{it}) &> r_t[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] \quad (45) \\ \Leftrightarrow \tilde{k}_{it} &< (f'^{-1}\left(r_t[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] - (1 - \delta)(1 - \kappa_{it} - \phi_{it})\right)) \end{aligned}$$

Finally, according to (44), \tilde{k}_{it} is increasing in \tilde{w}_{it} , so this inequality is satisfied for \tilde{w}_{it} lower than some $\tilde{w}^*(\kappa_{it}, \phi_{it}, r_t, r_t^L)$ and thus for w_t lower than some $w^*(A_{it}, \kappa_{it}, \phi_{it}, r_t, r_t^L)$.

In order to study how k is affected by ϕ , we differentiate Equation (44) with respect to it and find after rearranging

$$\frac{\partial \tilde{k}_{it}}{\partial \phi_{it}} = \frac{(1 - \delta)[f(\tilde{k}_{it}) - \tilde{k}_{it}f'(\tilde{k}_{it})] \left[f'(\tilde{k}_{it}) - r_t + (1 - \delta)(1 - \kappa_{it} + \kappa_{it}r_t/r_t^L) \right]}{-r_t f''(\tilde{k}_{it}) [1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] [f(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})\tilde{k}_{it}]}$$

As $f'' < 0$, the denominator is positive. The sign of the numerator depends then on $f'(\tilde{k}_{it}) - r_t + (1 - \delta)(1 - \kappa_{it} + \kappa_{it}r_t/r_t^L)$. Using (34), (35) and (39), we can establish:

$$\left[f'(\tilde{k}_{it}) - r_t + (1 - \delta)(1 - \kappa_{it} + \kappa_{it}r_t/r_t^L) \right] \beta E_t \gamma_{it+1} = \lambda_{it} r_t [1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]$$

When the constraint is binding, we have $\lambda_{it} > 0$. Besides, it is reasonable to assume that $1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t) > 0$ (it is sufficient that $\phi_{it} + \kappa_{it} \leq 1$). Therefore, $f'(\tilde{k}_{it}) - r_t + (1 - \delta)(1 - \kappa_{it} + \kappa_{it}r_t/r_t^L)$, so the numerator is positive as well, so $\partial \tilde{k}_{it}/\partial \phi_{it} > 0$. Following similar steps, we find $\partial \tilde{k}_{it}/\partial \kappa_{it} > 0$. Then k_{it} is also increasing in ϕ_{it} and κ_{it} .

Differentiating Equation (44) with respect to \tilde{w} , we find after rearranging

$$\frac{\partial \tilde{k}_{it}}{\partial \tilde{w}_{it}} = \frac{[f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})]^2}{-f''(\tilde{k}_{it}) [1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] [f(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})\tilde{k}_{it}]}$$

⁴³We can show that such a solution always exists in the Cobb-Douglas case.

Note that $k_{it} = A_{it}\tilde{k}_{it}$ and $w_t = A_{it}\tilde{w}_{it}$ so

$$\begin{aligned}\frac{\partial k_{it}}{\partial w_t} &= A_{it} \frac{\partial \tilde{k}_{it}}{\partial \tilde{w}_{it}} \frac{\partial \tilde{w}_{it}}{\partial w_t} = \frac{\partial \tilde{k}_{it}}{\partial \tilde{w}_{it}} > 0, \\ \frac{\partial k_{it}}{\partial A_{it}} &= \tilde{k}_{it} + A_{it} \frac{\partial \tilde{k}_{it}}{\partial \tilde{w}_{it}} \frac{\partial \tilde{w}_{it}}{\partial A_{it}} = \tilde{k}_{it} - \frac{\partial \tilde{k}_{it}}{\partial \tilde{w}_{it}} \tilde{w}_{it} \\ &= \tilde{k}_{it} - \frac{[f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})][f(\tilde{k}_{it}) - \tilde{k}_{it}f'(\tilde{k}_{it})]}{-f''(\tilde{k}_{it})[f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]}\end{aligned}$$

In the Cobb-Douglas case, we have

$$\frac{\partial k_{it}}{\partial A_{it}} = \frac{-(1-\alpha)^2(1-\delta)(1-\phi_{it}-\kappa_{it})f(\tilde{k}_{it})}{-f''(\tilde{k}_{it})[f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]} < 0$$

Proof of Proposition 1 Assume that the credit constraint is binding and that $r_t > r_t^L > 1$. Then the program of the firm is described by (40) and by the FOCs (41)-(44). We make the educated guess that there exists χ such that $c_{it} = (1-\chi)\Omega_{it}$. Combining our guess with (5), (7), (8), (10) and (13), we obtain

$$\chi\Omega_{it} = K_{it} + w_t l_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)K_{it} = A_{it}l_{it}[\tilde{k}_{it} + \tilde{w}_{it} - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)\tilde{k}_{it}]$$

Replacing \tilde{w}_{it} using (44) and rearranging, we obtain

$$\chi\Omega_{it} = A_{it}l_{it} \frac{[1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)][f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})}$$

As $\Omega_{it+1} = A_{it}l_{it}[f(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})\tilde{k}_{it}]$, we have

$$\chi\Omega_{it} = \frac{[1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]\Omega_{it+1}}{f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})} \quad (46)$$

Using (41) and (43) under log-utility $u(c) = \log(c)$, we obtain the following Euler equation

$$\frac{1}{c_{it}} [1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] = \beta E_t \left\{ \frac{1}{c_{it+1}} \right\} [f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})]$$

Given that shocks are known at the beginning-of-period, $c_{it+1} = \chi\Omega_{it+1}$ is known at the beginning-of-period, so the Euler equation can be written without the expectations operator

$$\frac{1}{c_{it}} [1 - (1-\delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)] = \beta \frac{1}{c_{it+1}} [f'(\tilde{k}_{it}) + (1-\delta)(1-\phi_{it}-\kappa_{it})]$$

Using our guess $c_{it} = \chi\Omega_{it}$ and $c_{it+1} = \chi\Omega_{it+1}$ to replace c_{it} and c_{it+1} , we obtain

$$\beta\Omega_{it} = \frac{[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]\Omega_{it+1}}{f'(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})} \quad (47)$$

Combining (46) and (47) yields $\chi = \beta$.

Combining $c_{it} = (1 - \beta)\Omega_{it}$ with the binding constraints (5), (7) and (10), we can easily derive equations (15)-(19) in Proposition 1.

Proof of Corollary 1 According to Equation (13), a decline in κ_{it} increases the cash ratio through a lower level of external liquid funds and through a lower capital-labor ratio. A decline in ϕ_{it} increases the cash ratio through a lower capital-labor ratio. A decline in A_{it} decreases the cash ratio through a higher capital-labor ratio.

According to Equation (15), the effect on labor depends directly on the effect on the financial multiplier Z_{it} . We can rewrite Z_{it} as follows:

$$Z_{it} = \frac{\beta}{w_t + A_{it}\tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]}$$

So the effect on Z_{it} depends on the effect on $X_{it} = \tilde{k}_{it}[1 - (1 - \delta)(\kappa_{it}/r_t^L + \phi_{it}/r_t)]$.

In the Cobb-Douglas case, we have

$$\frac{\partial X_{it}}{\partial \phi_{it}} = (1 - \delta)f(\tilde{k}_{it}) \frac{-\alpha(1 - \alpha)(1 - \phi_{it} - \kappa_{it})/r_t - (1 - \alpha)[1 - (1 - \delta)((1 - \kappa_{it})/r_t + \kappa_{it}/r_t^L)]}{|f''(\tilde{k}_{it})|[f(\tilde{k}_{it}) + (1 - \delta)(1 - \phi_{it} - \kappa_{it})\tilde{k}_{it}]} < 0$$

Similarly, we have $\partial X_{it}/\partial \kappa_{it} < 0$. Therefore, a decline in ϕ_{it} or κ_{it} decreases the financial multiplier Z_{it} and hence has a negative impact on labor. Note finally that, in the Cobb-Douglas case, k_{it} is decreasing in A_{it} as shown earlier. As a result, Z_{it} and l_{it} are increasing in A_{it} .

B The household's problem

The household has utility U_t with the discount factor β :

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s [v(c_{t+s}^h, l_{t+s-1})] \quad (48)$$

where c_t^h is households' consumption in the beginning-of-period, and l_{t-1} is the labor supplied by the households at the beginning of period t as well. However, note that l_{t-1} is agreed upon at the end of period $t - 1$.

The household maximizes this utility subject to her budget constraint

$$w_{t-1}l_{t-1} + R_{t-1}D_{t-1}^h + T_t = c_t^h + D_t^h \quad (49)$$

The household's Lagrangian writes then as follows:

$$\begin{aligned} \mathcal{L}_t^h = & E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \frac{\left(c_s^h - \bar{w} \frac{l_{s-1}^{1+1/\eta}}{1+1/\eta} \right)^{1-\sigma}}{1-\sigma} \right. \\ & \left. + \gamma_s^h [w_{t-1}l_{t-1} + R_{t-1}D_{t-1}^h + T_t - c_t^h - D_t^h] \right\} \end{aligned}$$

The household's program yields the following first-order conditions with respect to l_t , c_t^h , and D_t^h :

$$w_t E_t \gamma_{t+1}^h = \bar{w} l_t^{1/\eta} E_t \left(c_{t+1}^h - \bar{w} \frac{l_t^{1+1/\eta}}{1+1/\eta} \right)^{-\sigma} \quad (50)$$

$$\left(c_t^h - \bar{w} \frac{l_{t-1}^{1+1/\eta}}{1+1/\eta} \right)^{-\sigma} = \gamma_t^h \quad (51)$$

$$\gamma_t^h = \beta R_t E_t \gamma_{t+1}^h \quad (52)$$

where E_t is the expectation at the beginning-of-period. Combining (50) and (51), we obtain:

$$w_t = \bar{w} l_t^{1/\eta}$$

C Equilibrium with aggregate shocks only

Before characterizing the steady state, we establish the following Lemma:

Lemma 1 *If $r_t > r_t^L > 1$, there exists an increasing function $\Omega^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$ so that the credit constraint is binding whenever $\Omega_t < \Omega^*$. In that case the dynamics of K_t , M_t , D_t , l_t and Ω_{t+1} follow:*

$$l_t = Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) \Omega_t \quad (53)$$

$$K_t = k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) \Omega_t \quad (54)$$

$$M_t = [w_t - \kappa_t(1 - \delta)k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L)/r_t^L] Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) \Omega_t \quad (55)$$

$$D_t = \phi_t(1 - \delta)k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) \Omega_t / r_t \quad (56)$$

$$\begin{aligned}\Omega_{t+1} &= [(1 - \delta)(1 - \kappa_t - \phi_t)k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) \\ &\quad + A_t f[k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L)/A_t]Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L)]\Omega_t\end{aligned}\tag{57}$$

where

$$Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L) = \frac{\beta}{w_t + [1 - (1 - \delta)(\kappa_t/r_t^L + \phi_t/r_t)]k(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L)}$$

is the financial multiplier and

$$w_t = w(A_t, \kappa_t, \phi_t, r_t, r_t^L, \Omega_t)$$

is the equilibrium wage so that $w(A_t, \kappa_t, \phi_t, r_t, r_t^L, \Omega_t)$ is the solution to $l^s(w_t) = Z(w_t, A_t, \kappa_t, \phi_t, r_t, r_t^L)\beta r_t \Omega_t$.

Proof. Note that, as shown earlier, if $r_t > r_t^L > 1$, then Proposition 1 holds and the credit constraint is binding whenever $w_t < w^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$. Since we also have that the constrained equilibrium wage w is increasing in Ω_t , then there exists an increasing function Ω^* so that $w_t < w^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$ is equivalent to $\Omega_t < \Omega^*(A_t, \kappa_t, \phi_t, r_t, r_t^L)$. The rest of the Lemma derives from Proposition 1. ■

Using this Lemma, we can study the steady state. For all the constraints to be binding, so that Proposition 1 and Lemma 1 hold, we must have $r > r^L > 1$ in the steady state, and that the inequality (45) is satisfied. $r^L = 1 + \psi > 1$ is given by the assumption $\psi > 0$. According to (51) and (52), the stationarity of c^h and l implies that $R = 1/\beta$. Since $r = \tau R$, then $r > r^L$ is guaranteed by $\tau/\beta > 1 + \psi$

From Equation (57), we have that the steady-state wage must satisfy:

$$\tilde{w} + \tilde{k}[1 - (1 - \delta)(\kappa/r^L + \phi/r)] = \beta[f(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi)\tilde{k}]$$

Replacing \tilde{w} using (44) and rearranging:

$$1 - (1 - \delta)(\kappa/r^L + \phi/r) = \beta[f'(\tilde{k}) + (1 - \delta)(1 - \kappa - \phi)]$$

Since $r > r^L$, inequality (45) is satisfied as long as $1/\beta > r = \tau R$. Since $R = 1/\beta$, $1/\beta > r$ if and only if $\tau < 1$. Therefore, the constraints are binding in the steady state if $\beta < \tau < 1$ and $0 < \psi < \tau/\beta - 1$. These conditions implies that for small enough shocks, the equilibrium is constrained.

D Numerical method

The algorithm to compute the steady-state distribution of firms in Section 5 is as follows:

1. We first choose a grid of wealth Ω_{it} . Our grid is a 1000-value grid over $[5, 65]$. We use the Chebychev nodes to make the grid more concentrated on low values of Ω .
2. We allocate an initial uniform and independent distribution to the values of Ω_{i0} , κ_{i0} and A_{i0} , and make an initial guess on the equilibrium wage w_0 .
3. Given the initial distribution on Ω_{it} , κ_{it} and A_{it} and the initial equilibrium wage w_0 , we use Proposition 1 and the Markov Chain to compute the new distribution of Ω_{it+1} , κ_{it+1} and A_{it+1} . Using Proposition 1, we compute the corresponding distribution of labor demand l_{it+1} . We aggregate this labor demand $l_{t+1} = \sum_i l_{it+1} di$, and if $l_{t+1} > l^s(w_t)$ (if $l_{t+1} < l^s(w_t)$), then we update the equilibrium wage w_{t+1} upward (downward).
4. We repeat step 3 until the equilibrium wage is reached, i.e. when aggregate labor demand is fully satisfied.

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