

Technical Appendix to “Asymmetric Beta Comovement and Systematic Downside Risk”

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Abstract

This document provides supplementary material to the paper “Asymmetric Beta Co-movement and Systematic Downside Risk”. The document provides additional information regarding predictive variables and then it provides additional results for event analysis and trading strategies. Finally, results from subsample regression analysis are reported.

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1 Additional Information On Predictive Variables

1.1 Absorption Ratio

In this section, we provide details on our construction of the Absorption Ratio (AR). We carefully follow the approach described by [Kritzman, Li, Page, and Rigobon \(2011\)](#). In addition, to the use of a longer sample, we differ from their implementation by the use of Fama-French industry indices. Compared to MSCI industry indices, the coverage of Fama-French indices is broader and their sample is longer. The impact on the construction of the AR is however likely to be limited. For consistency with the construction of the SDR measure, we also used rolling windows of 3 years, whereas [Kritzman, Li, Page, and Rigobon \(2011\)](#) use 500 days. As in [Kritzman, Li, Page, and Rigobon \(2011\)](#), we then take the first 10 eigenvectors to calculate the AR. As reported in Table 1.1.1, the first 10 eigenvectors account for 80% of the variation in the 49 industry portfolio returns.

Figure 1.1.1 displays the evolution of the AR over the sample used by [Kritzman, Li, Page, and Rigobon \(2011\)](#), i.e., from 1998 to 2010, and with the same rolling window. As the figure illustrates, our index reproduces the index constructed by [Kritzman, Li, Page, and Rigobon \(2011\)](#) over the common sample (see their exhibit 6). The evolution of our AR over the entire sample is reported in Figure 4 of the main text.

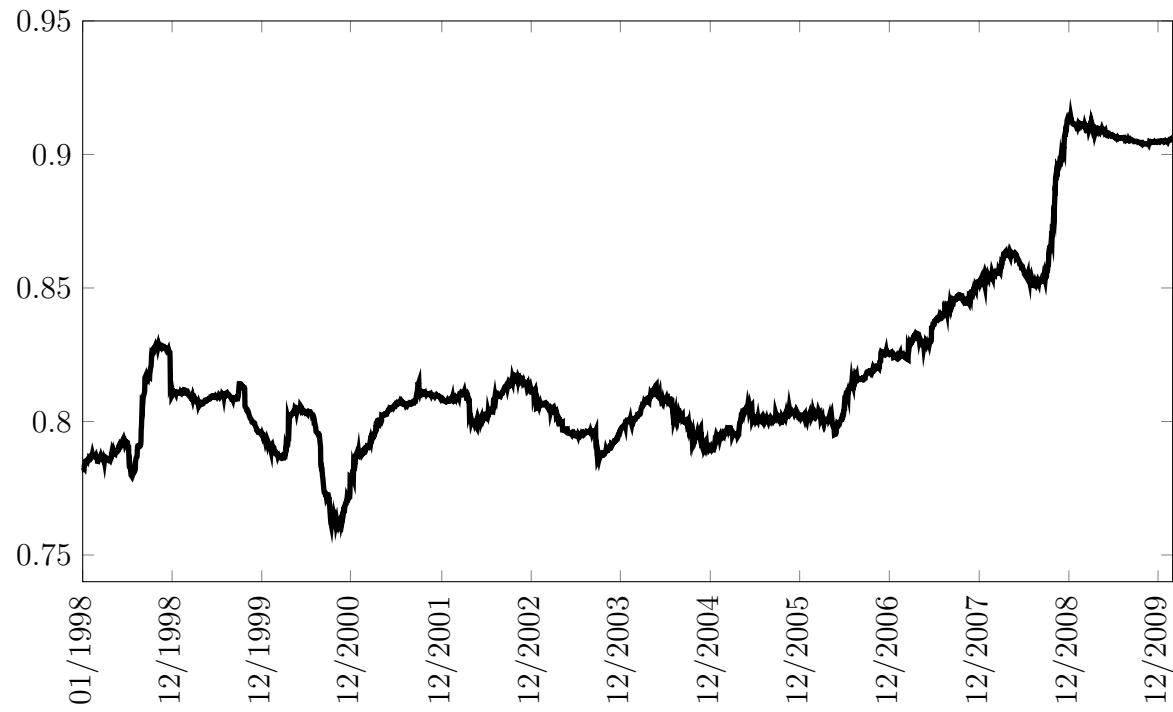
Table 1.1.1: Variance Fraction Explained by Eigenvectors

This table provides the cumulated fraction of the total variance of the 49 industry portfolio returns explained by the first 10 eigenvectors.

Fraction Explained (%)	
E1	52.648
E2	59.903
E3	64.817
E4	67.951
E5	70.517
E6	72.511
E7	74.474
E8	76.405
E9	78.239
E10	79.919

Figure 1.1.1: Absorption Ratio (1998-2010)

This figure presents the AR estimated from 49 industry portfolios using 2 year rolling windows, which is same as in [Kritzman, Li, Page, and Rigobon \(2011\)](#). Sample: 1st January 1998 through 31st January 2010.

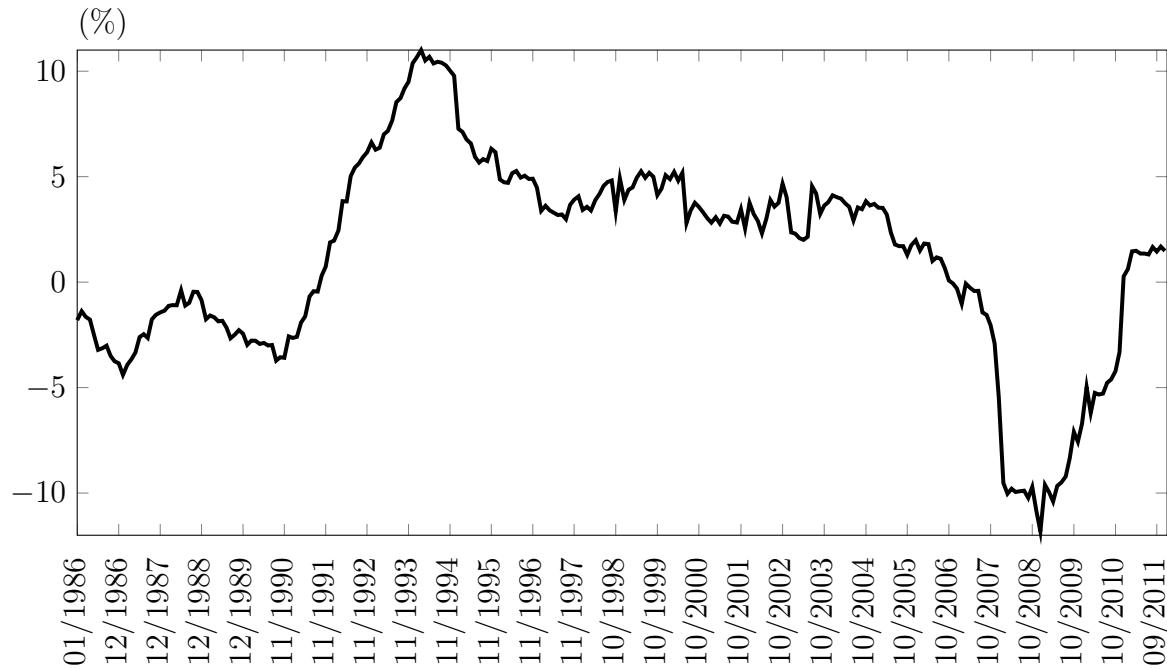


1.2 Net Equity Expansion

Figure 1.2.1 presents the dynamics of Net Equity Expansion. It is defined here as the ratio of the 3-year moving sum of net issues by NYSE-listed stocks divided by the total end-of-3-year market capitalization of NYSE stocks. This is the dollar amount (less dividends) of net equity issuing activity, such as IPOs, for NYSE listed stocks, computed from CRSP data. Our measure of Net Equity Expansion is similar as in [Goyal and Welch \(2008\)](#) except that they use 1-year moving sums and the total end-of-year market capitalization to obtain the Net Equity Expansion. We take 3 years to be consistent with the length of the window used for estimating the conditional betas.

Figure 1.2.1: Net Equity Expansion

This figure presents the dynamics of net equity expansion. Sample: 1st January 1986 through 30th December 2011.



1.3 Correlation between Variables

In this section, we report the correlation between the various measures used in the paper: the variables introduced in the paper, SDR and Downside-Beta Comovement ($Bcov_{Down}$); the variables proposed by [Ang and Chen \(2002\)](#) and [Ang, Chen, and Xing \(2006\)](#), Beta Asymmetry ($Beta_{Asy}$) and Downside Beta ($Beta_{Down}$); the Absorption Ratio (AR) proposed by [Kritzman, Li, Page, and Rigobon \(2011\)](#), and the Net Equity Expansion (NEE) studied by [Goyal and Welch \(2008\)](#) and [Ferreira and Santa-Clara \(2011\)](#). As expected, the NEE has a negative correlation with the other variables, as it is positively related to stock market growth. All the other indicators are positively correlated, although far from perfectly. The highest correlations are between SDR, $Bcov_{Down}$, and $Beta_{Down}$. The correlation between the SDR and the Absorption Ratio is 60% and the correlation between the SDR and the NEE is -26.2% .

If we compute correlations over the shorter period from January 1986 to January 2008, we observe that most correlations are reduced, although to a limited extent.

Table 1.3.1: Correlation

This table provides the correlation values between SDR, Downside-Beta Comovement ($Bcov_{Down}$), Beta Asymmetry ($Beta_{Asy}$), Downside Beta ($Beta_{Down}$), Absorption Ratio (AR), and Net Equity Expansion (NEE).

Jan. 86–Dec. 11	SDR	$Bcov_{Down}$	$Beta_{Asy}$	$Beta_{Down}$	AR	NEE
SDR	1					
$Bcov_{Down}$	0.857	1				
$Beta_{Asy}$	0.343	0.331	1			
$Beta_{Down}$	0.771	0.831	0.598	1		
AR	0.600	0.783	0.038	0.699	1	
NEE	-0.262	-0.421	0.100	-0.374	-0.689	1

Jan. 86–Jan. 08	SDR	$Bcov_{Down}$	$Beta_{Asy}$	$Beta_{Down}$	AR	NEE
SDR	1					
$Bcov_{Down}$	0.838	1				
$Beta_{Asy}$	0.314	0.310	1			
$Beta_{Down}$	0.812	0.868	0.573	1		
AR	0.627	0.815	-0.093	0.611	1	
NEE	-0.392	-0.502	0.145	-0.350	-0.639	1

2 Forecasting Large Movements of Market Return

In Section 4 of the paper on forecasting large movements of market return, we report results for the SDR, Downside-Beta Comovement ($Bcov_{Down}$), and Absorption Ratio (AR) indicators. In this section, we report additional results for the Beta Asymmetry ($Beta_{Asy}$) and Downside Beta ($Beta_{Down}$) indicators.

The first two figures (Figures 2.1 and 2.2) provide details on the ability of both measures to predict large movements of market return. They plot the mean values of Beta Asymmetry ($Beta_{Asy}$) and Downside Beta ($Beta_{Down}$) before and after the 1% and 5% extreme event dates, respectively.

We then document the investment strategies based on Beta Asymmetry and Downside Beta. Portfolio allocations are documented in Figures 2.3 and 2.4 for $\theta = 1$ and 0.5, respectively. The performances of both trading strategies are reported in Figure 2.5.

Figure 2.1: Standardized Beta Asymmetry and Downside Beta around 1% Extreme Market Movements

This figure presents the predictability of standardized Beta Asymmetry (Beta_{Asy}) and Downside Beta (Beta_{Down}) based on event study analysis. Plotted values are mean values in percentage. The extreme market movements are the 1% days with the lowest market return, largest market return, and most turbulent days over the whole sample period. Sample: 1st January 1986 through 30th December 2011.

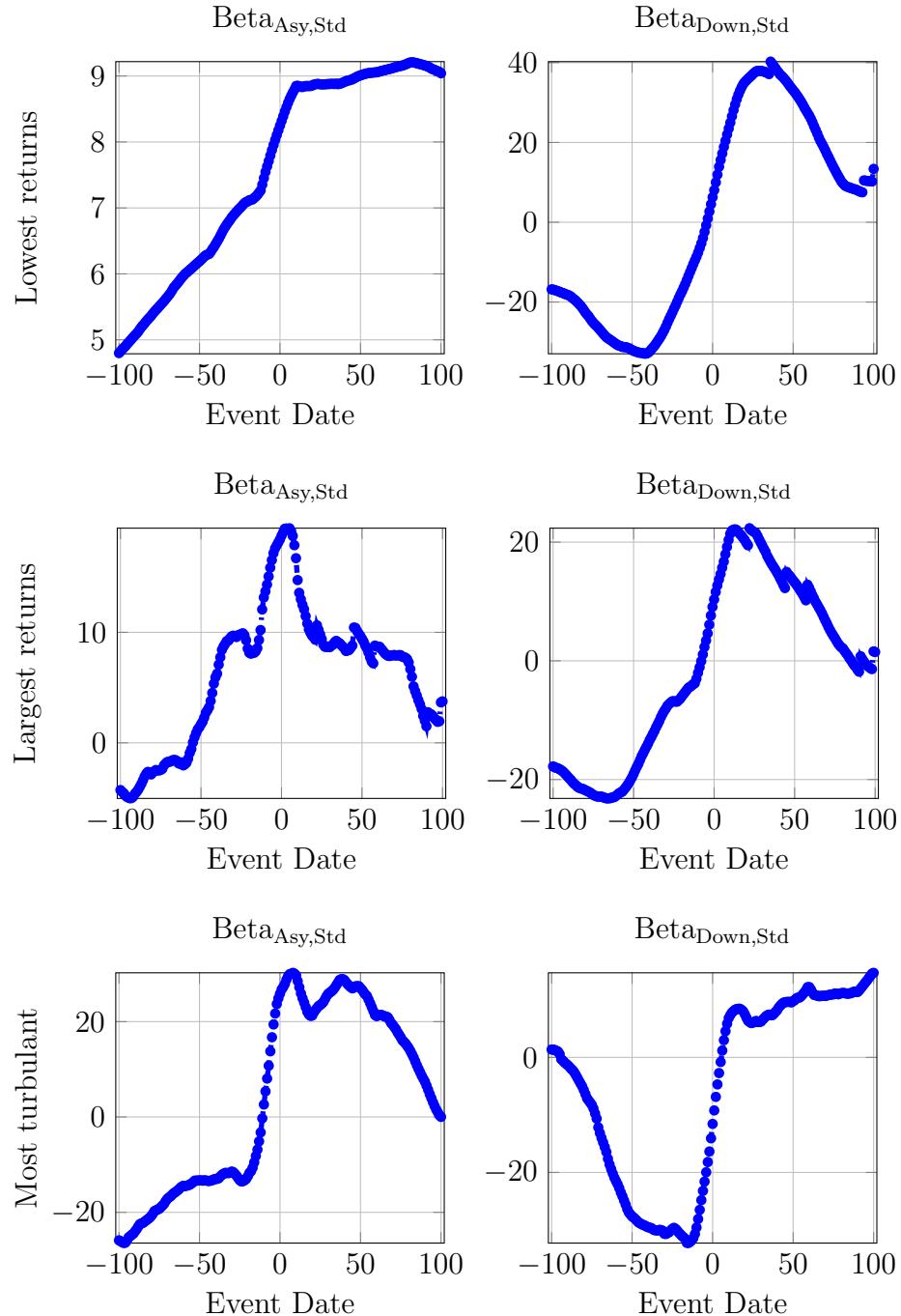


Figure 2.2: Standardized Beta Asymmetry and Downside Beta around 5% Extreme Market Movements

This figure presents the predictability of standardized Beta Asymmetry (Beta_{Asy}) and Downside Beta ($\text{Beta}_{\text{Down}}$) based on event study analysis. Plotted values are mean values in percentage. The extreme market movements are the 5% days with the lowest market return, largest market return, and most turbulent days over the whole sample period. Sample: 1st January 1986 through 30th December 2011.

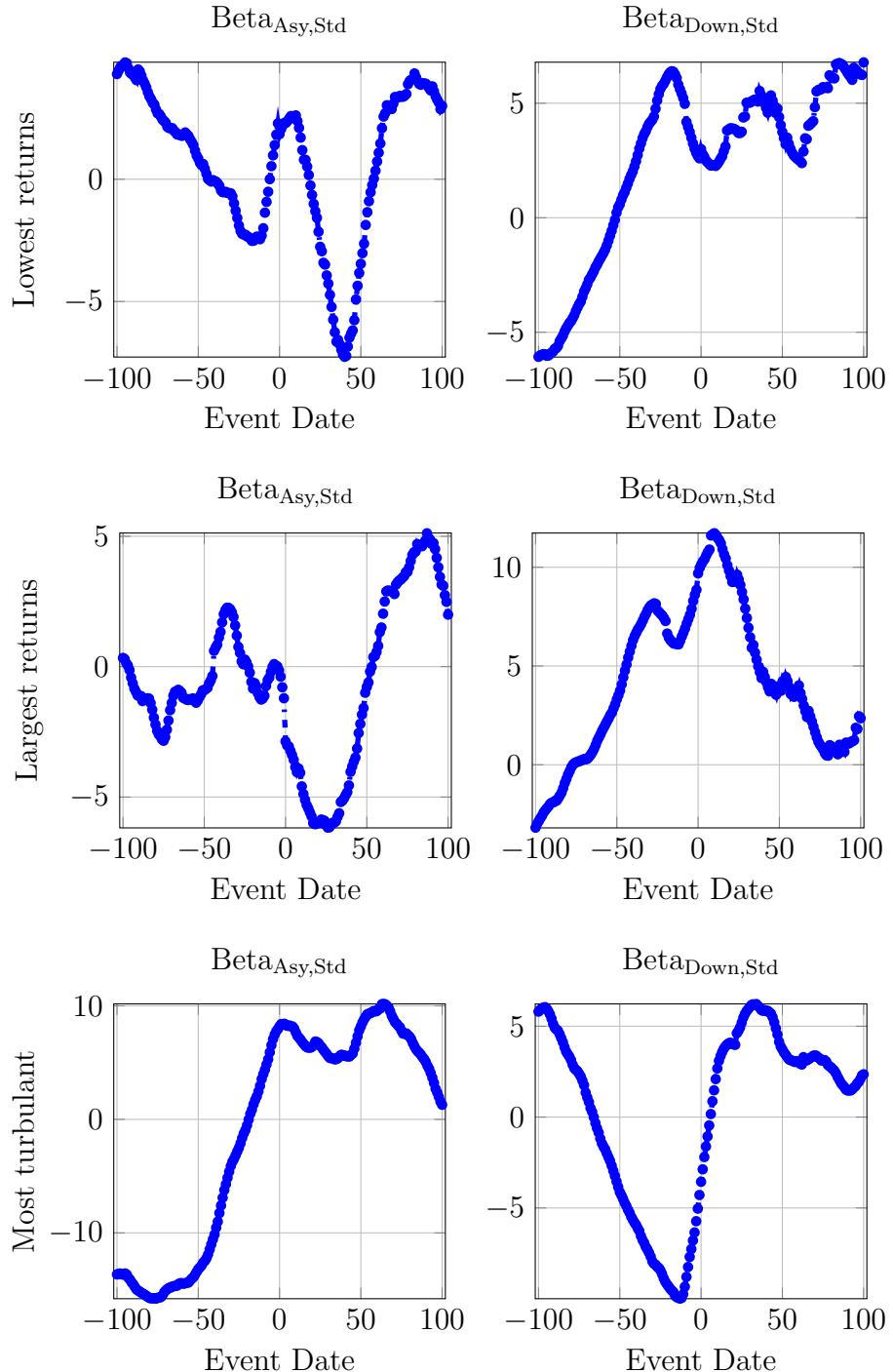


Figure 2.3: Trading Signal, Asset Allocation, and Market Index (for $\theta = 1$)

This figure presents trading signals (solid line) based on standardized Beta Asymmetry (Beta_{Asy}) and standardized Downside Beta ($\text{Beta}_{\text{Down}}$). It also presents investments in risky asset: S&P500 Index (circle-marked line) and risk-free asset (dash line). Trading signals and investment holdings are plotted on the left axis; S&P500 Index level is plotted in dot line on the right axis. Portfolios are rebalanced once the 3-month signal's absolute value (demeaned by its 2-year mean) is larger than its 2-year standard deviation. Sample: 1st January 1986 through 30th December 2011.

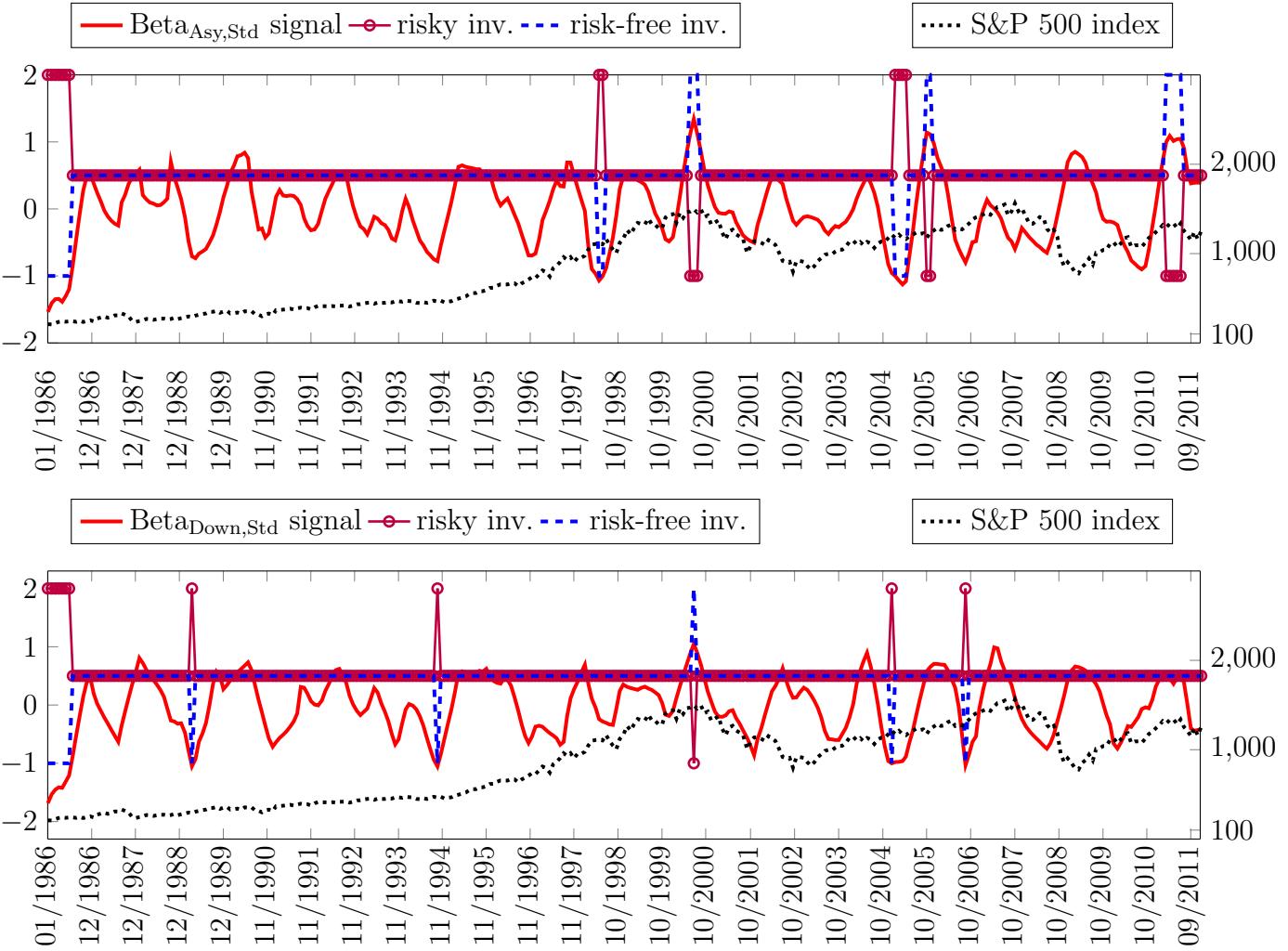


Figure 2.4: Trading Signal, Asset Allocation, and Market Index (for $\theta = 0.5$)

This figure presents trading signals (solid line) based on standardized Beta Asymmetry (Beta_{Asy}) and standardized Downside Beta ($\text{Beta}_{\text{Down}}$). It also presents investments in risky asset: S&P500 Index (circle-marked line) and risk-free asset (dash line). Trading signals and investment holdings are plotted on the left axis; S&P500 Index level is plotted in dot line on the right axis. Portfolios are rebalanced once the 3-month signal's absolute value (demeaned by its 2-year mean) is larger than half of its 2-year standard deviation. Sample: 1st January 1986 through 30th December 2011.

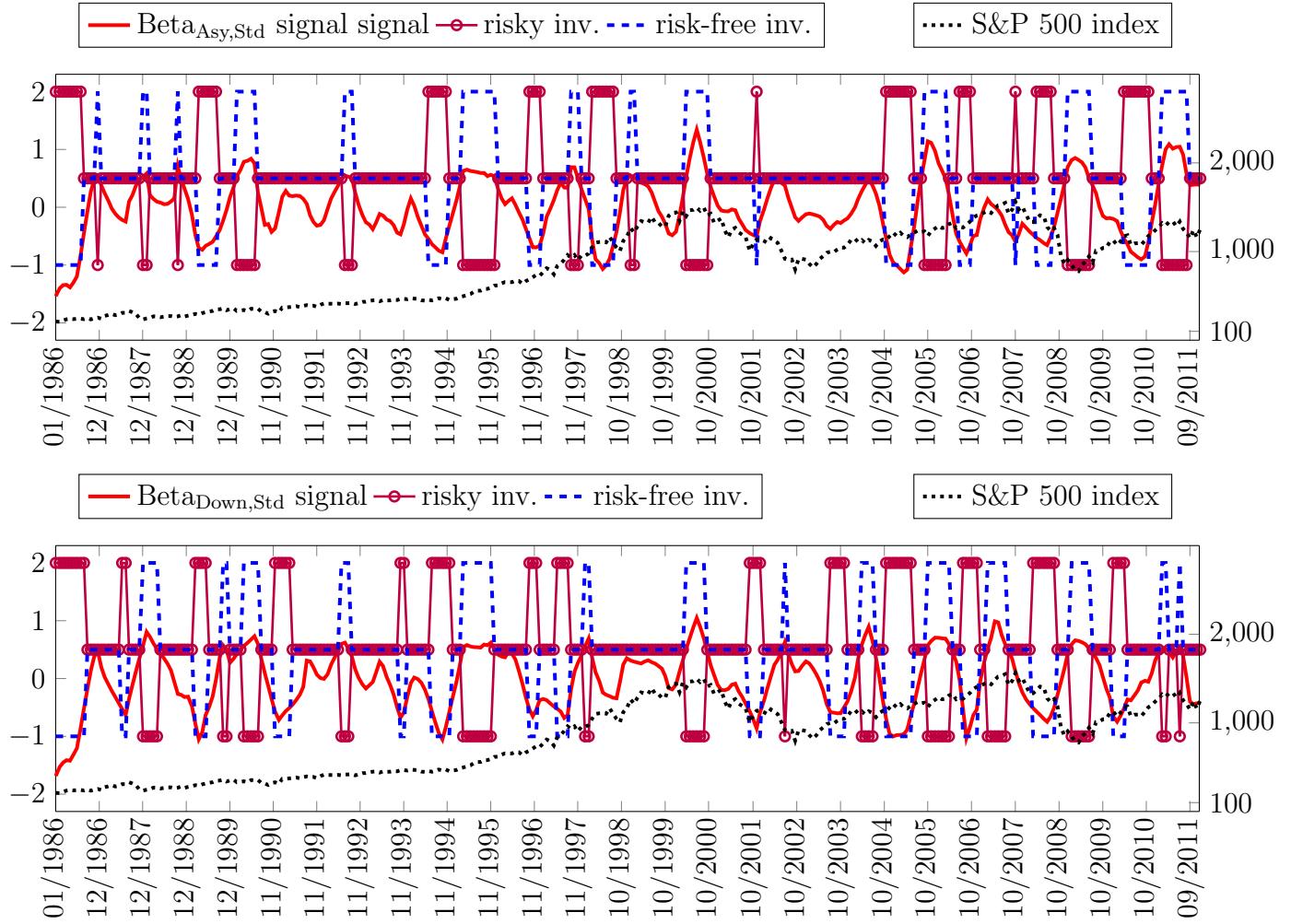
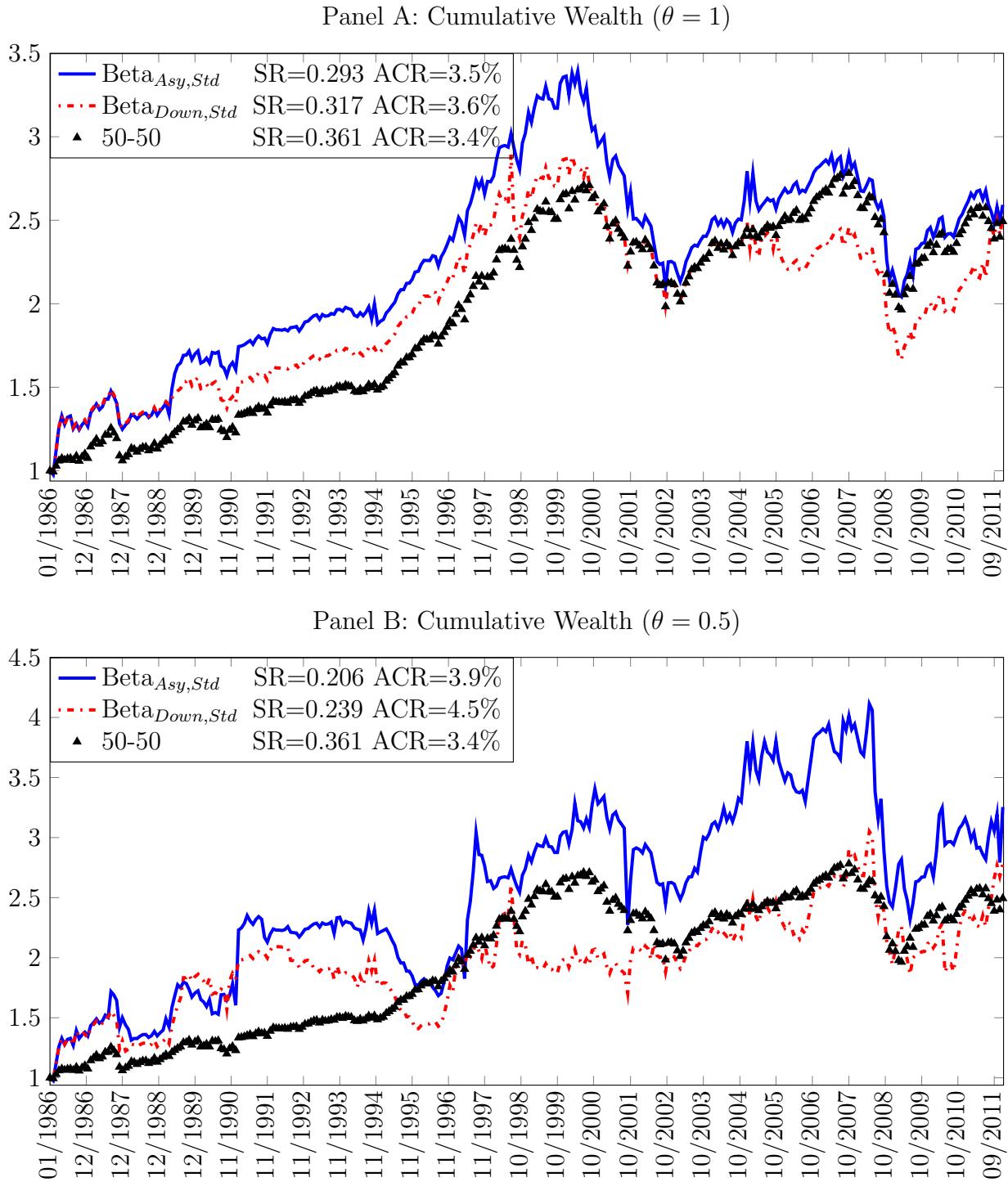


Figure 2.5: Cumulative Wealth of the Strategies Based on a Trading Signal

The figure presents cumulative wealth generated from trading strategies based on standardized Beta Asymmetry (Beta_{Asy}), standardized Downside Beta (Beta_{Down}), and 50/50 buy-and-hold. Panels A and B correspond to a trading signal based on $\theta = 1$ and 0.5, respectively. All strategies start with an initial investment of 1 dollar and have 1% transaction cost proportional to the unit of trading. Annualized Sharpe ratios (SR) and annualized cumulative return (ACR) for trading strategies are reported. Portfolios are rebalanced once the 3-month signal's value (demeaned by its 2-year mean) is larger than its 2-year standard deviation multiplied by θ . Panels A and B correspond to $\theta = 1$ and 0.5, respectively. Sample: 1st January 1986 through 30th December 2011.



3 Predictive Regressions Over Subsamples

In the main text, in Section 5 on predictive regressions, we report empirical results for the entire sample, from 1986 to 2011. In this section, we present results of a subsample analysis for the in-sample predictive regressions and the out-of-sample forecasting performance. We consider two subsamples (1986-1999 and 2000-2011) for forecasting future daily and monthly market returns.

Table 3.1: Daily Predictive Regressions of Market Return (In-Sample Analysis)

This table provides daily time-series regression results of market return on the SDR_{Std} , Downside-Beta Comovement ($Bcov_{Down,Std}$), Beta Asymmetry ($Beta_{Asy}$), Downside Beta ($Beta_{Down,Std}$), Absorption Ratio (AR_{std}) (all in standardized version) as well as Net Equity Expansion. Newey-West robust t-statistics are in parenthesis (10 lags for daily regression). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. The adjusted R-square is in percentage. Two Samples: 1st January 1986 through 31 December 1999; and January 3rd 2000 through 30th December 2011.

1986-99 Daily	I	II	III	IV	V	VI
Intercept	0.0012*** (3.9638)	0.0012*** (3.8623)	0.0012*** (3.838)	0.0012*** (3.7217)	0.0036 (1.1023)	0.0012*** (3.0655)
SDR_{Std}	-0.0007 (-1.4485)					
$Bcov_{Down,Std}$		-0.0006 (-1.2946)				
$Beta_{Asy,Std}$			-0.0003 (-0.641)			
$Beta_{Down,Std}$				-0.0004 (-1.0438)		
AR_{Std}					-0.0032 (-0.7125)	
Net Equity Expansion						-0.0009 (-0.1315)
$Adj.R^2$	0.042	0.044	-0.015	0.01	-0.008	-0.028
F_{stat}	2.475	2.552	0.473	1.356	0.703	0.024
2000-11 Daily	I	II	III	IV	V	VI
Intercept	0.0001 (0.3175)	0.0001 (0.3105)	0.0001 (0.1524)	0.0001 (0.1915)	-0.0036 (-0.4906)	0.0001 (0.2516)
SDR_{Std}	-0.0018* (-1.8092)					
$Bcov_{Down,Std}$		-0.0015** (-2.1178)				
$Beta_{Asy,Std}$			-0.0009 (-0.8449)			
$Beta_{Down,Std}$				0.0005 (0.8157)		
AR_{Std}					0.0046 (0.495)	
Net Equity Expansion						0.0037 (0.326)
$Adj.R^2$	0.202	0.191	-0.001	-0.008	-0.018	-0.024
F_{stat}	7.106	6.765	0.981	0.76	0.445	0.282

Table 3.2: Monthly Predictive Regressions of Market Return (In-Sample Analysis)

This table provides monthly time-series regression results of market return on the SDR_{Std} , Downside-Beta Comovement ($Bcov_{Down,Std}$), Beta Asymmetry ($Beta_{Asy}$), Downside Beta ($Beta_{Down,Std}$), Absorption Ratio (AR_{Std}) (all in standardized version) as well as Net Equity Expansion. Newey-West robust t-statistics are in parenthesis (6 lags for monthly regression). ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively. The adjusted R-square is in percentage. Two Samples: 1st January 1986 through 31st December 1999; and January 3rd 2000 through 30th December 2011.

1986-99 Monthly	I	II	III	IV	V	VI
Intercept	0.0152*** (6.7144)	0.0151*** (6.595)	0.0151*** (6.5899)	0.0147*** (6.3727)	0.0435* (1.9312)	0.0151*** (4.938)
SDR_{Std}	-0.0075* (-1.9555)					
$Bcov_{Down,Std}$		-0.0058 (-1.5489)				
$Beta_{Asy,Std}$			-0.0022 (-0.4809)			
$Beta_{Down,Std}$				-0.0048* (-1.8354)		
AR_{Std}					-0.0375 (-1.2214)	
Net Equity Expansion						0.0028 (0.0535)
<i>Adj.R²(%)</i>	0.682	0.522	0.02	0.56	0.245	-0.028
<i>F stat</i>	25.016	19.337	1.693	20.696	9.579	0.024
2000-11 Monthly	I	II	III	IV	V	VI
Intercept	0.001 (0.3671)	0.001 (0.3628)	0.0004 (0.134)	0.0005 (0.1652)	-0.0534 (-1.0565)	0.0009 (0.3079)
SDR_{Std}	-0.0242*** (-3.5211)					
$Bcov_{Down,Std}$		-0.017*** (-3.5766)				
$Beta_{Asy,Std}$			-0.0144* (-1.8449)			
$Beta_{Down,Std}$				0.0087* (1.8575)		
AR_{Std}					0.0665 (1.0553)	
Net Equity Expansion						0.0558 (0.6746)
<i>Adj.R²(%)</i>	4.513	3.045	0.77	0.683	0.285	0.186
<i>F stat</i>	141.287	94.208	24.031	21.406	9.493	6.54

Table 3.3: Predictive Regressions of Market Return (Out-of-Sample Performance)

This table reports out-of-sample R-square (in percentage), annualized Sharpe ratio gains, and annualized certainty equivalence gains for stock market return forecasts at daily and monthly frequencies from predictive regressions with expanding window. The explanatory variables are SDR_{Std} , Downside-Beta Comovement ($Bcov_{Down,Std}$), Absorption Ratio (AR_{Std}), Beta Asymmetry ($Beta_{Asy,Std}$), Downside Beta ($Beta_{Down,Std}$) (all in standardized version) as well as Net Equity Expansion. The endogenous variable is the future market return (simple return of S&P 500 index). The out-of-sample R-square (in percentage) compares the forecast error of the model with the forecast error of the historical mean. The Sharpe ratio gains and the certainty equivalent gains are portfolio gains of a trading strategy based on different return forecasts relative to the one with the historical mean return. Forecasts begin $s_0 = 5$ years after the sample starts. Two Samples: 1st January 1986 through 31st December 1999; and January 3rd 2000 through 30th December 2011.

Panel A: Forecasting Daily Market Return

1986-99 Daily	R^2 (%)	SR Gain	CE Gain
SDR_{Std}	0.224	0.880	-0.160
Downside $Bcov_{Std}$	0.262	0.919	0.049
$Beta_{Asy,Std}$	-0.094	0.090	-0.085
Downside $Beta_{Std}$	-0.134	-0.344	-0.093
AR_{Std}	-0.876	0.396	-3.101
Net Equity Expansion	-0.554	0.329	-1.389

2000-11 Daily	R^2 (%)	SR Gain	CE Gain
SDR_{Std}	-0.077	0.082	0.027
Downside $Bcov_{Std}$	-0.053	0.072	0.018
$Beta_{Asy,Std}$	-0.122	-0.086	-0.051
Downside $Beta_{Std}$	-0.165	-0.127	-0.069
AR_{Std}	-0.095	0.033	0.049
Net Equity Expansion	-0.200	-0.213	-0.117

Panel B: Forecasting Monthly Market Return

1986-99 Monthly	R^2 (%)	SR Gain	CE Gain
SDR_{Std}	7.360	1.297	0.212
Downside $Bcov_{Std}$	4.506	1.112	0.141
$Beta_{Asy,Std}$	-1.787	0.136	-0.094
Downside $Beta_{Std}$	-1.487	-0.117	-0.048
AR_{Std}	-14.783	0.401	-1.337
Net Equity Expansion	-7.593	0.416	-0.586

2000-11 Monthly	R^2 (%)	SR Gain	CE Gain
SDR_{Std}	-2.482	0.061	0.016
Downside $Bcov_{Std}$	-4.117	-0.058	-0.037
$Beta_{Asy,Std}$	-3.132	-0.176	-0.077
Downside $Beta_{Std}$	-3.007	-0.174	-0.070
AR_{Std}	-1.531	0.055	0.052
Net Equity Expansion	-1.424	-0.153	-0.076

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