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Reading PIBOR futures options smiles: The 1997 snap election

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Abstract

In this paper, we compare various methods that extract a Risk Neutral Density (RND) out of PIBOR interest-rate futures options and we investigate how traders react to a political event. Our benchmark model derives from A. Brace, D. Gątarek, M. Musiela [Mathematical Finance 7 (1997) 127–155]. We also consider a mixture of log-normals (as in W.R. Melik, C.P. Thomas, Journal of Financial and Quantitative Analysis 32 (1997) 91–116), an Hermite expansion (as in P. Abken, D.B. Madan, S. Ramamurtie, Estimation of risk-neutral and statistical densities by Hermite polynomial approximation: with an application to Eurodollar Futures Options, Federal Reserve Bank of Atlanta, 1996), and a method based on Maximum Entropy (according to P. Buchen, M. Kelly, Journal of Financial and Quantitative Analysis 31 (1996) 143–159). We take care of the early exercise feature and we show how to approximate RNDs for a fixed time to maturity. The various methods generate similar RNDs. A daily panel of options running from February 1997 to July 1997 reveals that operators expected the snap election a few days before the official announcement was made and that a substantial amount of political uncertainty subsisted even a month after the elections. Uncertainty evolved with polls forecasts of the future government. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Empirical studies of stock-options implied volatilities, such as Rubinstein (1994), demonstrate that options of a same maturity but different strike prices have different volatilities; this feature is called the *volatility smile*. This feature may arise when the underlying asset volatility changes in complicated ways, being in contradiction with the Black and Scholes (1973) and Merton (1973) assumptions. In this case, an option may not be perfectly hedged, which in turn implies that market operators' preferences play an important role. As shown by Jackwerth (1999), an abundant literature on how to read expectations for financial markets has emerged. This work adds to those few studies, such as the one by Söderlind and Svensson (1997), that assess the information content of interest-rate based derivatives. [2]

Under rather mild assumptions, there exists a Risk Neutral Density (RND) that allows option pricing as a conditional expectation (e.g., Harrison and Kreps, 1979; Harrison and Pliska, 1981). Breeden and Litzenberger (1978) observed that applying Leibnitz's rule to option prices also yields the RND. This RND is related to market participants' expectations of the future price process in a risk neutral environment. As shown by Campa et al. (1997), or Söderlind and Svensson (1997), once a RND is obtained it becomes possible to compute moments as well as confidence intervals whose evolution is indicative of market participants' perceptions of future. Clearly, because of a possibly time-varying risk premium, RNDs differ from real-life probability distributions. Nonetheless, RNDs play an important role as an indicator for Central Banks wishing to measure market expectations and variations thereof. RNDs are equally important to investors who need to measure other investors' expectations about future trends. For risk management RNDs also provide an objective measure of expected extreme variations in the underlying asset's price.

A first contribution of this paper is the study of options data that has attracted less attention than US or UK data. We investigate the information content of PIBOR (Paris Interbank Offered Rate) interest-rate futures options.

[2] Unlike some of the literature that has addressed the question of how to price options under non-constant volatility (e.g., Derman and Kani, 1994; Dumas et al., 1998; Dupire, 1994; Stein and Stein, 1991), we address the question of what the information content in options of various maturities is.

PIBOR instruments capture short-term interest rate movements. Both the option and its underlying futures contract are traded at MATIF (Marché à Terme International de France).

We further compare four RND extracting methods by applying them to actual options data. The first method, i.e., the benchmark model, is a special case of Brace et al. (1997), and assumes that the RND derives from a log-normal structure. The second method assumes that the RND is a mixture of log-normal densities as in Melick and Thomas (1997). The mixture of distributions model is also used by Bahra (1996) and Campa et al. (1997). The third method follows Abken et al. (1996) who implement the idea exposed in Madan and Milne (1994), which consists in approximating the underlying RND with Hermite polynomials. The fourth method approximates the RND with a functional having the highest Entropy conditional on being consistent with the data. This last approach has been developed by Buchen and Kelly (1996). To our knowledge, this paper is the first paper that assesses their method on interest-rate options. Within the entropy literature, Stutzer (1996) suggests a multistep Bayesian procedure, where the initial step involves a density estimation from historical prices of the underlying asset.

Further literature builds upon structural models by assuming jump diffusions such as Malz (1996, 1997) or stochastic volatility and jumps as in Bates (1991, 1996a,b). These methods have been successfully applied to foreign exchange and stock price data. See also Campa et al. (1998), as well as Jondeau and Rockinger (2001) for a comparison of various existing methods on exchange data.

Several other approaches yield RNDs. Rubinstein (1994) as well as Jackwerth and Rubinstein (1996) suggest a method based on binomial trees. Aït-Sahalia and Lo (1998) provide a non-parametric method based on binomial time-series analysis and kernel estimates. This observation has led Shimko (1993) to fit a second-order polynomial to the implied smile volatility in order to smooth the RND. Similarly, Neuhaus (1995) works with cumulative distribution functions. The objective of our paper is, however, to focus on the information content of options, for this reason we did not implement these methods.

Furthermore, we implement the early exercise feature of options. This is achieved by extending the model of Melick and Thomas (1997) to the models at hand. We find that the American-option feature does not play a role for the options considered.

We also provide the construction of *standardized* RNDs with a constant time to maturity. This construction is necessary since the traded options have fixed exercise dates. RNDs extracted for such options would evolve as the maturity date approaches, i.e., because of the term structure of volatility. We find that RNDs lacking this standardization cannot be compared and, thus, market participants' expectations cannot be gauged.

We compare the various methods on five selected dates surrounding the 1997 French snap election. Monday April 14, 1997, that is one week before the official announcement of the dissolution of the National Assembly, the trigger for snap elections. Monday April 21, the day after the official announcement by President Chirac. May 26, the first Monday after the first round of legislative elections. Next, we consider June 2, the first Monday after the second round of elections. At this stage, it was known that the Government of Alain Juppé was defeated and that the left-wing parties had made it back to government. Last, we choose June 9, which is a Monday, one week after the elections. Gemmill (1992) used options to investigate if markets were able to predict an election outcome in the UK.

By using daily data running from February 1997 to July 1997, we show that markets reacted strongly to the announcement of the election, even though information appeared to have trickled into the market before the announcement. Also before the first and second electoral rounds, markets appear being influenced by polls. Like the study by Bates (2000) on stock index options after the 1987 crash, or by Malz (1996) on foreign exchange data, we are able to exhibit a peso-problem: market participants attribute a significant probability to an event that has not occurred in recent history such as a very strong increase in interest rates.

The structure of this paper is as follows. In Section 2, we discuss the characteristics of the PIBOR market and of the PIBOR instruments. In Section 3, we present the benchmark model based on Brace et al. (1997) as well as several other models that allow us to construct RNDs in the situation where the benchmark model does not hold. In the same section, we also show how one can deal with the fact that PIBOR options are American and not European. Furthermore, we develop an ad hoc method to construct RNDs with fixed horizons. In Section 4, we first compare, for the five selected dates, the various methods that compute RNDs. Then we apply the Hermite polynomial method to all available options between February 1997 and July 1997, covering the 1997 snap election. In Section 5, conclusions are provided.

2. Facts concerning PIBOR options...

Since PIBOR options are written on PIBOR futures contracts, we first examine the futures market. There are five market makers for the 3-month PIBOR futures contract.[3] Along with a permanent presence on the floor, they have the duty to quote all the open strike prices at the request of either a

[3] BNP, Caisse Nationale du Crédit Agricole, CPR, Société Générale, and Transoptions Finance.

market participant or an exchange supervisor. For options for the first two expirations their quote must have a bid/ask spread of at most five basis points and they must be willing to trade up to 100 contracts for the quoted price. The nominal, N, is 5 million francs. If F is the forward PIBOR rate then the contract is quoted as $100(1-F)$. At maturity, if the spot 3-month PIBOR rate is r, the payoff in a long futures position is $N(F-r)/4$.

The market makers for the 3-months PIBOR futures-options are the same five as for the underlying contract. In theory the price of an option is quoted as a percentage of the nominal. In practice, options are quoted in terms of volatility. To obtain an option price, an inverted Black-type formula gets used. Later on, we show what kind of assumptions are involved, when proceeding in such a way. Since the rates are quoted in the $100(1-F)$ format if for instance, $F = 0.0366$, corresponding to a 3.66% rate, the futures price is 96.34. Strike prices are integer multiples of 10 basis points, e.g., the four strike prices closest to the futures price of 96.34 are 96.20, 96.30, 96.40, and 96.50.

There are at least quotes for the 15 options closest to the at-the-money option. At each point of time, there are four maturities with expiration month March, June, September, and December.[4] In-the-money options are automatically exercised on the last trading day. The last trading day is the 2nd business day preceding the 3rd Wednesday of delivery month at 11:00 a.m. (Paris time) for the expirations used in this study. The expiration date of the contracts is the same as of the underlying.

MATIF follows the CME and uses traditional compensation whereby the option premium must be paid at the time an option is brought. This differs from the LIFFE where options are futures-style margined.[5] Besides its role as clearing house, MATIF provides at the end of each trading day implicit volatilities for certain reference strike prices for compensation purposes. Given that PIBOR market-makers are compelled to give quotes with a reasonable spread, this insures that even if the trading volume of certain options is small there is information about market participants' expectations. In order to cover a wider range of strike prices, MATIF performs a linear interpolation between the range of reference strike prices. Outside the reference strikes, they assume a constant volatility by choosing the closest reference strike.

[4] This implies that, as time goes by, the maturity dates come closer. This fact creates difficulties and makes it necessary to find a way to construct information for options with constant time to maturity.

[5] A nice feature of futures-style margining is that the early exercise feature drops and an American option can be priced as a European one (see Chen and Scott, 1993).

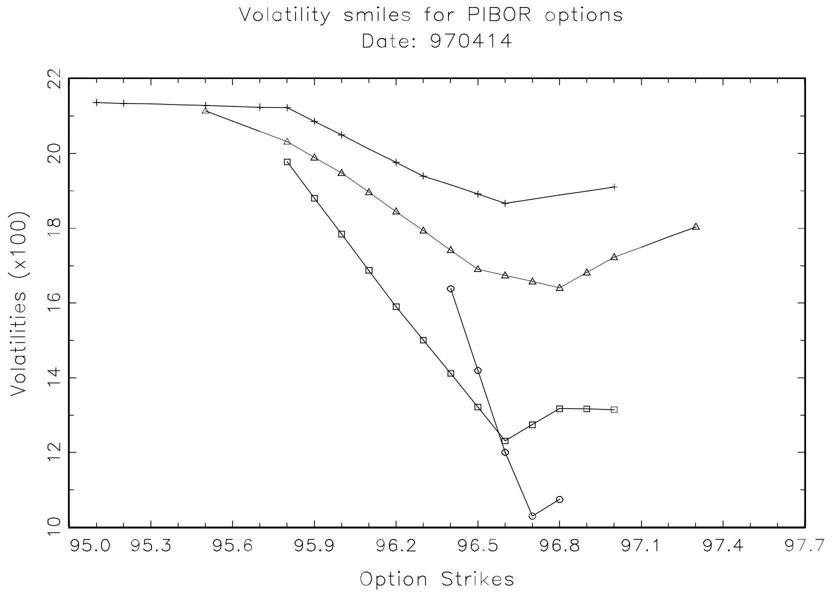

Fig. 1. Smiles for PIBOR options of several maturities on April 14, 1997. The symbols ○, □, △, and + correspond to options with 63, 154, 245, and 336 days to maturity.

Data covering the period from February 3, 1997 to July 30, 1997 on PIBOR futures-options as well as on the underlying futures contract has been kindly provided by MATIF. [6]

Figs. 1 and 2 display volatilities against strike prices for two selected dates. [7] Symbols on the various curves correspond to data provided by MATIF. Whenever symbols are on a straight line, they can be assumed to have been obtained with a linear interpolation. In other words, only symbols where the line has a kink are likely to correspond to actual information. We also notice that MATIF does not systematically provide interpolated volatilities. For instance, on April 14, 1997 for the PIBOR option with highest maturity, a quote was reported for the 96.0 and the 96.2 strike but not for the 96.1 strike. These observations suggest that the methods used to extract information should allow for possible options mispricing, and also that one should consider filters before using this raw data.

[6] Information on how the MATIF operates can be found under the web-page: http://www.matif.fr/.

[7] In order to simplify the figures, the extrapolations at constant volatility level to the left and to the right of the smile have already been discarded.

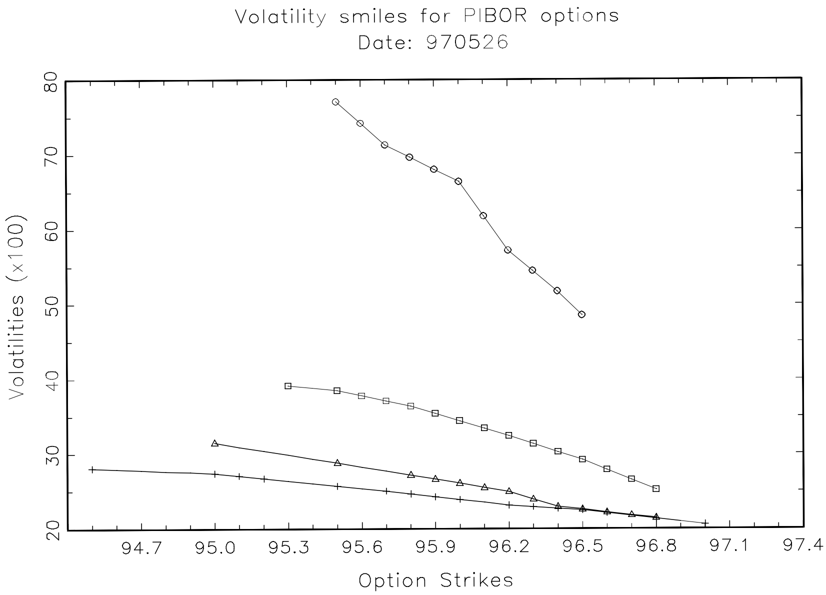

Fig. 2. Smiles for PIBOR options of several maturities on May 26, 1997. The symbols ○, □, △, and + correspond to options with 56, 147, 238, and 329 days to maturity.

These figures also have an economic implication. When we consider PIBOR options of a same maturity in Figs. 1 or 2, we notice that options with different strikes have different implied volatilities. This feature is precisely the *option smile*. We also notice the vertical shift of smiles as maturities change. This corresponds to the feature labeled in the literature as a *term structure of volatilities*. For April 14, 1997, a date one week before the official announcement of the snap election, higher maturities are associated with higher volatility. [8] We will refer to April 14 as the *normal* date. When we take May 26, the day after the first election round, we notice the reversal of the term structure of volatility, meaning that operators had lots of uncertainty concerning the short run.

3. The general framework

3.1. The Brace, Gatarek, and Musiela benchmark model

Many interest-rate models have been proposed in the literature. In a first category of models, the interest-rate curve and the discount factor are

[8] As shown later, on this date market makers did not yet anticipate the election.

described by state variables such as the short-term rate, the long-term rate, or the volatility of the short-term interest rate. [9]

Another more recent category of models shifts away from focusing on the dynamics of some state variables to modeling the dynamic for the whole term structure of interest rates. Heath et al. (1992) described the instantaneous forward rates by a normal structure. Brace et al. (1997) (BGM) have suggested a model of forward rates that allows the volatility of the forward LIBOR rate to depend on past volatilities and LIBOR for all maturities. This latter was chosen for our benchmark and is described in this section. [10]

The first step in their work is to define a term structure of interest rates in some alternative manner. Let $r(t,x)$ be the continuously compounded forward rate prevailing at time t over the time interval $[t+x, t+x+\mathrm{d}x]$. This rate relates to the Heath et al. (1992) forward rate $f(t,t+x) = r(t,x)$. The process

$$B(t,T) = \exp\left(-\int_0^{T-t} r(t,u)\,\mathrm{d}u\right) = \exp\left(-\int_t^T f(t,u)\,\mathrm{d}u\right)$$

defines for all $T > 0, 0 \leqslant t \leqslant T$, the price evolution of a zero-coupon bond with maturity T. BGM assume that the forward rate follows

$$\mathrm{d}r(t,x) = \frac{\partial}{\partial x}\left[\left(r(t,x) + \frac{1}{2}|\sigma(t,x)|^2\right)\mathrm{d}t + \sigma(t,x)\,\mathrm{d}W(t)\right],$$

where $W(t)$ represents a d-dimensional Brownian motion and where σ satisfies some regularity assumptions. Such a specification of the forward rate is desirable, since it yields a log-normal diffusion for the discount function.

The PIBOR rate is not a continuously compounded rate. Following Sandmann and Sondermann (1993), BGM introduce an effective rate defined as

$$L(t,x) = \frac{1}{\delta}\left[\exp\left(\int_x^{x+\delta} r(t,u)\,\mathrm{d}u\right) - 1\right]. \tag{1}$$

For a 3-month PIBOR rate, $\delta = 1/4$. Applying Ito's lemma to Eq. (1) one obtains that $\mathrm{d}L(t,x) = A(t,x)\,\mathrm{d}t + \delta^{-1}(1+\delta L(t,x))(\sigma(t,x+\delta) - \sigma(t,x))\,\mathrm{d}W(t)$. BGM assume furthermore that the interest rate follows a log-normal volatility structure, that is $\mathrm{d}L(t,x) = A(t,x)\,\mathrm{d}t + L(t,x)\gamma(t,x)\,\mathrm{d}W(t)$, where γ is a deterministic, bounded, and piecewise continuous function.

[9] This type of model was studied by Merton (1973), and is the equivalent to Black and Scholes (1973) for interest-rates derivatives. Further single factor models are by Vasicek (1977) and Cox et al. (1985), as well as by Hull and White (1990). Other studies assume two factors: Brennan and Schwartz (1982) proposed to add the long-term interest rate as state variable; Longstaff and Schwartz (1992) suggested the volatility of the short-term rate.

[10] The following presents an adaptation of the Brace et al. (1997) paper. Further details may be found in their work. We follow their notation.

This indicates that a log-normal volatility structure holds if and only if

$$\sigma(t,x+\delta) - \sigma(t,x) = \frac{\delta L(t,x)}{1+\delta L(t,x)}\gamma(t,x).$$

Under this assumption, BGM derived an alternative description of the term structure that allows them to obtain pricing formulas for caps and swaptions. Since we are interested in the value of a simple option on futures prices, their framework encompasses our needs. The BGM framework can, therefore, be used to price options on future rates. They determine that at time $t \leqslant T$ the price of a call option on a PIBOR contract with underlying forward price F_T and strike price K is given by

$$C(F,K,T,t) = B(t,T+\delta)\mathrm{E}^*_{T+\delta}[((100-K) - L(T,0))^+ | \mathscr{F}_t],$$

where $B(t,T)$, is the price of a zero-coupon bond with maturity T at date t. The operator X^+ is defined as $\max(X,0)$. Also, E^*_T represents the conditional expectation computed under the forward measure defined by El Karoui and Rochet (1989) and Musiela (1995). The conditioning involves the available information at time t, written as $\mathscr{F}_t$. Under this forward measure, and the assumed asset dynamics, BGM show that

$$\begin{aligned} C(F,K,T,t) = B(t,T+\delta)[&(100-K)\Phi(\zeta(t,T) - h(t,T)) \\ &- L(t,T-t)\Phi(-h(t,T))], \end{aligned} \quad (2)$$

where Φ is the cumulative distribution function of a zero mean, unit variance normal variate. Furthermore, the following definitions apply:

$$h(t,T) = \frac{1}{\zeta(t,T)}\left(\ln\left(\frac{L(t,T-t)}{K}\right) + \frac{1}{2}\zeta^2(t,T)\right),$$

$$\zeta^2(t,T) = \int_t^T |\gamma(s,T-s)|^2\,\mathrm{d}s.$$

Similar formulas hold for put options. Last, the function γ needs to be specified. If γ is a constant, then $\zeta^2(t,T) = (T-t)\gamma^2$ and Eq. (2) reduces to a classical Black (1976) formula.

From an empirical point of view, several remarks are of order. First, this very elegant derivation shows the assumptions implicitly made by PIBOR investors when they use Black's formula to derive the price for an option from volatility. Second, we notice that BGM's formula holds only under very special assumptions concerning the dynamics of the underlying rates. As a consequence, since in real-life applications the behavior of rates may deviate from BGM's general specification, it is possible that this theoretical model, and the constraints imposed on it for numerical tractability, be rejected in empirical applications. In particular, the observed non-constancy of volatility across smiles suggests that the structural assumption concerning the underlying asset is very complex.

Direct modeling of the RND in a non-parametric fashion may shed additional insights as well. These caveats being stated, the BGM model with simplifying assumptions provides a precious benchmark model, referred to as LN.

3.2. How to obtain a Risk Neutral Density

The results of Harrison and Kreps (1979), Harrison and Pliska (1981), or at textbook level of Duffie (1988, p. 115) remind us that there exists a RND $q(\cdot)$ such that the call and put option's price can be computed as a conditional expectation. In the PIBOR context, this implies, using Eq. (2):

$$C(F,K,T,t) = B(t,T)\int_{F_T=K}^{+\infty}(F_T-K)^+ q(F_T,\theta)\,\mathrm{d}F_T, \tag{3}$$

$$P(F,K,T,t) = B(t,T)\int_{0}^{F_T=K}(K-F_T)^+ q(F_T,\theta)\,\mathrm{d}F_T, \tag{4}$$

where θ is a vector of parameters characterizing the distribution.

There are different approaches to estimating the RND. Breeden and Litzenberger (1978) notice that, using Leibnitz' rule, the RND can be obtained by a second derivative of the option formula with respect to the strike price. There are alternative methods available, some of them described below. First, we indicate how these densities can be computed under the assumption that the options are European-style; then using Melick and Thomas' device, we show how the American character can be imbedded in this context. [11]

3.3. …using a mixture of log-normal densities…

The first natural way to approximate the RND is to follow Bahra (1996), Melick and Thomas (1997), or Söderlind and Svensson (1997) and describe it as a mixture of L weighted log-normal distributions, MIX. In this case, the call option price (3) can be written as

$$C = B(t,T)\sum_{j=1}^{L}\alpha_j\left(\exp\left(\mu_j+\frac{1}{2}\sigma_j^2\tau\right)\left[1-\Phi\left(\frac{\ln(K)-\mu_j-\sigma_j^2\tau}{\sigma_j\sqrt{\tau}}\right)\right] - K\left[1-\Phi\left(\frac{\ln(K)-\mu_j}{\sigma_j\sqrt{\tau}}\right)\right]\right),$$

where $\tau = T-t$ represents the time till maturity, and α_j is the weight put on distribution j. The parameters μ_j and σ_j are the mean and volatility for the jth

[11] Gauss code for this model as well as for all the other models is made available upon request. For this reason the presentation of all models is kept to a minimum.

distribution. Using a Lagrangian penalty we impose the constraint that, under risk-neutrality, the forward price be equal to the expected forward price at maturity

$$F_t = \sum_{j=1}^{L} \alpha_j \exp\left(\mu_j + \frac{1}{2}\sigma_j^2 \tau\right).$$

3.4. ...or using Hermite polynomials...

The theoretical foundations of this method, HER, are elaborated in Madan and Milne (1994) and applied in Abken et al. (1996) to Eurodollar futures options. Their idea is to project both the payoffs and the RND on a basis of Hermite polynomials. Given that series involving Hermite polynomials can approximate any given real valued function, they focus on a truncated expansion up to the fourth-order. Exploiting the orthogonality property, they approximate the RND with

$$q(x) = \left(1 + \frac{\pi_3}{\sqrt{6}B(t,T)}(x^3 - 3x) + \frac{\pi_4}{\sqrt{24}B(t,T)}(x^4 - 6x^2 + 3)\right)\phi(x).$$

We recognize here a Gram–Charlier density (see Jondeau and Rockinger, 2001). For such an expansion, it is known that skewness and excess kurtosis are given by $\sqrt{6}\pi_3/B(t,T)$ and $\sqrt{24}\pi_4/B(t,T)$. The parameters π_3 and π_4 are, therefore, directly related to skewness and kurtosis. For this reason they are named the price of skewness and kurtosis.

For such an RND, Madan and Milne (1994) derived that the call price of an option can be approximated as

$$C = \sum_{k=0}^{4} a_k \pi_k,$$

where the a_k are complicated functions of the mean, μ, and volatility, σ, of the underlying asset. The a_k also depend on the time to maturity τ, the strike price and the current forward price. The coefficient π_0 takes the values $B(t,T)$. Given that the mean and volatility are appearing explicitly in the coefficients a_k, it is necessary, for identification purposes, to impose $\pi_1 = \pi_2 = 0$. The parameters π_3 and π_4 have to be estimated. The truncation is up to the fourth-order. The reason why it is advisable to truncate after the fourth-order is based on numerical arguments: if one uses higher orders, because of multicolinearity of even and odd moments, the estimation becomes very unstable.

Abken et al. (1996) also show that for two times till maturity, τ and τ', the following restriction must hold:

$$\frac{\pi_k(\tau)/B(t,t+\tau)}{\tau^{k/2}} = \frac{\pi_k(\tau')/B(t,t+\tau')}{\tau'^{k/2}}. \tag{5}$$

Even though they reject this restriction, one can use it as an approximation to extrapolate information for options of a fixed horizon from given options.

3.5. …and using the maximum entropy principle

A method based on the principle of maximum entropy, ME, was introduced to finance by Buchen and Kelly (1996), and Stutzer (1996). It finds its origin in the work of Shannon (1948) who related the notion of Entropy of thermodynamics to information theory. Jaynes (1957, 1982) extended this concept to statistical inference. A textbook description of Entropy-based techniques can be found in Golan et al. (1996).

The general definition of entropy is

$$E = -\int_0^{+\infty} q(x) \ln(q(x))\, \mathrm{d}x, \tag{6}$$

where as before $q(\cdot)$ will represent a RND. The entropy can be viewed as a metric, whose maximization will give a RND with the *maximum* possible information content. [12]

With the notations introduced in Section 2 the constraints can be written as:

$$\int_0^{+\infty} q(x)\, \mathrm{d}x = 1, \tag{7}$$

$$\int_0^{+\infty} c_i(x) q(x)\, \mathrm{d}x = C_i / B(t,T), \quad i = 1, \ldots, m, \tag{8}$$

$$\int_0^{+\infty} x q(x)\, \mathrm{d}x = F_t. \tag{9}$$

Eq. (7) insures that q is a density. Eq. (8) relates the RND to the ith call option. The last equation is the martingale restriction.

Maximization of the entropy (6) subject to the constraints (7)–(9) is done by maximization of a Hamiltonian where the multipliers will be written as λ_i. Buchen and Kelly (1996) show that the RND is equal to

$$q(x) = \exp\left(\sum_{i=0}^{m} \lambda_i c_i(x)\right) \Bigg/ \int_0^{+\infty} \exp\left(\sum_{i=0}^{m} \lambda_i c_i(x)\right) \mathrm{d}x.$$

Clearly, the multipliers must be estimated numerically. This may be achieved elegantly following Agmon et al. (1979).

[12] Entropy is an intuitively appealing method since its maximization yields the uniform density if the only thing that we know is that the support is finite. If the mean is known, the method yields the exponential and if, furthermore, the variance is known, it gives the normal density.

3.6. The early exercise feature

As mentioned by Musiela and Rutkowski (1997, p. 357): "an efficient evaluation of American-style options, under uncertainty of interest rates, is a rather difficult problem". For this reason, even though a purely analytical solution is unavailable, it is interesting to dispose of techniques allowing for a correction. It is possible to (partially) correct a formula such as Eq. (2) for American-style options, following the approximation suggested by Barone-Adesi and Whaley (1987). Alternatively, it is possible to use the correction developed by Melick and Thomas (1997). The latters realized that the price of an American-style option is bounded from below by the European-style option price and from above by a bound derived by Chaudhary and Wei (1994). Therefore, the actual option price can be written as a convex combination of these two bounds. The bounds are:

$$
\begin{aligned}
C^u(F,K,T,t) &= \mathrm{E}_t^*[\max[0,F_T-K]],\\
C^l(F,K,T,t) &= \max[\mathrm{E}_t^*[F_T]-K, B(t,T)\mathrm{E}_t^*[\max[0,F_T-K]]],\\
P^u(F,K,T,t) &= \mathrm{E}_t^*[\max[0,K-F_T]],\\
P^l(F,K,T,t) &= \max[K-\mathrm{E}_t^*[F_T], B(t,T)\mathrm{E}_t^*[\max[0,K-F_T]]].
\end{aligned}
$$

Given that option pricing errors vary according to moneyness, Melick and Thomas (1997) distinguish the convex combination for in-the-money from the one for out-of-the-money options. This yields

$$
\begin{aligned}
C_t(K) &= w_{it}C_t^u(K) + (1-w_{it})C_t^l(K) + \varepsilon_{1t}(K),\\
P_t(K) &= w_{it}P_t^u(K) + (1-w_{it})P_t^l(K) + \varepsilon_{2t}(K),
\end{aligned}
$$

where $i=1$ for a call option with $\mathrm{E}_t^*[F_T] > K$ or for a put option with $\mathrm{E}_t^*[F_T] < K$ and $i=2$ otherwise. For any given day, t, the additional parameters can be easily estimated using non-linear least squares, if enough strike prices are available. Because all the expectations figuring in the bounds involve the same RND, we are able to extend their method in a direct way to all the models considered in this study.

3.7. RND for standardized options

As we will show in the empirical section, options are subject to a term structure of volatility as well as to complex changes of higher moments as time to maturity evolves. This complicates the comparison of RNDs across time, because both changing expectations and the term structure of volatility may deform them.

Butler and Davies (1998) suggest linear interpolation as the preferred way to obtain information for options with a constant time to maturity. [13] Formally, let q_{t,T_1} and q_{t,T_2} be the RNDs, extracted at day t, for the maturities T_1 and T_2. They suggest to construct a RND with maturity date T ($T_1 < T < T_2$) using a convex combination of the extracted RNDs where the weighting depends on the relative position of T between T_1 and T_2. Hence,

$$q_{t,T}(F) = \frac{T_2 - T}{T_2 - T_1} q_{t,T_1}(F) + \frac{T - T_1}{T_2 - T_1} q_{t,T_2}(F) \quad \forall F.$$

In theory, more complex combinations can be considered such as a quadratic interpolation. It should be noticed that this method must be used with care for extrapolations since it may yield negative densities.

Abken et al. (1996) derive, within their Hermite polynomial approach, Eq. (5) that links the parameters for different maturities. [14] In theory, it is even possible to make inference for options with maturity above the longest existing maturity and similarly for those below the shortest maturity. For our data, we noticed that, for a given day, the left- and right-hand side of Eq. (5) took different values as maturity changed. This suggests that it is possible to improve the extrapolation by using several estimations simultaneously. Formally, let $\theta_{t,T} = (F_{t,T}, \sigma_{t,T}, \pi_{3t,T}, \pi_{4t,T})$ be the row vector of the forward price at time t for maturity date T, $T \in \{T_1, \dots, T_m\}$, and the three parameters characterizing the RND under the Hermite approximation. We then estimate the OLS regression $\theta_{t,T} = A_t + B_t T + \mathrm{E}_t$ where A_t, B_t are (1×4) row vectors of parameters and E_t is a residual. Once A_t and B_t are estimated, we forecast the parameters $\theta_{t,T'}$ for time T' using $\theta_{t,T'} = A_t + B_t T'$. With these parameters, it is possible to construct RNDs with fixed time to maturity, holding $T' - t$ constant as t varies.

4. Empirical results

4.1. Comparison of methods to extract RNDs

In this section, we compare the various methods along several lines. For practical purposes, the method should be fast, should take into account the fact that the data is noisy, and should give results that are stable, i.e., in the numerical estimations, there should be no difficulties in finding a global optimum.

In the preliminary step, we implemented all the methods by considering them as American options and as European options. In Appendix A, we

[13] Even though Butler and Davies (1998) construct RNDs with the MIX approach, their technique can be used for all the methods considered in this study.

[14] It is also possible to use data on many maturities and options and to formally test Eq. (5).

compare estimates obtained with and without the early exercise feature, focusing on two randomly chosen dates. We find that the parameter estimates are essentially identical whether we consider the early exercise-feature or not.[15] Incorporation of the early-exercise feature slows the estimation procedure significantly. Furthermore, since more parameters need to be estimated for American options, for many dates, where only a limited number of strike prices are available, it is not possible to obtain a RND. For these various reasons from now on we will report results obtained without the early exercise feature.

4.1.1. Implementation of the various methods

We implement the benchmark model, (LN), and the Hermite expansion, (HER), in a non-linear least squares (NLLS) framework by minimizing separately for each day and maturity

$$\sum_{i=1}^{m}(C_t(K_i) - \widehat{C}_t(K_i))^2, \qquad (10)$$

where C_t is the actual price of a call option and $\widehat{C}_t$ is the price given by one of the two models. This means that we first transform put option prices into call option prices using the put–call parity.[16] Also when a put and a call option have a same strike, we use the out-the-money option.

In order to implement the model involving a mixture of log-normal distributions (MIX), we add to Eq. (10) a Lagrangian penalty involving the distance between the actual futures price and the expected one. The maximum entropy method, (ME), was implemented following Buchen and Kelly (1996).

To check the robustness of our parameter estimates, we estimate for each date and maturity each method with a set of 20 different starting values obtained as random variates with enough standard deviation to insure coverage of a large spectrum of possible initial values.[17] Concerning the MIX model, we report the difficulty to find a global minimum.

4.1.2. The fit of the models

In Table 1, we compare the mean squared error (defined as $\text{MSE} = (10^4/(m-n))\sum_{i=1}^{m}(C_i - \widehat{C}_i)^2$, where n is the number of parameters involved in the method) and the average relative error (defined as $\text{ARE} = (1/(m-n))\sum_{i=1}^{m}((C_i - \widehat{C}_i)/C_i)^2$). The MSE is expected to be larger if the option prices are

[15] This finding corroborates anecdotal evidence whereby interest-rate future options get exercised only very rarely.

[16] To check for robustness of our results, we also used separately out-the-money put and call options and experimented with the various methods. We obtained results similar to the ones reported. This confirms the results reported in Appendix A.

[17] Computations were performed on a Pentium II computer running at 300 MHz.

Table 1
Comparison of MSE and ARE for various methods and PIBOR options[a]

Date T	m	MSE			ARE		
		LN	MIX	HER	LN	MIX	HER
970414							
63	5	1.109	0.322	0.147	0.065	0.002	0.006
154	13	0.789	0.376	0.045	0.009	0.118	0.006
245	15	1.219	0.342	0.032	0.008	0.004	0.001
336	12	1.662	0.394	0.253	0.009	0.003	0.004
970421							
56	8	4.782	0.579	0.045	0.018	0.001	0.000
147	12	10.039	0.016	0.005	0.049	0.002	0.000
238	15	3.786	3.649	0.217	0.005	0.002	0.003
329	12	2.439	1.789	0.162	0.001	0.061	0.000
970526							
21	11	6.329	0.903	0.076	0.047	0.003	0.001
112	15	5.959	1.178	0.071	0.132	0.004	0.000
203	13	3.875	0.300	0.034	0.030	0.014	0.000
294	16	3.299	0.190	0.019	0.040	0.017	0.000
970602							
14	11	1.614	0.066	0.016	0.256	0.011	0.004
105	15	9.504	0.871	0.236	0.281	0.004	0.001
196	15	4.937	11.649	0.022	0.041	0.006	0.000
287	15	3.540	8.516	0.015	0.022	0.009	0.000
970609							
98	13	10.116	3.291	0.447	0.375	0.033	0.003
189	14	4.276	1.101	0.146	0.049	0.000	0.002
280	16	4.389	0.614	0.063	0.039	0.006	0.000

[a] Let $C_i, \hat{C}_i$ be the actual and the theoretical option prices for a given maturity and strike price K_i, $i = 1, \ldots, m$, then $\mathrm{MSE} = (10^4/(m-n)) \sum_{i=1}^{m} (C_i - \hat{C}_i)^2$, where n is the number of parameters involved in a given estimation, and $\mathrm{ARE} = (1/(m-n)) \sum_{i=1}^{m} ((C_i - \hat{C}_i)/C_i)^2$, where n takes the values 1, 4, 3 for LN, MIX, and HER, respectively.

larger and allows for comparison across methods for a given type of option. The ARE further allows for comparison across data sets by relating the error to the size of the option prices.

Table 1 does not contain the errors for the ME method since, by construction, this method is an exact one and there is no error term. We notice in both tables that the benchmark model, (LN), provides only a very poor fit. This fit improves when we shift to the mixtures of log-normal model. When we turn to HER, we find an even better fit both in the MSE and the ARE.

To shed further light on the relative contributions of the methods, in Table 2 we consider the patterns of volatility, skewness, and excess kurtosis for the PIBOR options. We notice a very similar magnitude of volatility across

Table 2
Volatility, skewness, and excess kurtosis for various methods applied to PIBOR options[a]

Date T	LN			MIX			HER			ME		
	σ	SK	KU	σ	SK	KU	σ	SK	KU	σ	SK	KU
970414												
63	0.390	−0.146	0.038	0.434	−1.367	4.873	0.423	−1.485	3.960	0.344	−0.039	1.352
154	0.467	−0.267	0.127	0.506	−1.351	6.147	0.497	−1.001	3.334	0.479	−1.253	5.463
245	0.631	−0.442	0.348	0.665	−1.247	3.739	0.660	−1.175	2.922	0.670	−1.258	3.820
336	0.744	−0.564	0.472	0.743	−0.905	1.308	0.732	−1.113	1.375	0.721	−0.984	1.152
970421												
56	1.500	−0.477	0.392	1.523	−1.539	2.791	1.465	−1.727	1.905	2.422	−1.378	1.482
147	1.047	−0.537	0.461	1.064	−1.794	4.143	1.075	−1.833	3.405	1.167	−1.790	4.387
238	0.921	−0.580	0.471	0.914	−1.193	2.727	0.939	−1.277	3.409	0.925	−1.089	2.781
329	0.918	−0.588	0.303	0.844	−0.364	1.518	0.852	−0.455	1.763	0.899	−0.732	1.637
970526												
21	2.371	−0.450	0.350	2.430	−1.629	3.301	2.373	−1.727	2.547	2.398	−1.336	2.453
112	1.241	−0.532	0.426	1.260	−1.367	2.153	1.247	−1.506	2.041	1.287	−1.390	3.139
203	0.931	−0.536	0.427	0.949	−1.353	2.621	0.963	−1.407	2.520	0.956	−1.233	3.079
294	0.899	−0.566	0.345	0.881	−1.078	1.523	0.883	−1.141	1.564	0.873	−0.936	1.639
970602												
14	1.442	−0.235	0.098	1.524	−1.632	4.146	1.506	−1.460	2.838	1.407	−0.999	4.532
105	1.145	−0.507	0.437	1.201	−1.732	3.830	1.206	−1.877	3.605	1.243	−1.936	4.915
196	0.898	−0.538	0.470	0.924	−0.997	1.699	0.954	−1.596	3.698	0.917	−1.434	3.982
287	0.820	−0.567	0.460	0.828	−0.874	1.395	0.824	−1.285	2.378	0.813	−1.149	2.383
970609												
98	0.932	−0.406	0.294	1.037	−2.199	7.210	1.014	−2.077	4.749	1.109	−2.631	8.827
189	0.833	−0.502	0.434	0.880	−1.522	3.927	0.883	−1.581	3.487	0.888	−1.534	4.039
280	0.786	−0.557	0.482	0.794	−1.379	2.768	0.801	−1.425	2.673	0.797	−1.216	2.836

[a] The moments involved in this table were obtained by numerical integration from the various RNDs.

methods. Skewness and kurtosis have different magnitudes across methods but for a given date, the skewness and kurtosis variation pattern across maturities is broadly similar for the MIX, HER and ME methods.

Further inspection of Table 2 shows that on the *normal* day, April 14, implied volatility increases with maturity but that skewness as well as kurtosis decrease, except for the first maturity for ME. If we associate *general* economic uncertainty to volatility and *occurrence of an extreme variation of the underlying asset* to kurtosis, this suggests that a variation of interest rates, that might be considered extreme in the short run, actually becomes usual in the longer run.

For agitated days, we notice that volatility systematically decreases with maturity, suggesting that shocks may occur within a few days, but that in the longer run they would be offset. For most agitated days and most methods, we find that skewness and kurtosis also decrease.

Besides, we notice, for all days and maturities, that skewness is always negative (and more negative than for the LN-benchmark). This suggests that operators expected an increase in interest rates rather than a decrease. This also holds true for April 14, suggesting that there is a peso problem in interest rates as well.

We further compare the various methods by displaying the various RNDs for April 14 and May 26 in Figs. 3 and 4. Careful inspection of these figures reveals that whatever the date, or the method we always obtain a similar RND shape. However, in all methods, we notice a strong deviation from lognormality.

In some cases, the more general approaches (MIX, HER, and ME) produce rather close RNDs. This is the case, for instance, for the third maturity of April 14 or for the second maturity of May 26. Conversely, for the last maturity of April 14, these methods give RNDs with rather different patterns. Note that, even for this case, the volatility, skewness and kurtosis are close in the various methods.

To conclude this section, we recognize the difficulty to select a best method among the various methods considered here. The rather popular method involving a mixture of log-normals, that appears to do a good job for exchange rate data (see Jondeau and Rockinger, 2000), has difficulties to converge to a global minimum for interest-rate data: its residual error is larger than the HER method. The ME method sometimes gives rise to densities with kinks. We pursue our investigations with HER only, which appears to be a quick and numerically robust method.

4.2. Market operators' expectations through time

In this section, we consider the Hermite polynomial approximation and the entire database in order to investigate how the expectations of traders in

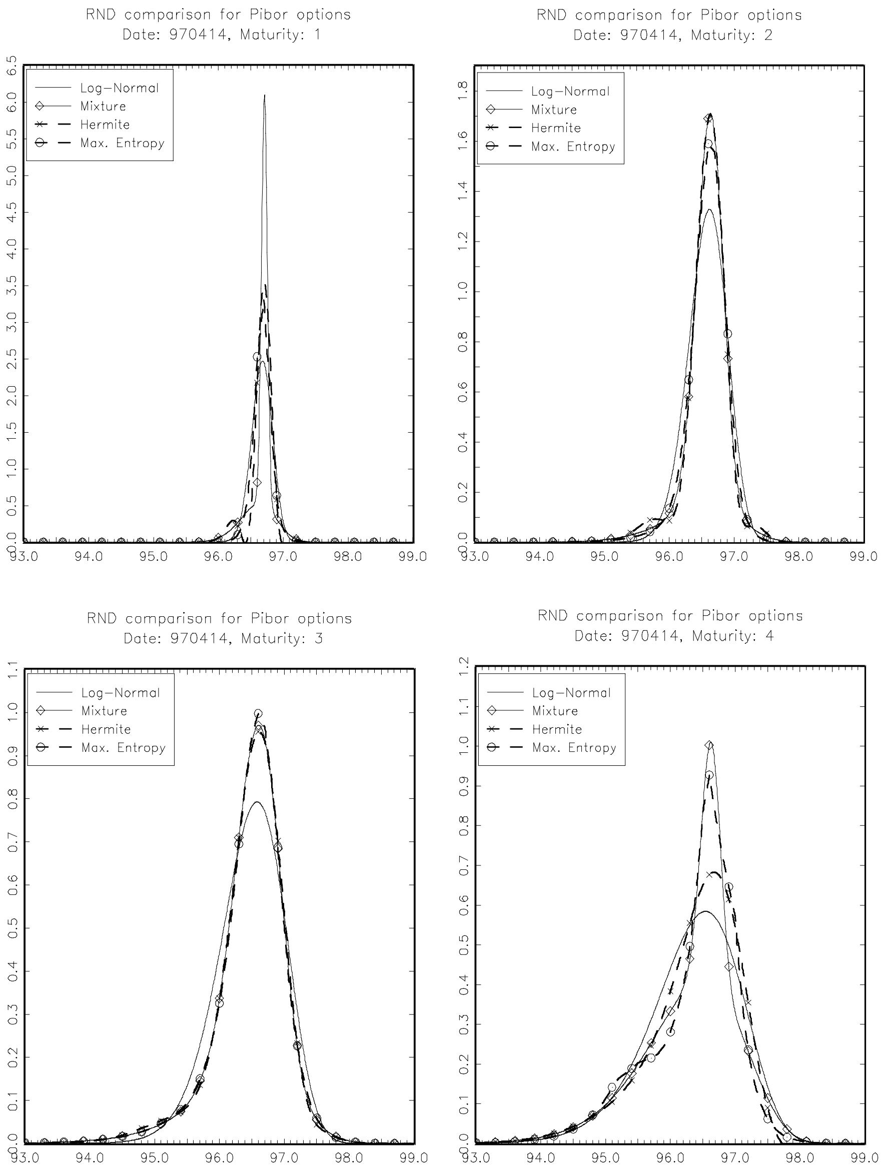

Fig. 3. RNDs for PIBOR options of several maturities on April 14, 1997, for the LN, MIX, HER, and ME approaches.

PIBOR options evolved through time. An event of great political importance occurs in our database, namely the 1997 snap election. For the UK, Gemmill (1992) investigated if the options market forecast an election outcome.

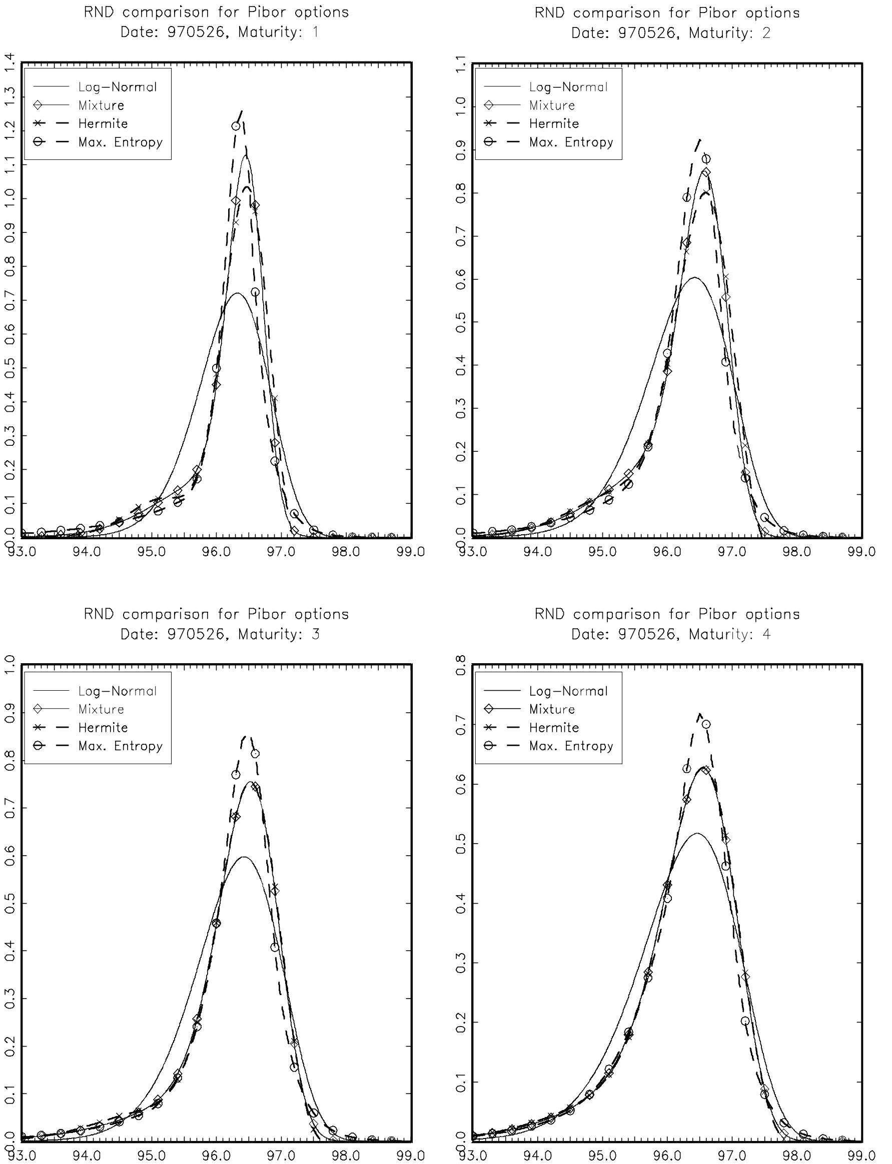

Fig. 4. RNDs for PIBOR options of several maturities on May 26, 1997, for the LN, MIX, HER, and ME approaches.

At this stage, we emphasize that we are interpreting moments derived from the RNDs as if they were obtained in the actual world. This is based on the remark by Rubinstein (1994) that: "… despite warnings to the contrary, we can justifiably suppose a rough similarity between the risk-neutral probabilities implied in option prices and subjective beliefs". Rubinstein justifies this with a numerical example. For the figures presented below this implies that any number should be considered as an approximation of the actual one. Thus, changes in the numbers are more informative than the actual levels.

Fig. 5 shows us how uncertainty of market operators evolved through time. We notice the relative calm in the market before April 14. Then volatility builds up in the week before the official announcement of the snap election occurred. After the official announcement on April 21, uncertainty remained constant. Then came the surprise on May 26 that the government might change. This uncertainty rose even further as polls revealed the possibility of a socialist victory. On June 2, it became clear that the socialists had won. As they held reassuring talks about their stance on the European Monetary Union and their general economic policy, markets calmed down rather quickly.

In Fig. 6, we turn to (the negative of) the price of skewness which further indicates that operators expected directional moves of interest rates. Clearly this measure has greater variability than volatility. After the official announcement, an increase in the short-term rate became more likely. But interestingly, even after the election and the sharp decrease in volatility, the price

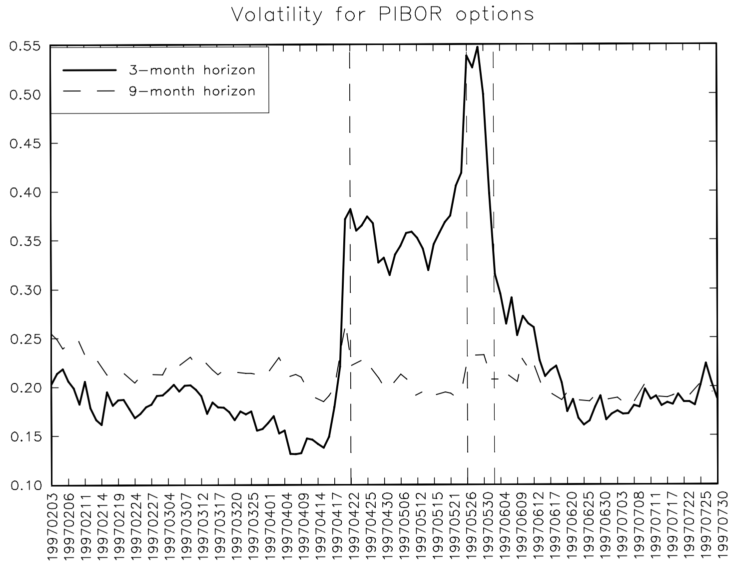

Fig. 5. Volatility for PIBOR options estimated with the HER model, from February 1997 to July 1997. A solid curve corresponds to the standardized 3-month maturity, the dashed curve corresponds to the standardized 9-month maturity. Vertical lines correspond to April 21, May 26, and June 2.

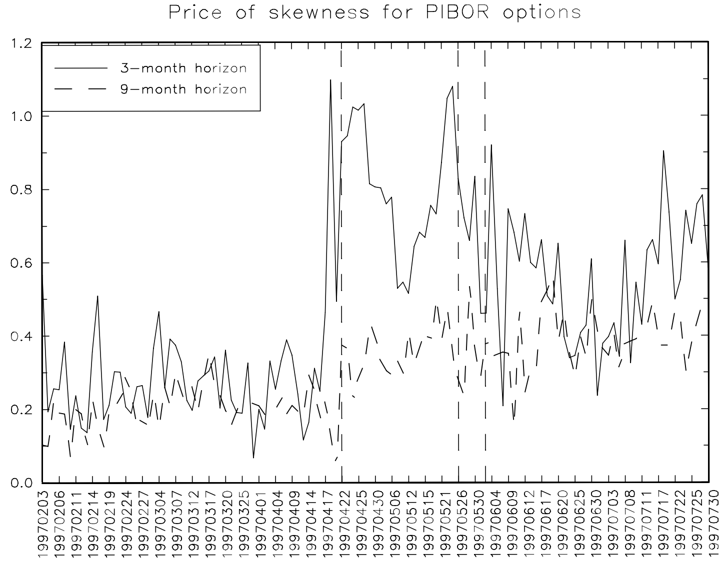

Fig. 6. Price of skewness of RNDs for PIBOR options estimated with the HER model, from February 1997 to July 1997. Curves and vertical lines correspond to April 21, May 26, and June 2.

of skewness remained rather high, indicating expectations of directional moves of interest rates.

In Fig. 7, we investigate the evolution through time of the price of kurtosis. Comparison with volatility, displayed in Fig. 5, shows that there is a kind of tradeoff between kurtosis and volatility. Dates distant from the election period had lower volatility but higher kurtosis and vice versa during the election period. Also, the trend towards higher skewness after the elections reveals that under the new government, traders were worried about variations towards abnormal levels of interest rates. During the election, global uncertainty increased and decreased after. The higher level of skewness towards the end of our sample suggests that, even by the end of July 1997, the new government was unable to dissolve all fears.

It is possible to analyze this information in an alternative manner by considering confidence intervals computed with RNDs. Let b_i and b_s be the 5- and 95-percentiles of a RND. We define a $b_s - b_i$ as *absolute deviations.* We also obtain percentage deviations with

$$b_{\text{inf}} = 100\left(\frac{F}{b_i} - 1\right), \quad b_{\text{sup}} = 100\left(\frac{b_s}{F} - 1\right), \quad \text{Range} = 100\left(\frac{b_s - b_i}{F}\right).$$

Percentage deviations are relative to the forward price and are indicative of the side towards which the RND tends.

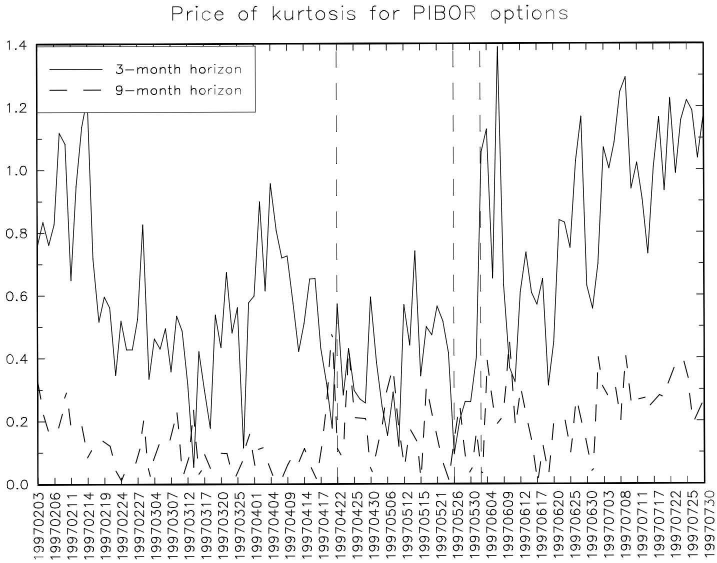

Fig. 7. Price of kurtosis of RNDs for PIBOR options as estimated by the HER approach, from February 1997 to July 1997. Curves and vertical lines correspond to April 21, May 26, and June 2.

Also, as discussed in Section 3.7, the availability of options with several maturities allows the construction of standardized options, i.e., options with a fixed time to maturity. We focus now on options arbitrarily standardized for 3 months (90 days) and 9 months (270 days). Table 3 displays confidence intervals as percentage deviations and as absolute deviations. Figs. 8 and 9 display daily confidence intervals. To make results as easy to interpret as possible, we transform the $100 - F$ quote into a forward rate F.

For the normal date, we find that for the 90 days to come, operators believed that with a 90% probability, interest rates would not go under 3.1% nor above 3.9%. This range increases substantially for the 9-month standardized option.

As news hit the market by April 21, the widening of the confidence intervals suggests fears of large movements of rates. These fears were the greatest after the first round of elections. One week after the second round, uncertainty had decreased but was still high.

Fig. 8 gives a more intuitive picture of this evolution. We notice that operators put a lower bound on rates ranging between 2.8% and 3.1%. Even though the forward rate does not move very much, we see huge variations in the upper bound. This is suggestive of fears of an increase in interest rates. After the elections, the upper bound decreases, however it remains at a high

Table 3
HER risk neutral confidence intervals for standardized PIBOR options[a]

Date T	F	Percentage deviation			Absolute deviation		
		b_{inf}	b_{sup}	Range	b_i	b_s	$b_s - b_i$
970414							
90	3.35	8.25	15.64	23.27	3.10	3.88	0.78
270	3.57	26.16	31.20	51.93	2.83	4.68	1.85
970421							
90	3.68	24.61	40.64	60.39	2.95	5.17	2.22
270	3.69	41.39	36.38	65.66	2.61	5.03	2.42
970526							
90	3.78	35.13	48.77	74.77	2.80	5.62	2.83
270	3.77	30.31	37.08	60.34	2.90	5.17	2.28
970602							
90	3.62	25.25	41.68	61.85	2.89	5.13	2.24
270	3.67	30.48	40.08	63.44	2.81	5.13	2.33
970609							
90	3.58	16.89	35.47	49.92	3.06	4.85	1.79
270	3.62	30.73	39.62	63.13	2.77	5.06	2.29

[a] Confidence intervals for the PIBOR forward rate. Let F be the forward rate, b_i and b_s the 5- and 95-percentiles of the RND, then $b_{\mathrm{inf}} = 100(F/b_i - 1)$, $b_{\mathrm{sup}} = 100(b_s/F - 1)$, Range $= 100(b_s - b_i)/F$. Confidence intervals are constructed for standardized options with 90 and 270 days to maturity.

level. This figure also confirms that the market knew about the election before the official announcement occurred.

Fig. 9 displays the confidence intervals for a 9-month horizon. We notice a slight widening from below and a large widening from above of the confidence interval. The interval has widened over the sample and shifted upwards.

This suggests that investigating RNDs can shed additional light on how financial markets operate and that they contain information that is not contained in traditional instruments such as a term structures of forward rates.

5. Conclusion

In this paper, we investigated the information contained in PIBOR futures options during a period when financial markets were subject to great political uncertainty, namely the 1997 snap election.

We first considered several methods for extracting risk neutral densities. Whatever the method, we obtained a better fit to options' prices than with

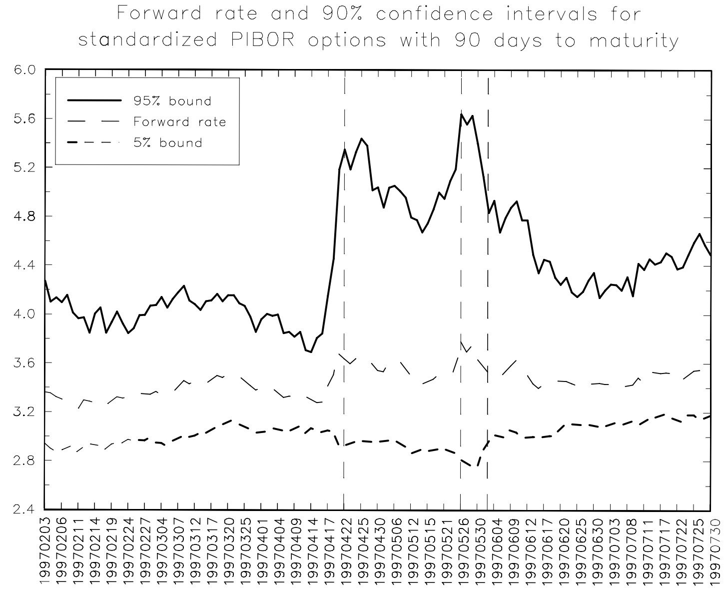

Fig. 8. Forward rate and 90% confidence intervals for PIBOR options obtained with the HER approach, from February 1997 to July 1997. The RNDs are calibrated to a standardized 3-month maturity. Vertical lines correspond to April 21, May 26, and June 2.

the traditional benchmark log-normal distribution. Eventually, we settled for an approximation based on Hermite polynomials developed originally by Madan and Milne (1994). This method yields accurate estimates and is able to deal with somewhat dirty data. It is numerically fast and stable. Also, this method can be used to obtain standardized options, i.e., with a fixed time till maturity.

Then we applied this method to data ranging from February 3, to July 30, 1997. For the options at hand, we found that the market anticipated an important event before the official announcement occurred. As polls confirm, a possible change of government before the first electoral round increased uncertainty. After the second round of the elections, even though uncertainty about the future decreased, fears of interest rate increases persisted. One rationale is that there exist several levels of uncertainty. At a first level there is the uncertainty about which political party is going to win and at an other level there is the program that the parties plan to implement. Right after the elections the uncertainty concerning the political party diminished but there remained the uncertainty about the future political program. By the end of July 1997, we found that the new government had not yet reassured fully the financial market about its intentions.

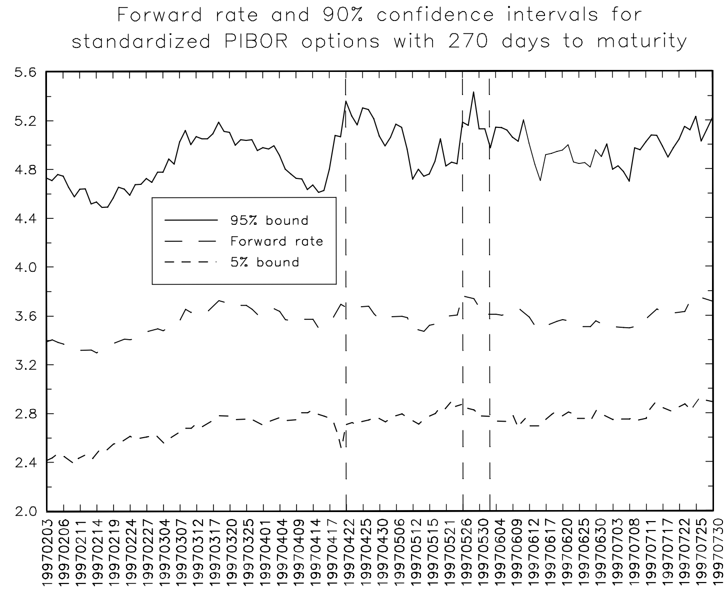

Fig. 9. Forward rate and 90% confidence intervals for PIBOR options obtained with the HER approach, from February 1997 to July 1997. The RNDs are calibrated to a standardized 9-month maturity.

Acknowledgements

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Appendix A

In this section, we show that parameter estimates obtained for the two selected dates are essentially identical whether we consider European-style option prices or American-style option prices. For each date and each maturity, Tables 4–6 report for various methods the parameter estimates and sum of

Table 4
Benchmark model (LN)

Date T	m	Parameter estimates		SSR_1	SSR_2
		γ	w_1		
970414					
63	5	0.1171		1.0464	4.4334
		0.1171	0.9995	1.0464	4.4334
154	13	0.1364		2.5955	9.4630
		0.1364	1.0000	2.5955	9.4630
245	15	0.1779		0.6665	17.0046
		0.1779	1.0000	0.6665	17.0046
336	12	0.2023		0.2626	18.1237
		0.2023	1.0000	0.2626	18.1237
970526					
21	11	0.6173		0.9346	66.7707
		0.6173	0.9998	0.9346	66.7708
112	15	0.3284		2.2856	83.3404
		0.3284	1.0000	2.2856	83.3404
203	13	0.2467		0.8774	46.4250
		0.2467	1.0000	0.8774	46.4250
294	16	0.2382		1.2828	49.1399
		0.2382	1.0000	1.2828	49.1399

Table 5
Mixture of log-normal distributions (MIX)

Date T	m	Parameter estimates						SSR_1	SSR_2
		α_1	μ_1	μ_2	σ_1	σ_2	w_1		
970414									
63	5	0.6381	1.1905	1.2211	0.0333	0.1970		0.0000	0.0000
		0.6392	1.1905	1.2211	0.0333	0.1970	0.5003	0.0000	0.0000
154	13	0.7969	1.2174	1.2563	0.0926	0.2525		0.1416	0.3365
		0.7969	1.2174	1.2563	0.0926	0.2525	0.9999	0.1416	0.3365
245	15	0.5250	1.2734	1.2245	0.2290	0.0919		0.0434	0.0521
		0.5250	1.2734	1.2245	0.2290	0.0919	0.9999	0.0434	0.0521
336	12	0.7710	1.2975	1.2174	0.2202	0.0461		0.0109	0.3164
		0.7581	1.2973	1.2235	0.2202	0.0497	0.4269	0.0107	0.3108
970526									
21	11	0.7068	1.2737	1.4462	0.3210	0.7633		0.0073	1.2082
		0.7068	1.2737	1.4462	0.3210	0.7633	0.9999	0.0073	1.2082
112	15	0.7068	1.2366	1.4816	0.1912	0.3321		0.0367	1.2864
		0.7249	1.2401	1.4891	0.1940	0.3313	0.0001	0.0361	1.2586
203	13	0.8106	1.2578	1.5164	0.1762	0.2936		0.0019	0.2784
		0.8286	1.2622	1.5240	0.1769	0.3000	0.0001	0.0009	0.2002
294	16	0.8249	1.2560	1.5639	0.1786	0.2386		0.0095	0.2243
		0.8249	1.2560	1.5639	0.1786	0.2386	0.9999	0.0095	0.2243

Table 6
Hermite polynomial expansion (HER)

Date T	m	Parameter estimates				SSR_1	SSR_2
		σ	π_3	π_4	w_1		
970414							
63	5	0.1233	0.4787	0.7193		0.0485	0.2909
		0.1231	0.4776	0.7226	0.5001	0.0489	0.2925
154	13	0.1419	0.1610	0.5230		0.2626	0.4410
		0.1419	0.1610	0.5230	0.9999	0.2626	0.4411
245	15	0.1798	0.1731	0.3078		0.0286	0.3589
		0.1798	0.1731	0.3078	0.9999	0.0286	0.3589
336	12	0.1912	0.2462	0.0075		0.0890	2.1988
		0.1912	0.2462	0.0076	0.9999	0.0890	2.1988
970526							
21	11	0.5738	0.5758	0.2265		0.0072	1.4428
		0.5738	0.5758	0.2265	0.9999	0.0072	1.4429
112	15	0.3074	0.4616	0.0635		0.0157	1.3301
		0.3074	0.4616	0.0635	0.9999	0.0157	1.3301
203	13	0.2447	0.3180	0.2413		0.0036	0.4768
		0.2433	0.3051	0.2774	0.0001	0.0027	0.4022
294	16	0.2308	0.2387	0.1415		0.0105	0.4076
		0.2308	0.2387	0.1415	0.9999	0.0105	0.4076

squared residuals for two methods: first, the model is estimated assuming European-style options; second, the model is estimated assuming American-style options. The early exercise feature is taken account off with the Melick and Thomas (1997) device (e.g., Section 3.6). The sum of squared residuals is defined as $SSR_1 \equiv \sum_{i=1}^{m}(C_i - \hat{C}_i)^2$ and in percentage terms as $SSR_2 \equiv \sum_{i=1}^{m}((C_i - \hat{C}_i)/\hat{C}_i)^2$.

In the benchmark model (LN), we estimate the volatility, γ. In the mixture of log-normal distributions (MIX), we estimate the weight, α_1, the annualized means, μ_1 and μ_2, and the annualized volatilities, σ_1 and σ_2. Last, in the Hermite polynomial expansion (HER), we estimate the volatility σ, the price of skewness, π_3, and the price of kurtosis, π_4. Note that when $w_1 = 1$, the American-style option price is equal to the European-style option price.

In most cases, we obtain that the parameter estimates are equal whether options are assumed American-style or European-style ($w_1 = 1$). For the first maturity of April 14, results are not comparable for the MIX and the HER approaches, because there are not enough strikes. Even when we obtain an estimate of w_1 different from 1, parameter estimates obtained assuming American-style options are close to parameter estimates obtained assuming European-style options. Moreover, pricing errors are very close to each other.

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