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Does Correlation Between Stock Returns Really Increase During Turbulent Periods?

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Correlations betwen international equity markets are often claimed to increase during periods of high volatility. Therefore the benefits of international diversification are reduced when they are most needed, i.e. during turbulent periods. This paper investigates the relationship between international correlation and stock-market turbulence. We estimate a multivariate Markov-switching model, in which the correlation matrix varies across regimes. Subsequently, we test the null hypothesis that correlations are regime-independent. Using weekly stock returns for the S&P, the DAX and the FTSE over the period 1988–99, we find that international correlations significantly increased during turbulent periods.

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1. Introduction

Correlations between international equity markets are often claimed to increase during turbulent periods. This issue is truly important for both portfolio managers and regulators, since international diversification benefits seem to decrease when they are most needed, i.e. during periods of market turbulence. Since the seminal work of Markowitz (1952), modern portfolio theory underlines that not only returns and volatilities are important in the portfolio selection process, but also that correlations between assets are really a key to a good asset allocation. Therefore, to perform an optimal allocation, one needs to determine correlations between assets precisely. But, if correlations increase during turbulent periods, then standard portfolio diversification cannot decrease the risk during these periods of high volatility.

Thus, as pointed out by Odier and Solnik (1993), Lin *et al*. (1994), or Ang and Bekaert (1999), the increase in correlation may partly explain the home

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bias puzzle, one of the most challenging puzzles in international finance. The home bias puzzle states that investors tend to diversify far less internationally than what theory would predict.[1]

The existing literature actually found rather mixed empirical evidence on the link between international correlation and stock-market turbulence. A first approach examined the stability of the correlation between returns over different periods of time. Kaplanis (1988), for instance, did not reject the null hypothesis of constant correlation of monthly returns of 10 markets over the 1967–82 period. Ratner (1992) obtained a similar result over the 1973–89 period. Koch and Koch (1991) obtained a growing market interdependence in 1980 and 1987 as compared to 1972. Some papers focused more precisely on the effect of the 1987 crash: Bertero and Mayer (1990), King and Wadhwani (1990), Lee and Kim (1993) claimed that correlations increased significantly after the US stock-market crash. Similarly, King *et al*. (1994) found that the increase in correlation is only a transitory effect caused by the 1987 crash. See also Roll (1989) for a survey. Most papers cited above consider changes in correlation by comparing unconditional correlation across different subperiods. However, the breakpoint is generally exogenously selected. This approach implies that two subperiods corresponding to low and high volatilities have to be identified *a priori*. However, recent evidence by Boyer *et al*. (1997) as well as Forbes and Rigobon (1999) has shown that testing unconditional correlation coefficient may be misleading. Indeed, this coefficient is biased when volatility shifts over time.

Another strand of the literature relies on the autoregressive conditional heteroskedasticity (ARCH) framework: Hamao *et al*. (1990) estimated a two-step multivariate GARCH model allowing them to measure interdependence of returns and volatilities across the New York, Tokyo and London stock markets. When they include the October 1987 crash period in the data set, they obtained significant spillovers in almost all directions, in terms of both return and volatility. Using a similar GARCH approach to study the interrelation between the New York and London stock markets, Susmel and Engle (1994) focused on hourly data. Even for the period including the 1987 crash, they did not find strong evidence of international volatility spillovers. Longin and Solnik (1995) specifically tested the hypothesis of a constant international conditional correlation between a large number of monthly stock returns. Using bivariate GARCH models, they explored several potential sources of deviation from the constant conditional correlation model. In particular, they tested the hypothesis of higher international correlation during turbulent periods. They found that correlation generally rises in periods of high volatility. Bera and Kim (1996) suggested an Information Matrix (IM) test for the null hypothesis of a constant

[1] For instance, French and Poterba (1991) report that, at the end of the 1980s, domestic ownership shares in the stock market were 94 per cent for the USA, 98 per cent for Japan and 82 per cent for the UK.

correlation in a GARCH model. Using daily stock returns, Bera and Kim rejected strongly the constancy of the conditional correlation between the US market and other major stock markets. More recently, Tse (2000) proposed a test for the constant-correlation hypothesis based on the Lagrange Multiplier approach. Tse found that, under non-normality, the IM test rejects the constant-correlation hypothesis too often. Using the LM test, he obtains less evidence that correlation between stock returns is time-varying.

The GARCH approach improves clearly the measure of time-varying volatility and, for some parameterizations, the measure of time-varying correlations.[2] In many empirical studies, however, stock-market volatility is found to be too persistent, implying an explosive conditional variance. For instance, some estimations performed by Hamao *et al*. (1990), Hamilton and Susmel (1994), or Susmel and Engle (1994), display excessive volatility persistence. Lamoureux and Lastrapes (1990) argued that the excessive persistence generally found in ARCH models may be due to the occurrence of structural breaks, like the 1987 crash. As highlighted by Hamilton and Susmel (1994), after a large shock on the stock market such as the 1987 crash, the volatility forecast decreased much more slowly than the true volatility (as measured, for instance, by implied volatility extracted from stock-option prices).

An alternative way to study the effect of turbulent period on international correlation is the Markov-switching (MS) model, introduced by Hamilton (1989). An interesting empirical feature of MS models is that thus-estimated volatility appears to be significantly less peristent than standard GARCH-model estimated volatility (Sola and Timmermann, 1994). Ramchand and Susmel (1998) develop a multivariate MS model to test the hypothesis of a constant international conditional correlation between stock markets. They assume for each regime a constant-correlation bivariate ARCH model, in which volatility shifts are captured by a scale parameter. Correlation is assumed to depend only on the regime of the domestic (US) return. The conditional correlation between the US market and four other major stock markets is found to be constant in two of the four cases. In a more general setting, Ang and Bekaert (1999) estimated several MS models for US, UK, and German stock markets. They obtained evidence of the presence of a high-volatility and high-correlation regime and a low-volatility and low-correlation regime.

[2] For numerical tractability, multivariate GARCH models often assume constant conditional correlation (Bollerslev, 1990), or strong restrictions on the volatility dynamics (as in the diagonal GARCH, originally suggested by Bollerslev *et al*. (1988). The constant-correlation model appears to be too restrictive to study the effect of a bear market on international correlation. The BEKK parameterization (Engle and Kroner, 1995) does not impose any restriction on the dynamics of conditional second moments, including conditional correlations. However, it is often very difficult to estimate, because the number of unknown parameters increases rapidly with the number of markets.

The aim of this paper is to investigate the relationship between international correlation and stock-market turbulence. We assess empirically whether the claim that correlations increase during turbulent period is true. We focus on US, German, and UK weekly stock-market returns, over the 1988–99 period. First, we estimate several MS models for stock-market returns. We assume that volatilities shift in all markets at the same date. This assumption allows us to distinguish unambiguously between calm and turbulent regimes. We effectively obtain evidence of a regime with low volatilities and low correlations and a regime with high volatilities and high correlations. Then, we test the null hypothesis that correlations are equal in both regimes. We find that the null hypothesis is strongly rejected, whatever the MS model. Therefore, we conclude that turbulent periods are associated with higher correlations than calm periods.

The outline of the paper is as follows. In section 2, we describe the data used and provide some preliminary evidence on unconditional correlation between stock markets. Section 3 is devoted to the econometric methodology. We briefly present multivariate two-regime MS models and we indicate how to test the null hypothesis of constant conditional correlation. Empirical results and economic implications are presented in section 4. Our conclusions are summarized in Section 5.

2. Data and Preliminary Evidence

This section describes the data used and provides some preliminary evidence on unconditional correlation between stock markets.

2.1. Data

We use weekly (from Friday to Friday) stock returns for New York, Frankfurt and London stock markets.[3] For New York, we use observations from the Standard and Poor's 500 Composite Index (S&P). The index represents approximately 75 per cent of the investment-grade stocks held by most institutional investors. For Frankfurt, we use the DAX Share Index, which includes 30 of the most heavily traded stocks listed on the Frankfurt Stock Exchange, representing over 75 per cent of the total turnover in German equities. For London, we use the Financial Times 100 Share Index (FTSE), which also represents about 75 per cent of the total equity turnover in the UK.

[3] We prefer weekly returns to daily returns, because weekly data is less noisy than daily data. Moreover, we did not consider monthly data, because it requires a much longer period of time, questioning the hypothesis that regimes are perfectly correlated across countries.

The three indices are capitalization-weighted. The data cover the period from January 1988 to December 1999, and consist of 620 observations. Unlike most previous studies, our sample period excludes the October 1987 crash. The 1987 crash has been shown to have dramatically affected stock markets and increased, at least transitory, international correlations (King and Wadhwani, 1990; Hamilton and Susmel, 1994).

Let r_{it}, $t = 1, \ldots, T$, denote the weekly stock (log) return of market i. As a preliminary look at the data, Table 1 reports summary statistics on stock returns, including the mean, standard deviation, skewness and excess kurtosis.

The average weekly return is positive, ranging from 0.22 per cent to 0.28 per cent for the three stock returns. Standard deviations are ranging from 1.88 per cent for the S&P to 2.65 per cent for the DAX. Skewness (*Sk*) is a measure of the distribution's asymmetry of returns. US and German stock returns are negatively skewed, indicating that crashes are more likely to occur than booms. For UK stock returns, conversely, the skewness is positive. When standard

Table 1: Summary Statistics on Weekly Stock-market Returns

	S&P	DAX	FTSE
Mean	0.284	0.306	0.221
Std error	0.066	0.108	0.080
Std deviation	1.879	2.654	1.991
Std error	0.090	0.158	0.083
Sk	−0.171	−0.454	0.106
Std error	0.144	0.139	0.150
XKu	0.903	1.977	0.919
Std error	0.347	1.170	0.332
Wald test statistic	6.773	10.678	8.833
p-value	0.034	0.005	0.012
$LM(4)$	42.556**	73.114**	13.149**
$LM(8)$	45.343**	92.403**	26.072**
$LB(4)$	22.057	0.682	3.225
$LB(8)$	26.253**	6.490	5.461
$LB_c(4)$	13.048*	0.467	2.387
$LB_c(8)$	16.771*	4.855	4.848

Notes: This tables reports over the period from January 1988 through December 1999, for a total of 620 observations.
Sk = skewness and XKu = excess kurtosis.
Standard errors of moments are computed using the Generalized-Method-of-Moments procedure proposed by Richardson and Smith (1993), with 8 lags.
The Wald test statistics for normality corresponds to the null hypothesis that the coefficients of skewness and excess kurtosis are jointly equal to zero. Under the null hypothesis of normality, it is asymptotically distributed as a χ^2_2.
$LM(K)$ is the Engle test statistic associated with the null hypothesis of no serial correlation (up to K lags) of the squared change in return.
$LB(K)$ denotes the Ljung–Box portmanteau statistics associated with the null hypothesis of no serial correlation (up to K lags) of the change in return.
$LB_c(K)$ is the Ljung–Box statistics corrected for heteroskedasticity.
These three statistics are distributed as a χ^2 with K degrees of freedom.
* and ** indicate that the statistic is significant at the 5% and 1% level respectively.

errors are computed using Generalized Method of Moments – as suggested by Richardson and Smith (1993) – skewness is significantly different from zero for Germany only. Excess kurtosis measures the heaviness of distribution's tails compared to the normal one. The excess kurtosis is positive for all markets, therefore the empirical distribution has fatter tails than the normal one. We also perform the Wald test for normality of returns. The null hypothesis of this test is that skewness and excess kurtosis are jointly zero. Under the null, the Wald test statistic is distributed as a χ^2 with 2 degress of freedom. Normality is rejected at the 5% significance level for each market. Those preliminary statistics confirm some widespread results in the financial literature on stock returns: positive return, negative skewness and fat tails.

We next consider heteroskedasticity by regressing squared returns on past squared returns (up to 4 and 8 lags). The TR^2 Engle statistic, where R^2 is the coefficient of determination, is distributed as a χ^2_K under the null hypothesis of homoskedasticity ($K = 4$ and $K = 8$ respectively). The Engle statistic takes very large values for each market, indicating strong nonlinear (second-moment) dependencies. Therefore, we conclude that there is a fair amount of heteroskedasticity in the data.

We now wish to test for the presence of return serial correlation. Given the high level of heteroskedasticity, we consider the usual Ljung–Box statistic as well as a version of the Ljung–Box statistic which corrects for heteroskedasticity (White, 1980). For 4 (resp. 8) lags, the Ljung–Box statistic (LB) and the corrected Ljung–Box statistic (LB_c) are distributed as a χ^2_4 (resp. χ^2_8). LB and LB_c statistics for returns do not indicate significant linear dependencies of returns, for all markets investigated.

2.2. Preliminary Evidence on International Correlations

Table 2 reports unconditional correlation coefficients between stock returns estimated over the whole sample 1988–99. Correlation between stock returns is quite high: the lowest correlation is 0.45 (between S&P and DAX), whereas the highest correlation is 0.57 (between DAX and FTSE).

For some additional insight on international correlation, Figure 1 displays unconditional variances and unconditional correlations across markets (S&P–DAX, S&P–FTSE and DAX–FTSE). Variances and correlations are computed over a sliding window of one year.[4] The first subperiod (1988–91) has been affected by the German reunification in mid-1990 and the Gulf war at the beginning of 1991. S&P and DAX variances appear to be very low over the 1992–95 subperiod. The major financial event occurring during this subperiod is the EMS crisis, in mid-1992, which appears to have strongly boosted the

[4] We notice that such a computation allows us to identify large swings in variance as well as in correlation, but not structural breaks in the series, since the series are smoothed.

Table 2: Unconditional Correlation Matrices and Variances Over Various Subperiods

	Correlation matrix			Variance
	S&P	DAX	FTSE	
1988–99				
S&P	1.000	0.454	0.528	3.535
DAX	0.454	1.000	0.573	7.054
FTSE	0.528	0.573	1.000	3.971
1988–91				
S&P	1.000	0.296	0.509	4.034
DAX	0.296	1.000	0.468	7.112
FTSE	0.509	0.468	1.000	4.003
1992–95				
S&P	1.000	0.303	0.367	1.381
DAX	0.303	1.000	0.508	4.449
FTSE	0.367	0.508	1.000	3.114
1996–99				
S&P	1.000	0.628	0.623	5.201
DAX	0.628	1.000	0.693	9.606
FTSE	0.623	0.693	1.000	4.817

Notes: The sample period is January 1988 through December 1999, with a total of 620 observations.

FTSE. The last subperiod is associated with a strong S&P volatility increase. The increase took place in Germany and the UK in mid-1997. Two major events have affected stock markets, the South-East Asian crisis in mid-1997 and the Russian crisis in mid-1998. Therefore, at first glance, the second subperiod can be seen as a calm period, whereas the first and last periods can be seen as turbulent periods.

Correlations present a somewhat different pattern. First, S&P–DAX and S&P–FTSE correlations attain a minimum in 1994, during the so-called calm period. Moreover, correlations are rather high during the last subperiod, especially the S&P–DAX correlation. However, an increase in correlation cannot be systematically related to an increase in variance in our data sample, as illustrated by the two following subperiods. First, the S&P–DAX correlation strongly decreased between 1993 and 1994 (from about 0.4 to 0.1). Second, the S&P–FTSE correlation peaked markedly in 1995 (from 0 to 0.6). None of these events appears to be related to particular shocks on the variance of stock markets.

Table 2 also reports unconditional correlation matrices and variances computed over the three identified subperiods (1988–91, 1992–95 and 1996–99). The first and last subperiods can be seen as high-volatility episodes, whereas the second subperiod is characterized by a low volatility. Therefore, testing for a constant unconditional correlation over these subperiods can be interpreted as a test of the link between correlation increase and stock-market

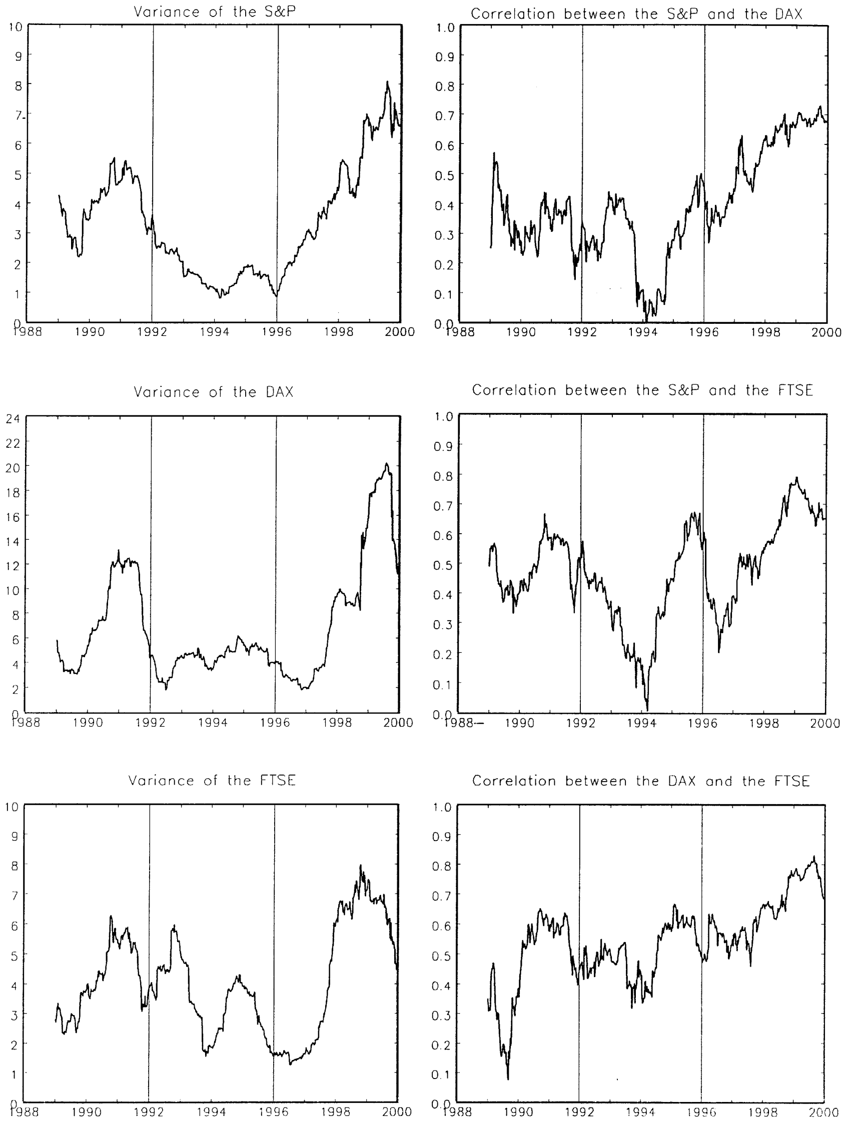

Figure 1: Unconditional Variances over Time and Correlations over Time
Note: This figure illustrates the evolution of unconditional variance and correlation across the markets. They are computed over a sliding window of one year. Beginning of 1992 and 1995 are marked with vertical lines.

turbulence. A formal test for a constant unconditional correlation can be performed using the Jennrich (1970) test of equality of two correlation matrices computed over independent subsamples. This test has been performed for instance by Kaplanis (1988), Ratner (1992) and Longin and Solnik (1995). It is briefly presented in the Appendix. Table 3 reports results of the Jennrich test. For an $(n \times n)$-dimensional correlation matrix, the test statistic is distributed as a χ^2 with $n(n-1)/2$ degrees of freedom. Each subsample contains 206 observations. First, the null hypothesis cannot be rejected over the 1988–91 and 1992–95 subperiods. Even if we consider pairwise correlations, none is found to have significantly changed. Second, the correlation matrices estimated over the 1992–95 and 1996–99 subperiods are found to be significantly different at any usual level. Over the last subperiod (1996–99), international linkages increased dramatically. Correlations are higher than 0.6 for the three stock markets under study. The correlation between DAX and FTSE returns even reached 0.7. These results confirm the empirical evidence by Kaplanis (1988) and Longin and Solnik (1995), who found lower p-values for the Jennrich test over the more recent period. This increase in correlations may be indicative of a growing integration between stock markets.[5]

Finally, our results confirm the presumed relationship between international correlation and stock-market turbulence only partially. On one hand, the agitated period beginning in 1997 has rightly led to a significant increase in correlation. But, on the other hand, the decrease in volatility in 1992 has not been accompanied by a signifiant decrease in correlation.

Recently, Boyer *et al*. (1997), Loretan and English (1999), and Forbes and Rigobon (1999) argued that the test of unconditional correlation constancy

Table 3: Jennrich Test of Equality of Correlation Matrices over Various Subperiods

Model	Degree of freedom	1988–91 compared to 1992–95		1992–95 compared to 1995–99	
		Statistics	p-value	Statistics	p-value
S&P–DAX–FTSE	3	4.006	0.2608	20.802	0.0001
S&P–DAX	1	0.024	0.8758	16.726	0.0000
S&P–FTSE	1	2.659	0.1030	11.202	0.0008
DAX–FTSE	1	0.358	0.5497	9.253	0.0024

Notes: Correlation matrices of weekly stock returns for the S&P the DAX and the FTSE are computed over various subperiods. As shown in the Appendix, the Jennrich test statistic is asymptotically distributed as a χ^2 with a degree of freedom equal to the number of independent correlation coefficients.

[5] However, it is worth noting that the period studied by Longin and Solnik (1995) ended with the 1987 crash, whereas our sample ended with the 1997–98 South-East Asian and Russian crises. These events may be largely responsible for the increasing international correlation obtained in both papers.

across various subperiods may be misleading. This is because the unconditional correlation estimate is biased in case of variance shift. Therefore, even when the breaking data is assumed to be known (corresponding to a well-established crash, for instance), unconditional correlation estimates have to be corrected before any testing procedure. Moreover, as pointed out by Boyer *et al*. (1997), when the breaking date cannot be considered as a clear structural break, "changes in correlations over time or across 'regimes' cannot be detected reliably by splitting a sample according to the realized values of the data." This result is a consequence of the selection bias that occurs when subsamples are chosen *a priori*, according to the data. To test for a change in correlation, it is therefore necessary

1. to use a data generating process allowing for the possibility of structural changes,
2. to estimate the model's parameters, and
3. to test changing correlations (and possibly other structural breaks).

In section 3, we test the null hypothesis of a constant conditional correlation in a model where the variance regime is determined endogenously. More precisely, we test whether a change in variance regime (from a calm regime to a turbulent regime) can affect significantly the conditional correlation between stock returns.[6]

3. The Multivariate Markov-switching (MS) Model

The MS model has been developed by Hamilton (1988, 1989). In this model, time series are assumed to have different values of the mean and variance in a small number of regimes. Let $r_t = \{r_{1t}, \ldots, r_{nt}\}$ denote the $(n \times 1)$ vector of returns. In the following multivariate MS model, the conditional distribution of the r_t process depends on the underlying regime S_t

$$r_t = \mu(S_t) + H(S_t)^{\frac{1}{2}}\varepsilon_t$$

$$\varepsilon_t / S_t \sim \text{iid}(0, I_n)$$

The conditional mean is captured by $\mu(S_t)$. It is assumed to be constant within regime, with $\mu(S_t = k) = \mu^k$, $k = 0, 1$. The conditional covariance matrix is given by $H(S_t)$. Conditional variances are set up such that $H_{ii}(S_t = k) = h_i^k$ and conditional covariances are set up such that

$$H_{ij}(S_t = k) = \rho_{ij}^k \sqrt{h_i^k h_j^k}$$

[6] Another way to compute correlations conditional to the regime has been advocated by Longin and Solnik (2000), in the context of the multivariate extreme value theory.

with $\rho_{ij}^k = \rho_{ij}(S_t = k)$. Last, S_t denotes the unobserved regime of the system. S_t is assumed to follow a two-state Markov process, with transition probability matrix given by

$$\begin{pmatrix} p & 1-p \\ 1-q & p \end{pmatrix}$$

with

$$p = \Pr[S_t = 0 | S_{t-1} = 0]$$

and

$$q = \Pr[S_t = 1 | S_{t-1} = 1].$$

As pointed out by Sola and Timmermann (1994), such a model, although very simple, is able to generate persistence in the aggregated over regimes conditional variance process. Aggregated over regimes mean and covariance matrix at time t are defined respectively as

$$\mu_t = E[r_t | I_{t-1}] = \pi_t \mu^0 + (1 - \pi_t)\mu^1 \tag{1}$$

and

$$\begin{aligned} H_t &= E[r_t r_t' | I_{t-1}] - E[r_t | I_{t-1}] E[r_t | I_{t-1}]' \\ &= \pi_t(\mu^0 \mu^{0\prime} + H^0) + (1 - \pi_t)(\mu^1 \mu^{1\prime} + H^1) - \mu_t \mu_t' \end{aligned} \tag{2}$$

where $\pi_t = \Pr[S_t = 0 | I_{t-1}]$ is the conditional probability of being in regime 0. Now, assume that r_t depends on two regimes, one characterized by a low variance and the other by a high variance. Then, according to (2), if regimes are persistent, this model is sufficient to obtain persistence in volatility. On the contrary, a one-regime GARCH model is not capable of capturing the persistence of regimes. It will therefore imply a strong volatility persistence, even if volatility is constant within regime.

Note that we also estimated a MS-ARCH(1) parameterization for the conditional covariance matrix, with

$$H_{ii}(S_t = k) = \alpha_i^k + \beta_i^k \varepsilon_{it-1}^2$$

This parameterization is very close to the one proposed by Hamilton and Susmel (1994) and used by Ramchand and Susmel (1998). However, as in Ramchand and Susmel, we were unable to obtain a significant ARCH effect with our data. To save space we do not report the results obtained with the MS-ARCH(1) model but they can be found in Chesnay and Jondeau (2000).

Stock returns are assumed to be characterized by two regimes. Two regimes, one with high volatility and the other one with low volatility, are generally found to be sufficient to describe time-variability in first and second moments. Moreover, increasing the number of regimes will induce an excessive computational burden. Note also that we do not focus on crashes, in which case a third regime would probably be relevant. Instead, we intend to test the positive relationship between volatilities and correlations. Therefore, two regimes are sufficient for our test. Stock markets are assumed to switch from one regime to the other at the same time, so that regimes in various markets are perfectly correlated and transition probabilities are identical for all stock returns. Under this assumption, calm and turbulent regimes are identified unambiguously. Weakening this assumption would increase the number of parameters to be estimated excessively.[7]

Estimation of a MS model is performed using (quasi) maximum likelihood (QML) estimation. The sample log-likelihood function of the multivariate MS model is

$$\begin{aligned} \ln L(\theta) &= \sum_{t=1}^{T} \ln(f(r_t|I_{t-1})) \\ &= \sum_{t=1}^{T} \ln\left(\sum_{k=0}^{1} f(r_t|S_t = k, I_{t-1}) \Pr[S_t = k|I_{t-1}]\right) \\ &= \sum_{t=1}^{T} \ln\left(\sum_{k=0}^{1} g_t^k \pi_t\right) \end{aligned}$$

where θ is the vector of parameters to be estimated. The expression $\pi_t = \Pr[S_t = 0|I_{t-1}]$ is computed as

$$\pi_t = (1-q)\frac{g_{t-1}^1(1-\pi_{t-1})}{g_{t-1}^0\pi_{t-1} + g_{t-1}^1(1-\pi_{t-1})} + p\frac{g_{t-1}^0\pi_{t-1}}{g_{t-1}^0\pi_{t-1} + g_{t-1}^1(1-\pi_{t-1})}$$

and the conditional density $g_t^k = f(r_t|S_t = k, I_{t-1})$ is computed as

$$g_t^k = (2\pi)^{-\frac{n}{2}}|H^k|^{-\frac{1}{2}}\exp(-\tfrac{1}{2}(r_t-\mu^k)'(H^k)^{-1}(r_t-\mu^k)) \qquad k = 0, 1$$

The log-likelihood function can be computed recursively. Reported standard errors are robust to heteroskedasticity and serially correlated errors. Additional details on the estimation method for the MS models can be found in Gray (1996). The log-likelihood function is maximized using the BHHH algorithm (Berndt *et al*. 1974) with numerical derivatives.

To account for non-normality of the residual distribution, we also estimate a model, in which standardized innovations, defined as $\varepsilon_{it}/\sqrt{h_{it}}$, are assumed

[7] Alternatively, in a bivariate setting, Ramchand and Susmel (1998) consider a model with four regimes. But to keep the system tractable, they assume that correlations only depend on the state of the US return. Ang and Bekaert (1999) estimate two models, the first one with perfectly correlated regimes between the US and UK stock returns, the second one allowing non-contemporaneous regimes. They obtain that the US and UK face essentially the same regime shifts.

to be drawn from a Student-t distribution with ν degrees of freedom (Bollerslev, 1987). Therefore, the log-likelihood function for such a model is

$$\ln L(\theta) = \sum_{t=1}^{T} \ln\left(\sum_{k=0}^{1} \tilde{g}_t^k \pi_t\right)$$

where $\tilde{g}_t^k$ is computed as

$$\tilde{g}_t^k = \Gamma\left(\frac{\nu+n}{2}\right)\left[\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)\right]^{-n} \left(1+\frac{(r_t-\mu^k)'(H^k)^{-1}(r_t-\mu^k)}{(\nu-2)}\right)^{-\frac{\nu+n}{2}} |H^k|^{-\frac{1}{2}}$$

Normality is attained when $\nu \to +\infty$. When innovations are assumed to be Student-t distributed, the degree of freedom, ν, is added to the parameter vector.

The MS model described above is designed to test the null hypothesis of a conditional correlation constant across regimes. Indeed, it is generally possible to identify a low-volatility regime and a high-volatility regime.[8] In this case, one only has to compare conditional correlations obtained for both regimes. The test of the null hypothesis of a conditional correlation matrix constant across regimes is based on the Likelihood Ratio (LR) test (Ramchand and Susmel, 1998; Ang and Bekaert, 1999). The LR test statistic is

$$\xi = 2(\ln\ L(\theta) - \ln\ L(\tilde{\theta}^0))$$

where $\tilde{\theta}^0$ corresponds to the vector of parameters under the null hypothesis, assuming

$$\rho_{ij}^0 = \rho_{ij}^1 \qquad i, j = 1, \ldots, n, j > i$$

Under the null, the test statistic ξ is distributed as a χ^2 with $n(n-1)/2$ degrees of freedom. Note that, unlike most previous tests based on unconditional correlations (see references in the Introduction), this test procedure is not based on data-mining. With such an approach, low-volatility and high-volatility regimes are determined endogenously during the estimation. Moreover, unlike the test procedures suggested by Bera and Kim (1996) and Tse (2000), this procedure is not designed to test the constant correlation hypothesis against very general alternatives, but specifically against the alternative that correlations are regime-dependent. Therefore, against this specific alternative hypothesis, our procedure is likely to be more powerful.

[8] Note, however, that, in a multivariate context, it is not always possible to identify low- and high-volatility regimes, since all stock returns have not necessarily their low volatility in the same regime.

4. Empirical Results

This section presents empirical results and considers the economic implications.

4.1. The Significance of the Regime Switching

A first interesting feature is to test the statistical significance of the regime switching. This cannot be performed using a standard LR test, because parameters associated with the second regime are not identified under the null hypothesis of no regime switching. Therefore, regularity conditions justifying the χ^2 approximation to the LR test do not hold. Hansen (1992, 1996) has proposed a LR test procedure that overcomes this problem. But, even for simple models, the computational burden is very important. Therefore, we adopt the strategy proposed by Ang and Bekaert (1999), which is based on Monte-Carlo simulations to obtain the small sample distribution of the LR test statistic. We consider the baseline MS model, in which only variances are assumed to be regime-dependent. Since volatility is the more likely to be regime-dependent, it seems to be sufficient to test the null hypothesis of no regime switching. Under the null hypothesis of no regime switching, conditional moments are all constant over time.

The small-sample distribution of the associated LR test statistic is obtained as follows: First, we estimate the one-regime model using the data set. Estimated mean and covariance matrix are denoted $\hat{\mu}$ and $\hat{H}$ respectively. Then, we simulate N samples of T returns, using the estimated one-regime model

$$\tilde{r}_t^{(i)} = \hat{\mu} + \hat{H}^{\frac{1}{2}}\tilde{\varepsilon}_t^{(i)} \qquad i = 1, \ldots, N$$

where $\tilde{\varepsilon}_t^{(i)}$ is drawn from a normal distribution with zero mean and unit variance. The length of simulated samples (T) is the same as the data sample. Next, for each simulated samples, we estimate the MS model with regime-dependent variances, and we compute the LR test statistics. Last, the small-sample distribution of the LR test statistic is computed over the N samples. Since the estimation of the MS model for simulated samples is very computationally intensive, we choose $N = 500$ and test the hypothesis of no regime switching for the baseline model only.

Table 4 reports descriptive statistics for the small-sample distribution of the LR test statistic of no regime switching. The null hypothesis is overwhelmingly rejected, since all LR test statistics obtained wtih simulated samples are smaller than the sample LR test statistics. The largest test statistic generated under the null is 22.1, while the sample test statistic is 139.1.

Table 4: Test of Regime-switching

Sample LR statistic	139.100
Small-sample distribution	
Number of samples (N)	500
Mean	5.107
Std deviation	4.195
Minimum	0.000
5%	0.964
50%	4.195
95%	11.216
Maximum	22.124

Notes: Samples of length 620 from the estimated one-regime model are generated. The regime-switching model is estimated on the simulated data and the sample LR statistic is computed. The procedure is repeated 500 times.

4.2. Model Estimates

Now, we examine results obtained with several multivariate MS models. In all models, conditional variances are assumed to vary across regimes. Conditional means and conditional correlations are assumed to be constant across regimes as well as regime-dependent. Each model is estimated assuming Gaussian or Student-t distributed innovations. Summary statistics are reported in Table 5. The statistics include the log-likelihood as well as the model selection statistics proposed by Akaike (1974) and Schwartz (1978). We also indicate the degree of freedom for the Student-t distribution.

We first consider different specification tests so as to rank our models. First, we test the null hypothesis that conditional mean is constant across regimes ($\mu_i^0 = \mu_i^1$, $\forall i = 1, 2, 3$). This test is based on the LR statistic, which is distributed as a χ^2 with $n = 3$ degrees of freedom. At any usual significance level, we do not reject the null hypothesis, whatever the specification. The degree of freedom in the Student-t distribution is large, but $1/\nu$ is found to be significantly different from 0. The LR test overwhelmingly rejects the Gaussian distribution in favour of the Student-t distribution. The AIC and Schwartz criteria also reject the Gaussian formulation in favour of the Student-t distribution.

Maximum-likelihood parameter estimates of the MS model with Student-t innovations are reported in Table 6. For aim of comparison, the first column reports the parameter estimates of the one-regime constant-variance model. The conditional means and variances are very close to the unconditional means and variances shown in Table 1. The conditional correlations are also very close to the unconditional correlation coefficients reported in Table 2. The

Table 5: Summary Statistics for Various MS Models

	No. of parameters (k)	Log-likelihood (L^*)	AIC criterion	Schwartz criterion	Degree of freedom (ν)	LR test statistics			
						about means $H_0^{(M)}: \mu(S_t) = \mu$		about correlations $H_0^{(C)}: \rho(S_t) = \rho$	
						Statistic	p-value	Statistic	p-value
One-regime model with constant variance									
Gaussian innovations	9	−3845.23	−3854.23	−3874.20	—	—		—	
Student-t innovations	10	−3814.62	−3824.62	−3846.81	8.764	—		—	
MS model with regime-independent correlations									
Gaussian innovations									
regime-ind. returns	14	−3775.68	−3789.68	−3820.74	—	6.63	0.08	15.36	0.00
regime-dep. returns	17	−3772.36	−3789.36	−3827.08	—	—		10.39	0.02
Student-t innovations									
regime-ind. returns	15	−3770.42	−3785.42	−3818.70	17.953	0.40	0.94	17.04	0.00
regime-dep. returns	18	−3770.22	−3788.22	−3828.16	18.315	—		16.93	0.00
MS model with regime-dependent correlations									
Gaussian innovations									
regime-ind. returns	17	−3767.99	−3784.99	−3822.72	—	1.66	0.65		
regime-dep. returns	20	−3767.16	−3787.17	−3831.54	—	—			
Student-t innovations									
regime-ind. returns	18	−3761.90	−3779.90	−3819.84	17.637	0.29	0.96		
regime-dep. returns	21	−3761.75	−3782.75	−3829.35	17.953	—			

Notes: AIC and Schwartz model selection criteria are computed as $L^* - k$ and $L^* - 0.5k\ \ln(T)$ respectively, where k is the number of parameters and T the number of observations. The degree of freedom parameter is the estimate of ν for the Student-t distribution. Both LR test statistics are distributed, under the null hypothesis, as a χ^2 with $n = 3$ degrees of freedom.

Table 6: Parameter Estimates for One-regime and Two-regime Models

Parameter	One-regime		Two-regime with regime-independent correlations		Two-regime with regime-dependent correlations	
	Estimates	Student	Estimates	Student	Estimates	Student
μ_1	0.305	4.215	0.299	4.392	0.298	4.467
h_1^0	3.594	14.337	2.266	11.331	2.102	11.192
h_1^1	—	—	5.640	9.297	6.302	8.423
μ_2	0.391	3.795	0.364	3.888	0.361	3.954
h_2^0	6.988	15.170	4.072	12.554	3.802	11.728
h_2^1	—	—	11.471	10.228	12.910	9.606
μ_3	0.237	3.101	0.226	2.935	0.227	3.078
h_3^0	3.976	14.273	3.095	14.514	2.802	13.802
h_3^1	—	—	5.088	10.791	5.915	9.031
ν	8.766	5.374	17.963	3.103	17.645	3.119
ρ_{12}^0	0.453	5.272	0.417	4.829	0.342	2.767
ρ_{12}^1	—	—	—	—	0.530	3.673
ρ_{13}^0	0.526	5.724	0.505	5.594	0.421	3.597
ρ_{13}^1	—	—	—	—	0.624	4.078
ρ_{23}^0	0.572	6.644	0.544	6.409	0.462	3.958
ρ_{23}^1	—	—	—	—	0.665	4.737
p	—	—	0.991	6.656	0.991	6.541
q	—	—	0.989	6.136	0.991	6.068
Log likelihood	−3814.625		−3770.419		−3761.897	

continued overleaf

Table 6: (*continued*)

	One-regime		Two-regime with regime-independent correlations		Two-regime with regime-dependent correlations	
	Statistics	p-value	Statistics	p-value	Statistics	p-value
$LM(4)$ for r_{1t}	42.556	0.000	9.246	0.055	4.755	0.313
$LM(4)$ for r_{2t}	73.114	0.000	18.637	0.001	12.540	0.014
$LM(4)$ for r_{3t}	13.149	0.011	5.091	0.278	4.128	0.389
$LB_c(4)$ for r_{1t}	13.048	0.011	14.006	0.007	14.090	0.007
$LB_c(4)$ for r_{2t}	0.467	0.977	0.982	0.913	1.030	0.905
$LB_c(4)$ for r_{3t}	2.387	0.665	2.959	0.565	3.007	0.557

Notes: $LM(4)$ is the TR^2 test statistic for conditional heteroskedasticity obtained by regressing squared returns on 4 lags.
$LB_c(4)$ is the Ljung–Box test statistic corrected for heteroskedasticity.
These test statistics are distributed under the null hypothesis as a χ^2 with 4 degrees of freedom.

degree of freedom for the Student-t distribution is equal to 8.8. All parameters are strongly significant.

The second column of Table 6 reports parameter estimates for the MS model with constant correlations. Since we did not reject the null hypothesis that conditional means are constant across regimes, we report results obtained with regime-independent means only. The first regime is characterized by low variances, the second regime by high variances. Indeed, the regime-1 variances are 2–3 times the regime-0 variances. The two regimes are strongly persistent since the transition probabilities p and q are very large, at 0.991 and 0.989 respectively. Both regimes would be expected to last on average for $(1 - p)^{-1} = 100$ weeks.

Last, column 3 reports parameter estimates for the model with regime-dependent correlations. Estimates for conditional means and variances are essentially unaltered as compared to the model with correlations constant across regimes. Estimated conditional correlations are (0.42; 0.51; 0.54). The corresponding correlations in the MS model with correlations varying across regimes are (0.34; 0.42; 0.46) in regime 0 and (0.53; 0.62; 0.67) in regime 1. Correlation coefficients increase by about 0.2 from regime 0 to regime 1. For instance, the conditional correlation between the DAX and FTSE returns is 0.46 during calm periods and 0.67 during turbulent periods. The LR test statistic for regime-independent correlations is equal to 17.04. Since it is distributed as a χ^2 with 3 degrees of freedom, the null hypothesis is strongly rejected at any usual significance level.

Summary statistics indicate that, at least for the DAX, residuals display heteroskedasticity. Therefore, volatility persistence is shown to have two sources: persistence of regimes, which is modelled with MS model; and within-regime volatility clustering, a feature which is not incorporated in this model.

4.3. The Economic Importance of Switching Models

At this stage, we get a further insight into the economic importance of switching models, in particular by studying international correlations. Figure 2 contains plots of the *ex-ante* probabilities $\Pr[S_t = 0|I_{t-1}]$ and the smoothed probabilities $\Pr[S_t = 0|I_T]$ for the MS model. These probabilities are computed as derived in Gray (1995), whose smoothing algorithm relates *ex-ante* probabilities and corresponding smoothed probabilities. The high-volatility regime (regime 1) can be associated with four periods: the very beginning of 1988; from end-1989 to mid-1991; from beginning of 1997 to beginning of 1998; and since the end of 1998.

The first period can be associated with the end of the October 1987 crash. The second period (from October 1989 to May 1991) begins with the mini-crash of 13 October 1989 in the US and also corresponds to the Kuwait crisis from Iraq's invasion on 2 August 1990 through the conclusion of the Gulf war

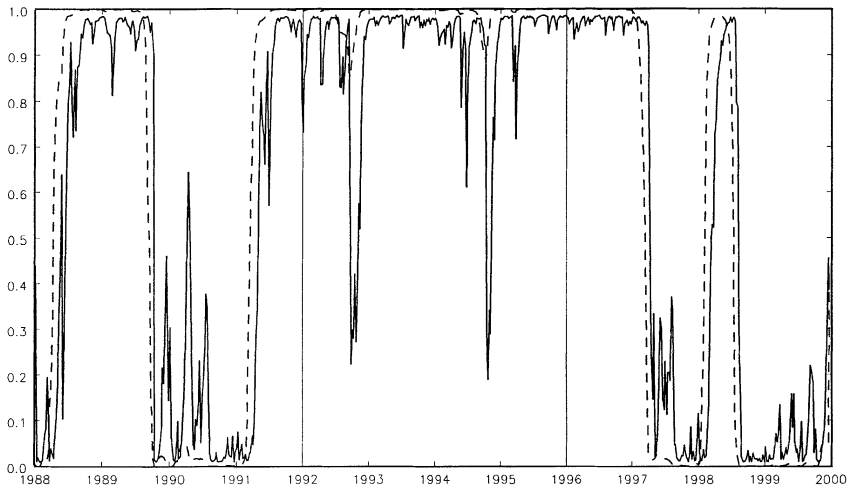

Figure 2: *Ex-ante* and Smoothed Probabilities
Note: This figure contains a time series of the *ex-ante* and smoothed probabilities that stock returns are in low-volatility regime (regime 0) at time t according to the MS model with regime-independent returns and regime-dependent correlations.

on 3 March 1991. The third period (April 1997–March 1998) is clearly driven by the South-East Asian crisis, which started in June 1997. The last period (from August 1998 to the end of the sample) has been clearly initiated by the Russian crisis, which started with the collapse of the bond market at the beginning of August. We also note a short-lasting spike in September 1992 corresponding the EMS crisis, which implied a strong increase in the FTSE volatility.

Smoothed probabilities are event more clear-cut, since the whole period is characterized with only two regime shifts. A first, turbulent, subperiod ends at the end of 1990, after the invasion of Kuwait by Iraq. The second shift occurred at the beginning of 1996, more then one year before the South-East Asian crisis started. This is due to the fact that the smoothed probability at date t is computed using information on the whole sample, so that smoothed probabilities seem to precede *ex-ante* probabilities.

Aggregated over regimes correlations implied by the MS model with regime-independent returns are plotted in Figure 3. For each stock market, we display correlations estimated using the MS model with regime-dependent as well as regime-independent correlations. Since correlations are assumed to be constant within regime, there are only two possible levels of correlation and therefore the conditional correlation mimics the *ex-ante* regime probabilities. The conditional correlations are much less dispersed than unconditional correlations computed over a sliding window, as plotted in Figure 1b. They

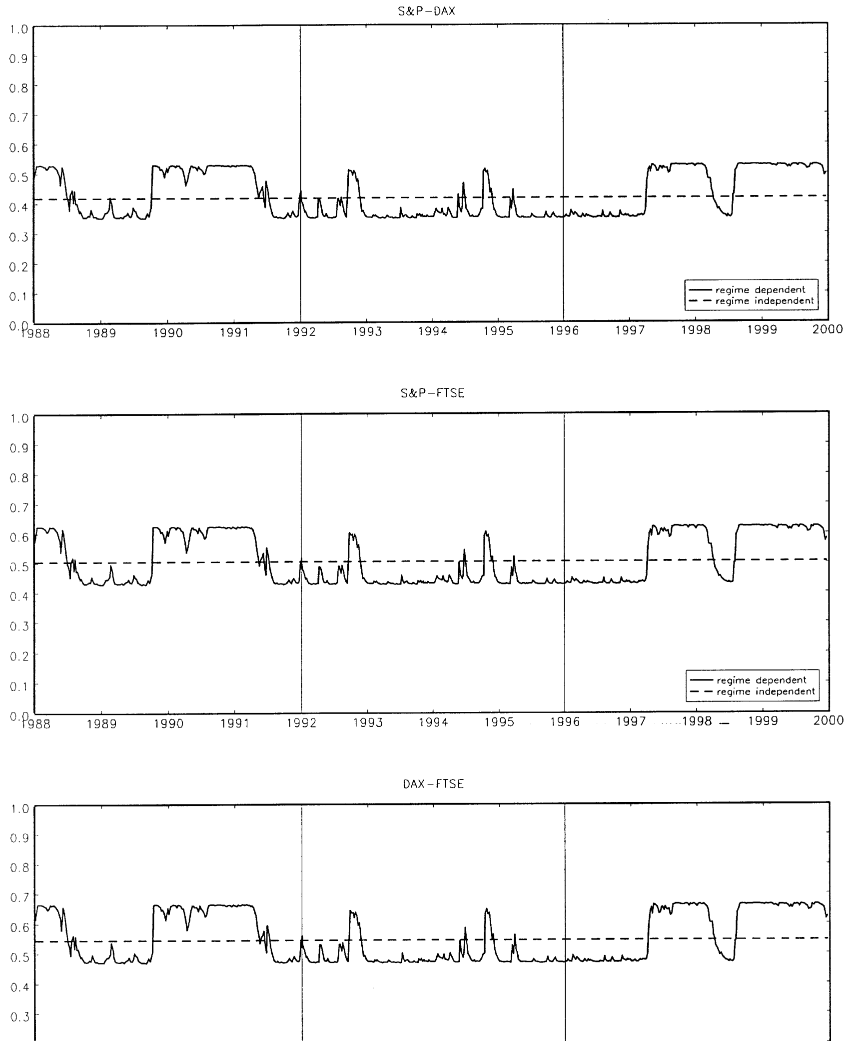

Figure 3: Conditional Correlation Estimates
Note: This figure contains a time series plot of conditional correlation across markets, with parameter estimates based on the MS model with regime-independent returns with regime-dependent correlations.

display a similar pattern, however, with peaks in 1988, 1990–91 and 1998–99. Note that, conditionally to the existence of two regimes driven by volatility, stock-market returns in 1995 are not found to be as strongly correlated as in Figure 1b.

5. Conclusion

This paper considers the relationship between international correlation and stock-market turbulence. We assume that stock markets are driven by two regimes, characterized by a low volatility and a high volatility. We estimate a multivariate Markov-switching model and we then test the null hypothesis that correlations are constant across regimes.

Using weekly stock return series for three of the largest stock markets, we find that MS models offer a good statistical fit to the data. Turning to the hypothesized relationship between international correlation and stock-market turbulence, we effectively obtain that returns are more highly correlated during the high-volatility regime than during the low-volatility regime. We perform a LR test which confirms that an increase in volatility is usually associated with an increase in correlation. Broadly speaking, our sample can be split into three subperiods corresponding to different levels of volatility. Before 1992, stock markets faced a high-volatility regime, associated in particular with the Gulf crisis. The second agitated period started in 1997 and was characterized by the South-East Asian crisis and the Russian crisis. The 1992–96 period is found to be a low-volatility regime.

Our test procedure improves previous tests based on a data-driven selection of high- and low-volatility subperiods. Unlike these tests, which have been shown to be biased because of regime selection (Boyer *et al*. 1997), our test is based on regimes determined endogenously and consistently with the data generating process. Therefore, our test procedure does not suffer from any selection bias.

Our work may be extended in two ways. First, the econometric model may be improved to incorporate further statistical features of stock returns. Markov-switching models can be designed to allow transition probabilities to be different across markets and/or to vary over time. Hamilton and Lin (1996) incorporated the first extension and Gray (1996) incorporated the second one. However, in a multivariate framework, such extensions would increase the computational burden dramatically.

Second, our test for a regime-independent conditional correlation may be performed for other groups of markets. In particular, it would be interesting to assess whether correlation between emerging-market returns really increased during the well-documented Mexican (1994), South-East Asian (1997) and Russian (1998) crises. Many authors have focused on these episodes – for instance, Baig and Goldfajn (1999) and Forbes and Rigobon (1999) – but most of them considered unconditional correlations computed over subperiods selected *ex post* and therefore incorporating all information about past crises.

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Appendix

Test of Constant Unconditional Correlation Matrix

A convenient way to test the null hypothesis of a constant unconditional correlation matrix is to test for the equality of the correlation matrices computed over two subsamples. Different test statistics have been proposed in the literature to perform such a test. One of the most popular is the test developed by Jennrich (1970), based on the normalized difference between the two correlation matrices.[9] The test for the equality of the correlation matrices, denoted R_1 and R_2 over two independent subsamples of equal size $n_1 = n_2 = n$ is based on the statistics:

$$\chi^2 = \tfrac{1}{2}\,\mathrm{tr}(Z^2) - \mathrm{diag}(Z)' S^{-1}\,\mathrm{diag}(Z)$$

where

$$Z = \sqrt{\tfrac{n}{2}} R^{-1}(R_1 - R_2)$$

[9] Box (1949) also proposed a statistic for testing the equality of two covariance matrices. However, his test cannot be adapted for testing the equality of two correlation matrices.

$R = \frac{1}{2}(R_1 + R_2)$ is the average correlation matrix over the two subsamples $S = (\delta_{ij} + r_{ij} r^{ij})$ with $R = (r_{ij})$, $R^{-1} = (r^{ij})$ and

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

and diag(X) denotes the diagonal of the square matrix X in a column form.

The Jennrich test statistic has an asymptotic χ^2 distribution with $p(p-1)/2$ degrees of freedom, if the correlation matrix is computed for p variables.

It is noteworthy that the test statistic for constant correlation between two variables ($p = 2$) is simply

$$\chi^2 = \frac{n}{2} \frac{(r_1 - r_2)^2}{(1 - r^2)^2}$$

where r_1 and r_2 are the estimated correlation over the two subsamples and $r = \frac{1}{2}(r_1 + r_2)$.

Non-technical Summary

Correlations between international equity markets are often claimed to increase during turbulent periods. This issue is truly important for both portfolio managers and regulators, since international diversification benefits seem to decrease when they are most needed, i.e. during periods of market turbulence. Modern portfolio theory underlines that not only returns and volatilites are important in the portfolio selection process, but also that correlations between assets are really a key to a good asset allocation. But, if correlations increase during turbulent periods, then standard portfolio diversification will not be able to decrease the risk during these periods of high volatilities.

The link between international correlation and stock-market turbulence has been studied using different approaches. A first approach examined the stability of the correlation between returns over different periods of time. Most papers consider changes in correlation by comparing unconditional correlation before and after a crash. The breakpoint being exogenously selected, recent evidence has shown that testing unconditional correlation coefficient may be misleading. This is because the correlation coefficient is biased when volatility shifts over time.

Another strand of the literature is based on the autoregressive conditional heteroskedasticity (ARCH) framework. For instance, using multivariate GARCH models, Hamao *et al.* (1990) and Susmel and Engle (1994) measured the interdependence of returns and volatilities across major stock markets. Longin and Solnik (1995) tested specifically the hypothesis of a constant conditional correlation between monthly returns of a large number of stock indices. They explored several potential sources of deviation from the constant conditional correlation model, and they found that correlation generally rises in periods of high volatility. Bera and Kim (1996) and Tse (2000) also proposed a test for the constant-correlation hypothesis. The GARCH approach improves clearly the measure of time-varying volatility and, for some parameterizations, the measure of time-varying correlations. In many empirical studies, however, stock-market volatility is found to be too persistent, implying an explosive conditional variance. As argued by Lamoureux and Lastrapes (1990), the excessively persistent volatility found in ARCH models may be due to the occurrence of structural breaks, such as the October 1987 crash.

An alternative, and more appealing, way to study the effect of the bear market on international correlation is the Markov-switching (MS) model, which provides a far less persistent volatility than the GARCH model. Two recent studies adopted this approach. Ramchand and Susmel (1998) developed a bivariate MS model to test the hypothesis of a constant conditional correlation between stock markets. The conditional correlation between the US market and other major stock markets is found to be constant in two over the four cases. In a more general setting, Ang and Bekaert (1999) obtained evidence of a high-volatility and high-correlation regime and a low-volatility and low-correlation regime.

This paper investigates the relationship between international correlation and stock-market turbulence, assessing empirically whether the claim that correlations increase during turbulent period is true. The focus is on US, German and UK weekly stock-market returns, over the 1988–99 period. First, we estimate several multivariate MS models for stock-market returns. We assume that volatilities shift in all markets at the same date. This assumption allows calm and turbulent regimes to be identified unambiguously. We effectively obtain evidence of a regime with low volatilities and low correlations and a regime with high volatilities and high correlations. Broadly speaking, our sample can be split into four subperiods. After a calm period in 1988–89, stock markets experienced a high-volatility regime in 1990–91, associated in particular with the Gulf crisis. The 1992–96 period to be a low-volatility regime. The South-East Asian crisis and the Russian crisis occurred during the last, agitated period, which started in 1997. Correlation coefficients are increased by about 0.2 from the low-volatility regime to the high-volatility regime. For instance, the conditional correlation between the German and the UK returns is 0.46 during calm periods and 0.67 during turbulent periods.

Then, we test the null hypothesis that correlations are equal in both regimes, using a LR test procedure. This test procedure improves previous tests based on a data-driven selection of high- and low-volatility subperiods. Unlike these tests, the LR test is based on regimes determined endogenously and consistently with the data generating process. Therefore, it does not suffer from any selection bias. We obtain that the null hypothesis is strongly rejected, whatever the MS model. Therefore, we conclude that turbulent periods are associated with significantly higher correlations than calm periods.

As pointed out by Odier and Solnik (1993) and Lin *et al.* (1994), the increase in correlation may partly explain why investors tend to diversify far less internationally than theory would predict. This may solve the home bias puzzle, one of the most challenging puzzles in international finance.