Technical Appendix (Online)

This document provides supplementary material to the paper "Average Skewness Matters." It provides (1) additional details on the theoretical model described in Section 2 of the main text (Section A), (2) additional regression results based on alternative definitions of the variables (Section B), and (3) additional regression results based on alternative specifications of the regressions (Section C).

A Theoretical Results

A.1 Source of Predictability in Average Skewness

To clarify the theoretical link between the market return at date $t+1$ and the average skewness at date t, we assume a simple model. In month $t+1$, the return of firm i is described by $R_{i,t+1} = \mu_{i,t+1} + \varepsilon_{i,t+1}$, where $\mu_{i,t+1} = E_t[R_{i,t+1}]$ denotes the expected return conditional on the information available at time t and $\varepsilon_{i,t+1}$ denotes the unexpected return. The unexpected return has two components:

$$\varepsilon_{i,t+1} = \beta_i \ \varepsilon_{m,t+1} + z_{i,t+1},$$

where $\varepsilon_{m,t+1} = R_{m,t+1} - E_t[R_{m,t+1}]$ is the aggregate innovation and $z_{i,t+1}$ is a purely idiosyncratic innovation. Both innovations are allowed to have an asymmetric distribution, with conditional variance denoted by $V_t[\varepsilon_{m,t+1}]$ and $V_t[z_{i,t+1}]$ and conditional skewness denoted by $Sk_t[\varepsilon_{m,t+1}]$ and $Sk_t[z_{i,t+1}]$, respectively. Innovations $\varepsilon_{m,t+1}$ and $z_{i,t+1}$ are independent from each other and $z_{i,t+1}$ and $z_{j,t+1}$ are independent from each other for all i and j.[1]

Given the data generating process described above, the individual variance and skewness have the following expressions:

$$\begin{aligned} V_t[\varepsilon_{i,t+1}] &= E_t[\varepsilon_{i,t+1}^2] = \beta_i^2 \ V_t[R_{m,t+1}] + V_t[z_{i,t+1}], \\ Sk_t[\varepsilon_{i,t+1}] &= E_t[\varepsilon_{i,t+1}^3] = \beta_i^3 \ Sk_t[R_{m,t+1}] + Sk_t[z_{i,t+1}]. \end{aligned}$$

The first order condition for the portfolio choice problem is the Euler equation:

$$E_t[(1 + R_{i,t+1})m_{t+1}] = 1 \quad \text{for all } i,$$

[1] Alternatively, we may assume a model with aggregate and idiosyncratic jumps, which would generate asymmetry in the distribution of the unexpected return. This case is investigated by Maheu and McCurdy (2004) and Maheu, McCurdy, and Zhao (2013).

where m_{t+1} is the intertemporal marginal rate of substitution between t and $t+1$, which also represents the pricing kernel for risky assets.[2] In the three-moment CAPM, the pricing kernel is quadratic in the market return (Harvey and Siddique, 2000; Dittmar, 2002):

$$m_{t+1} = \Lambda_0 + \Lambda_m \; R_{m,t+1} + \Psi_m \; R^2_{m,t+1},$$

where the expression for parameters Λ_m and Ψ_m can be derived from a model of the investors' preferences. This pricing kernel yields the following expressions for the stock and market risk premia:

$$E_t[R_{i,t+1}] - R_{f,t} = \tilde{\lambda}_{m,t} \; Cov_t[R_{i,t+1}, R_{m,t+1}] + \tilde{\psi}_{m,t} \; Cov_t[R_{i,t+1}, R^2_{m,t+1}], \tag{A.1}$$

$$E_t[R_{m,t+1}] - R_{f,t} = \tilde{\lambda}_{m,t} \; V_t[R_{m,t+1}] + \tilde{\psi}_{m,t} \; Sk_t[R_{m,t+1}]. \tag{A.2}$$

The expressions for the market prices of risk $\tilde{\lambda}_{m,t}$ and $\tilde{\psi}_{m,t}$ are given for instance in Harvey and Siddique (2000).

When investors have preference both for systematic and individual skewness, the pricing kernel depends on all sources of risk, including individual innovations. A typical approach consists in writing the pricing kernel as linear in the underlying sources of risk (Aït-Sahalia and Lo, 1998; Bates, 2008; Christoffersen, Jacobs, and Ornthanalai, 2012). In our context with quadratic terms, the pricing kernel writes:

$$m_{t+1} = \Lambda_0 + \Lambda_m \; R_{m,t+1} + \Psi_m \; R^2_{m,t+1} + \sum_{i=1}^{N} \Lambda_i \; \varepsilon_{i,t+1} + \sum_{i=1}^{N} \Psi_i \; \varepsilon^2_{i,t+1},$$

where parameters Λ_i and Ψ_i reflect investors' aversion for individual variance and preference for individual skewness, respectively.

This pricing kernel gives the following expression for the expected excess return on asset i:

$$\begin{aligned} \frac{E_t[R_{i,t+1}] - R_{f,t}}{1 + R_{f,t}} &= -\Lambda_m \; Cov_t[R_{i,t+1}, R_{m,t+1}] - \Psi_m \; Cov_t[R_{i,t+1}, R^2_{m,t+1}] \\ &\quad - \sum_{j=1}^{N} \Lambda_j \; Cov_t[R_{i,t+1}, \varepsilon_{j,t+1}] - \sum_{j=1}^{N} \Psi_j \; Cov_t[R_{i,t+1}, \varepsilon^2_{j,t+1}]. \end{aligned}$$

[2] The Euler equation implies that $1 + E_t[R_{i,t+1}] = \frac{1}{E_t[m_{t+1}]} - \frac{Cov_t[R_{i,t+1}, m_{t+1}]}{E_t[m_{t+1}]}$. As this relation applies to the risk-free rate, we have: $1 + R_{f,t} = \frac{1}{E_t[m_{t+1}]}$, so that the Euler equation can also be written as: $E_t[R_{i,t+1}] - R_{f,t} = -\frac{Cov_t[R_{i,t+1}, m_{t+1}]}{E_t[m_{t+1}]}$.

This expression generalizes Equation (A.1) with

$$E_t[R_{i,t+1}] - R_{f,t} = \tilde{\lambda}_{m,t}\ Cov_t[R_{i,t+1}, R_{m,t+1}] + \tilde{\psi}_{m,t}\ Cov_t[R_{i,t+1}, R^2_{m,t+1}] + \sum_{j=1}^{N} \tilde{\lambda}_{j,t}\ Cov_t[R_{i,t+1}, \varepsilon_{j,t+1}] + \sum_{j=1}^{N} \tilde{\psi}_{j,t}\ Cov_t[R_{i,t+1}, \varepsilon^2_{j,t+1}], \quad (A.3)$$

where $\tilde{\lambda}_{m,t} = -(1+R_{f,t})\Lambda_m$, $\tilde{\psi}_{m,t} = -(1+R_{f,t})\Psi_m$, $\tilde{\lambda}_{j,t} = -(1+R_{f,t})\Lambda_j$, and $\tilde{\psi}_{j,t} = -(1+R_{f,t})\Psi_j$.

We further assume that market prices of risk are the same across firms, i.e., $\Lambda_i = \Lambda_I$, and $\Psi_i = \Psi_I$ for all i. By aggregation we obtain the expected excess market return as:

$$E_t[R_{m,t+1}] - R_{f,t} = \tilde{\lambda}_{m,t}\ V_t[R_{m,t+1}] + \tilde{\psi}_{m,t}\ Sk_t[R_{m,t+1}] + \tilde{\lambda}_{I,t}\ \sum_{i=1}^{N} w_i \sum_{j=1}^{N} Cov_t[R_{i,t+1}, \varepsilon_{j,t+1}] + \tilde{\psi}_{I,t}\ \sum_{i=1}^{N} w_i \sum_{j=1}^{N} Cov_t[R_{i,t+1}, \varepsilon^2_{j,t+1}],$$

where w_i denotes the relative market capitalization of firm i. In our data generating process, we have the following equalities: $\sum_{j=1}^{N} Cov_t[R_{i,t+1}, \varepsilon_{j,t+1}] = \sum_{j\neq i}^{N} \beta_i\beta_j\ V_t[R_{m,t+1}] + V_t[\varepsilon_{i,t+1}]$ and $\sum_{j=1}^{N} Cov_t[R_{i,t+1}, \varepsilon^2_{j,t+1}] = \sum_{j\neq i}^{N} \beta_i\beta_j^2\ Sk_t[R_{m,t+1}] + Sk_t[\varepsilon_{i,t+1}]$. This relation simplifies to:

$$E_t[R_{m,t+1}] - R_{f,t} = \lambda_{m,t}\ V_t[R_{m,t+1}] + \psi_{m,t}\ Sk_t[R_{m,t+1}] + \lambda_{I,t}\ V_{w,t} + \psi_{I,t}\ Sk_{w,t}, \quad (A.4)$$

where $V_{w,t} = \sum_{i=1}^{N} w_i\ V_t[\varepsilon_{i,t+1}]$ and $Sk_{w,t} = \sum_{i=1}^{N} w_i\ Sk_t[\varepsilon_{i,t+1}]$ denote the expected average variance and skewness across firms, respectively. Parameters are defined as $\lambda_{m,t} = -(1+R_{f,t})\left(\Lambda_m + \Lambda_I \sum_{i=1}^{N}\beta_i\left(\sum_{j=1}^{N}\beta_j - \beta_i\right)\right)$, $\psi_{m,t} = -(1+R_{f,t})\left(\Psi_m + \Psi_I \sum_{i=1}^{N}\beta_i\left(\sum_{j=1}^{N}\beta_j^2 - \beta_i^2\right)\right)$, $\lambda_{I,t} = -(1+R_{f,t})\Lambda_I$, and $\psi_{I,t} = -(1+R_{f,t})\Psi_I$. This relation corresponds to Equation (1) in the main text.

The aggregate expected return is driven by the market variance and skewness but also by the cross-sectional average variance and skewness. The magnitude and significance of the parameters associated to these various predictors in principle depend on investors' preferences. We note that because investors have preference for individual skewness, they are likely to under-diversify their portfolio and therefore individual terms may not vanish in the expected excess market return. In the last equation, the role played by the individual variance and skewness can be decomposed into two components: (1) the idiosyncratic variance and skewness ($V_t[z_{i,t+1}]$ and $Sk_t[z_{i,t+1}]$) and (2) the covariance and coskewness terms ($\sum_{j=1}^{N} \beta_i\beta_j\ V_t[R_{m,t+1}]$ and $\sum_{j=1}^{N} \beta_i\beta_j^2\ Sk_t[R_{m,t+1}]$).

A.2 Relation between Market Moments and Co-moments

Market moments and average moments convey different types of information about the dependencies between stock returns. It is well known that market variance is mainly driven by the average correlation. We consider N stocks in month t, with D_t trading days, $d = 1, \cdots, D_t$. We denote by $\tilde{r}_{m,d} = r_{m,d} - \bar{r}_{m,t}$ the centered daily market excess return and $\tilde{r}_{i,d} = r_{i,d} - \bar{r}_{i,t}$ the centered daily excess stock return. The market variance in month t is given by:

$$
\begin{aligned}
V_{m,t} &= \frac{1}{D_t}\sum_{d=1}^{D_t}\tilde{r}^2_{m,d} = \frac{1}{D_t}\sum_{d=1}^{D_t}\left(\frac{1}{N}\sum_{i=1}^{N}\tilde{r}_{i,d}\right)^2 = \frac{1}{D_t}\sum_{d=1}^{D_t}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{1}{N^2}\tilde{r}_{i,d}\tilde{r}_{j,d}\right) \\
&= \frac{1}{N^2}\sum_{i=1}^{N}\sigma_{ii,t} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j\neq i,1}^{N}\sigma_{ij,t} = \frac{1}{N}\bar{\sigma}^2 + \frac{(N-1)}{N}c\bar{o}v,
\end{aligned}
$$

where $\bar{\sigma}^2$ and $c\bar{o}v$ denote the average variance and the average covariance, respectively. As the number of stocks N goes to infinity, the average variance does not play any role and only $c\bar{o}v$ contributes to the market variance.

A similar result holds for skewness: for large N, the market skewness is not driven by the average skewness, but by the co-skewness terms (see Conine and Tamarkin, 1981):

$$
\begin{aligned}
Sk_{m,t} &= \frac{1}{D_t}\sum_{d=1}^{D_t}\tilde{r}^3_{m,d} = \frac{1}{D_t}\sum_{d=1}^{D_t}\left(\frac{1}{N}\sum_{i=1}^{N}\tilde{r}_{i,d}\right)^3 = \frac{1}{D_t}\sum_{d=1}^{D_t}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(\frac{1}{N^3}\tilde{r}_{i,d}\tilde{r}_{j,d}\tilde{r}_{k,d}\right) \\
&= \frac{1}{N^3}\sum_{i=1}^{N}sk_{iii,t} + \frac{3}{N^3}\sum_{i=1}^{N}\sum_{j\neq i,1}^{N}sk_{ijj,t} + \frac{1}{N^3}\sum_{i=1}^{N}\sum_{j\neq i,1}^{N}\sum_{k\neq\{i,j\},1}^{N}sk_{ijk,t} \\
&= \frac{1}{N^2}s\bar{k} + \frac{3(N-1)}{N^2}cos\bar{k}_1 + \frac{(N-1)(N-2)}{N^2}cos\bar{k}_2,
\end{aligned}
$$

where $sk_{ijk,t} = \frac{1}{D_t}\sum_{d=1}^{D_t}\tilde{r}_{i,d}\tilde{r}_{j,d}\tilde{r}_{k,d}$, $s\bar{k} = \frac{1}{N}\sum_{i=1}^{N}sk_{iii,t}$, $cos\bar{k}_1 = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j\neq i,1}^{N}sk_{ijj,t}$, and $cos\bar{k}_2 = \frac{1}{N(N-1)(N-2)}\sum_{i=1}^{N}\sum_{j\neq i,1}^{N}\sum_{k\neq\{i,j\},1}^{N}sk_{ijk,t}$.

As N goes to infinity, the market skewness reflects the average co-skewness across three different firms ($cos\bar{k}_2$), which conveys a different type of information compared to the average skewness.

B Additional Results Based on the Definition of the Variables

B.1 Demeaning of Daily Returns

B.1.1 Firm's Data

As directly posting the data would be a violation of the "WRDS Data User License Agreement", we describe the procedure of data request, with the following data cleaning process:

1. We download firm-level data for Price (PRC), Trading Volume (VOL), Shares outstanding (SHROUT), and Return (RET), for firms with a share code (SHRCD) equal to 10 or 11. Prices and Shares outstanding are used to compute the market value of the firms.

2. Prices equal to zero are treated as missing values.

3. All prices are converted to absolute values.

4. Negative trading volume values are treated as missing values.

5. Returns equal to -66, -77, -88, and -99 are treated as missing values.

B.1.2 Demeaning Returns

In Equations (2) and (3) of the main text, we define the average stock variance and skewness using centered daily returns, which are obtained by subtracting the average daily return within each month t. It is usually assumed that demeaning daily returns has no material impact on predictability. In fact, the way that returns are centered is not necessarily innocuous because of the turn-of-the-month effect in stock returns identified by Lakonishok and Smidt (1988). They report that the four days at the turn of the month account for all of the positive return to the Dow Jones index over the period 1897–1986. McConnell and Xu (2008) confirm their results over the period 1987–2005 and find that the daily value-weighted CRSP market return is the highest during the last two days and the first two days of the month. In our sample (August 1963–December 2016), we obtain a similar result: the value-weighted CRSP daily market return for the last two days and the first two days of the month are equal to 0.08%, 0.064%, 0.122%, and 0.131%, while the average daily return for the other days is only0.027%. Using equal weights, the average daily market return for the same four days are equal to 0.16%, 0.424%, 0.173%, and 0.30%, while the average daily return for the other days is 0.11%.

This turn-of-the-month effect also affects the dynamics of the daily returns of individual firms, in particular, the correlation between consecutive days at the turn of the month: whereas the average correlation of order 1 is equal to -6% when all days are considered, the average correlation between the return of the last day of the month and the return of the first day of the subsequent month is much larger in magnitude (-13%). This stronger correlation between daily returns at the turn of the month alters the correlation between monthly variables. When we center daily returns with the average return involving all the days of the month, the serial correlation of order 1 is equal to 6.6% and 41% for the value-weighted and the equal-weighted average skewness, respectively. When the average return excludes the last two days of the month, the correlation of order 1 is reduced to 5.3% and 25%, respectively. In contrast, when we center daily returns with the average return involving all the days of the month, the correlation with the market return in month $t+1$ is equal to -4.2% and -4% for the value-weighted and the equal-weighted average skewness, respectively. When the average return excludes the last two days of the month, the correlation with the market return in month $t+1$ almost decreases to -11.6% and -9.7%, respectively.

To avoid these issues related to the correlation between daily returns at the turn of the month, we center daily returns using $\bar{r}_{i,t} = \sum_{d=1}^{D_t-2} r_{i,d}$.

B.1.3 Alternative Demeaning of Daily Returns

To further investigate the impact of the average daily return on predictive regressions, we consider an alternative case in which, instead of subtracting the average daily stock return, we subtract the average daily market return. The intuition is that in measuring the idiosyncratic return with the market model, the estimation of the beta parameter is usually noisy. Imposing a sensitivity of the stock return to the market return equal to 1 introduces a bias but may render the demeaning more robust. In this case, the monthly variance and skewness of stock i in month t are defined as follows:

$$V_{i,t}^{(\text{market})} = \sum_{d=1}^{D_t} (r_{i,d} - \bar{r}_{m,t})^2 + 2\sum_{d=2}^{D_t} (r_{i,d} - \bar{r}_{m,t})\ (r_{i,d-1} - \bar{r}_{m,t}), \tag{A.5}$$

$$Sk_{i,t}^{(\text{market})} = \sum_{d=1}^{D_t} \tilde{r}_{i,d}^3, \tag{A.6}$$

where $\tilde{r}_{i,d} = (r_{i,d} - \bar{r}_{m,t})/\sigma_{i,t}$ with $\sigma_{i,t}^2 = \sum_{d=1}^{D_t} (r_{i,d} - \bar{r}_{m,t})^2$ and $\bar{r}_{m,t}$ is the average daily market excess return in month t (based on days $d = 1, \cdots, D_t - 2$). The value-weighted average of monthly skewness across firms is denoted by $Sk_{vw,t}^{(\text{market})} = \sum_{i=1}^{N_t} w_{i,t}\ Sk_{i,t}^{(\text{market})}$.

Table A1 reports the results for the predictive regressions in which the average skewness are based on Equations (A.5) and (A.6). The parameter estimate of the average skewness is equal to -0.124 (p-value equal to 0.1%) and the adjusted R^2 is equal to 1.22%. For the 1990–2016 sample, the parameter estimate is equal to -0.126 and the adjusted R^2 is equal to 1.41%. As in the baseline case, the equal-weighting scheme (not reported here) is associated with a weaker effect of the average skewness.

[Insert Table A1 here]

B.2 Analysis of Market Conditions

In Section 4.1, we consider the case where future market excess return is driven by the combination of market excess return and average skewness. More precisely, we find that the market excess return is on average equal to 1.14% for the months following a month with a market excess return above its mean and an average skewness below its mean. The market excess return is on average equal to -0.19% for the months following a month with a market excess return below its mean and an average skewness above its mean.

To characterize these market conditions, we have analyzed two particular cases: Does the case with high market return and low average skewness correspond to expansions or recessions? And does it correspond to liquid or illiquid market conditions? We proceed as follows: We take all the days corresponding to a recession (according to NBER) and compute the frequency of these days with (market return above its mean, skewness below its mean), with (market return below its mean, skewness above its mean), etc. For liquid market conditions, we take all the days corresponding to the aggregate measure of illiquidity being below its mean (and its unexpected component) and compute the frequency of these days with (market return above its mean, skewness below its mean), with (market return below its mean, skewness above its mean), etc.

Table A2 (Panel A) reports the probability of occurrence of each of the states and the average market return of the month after having been in a given state. Panel B reports the probability of a given event to occur in a given state. For instance, 32.2% of NBER recessions take place in the (Low return, High skewness) state but only 18.9% take place in the (High return, Low skewness) state. These results suggest that, compared to the case with low market return and high average skewness, the case with high market return and low average skewness correspond to expansionary conditions.

We also computed the probability of occurrence of an aggregate illiquidity measure above its mean in a given state. The construction of the aggregate illiquidity measure is described in Section 4.2. For instance, 31.8% of low aggregate illiquidity measure take place in the (High return, Low skewness) state but only 19.6% take place in the (Low return, High skewness) state. A similar result holds for the measure of unexpected illiquidity. This result suggests that on average, periods with high market return and low average skewness correspond to periods of relatively liquid market conditions.

[Insert Table A2 here]

B.3 Return on S&P 500 Index Futures Contracts

As our skewness measure is constructed using daily stock returns, one concern may be that the predictability of average skewness is due to lead-lag effects in the aggregation across stocks. To address this issue, we follow the approach of Antonakakis, Floros, and Kizys (2016) and use the monthly return from S&P 500 index futures contracts as the dependent variable in the predictive regressions. We test whether skewness still predicts the return on index futures contracts.

Returns are calculated as the continuously compounded day-to-day capital gain on the futures index. The sample spans the period from May 1982 to December 2016 (near-time delivery futures contracts are considered). The standard S&P 500 futures contract size is 250 US dollars per index point of the underlying. In their study on the relation between the S&P 500 stock index and futures markets, In and Kim (2006) provide details of the S&P 500 market characteristics. In particular, they argue that the stock index futures are heavily traded in the last three months before expiration. As a futures contract approaches its expiration, investors close their positions and open new positions in the next near contract. The S&P 500 stock index futures contracts have maturity dates in March, June, September, and December and are settled in cash.

Table A3 reveals that the predictive power of the average skewness in forecasting index future returns is still significant with p-values equal to 0.8% over the 1982–2016 sample and 2.9% over the 1990–2016 sample. The adjusted R^2 values are equal to 1.17% and 1%, respectively.

[Insert Table A3 here]

B.4 Square Root and Log of Variance

Table A4 reports regression results when we use the square root and the log transform of the average variance. The coefficients of the square root and log of the variance are insignificant for both samples. The significance of the average skewness is not affected by the transformation of the average variance.

[Insert Table A4 here]

B.5 Median Measures

Because the average skewness may be sensitive to outliers in the cross-sectional distribution of the monthly variance or skewness, we consider the median as a more robust estimator. It is defined as the monthly stock skewness such that 50% of the stocks have a smaller skewness and 50% of the stocks have a larger skewness. As is well known, the median is less sensitive to outliers than the average and can thus be viewed as a more robust estimator of aggregate risk.

Table A5 reports the regression results based on the median of the cross-sectional distribution of the variance and skewness. When the median skewness is introduced alone, its parameter estimate is highly significant, with a p-value equal to 0.8% and an adjusted R^2 equal to 0.83%. When introduced with other predictors, the predictability of median skewness still remain significant. For instance in Column IV, the coefficient of the median skewness is significant with p-values equal to 0.6% for the 1963–2016 sample and 8% for the 1990–2016 sample.

[Insert Table A5 here]

B.6 Cross-sectional Measures

The monthly average variance and skewness used in the paper are based on the demeaned daily returns within a month. These measures may be sensitive to outliers in the time series of daily returns. As in Goyal and Santa-Clara (2003) and Bali, Cakici, Yan, and Zhang (2005), we also investigate average risk measures based on the cross-sectional distribution of monthly returns. For this purpose, we define the average variance as the cross-sectional variance of monthly excess returns as below:

$$V_{ew,t}^{(CS)} = \frac{1}{N_t} \sum_{i=1}^{N_t} (r_{i,t} - r_{m,t})^2,$$

where $r_{i,t}$ is monthly excess return of firm i and $r_{m,t}$ is the monthly market excess return. We define the cross-sectional skewness as

$$Sk_{ew,t}^{(CS)} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left(\frac{r_{i,t} - r_{m,t}}{\sqrt{V_{ew,t}^{(CS)}}} \right)^3 .$$

This definition of skewness based on the cross-sectional average also allows us to verify that measuring the monthly skewness using daily data does not generate any sizable finite-sample bias.

Table A6 reports the results for the predictive regressions with cross-sectional average variance and skewness. As we use the cross-section of monthly stock returns, we can use a longer sample of CRSP market return, starting in July 1926. We first note that the predictive power of the cross-sectional skewness is not substantially affected compared with the regression with the average skewness based on daily returns. For instance, for the regression based on 1963–2016 sample, the adjusted R^2 slightly decreases from 1.18% to 0.96% (average skewness alone) and decreases from 1.726% to 1.137% (with market return). In addition, the coefficients of the cross-sectional skewness are still highly significant, with p-values below 3%. For the longer sample, starting in 1926, the results are essentially unaltered: the skewness parameter is negative and highly significant, with p-values around 3%, even when skewness is introduced alone in the regression.

[Insert Table A6 here]

B.7 Robust Skewness

Some recent papers (Garcia, Mantilla-Garcia, and Martellini, 2014, and Ghysels, Plazzi, and Valkanov, 2016) consider alternative measures of skewness based on the quantiles of the cross-distribution of returns to circumvent the sensitivity of standard measures to outliers. Hinkley (1975)'s robust coefficient of asymmetry (see also Groeneveld and Meeden, 1984) is defined as

$$RSk_t^{(H)} = \frac{q_t(1-\alpha) + q_t(\alpha) - 2q_t(0.5)}{q_t(1-\alpha) - q_t(\alpha)}, \tag{A.7}$$

where $q_t(\alpha) = F_t^{-1}(\alpha)$ is the α-quantile of the cross-sectional distribution of returns on month t and $q_t(0.5)$ is the median of the cross-sectional distribution of the monthly return. The measure $RSk_t^{(H)}$ is defined over $(-1, 1)$; therefore, its magnitude is not directly comparable to the usual measure of standardized skewness. We select values of α equal to 0.01 and 0.05, which satisfy Groeneveld and Meeden (1984)'s properties for reasonable skewness coefficients. We also define the average measure proposed by Groeneveld and Meeden (1984):

$$RSk_t^{(GM)} = \frac{\int_0^{0.5} [q_t(1-\alpha) + q_t(\alpha) - 2q_t(0.5)]d\alpha}{\int_0^{0.5} [q_t(1-\alpha) - q_t(\alpha)]d\alpha} = \frac{r_{m,t} - q_t(0.5)}{E[|r_t - q_t(0.5)|]}. \tag{A.8}$$

The numerator is the difference between the cross-sectional mean and median in month t, whereas the denominator is a measure of the dispersion.

Table A7 reports the results for the predictive regressions based on the robust measures of skewness. We observe that the robust skewness is still a significant predictor of the future market return, although we only use monthly data and trim the tails of the distribution. The parameter estimates of the robust skewness are all negative, with values between -0.08 and -0.038. The p-values of the skewness parameters range between 3.2% and 11.3% for the 1963–2016 sample. These estimates contrast with the results reported by Garcia, Mantilla-Garcia, and Martellini (2014), who find a positive, yet weakly significant, effect of the robust measure of skewness.[3] We note however that, for the 1990–2016 sample, the robust measure of skewness fails to predict future market returns.

[Insert Table A7 here]

C Additional Regression Results

C.1 Controlling for Firm Size and Liquidity

To determine whether the significance of the average skewness is driven by small or illiquid stocks, we report in Table A8 the results of the predictive regression of the CRSP market excess return on the average skewness computed using NYSE/AMEX, NYSE, and NASDAQ stocks separately. Over the 1963–2016 period (Panel A), the effect of average skewness on future market return is as strong as previously reported when average skewness is based on NYSE/AMEX stocks or NYSE stocks (p-values below 0.5%) and slightly weaker when it is based on NASDAQ stocks (p-value equal to 2.5%). Over the recent 1990–2016 period (Panel B), average skewness is also significant for most stock exchanges used to measure average skewness. These results clearly indicate that the predictability of average skewness is not only driven by small firms.

The use of all of the stocks traded on the NYSE, AMEX, and NASDAQ to measure average skewness may also generate some bias due to outliers. In particular, firms with small market capitalization, low price, or large bid-ask spread may generate microstructure noise in individual skewness measures, thus affecting the aggregate skewness. To control for these possible biases, we measure skewness using an additional filter for size, liquidity, and price. We exclude the smallest firms (firms with a market capitalization smaller than the smallest NYSE size decile), the least liquid stocks (firms with a number of shares traded lower than the smallest NYSE decile for the number of shares traded), and the lowest-priced stocks (stocks with a price less than \$5).

Columns VII and VIII present the predictive regressions when firms are selected based on a screen for size, liquidity, and price. The estimates are similar to those reported in Table 2. The average skewness has highly significant parameter estimates, with adjusted R^2 values equal to 1.21% and 1.16% for the 1963–2016 and 1990–2016 samples, respectively. These results confirm that the significance of the average skewness is not due to small and illiquid firms but that it

[3] It should be noted that they use a non-conventional definition of Hinkley (1975)'s robust coefficient of asymmetry, with $q_t(1-\alpha)+q_t(\alpha)$ in the denominator of Equation (A.7).

is mostly driven by medium and large firms.[4]

[Insert Table A8 here]

C.2 Results with a Dummy for Economic Conditions

We know from Figures 1 and 2 that the average variance and skewness are affected by economic recessions as identified by the NBER. To test whether our main results are robust to controlling for economic recessions, we added to the baseline regression a dummy variable equal to 1 for NBER-dated economic recessions and 0 otherwise. As Table A9 demonstrates, after introducing the recession variable, the coefficients of average skewness still remain highly significant with p-values below 0.3% in the 1963–2016 sample and below 4.2% in the 1990–2016 sample. We also added a dummy variable for economic expansion, and the results remain similar. These results suggest that the strong predictive power of average skewness is not due to specific economic conditions.

[Insert Table A9 here]

C.3 Results with Longer Investment Horizons

In all our estimates, we report one month ahead predictive regressions. Several previous papers have identified variables with a substantial predictive ability for longer horizons. For instance, Moskowitz, Ooi, and Pedersen (2012) find that most financial markets exhibit persistence in returns for horizons up to 12 months. For equity index futures, they obtain the highest abnormal performance for horizons 9 and 12 months.

In Tables A10 and A11, we report additional regression results with horizons equal to 1, 3, and 6 months, respectively. Table A10 corresponds to (value-weighted and equal-weighted) average variance and skewness. We find that for all the horizons the average skewness measures are the best predictors of future market excess returns. For the 1-month horizon, the value-weighted measure performs the best: for the 3-month horizon, the value-weighted and equal-weighted measures have similar performance; for the 6-month horizon, the equal-weighted measure dominates. We also note that for the recent period (1990–2016), the adjusted R^2 values are usually higher than for the whole period (1963–2016).

Table A11 corresponds to financial competitors, including average correlation (AC), tail risk measure (TR), the aggregate short interest index (SII), the VIX, and the variance (VRP) and

[4] In Technical Appendix C.5, we report results for the case where skewness is measured using firms sorted into terciles based on their market capitalization. Again, the average skewness remains highly significant for the tercile corresponding to large firms.

tail (TRP) risk premia. The table reveals that the average skewness is a significant predictor of market excess returns for all horizons and all samples that we consider. The p-values usually increase and the adjusted R^2 values decrease for the 6-month horizons. Average skewness dominates the tail risk measure, the VIX and the tail risk premium for the one-month and three-month horizons. However, the average correlation, short interest index, and the variance risk premium generate higher adjusted R^2 values for the three-month and six-month horizons.

[Insert Tables A10 and A11 here]

C.4 Contemporaneous and Lagged Relation between Market Return and Average Skewness

Figure A1 displays the scatterplot of the contemporaneous relation between market return and value-weighted average skewness. Figure A2 displays the same relation when market returns below -15% and value-weighted average skewness below -10% are winsorized. As Table A12 (Panel A) reports, winsorization has limited impact on the contemporaneous relation between market return and average skewness.

Figures A3 and A4 display the scatterplot of the lagged relation between market return and value-weighted average skewness, without and with winsorization, respectively. Figure A3 corresponds precisely to the regression reported in Column V on Table 2 of the paper. Panel B of Table A12 reports the correlation, parameter estimates, and adjusted R^2 corresponding to Figures A3 and A4. Figure A5 plots the time series of market return and its prediction when only value-weighted average skewness is used as a predictor. Again, winsorization barely affects the relation between current market return and lagged average skewness.

[Insert Table A12 and Figures A1 to A5 here]

C.5 Results with Firm Size Terciles

Table A13 reports regression results when firms are sorted into terciles according to their market capitalization (size) from small (Tercile 1) to large firms (Tercile 3). The average skewness significantly predicts next-month market returns for firms with different market capitalization. For the whole sample period 1963-2016, the parameter associated with the average skewness has a low p-value (below 2.3%) for the three size terciles. The adjusted R^2 is also high for all firm-size categories. As in Section C.1, these results confirm that the predictive ability of average skewness is not driven by small firms.

[Insert Table A13 here]

C.6 Skewness Factor Model Estimates

Following the approach of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) for idiosyncratic volatility, we estimate factor regression models of firm-level skewness. For both individual skewness and standardized skewness, we regress the skewness on alternative skewness factors: $Sk_{i,t} = \phi_0 + \phi_1 Sk_{\text{factor},t} + e_{i,t}$, $t = 1, \cdots, T$. We consider three definitions of factors: the equal-weighted average skewness (Sk_{ew}), the principal component of individual skewness (Sk_{PC}), and the principal component of standardized skewness ($Sk_{PC}^{(Std)}$). Specifically, we estimate the average of the first three principal components of the individual skewness and the standardized individual skewness. Then, we estimate the individual and standardized individual skewness exposure to these common factors.

Table A14 (Panel A) reports parameter estimates of the skewness factor model for monthly individual skewness. Columns correspond to the various form of common factors. The average univariate time-series adjusted R^2 ranges from 11.62% for the factor defined as the equal-weighted average skewness to 12.26% for the principal component of individual and standardize individual skewness. This evidence suggests that there is a common factor in firm-level skewness values.

Panel B reports results of predictive regressions using the average of the first three principal components of individual skewness and standardized skewness as predictors. The average of the first three principal components of individual skewness and standardized skewness has significant predictive power for market risk premium. When the skewness factor is considered alone, the p-values are equal to 3.2% and 2.5%, with adjusted R^2 values equal to 0.50% and 0.52%, respectively. In combination with market return, the p-values are equal to 0.3% and 0.1%, with adjusted R^2 values equal to 1.39% and 1.49%, respectively.

[Insert Table A14 here]

C.7 Time Series Momentum Strategy

In Section 4.1 of the main text, we report that at the market level, a combination of high return and low skewness in month t is, on average, followed by a high market return in month $t+1$. Conversely, a combination of low return and high skewness in month t is, on average, followed by a low market return in month $t+1$. This result is akin to the time series momentum, as illustrated by Moskowitz, Ooi, and Pedersen (2012), who show that most financial markets exhibit persistence in returns for horizons up to 12 months. As we discuss in Technical Appendix

C.3, when value-weighted average skewness is used as a predictor of market returns, the best performance is obtained for a horizon of one to three months. We follow this result and design a trading strategy based on a one-month horizon.

We investigate whether a simple trading strategy with both a time series momentum and skewness effects can generate superior performance at the market level. The strategy is designed as follows: when the market return is above its mean and the average skewness is below its mean, the investor borrows and invests in the stock market (2 dollars in the stock market, -1 dollar in the risk-free asset); when the market return is below its mean and the average skewness is above its mean, the investor shorts the stock market (-1 dollar in the stock market, 2 dollars in the risk-free asset); and otherwise, the investor only invests in the stock market (1 dollar in the stock market). We compare this strategy with the strategy considering only the time series momentum effect, which is designed as follows: when the market return is above its mean, the investor invests 1 dollar in the stock market; when the market return is below its mean, the investor invests 1 dollar in the risk-free asset.

In Table A15, we report some summary statistics on the three time series momentum strategies over the sample August 1963 to December 2016. Figure A6 shows the cumulative return of these strategies. The strategy that combines both market return and (value- or equal-weighted) skewness dominates the one that considers only the time series momentum effect. With the combined strategy, the annualized average return over the period is 14.6% (with a Sharpe ratio of 0.57) for value-weighted average skewness and 13.3% (with a Sharpe ratio of 0.52) for equal-weighted average skewness. With the pure time series momentum strategy, the annualized return is 9.5% (with a Sharpe ratio of 0.52). In addition, the number of rebalancing trades is similar under the combined strategy and the pure momentum strategy (389 vs. 309 trades over 641 months). Taking transaction fees into account would not substantially alter the difference between the performances of these strategies.

[Insert Table A15 and Figure A6 here]

C.8 One-day Regressions

Instead of forecasting market returns using end-of-month data, we also investigate the impact of cumulating daily returns starting on a different day in the month. To do so, we forecast monthly market returns on each of trading days in the month using the information available up to that trading day. For instance, for "Day 2", we define $r_{m,t}$ from day 2 of month t to day 1 of month $t+1$, we define $r_{m,t+1}$ from day 2 of month $t+1$ to day 1 of month $t+2$, etc.

Results reported in Table A16 indicate that there is some heterogeneity (maybe seasonality related to the turn-of-the-month discussed in Technical Appendix B.1) in the parameter estimates and adjusted R^2 of the regression. The parameter associated with market return is in general positive and at best weakly significant. The average skewness parameter is almost always negative. It is statistically significant at the beginning of the month (Days 1 to 6) and at the end of the month (Days 20 to 21). The adjusted R^2 is usually large for the same days. It is the highest on Day 2 with average skewness alone (1.19%) and on Day 1 in combination with market return (1.73%).

Similarly, in Table A17, the out-of-sample allocation analysis demonstrates that strategies based on average skewness or on market return and average skewness perform better when "months" are constructed close to the usual way (Days 1 to 2 and 20 to 21).

[Insert Tables A16 and A17 here]

References

Aït-Sahalia, Yacine, and Andrew W. Lo, 1998, Nonparametric estimation of state-price densities implicit in financial asset prices, *Journal of Finance* 53, 499–547.

Antonakakis, Nikolaos, Christos Floros, and Renatas Kizys, 2016, Dynamic spillover effects in futures markets: UK and US evidence, *International Review of Financial Analysis* 48, 406–418.

Bali, Turan G., Nusret Cakici, Xuemin Sterling Yan, and Zhe Zhang, 2005, Does idiosyncratic risk really matter?, *Journal of Finance* 60, 905–929.

Bates, David S., 2008, The market for crash risk, *Journal of Economic Dynamics and Control* 32, 2291–2321.

Christoffersen, Peter, Kris Jacobs, and Chayawat Ornthanalai, 2012, Dynamic jump intensities and risk premiums: Evidence from S&P 500 returns and options, *Journal of Financial Economics* 106, 447–472.

Clark, Todd E., and Michael W. McCracken, 2001, Tests of equal forecast accuracy and encompassing for nested models, *Journal of Econometrics* 105, 85–110.

Conine, Thomas E., and Maurry J. Tamarkin, 1981, On diversification given asymmetry in returns, *Journal of Finance* 36, 1143–1155.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal, 2009, Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?, *Review of Financial Studies* 22, 889–908.

Dittmar, Robert F., 2002, Nonlinear pricing kernels, kurtosis preference, and evidence from the cross-section of equity returns, *Journal of Finance* 57, 369–403.

Garcia, René, Daniel Mantilla-Garcia, and Lionel Martellini, 2014, A model-free measure of aggregate idiosyncratic volatility and the prediction of market returns, *Journal of Financial and Quantitative Analysis* 49, 1133–1165.

Ghysels, Eric, Alberto Plazzi, and Rossen Valkanov, 2016, Why invest in emerging markets? the role of conditional return asymmetry, *Journal of Finance* 71, 2145–2192.

Goyal, Amit, and Pedro Santa-Clara, 2003, Idiosyncratic risk matters!, *Journal of Finance* 58, 975–1008.

Groeneveld, Richard A., and Glen Meeden, 1984, Measuring skewness and kurtosis, *The Statistician* 33, 391–399.

Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295.

Harvey, David I., Stephen J. Leybourne, and Paul Newbold, 1998, Tests for forecast encompassing, *Journal of Business and Economic Statistics* 16, 254–259.

Herskovic, Bernard, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh, 2016, The common factor in idiosyncratic volatility: Quantitative asset pricing implications, *Journal of Financial Economics* 119, 249–283.

Hinkley, David V., 1975, On power transformations to symmetry, *Biometrika* 62, 101–111.

In, Francis, and Sangbae Kim, 2006, The hedge ratio and the empirical relationship between the stock and futures markets: A new approach using wavelet analysis, *Journal of Business* 79, 799–820.

Lakonishok, Josef, and Seymour Smidt, 1988, Are seasonal anomalies real? A ninety-year perspective, *Review of Financial Studies* 1, 403–425.

Lo, A.W., 2002, The statistics of Sharpe ratios, *Financial Analysts Journal* 58, 36–52.

Maheu, John M., and Thomas H. McCurdy, 2004, News arrival, jump dynamics, and volatility components for individual stock returns, *Journal of Finance* 59, 755–794.

———, and Xiaofei Zhao, 2013, Do jumps contribute to the dynamics of the equity premium?, *Journal of Financial Economics* 110, 457–477.

McConnell, John J., and Wei Xu, 2008, Equity returns at the turn of the month, *Financial Analysts Journal* 64, 49–64.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Table A1: Predictive Regressions of Market Return – Demeaning Daily Returns with Monthly Market Return

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $V_{vw,t}^{(\text{market})}$ and $Sk_{vw,t}^{(\text{market})}$ are the value-weighted average variance and skewness when daily stock returns are demeaned by the average daily excess market return. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI	VII
Panel A: 1963–2016							
Constant	0.0047	0.0095	0.0109	0.0141	0.0145	0.014	0.0105
	(0.011)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$r_{m,t}$	0.0725	–	–	–	–	0.0458	0.0738
	(0.089)					(0.242)	(0.077)
$V_{m,t}$	–	–	–	–	−20.3819	−15.4904	–
					(0.334)	(0.471)	
$Sk_{m,t}$	–	–	–	–	0.0039	0.0035	
					(0.256)	(0.307)	
$V_{vw,t}^{(\text{market})}$	–	−0.4799	–	−0.4058	−0.0207	−0.0595	–
		(0.024)		(0.063)	(0.963)	(0.892)	
$Sk_{vw,t}^{(\text{market})}$	–	–	−0.1240	−0.1131	−0.1506	−0.1475	−0.1249
			(0.001)	(0.006)	(0.001)	(0.001)	(0.001)
Adj. R^2	0.370%	0.694%	1.222%	1.665%	1.782%	1.813%	1.613%
Panel B: 1990–2016							
Constant	0.0060	0.0124	0.0116	0.0151	0.0151	0.0147	0.0110
	(0.020)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
$r_{m,t}$	0.0730	–	–	–	–	0.0222	0.0637
	(0.260)					(0.708)	(0.314)
$V_{m,t}$	–	–	–	–	−20.2300	−18.1914	–
					(0.410)	(0.470)	
$Sk_{m,t}$	–	–	–	–	0.0028	0.0025	–
					(0.539)	(0.594)	
$V_{vw,t}^{(\text{market})}$	–	−0.5370	–	−0.4184	−0.0619	−0.0769	–
		(0.038)		(0.137)	(0.896)	(0.869)	
$Sk_{vw,t}^{(\text{market})}$	–	–	−0.1255	−0.0995	−0.1299	−0.1258	−0.1209
			(0.009)	(0.085)	(0.029)	(0.034)	(0.016)
Adj. R^2	0.230%	1.324%	1.406%	2.020%	1.917%	1.650%	1.508%

Table A2: Characterization of Market Return and Average Skewness Conditions

This table provides information about the market conditions, corresponding to different states defined by the value of market return and average skewness. For instance, (High return, Low skewness) corresponds to months when the market excess return is above its mean and average skewness is below its mean. Panel A reports the probability of occurrence of each of the four states and the average mean market return of the month after having been in a given state. Panel B reports the probability of a given event (for instance, an NBER recession) when the market is in a given state (for instance, (High return, Low skewness)).

	I	II	III	IV
	High return Low skew.	Low return High skew.	Low return Low skew.	High return High skew.
Panel A: Information about each state				
Probability of occurrence	27.10%	21.34%	23.68%	27.88%
Average return of next month	1.14%	−0.19%	0.64%	0.33%
Panel B: Probability of a given event to occur in a given state				
NBER recession	18.89%	32.22%	25.56%	23.33%
NBER expansion	25.21%	22.06%	22.06%	30.67%
Aggregate illiquidity high	20.45%	23.86%	26.52%	29.17%
Aggregate illiquidity low	31.75%	19.58%	21.69%	26.98%
Unexpected illiquidity high	20.75%	26.18%	28.54%	24.53%
Unexpected illiquidity low	32.50%	18.13%	21.46%	27.92%

Table A3: Predictive Regressions of S&P 500 Index Futures Return

This table reports results of the one-month-ahead predictive regressions of the excess market return $r_{m,t+1}^{(Fut)}$. The market return is the monthly return from S&P 500 index futures contracts. $V_{vw,t}$ and $Sk_{vw,t}$ are the value-weighted average variance and skewness. $V_{m,t}^{(Fut)}$ and $Sk_{m,t}^{(Fut)}$ are market variance and skewness based on futures index. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: May 1982 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI	VII
Panel A: 1982–2016							
Constant	0.0067	0.0129	0.0107	0.0157	0.0142	0.0136	0.0105
	(0.006)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$r_{m,t}^{(Fut)}$	0.0530	–	–	–	–	0.0393	0.0752
	(0.389)					(0.471)	(0.206)
$V_{vw,t}$	–	-0.6089	–	-0.5622	-0.2760	-0.2643	–
		(0.008)		(0.014)	(0.439)	(0.461)	
$Sk_{vw,t}$	–	–	-0.1254	-0.1137	-0.1074	-0.1092	-0.1385
			(0.008)	(0.035)	(0.075)	(0.074)	(0.005)
$V_{m,t}^{(Fut)}$	–	–	–	–	-9.0383	-7.8269	–
					(0.125)	(0.173)	
$Sk_{m,t}^{(Fut)}$	–	–	–	–	-0.0020	-0.0025	–
					(0.527)	(0.425)	
Adj. R^2	0.032%	1.440%	1.171%	2.357%	2.269%	2.155%	1.470%
Panel B: 1990–2016							
Constant	0.0058	0.0118	0.0094	0.0135	0.0133	0.0128	0.0092
	(0.028)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)
$r_{m,t}^{(Fut)}$	0.0599	–	–	–	–	0.0487	0.0755
	(0.404)					(0.450)	(0.255)
$V_{vw,t}$	–	-0.5578	–	-0.4714	-0.3378	-0.3544	–
		(0.043)		(0.103)	(0.440)	(0.405)	
$Sk_{vw,t}$	–	–	-0.1170	-0.0952	-0.1050	-0.1068	-0.1263
			(0.029)	(0.132)	(0.124)	(0.120)	(0.017)
$V_{m,t}^{(Fut)}$	–	–	–	–	-6.5235	-3.0715	–
					(0.727)	(0.871)	
$Sk_{m,t}^{(Fut)}$	–	–	–	–	0.0009	0.0003	–
					(0.807)	(0.945)	
Adj. R^2	0.034%	1.214%	1.004%	1.742%	1.197%	1.076%	1.236%

Table A4: Predictive Regressions of Value-Weighted Market Return – Square Root of Variance and Log Variance

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $V_{vw}^{1/2}$ and $\log V_{vw}$ are the square root and log of the value-weighted variance, respectively. Sk_{vw} is the value-weighted skewness. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI	VII	VIII	IX
Panel A: 1963–2016									
Constant	0.0140	−0.0147	0.0091	0.0167	−0.0078	0.0101	0.0221	0.0100	0.0204
	(0.010)	(0.441)	(0.000)	(0.002)	(0.687)	(0.153)	(0.301)	(0.148)	(0.321)
$r_{m,t}$	–	–	–	–	–	–	–	0.0568	0.0567
								(0.157)	(0.158)
$V_{vw,t}^{1/2}$	−0.1029	–	–	−0.0900	–	0.0286	–	0.0217	–
	(0.134)			(0.189)		(0.769)		(0.818)	
$\log V_{vw,t}$	–	−0.0040	–	–	−0.0034	–	0.0019	–	0.0017
		(0.275)			(0.354)		(0.629)		(0.654)
$Sk_{vw,t}$	–	–	−0.1259	−0.1206	−0.1226	−0.1536	−0.1533	−0.1569	−0.1569
			(0.002)	(0.004)	(0.003)	(0.001)	(0.002)	(0.001)	(0.001)
$V_{m,t}$	–	–	–	–	–	−25.4251	−25.6273	−20.3084	−20.9516
						(0.095)	(0.026)	(0.194)	(0.083)
$Sk_{m,t}$	–	–	–	–	–	0.0034	0.0034	0.0032	0.0031
						(0.319)	(0.327)	(0.349)	(0.356)
Adj. R^2	0.339%	0.105%	1.179%	1.402%	1.212%	1.778%	1.797%	1.910%	1.929%
Panel B: 1990–2016									
Constant	0.0192	−0.0259	0.0097	0.0200	−0.0178	0.0141	0.0116	0.0136	0.0109
	(0.005)	(0.308)	(0.002)	(0.002)	(0.502)	(0.097)	(0.667)	(0.109)	(0.675)
$r_{m,t}$	–	–	–	–	–	–	–	0.0467	0.0465
								(0.439)	(0.443)
$V_{vw,t}^{1/2}$	−0.1367	–	–	−0.1153	–	−0.0134	–	−0.0153	–
	(0.109)			(0.187)		(0.905)		(0.889)	
$\log V_{vw,t}$	–	−0.0067	–	–	−0.0056	–	−0.0003	–	−0.0003
		(0.175)			(0.263)		(0.954)		(0.952)
$Sk_{vw,t}$	–	–	−0.1168	−0.0995	−0.1041	−0.1241	−0.1251	−0.1279	−0.1291
			(0.029)	(0.103)	(0.076)	(0.074)	(0.074)	(0.067)	(0.067)
$V_{m,t}$	–	–	–	–	–	−22.2080	−23.2246	−18.6153	−19.8593
						(0.260)	(0.150)	(0.366)	(0.249)
$Sk_{m,t}$	–	–	–	–	–	0.0024	0.0024	0.0021	0.0021
						(0.619)	(0.616)	(0.664)	(0.660)
Adj. R^2	0.959%	0.635%	1.016%	1.584%	1.356%	1.640%	1.635%	1.520%	1.513%

Table A5: Predictive Regressions of Market Return – Median Variance and Skewness

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $V_{md,t}$ and $Sk_{md,t}$ are the median variance and skewness. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI
Panel A: 1963–2016						
Constant	0.0085	0.0127	0.0145	0.0108	0.0107	0.0127
	(0.001)	(0.000)	(0.000)	(0.022)	(0.021)	(0.000)
$r_{m,t}$	–	–	–	–	0.0508	0.0799
					(0.197)	(0.050)
$V_{md,t}$	−0.3818	–	−0.2729	0.7962	0.7303	–
	(0.191)		(0.362)	(0.262)	(0.297)	
$Sk_{md,t}$	–	−0.1600	−0.1461	−0.1792	−0.1829	−0.1691
		(0.008)	(0.026)	(0.006)	(0.004)	(0.004)
$V_{m,t}$	–	–	–	−46.0602	−40.0073	–
				(0.083)	(0.141)	
$Sk_{m,t}$	–	–	–	0.0000	−0.0002	–
				(0.989)	(0.949)	
Adj. R^2	0.230%	0.834%	0.869%	1.519%	1.591%	1.316%
Panel B: 1990–2016						
Constant	0.0109	0.0122	0.0142	0.0106	0.0104	0.0122
	(0.001)	(0.003)	(0.000)	(0.059)	(0.065)	(0.003)
$r_{m,t}$	–	–	–	–	0.0354	0.0814
					(0.540)	(0.176)
$V_{md,t}$	−0.4322	–	−0.3168	0.9363	0.8980	–
	(0.243)		(0.448)	(0.336)	(0.351)	
$Sk_{md,t}$	–	−0.1390	−0.1073	−0.1690	−0.1721	−0.1499
		(0.067)	(0.271)	(0.080)	(0.075)	(0.045)
$V_{m,t}$	–	–	–	−53.3251	−49.4766	–
				(0.147)	(0.189)	
$Sk_{m,t}$	–	–	–	0.0001	−0.0001	–
				(0.977)	(0.975)	
Adj. R^2	0.421%	0.503%	0.544%	1.707%	1.505%	0.861%

Table A6: Predictive Regressions of Value-Weighted Market Return – Cross-sectional Measures

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $V_{ew,t}^{(CS)}$ and $Sk_{ew,t}^{(CS)}$ are the cross-sectional equal-weighted average variance and skewness. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: July 1926 to December 2016 (Panel A), August 1963 to December 2016 (Panel B) and January 1990 to December 2016 (Panel C).

	I	II	III	IV	V	VI	VII
Panel A: 1926–2016							
Constant	0.0058	0.0046	−0.0024	−0.0037	−0.0020	−0.0029	−0.0022
	(0.000)	(0.031)	(0.595)	(0.426)	(0.665)	(0.521)	(0.592)
$r_{m,t}$	0.1098	–	–	–	–	0.0961	0.1060
	(0.051)					(0.100)	(0.057)
$V_{ew,t}^{(CS)}$	–	2.7175	–	2.5617	4.9164	3.4828	–
		(0.335)		(0.354)	(0.048)	(0.113)	
$Sk_{ew,t}^{(CS)}$	–	–	−0.0278	−0.0263	−0.0223	−0.0221	−0.0250
			(0.031)	(0.053)	(0.091)	(0.075)	(0.032)
$V_{m,t}$	–	–	–	–	−20.3356	−8.9997	–
					(0.300)	(0.683)	
$Sk_{m,t}$	–	–	–	–	−0.0024	−0.0034	–
					(0.418)	(0.229)	
Adj. R^2	1.114%	0.341%	0.357%	0.649%	0.969%	1.654%	1.386%
Panel A: 1963–2016							
Constant	0.0047	0.0048	−0.0065	−0.0076	−0.0045	−0.0044	−0.0058
	(0.011)	(0.060)	(0.276)	(0.207)	(0.460)	(0.452)	(0.299)
$r_{m,t}$	0.0725	–	–	–	–	0.0338	0.0582
	(0.089)					(0.381)	(0.157)
$V_{ew,t}^{(CS)}$	–	0.2810	–	1.1608	3.9532	3.5091	–
		(0.934)		(0.709)	(0.144)	(0.180)	
$Sk_{ew,t}^{(CS)}$	–	–	−0.0369	−0.0379	−0.0289	−0.0282	−0.0339
			(0.025)	(0.020)	(0.074)	(0.077)	(0.032)
$V_{m,t}$	–	–	–	–	−23.8098	−20.4969	–
					(0.025)	(0.062)	
$Sk_{m,t}$	–	–	–	–	−0.0013	−0.0016	–
					(0.669)	(0.613)	
Adj. R^2	0.370%	−0.154%	0.960%	0.843%	1.277%	1.219%	1.137%
Panel B: 1990–2016							
Constant	0.0058	0.0088	−0.0095	−0.0086	−0.0060	−0.0058	−0.0088
	(0.026)	(0.030)	(0.244)	(0.287)	(0.458)	(0.460)	(0.249)
$r_{m,t}$	0.0723	–	–	–	–	0.0167	0.0421
	(0.265)					(0.762)	(0.477)
$V_{ew,t}^{(CS)}$	–	−2.3141	–	−0.5395	1.6383	1.4220	–
		(0.585)		(0.888)	(0.614)	(0.646)	
$Sk_{ew,t}^{(CS)}$	–	–	−0.0498	−0.0490	−0.0405	−0.0398	−0.0468
			(0.022)	(0.018)	(0.038)	(0.039)	(0.022)
$V_{m,t}$	–	–	–	–	−19.3836	−17.9296	–
					(0.200)	(0.272)	
$Sk_{m,t}$	–	–	–	–	−0.0014	−0.0016	–
					(0.732)	(0.697)	
Adj. R^2	0.213%	−0.102%	1.956%	1.661%	1.782%	1.495%	1.821%

Table A7: Predictive Regressions of Market Return – Robust Skewness

This table reports results of the one-month-ahead predictive regressions of the excess value-weighted market return $r_{m,t+1}$. Value-weighted market return is calculated based on AMEX, NYSE and NASDAQ stocks. $RSk_t^{(H1)}$ and $RSk_t^{(H2)}$ are the Hinkley (1975) robust measures of skewness (Equation (A.7)) defined with $\alpha = 0.01$ and 0.05, respectively. $RSk_t^{(GM)}$ is the Groeneveld and Meeden (1984) robust measure of skewness (Equation (A.8)). Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI
Panel A: 1963–2016						
Constant	−0.0642	−0.0548	−0.0341	−0.0280	−0.0241	−0.0196
	(0.053)	(0.088)	(0.130)	(0.189)	(0.152)	(0.222)
$r_{m,t}$	–	0.0554	–	0.0587	–	0.0598
		(0.185)		(0.155)		(0.153)
$RSk_t^{(H1)}$	−0.0803	−0.0691	–	–	–	–
	(0.032)	(0.059)				
$RSk_t^{(H2)}$	–	–	−0.0540	−0.0452	–	–
			(0.071)	(0.113)		
$RSk_t^{(GM)}$	–	–	–	–	−0.0456	−0.0381
					(0.068)	(0.113)
Adj. R^2	0.616%	0.753%	0.425%	0.599%	0.381%	0.569%
Panel B: 1990–2016						
Constant	−0.0629	−0.0459	−0.0184	−0.0087	−0.0164	−0.0089
	(0.369)	(0.483)	(0.676)	(0.824)	(0.631)	(0.772)
$r_{m,t}$	–	0.0562	–	0.0658	–	0.0644
		(0.353)		(0.263)		(0.285)
$RSk_t^{(H1)}$	−0.0785	−0.0588	–	–	–	–
	(0.315)	(0.421)				
$RSk_t^{(H2)}$	–		−0.0332	−0.0196	–	–
			(0.565)	(0.703)		
$RSk_t^{(GM)}$	–	–	–	–	−0.0340	−0.0221
					(0.490)	(0.618)
Adj. R^2	0.171%	0.147%	−0.134%	−0.040%	−0.087%	−0.008%

Table A8: Predictive Regressions of Market Return – Firm's Size and Liquidity

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $Sk_{vw,t}$ is the value-weighted average skewness. In Columns I to VI, the average skewness is calculated based on NYSE/AMEX stocks, NYSE stocks, and NASDAQ stocks, respectively. In Columns VII and VIII, the average skewness is calculated based on stocks excluding the smallest firms, the least liquid stocks, and the lowest-priced stocks. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI	VII	VIII
	NYSE/AMEX		**NYSE**		**NASDAQ**		**With screening**	
Panel A: 1963–2016								
Constant	0.0084	0.0082	0.0083	0.0081	0.0086	0.0084	0.0079	0.0077
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)
$R_{m,t}$	–	0.0819	–	0.0817	–	0.0800	–	0.0812
		(0.050)		(0.051)		(0.050)		(0.053)
$Sk_{vw,t}$	−0.1095	−0.1167	−0.1085	−0.1156	−0.0908	−0.0963	−0.1069	−0.1122
	(0.003)	(0.002)	(0.004)	(0.002)	(0.025)	(0.017)	(0.002)	(0.001)
Adj. R^2	0.953%	1.466%	0.944%	1.454%	0.774%	1.257%	1.212%	1.716%
Panel B: 1990–2016								
Constant	0.0087	0.0083	0.0087	0.0083	0.0113	0.0112	0.0089	0.0085
	(0.003)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)
$R_{m,t}$	–	0.0820	–	0.0820	–	0.0917	–	0.0858
		(0.175)		(0.176)		(0.121)		(0.150)
$Sk_{vw,t}$	−0.0903	−0.0973	−0.0904	−0.0974	−0.1263	−0.1386	−0.1064	−0.1140
	(0.057)	(0.041)	(0.057)	(0.041)	(0.035)	(0.019)	(0.025)	(0.016)
Adj. R^2	0.579%	0.948%	0.584%	0.953%	1.310%	1.843%	1.156%	1.589%

Table A9: Predictive Regressions of Market Return – Baseline Case with Recession (Expansion) Dummy

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. $Sk_{vw,t}$ is the value-weighted average skewness. *Recession* (*Expansion*) is a monthly dummy variable that equals 1 when the current month is an NBER-dated economic recession (expansion) and equals to 0 otherwise. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV
Panel A: 1963–2016				
Constant	0.0101	0.0098	0.0094	0.0093
	(0.000)	(0.000)	(0.048)	(0.037)
$r_{m,t}$	–	0.0762	–	0.0840
		(0.059)		(0.043)
$Sk_{vw,t}$	−0.1199	−0.1285	−0.1254	−0.1337
	(0.003)	(0.002)	(0.002)	(0.001)
Recession	−0.0088	−0.0076	–	–
	(0.280)	(0.325)		
Expansion	–	–	−0.0004	−0.0006
			(0.929)	(0.895)
Adj. R^2	1.502%	1.917%	1.026%	1.575%
Panel B: 1990–2016				
Constant	0.0112	0.0108	0.0103	0.0099
	(0.000)	(0.000)	(0.046)	(0.046)
$r_{m,t}$	–	0.0716	–	0.0862
		(0.187)		(0.150)
$Sk_{vw,t}$	−0.1120	−0.1204	−0.1150	−0.1247
	(0.042)	(0.030)	(0.030)	(0.019)
Recession	−0.0142	−0.0127	–	–
	(0.305)	(0.335)		
Expansion	–	–	−0.0010	−0.0009
			(0.856)	(0.869)
Adj. R^2	1.826%	2.023%	0.719%	1.157%

Table A10: Long-Horizon Predictive Regressions of Market Return – Moment Variables

This table reports results of the one-month, three-month, and six-month ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. Predictors are: the value-weighted average variance $V_{vw,t}$ and skewness $Sk_{vw,t}$, and the equal-weighted average variance $V_{ew,t}$ and skewness $Sk_{ew,t}$. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panels A to C) and January 1990 to December 2016 (Panels D to F).

	I $R_{m,t}$	II $V_{vw,t}$	III $V_{ew,t}$	IV $Sk_{vw,t}$	V $Sk_{ew,t}$
			1963–2016		
Panel A: 1 month					
Constant	0.0047	0.0093	0.0075	0.0091	0.0135
	(0.011)	(0.000)	(0.025)	(0.000)	(0.000)
X_t	0.0725	−0.5044	−0.1156	−0.1259	−0.1559
	(0.089)	(0.023)	(0.487)	(0.002)	(0.008)
Adj. R^2	0.370%	0.676%	−0.008%	1.179%	0.792%
Panel B: 3 months					
Constant	0.0148	0.0209	0.0189	0.0226	0.0339
	(0.005)	(0.001)	(0.038)	(0.000)	(0.000)
X_t	0.0553	−0.6932	−0.1827	−0.2353	−0.3459
	(0.474)	(0.327)	(0.676)	(0.002)	(0.008)
Adj. R^2	−0.062%	0.332%	−0.042%	1.291%	1.288%
Panel C: 6 months					
Constant	0.0297	0.0343	0.0294	0.0375	0.0576
	(0.002)	(0.001)	(0.030)	(0.000)	(0.000)
X_t	0.0771	−0.4996	0.0314	−0.2309	−0.5032
	(0.448)	(0.611)	(0.958)	(0.061)	(0.019)
Adj. R^2	−0.069%	−0.037%	−0.156%	0.503%	1.287%
			1990–2016		
Panel D: 1 month					
Constant	0.0060	0.0121	0.0116	0.0097	0.0134
	(0.020)	(0.000)	(0.007)	(0.002)	(0.002)
X_t	0.0730	−0.5690	−0.1966	−0.1168	−0.1432
	(0.261)	(0.038)	(0.310)	(0.029)	(0.055)
Adj. R^2	0.230%	1.293%	0.339%	1.016%	0.527%
Panel E: 3 months					
Constant	0.0187	0.0339	0.0352	0.0288	0.0417
	(0.010)	(0.000)	(0.001)	(0.000)	(0.000)
X_t	0.0843	−1.4753	−0.6097	−0.3516	−0.4643
	(0.501)	(0.038)	(0.196)	(0.000)	(0.007)
Adj. R^2	−0.093%	3.002%	1.611%	3.370%	2.374%
Panel F: 6 months					
Constant	0.0380	0.0546	0.0514	0.0475	0.0661
	(0.004)	(0.000)	(0.003)	(0.000)	(0.000)
X_t	0.0372	−1.636	−0.5024	−0.3350	−0.5699
	(0.811)	(0.130)	(0.438)	(0.015)	(0.026)
Adj. R^2	−0.296%	1.618%	0.305%	1.249%	1.574%

Table A11: Long-Horizon Predictive Regressions of Market Return – Financial Variables

This table reports results of the one-month, three-month, and six-month ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. Predictors are: the value-weighted average skewness $Sk_{vw,t}$, the average correlation AC, the tail risk measure TR, the aggregate short interest index SII, the VIX, the variance risk premium VRP and the tail risk premium TRP. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods are in bold.

	I	II	III	IV	V	VI
	$Sk_{vw,t}$	AC_t	TR_t		$Sk_{vw,t}$	SII_t
	1963:08–2016:12				**1973:01–2014:12**	
Panel A: 1 month						
Constant	0.0091	−0.0006	−0.0258		0.0090	0.0053
	(0.000)	(0.890)	(0.061)		(0.000)	(0.013)
X_t	−0.1259	0.0225	0.0723		−0.1257	0.0052
	(0.002)	(0.213)	(0.019)		(0.004)	(0.011)
Adj. R^2	1.179%	0.231%	0.648%		1.078%	1.058%
Panel B: 3 months						
Constant	0.0226	−0.0100	−0.0400		0.0238	0.0162
	(0.000)	(0.314)	(0.280)		(0.000)	(0.006)
X_t	−0.2353	0.1002	0.1290		−0.2651	0.0173
	(0.002)	(0.012)	(0.122)		(0.001)	(0.005)
Adj. R^2	1.291%	2.236%	0.639%		1.581%	4.194%
Panel C: 6 months						
Constant	0.0375	−0.0161	−0.0858		0.0405	0.0332
	(0.000)	(0.292)	(0.205)		(0.000)	(0.002)
X_t	−0.2309	0.1844	0.2716		−0.2590	0.0337
	(0.060)	(0.001)	(0.073)		(0.049)	(0.004)
Adj. R^2	0.503%	3.697%	1.524%		0.617%	7.805%
	$Sk_{vw,t}$	VIX_t		$Sk_{vw,t}$	VRP_t	TRP_t
	1990:01–2015:12			**1996:01–2013:08**		
Panel D: 1 month						
Constant	0.0097	0.0035		0.0080	0.0000	0.0040
	(0.002)	(0.714)		(0.058)	(0.997)	(0.348)
X_t	−0.1168	0.0005		−0.1010	0.0561	0.0328
	(0.029)	(0.814)		(0.105)	(0.088)	(0.785)
Adj. R^2	1.016%	−0.271%		0.471%	2.109%	−0.414%
Panel E: 3 months						
Constant	0.0288	0.0010		0.0274	−0.0001	0.0072
	(0.000)	(0.966)		(0.005)	(0.992)	(0.602)
X_t	−0.3516	0.0030		−0.4224	0.1676	0.2362
	(0.000)	(0.512)		(0.000)	(0.000)	(0.539)
Adj. R^2	3.370%	0.396%		4.430%	6.310%	0.506%
Panel F: 6 months						
Constant	0.0475	−0.0036		0.0418	0.0054	−0.0007
	(0.000)	(0.893)		(0.029)	(0.769)	(0.976)
X_t	−0.3350	0.0071		−0.3998	0.2626	0.8752
	(0.015)	(0.190)		(0.020)	(0.000)	(0.033)
Adj. R^2	1.249%	1.554%		1.501%	7.133%	5.759%

Table A12: Relation between Market Return and Average Skewness

Market return is the value-weighted CRSP market excess return $r_{m,t}$. Average skewness is the value-weighted average stock skewness $Sk_{vw,t}$. "Winsorization" corresponds to the case when market returns below −15% and value-weighted average skewness below −10% are winsorized. Sample period: August 1963 to December 2016.

	Correlation	Constant (p-value)	β (p-value)	Adj. R^2
Panel A: Contemporaneous relation				
No winsorization	0.0927	0.0018	0.1000	0.703%
		(0.452)	(0.079)	
Winsorization	0.0774	0.0025	0.0861	0.443%
		(0.286)	(0.109)	
Panel B: Lagged relation				
No winsorization	−0.1155	0.0091	−0.1259	1.179%
		(0.000)	(0.002)	
Winsorization	−0.1154	0.0094	−0.1266	1.142%
		(0.000)	(0.002)	

Table A13: Predictive Regressions of Market Return – Baseline Case within Firm Size Terciles

This table reports results of the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. To compute the average skewness, firms are sorted according to their market capitalization into terciles from small to large, i.e., denoted by Terciles 1, 2, and 3 (with thresholds corresponding to 30th and 70th percentiles) above the column. Within each size tercile, $Sk_{vw,t}^{(tercile)}$ is the value-weighted average skewness. In each panel, rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods: August 1963 to December 2016 (Panel A) and January 1990 to December 2016 (Panel B).

	I	II	III	IV	V	VI
	Tercile 1		**Tercile 2**		**Tercile 3**	
Panel A: 1963–2016						
Constant	0.0142	0.0148	0.0110	0.0114	0.0087	0.0085
	(0.001)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
$r_{m,t}$	–	0.0847	–	0.0839	–	0.0832
		(0.042)		(0.040)		(0.046)
$Sk_{vw,t}^{(tercile)}$	−0.1472	−0.1640	−0.1159	−0.1304	−0.1201	−0.1279
	(0.020)	(0.007)	(0.022)	(0.009)	(0.002)	(0.001)
Adj. R^2	0.659%	1.214%	0.530%	1.070%	1.146%	1.681%
Panel B: 1990–2016						
Constant	0.0195	0.0209	0.0099	0.0100	0.0095	0.0092
	(0.000)	(0.000)	(0.006)	(0.006)	(0.002)	(0.002)
$r_{m,t}$	–	0.0989	–	0.0816	–	0.0861
		(0.108)		(0.183)		(0.148)
$Sk_{vw,t}^{(tercile)}$	−0.2261	−0.2610	−0.0780	−0.0904	−0.1140	−0.1230
	(0.016)	(0.004)	(0.205)	(0.132)	(0.029)	(0.018)
Adj. R^2	1.145%	1.798%	0.068%	0.423%	1.024%	1.460%

Table A14: Skewness Factor Estimates and Predictive Regressions

This table reports annual skewness one-factor regression models and the results of predictive regression using skewness factor as predictor for the one-month-ahead predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$. In Panel A, skewness factor models are estimated as: $Sk_{i,t} = \phi_0 + \phi_1 Sk_{\text{factor},t} + e_{i,t}$. Columns from left to right report various forms of skewness common factor: the equal-weighted cross-sectional average of firm skewness $Sk_{ew,t}$; the average of the first three principal components of individual skewness $Sk_{PC,t}$; and the average of the first three principal components of standardized skewness $Sk_{PC,t}^{(Std)}$. In Panel B, the regressors correspond to $Sk_{PC,t}$ and $Sk_{PC,t}^{(Std)}$. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample period: August 1963 to December 2016.

	I	II	III	IV
Panel A: Individual skewness factor model				
	Sk_{ew}	$Sk_{PC,t}$	$Sk_{PC,t}^{(Std)}$	
Constant	0.0023	0.0558	0.0558	
Std. dev.	(0.005)	(0.011)	(0.011)	
$Sk_{\text{factor},t}$	0.9563	0.0013	0.0199	
Std. dev.	(0.081)	(0.001)	(0.011)	
Adj. R^2	11.615%	12.262%	12.262%	
Panel B: Predictive regression				
Constant	0.0051	0.0045	0.0051	0.0045
	(0.007)	(0.012)	(0.007)	(0.012)
$r_{m,t}$	–	0.1070	–	0.1124
		(0.009)		(0.007)
$Sk_{PC,t}$	−0.0332	−0.0465	–	–
	(0.032)	(0.003)		
$Sk_{PC,t}^{(Std)}$	–	–	−0.6521	−0.9510
			(0.025)	(0.001)
Adj. R^2	0.503%	1.392%	0.516%	1.486%

Table A15: Out-of-Sample Performances of Time Series Momentum Strategies

This table reports the out-of-sample performance of the following variables: the value-weighted CRSP market excess return ($r_{m,t}$), the value-weighted ($Sk_{vw,t}$) and equal-weighted ($Sk_{ew,t}$) average skewness. Performance measures are: the average market weight, $\bar{w}_m$; the annualized average return, volatility, and skewness of the portfolio; the annualized Sharpe ratio (SR); the annual transaction fee, obtained by assuming an $f = 10$ basis point fee; and the number of rebalancing trades. The risk-aversion parameter λ is equal to 2. Critical values for the encompassing test statistics are from Clark and McCracken (2001) (Table 1). The asymptotic distribution for the test of the null hypothesis that the SR of a given strategy is equal to 0 is given in Lo (2002). The asymptotic distribution for the test of the null hypothesis that the SR of a given strategy is equal to the SR of the Buy-and-Hold strategy is given in DeMiguel, Garlappi, and Uppal (2009). * denotes significance at the 5% significance level. The out-of-sample period is from August 1963 to December 2016.

	I	II	III	IV	V		VI	VII
	Market weight ($\bar{w}_m$)	Annual. return (%)	Annual. volatility (%)	Skew-ness	Annual. SR (%)		Annual fee	Number of rebalancing trades
Buy-&-Hold	1.00	10.11	15.29	−0.50	0.40	*	0.00	–
$r_{m,t}$	0.50	9.47	9.19	0.07	0.52	*	0.57	309
$(r_{m,t}; Sk_{vw,t})$	0.84	14.57	19.06	−0.04	0.57	*	1.23	389
$(r_{m,t}; Sk_{ew,t})$	0.77	13.29	18.39	−0.04	0.52	*	1.19	370

Table A16: Predictive Regressions by Day of the Month

This table reports results based on rolling 1 day in each month, and obtain monthly predictors to forecast next monthly market return. For instance, for "Day 2", we define $r_{m,t}$ from day 2 of month t to day 1 of month $t+1$. Then we define $r_{m,t+1}$ from day 2 of month $t+1$ to day 1 of month $t+2$. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p-values based on Newey-West adjusted t-statistics. The last column presents the adjusted R^2 values. Sample period: August 1963 to December 2016.

	I	II	III	IV	V	VI	VII
	Constant	p-value	$r_{m,t}$	p-value	$Sk_{vw,t}$	p-value	Adj. R^2 (%)
Day 1	0.0091	(0.000)	–	–	−0.1259	(0.002)	1.179
	0.0089	(0.000)	0.0839	(0.044)	−0.1344	(0.001)	1.726
Day 2	0.0092	(0.000)	–	–	−0.1238	(0.005)	1.189
	0.0091	(0.000)	0.0786	(0.043)	−0.1327	(0.003)	1.648
Day 3	0.0075	(0.002)	–	–	−0.0714	(0.100)	0.300
	0.0076	(0.001)	0.0434	(0.227)	−0.0805	(0.068)	0.325
Day 4	0.0076	(0.002)	–	–	−0.0764	(0.067)	0.308
	0.0076	(0.001)	−0.0051	(0.893)	−0.0750	(0.078)	0.154
Day 5	0.0065	(0.006)	–	–	−0.0432	(0.363)	−0.013
	0.0064	(0.007)	−0.0265	(0.517)	−0.0354	(0.461)	−0.104
Day 6	0.0075	(0.001)	–	–	−0.0794	(0.082)	0.332
	0.0075	(0.001)	−0.0333	(0.360)	−0.0716	(0.112)	0.283
Day 7	0.0067	(0.003)	–	–	−0.0536	(0.185)	0.065
	0.0067	(0.004)	−0.0457	(0.222)	−0.0440	(0.286)	0.110
Day 8	0.0038	(0.140)	–	–	0.0473	(0.409)	0.012
	0.0038	(0.146)	−0.0145	(0.684)	0.0504	(0.397)	−0.125
Day 9	0.0034	(0.163)	–	–	0.0607	(0.258)	0.141
	0.0034	(0.165)	−0.0058	(0.882)	0.0618	(0.265)	−0.012
Day 10	0.0060	(0.004)	–	–	−0.0288	(0.433)	−0.089
	0.0060	(0.004)	0.0198	(0.619)	−0.0321	(0.399)	−0.208
Day 11	0.0048	(0.031)	–	–	0.0105	(0.794)	−0.147
	0.0048	(0.029)	0.0170	(0.654)	0.0073	(0.857)	−0.276
Day 12	0.0033	(0.197)	–	–	0.0571	(0.215)	0.122
	0.0033	(0.192)	0.0268	(0.488)	0.0529	(0.237)	0.035
Day 13	0.0025	(0.353)	–	–	0.0792	(0.117)	0.482
	0.0024	(0.350)	0.0372	(0.394)	0.0740	(0.132)	0.462
Day 14	0.0045	(0.032)	–	–	0.0175	(0.669)	−0.120
	0.0045	(0.029)	0.0528	(0.194)	0.0088	(0.844)	−0.006
Day 15	0.0060	(0.002)	–	–	−0.0334	(0.451)	−0.048
	0.0060	(0.001)	0.0412	(0.357)	−0.0418	(0.365)	−0.042
Day 16	0.0060	(0.004)	–	–	−0.0350	(0.411)	−0.047
	0.0061	(0.003)	0.0533	(0.283)	−0.0460	(0.296)	0.070
Day 17	0.0065	(0.005)	–	–	−0.0507	(0.290)	0.083
	0.0065	(0.004)	0.0349	(0.544)	−0.0585	(0.217)	0.043
Day 18	0.0063	(0.004)	–	–	−0.0453	(0.258)	0.030
	0.0064	(0.003)	0.0639	(0.159)	−0.0583	(0.181)	0.267
Day 19	0.0071	(0.001)	–	–	−0.0682	(0.145)	0.231
	0.0071	(0.001)	0.0377	(0.325)	−0.0754	(0.133)	0.212
Day 20	0.0084	(0.000)	–	–	−0.1079	(0.008)	0.872
	0.0084	(0.000)	0.0329	(0.378)	−0.1139	(0.008)	0.822
Day 21	0.0083	(0.000)	–	–	−0.1030	(0.013)	0.691
	0.0083	(0.000)	0.0731	(0.063)	−0.1169	(0.006)	1.056

Table A17: Out-of-Sample Performances based on Predictive Regressions of Market Return by Day of the Month

This table reports the following results: the adjusted out-of-sample R^2, $\bar{R}^2_{OOS}$; the ENC statistics, which is the encompassing test of Harvey, Leybourne, and Newbold (1998) and Clark and McCracken (2001); the annualized Sharpe ratio (SR) and the annualized certainty equivalent (CE) for market return forecasts at monthly frequency; the annual transaction fee, obtained by assuming a 10 basis point fee; the annualized average return over the period. The risk-aversion parameter λ is equal to 2. The out-of-sample period is from January 1985 to December 2016. Critical values for the encompassing test statistics are from Clark and McCracken (2001) (Table 1). The asymptotic distribution for the test of the null hypothesis that the SR (CE) of a given strategy is equal to the SR (CE) of the strategy based on the historical mean of the market return is given in DeMiguel, Garlappi, and Uppal (2009). * denotes significance at the 5% level.

	$\bar{R}^2_{OOS}$ (%)	ENC	Market weight ($\bar{w}_m$)	Annual. return (%)	Annual. volat.	Skew ness	Annual. SR	Annual. CE (%)	Annual fee (%)
Day 1 $Sk_{vw,t}$ alone	0.93	6.16 *	1.13	15.80	21.62	−0.82	0.62 *	8.82 *	0.91
$(r_m, Sk_{vw,t})$	1.16	8.44 *	1.19	14.80	20.97	−0.90	0.60 *	8.08 *	0.89
Day 2 $Sk_{vw,t}$ alone	1.17	4.92 *	1.16	14.57	21.71	−0.35	0.57 *	7.68 *	0.80
$(r_m, Sk_{vw,t})$	0.60	5.93 *	1.19	14.73	19.15	−0.41	0.63 *	8.32 *	0.87
Day 3 $Sk_{vw,t}$ alone	−0.04	0.91	1.16	10.14	21.00	−0.56	0.39	3.87	0.45
$(r_m, Sk_{vw,t})$	−0.84	0.52	1.16	10.31	19.00	−1.00	0.42	4.43	0.67
Day 4 $Sk_{vw,t}$ alone	−0.26	1.39	1.12	11.84	21.13	−0.19	0.46	5.35	0.70
$(r_m, Sk_{vw,t})$	−1.05	0.41	1.11	11.07	21.31	−0.21	0.43	4.62	0.71
Day 5 $Sk_{vw,t}$ alone	−0.95	−0.55	1.13	8.34	22.30	−1.29	0.32	2.06	0.57
$(r_m, Sk_{vw,t})$	−1.70	−1.45	1.10	7.81	22.31	−1.29	0.29	1.57	0.61
Day 6 $Sk_{vw,t}$ alone	−0.32	1.00	1.10	9.50	21.32	−0.91	0.37	3.30	0.65
$(r_m, Sk_{vw,t})$	−0.95	0.39	1.08	9.22	21.93	−0.77	0.35	2.90	0.68
Day 7 $Sk_{vw,t}$ alone	−0.28	0.52	1.04	7.72	21.11	−1.16	0.29	1.71	0.49
$(r_m, Sk_{vw,t})$	−0.84	0.21	1.01	8.22	22.06	−0.91	0.31	1.97	0.63
Day 8 $Sk_{vw,t}$ alone	−0.84	−0.63	1.01	7.02	19.98	−2.03	0.27	1.39	0.40
$(r_m, Sk_{vw,t})$	−1.41	−1.23	1.00	6.13	20.34	−2.02	0.23	0.49	0.46
Day 9 $Sk_{vw,t}$ alone	−0.86	−0.22	0.97	6.46	21.09	−1.61	0.24	0.62	0.52
$(r_m, Sk_{vw,t})$	−1.45	−0.76	0.98	6.04	20.01	−2.06	0.22	0.44	0.55
Day 10 $Sk_{vw,t}$ alone	−0.66	−0.61	1.08	7.57	21.05	−1.47	0.29	1.62	0.28
$(r_m, Sk_{vw,t})$	−1.22	−0.92	1.11	7.78	19.90	−1.46	0.30	2.00	0.43
Day 11 $Sk_{vw,t}$ alone	−0.88	−0.98	1.10	8.12	19.87	−0.98	0.31	2.27	0.27
$(r_m, Sk_{vw,t})$	−1.48	−1.51	1.11	7.31	18.77	−1.17	0.28	1.73	0.40
Day 12 $Sk_{vw,t}$ alone	−1.01	0.33	1.05	10.87	18.99	0.12	0.45	4.88	0.73
$(r_m, Sk_{vw,t})$	−1.59	0.07	1.07	9.97	18.10	−0.20	0.42	4.23	0.80
Day 13 $Sk_{vw,t}$ alone	−0.62	1.69	1.04	9.90	19.70	0.09	0.39	3.86	0.79
$(r_m, Sk_{vw,t})$	−1.92	1.28	1.07	9.54	18.82	0.14	0.39	3.70	1.00
Day 14 $Sk_{vw,t}$ alone	−1.30	−1.52	1.12	7.58	20.57	−1.58	0.29	1.71	0.23
$(r_m, Sk_{vw,t})$	−3.05	−2.03	1.13	8.76	17.65	−0.53	0.36	3.22	0.78
Day 15 $Sk_{vw,t}$ alone	−0.54	−0.45	1.10	8.44	18.80	−1.07	0.34	2.78	0.23
$(r_m, Sk_{vw,t})$	−2.50	−1.80	1.08	8.94	16.84	−0.63	0.38	3.53	0.79
Day 16 $Sk_{vw,t}$ alone	−0.58	−0.16	1.12	10.08	20.03	−1.15	0.40	4.09	0.45
$(r_m, Sk_{vw,t})$	−2.01	−0.63	1.11	9.44	18.83	−1.19	0.39	3.70	0.85
Day 17 $Sk_{vw,t}$ alone	−0.39	0.53	1.11	10.85	20.53	−1.49	0.44	4.75	0.54
$(r_m, Sk_{vw,t})$	−1.94	−0.54	1.10	9.25	20.64	−2.01	0.37	3.33	0.84
Day 18 $Sk_{vw,t}$ alone	−0.37	0.38	1.15	10.90	19.82	−1.78	0.45	4.95	0.50
$(r_m, Sk_{vw,t})$	−1.39	0.92	1.17	11.13	19.46	−0.90	0.46	5.10	0.81
Day 19 $Sk_{vw,t}$ alone	−0.29	1.54	1.13	10.69	20.87	−2.10	0.43	4.66	0.66
$(r_m, Sk_{vw,t})$	−1.68	0.84	1.16	11.68	19.88	−0.74	0.48	5.51	0.80
Day 20 $Sk_{vw,t}$ alone	0.88	4.10 *	1.07	11.48	21.03	−2.13	0.47 *	5.38 *	0.71
$(r_m, Sk_{vw,t})$	−0.56	3.46 *	1.11	13.02	19.23	−0.07	0.55 *	6.79 *	0.86
Day 21 $Sk_{vw,t}$ alone	0.61	2.86 *	1.09	11.39	21.44	−0.92	0.45	4.99	0.59
$(r_m, Sk_{vw,t})$	0.16	4.04 *	1.14	9.96	20.82	−1.08	0.39	3.81	0.85

Figure A1: Contemporaneous Relation between Average Skewness and Market Return – No Winsorization

Market return is the value-weighted CRSP market excess return $r_{m,t}$. Average skewness is the value-weighted average stock skewness $Sk_{vw,t}$. Sample period: August 1963 to December 2016.

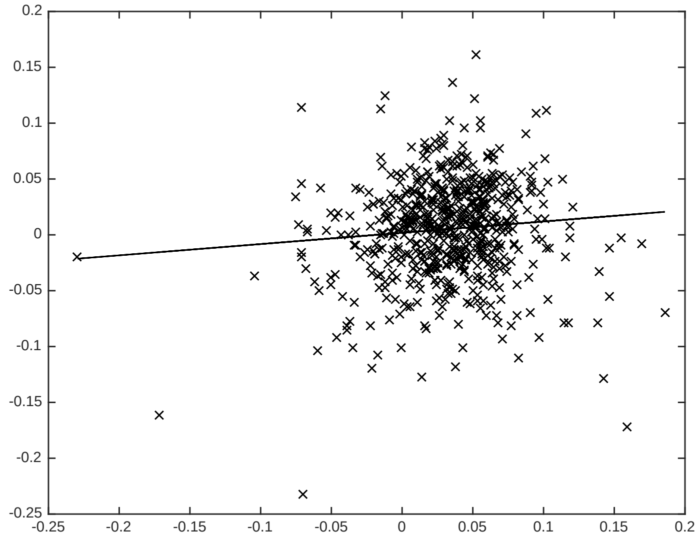

Figure A2: Contemporaneous Relation between Average Skewness and Market Return – With Winsorization

Market return is the value-weighted CRSP market excess return $r_{m,t}$. Average skewness is the value-weighted average stock skewness. We removed market returns below -15% and average skewness below -10%. Sample period: August 1963 to December 2016.

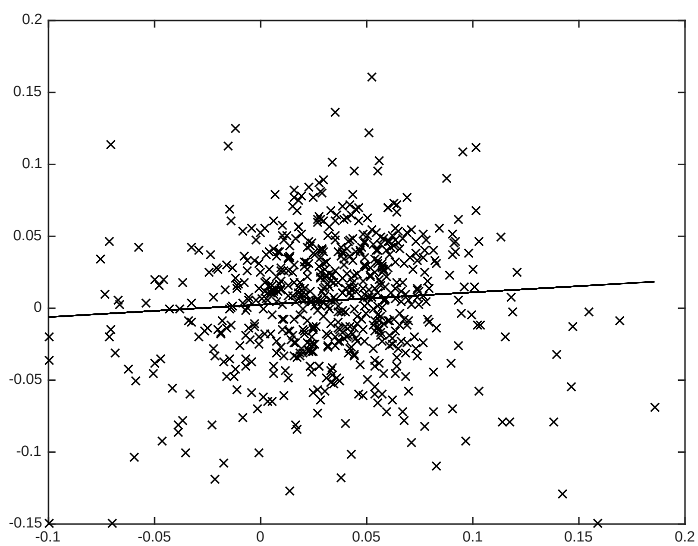

Figure A3: Lagged Relation between Average Skewness and Market Return – No Winsorization

Market return is the value-weighted CRSP market excess return $r_{m,t}$. Average skewness is the value-weighted average stock skewness $Sk_{vw,t}$. Sample period: August 1963 to December 2016.

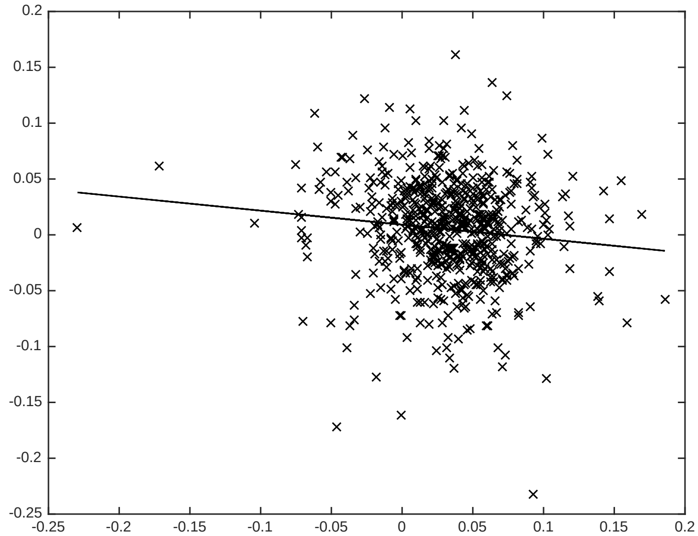

Figure A4: Lagged Relation between Average Skewness and Market Return – With Winsorization

Market return is the value-weighted CRSP market excess return $r_{m,t}$. Average skewness is the value-weighted average stock skewness $Sk_{vw,t}$. We removed market returns below -15% and average skewness below -10%. Sample period: August 1963 to December 2016.

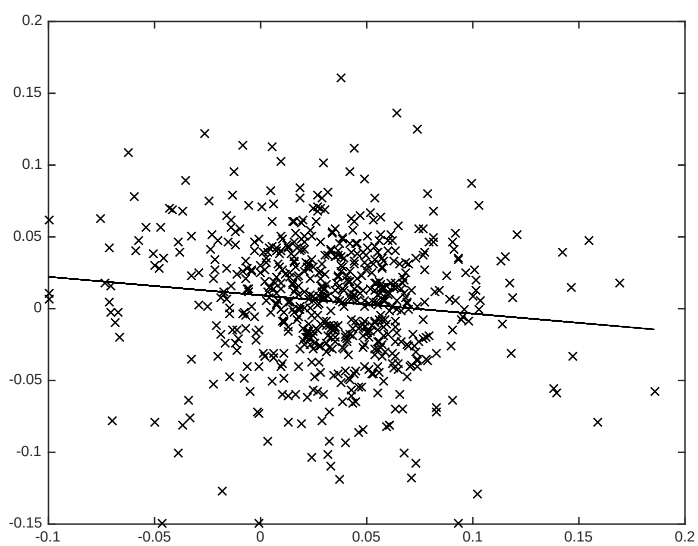

Figure A5: Market Return and Its Prediction

Market return is the value-weighted CRSP market excess return $r_{m,t}$. The prediction is based on the average skewness defined as the value-weighted average stock skewness $Sk_{vw,t}$. Sample period: August 1963 to December 2016.

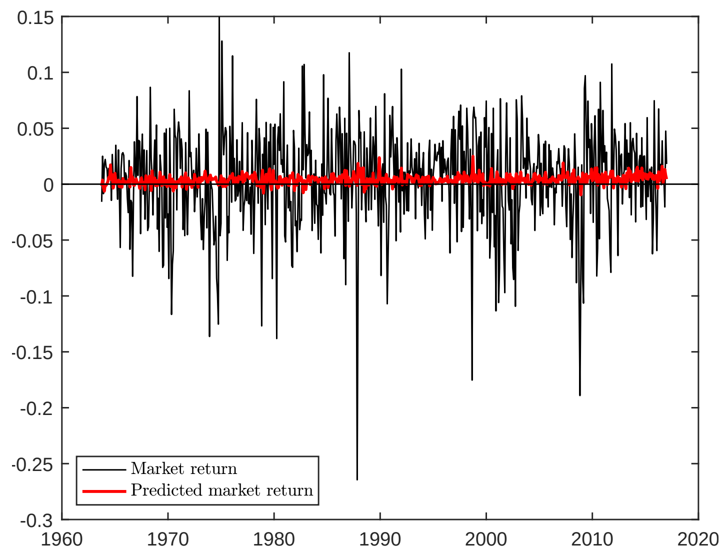

Figure A6: Cumulative Returns of Trading Strategies with Time Series Momentum and Skewness Effect

This figure presents the cumulative return generated by implementing two trading strategies. Trading strategies are formed by signals based on time series momentum alone (red-dotted line), based on time series momentum and value-weighted average skewness $Sk_{vw,t}$ (black-solid line), or time series momentum and equal-weighed average skewness $Sk_{ew,t}$ (blue-dash line). Sample period: August 1963 to December 2016. NBER recessions are represented by shaded bars.