International Portfolio Choice with Frictions: Evidence from Mutual Funds

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Abstract

Using data on international equity portfolio allocations of US mutual funds, we estimate a simple portfolio expression derived from a standard Markowitz mean-variance portfolio model extended with portfolio frictions. The optimal portfolio depends on two benchmark portfolios, the previous month and the buy-and-hold portfolio shares, and a present discounted value of expected future excess returns. We show that equity return differentials are predictable and use the expected return differentials in the mutual fund portfolio regressions. The estimated reduced form parameters are related to the structural model parameters. The estimates imply significant portfolio frictions and a modest rate of risk-aversion. While mutual fund portfolios respond significantly to expected returns, portfolio frictions lead to a weaker and more gradual portfolio response to changes in expected returns. We also document heterogeneity across funds. Global and larger funds face bigger portfolio frictions, while more active funds give relatively less weight to the buy-and-hold portfolio (rebalance more aggressively).
1 Introduction

An extensive literature has introduced frictions into models of portfolio choice that lead to deviations from the standard Markowitz mean-variance portfolio. This is supported by micro evidence of sluggish portfolio decisions by households and helps explain various asset pricing facts. In this paper we focus on international portfolio decisions. The objective is to provide evidence on how US mutual funds allocate their equity portfolios across countries, and specifically to what extent this is affected by portfolio frictions that lead to a weaker and more gradual response to changes in expected returns. It has frequently been suggested that global investors are slow to adjust their portfolios in response to new information. In the context of US external equity investments, Bohn and Tesar (1996) comment that “we suspect that investors may adjust their portfolios to new information gradually over time, resulting in both autocorrelated net purchases and a positive linkage with lagged returns.” Froot et al. (2001) provide similar evidence. Froot and Thaler (1990), in attempting to explain the forward discount puzzle of excess return predictability in the foreign exchange market, hypothesize that “...at least some investors are slow in responding to changes in the interest differential.” More formally, Bacchetta and van Wincoop (2010, 2019) and Bacchetta, van Wincoop, and Young (2020) show that open economy models with portfolio frictions can explain a variety of evidence related to excess return predictability in foreign exchange and equity markets as well as various data moments involving aggregate US equity portfolios. Nonetheless none of the existing literature has provided direct evidence of portfolio frictions in international portfolio allocation data. This paper aims to fill that gap.

Our evidence is based on 15 years of monthly equity portfolio allocation data across 35 countries for 316 US mutual funds that report to EPFR (Emerging Port-

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2 Although our focus is on international portfolio choice, this ultimately has a significant effect on global capital flows, which in turn can have a significant effect on business cycles. While there are many models of international capital flows driven by portfolio choice, these tend to abstract from portfolio frictions considered here. Examples of recent DSGE models of capital flows based on portfolio choice include Benhima and Cordonier (2020), Davis and van Wincoop (2018), Devereux and Sutherland (2007, 2010), Didier and Lowenkron (2012), Evans and Hnatkovska (2012, 2014), Gabaix and Maggiori (2015), Hnatkovska (2010) and Tille and van Wincoop (2010a, b, 2014).
folio Fund Research). Mutual funds are the most important players in external US equity holdings, accounting for 61 percent of all US foreign equity holdings at the end of 2018. To structure the analysis, we present a portfolio choice model that enables us to derive a simple and testable portfolio equation. While the standard Markowitz mean-variance portfolio is embedded as a special case, the model allows for deviations from the Markowitz portfolio as a result of portfolio frictions that involve costs of deviating from two benchmark portfolios. The optimal portfolio share then depends on both of these benchmark portfolios and a present discounted value of expected future excess returns. We first document that international differences in stock returns are predictable and that predictability improves over longer horizons. We then use estimates of these expected excess returns in our portfolio regressions. We find that portfolios respond to expected return differentials, but deviate gradually from benchmark portfolios. The results from the portfolio regressions are used to obtain estimates of the structural parameters of the model, such as the two portfolio frictions and risk-aversion.

The simple theoretical portfolio choice model that structures the empirical analysis is analogous to Gärleanu and Pederson (2013). It assumes that funds (investors) maximize the present discounted value of risk-adjusted portfolio returns minus quadratic costs of deviating from two benchmark portfolios. The first portfolio friction is a cost of deviating from the portfolio share during the previous month, which is the portfolio under complete rebalancing (like an index fund). The second is a cost of deviating from a buy-and-hold portfolio. The more important these portfolio frictions are, the more the optimal portfolio share depends on the two benchmark portfolios and the less it depends on expected excess returns. In addition, the portfolio frictions imply that the optimal portfolio depends not just on expected excess returns over the next period (as in the Markowitz portfolio), but on a present discounted value of future excess returns. The frictions lead to a more gradual response of portfolio shares to changes in expected returns. One can think of the frictions as related to both trading costs and decision making costs, though we do not take a position on this. A parameter that captures the sum of the two portfolio frictions is referred to as the aggregate portfolio friction. For a given aggregate portfolio friction, an increase in the relative size of the first friction implies a higher relative weight on the lagged portfolio share and therefore more.

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3See Exhibit 19 in “Portfolio Holdings of Foreign Securities,” October 2019, Department of the Treasury.
extensive rebalancing.

We find that the funds respond to the discounted expected excess return with strong statistical significance. We also find that both benchmark portfolios are important, especially the portfolio share from the previous month. Our estimates imply a humped shaped portfolio response to an expected excess return innovation. The initial portfolio response is weaker than in the absence of portfolio frictions, while the portfolio response builds gradually as a result of the frictions. The regression estimates imply a plausible rate of risk aversion between 1 and 2. The aggregate portfolio friction is precisely estimated. While we estimate a somewhat higher friction of deviating from the lagged portfolio than the buy-and-hold portfolio, we cannot systematically reject that these frictions are identical.

We also consider various types of fund heterogeneity: large versus small funds; active versus passive funds; global versus regional and emerging market funds. The results imply that global and large funds face bigger aggregate portfolio frictions, while active funds have a much larger relative friction of deviating from the previous portfolio share. The latter is consistent with very active rebalancing by the active funds. Global funds are found to have a higher risk parameter than more specialized funds such as emerging market and regional funds, reflecting either higher risk aversion or higher perceived uncertainty about excess returns.

The paper is related to various strands of literature. The first is the literature on excess return predictability. While the evidence we report on the predictability of international stock return differentials is new, the evidence on the predictability of international short term bond return differentials (UIP deviations) is known since Fama (1984). Predictability has also been widely documented in the context of country or individual stock returns or the excess of stock returns over bond returns (for a textbook discussion, see Campbell, Lo and MacKinlay, 1997). While the latter literature focuses mainly on the US, some papers document stock return predictability in other countries or show, by pooling the data, that there is global predictability (Hjalmarsson, 2010). Cenedese et al. (2016) consider the profitability of trading strategies that exploit international equity return differentials. They sort countries into various “bins” based on the realization of variables like the dividend yield that are likely to predict future equity returns. They do not estimate portfolio expressions or excess returns, but find substantial Sharpe ratios from trading strategies that exploit in which bin countries are located.

In terms of estimation of portfolio regressions, the open economy literature is
very limited, which is the motivation for doing this project. Frankel and Engel (1984) invert the portfolio expression obtained from a simple frictionless mean variance portfolio model, relating expected returns on various currencies to asset supplies. They strongly reject the model. Also relevant is recent work by Koijen and Yogo (2019). For different asset classes they consider aggregate bilateral portfolio shares (as opposed to portfolio shares of individual funds as in this project). They do not regress portfolio shares on expected excess returns, but directly regress on a variety of variables that may or may not affect expected excess returns, including asset prices and macro variables like inflation, trade, GDP, distance, etc. Some papers have investigated the link between international capital flows (as opposed to portfolio shares) and past returns as well as expected future returns (e.g. Bohn and Tesar (1996), Froot et al. (2001), Didier and Lowenkron (2012)).

There is also a literature that has investigated portfolio allocation using mutual fund data. For example, Raddatz and Schmukler (2012) and Raddatz et al. (2017) use EPFR data to regress portfolio allocation of funds across countries on variables such as lagged portfolio shares, buy-and-hold portfolio shares and valuation effects. The impact of valuation effects and portfolio rebalancing is analyzed by Camanho et al. (2018) using another source of mutual fund data. However, the regressions in this literature do not have a clear theoretical foundation. In particular, there is no forward-looking dimension of portfolios. Expected excess returns are not included.\footnote{Curcuru et al. (2014) stress the role of future returns, but use \textit{ex post} realized returns in their regressions.}

Outside the open economy literature, there is a literature on individual portfolio choice that has documented significant portfolio inertia. This literature (e.g. Ameriks and Zeldes (2004), Bilias et al. (2010), Brunnermeier and Nagel (2008), Mitchell et al. (2006)) uses data on portfolio allocation by individual households. It is consistent with gradual portfolio adjustment, although (like the other literatures discussed above) it does not relate portfolio allocation to expected excess returns as in standard portfolio theory. An exception is the recent paper by Giglio et al. (2019), that relates equity portfolio shares to expected returns based on survey data of US-based Vanguard investors. They find that portfolio shares depend positively on reported equity return expectations, but that responsiveness to expected equity returns is too weak to make sense in the context of the frictionless mean-variance portfolio choice model (implied risk-aversion is excessive).
They further provide evidence that changes in expected returns have limited explanatory power for when investors trade, but help predict the direction and the magnitude of trading conditional on its occurrence. They argue that the evidence is consistent with infrequent trading.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the predictability of international equity return differentials. Section 4 presents results from estimating the fund-level portfolio regressions. Section 5 considers various types of fund heterogeneity. Section 6 concludes.

2 A Model of Portfolio Allocation with Financial Frictions

Our portfolio theory focuses on just two assets and therefore one portfolio share. There are several reasons for doing so. First, even though our mutual funds invest in many countries, one can always think of the optimal portfolio share invested in a particular country, conditional on the relative allocation to the other countries. Second, inverting a large variance matrix of portfolio returns can make results very sensitive to small measurement errors of numerous covariance terms.\(^5\) A related problem is that it is hard to introduce heterogeneity of perceived asset return risk across funds when the risk involves so many variance and covariance terms simultaneously. Finally, such an approach would involve a large matrix \(A\) of portfolio friction parameters in the portfolio cost \(x'Ax\), where \(x\) is a vector of deviations from benchmark portfolios. Our approach has the advantage of leading to a simple optimal portfolio expression that can easily be brought to the data.

Consider a mutual fund allocating its portfolio to countries 1 and 2, with gross returns \(R_{1,t+1}\) and \(R_{2,t+1}\) from \(t\) to \(t + 1\). When applying this to the data below, the mutual fund is a US fund, country 1 is a specific foreign country, and country 2 is the aggregate of all other foreign countries. \(R_{2,t+s}\) therefore represents the overall return in relevant countries outside of country 1.

The share invested in country 1 is \(z_t\) and the excess return of that country is \(er_{t+1} = R_{1,t+1} - R_{2,t+1}\). We consider a structure similar to Gärleanu and Peder- 

\(^5\)A good example of this is Adler and Dumas (1983), who consider the optimal equity portfolio allocation across 9 countries and find quite extreme and implausible allocations that are inconsistent with equilibrium.
sen (2013), where funds maximize the present discounted value of risk-adjusted portfolio returns, but face costs of deviating from passive benchmarks. There may be costs of transactions, information gathering or decision making. Mutual funds may also be evaluated against passive benchmarks by investors (e.g., see Berk and Green, 2004), making them reluctant to deviate from these benchmarks.

We assume a quadratic cost of deviating from two types of benchmarks. The first is the past portfolio share \( z_{t-1} \), with a quadratic deviation cost \( 0.5 \lambda_1 (z_t - z_{t-1})^2 \). The second benchmark is the buy-and-hold portfolio \( z_{bh}^t \), with a deviation cost of \( 0.5 \lambda_2 (z_t - z_{bh}^t)^2 \). The only difference between the two benchmarks is the valuation effect between \( t-1 \) and \( t \). The extent of financial frictions is given by \( \lambda_1 \) and \( \lambda_2 \), which should be determined empirically. We will refer to the aggregate portfolio friction as \( \theta = \lambda_1 + \lambda_2 \).

The fund’s objective function is:

\[
\sum_{s=0}^{\infty} \beta_s E_t \left( z_{t+s} R_{1,t+s+1} + (1 - z_{t+s}) R_{2,t+s+1} \right) - 0.5 \gamma \sum_{s=0}^{\infty} \beta_s \left( z_{t+s}^2 \sigma_1^2 + (1 - z_{t+s})^2 \sigma_2^2 + 2z_{t+s}(1 - z_{t+s})\sigma_{12} \right) - 0.5 \sum_{s=0}^{\infty} \beta_s E_t \left( \lambda_1 (z_{t+s} - z_{t+s-1})^2 + \lambda_2 (z_{t+s} - z_{bh}^t)^2 \right) \tag{1}
\]

where \( \gamma \) measures risk aversion and for any \( s \geq 0 \): \( \sigma_i^2 = var(R_{i,t+s+1}) \) and \( \sigma_{12} = cov(R_{1,t+s+1}, R_{2,t+s+1}) \). The term in brackets in the second line is the variance of the portfolio return at \( t + s + 1 \). The fund therefore maximizes the present discounted value of the certainty equivalent of future portfolio returns minus the costs of deviating from the benchmarks.

The Appendix shows that the optimal portfolio is given by:

\[
z_t = a_1 + a_2 \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} z_{t-1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} z_{bh}^t \right) + a_3 \sum_{s=1}^{\infty} \delta^{s-1} E_t er_{t+s} \tag{2}
\]

The linearized buy-and-hold portfolio is \( z_{bh}^t = z_{t-1} + \bar{z}(1 - \bar{z})er_t \), where \( \bar{z} \) is the steady-state portfolio share in country 1. Denoting the variance of the excess return \( er_{t+1} \) as \( \sigma_{er}^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \), the Appendix shows that the parameters are\(^6\)

\[
a_2 = \omega; \quad a_3 = \frac{\omega}{\theta}; \quad \delta = \beta \omega
\]

\(^6\)Here we abstract from an additional term in the expression for \( a_3 \) that numerically is very close to zero.
\[
\omega = \frac{2\theta}{\Gamma + (1 + \beta)\theta + \sqrt{\Gamma^2 + (1 - \beta)^2\theta^2 + 2(1 + \beta)\Gamma\theta}}
\]

where

\[
\Gamma = \gamma \sigma_{e}^2
\]

is what we refer to as a risk parameter that depends on both risk and risk-aversion. It is easily checked that \( \omega < 1 \). In the simple case where \( \beta = 0 \) and agents are myopic, this becomes

\[
z_t = a_1 + \frac{\theta}{\Gamma + \theta} \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} z_{t-1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} z_{bh} \right) + \frac{1}{\Gamma + \theta} E_t e_{r, t+1}
\]

The optimal portfolio depends on a weighted average of the two benchmark portfolios and a present discounted value of future expected excess returns. The relative weight on the two benchmark portfolios depends on the relative size of the two portfolio frictions. In the absence of portfolio adjustment costs, the optimal portfolio is independent of \( \beta \) and is simply given by

\[
z_t = a_1 + \frac{E_t e_{r, t+1}}{\Gamma}
\]

It then does not depend on the benchmark portfolios and only depends on the expected excess return over the next period. Portfolio adjustment costs lead the portfolio today to depend on expected excess returns further into the future as investors wish to smooth portfolio changes in anticipation of changes in expected excess returns.\(^7\)

The aggregate portfolio friction \( \theta \) and risk parameter \( \Gamma \) critically drive the sensitivity to the benchmark portfolios and expected excess return in (2). Investors care about maximizing expected returns, minimizing portfolio risk and minimizing deviations from the benchmark portfolios. The relative weight of these three objectives depends on the parameters \( \theta \) and \( \Gamma \). An increase in the aggregate portfolio friction \( \theta \) naturally gives more weight to the benchmark portfolios and less weight to expected excess returns, raising \( a_2 \) and lowering \( a_3 \). An increase in the risk parameter \( \Gamma \) naturally gives more weight to minimizing portfolio risk, lowering the

\(^7\)Bacchetta, van Wincoop, and Young (2020) obtain a similar dependence of the portfolio on the present value of expected future excess returns in a framework where instead of a cost of portfolio adjustment there is a given probability \( p \) of changing the portfolio each period, analogous to Calvo price setting.
weight $a_2$ on the benchmark portfolios and the sensitivity $a_3$ to expected excess returns. We can also see that investors give more weight to expected excess returns further into the future (higher $\delta$) when the time discount rate $\beta$ is higher, the aggregate portfolio friction $\theta$ is higher and the risk parameter $\Gamma$ is lower.

In connecting reduced form parameter estimates from portfolio regressions more clearly to the structural parameters, it helps to write the portfolio share as a function of the average and the difference of the two benchmark portfolios. Specifically, in Section 4 we estimate regressions of the form

$$z_t = b_0 + b_1 \frac{z_{t-1} + z_{bh}^t}{2} + b_2 (z_{t-1} - z_{bh}^t) + b_3 \sum_{s=1}^{\infty} \delta^{s-1} E_t r_{t+s}$$

An increase in the aggregate portfolio friction $\theta$ raises the overall weight $b_1$ on the benchmark portfolios and reduces the weight $b_3$ on the expected excess returns. An increase in the risk parameter $\Gamma$ reduces both the overall weight $b_1$ on the benchmark portfolios and the weight $b_3$ on expected excess returns. Finally, a positive (negative) $b_2$ signifies that the portfolio friction $\lambda_1$ is larger (smaller) than the portfolio friction $\lambda_2$. Mathematically, the structural parameters are linked to these reduced form parameters as follows:

$$\theta = \lambda_1 + \lambda_2 = \frac{b_1}{b_3}$$

$$\lambda_1 - \lambda_2 = \frac{2b_2}{b_3}$$

$$\Gamma = \frac{1}{b_3} (1 - b_1)(1 - \beta b_1)$$

We also have $\delta = \beta b_1$. To estimate (6), we first need estimates of expected return differentials. We turn to this in the next section.

### 3 Predicting Cross-Country Equity Return Differentials

This section describes how we construct estimates for expected return differentials. After reviewing the empirical strategy, we show that return differentials can be predicted by standard variables: dividend-price, earnings-price and momentum. We present results for pooled linear regressions and High-Minus-Low trading strategies.
3.1 Outline

The previous section focused on the portfolio share allocated to country 1, with the remainder allocated to the aggregate of all other foreign countries. In the empirical analysis we focus on the portfolio share allocated by a US based fund to country \( n \), with the remainder invested in other ”relevant” foreign countries. The excess return is then fund specific. It is the equity return in country \( n \) minus a weighted average of the equity returns in other foreign countries, with the weights corresponding to the portfolio shares of the fund.

In this section, rather than considering excess return predictability for individual funds, we consider return differentials relative to the US. Specifically, the excess return for country \( n \) at \( t + s \) is \( er_{n,t+s} = R_{n,t+s} - R_{US,t+s} \), where \( R_{n,t+s} \) and \( R_{US,t+s} \) are the equity returns of country \( n \) and the US at \( t + s \). As discussed further in the next section, the expected excess return for a specific fund can easily be computed once we know the expected excess returns relative to the US for individual countries. For a fund \( i \) it is simply the expected excess return \( E_t er_{n,t+s} \) for country \( n \) minus the weighted average of expected excess returns \( E_t er_{m,t+s} \) of other foreign countries, using the portfolio shares of fund \( i \) for the weights.

In the theory, portfolio shares depend on a present discounted value of expected excess returns at all future dates. We will indeed use such present values when applying the theory to US mutual fund portfolio shares in the next section. But in this section we consider either the predictability of excess returns \( er_{n,t+1} \) over the next month or cumulative excess returns \( er_{n,t,t+k} = er_{n,t+1} + \ldots + er_{n,t+k} \) over the next \( k \) months. This section starts by showing that equity return differentials \( er_{n,t,t+k} \) are indeed predictable at different horizons. We first consider a large number of 73 countries that have data in the MSCI database. Predictability from regressions is confirmed by sorting countries according to the value of predictive variables and using High-Minus-Low (HML) trading strategies. While this section reports results using the entire sample period, when we turn to portfolio analysis we use recursive regressions to compute true forecasts.

3.2 Panel Regressions

We use pooled regressions over 73 countries with monthly data from January 1970 to March 2019. All data in the baseline regressions come from MSCI, using the last trading day of the month. Since data availability starts later for many countries,
this gives us an unbalanced panel with more than 22,000 observations.\(^8\) Returns are computed from the MSCI total return index. We consider the following benchmark regression:

\[
er_{n,t,t+k} = \alpha + \alpha_n + \beta' X_{n,t} + \varepsilon_{n,t}
\]  

(10)

where \(X_{n,t}\) is a set of explanatory variables known at time \(t\). Following Petersen (2009), we include a country fixed effect and cluster standard errors by month.\(^9\) Using pooled data and assuming a common coefficient \(\beta\) allows us to get more precise estimates.\(^10\)

The explanatory variables in the benchmark specification are standard in the literature on stock return predictability,\(^11\) but here we consider the differential with the US. These variables are the differential in the log earning-price ratio \(dep_{n,t} = \ln(E/P)_{n,t} - \ln(E/P)_{US,t}\); the differential in the log dividend-price ratio \(ddp_{n,t} = \ln(D/P)_{n,t} - \ln(D/P)_{US,t}\); and momentum, measured by the current return differential \(er_{n,t-1,t}\). Since we take the log of the earning-price ratio, we omit the periods where it takes a negative value.\(^12\)

Table 1 shows the results of regression (10) for one-period ahead returns \(er_{n,t,t+1}\). We see that the three variables are strongly significant and have the expected sign. From the first column, it is interesting to notice that the small coefficient of 0.0426 on momentum implies that excess returns are not very persistent. In line with the literature on return predictability, the \(R^2\) is extremely low for short-horizon predictions.

The fit of equation (10) significantly improves when the horizon increases. Table 2 shows the results for one month (as in Table 1), 12 months, 24 months, and 60 months excess returns, using the three variables in the regression. We see that coefficient values increase with the horizon. Moreover, the \(R^2\) increases significantly, reaching 18.5% at the 60-month horizon.

The results in Tables 1 and 2 show that there is indeed predictability of stock market return differentials and that it is particularly strong at longer horizons.

\(^8\)There are 18 developed countries in 1970, increasing to 35 in 1988, 44 in 1993, etc.
\(^9\)Results do not change much if we include time fixed effects. We notice, however, that since we consider return differentials, global stock market shocks should not matter much.
\(^10\)Hjalmarsson (2010) shows that pooling across countries gives superior predictability.
\(^11\)See for example Koijen and Van Nieuwerburgh (2011) or Rapach and Zhou (2013) for surveys.
\(^12\)Negative values are observed during the Asian and the Scandinavian financial crises.
Table 1: Regressions One-Month Return Differential $r_{n,t,t+1}$

<table>
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<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td>Momentum</td>
<td>0.0426**</td>
<td>0.0426**</td>
<td>0.0439**</td>
<td>0.0441**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0167)</td>
<td>(0.0177)</td>
<td>(0.0173)</td>
<td>(0.0178)</td>
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<tr>
<td>Dividend-Price</td>
<td>0.00695***</td>
<td></td>
<td>0.00757***</td>
<td>0.00595***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00210)</td>
<td></td>
<td>(0.00208)</td>
<td></td>
<td>(0.00204)</td>
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</tr>
<tr>
<td>Earning-Price</td>
<td></td>
<td>0.00660***</td>
<td>0.00716***</td>
<td>0.00459**</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.00197)</td>
<td>(0.00196)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.000846</td>
<td>-0.00198</td>
<td>-0.00177</td>
<td>-0.00189</td>
<td>-0.00218</td>
<td>-0.00298*</td>
</tr>
<tr>
<td></td>
<td>(0.00146)</td>
<td>(0.00153)</td>
<td>(0.00149)</td>
<td>(0.00148)</td>
<td>(0.00152)</td>
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<tr>
<td>Observations</td>
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<td>22873</td>
<td>22033</td>
<td>22021</td>
<td>22856</td>
<td>21908</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
</tr>
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Notes: Table 1 reports regressions with 73 countries over the interval 1970:01-2019:02. All regressions include a country fixed effect.

Table 2: Regressions Return Differential - Different Horizons

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
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</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>0.0441**</td>
<td>0.332***</td>
<td>0.493***</td>
<td>0.902***</td>
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<td></td>
<td>(0.0178)</td>
<td>(0.0683)</td>
<td>(0.113)</td>
<td>(0.311)</td>
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<tr>
<td>Dividend-Price</td>
<td>0.00595***</td>
<td>0.0994***</td>
<td>0.229***</td>
<td>0.759***</td>
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<tr>
<td></td>
<td>(0.00204)</td>
<td>(0.00972)</td>
<td>(0.0198)</td>
<td>(0.0679)</td>
</tr>
<tr>
<td>Earning-Price</td>
<td>0.00459**</td>
<td>0.0372***</td>
<td>0.0935***</td>
<td>0.345***</td>
</tr>
<tr>
<td></td>
<td>(0.00196)</td>
<td>(0.00903)</td>
<td>(0.0161)</td>
<td>(0.0564)</td>
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<tr>
<td>Constant</td>
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<td>-0.133***</td>
</tr>
<tr>
<td></td>
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<td>(0.00756)</td>
<td>(0.0143)</td>
<td>(0.0359)</td>
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<td>20254</td>
<td>17676</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.009</td>
<td>0.064</td>
<td>0.104</td>
<td>0.185</td>
</tr>
</tbody>
</table>

Notes: Table 2 reports regressions with 73 countries over the interval 1970:01-2019:02. All regressions include a country fixed effect.
3.3 Trading Strategies

To evaluate the prediction performance and estimate the economic significance of predictability, we follow the literature in building trading strategies based on the three predictors used in the previous regressions. The analysis is close to Cenedese et al. (2016). For each month, we sort countries into quintiles based on their values of momentum, dividend-price differential, or earning-price differential. The one fifth of countries whose predictors have the lowest value are allocated to the first quintile Q1, the next fifth to the second quintile Q2, and so on. Thus, Q1 should contain low excess returns and Q5 high excess returns. For each pair month-quintile, we take the equally weighted average equity return differential with the US. Then, for each predictor variable we form a long-short HML portfolio, obtained by going long on Q5 and short on Q1. The sample is January 1970 to February 2019.

Table 3 reports the average annualized equity return by quintile and the portfolio return when the predictor is momentum, the dividend-price ratio, or the earning-price ratio. The table shows that returns tend to be higher for higher quintiles, i.e., higher values of momentum, dividend-price, or earning-price are associated with higher returns. This is confirmed by the results in the last column that show large returns from HML portfolios. These results therefore demonstrate the economic significance of equity return predictability, which justifies that time-varying expected excess returns are taken into account in actual portfolio allocations.

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>.69</td>
<td>1.01</td>
<td>-.28</td>
<td>2.33</td>
<td>9.56</td>
<td>8.87</td>
</tr>
<tr>
<td>Dividend-Price</td>
<td>-2.02</td>
<td>2.62</td>
<td>.86</td>
<td>1.2</td>
<td>4.42</td>
<td>6.44</td>
</tr>
<tr>
<td>Earning-Price</td>
<td>.06</td>
<td>-1.1</td>
<td>.11</td>
<td>3.31</td>
<td>5.49</td>
<td>5.43</td>
</tr>
</tbody>
</table>

Notes: Table 3 reports mean annualized equity excess returns relative to the US by sorting countries-months in quintiles based on their values for momentum, dividend-price and earning-price. HML shows the return from borrowing in Q1 and investing in Q5. Sample: 73 countries over the horizon 1970:01-2019:02.

13It is also possible to build strategies based on a combination of the three variables.
14These results are in line with Cenedese et al. (2016) who use a more restricted sample.
4 International Equity Portfolios: Fund-Level Analysis

4.1 Basic Framework

In this section we analyze the allocations of portfolios across countries using the model developed in Section 2. We use panel data of US-based equity funds that report country allocations to EPFR and that have more than 5 million USD in assets under management (AUM) at the end of the sample. We restrict the sample to funds that are reporting at least 12 months. This leaves us with a total of 316 funds. We consider the main 35 investment countries. These countries represent on average 90% of international equity portfolios of our US mutual funds. At the fund level, we drop those countries in which the fund invests on average less than 2%. The sample runs from January 2002 to July 2016. We only consider observations where fund $i$ positively invests in country $n$ both at time $t$ and $t-1$. This results in 316,732 observations.

For each fund $i$ we compute $z_{i,n,t}$ as the share invested in country $n$ out of the 35 non-US countries at time $t$. We construct the expected return differential relevant to fund $i$ as

$$E_t E_{i,n,t,t+k} = \sum_{s=1}^{k} \delta^{s-1} E_t \left( R_{n,t+s} - \sum_{m \neq n} z_{i,m,-n} R_{m,t+s} \right)$$  \hspace{1cm} (11)

The term in brackets is the return in country $n$ at $t+s$ minus the weighted average of the returns in the other countries, with the weights corresponding to the average portfolio weights of fund $i$. Specifically, $\bar{z}_{i,m,-n}$ is the average portfolio share of fund $i$ in country $m$ of its foreign equity holdings outside of country $n$. We then take a present discounted value of these excess returns with discount rate $\delta$. While in the theory $k = \infty$, in the empirical applications $k$ is necessarily finite. We consider

---

15 The countries are: Australia, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Finland, France, Germany, Hong-Kong, India, Indonesia, Ireland, Israel, Italy, Japan, Malaysia, Mexico, Netherlands, Norway, Peru, Philippines, Poland, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, and U.K.. We dropped those countries in which less than 10 funds invest, as well as Argentina, the Russian Federation and the United Arab Emirates because of limited data coverage.

16 The Online Appendix shows that results are virtually identical when we replace $\bar{z}_{i,m,-n}$ with the contemporaneous weights $z_{i,m,-n,t}$. 
The valuation effect in the buy-and-hold portfolio is computed using fund-specific EPFR data. We have
\[ z_{i,n,t}^{bh} = z_{i,n,t-1} + \bar{z}_{i,n}(1 - \bar{z}_{i,n})(R_{n,t} - R_{i,p,t}), \]
where \( R_{n,t} \) is country \( n \) stock market return computed from MSCI and \( R_{i,p,t} \) is the portfolio return of fund \( i \) given in EPFR data.

Consistent with the theoretical portfolio expression (6), we consider the following regression:
\[ z_{i,n,t} = b_0 + b_{in} + b_1 \frac{z_{i,n,t-1} + z_{i,n,t}^{bh}}{2} + b_2 \left( z_{i,n,t-1} - z_{i,n,t}^{bh} \right) + b_3 E_t \epsilon_{i,n,t}^{r,t+k} + \epsilon_{i,n,t} \]  

In the benchmark case, we assume a fund-country fixed effect because the mean portfolio share varies across fund-country combinations. For example, as shown in the Appendix, the intercept is higher when the perceived excess return risk \( \sigma_{er}^2 \) is lower, which is fund-country dependent. The other parameters are assumed to be the same across funds, but we examine various sources of heterogeneity in Section 5.

4.2 Benchmark Results

Table 4 presents the benchmark estimation of equation (13) with expected return differentials at different horizons. The last column is a bit different as it is based on a portfolio regression for aggregate EPFR data, which we will discuss at the
end of this subsection. Column (1) does not include fund-country fixed effects, while the other columns for fund-level portfolio regressions do. We will link the regression estimates in Table 4 to the structural parameters $\lambda_1$, $\lambda_2$ and $\Gamma$ from the theory. We will also explain why a discount rate of $\delta = 0.9$ is appropriate based on the theory. But before doing so, we first discuss the key takeaways from Table 4.

**Table 4: Portfolio Regressions, Benchmark**

<table>
<thead>
<tr>
<th></th>
<th>Fund-Level</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$(z_{i,n,t-1} + z_{bh,i,n,t})/2$</td>
<td>0.992***</td>
<td>0.926***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$(z_{i,n,t-1} - z_{bh,i,n,t})$</td>
<td>0.196**</td>
<td>0.223***</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.079)</td>
</tr>
<tr>
<td>$E_{t}er_{i,n,t,t+1}^{0.9}$</td>
<td>0.162***</td>
<td>0.674***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>$E_{t}er_{i,n,t,t+24}^{0.9}$</td>
<td></td>
<td>1.650***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.292)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fund-Country FE</th>
<th>No</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes</th>
<th>Yes$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>316732</td>
<td>316732</td>
<td>316732</td>
<td>196828</td>
<td>5918</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.986</td>
<td>0.987</td>
<td>0.987</td>
<td>0.988</td>
<td>0.999</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors clustered by month in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Table 4 estimates equation (13) with 35 countries over the horizon 2002:01-2016:07. The constant is included in all regressions but not shown. Columns (1)-(4) show the results at the fund level, while column (5) aggregates funds positions. $^a$In column (5), we use a country fixed effect.

There are three lessons from Table 4. First, expected return differentials are strongly significant for all the horizons considered. The coefficient on the five-year ahead return is higher (column (4)), but is not fully comparable since the number of observations is significantly smaller.\(^{17}\) Second, the coefficient on the average of

\(^{17}\)The reason is that we lose about three years of portfolio data at the end of the sample, where there are more observations.
the two benchmark portfolios is strongly significant, with a very small standard error. With the exception of column (1), the coefficient is about 0.928 with a standard error in the range of 0.004 to 0.006. The coefficient is very close to 1 in column (1). This compensates for the absence of fund-country fixed effects, which shows that it is important to include these fixed effects. Third, the coefficient on $z_{i,n,t-1} - z_{i,n,t}^bh$ is positive, indicating that there is a larger cost of deviating from the lagged portfolio share than the buy-and-hold portfolio share. However, the standard error is much higher here and in column (3) we cannot reject at the 5 percent significance level that the two frictions are identical. So while the evidence strongly indicates that the portfolio frictions are important, it tells us less about the relative importance of the two frictions.

Various robustness tests, presented in the Online Appendix, confirm these results. In particular, the results are similar when we focus on the latter part of the sample, starting in 2010 or 2012, where there is a larger number of funds. Similar results also apply when we focus on funds that report for the whole sample or for at least 24 months. We also examine regressions using different predictors for return differentials.

In connecting the estimates in Table 4 to the structural parameters from the theory, we will assume a time discount rate of $\beta = 0.97$. The time discount rate is not identified by the reduced form parameter estimates. We consider alternative values of $\beta$ in the Online Appendix. While $\beta = 0.97$ may seem low with monthly data, it is important to keep in mind that the average turnover of portfolio managers is 2 percent per month (see Kostovetsky and Warner (2015)). An even lower $\beta$ may need to be assumed if we take into account that many funds have short lives. We will focus on column (3) of Table 4. Column (1) is mainly there to show the importance of fund-country fixed effects. Column (2) assumes myopic agents, where $\beta = 0$. Column (4) assumes a 60-month instead of a 24-month horizon for the present value of expected excess returns. But since we use $\delta = 0.9$ (discussed below), almost all of the weight is on expected excess returns over the first 24 months, even if we use a 60-month horizon. Adding a longer horizon has the drawback that it leads to a significant reduction in the number of observations.

We find $\delta = 0.9$ as follows. We re-estimate (13) for a wide range of $\delta$. Then we use that the theory implies $\delta = \beta b_1$ and choose $\delta$ such that the assumed value in computing the present value of expected excess returns corresponds to $\beta b_1$ implied by the portfolio regression. It turns out that $b_1$ is not very sensitive to the assumed
δ and always close to 0.928 as in column (3). This implies \( \delta = 0.97 \times 0.928 = 0.90 \).

Next we use equations (7), (8) and (9) to determine the structural parameters from the estimates of \( b_1, b_2 \) and \( b_3 \). The results, together with standard errors and 95 percent confidence intervals, are reported in Table 5. It also reports a measure of the overall cost of portfolio adjustment that is discussed below. Several conclusions can be drawn regarding the portfolio friction parameters. First, there are costs of deviating from both benchmark portfolios: \( \lambda_1 \) and \( \lambda_2 \) are both positive and statistically significant and so is the aggregate portfolio friction \( \theta = \lambda_1 + \lambda_2 \). Second, while the point estimates imply \( \lambda_1 - \lambda_2 > 0 \), we cannot reject \( \lambda_1 = \lambda_2 \). Third, we can compute the overall cost of portfolio frictions from (1). This is the cost in terms on an equivalent drop in the expected portfolio return. This cost, when annualized and expressed in basis points, is

\[
Cost = 60000 \left( \lambda_1 \text{var}(z_{i,n,t} - z_{i,n,t-1}) + \lambda_2 \text{var}(z_{i,n,t} - z_{bh}^{i,n,t}) \right)
\]  

(14)

Based on all observations, \( \text{var}(z_{i,n,t} - z_{i,n,t-1}) \) and \( \text{var}(z_{i,n,t} - z_{bh}^{i,n,t}) \) are respectively 0.0001258 and 0.0001437. The cost of portfolio frictions is reported at the bottom of Table 5. The overall cost is 7.8 basis points, with a 95 percent confidence interval from 6 to 11 basis points. One should keep in mind though that this is the cost of changing just one portfolio share in one country. Funds face additional costs associated with changing the allocation across the remaining countries that they invest in.

### Table 5: Estimated Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate (s.e.)</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 )</td>
<td>0.680 (0.074)</td>
<td>[0.550, 0.840]</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.314 (0.144)</td>
<td>[0.102, 0.661]</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>0.0077 (0.0012)</td>
<td>[0.0059, 0.0105]</td>
</tr>
<tr>
<td>( \theta = \lambda_1 + \lambda_2 )</td>
<td>0.994 (0.156)</td>
<td>[0.775, 1.380]</td>
</tr>
<tr>
<td>( \lambda_1 - \lambda_2 )</td>
<td>0.366 (0.168)</td>
<td>[-0.020, 0.640]</td>
</tr>
<tr>
<td>Cost</td>
<td>7.84 (1.31)</td>
<td>[5.99, 11.08]</td>
</tr>
</tbody>
</table>

Notes: Table 5 reports the estimates, standard errors and 95 percent confidence intervals of the structural parameters implied by the regression reported in Table 4, column 3. It is based on 100,000 draws from the distribution of the reduced form parameters \([b_1, b_2, b_3]\). Cost is equal to the overall cost of portfolio frictions in terms of an equivalent drop in the expected portfolio return, annualized and in basis points.
Table 5 also reports $\Gamma = \gamma \sigma_{e}^{2}$. The point estimate is 0.0077, with a standard error of 0.0012. The mean variance of the excess return in the data is 0.0053. This implies a rate of risk aversion of $\gamma = 1.45$, which is quite reasonable. By contrast, if we just regress on the one-period expected excess return (plus the fund-country fixed effects), as would be appropriate in the absence of portfolio frictions, we obtain a coefficient of 1.8 (s.e.=0.12). Since the coefficient on the one-period expected excess return in the frictionless model is $1/\Gamma$, it would imply $\Gamma = 0.556$. With a variance of the excess return of 0.0053, this implies $\gamma = 105$, which is clearly excessive.\footnote{Giglio et al. (2019) also make the point that excessive risk aversion is needed to account for the response of portfolios to expected returns.} For a more reasonable, lower, level of risk aversion, the coefficient on the expected excess return would be far higher in the frictionless model. Therefore the estimates of $\lambda_1$, $\lambda_2$ and $\Gamma$ all provide evidence of the importance of portfolio frictions.

Finally, consider the last column of Table 4. Here we run a portfolio regression based on portfolio shares in the 35 countries of the aggregate of all EPFR funds in our data. This is done for the same expected excess return variable as in column (3), but using aggregate positions to weigh reference countries. There remains a large and significant weight on the average of the benchmark portfolios and a statistically significant coefficient on the expected excess return. However, the coefficients have a very different magnitude than the comparable coefficients in column (3) based on fund-level regressions. With aggregate portfolio data we see that the coefficient on the average benchmark portfolio share is much higher (0.997 versus 0.928), the lagged portfolio share is less important than the buy-and-hold portfolio (opposite to fund-level data), while the expected excess return coefficient is much smaller than for fund-level data.

We discuss in the Online Appendix what drives these differences between aggregate portfolio regressions versus fund-level regressions. Some explanations have to do with the fact that aggregating the fund-level portfolio expressions (13) over all funds delivers a different portfolio expression than simply applying (13) directly to aggregate portfolio data, even if the coefficients are the same for all funds. For example, even if all funds are equally responsive to the expected excess return, the aggregate portfolio share is less responsive simply because only a limited fraction of funds invests in a particular country. Another set of explanations is associated with heterogeneity of the coefficients across funds, a topic that we address in the
next section.

4.3 Portfolio Dynamics

It is useful to consider the implication of the results above for the dynamic response of portfolios to an expected excess return innovation and compare the case with the estimated portfolio frictions to the frictionless case. For the case with frictions, the expected excess return variable is \( y_{i,n,t} = E_t \text{er}_{i,n,t,t+24}^{0.9} \). We estimate an AR(1) process for \( y_{i,n,t} \) for each \((i,n)\), and then average the AR coefficient and standard deviation of the innovation across all \((i,n)\). The AR coefficient is 0.856 and the standard deviation of the expected excess return innovation is 0.00912. In the frictionless case the expected excess return is \( E_t \text{er}_{i,n,t,t+1} \), which has an average AR coefficient of 0.535 and standard deviation 0.00188 of the expected excess return innovation.

We make two additional assumptions. First, for the purpose of this exercise we only include the lagged portfolio share in the regression in order to abstract from valuation effects in the buy-and-hold portfolio. This regression is shown in the Online Appendix. The coefficient on the lagged portfolio share is 0.926 and the coefficient on the expected excess return variable is 1.253. Second, we need to make an assumption about the portfolio response in the frictionless case. We cannot use the estimated response when regressing \( z_{i,n,t} \) on \( E_t \text{er}_{i,n,t,t+1} \) as that is based on data that provide strong evidence of portfolio frictions. As shown in (5), in the frictionless case the coefficient \( b_3 \) is equal to \( 1/(\gamma \sigma^2_{er}) \). We will use the average variance of the excess return in the data of 0.0053 and assume a rate of risk aversion of \( \gamma = 10 \). This may seem rather high, but simplifies a visual comparison of the portfolio response in the two cases. If instead \( \gamma = 2 \), the impulse response in the frictionless case is simply 5 times as high as shown.

Figure 1 shows the results. The initial portfolio response to a one standard deviation expected excess return innovation is much larger in the frictionless case. For a lower rate of risk aversion it would have been even much higher. We also see significant portfolio persistence with the estimated portfolio friction. The portfolio response peaks after 9 months, while in the frictionless case it peaks at the time of the shock and dies out quickly. Bacchetta and van Wincoop (2019) refer to the initial portfolio response as return sensitivity and the gradual portfolio response as portfolio persistence. They show in a model for the foreign exchange mar-
ket with portfolio frictions that both diminished return sensitivity and increased portfolio persistence are key to accounting for a variety of currency excess return predictability puzzles.

Figure 1: Impulse Response Portfolio Share to Expected Excess Return Shock

Notes: Figure 1 shows the impulse response of portfolio share to an expected excess return shock.

5 Heterogeneity

So far we have assumed that structural and reduced form parameters are the same for all funds and countries. In this section we consider two types of heterogeneity. The first is heterogeneity across fund-country pairs that leads to differences in the average portfolio shares $\bar{z}_{in}$. The second is heterogeneity across funds of various types. In both cases we investigate how differences in reduced form parameters are associated with differences in the structural parameters.

5.1 Frictions by Size of Portfolio Shares

The average portfolio share $\bar{z}_{in}$ in the sample depends on the fund-country combination and varies considerably. Panel A of Figure 2 shows the cumulative di-

20
tribution of all $\bar{z}_{in}$ observations. In the lowest decile of $\bar{z}_{in}$ observations funds invest between 2 and 2.9 percent in a country, while in the largest decile funds invest more than 21 percent in a country. The regressions reported above, and the implied structural parameters, combine data on all portfolio shares, large and small. But the structural parameters may well vary with the size of the portfolio share. Specifically, a higher perceived risk $\sigma_{er}^2$ implies a smaller mean portfolio share. We can therefore expect a higher risk parameter $\Gamma$ for smaller portfolio shares. Perceived risk may vary not just by country. Different funds may be more or less informed about individual countries and therefore perceive the risk to be different.

Figure 2: Cumulative Distribution $\bar{z}_{i,n}$

A. All Funds

B. By Activeness

C. By Size

D. By Strategy

Notes: Figure 2 plots the cumulative distribution of $\bar{z}_{i,n}$ for all funds (Panel A), active and passive funds (Panel B), large and small funds (Panel C) and global, emerging and regional funds (Panel D).

It is also possible that the portfolio frictions $\lambda_1$ and $\lambda_2$ vary with $\bar{z}_{in}$. Gârleanu
and Pedersen (2013) make an argument for a specification where the portfolio frictions are proportional to $\sigma^2_{er}$. In that case our portfolio friction parameters would be smaller for larger $\bar{z}_{in}$ as a larger $\bar{z}_{in}$ is associated with less perceived risk. It could also be that the portfolio cost depends on the percentage changes in portfolio shares rather than the absolute changes in portfolio shares. In that case our portfolio friction parameters would also be smaller for larger $\bar{z}_{in}$.

In order to consider the role of the size of portfolio shares, we add interactions of the dependent variables with $\bar{z}_{in}$. The results are reported in Table 6. The first column shows that interactions of $\bar{z}_{in}$ with the average and difference of the benchmark portfolios gives coefficients that are not statistically significant. We therefore focus on the second column, where we only add the interaction with the expected excess return variable. A higher $\bar{z}_{in}$ implies a significantly higher coefficient on the expected excess return. The point estimate in the second column implies that the coefficient on the expected excess return varies from 0.37 when $\bar{z}_{in} = 0.02$ to 1.63 when $\bar{z}_{in} = 0.2$.

Figure 3 reports structural coefficients as a function of $\bar{z}_{in}$, and the associated 95 percent confidence bands. Rather than $\lambda_1$ and $\lambda_2$, we only report their sum and difference. We also report $\Gamma$ and the cost (14) of portfolio frictions. To compute the cost of portfolio frictions, we first regress $\text{var}(z_{i,n,t} - z_{i,n,t-1})$ and $\text{var}(z_{i,n,t} - z_{bh,i,n,t})$ on a constant, $\bar{z}_{in}$ and $\bar{z}_{in}^2$ and then compute the cost in (14) given $\lambda_1$ and $\lambda_2$ and the fitted variances as functions of $\bar{z}_{in}$.

Panel A of Figure 3 shows that as conjectured above, the aggregate portfolio friction $\theta = \lambda_1 + \lambda_2$ is smaller for larger $\bar{z}_{in}$. Panel 3B shows that we can barely reject that $\lambda_1 = \lambda_2$ at 95 percent confidence. Panel 3C shows that the risk parameter $\Gamma$ tends to be smaller when $\bar{z}_{in}$ is larger. It is exactly this lower risk that is responsible for higher portfolio shares in the first place. Finally, Panel 3D shows that the cost of portfolio frictions does not depend on $\bar{z}_{in}$ in a way that is statistically significant. While the aggregate portfolio friction $\theta$ is smaller for larger $\bar{z}_{in}$ (Panel 3A), the deviations from the benchmark portfolios measured by the variance of $z_{i,n,t} - z_{i,n,t-1}$ and $z_{i,n,t} - z_{bh,i,n,t}$ are naturally larger for larger $\bar{z}_{in}$. The cost of portfolio adjustment is mostly between 6 and 15 basis points.
Table 6: Portfolio Regressions, Interaction with $\bar{z}_{in}$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(z_{i,n,t-1} + z_{ih,i,n,t})/2$</td>
<td>0.916***</td>
<td>0.927***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$(z_{i,n,t-1} - z_{ih,i,n,t})$</td>
<td>0.142</td>
<td>0.179**</td>
</tr>
<tr>
<td></td>
<td>(0.159)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>$\bar{z}<em>{i,n} \times (z</em>{i,n,t-1} + z_{ih,i,n,t})/2$</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td></td>
</tr>
<tr>
<td>$\bar{z}<em>{i,n} \times (z</em>{i,n,t-1} - z_{ih,i,n,t})$</td>
<td>0.139</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td></td>
</tr>
<tr>
<td>$E_{t}er_{i,n,t,t+24}^{0.9}$</td>
<td>0.239</td>
<td>0.229*</td>
</tr>
<tr>
<td></td>
<td>(0.158)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>$\bar{z}<em>{i,n} \times E</em>{t}er_{i,n,t,t+24}^{0.9}$</td>
<td>6.812***</td>
<td>6.986***</td>
</tr>
<tr>
<td></td>
<td>(1.502)</td>
<td>(1.645)</td>
</tr>
</tbody>
</table>

Observations: 316732 316732  
$R^2$: 0.987 0.987

Standard errors clustered by month in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Table 6 reports regressions for 35 countries over the interval 2002:01-2016:07. All regressions include a fund-country fixed effect. The constant is included but not shown.
Figure 3: Structural Parameters as a function of $\bar{z}_{i,n}$

A. Aggregate Portfolio Frictions

B. Relative Portfolio Frictions

C. Risk Parameter $\Gamma$

D. Cost Portfolio Frictions (basis points)

Notes: The charts of Figure 3 show point estimates and 95 percent confidence bands for the structural parameters. Aggregate Portfolio Frictions (panel A) correspond to $\theta = \lambda_1 + \lambda_2$. Relative Portfolio Frictions (Panel B) correspond to $\lambda_1 - \lambda_2$. Panel D shows the cost of portfolio friction in basis points. This is based on 100,000 draws from the distribution of the estimated reduced form parameters.
5.2 Fund Heterogeneity

So far we have considered all funds in the sample. We now examine fund heterogeneity. We split the sample in three ways: active versus passive funds, small versus large funds and global versus regional and emerging markets funds. We investigate to what extent frictions are different for these different fund categories.

Panel A of Figure 4 shows a measure of how active funds are. We measure portfolio volatility of fund $i$ as $V^i = \frac{1}{T} \sum_t \sum_{n=1}^{35} |z_{i,n,t} - z_{bh,i,n,t}^{bh}|$, where $T$ is the number of months over which we observe portfolio shares for the fund. The portfolio volatility measure $V^i$ captures active trading. We see that volatility is particularly high for a small number of funds. We will refer to the top decile of funds in terms of $V^i$ as active funds and the remainder as passive funds. Panel B of Figure 4 shows how funds differ in size, measured by average AUM over the period. The Figure uses a semi-log scale as the difference in fund size is substantial. We refer to the bottom 50 percent of funds in terms of size as small funds and the others as large funds. The final categorization of funds, as global, regional or EM funds, relates more to the geographic orientation of the funds.

**Figure 4: Fund Heterogeneity by Portfolio Volatility and Size**

**A. Portfolio Volatility**

**B. Distribution of Fund Size**

*Notes:* Figure 4 shows the heterogeneity of funds. Panel A shows a measure of how active funds are. We measure portfolio volatility of fund $i$ as $V^i = \frac{1}{T} \sum_t \sum_{n=1}^{35} |z_{i,n,t} - z_{bh,i,n,t}^{bh}|$, where $T$ is the number of months over which we observe portfolio shares for the fund. Panel B shows how funds differ in size, measured by average AUM over the period. This chart uses a semi-log scale as the difference in fund size is substantial.

We first investigate if there is a systematic difference between the type of funds
and the size of portfolio shares. For example, a fund that invests in more countries is likely to have smaller portfolio shares, which we found to be associated with a higher risk parameter $\Gamma$ and higher aggregate portfolio friction $\theta$. Panels B to D of Figure 2 show the cumulative distribution of all $z_{in}$ by fund type. The differences are quite minor. There is virtually no difference between large and small funds. Active funds have slightly smaller portfolio shares than passive funds, but the difference is very small. Global funds have somewhat lower portfolio shares than the other funds. The difference relative to emerging market funds is again slight, only 0.01 on average. Regional funds tend to be a bit more specialized and have somewhat larger portfolio shares, on average 0.035 above that of global funds (0.116 versus 0.081). This would suggest slightly smaller $\Gamma$ and $\theta$ for regional funds, although the difference is probably not statistically significant based on Figure 3. We conclude that any differences in structural parameters across fund types is not likely to be associated with differences in the size of portfolio shares.

Table 7 shows reduced form regressions, where we add interactions of the three variables with dummies for fund categories. In column (1) the dummy $A$ is 1 when the fund is active and zero otherwise. In column (2) the dummy $L$ is 1 when the fund is large and 0 otherwise. In column (3) the dummy $R$ is 1 when it is a regional fund and zero otherwise and the dummy $EM$ is 1 when it is an emerging market fund and zero otherwise. We include all interactions of the dummies with the original three variables, except for column (1), where we have removed the interaction of $A$ with the expected excess return variable as it was insignificant.

By considering the interaction with the dummies, we can immediately evaluate differences in the reduced form parameters across fund types. First, we see that active, small and regional funds give relatively more weight to the lagged portfolio share and less weight to the buy-and-hold portfolio. The difference is especially large for active funds. Second, we see that the weight on the average benchmark portfolio is significantly lower for active, small and global funds. Finally, global and large funds are less responsive to expected excess returns.

The higher relative weight on the lagged portfolio share than the buy-and-hold portfolio share by active, small and regional funds suggests that these funds do more active rebalancing. In order to rebalance to keep the portfolio share unchanged, funds need to actively sell (buy) equity of countries whose relative price rises (falls). To investigate if these funds indeed rebalance more aggressively,
### Table 7: Portfolio Regressions, Interaction With Dummies

<table>
<thead>
<tr>
<th></th>
<th>(1) By Activeness</th>
<th>(2) By Size</th>
<th>(3) By Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>[(z_{i,n,t-1} + z_{i,n,t})/2]</td>
<td>0.941***</td>
<td>0.917***</td>
<td>0.905***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>[(z_{i,n,t-1} - z_{i,n,t})]</td>
<td>0.019</td>
<td>0.232**</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.095)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>[E_{t}r_{i,n,t+24}^{0.9}]</td>
<td>0.750***</td>
<td>1.267***</td>
<td>0.476***</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.192)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>[A \times (z_{i,n,t-1} + z_{i,n,t})/2]</td>
<td>-0.069***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[A \times (z_{i,n,t-1} - z_{i,n,t})]</td>
<td>0.365***</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.051)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[L \times (z_{i,n,t-1} + z_{i,n,t})/2]</td>
<td></td>
<td>0.030***</td>
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<tr>
<td></td>
<td></td>
<td>(0.007)</td>
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<tr>
<td>[L \times (z_{i,n,t-1} - z_{i,n,t})]</td>
<td></td>
<td>-0.240**</td>
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<td></td>
<td></td>
<td>(0.099)</td>
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<td>[L \times E_{t}r_{i,n,t+24}^{0.9}]</td>
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<td>-0.711***</td>
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<tr>
<td></td>
<td></td>
<td>(0.184)</td>
<td></td>
</tr>
<tr>
<td>[R \times (z_{i,n,t-1} + z_{i,n,t})/2]</td>
<td></td>
<td></td>
<td>0.041***</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>[EM \times (z_{i,n,t-1} + z_{i,n,t})/2]</td>
<td></td>
<td></td>
<td>0.031***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>[R \times (z_{i,n,t-1} - z_{i,n,t})]</td>
<td></td>
<td></td>
<td>0.105***</td>
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<td></td>
<td>(0.038)</td>
</tr>
<tr>
<td>[EM \times (z_{i,n,t-1} - z_{i,n,t})]</td>
<td></td>
<td></td>
<td>0.218</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.140)</td>
</tr>
<tr>
<td>[R \times E_{t}r_{i,n,t+24}^{0.9}]</td>
<td></td>
<td></td>
<td>0.782***</td>
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<td></td>
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<td>(0.217)</td>
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<tr>
<td>[EM \times E_{t}r_{i,n,t+24}^{0.9}]</td>
<td></td>
<td></td>
<td>0.601***</td>
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<td></td>
<td>(0.197)</td>
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<table>
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<th>Observations</th>
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<th>316732</th>
<th>316732</th>
</tr>
</thead>
<tbody>
<tr>
<td>[R^2]</td>
<td>0.987</td>
<td>0.987</td>
<td>0.987</td>
</tr>
</tbody>
</table>

Standard errors clustered by month in parentheses. * \(p < 0.10\), ** \(p < 0.05\), *** \(p < 0.01\).

Notes: Table 7 reports regressions for 35 countries over the interval 2002:01-2016:07. All regressions include a fund-country fixed effect. The constant is included but not shown.
Table 8 reports results for the following regression:

\[ z_{i,n,t} - z_{bh,i,n,t} = \alpha_0 + \alpha_{in} + \alpha_1 (z_{i,n,t-1} - z_{bh,i,n,t}) \]  

(15)

The variable \( z_{i,n,t-1} - z_{bh,i,n,t} \) is associated with valuation effects. It is more negative the higher the increase in the relative price of country \( n \) equity. The dependent variable is the change in the portfolio share associated with active portfolio reallocation. If \( \alpha_1 = 1 \) there is perfect rebalancing. Funds rebalance completely to keep the portfolio share the same as one month ago. When \( \alpha_1 = 0 \) there is no rebalancing. Table 8 confirms that indeed active, small and regional funds are more actively rebalancing. For active funds there is almost perfect rebalancing, with \( \alpha_1 = 0.942 \) (s.e.=0.05). For passive funds \( \alpha_1 = 0.52 \). We also see that the \( R^2 \) is 0.32 for active funds, considerably higher than for passive funds. We should emphasize though that the error term from the regression, which captures portfolio reallocation unrelated to rebalancing, is also significantly more volatile for active than for passive funds. Active funds engage in more active portfolio reallocation for reasons both related and unrelated to rebalancing. For the other funds especially small funds and EM funds stand out, each rebalancing about 75 percent of portfolio share changes due to valuation effects. Global and large funds rebalance less, though still close to 50 percent.

Table 8: Rebalancing Dynamics

<table>
<thead>
<tr>
<th>By Activeness</th>
<th>By Size</th>
<th>By Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Passive</td>
<td>(3) Small</td>
<td>(5) Global</td>
</tr>
<tr>
<td>(2) Active</td>
<td>(4) Large</td>
<td>(6) EM</td>
</tr>
<tr>
<td>(7) Regional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(zi,n,t-1 - z_{bh,i,n,t})</td>
<td>0.520***</td>
<td>0.752***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.103)</td>
</tr>
<tr>
<td></td>
<td>0.942***</td>
<td>0.493***</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.034)</td>
</tr>
<tr>
<td></td>
<td>0.752***</td>
<td>0.493***</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.034)</td>
</tr>
<tr>
<td></td>
<td>0.555***</td>
<td>0.761***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.146)</td>
</tr>
<tr>
<td></td>
<td>0.555***</td>
<td>0.761***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.146)</td>
</tr>
<tr>
<td></td>
<td>0.635***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
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<td>157954</td>
</tr>
<tr>
<td></td>
<td>23930</td>
<td>158778</td>
</tr>
<tr>
<td>R^2</td>
<td>0.101</td>
<td>0.205</td>
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<tr>
<td></td>
<td>0.323</td>
<td>0.089</td>
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<td></td>
<td>0.205</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>0.055</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>0.055</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>0.218</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table 8 estimates equation (15). All regressions include a fund-country fixed effect. The constant is included but not shown.

Next we turn to the structural parameters. It is useful to recall some key insights discussed in Section 2 regarding the role of the aggregate portfolio friction \( \theta \) and risk parameter \( \Gamma \). A higher \( \theta \) implies higher weight on the average benchmark
portfolio and less responsiveness to expected excess returns. A higher $\Gamma$ implies both less weight on the average benchmark portfolio and less responsiveness to expected excess returns. Figure 5 reports structural parameters and their 95 percent confidence intervals for all fund categories using the results from Table 7 and the associated variance of the vector of reduced form parameter estimates.

Figure 5: Fund Heterogeneity and Structural Parameters

A. Aggregate Portfolio Frictions

B. Relative Portfolio Frictions

C. Risk Parameter $\Gamma$

D. Cost Portfolio Frictions (basis points)

Notes: The charts of Figure 5 show point estimates and 95 percent confidence bands for the structural parameters for different groups of funds. Aggregate Portfolio Frictions (panel A) correspond to $\theta = \lambda_1 + \lambda_2$. Relative Portfolio Frictions (Panel B) correspond to $\lambda_1 - \lambda_2$. Panel D shows the cost of portfolio friction in basis points. This is based on 100,000 draws from the distribution of the estimated reduced form parameters.

For the aggregate portfolio friction $\theta$ we see from panel A that large funds and global funds stand out as having a larger friction. This is mainly inferred from the much weaker response of these funds to expected excess returns. For the risk parameter $\Gamma$, we see in panel C that active and global funds stand out as having significantly higher risk-aversion or perceived risk. This is inferred both from the lower weight on the benchmark portfolios and (in the case of global funds) the
weaker response to expected excess returns. For small, active and regional funds we see from panel B that $\lambda_1 > \lambda_2$. This is particularly the case for active funds. The higher friction associated of deviating from the lagged portfolio share leads to more active rebalancing by these funds, as also confirmed in Table 8.

Finally, Panel D reports the cost of portfolio frictions. Here especially active funds stand out as having a high cost, and to a lesser extent global funds as well. To understand this, Figure 6 reports the variance of $z_{i,n,t} - z_{i,n,t-1}$ and $z_{i,n,t} - z_{i,n,t}^{bh}$ by fund type. Active funds naturally stand out as having the highest deviations from both benchmark portfolios. Even though the aggregate portfolio friction $\theta$ is virtually the same for active and passive funds (Panel A of Figure 5), active funds bear a much higher cost by being more active. Our estimates indicate that this cost is in the range of 25 to 41 basis points. We do not find evidence that this higher cost of active portfolio management is compensated by correspondingly higher expected portfolio returns.$^{19}$

Figure 6: Variance Portfolio Deviations from Benchmarks

![Graph showing variance portfolio deviations from benchmarks](image)

**Notes:** Figure 6 reports the variance of $z_{i,n,t} - z_{i,n,t-1}$ and $z_{i,n,t} - z_{i,n,t}^{bh}$ by fund type.

$^{19}$To this end we consider the increase in the average portfolio return when replacing the mean portfolio share $\bar{z}_{i,n}$ with the actually observed portfolio share $z_{i,n,t}$, while holding the reference portfolio shares equal to their mean $\bar{z}_{i,m,n}$. The average portfolio return gain is 5.7 annualized basis points for active funds versus 5.0 basis points for passive funds.
6 Conclusion

The objective of the paper was to provide empirical evidence on international portfolio choice and specifically the role of portfolio frictions. We developed a simple optimal portfolio expression that relates portfolio choice to the present discounted value of expected excess returns and two benchmark portfolios, the lagged portfolio share and the buy-and-hold portfolio. We estimated the reduced form parameters of the portfolio expression with international equity portfolio data from US mutual funds. We find that portfolio shares of US mutual funds depend significantly on the average of the two benchmark portfolios, with a weight of 0.928 that is very precisely estimated, with a standard error of only 0.004. Not surprisingly, we find a large and significant estimate of the aggregate portfolio friction (the sum of the two portfolio frictions).

We also find that international equity return differentials are predictable and that mutual fund portfolios respond to expected excess returns. The results are consistent with modest risk-aversion of about 1.5. While the responsiveness to the present value of expected excess returns is strongly statistically significant, we also find that quantitatively the portfolio response to expected returns is much smaller than it would be in a frictionless portfolio model. Portfolio frictions make the response to changes in expected returns smaller initially and more gradual.

We have also documented heterogeneities across funds and fund-country pairs. We find that when portfolio shares are larger, for example as a result of lower perceived risk, they are more sensitive to expected excess returns. This is found to be consistent with lower risk and a lower aggregate portfolio friction. Several types of fund heterogeneity are identified. Global and large funds have a larger aggregate portfolio friction. Active funds have a higher relative friction of deviating from the lagged portfolio relative to the buy-and-hold portfolio, consistent with more active rebalancing. While the aggregate portfolio friction is about the same for active and passive funds, the cost of portfolio frictions is considerably higher for active funds due to their more aggressive deviations from the benchmark portfolios.

There is a clear need to introduce these portfolio frictions into open economy models. It has significant implications for exchange rates and other asset prices, as well as international capital flows. A portfolio response to expected returns that is weaker and more gradual implies excess return predictability in both the foreign exchange market and global equity markets. It also implies a much larger impact of
exogenous portfolio shocks, such as foreign exchange intervention, on asset prices and capital flows. The importance of such financial shocks for exchange rates and capital flows has recently been emphasized by Gabaix and Maggiori (2015) and Itskhoki and Muhkin (2019) and, as these papers emphasize, is consistent with a variety of evidence.
Appendix: A Simple Model of Portfolio Allocation

Consider the portfolio model in Section 2. Agents maximize (1), where the buy and hold portfolio is

$$z_{bh}^{t+s} = z_{t+s-1} + \frac{1 + R_{1,t+s}}{1 + z_{t+s-1}R_{1,t+s} + (1 - z_{t+s-1})R_{2,t+s}}$$  \hspace{1cm} (A.1)

Linearizing around returns of $\bar{R} = 0$ and an asset 1 portfolio share of $\bar{z}$ (the mean portfolio share for the fund), this can be approximated as

$$z_{bh}^{t+s} = z_{t+s-1} + \bar{z}(1 - \bar{z})er_{t+s}$$  \hspace{1cm} (A.2)

where $er_{t+s} = R_{1,t+s} - R_{2,t+s}$ is the excess return.

Substituting (A.2) into (1), maximization with respect to $z_t$ gives

$$E_t er_{t+1} - \gamma z_t \sigma_{er}^2 + \gamma (\sigma_2^2 - \sigma_{12}) - \lambda_1 (z_t - z_{t-1}) - \lambda_2 (z_t - z_{t-1} - \bar{z}(1 - \bar{z})er_t) + \beta \lambda_1 (E_t z_{t+1} - z_t) + \beta \lambda_2 (E_t z_{t+1} - z_t - \bar{z}(1 - \bar{z})er_t) = 0$$

Collecting terms, this is

$$Dz_t = E_t er_{t+1} + \gamma (\sigma_2^2 - \sigma_{12}) + \theta z_{t-1} + \beta \theta E_t z_{t+1} + \lambda_2 \bar{z}(1 - \bar{z})er_t - \beta \lambda_2 \bar{z}(1 - \bar{z})E_t er_{t+1}$$  \hspace{1cm} (A.3)

where

$$D = \Gamma + \theta (1 + \beta)$$  

$$\theta = \lambda_1 + \lambda_2$$  

$$\Gamma = \gamma \sigma_{er}^2$$

This can be written as

$$
\left( L^{-2} - \frac{D}{\beta \theta} L^{-1} + \frac{1}{\beta} \right) z_{t-1} = -\frac{1}{\beta \theta} E_t er_{t+1} - \frac{\gamma}{\beta \theta} (\sigma_2^2 - \sigma_{12}) - \frac{1}{\beta \theta} \lambda_2 \bar{z}(1 - \bar{z})er_t + \frac{1}{\theta} \lambda_2 \bar{z}(1 - \bar{z})E_t er_{t+1}
$$

where $L^{-2}z_{t-1} = E_t z_{t+1}$ and $L^{-1}z_{t-1} = z_t$. Factoring gives

$$
(L^{-1} - \omega_1)(L^{-1} - \omega_2)z_{t-1} = -\frac{1}{\beta \theta} E_t er_{t+1} - \frac{\gamma}{\beta \theta} (\sigma_2^2 - \sigma_{12}) - \frac{1}{\beta \theta} \lambda_2 \bar{z}(1 - \bar{z})er_t + \frac{1}{\theta} \lambda_2 \bar{z}(1 - \bar{z})E_t er_{t+1}
$$
where \( \omega_1 \) and \( \omega_2 \) are the roots of the characteristic equation

\[
\omega^2 - \frac{D}{\beta\theta} \omega + \frac{1}{\beta} = 0 \tag{A.4}
\]

These roots are

\[
\omega = 0.5 \left( \frac{D}{\beta\theta} \pm \sqrt{\left( \frac{D}{\beta\theta} \right)^2 - \left( 4/\beta \right)} \right) \tag{A.5}
\]

It is easily checked that the larger root, \( \omega_2 \), is explosive (larger than 1), and the smaller root, \( \omega_1 \), is stable (less than 1).

Now write the solution as

\[
(L^{-1} - \omega_1)z_{t-1} = -\frac{1}{\beta\theta}(L^{-1} - \omega_2) - \frac{\gamma}{\beta\theta(\omega_2 - 1)} \left( \frac{1}{\omega_2} - \beta \right) \sum_{s=1}^{\infty} E_{t,s} \left( \frac{1 - \bar{z}}{\omega_2} \right) E_{t,s+1} + \frac{1}{\beta\theta(\omega_2 - 1)} E_{t,s+1} + \frac{\lambda_2 \bar{z}(1 - \bar{z})}{\beta\theta \omega_2} \frac{E_{t,s+1}}{L^{-1} - \omega_2}
\]

This implies

\[
z_t = \omega_1 z_{t-1} + \frac{1}{\beta\theta \omega_2} \sum_{s=1}^{\infty} \omega_2^{1-s} E_{t,s} + \frac{\gamma}{\beta\theta(\omega_2 - 1)} \left( \frac{1}{\omega_2} - \beta \right) \sum_{s=1}^{\infty} \omega_2^{1-s} E_{t,s+1} + \frac{\lambda_2 \bar{z}(1 - \bar{z})}{\beta\theta \omega_2} \frac{E_{t,s+1}}{L^{-1} - \omega_2}
\]

To summarize, we have

\[
z_t = a_1 + a_2 z_{t-1} + a_3 \sum_{s=1}^{\infty} \omega_2^{1-s} E_{t,s} + a_4 E_{t,s+1} \tag{A.6}
\]

where

\[
a_1 = \frac{\gamma}{\beta\theta(\omega_2 - 1)} (\omega_2^2 - \sigma_{12})
\]

\[
a_2 = \omega_1
\]

\[
a_3 = \frac{1}{\beta\theta \omega_2} + \frac{\lambda_2 \bar{z}(1 - \bar{z})}{\beta\theta \omega_2} \left( \frac{1}{\omega_2} - \beta \right)
\]

\[
a_4 = \frac{\lambda_2 \bar{z}(1 - \bar{z})}{\beta\theta \omega_2}
\]

We can also write the solution for \( z_t \) as a function of the lagged and buy and hold portfolio. For this, use that \( z_{t,bh} = z_{t-1} + \bar{z}(1 - \bar{z})E_r \), so that

\[
E_r = \frac{z_{t,bh} - z_{t-1}}{\bar{z}(1 - \bar{z})}
\]
We then have
\[ z_t = a_1 + \left( a_2 - \frac{a_4}{\bar{z}(1 - \bar{z})} \right) z_{t-1} + \frac{a_4}{\bar{z}(1 - \bar{z})} \bar{z}^{bh} + a_3 \sum_{s=1}^{\infty} \omega_2^{1-s} E_t er_{t+s} \] (A.7)

Use that \( \omega_2 = 1/(\beta \omega_1) = 1/(\beta a_2) \). This gives
\[ z_t = a_1 + a_2 \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} z_{t-1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} \bar{z}^{bh} \right) + a_3 \sum_{s=1}^{\infty} \omega_2^{1-s} E_t er_{t+s} \] (A.8)

It is easily checked that \( a_2 = \omega_1 \) can be written as the parameter \( \omega \) in Section 2. The second term in the expression for \( a_3 \) is numerically very close to zero. As mentioned in footnote 5, we therefore abstract from it, so that \( a_3 = 1/(\beta \theta \omega_2) = a_2/\theta \). Finally, the discount rate is \( \delta = 1/\omega_2 = \beta a_2 \). (A.8) therefore corresponds to (2) with the coefficients as shown below that.

Finally, we can also write it including the valuation effect \( er_t \) and the buy-and-hold portfolio. To this end, substitute \( z_{t-1} = z^{bh}_t - \bar{z}(1 - \bar{z}) er_t \) which gives
\[ z_t = a_1 + a_2 z^{bh}_t - \tilde{a}_2 er_t + a_3 \sum_{s=1}^{\infty} \omega_2^{1-s} E_t er_{t+s} \] (A.9)

where \( \tilde{a}_2 = a_2 \bar{z}(1 - \bar{z}) \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} \right) \).
References


