Forecasting French and German Long-Term Rates Using a Rational Expectations Model

By

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I. Introduction

According to the expectations hypothesis of the term structure (EH), the long-term rate is equal to the weighted average of expected short-term rates over the life of the bond, plus possibly a constant risk premium. This framework has been used in many different ways for empirical purposes, such as extracting information from market expectations concerning future changes in interest rates or changes in inflation (Söderlind and Svensson 1997). More recently implications of the EH have been used in many macroeconomic models to represent the dynamics of the long-term interest rates (see for instance Brayton and Tinsley 1996 for the FRB/US model developed at the Fed or Côté and Macklem 1995 for the QPM model developed at the Bank of Canada). Since future short-term rates are unobserved, their expected path is generally modeled through a monetary reaction function.

The implications of the EH, however, have long been contested by empirical work. The EH has been rejected on the basis of US data both for long-term interest rates (see Shiller et al. 1983; Campbell and Shiller 1991) and for short-term interest rates (see Mankiw and Miron 1986; Evans and Lewis 1994).[1] This failure has been interpreted as the result

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[1] The expectations hypothesis has not been rejected using data from other countries for long-term rates (Hardouvelis 1994; Gerlach 1996) or short-term rates (Gerlach and Smets 1997 for an international comparison; Jondeau and Ricart 1996 for French data).

of the Fed's high degree of credibility (see Mankiw and Miron 1986; Rudebusch 1995; Balduzzi et al. 1998). If the monetary authorities act in a way that is deemed to be credible, private agents will expect short-term interest rates to remain stable at their current levels. The term spread would therefore not contain any information about the future rates. Hence, it would not be able to improve long-term interest rate forecasts, leading to a rejection of the EH.

The way the short-term rate is modeled appears to be crucial. In empirical work, a first approach consists in using a univariate forecasting model. However, the terminal boundary of such a model generally does not appear relevant. In fact, for a stationary representation, the long-run level of the target is constant over time. Conversely, in a nonstationary approach, the terminal boundary changes with the last observation of the short-term interest rate. In both cases, the pattern of the terminal boundary is unsatisfactory and other alternatives have to be examined.

The estimation of a monetary reaction function is one of the lines of research that has been followed. For instance, Rudebusch (1995) proposes different models for the Fed reaction functions. Using simulations, he shows that these specifications make the EH consistent again. Fuhrer (1996) highlights that, using time-varying weights for the main targets of the central bank (inflation and growth) provides an estimated long-term rate based on the EH that is close to the observed long rate.

In this paper, we study another approach, in which the short-term rate dynamics depends on a long-run time-varying boundary value (also called a moving endpoint). This endpoint is supposed to reflect the ex-ante expectations of private agents. This approach relies upon the fact that the long-run expectations of the short-term rate are important for the determination of the long-term rate. Then, such market expectations make it possible to circumvent the estimation of a reaction function in order to forecast long-run interest rates within the EH framework. This way of modeling interest rates has already been implemented at the Fed (see Kozicki et al. 1995; Kozicki and Tinsley 1996) but the technique has not yet been applied to European data.

We compare the ability of alternative models to forecast French and German long-term rates over the period 1960–1996. The remainder of the paper is organized as follows: In Section II, we briefly develop the theoretical underpinnings of the EH. In Section III, we study different univariate forecasting models for the short-term rate and we shed some light on the implications of both stationary and nonstationary short-term models for forecasting long-term rates. In Section IV, we model the short-term rate using a moving endpoint approach. This endpoint, which embodies the expectations of private agents, is estimated using a long-horizon forward rate. In Section V, we check the predictive ability of these different models and we compare them to a usual "benchmark", the bivariate VAR model developed by Campbell and Shiller (1987, 1988). Section VI gives the main conclusions.

II. The Expectations Hypothesis

The expectations hypothesis relies on the joint hypothesis of no arbitrage opportunities and rational expectations. Accordingly, the term structure is given by:

$$R_{z,t} = \frac{1}{n} \sum_{i=0}^{n-1} E_t\, r_{t+i} + \varphi_z\,, \qquad (1)$$

where $R_{z,t}$ denotes the yield to maturity of the n-period zero-coupon bond at time t; r_t is the one-period rate; the risk premium φ_z is constant over time but can be maturity-dependent. $E_t x_{t+i}$ denotes the linear projection of x_{t+i} on the information set available at time t, noted Ω_t, that is $E_t x_{t+i} = E(x_{t+i} \mid \Omega_t)$.

In the case of a coupon-bearing bond, the long-term rate is a weighted average of expected future short-term rates (Shiller 1979):

$$R_t = \frac{1-\beta}{1-\beta^n} \sum_{i=0}^{n-1} E_t\, \beta^i\, r_{t+i} + \varphi, \qquad (2)$$

where $\beta = 1/(1+\bar{R})$ is the discount factor, and $\bar{R}$ can be evaluated as the sample average of the long-term rate. The risk premium φ is supposed to be constant over time.

We note that in the definition of the long-term rate in (1), expected short-term rates for the far horizon (say $t+n-1$) have the same weight as current short-term rates. In the case of a coupon-bearing bond (equation (2)), the weights given to future rates decrease exponentially with the maturity. However, the lower the average rate $\bar{R}$, the slighter the decrease in these weights.[2] The choice of a relevant forecasting model and, more specifically, the choice of an adapted long-run boundary for the short-term rate is crucial.

[2] For instance, for $\bar{R} = 7.5\%$ (average of the German rates over the period), the weight given to the short-term rate at time $t+i$, r_{t+i}, is 0.074 for $i=0$, 0.017 for $i=20$, and 0.004 for $i=40$ using quarterly data.

Many studies tested the EH on the basis of (1) or (2), using realized short-term rates rather than expected rates, assuming that agents are rational and then that the forecasting error is white noise. However, this assumption appears to be too strong to explain the movement of long-term rates in some countries (see, e.g., Hardouvelis 1994). Indeed, the result generally obtained is that the theoretical long-term rate (substituting expected short-term rates for realized rates into relation (2) is too smooth compared to the realized long-term rate. This excess volatility in long-term rates relative to the predictions of the EH has been highlighted by Shiller (1979), among others. The EH can be weakened, assuming purely autoregressive dynamics for the short-term rate. In this univariate approach, the information set is restricted to past short-term rates only.

III. The Univariate Approach

This approach however is not without drawbacks. The main difficulty lies in characterizing the long-run behavior of interest rates. This is because a univariate forecast of the short-term rate implies a choice between a level specification (stationary representation) and a first-difference specification (nonstationary representation). Most of the theoretical models (as in Vasicek 1977 and in Cox et al. 1985) assume stationary interest rates, whereas empirical studies generally conclude that interest rates are nonstationary (Campbell and Shiller 1987; Hall et al. 1992). We outline both representations in this section and focus on their relevance for French and German data. Appendix 1 provides some technical details in computing long-term rate forecast using these models. It is noteworthy that both models cannot be presented in a unified approach.

1. The Stationary Representation

The univariate stationary (or mean-reverting) representation can be written as:

$$b(L)\Delta r_t = \mu_1 + \alpha r_{t-1} + e_t, \tag{3}$$

where $\alpha < 0$ and $b(L)$ a $(p-1)$-order polynomial in the lag operator L.

First, we note that the autoregression (3) can be rewritten as the first-order companion system: $z_t = A z_{t-1} + \mu + \upsilon_t$, where the p-vector $z_t = (r_t \dots r_{t-p+1})'$ summarizes the agents' information set, $\mu = (\mu_1\ 0 \dots 0)'$ and $\upsilon_t = (e_t\ 0 \dots 0)'$. A is the (p,p) companion matrix of the VAR.

We define h_r the $(p, 1)$ vector selecting the current short-term rate in z_t, such that $r_t = h_r' z_t$. The multiperiod forecast for the short-term rate r_t is given by:

$$E[r_{t+i} \mid I_{t-1}] = h_r'\rho + h_r'A^{i+1}[z_{t-1} - \rho], \tag{4}$$

where $\rho = (Id_p - A)^{-1}\mu$. $I_t = \{r_t, \ldots, r_{t-p+1}\}$ is the information set (included in Ω_t) available to the econometrician, restricted to present and past interest rates. Id_p denotes the (p, p) identity matrix.

Equation (4) shows that the short-term rate tends to the following endpoint: $\lim_{i\to\infty} E[r_{t+1} \mid I_{t-1}] = h_r'\rho \equiv r^{(\infty)}$, when the horizon goes to infinity. It is noteworthy that the short-term rate endpoint, $r_{t-1}^{(\infty)}$, is constant over time. It is simply equal to the sample mean of the short-term rate.

Now substituting (4) into the term structure relation (2) yields the following one-period optimal forecast of the long-term rate consistent with the EH:

$$\begin{aligned} R_{t/t-1} &\equiv E[R_t \mid I_{t-1}] = \frac{1-\beta}{1-\beta^n} \sum_{i=0}^{n-1} \beta^i E[r_{t+i} \mid I_{t-1}] + \varphi \\ &= h_r'\rho + \theta'[z_{t-1} - \rho] + \varphi \end{aligned} \tag{5}$$

where

$$\theta' = \frac{1-\beta}{1-\beta^n} h_r'(Id_p - (\beta A)^n)(Id_p - \beta A)^{-1} A.$$

Once model (3) has been estimated, short-term rate multiperiod forecasts are given by (4), and (5) yields the expression of the term structure for a stationary model consistent with the EH. The forecast of the long-term rate k periods ahead is given by:

$$R_{t+k/t-1} = h_r'\rho + \theta' A^k [z_{t-1} - \rho] + \varphi.$$

So the further away the forecast horizon, the closer the forecast is to the short-term rate endpoint, $h_r'\rho = r^{(\infty)}$, apart from the risk premium φ.

2. The Nonstationary Representation

The nonstationary representation for the short-term rate is written as an autoregressive model for the differenced short-term rate:

$$b(L)\Delta r_t = \mu_1 + e_t, \tag{6}$$

where $b(L)$ a $(p-1)$-order polynomial in the lag operator. As shown in Appendix 1, the first-order companion form of (6) is then expressed as:

$\Delta z_t = \tilde{A}\Delta z_{t-1} + \mu + v_t$. The short-term rate multiperiod forecast is given by:

$$E[r_{t+i} \mid I_{t-1}] = h_r' z_{t-1} + h_r'(Id_{p-1} - \tilde{A}^{i+1})(Id_{p-1} - \tilde{A})^{-1} \cdot \tilde{A}(\Delta z_{t-1} - \tilde{\rho}) + (i+1) h_r' \tilde{\rho}\,, \qquad (7)$$

where $\tilde{\rho} = (Id_{p-1} - \tilde{A})^{-1} \mu$. We assume in the following that $\mu_1 = 0$ and then $\tilde{\rho} = 0$. This assumption ensures that the forecast rate for an infinite horizon does not tend to infinity ($-\infty$ if $\mu_1 < 0$ or $+\infty$ if $\mu_1 > 0$).[3]

The endpoint for the short-term rate is now time-varying, since it can be expressed as a moving average of the most recent short-term rates: $\lim_{i \to \infty} E[r_{t+i} \mid I_{t-1}] = h_r' z_{t-1} + h_r'(Id_{p-1} - \tilde{A})^{-1} \tilde{A}\Delta z_{t-1} \equiv r_{t-1}^{(\infty)}$.

As in the stationary case, the optimal long-term rate forecast consistent with the EH is now:

$$R_{t/t-1} = h_r' z_{t-1}^{(\infty)} + \tilde{\theta}'[z_{t-1} - z_{t-1}^{(\infty)}] + \varphi\,, \qquad (8)$$

where $z_{t-1}^{(\infty)} = z_{t-1} + (Id_{p-1} - \tilde{A})^{-1} \tilde{A}\Delta z_{t-1}$ and

$$\tilde{\theta}' = \frac{1-\beta}{1-\beta^n} h_r'(Id_{p-1} - (\beta\tilde{A})^n)(Id_{p-1} - \beta\tilde{A})^{-1}.$$

The multiperiod forecast is now written as:

$$R_{t+k/t-1} = h_r' z_{t-1}^{(\infty)} + \tilde{\theta}'\tilde{A}^k[z_{t-1} - z_{t-1}^{(\infty)}] + \varphi\,.$$

So contrary to the stationary case, the short-term rate endpoint and the long-term rate forecast are directly related to the most recent short-term rates.

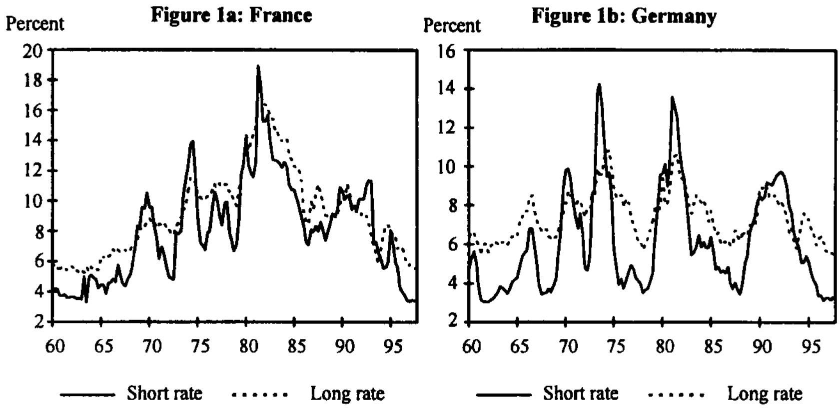
3. Using Univariate Models to Forecast the Long-Term Rate

We use quarterly data over the period from 1960 to 1996. The short-term rate is the 3-month interbank rate. The long-term rate is the average yield on long-term public bonds: for France, the average yield on public sector and semi-public sector bonds; for Germany, the average yield on 7- to 15-year public bonds. The data are drawn from the OECD database. Figures 1a and 1b display the 3-month rate and the 10-year rate for France and Germany, respectively.

Table 1 reports the estimates of the stationary model (3) and the nonstationary model (6). The autoregressive lag number is selected with

[3] The estimates show that μ_1 is not significantly different from zero (cf. Table 1).

Figure 1 – *Interest Rates*



the BIC criterion. Both approaches provide similar results. This is because α is generally close to 0 (or, alternatively, the root of the autoregressive process is close to 1).[4] However, the long-run forecasts of the short-term rates obtained from both models differ dramatically. In the first case, the short-term rate forecast tends to the average $-\mu_1/\alpha$. In the second case, the short-term rate forecast essentially moves like the most recent rates. Specification tests indicate that residuals are serially uncorrelated (LM test) and homoskedastic (ARCH-effect test). However the normality restriction is clearly violated (Jarque-Bera test).

Figures 2a and 2b show the long-term rate and its 3-month forecast using the stationary model for France and Germany. Similarly, Figures 3a and 3b display the long-term rate and its forecast in the nonstationary model. The long-term rate forecasts appear to be too smooth in the stationary case, but too volatile in the nonstationary case. The underlying endpoint does not seem to reflect market expectations. Indeed, the revision of expectations from one period to the next is too weak or too volatile, except for the stationary model in Germany.

For both countries, Figures 2 and 3 give different interpretations. In the case of France, the choice of a stationary model for the short-term rate is clearly misleading, since it gives large over-estimates of long-term rates (for the first 10 years of the sample) or large under-estimates

[4] Using relation (3) of Table 1, the t-statistics for α are –2.5 for France and –3.3 Germany, which can be compared to –2.9 at the 5 percent significance level and –2.6 at the 10 percent significance level (Fuller 1976). DF tests then indicate that we cannot reject nonstationarity for France, but we clearly reject nonstationarity for Germany.

Table 1 – *Univariate Models of the Short-Term Rate, 1960–1996*

	France		Germany	
	Stationary model (equation (3))			
μ_1	0.5927	(0.256)	0.5486	(0.181)
α	–0.0717	(0.029)	–0.0890	(0.027)
b_1	0.1916	(0.083)	0.4443	(0.076)
$\bar{R}^2$	0.052		0.207	
see	1.102		0.837	
		p-value		p-value
LM(12)	2.001	(99.9%)	10.223	(59.6%)
ARCH(12)	3.433	(99.2%)	13.372	(34.3%)
JB	452.9	(0%)	735.1	(0%)
	Nonstationary model (equation (6))			
μ_1	–0.0016	(0.094)	–0.0059	(0.072)
b_1	0.1551	(0.083)	0.3966	(0.077)
$\bar{R}^2$	0.017		0.152	
see	1.122		0.865	
		p-value		p-value
LM(12)	4.932	(96.0%)	12.138	(43.5%)
ARCH(12)	4.307	(97.7%)	21.874	(3.9%)
JB	304.4	(0%)	370.2	(0%)
	Moving endpoint model (equation (9))			
μ_1	–0.0705	(0.091)	–0.0423	(0.070)
α	–0.2332	(0.061)	–0.1352	(0.036)
b_1	0.2568	(0.084)	0.3862	(0.080)
b_2	-		0.1949	(0.086)
$\bar{R}^2$	0.103		0.225	
see	1.072		0.827	
		p-value		p-value
LM(12)	2.423	(99.8%)	6.265	(90.2%)
ARCH(12)	4.237	(97.9%)	17.729	(12.4%)
JB	205.1	(0%)	455.5	(0%)

Note: The lag number of the autoregressive polynomial is selected to minimize the BIC criterion. LM(12) represents the LM statistics for serial correlation obtained by regressing residuals on 12 lags. ARCH(12) represents the ARCH statistics for heteroskedasticity obtained by regressing squared residuals on 12 lags. Under their respective null hypothesis, these statistics are distributed as a $\chi^2(12)$. JB is the Jarque-Bera statistics for normality. Under the null hypothesis, this statistics is distributed as a $\chi^2(2)$.

Figure 2 – *Long-Term Rate Forecast Using a Stationary AR for the Short-Term Rate*

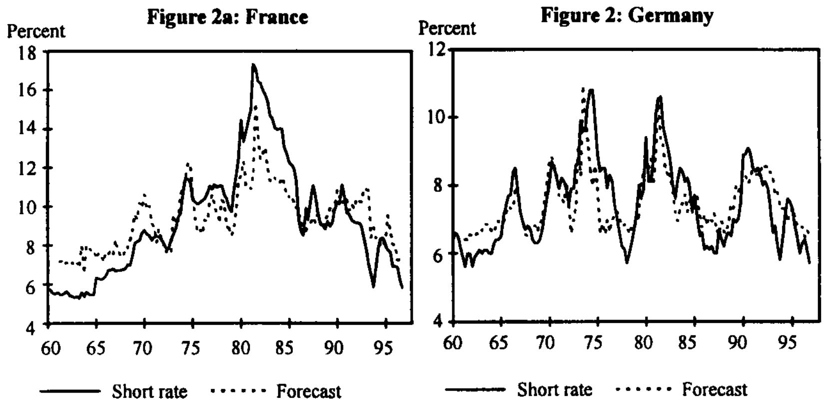
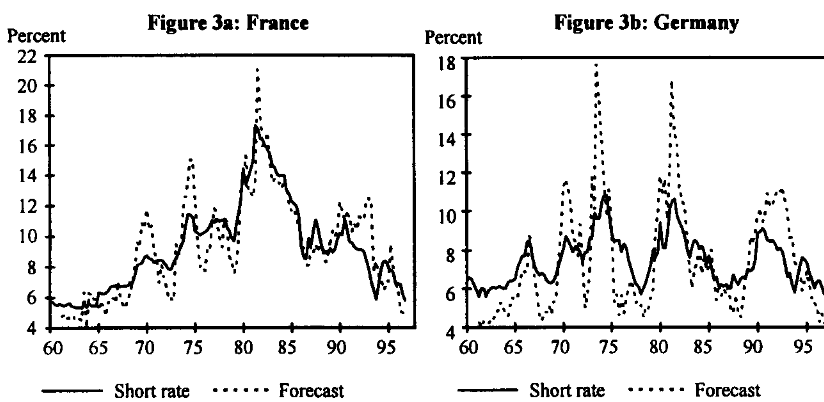


Figure 3 – *Long-Term Rate Forecast Using a Nonstationary AR for the Short-Term Rate*



(between 1974 and 1985). On the contrary, the nonstationary model appears more suitable for explaining long-term rate movements. For Germany, the nonstationary specification for the short-term rate leads to movements in the long-term rate that are too exaggerated during monetary policy tightening (in 1973–74 and 1980–82 in particular), whereas the stationary process tracks the observed long-term rate more closely. This outcome is clearly consistent with the results of the tests briefly shown in Footnote 4.

The lack of an anchor for the expected short-term rate in the long run is the main drawback for both univariate specifications in this section. The stationary approach has a constant endpoint, clearly not suited to French data whereas the endpoints of the nonstationary approach are too volatile. It therefore seems necessary to adopt an alternative endpoint representation for the short-term rate.

IV. An Approach Based on Market Expectations

Using a short-term rate endpoint based on market expectations is a way to address most of the difficulties raised above. This formulation has two main advantages. First, it is no longer necessary to stipulate whether interest rates are stationary or nonstationary, since the long-run dynamics is defined by the endpoint. Second, it avoids estimating a reaction function and then selecting macroeconomic fundamentals: all the information contained in the fundamentals is supposed to be summarized in the market-expectations variable.

This approach can be related to the literature on factor models of the term structure. As a first step, the term structure was described using one factor only, the short-term rate (Vasicek 1977; Cox et al. 1985). These models were rapidly extended to two factors. Several definitions of the second factor have been proposed: the long-term rate (Brennan and Schwartz 1982), the term spread (Schaefer and Schwartz 1984) or the conditional volatility of the short-term rate (as in Longstaff and Schwartz 1992).

Campbell and Shiller (1987, 1988) have studied the joint dynamics of the short-term and the long-term rates. In particular, they consider the effect of the nonstationarity of the short-term rate on this relation. They show that, in this case, the long-term rate is nonstationary too, but the term spread is stationary. They then study a model with two factors, the short-term rate and the term spread. When they use a bivariate VAR model of the change in the short-term rate and the spread to forecast the long-term rate, they always reject the EH. But the realized and estimated long-term rates move very closely, indicating that the EH may be pertinent from an economic point of view. In the next section, we will compare our results to those obtained with a bivariate VAR model. Balduzzi et al. (1997) recently proposed a related formulation, based on the central tendency of the short-term rate. This tendency is directly related to the long-term rates.

Lastly, Kozicki and Tinsley (1996) choose a formulation in which the second factor is the short-term rate expected at an infinite horizon. This endpoint is evaluated using a long-horizon forward rate. If the EH is true, it reflects market expectations of the short-term rate in the long run, apart from a constant risk premium. The main advantage of this model is that it explicitly identifies the "long-run expectations" component, which is essential for forecasting long-term rates.

1. The Long-Run Expectation of the Short-Term Rate

Following Kozicki et al. (1995) (see also Kozicki and Tinsley 1996), the model of the short-term rate includes a moving endpoint, $\hat{r}_{t-1}^{(\infty)} = \lim_{\tau \to \infty} E(r_{t+\tau} \,|\, I_{t-1})$:

$$b(L)\,\Delta r_t = \mu_1 + \alpha(r_{t-1} - \hat{r}_{t-1}^{(\infty)}) + e_t \tag{9}$$

where

$$E(\hat{r}_t^{(\infty)} \,|\, I_{t-1}) = \hat{r}_{t-1}^{(\infty)} . \tag{10}$$

Equation (9) looks like an error-correction model: the term $(r_{t-1} - \hat{r}_{t-1}^{(\infty)})$ plays the role of an error-correcting term. For a given date t, the short-term rate forecast will tend to the endpoint $\hat{r}_{t-1}^{(\infty)}$. Equation (10) points out that the endpoint forecast is constant from one period to another.

As in the stationary model, (9) can be rewritten as: $E_{t-1}\, z_t = A\, z_{t-1} + \mu_t$, where z_t and A are defined as previously and $E(\mu_t \,|\, I_{t-1}) = (\mu_1 - \alpha\, \hat{r}_{t-1}^{(\infty)}$ $0 \ldots 0)' \equiv \mu_t^{(t-1)}$. The multiperiod forecast of the short-term rate is now given by:

$$\begin{aligned} E[r_{t+i} \,|\, I_{t-1}] &= h_r' (I d_p - A)^{-1} \mu_t^{(t-1)}\, h_r' A^{i+1} (z_{t-1} - (I d_p - A)^{-1} \mu_t^{(t-1)}) \\ &= h_r' \hat{z}_{t-1}^{(\infty)} + h_r' A^{i+1} (z_{t-1} - \hat{z}_{t-1}^{(\infty)}) , \end{aligned} \tag{11}$$

which yields the short-term rate endpoint: $\lim_{i \to \infty} E[r_{t+i} \,|\, I_{t-1}] = h_r' \hat{z}_{t-1}^{(\infty)} \equiv \hat{r}_{t-1}^{(\infty)}$, where the last equality is true if the estimated intercept is zero ($\mu_1 = 0$). The forecast of the long-term rate, consistent with the EH, is then given by:

$$R_{t/t-1} = h_r' \hat{z}_{t-1}^{(\infty)} + \theta' [z_{t-1} - \hat{z}_{t-1}^{(\infty)}] + \varphi \tag{12}$$

where

$$\theta' = \frac{1-\beta}{1-\beta^n}\, h_r' (I d_p - (\beta A)^n)(I d_p - \beta A)^{-1} A .$$

See Appendix 1 for further details. This formulation clearly indicates that the choice of the moving endpoint $\hat{r}_{t-1}^{(\infty)}$ is open, since (10) does not specify the endpoint. Kozicki et al. (1995) suggest to measure market expectations from the 3-month forward rate for a rather distant date.

The forecast system (9)–(10), associated with the EH equation (2), can be interpreted as a two-factor model of the term structure: the first factor is the short-term rate, the second factor is the deviation of the short-term rate from its endpoint. This representation can be seen as an extension of the univariate approaches that can be compared to one-factor models (the short-term rate).

It is interesting to note that the univariate stationary model corresponds to the case $\hat{r}_t^{(\infty)} = \bar{r}$ where $\bar{r}$ is the sample mean of the short-term rate; the nonstationary model can be seen as a special case, where $\hat{r}_t^{(\infty)} = r_t$ such that (9) reduces to (6).

2. Estimation of the Forward Rate

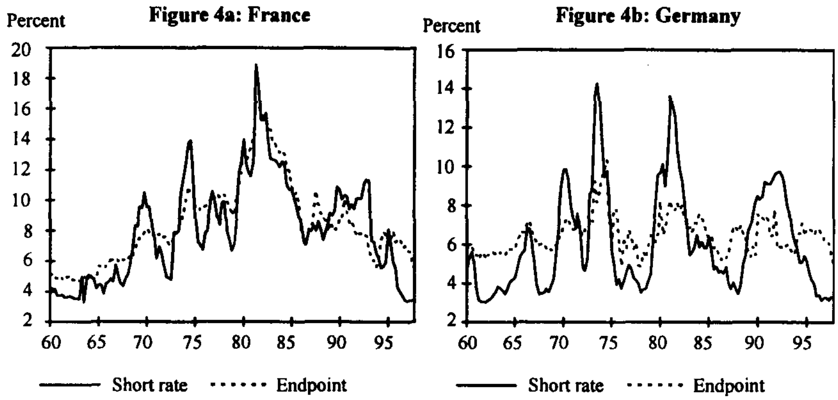
In the following, we use the 3-month forward rate in 10 years to define the moving endpoint. We evaluate this forward rate using zero-coupon term structures interpolated by the Banque de France over the period from 1980 to 1996 for France (Jondeau and Ricart 1997) and from 1972 to 1996 for Germany (see Deutsche Bundesbank 1995 and Schich 1996). This interpolation is based on the Nelson and Siegel (1987) approach. Data on French public bonds have been obtained from the "Cote Officielle" for the last day of each month. German public bond prices have been kindly provided by the Bundesbank. The interpolation for each month of zero-coupon curves enables a data set of fixed-term rates to be extracted. Appendix 2 gives further details on the data used for both countries.

From the zero-coupon curve, it is possible to compute the forward rate of maturity m for date $(t+n)$ as: $f_t^{(n)} = ((n+1)\, R_{z,t}^{(n+1)} - n\, R_{z,t}^{(n)})$. We then obtain an unbiased estimate of the short-term rate moving endpoint by subtracting risk premia: $\hat{r}_t^{(\infty)} = f_t^{(n)} - ((n+1)\, \varphi_z^{(n+1)} - n\, \varphi_z^{(n)})$. In practice, if $f_t^{(40)}$ denotes the 3-month forward rate in 10 years, the endpoint is given by: $r_t^{(\infty)} = f_t^{(40)} + \bar{r} - \bar{f}$, where $\bar{r}$ and $\bar{f}$ are the sample average short-term rate and the sample average forward rate, respectively.[5]

Figures 4a and 4b display the 3-month rate and its endpoint $r_t^{(\infty)}$. These figures clearly show that the change in the short-term rate end-

[5] Over the period from 1960 to 1979 for France and from 1960 to 1971 for Germany, we do not have any zero-coupon curves and we are not able to estimate endpoints in this way. We therefore use the following result: for a sufficiently distant horizon, forward rates and zero-coupon rates are the same. Indeed, when n is large enough, one has $R_{z,t}^{(n+1)} \approx R_{z,t}^{(n)}$ and then $f_t^{(n)} \approx R_{z,t}^{(n)}$. We therefore use the 10-year rate to approximate the 3-month forward rate in 10 years for the start of the samples ($f_t^{(40)} \approx R_t^{(40)}$, by correcting as before for the sample average spread).

Figure 4 – *Short-Term Rate Endpoint*



point is basically the same as that in the 10-year rate, even when the 10-year rate has not been used to evaluate the forward rate.

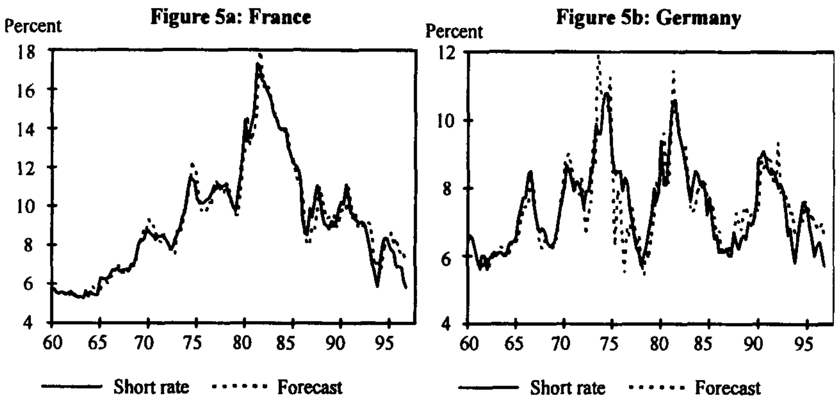
3. Estimates of Models and Long-Term Rate Forecasts

Estimates of the market expectations model (Table 1, part 3) highlight the crucial role played by the endpoint in explaining the dynamics of the short-term rate. Indeed, the error-correcting term is equal to $\alpha = -0.23$ for France and $\alpha = -0.14$ for Germany, with a t-statistic equal to 3.8 in both cases. The *see* is lower than those computed with previous univariate approaches.

Figures 5a and 5b show long-term rate forecasts as evaluated using the market expectations approach. In the case of France, forecasts made with this representation are much closer to the observed rates than forecasts made with the other two representations. In the case of Germany, the market expectations approach provides results similar to the stationary approach.

The 3-month forecast of the French long-term rate seems to lag the actual rate by three months (see Figure 5a). We could conclude that the forward rate is of no use in forecasting the long-term rate. This result, which is specific to French data, can be explained in two ways. First, the error-correcting term is much larger for France than for Germany; the adjustment toward the endpoint is thus much more rapid for French rates than for German rates. Second, the 3-month forward rate in 10 years is generally closer to the long-term rate in France than in Germany. These two reasons explain why the French forecast rate appears to

Figure 5 – *Interest Rates*



move before the observed long-term rate. However, it is worth distinguishing this result from what one would obtain using a simple random walk for the long-term rate. Indeed, our forecasts are consistent with the EH, whereas this is not the case with a random walk for the long-term rate. Moreover, at the end of the sample the information contained in forward rates cannot be summed up as the past of long-term rates, since there is a large gap between the long-term rate and its estimates.

V. Analysis of the RMSE and Theil Statistics

We are interested in the ability of the various short-term rate models to forecast long-term rates in the context of the EH. We use two statistics: the RMSE, which measures the standard deviation of the forecast error at a given horizon; and the Theil statistic, which is equal to the ratio of this RMSE to the RMSE of a reference model.[6] We select the univari-

[6] These statistics are evaluated within the sample: the various models are estimated over the whole sample period 1960-96; for each date, a 1-quarter, 2-quarter, ..., 12-quarter forecast is made, i.e., for a given model i:

$$R_{i,t+k/t-1} = h_r' z_{i,t-1}^{(\infty)} + \theta_i' A_i^k \left[z_{i,t-1} - z_{i,t-1}^{(\infty)} \right] + \varphi_i \qquad k = 1, \dots, 12,$$

and the corresponding forecast errors, $\varepsilon_{i,t+k/t-1} = R_{t+k} - R_{t+k/t-1}$, $k = 1, 2, \dots, 12$, are computed. It is then possible to estimate the RMSE, that is the standard deviation of these forecast errors, and the Theil statistic:

$$RMSE_i(k) = \sqrt{\frac{1}{T-k} \sum_{t=1}^{T} \varepsilon_{i,t+k/t-1}^2} \quad \text{and } Theil_i(k) = \frac{RMSE_i(k)}{RMSE^*(k)}$$

where $RMSE^*(k)$ is the RMSE at horizon k for the reference model.

Table 2 – *RMSE and Theil Statistics over the Period 1960–1996*

Horizon in quarters	Nonstat. model	Stationary model		Moving endpoint model		Bivariate VAR model	
	RMSE	RMSE	Theil	RMSE	Theil	RMSE	Theil
				France			
1	1.499	1.673	1.116	0.625	0.417	0.532	0.355
2	1.716	1.814	1.057	0.905	0.528	0.939	0.546
3	1.907	1.960	1.028	1.156	0.606	1.340	0.699
4	2.091	2.092	1.000	1.364	0.652	1.686	0.799
6	2.354	2.271	0.965	1.646	0.699	2.174	0.915
8	2.631	2.409	0.915	1.898	0.724	2.535	0.964
10	2.896	2.506	0.865	2.129	0.735	2.845	0.979
12	3.083	2.560	0.831	2.334	0.757	3.071	0.987
				Germany			
1	1.992	0.806	0.404	0.680	0.342	0.578	0.290
2	2.035	0.851	0.418	0.795	0.391	0.817	0.401
3	2.101	0.916	0.436	0.922	0.439	1.158	0.549
4	2.203	0.994	0.451	1.040	0.472	1.463	0.660
6	2.461	1.114	0.452	1.266	0.514	2.009	0.808
8	2.739	1.193	0.435	1.420	0.518	2.445	0.883
10	2.919	1.220	0.418	1.517	0.520	2.731	0.926
12	3.077	1.230	0.400	1.638	0.532	2.952	0.949

Note: The reference model for the Theil statistics is the nonstationary model.

ate nonstationary model as the reference model. Since this model is almost always the worst model in terms of RMSE, the Theil statistics can be interpreted as measuring the improvement, in percentage, of the other approaches vis-à-vis this model. We notice that, since the Theil statistics are used to compare different models, this normalization has been chosen for convenience.

Table 2 contains the RMSE and Theil statistics for both univariate representations (stationary and nonstationary models) and for the market expectations model, for horizons from 1 to 12 quarters. We also consider the bivariate VAR model, as suggested by Campbell and Shiller (1987, 1988) in the case of nonstationary but cointegrated interest rates. In this model, the short-term rate and the term spread are assumed to be a stationary vector process. The future changes in the short-term rate are then forecast using past changes in the short-term rate and past term spreads. Under the EH, this information set contains the whole information useful to predict short-term rates. The forecast of the long-term rate, consistent with the EH, is then obtained in a way similar to the other approaches. In our comments, we put the emphasis more specifically on short (1 quarter), medium (4 quarters) and long horizons (12 quarters).[7]

In the case of France, the stationary and nonstationary models are very similar for short and intermediate horizons: the 1-quarter RMSE are equal to 1.5 basis points for the nonstationary model and to 1.7 bp for the stationary model; the 4-quarter RMSE are equal to 2.1 bp for both models. For more distant horizons, the long-term rate forecasts of the stationary model track the historical rates more closely (with a 12-quarter RMSE equal to 2.6 compared with 3.1). The predictive capability of the market expectations model is clear: For short horizons, the RMSE is more than twice as small as that obtained with univariate models: it is equal to 0.6 at a 1-quarter horizon, against 1.5 for the nonstationary model, giving a Theil statistic of 0.42. For a 1-quarter horizon, the market expectations model is only dominated by the bivariate VAR model. For this model the Theil statistic is as low as 0.36. For longer horizons, however, the market expectations model clearly appears as the best model among those considered in this paper.

As far as Germany is concerned, the nonstationary model is clearly dominated by the stationary model for all horizons: the 1-quarter RMSE is equal to 2 for the nonstationary model and to 0.8 for the stationary model. The 12-quarter RMSE are 3.1 and 1.2, respectively. So, whatever the horizon, the Theil statistic is always smaller than 0.5. The market expectations model dominates the other models for short horizons. The 1-quarter Theil statistic is equal to 0.34 against 0.40 for the stationary model. As for France, the best results for the 1-quarter horizon are obtained with the bivariate VAR model, with a Theil statistic equal to 0.29. However, the stationary model performs much better than other models for medium-term and long horizons.

We note that the nonstationary model gives similar RMSE for France and Germany. On the contrary, the stationary model is clearly more appropriate for explaining the dynamics of German rates: for long horizons, the best model for Germany gives a RMSE that is twice as small as that obtained with the best model for France. The market expectations model gives more balanced results, since the RMSE are generally of the same order of magnitude for France and Germany, except for the longest horizons.

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[7] The different models could also be compared by analyzing out-of-sample RMSE and Theil statistics. We performed such an exercise in which RMSE and Theil statistics were estimated over the 1975–96 period, with the 1960–74 pre-sample being used as the initial conditions. The results (not reported here) are very close to those obtained within the sample. In particular, it does not change the hierarchy of the models. We then chose to analyze the RMSE and Theil statistics estimated within the sample.

Lastly, the best results for the 1-quarter horizon are obtained with the bivariate VAR model, but the market expectations model is the more appropriate representation for France at all other horizons and for Germany at short horizons. The stationary model dominates the other representations for long horizons in Germany. It is noteworthy that this result confirms the interpretation outlined in Figure 2b concerning the comparison between stationary and nonstationary models in Germany.

VI. Conclusion

In this paper, we have studied the ability of different short-term rate models to forecast long-term rates in the context of the expectations hypothesis of the term structure. We first considered univariate autoregressive models for short-term rates (assuming both stationarity and non-stationarity). The poor forecasting ability of these models appears to be due, at least in France, to the absence of a relevant endpoint for the short-term rate. Modeling the short-term rate endpoint using a reaction function should significantly improve the accuracy of forecasts. However, modeling such a reaction function means that the macroeconomic fundamentals of the central bank, the regime shifts or structural breaks, etc., have to be identified in order to obtain satisfying forecasts.

The use of market expectations allows to construct a short-term rate forecasting model in which the endpoint is allowed to vary over time. This kind of model has three advantages. First, long-run market expectations are clearly identified (which is not usually the case with factor models) and they implicitly adjust to changes in monetary policy. Second, these models are quite easy to use, since we mainly need zero-coupon yield curves over a long period of time. Last, the empirical results for France and Germany are encouraging. For the shortest horizons, the market expectations approach is the most appropriate for both countries as compared to the univariate autoregressive models. This indicates that the short-term rate endpoint contains most of the information needed to forecast short-term and long-term rates. For a 1-quarter horizon, better results can be obtained from a bivariate VAR model, for which the information set includes past changes in short-term rates and past term spreads. As far as German rates are concerned, it is possible to obtain reasonable forecasts using a univariate autoregressive stationary model for the short-term rate. However, it is worth noting that this conclusion in favor of the stationarity of German short-term rates mainly stems from the sample used for the estimation.

Lastly, we find that an appropriate representation of market expectations concerning future changes in short-term rates means that the expectations hypothesis largely recovers its empirical relevance.

Appendix 1. Computations for the Stationary and Nonstationary Representations and for the Market Expectations Approach

a. The Stationary Representation

We begin with the following univariate stationary representation:

$$b(L)\Delta r_t = \mu_1 + \alpha r_{t-1} + e_t, \tag{13}$$

where $\alpha < 0$ and $b(L)$ a $(p-1)$-order polynomial in the lag operator L. This model can be rewritten as the first-order companion system:

$$z_t = A z_{t-1} + \mu + v_t, \tag{14}$$

where $z_t = (r_t \dots r_{t-p+1})'$, $\mu = (\mu_1\ 0 \dots 0)'$ and $v_t = (e_t\ 0 \dots 0)'$ and A is the (p,p) companion matrix of the VAR:

$$A = \begin{pmatrix} 1+\alpha+b_1 & b_2-b_1 & \cdots & b_{p-1}-b_{p-2} & -b_{p-1} \\ 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & \ddots & 0 & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{pmatrix}. \tag{15}$$

Since relation (14) has a first-order structure, the i-quarter ahead forecast of z_t is generated by:

$$E[z_{t+1} \mid I_{t-1}] = A^{i+1} z_{t-1} + (Id_p - A^{i+1})(Id_p - A)^{-1}\mu, \tag{16}$$

where $I_t = \{r_t, \dots, r_{t-p+1}\}$ is the information set (included in Ω_t) available to the econometrician. We define h_r the $(p, 1)$ vector selecting the current short-term rate z_t, such that $r_t = h_r' z_t$. Equation (16) then gives the multiperiod forecast for the short-term rate r_t:

$$E[r_{t+1} \mid I_{t-1}] = h_r' E[z_{t+i} \mid I_{t-1}] = h_r'\rho + h_r' A^{i+1}[z_{t-1} - \rho], \tag{17}$$

where $\rho = (Id_p - A)^{-1}\mu$. Equation (17) indicates that the short-term rate tends to the following endpoint:

$$\lim_{i\to\infty} E[r_{t+i} \mid I_{t-1}] = h_r'\rho \equiv r^{(\infty)},$$

when the horizon goes to infinity. This short-term rate endpoint, $r^{(\infty)}$, is constant over time.

Now substituting (17) into the term structure relation (2) yields the following one-period optimal forecast of the long-term rate consistent with the EH:

$$R_{t/t-1} \equiv E[R_t \,|\, I_{t-1}] = \frac{1-\beta}{1-\beta^n} \sum_{i=0}^{n-1} \beta^i E[r_{t+i} \,|\, I_{t-1}] + \varphi$$

$$= h_r' \rho + \theta'[z_{t-1} - \rho] + \varphi, \tag{18}$$

where

$$\theta' = \frac{1-\beta}{1-\beta^n} h_r'(Id_p - (\beta A)^n)(Id_p - \beta A)^{-1} A. \tag{19}$$

Note that forecast of the long-term rate k periods ahead is given by:

$$R_{t+k/t-1} = h_r' \rho + \theta' A^k [z_{t-1} - \rho] + \varphi. \tag{20}$$

b. The Nonstationary Representation

Let us consider now the nonstationary representation for the short-term rate, which can be written as an autoregressive model for the first-difference short-term rate:

$$b(L)\,\Delta r_t = \mu_1 + e_t, \tag{21}$$

where $b(L)$ is a $(p-1)$-order polynomial in the lag operator. The first-order companion form of (21) is then expressed as:

$$\Delta z_t = \tilde{A}\,\Delta z_{t-1} + \mu + \upsilon_t,$$

where $z_t = (r_t \ldots r_{t-p+2})'$, μ and υ_t are defined as before, and

$$\tilde{A} = \begin{pmatrix} b_1 & b_2 & \cdots & 0 & b_{p-1} \\ 1 & 0 & \ldots & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & 1 & 0 \end{pmatrix}.$$

Since the multiperiod forecast of the differenced short-term rate is:

$$E[\Delta r_{t+j} \,|\, I_{t-1}] = h_r' \tilde{A}\,\Delta z_{t-1} + h_r'(Id_{p-1} - \tilde{A}^{j+1})\,(Id_{p-1} - \tilde{A})^{-1} \mu,$$

the short-term rate multiperiod forecast is given by:

$$E[r_{t+i} \,|\, I_{t-1}] = r_{t-1} + \sum_{j=0}^{i} E[\Delta r_{t+j} \,|\, I_{t-1}]$$

$$E[r_{t+i}|I_{t-1}] = h_r' z_{t-1} + h_r'(Id_{p-1} - \tilde{A}^{i+1})(Id_{p-1} - \tilde{A})^{-1}\tilde{A}(\Delta z_{t-1} - \tilde{\rho}) + (i+1)h_r'\tilde{\rho}, \quad (22)$$

where $\tilde{\rho} = (Id_{p-1} - \tilde{A})^{-1}\mu$. The endpoint for the short-term rate is now time-varying, since it can be expressed as a moving average of the most recent short-term rates:

$$\lim_{i\to\infty} E[r_{t+1} | I_{t-1}] = h_r' z_{t-1} + h_r'(Id_{p-1} - \tilde{A})^{-1}\tilde{A}\,\Delta z_{t-1} \equiv r_{t-1}^{(\infty)}.$$

The optimal long-term rate forecast consistent with the EH is now:

$$R_{t/t-1} = h_r' z_{t-1}^{(\infty)} + \tilde{\theta}'[z_{t-1} - z_{t-1}^{(\infty)}] + \varphi, \quad (23)$$

where

$$z_{t-1}^{(\infty)} = z_{t-1} + (Id_{p-1} - \tilde{A})^{-1}\tilde{A}\Delta z_{t-1}$$

and

$$\tilde{\theta}' = \frac{1-\beta}{1-\beta^n} h_r'(Id_{p-1} - (\beta\tilde{A})^n)(Id_{p-1} - \beta\tilde{A})^{-1}.$$

The multiperiod forecast is simply:

$$R_{t+k/t-1} = h_r' z_{t-1}^{(\infty)} + \tilde{\theta}'\tilde{A}^k[z_{t-1} - z_{t-1}^{(\infty)}] + \varphi. \quad (24)$$

Contrary to the stationary case, the short-term rate endpoint and the long-term rate forecast are directly related to the most recent short-term rates.

c. The Approach Based on Market Expectations

Following Kozicki et al. (1995), the model of the short-term rate includes a moving endpoint, $\hat{r}_{t-1}^{(\infty)} = \lim_{i\to\infty} E(r_{t+\tau} | I_{t-1})$:

$$b(L)\Delta r_t = \mu_1 + \alpha(r_{t-1} - \hat{r}_{t-1}^{(\infty)}) + e_t, \quad (25)$$

where

$$E(\hat{r}_t^{(\infty)} | I_{t-1}) = \hat{r}_{t-1}^{(\infty)}. \quad (26)$$

As previously, equation (25) can be rewritten as: $z_t = A z_{t-1} + \mu_t + \upsilon_t$, where $z_t = (r_t \ldots r_{t-p+1})'$, $E[\mu_t | I_{t-1}] = (\mu_1 - \alpha\hat{r}_{t-1}^{(\infty)}\ 0 \ldots 0)' = \mu_t^{(t-1)}$, $\upsilon_t = (e_t\ 0 \ldots 0)'$, and A is defined as in (15).

The multiperiod forecast of the short-term rate is now given by:

$$E[r_{t+i} | I_{t-1}] = h_r'(Id_p - A)^{-1}\mu_t^{(t-1)}\, h_r' A^{i+1}(z_{t-1} - (Id_p - A)^{-1}\mu_t^{(t-1)}) = h_r'\hat{z}_{t-1}^{(\infty)} + h_r' A^{i+1}(z_{t-1} - \hat{z}_{t-1}^{(\infty)}), \quad (27)$$

which yields the short-term rate endpoint:

$$\lim_{i \to \infty} E[r_{t+i} \mid I_{t-1}] = h_r' \hat{z}_{t-1}^{(\infty)} \equiv \hat{r}_{t-1}^{(\infty)},$$

where the last equality is true if the estimated intercept is zero ($\mu_1 = 0$). The forecast of the long-term rate, consistent with the EH, is then given by:

$$R_{t/t-1} = h_r' \hat{z}_{t-1}^{(\infty)} + \theta' [z_{t-1} - \hat{z}_{t-1}^{(\infty)}] + \varphi,$$

where

$$\theta' = \frac{1-\beta}{1-\beta^n} h_r' (Id_p - (\beta A)^n)(Id_p - \beta A)^{-1} A.$$

Appendix 2. Data

a. The Data Used for France

Until the mid-eighties, the French public bond market consisted of old classes of securities, with low levels of both liquidity and homogeneity. These classes were not themselves homogenous, with several clauses included at the time of the issue (possibility for the Treasury to modify the coupon rate, ability to postpone repayment). Last, for some securities, the Treasury modified certain characteristics after issue (as the repayment before due date). Therefore, the yields to maturity of these different classes of issues were not always comparable. Zero-coupon curves used in this paper were estimated using fixed-rate French-franc denominated issues.[8] We did not introduce in the estimation consol bonds (*rentes perpétuelles*), some old-fashioned government bonds (*emprunts d'Etat*) with repayment by drawing lots and fungible Treasury bonds (*obligations assimilables du Trésor*) with exchange options, because of the difficulty in evaluating an ex-post yield to maturity. With regard to the beginning of the eighties (from 1980 to 1983), there was an insufficient number of issues with short residual maturities. We therefore included interbank market rates in the estimation, and we constrained the short-end of the yield curve to pass through the overnight rate. From 1984 onwards, interbank market rates were no longer used.[9]

[8] This definition does not include Treasury bills, however. Indeed, we could not find a pre-1990 historical data set of Treasury bill prices, although they were introduced in 1986. Moreover, to ensure homogeneity with the beginning of the sample, we do not use bills after 1990.

[9] Nevertheless, we used the overnight rate to anchor the curve to the shortest-term rate.

The number of issues used for the estimation rise dramatically over time: on average, 10 issues were used between 1980 and 1983, 18 between 1984 and 1989 and 20 between 1990 and 1996. The yield curves estimated for the start of the eighties should be used with caution.

b. The Data Used for Germany

The German public debt securities include bonds issued by the Federal Republic of Germany, the 'German Unity' Fund, the ERP Special Fund, the Treuhand agency, the German Federal Railways and the German Federal Post Office (*Anleihen*), five-year special Federal bonds, five-year special Treuhand agency bonds, special bonds issued by the German Federal Post Office (*Obligationen*), treasury bonds issued by the German Federal Railways and the German Federal Post Office and Federal treasury notes (*Schatzanweisungen*) (see Deutsche Bundesbank 1995 for additional details). The data set has been kindly provided by the Bundesbank. In constructing German zero-coupon yield curves, we eliminated bonds issued by the German Federal Railways and the German Federal Post Office from the data set because they pay an additional premium compared to other public debt securities. Moreover, we selected securities with a fixed maturity and an annual coupon. The last bonds with semi-annual coupon payments matured in December 1980.

The number of securities used for the estimations appears to be large enough: from 60 in 1980 to about 100 after 1984. The number of securities with a short residual maturity is rather low at the beginning of the 1980s, but never as low as for French data. Therefore we do not include interbank rates in the estimation.

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Abstract: Forecasting French and German Long-Term Rates Using a Rational Expectations Model. – In this paper, the authors study a forecasting model for long-term rates based on the expectations hypothesis of the term structure. The long-term rate is expressed as an average of expected short-term rates, which are modelled using three models: two univariate models (with stationary and nonstationary rates) and one model in which the short-term rate terminal boundary is specified as a function of agents' expectations. These approaches are used to forecast French and German long-term rates from 1960 to 1996. The authors find that the model based on agents' expectations gives the best forecasts, especially for short horizons. JEL no. E43

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Zusammenfassung: Zur Prognose langfristiger Zinssätze in Frankreich und Deutschland unter Verwendung eines Modells rationaler Erwartungen. – Die Verfasser untersuchen ein Modell zur Vorausschätzung langfristiger Zinssätze, das auf der Erwartungshypothese der Zinsstruktur basiert. Der langfristige Zinssatz wird als Durchschnitt erwarteter kurzfristiger Zinssätze ausgedrückt, die auf dreierlei Art modelliert werden: zwei eindimensionale Modelle (mit stationären und nichtstationären Zinssätzen) und ein Modell, in dem die Wirtschaftssubjekte die kurzfristigen Zinssätze am Ende des jeweils modellierten Zeithorizontes erwarten. Diese Ansätze werden benutzt, um die langfristigen Zinssätze in Frankreich und Deutschland für die Periode 1960–1996 vorauszuschätzen. Die Verfasser kommen zu dem Schluß, daß das Modell, das auf den Erwartungen der Wirtschaftssubjekte basiert, die besten Vorausschätzungen ermöglicht, insbesondere auf kurze Sicht.