REINSURANCE

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In a *reinsurance contract*, one party (the *reinsurer*) for a certain premium agrees to indemnify another party (called the *reinsured*, the *first-line insurer* or also the *ceding company*) for specified parts of its underwritten insurance risk. Hence reinsurance is a form of *risk sharing* between insurance companies (cf. [3]). Among the main motivations for an insurance company to buy reinsurance is to reduce the probability of suffering losses that it can not easily cope with, in particular the appearance of excessively large or unusually many claims from the underwritten policies. Also, reinsurance is a way to increase the underwriting capacity (which is particularly but not exclusively relevant for smaller insurance companies) and can be interpreted as a (virtual) increase of the solvency (see eqf21/011) capital of the ceding company. Replacing part of the random costs (claims) by fixed costs (premiums) is the core insurance activity and helps the policy-holders to stabilize their business. But this also applies to the first-line insurer as a reinsurance customer and reinsurance can increase the effectiveness of the marketplace. For the same reason, a reinsurance company may itself, again for some suitable premium, pass on parts of the reinsured risks to another reinsurer (both domestic and international), which is then called *retrocession*. For the general concept of *need for reinsurance* see e.g. Gerber [16] and Schnieper [25].

The type and amount of reinsurance coverage may be negotiated for individual risks in the portfolio (*facultative reinsurance*). But predominantly *reinsurance treaties* are signed, which are obligatory agreements that specify the type and amount of reinsurance coverage for an entire portfolio of risks within an insurance branch (usually for a time horizon of one year).

Among the crucial questions in the context of reinsurance are:

- Which type of reinsurance form is appropriate in a given situation?

- How much reinsurance should be purchased?
- For which premium?

Forms of Reinsurance

Let $X(t) := \sum_{i=1}^{N(t)} X_i$ denote the aggregate claim amount in an insurance portfolio up to time t, where the counting process N_t specifies the number of claims up to time t and the random variables X_i denote the individual claim sizes. In every reinsurance treaty, this amount is split into $X(t) = D(t) + R(t)$, where $D(t)$ is the amount that the insurance company retains for itself (the *deductible*) and $R(t)$ is the *reinsured* amount to be covered by the reinsurance company.

There are two types of proportional reinsurance forms:

Quota-Share Reinsurance

A very common and simple reinsurance form is the Quota-Share (QS) treaty, where one has

$$R(t) := a\,X(t), \qquad D(t) := (1-a)\,X(t)$$

for some proportionality factor $0 < a < 1$. In this case it is straight-forward to calculate the distribution of $R(t)$ and $D(t)$, once the distribution of $X(t)$ is known.

Surplus Reinsurance

Whereas in a QS treaty also small claims are reinsured (which the first-line insurer often wants to keep for himself), a surplus treaty only reinsures claims X_i whose corresponding insured sum Q_i exceeds some retention M. In the latter case the first line insurer deducts a proportion M/Q_i and shifts the remaining part to the reinsurer. Consequently,

$$R(t) = \sum_{i=1}^{N(t)} \left(1 - \frac{M}{Q_i}\right) X_i\, 1_{\{Q_i \geq M\}},\; D(t) = \sum_{i=1}^{N(t)} \left(X_i I(Q_i \leq M) + M \frac{X_i}{Q_i} 1_{\{Q_i > M\}} \right) .$$

Surplus treaties are quite popular in practice, in particular in fire, marine and storm insurance.

Other types of reinsurance forms are of non-proportional nature:

Excess-of-Loss Reinsurance

In an Excess-of-Loss (XL) treaty, for each individual claim the excess over some retention M is paid by the reinsurer:

$$R(t) := \sum_{i=1}^{N(t)} (X_i - M)^+, \quad D(t) := \sum_{i=1}^{N(t)} \min(X_i, M) \, .$$

XL covers are for instance popular in casualty and windstorm insurance. In most situations there is an upper limit L of the XL cover of each risk as well, resulting in the treaty

$$R(t) := \sum_{i=1}^{N(t)} \min\{(X_i - M)^+, L\},$$

where L is then the size of the so-called *XL reinsurance layer* (the usual notation is then $L \; xs \; M$). It is also common to have an upper limit $k \cdot L$ on the total reinsured amount $R(t)$ for some integer k. In the latter case one says to have *k reinstatements* and, depending on the contract, the premium for a higher reinstatement often only has to be paid once the lower reinstatement is used up.
A variant of XL reinsurance is the *per-event XL cover* (or sometimes called *catastrophe XL cover*), where all claims due to some external event such as a storm or an earthquake are first added and then treated as a single claim in an XL contract (avoiding that for many small claims the XL treaty may not provide sufficient protection).

Stop-Loss Reinsurance

The Stop-Loss (SL) treaty acts on the aggregate claim amount $X(t)$ of the portfolio:

$$R(t) := \left\{ \sum_{i=1}^{N(t)} X_i - C \right\}^+, \quad D(t) := \min(X(t), C),$$

where C is some retention. In general there is again an additional upper limit on $R(t)$.

In practice, often combinations of the above reinsurance forms are employed (together with upper limits on the total reinsurance cover).

Large Claim Reinsurance

Let $\{X_1^*, X_2^*, \ldots, X_{N(t)}^*\}$ denote the order statistics of the claims ordered according to their size. Then the *largest claims reinsurance* is defined through

$$R(t) := \sum_{i=1}^{r} X_{N(t)-i+1}^*, \quad D(t) := \sum_{i=1}^{N(t)-r} X_i^*,$$

i.e. the reinsurer covers the r largest claims of the portfolio that have occurred up to time t (which again usually is one year). Another variant called ECOMOR was introduced by Thépaut [27] with a reinsured amount of the form

$$R(t) := \sum_{i=1}^{N(t)} \left\{X_i - X_{N(t)-r}^*\right\}^+,$$

so that in this case the reinsurer covers only that part of the r largest claims that overshoots the random retention $X_{N(t)-r}^*$. Such a contract gives the reinsurer protection against claim inflation, since if the claim amounts increase, the retention will also increase.

Although theoretically appealing, large claim reinsurance treaties are rarely applied in practice.

Reinsurance Premiums

In the first place, reinsurance is a specific form of insurance. Consequently, the principles of premium calculation (see eqf21/003) can to some extent also be applied in the reinsurance setting, once the moments or even the distribution of the number and the size of the reinsured claims are available (although costs and loading factors will in general be different). Extreme value statistics is of vital importance in this context, since typically the reinsured claims are heavy-tailed and their distribution has to be estimated, often on the basis of only few data points (see eqf21/008 and [14]).

Whereas for proportional reinsurance the actuarial reinsurance premium will simply be the corresponding proportion of the premium that the first-line insurer has received for the underlying risks (minus some aquisition and administration costs), the situation for non-proportional reinsurance is more involved.

The claim number process for the reinsured claims in an XL treaty can be interpreted as an independent thinning of the point process (see eqf02/007) that generates the claims for the first-line insurer. For many practically relevant examples, the resulting claim number process for the reinsurer is then of the same form with just some parameters modified (see e.g. [4]). For the premium calculation of an XL treaty with retention M, under the assumption of independence between the number and the sizes of the claims as well as identically distributed claim sizes often the *reduction effect*

$$r(M) := \frac{E(D(t))}{E(X(t))} = \frac{1}{E(X)} \int_0^M (1 - F(w))dw$$

is used, which turns out to be the *equilibrium distribution function* of the distribution function F of an individual claim X. In practice one distinguishes between *exposure rating* where the reinsurer uses the portfolio and premium information of the first-line insurer, and *experience rating* which is mainly based on the previous claim experience of the XL layer itself. Especially the latter method often faces the challenge of having only few data points available, in which case credibility estimates (see eqf21/019) may be used. Another important aspect in XL treaties is potential claims inflation, which may be substantial (as the time until settlement of a claim may be large) and would often just affect the reinsurer. Hence, typically some *index clause* is arranged which splits the amount of claims inflation between the first-line insurer and the reinsurer.

For SL treaties, the aggregate claim distribution (see eqf21/013) of the first-line insurance portfolio will be the crucial quantity to determine suitable reinsurance premiums, using for instance generalized stop-loss transforms to determine moments of the reinsured claim amount (for numerical and algorithmic issues see Kaas [20]). Premium calculations for large claim reinsurance contracts are more involved, but several asymptotic results using extreme value theory are available (see [4] for a survey). Note that for all these treaties it is also important to adequately assess the possible dependence among the risks, as this may drastically influence the appropriate premium (see for instance Denuit et al. [12]).

In addition to the resulting actuarial premium, the actual reinsurance premium will also be influenced by other factors like competition, volume of the portfolio and perhaps a loading that accounts for potential *moral hazard* and *adverse selection*. Here moral hazard refers to the tendency of the reinsured not to be as strict in the claim settlement process as without reinsurance (see e.g. [13]), whereas adverse selection accounts for the asymmetry of information of the cedant and the reinsurer on the underlying risk in general (see [19]). Also, for contracts like the XL treaty, the reinsurer typically has no insight into the number and sizes of the claims below the retention and needs to get information from other sources, which also increases the costs (see e.g. [17, 18]). General considerations on planning in reinsurance can be found in Straub [26].

Robustness questions of reinsurance premiums are discussed in Brazauskas [8]. In addition, the economic environment influences the pricing of reinsurance risks (see Aase [1, 2]).

Choice of Reinsurance

Each reinsurance form has its particular advantages and disadvantages in terms of the type of protection it provides (frequency risk, large claim risk), premium calculation, practical handling, administration and processing of loss estimation (including issues like moral hazard and adverse selection). An insurer may have a concrete objective such as to maximize expected profit after reinsurance (subject to a certain security level condition) or to minimize the probability of ruin after reinsurance (see eqf21/001). There are a number of theoretical results available which under specific assumptions on the underlying premium principle identify the optimal reinsurance form with respect to given maximization criteria and constraints (see [4] for an overview). A natural and close connection of such optimality questions with utility theory (see eqf03/007) is e.g. discussed in Borch [7] and Bühlmann and Jewell [9]. For instance, under an expected value premium principle of the reinsurer and a given reinsurance premium, a SL contract maximizes the expected utility of terminal wealth for any risk-averse utility function (a result attributed to K. Arrow, see eqf03/007). As a consequence, in this setting a SL contract also minimizes the retained variance $\mathrm{Var}(D)$ of the first-line insurer. On the other hand, under a variance or standard deviation premium principle and a fixed value of $\mathrm{Var}(R)$, a QS contract minimizes $\mathrm{Var}(D)$ [5, 23]. Already these two examples illustrate that the selected reinsurance premium principle plays a decisive rule for the optimal decision. Also, the optimality results usually heavily depend on the type and amount of involved transaction costs. In recent years many refinements of such optimality results have been developed (see [4, 21] for a general survey on this topic). For a dual approach in the framework of solvency in finance and insurance, see Yaari [29]. In the collective model, it is also quite common to look for reinsurance strategies that minimize the Lundberg adjustment coefficient (see eqf21/002), see e.g. Gerber [15] and Centeno [10]. An overview on optimal dynamic reinsurance strategies to minimize the probability of ruin can be found in Schmidli [24] (see also eqf21/001 and eqf21/023). For other optimization criteria, see [30].

In practice, the concrete form and amount of reinsurance choice for a certain portfolio often also will be influenced by experience, availability of reinsurance offers and current market prices.

Reinsurance and Finance

The financial component in any insurance and reinsurance activity has become more important over the last years, not the least due to higher interest rates, longer time horizons for claim settlement and the general need to combine the analysis and management of assets and liabilities (see e.g. Wüthrich et al. [28]). Financial pricing of reinsurance contracts in various contexts is discussed in [11] (see eqf21/006, and note that a (re)insurance market is incomplete, (see eqf04/006)). Since reinsurance is a global business, foreign exchange risk (see eqf06/001) also has to be considered (cf. Blum et al. [6]).

Nowadays classical reinsurance is often complemented by other mechanisms to cope with risk (summarized under the name *alternative risk transfer*) that have a certain financial flavour such as *captives, finite risk reinsurance, multi-trigger products* and *contingent capital* (cf. [22]). Further bridging the gap between insurance and finance, insurance companies also started to transfer some of their risk directly to the capital market (see eqf20/001). This includes *reinsurance sidecars*, where investors act like a QS reinsurer, as well as the issuance of catastrophe bonds (see eqf21/005). Recently, also some structured insurance-linked products such as *catastrophe* CDOs (see eqf10/012) have been issued on the financial market, which sometimes receive quite competitive credit ratings for their tranches (see eqf10/041). At the same time, some of the traded catastrophe CDOs offer loss coverage that can be significantly cheaper than in classical reinsurance contracts.

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Abstract

In this chapter, the notion of reinsurance as a means for a first-line insurer to pass on part of his underwritten risk for some appropriate premium payment is defined and motivated. Various forms of common reinsurance treaties of proportional and non-proportional type are described, including Quota-Share, Surplus, Excess-of-Loss, Stop-Loss and Large-Claim treaties. Some actuarial and practical challenges in connection with the determination of suitable premiums for the reinsured risk are discussed. Subsequently, main obstacles in deciding upon the choice of the appropriate reinsurance form in a given situation are treated, which often can be expressed as an optimization problem with an objective function under constraints. Finally, connections between reinsurance and the realm of finance are discussed. Throughout the chapter, references to the literature and links to other chapters are provided.

Keywords: insurance risk, premium principles, large claims, aggregate claim distribution, reinsurance forms, optimization, utility theory, extreme value theory