Are Overconfident Workers More Likely to Win Bonuses or Be Promoted?

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Abstract

This paper investigates whether overconfident workers are more likely to win bonuses or be promoted. To answer this question I consider a tournament between an overconfident worker and an unbiased rival. The overconfident worker overestimates his productivity of effort and, therefore, his winning probability. The unbiased worker knows about the rival's bias and optimally reacts to it. I show that if an overconfident worker perceives his marginal output is increasing with self-confidence, then too much overconfidence jeopardizes his chances of being promoted but a little overconfidence helps. The paper also shows an underconfident worker always has a smaller chance of being promoted than an unbiased rival. These findings clarify the conditions under which higher male self-confidence leads men to outperform women in tournaments.

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1 Introduction

Competition between workers in internal labor markets often takes the form of tournaments. Firms use tournaments to incentivize effort provision–a bonus or free vacation for the top salesperson–and to promote staff–workers compete to become managers and managers to become CEOs (Malcomson 1986, Gibbons and Murphy 1990, Baker et al. 1994, Murphy et al. 2004, Harbring and Lünser 2008). In academia, institutions often use tournaments to incentivize publications–a monetary bonus or a teaching reduction for the top researchers–and to promote employees–assistant professors compete to become tenured professors.

In a typical labor market tournament, workers compete by choosing effort. Higher effort leads to higher output and the worker with the highest output wins the tournament. In equilibrium, workers should increase their effort level up to the point where the marginal benefit of doing so—the marginal probability of winning the tournament times the utility differential between winning and losing—equals its incremental cost—the marginal disutility of effort (Lazer and Rosen 1981, Nalebuff and Stiglitz 1983).

The benchmark tournament model assumes workers have rational (or unbiased) perceptions of their winning probabilities. However, workers' perceptions may be biased due to misperceptions of their productivity of effort. In fact, evidence from psychology and economics shows that humans tend to be overconfident.[1] A majority of people believe they are better than others in a wide variety of positive traits and skills (Myers 1996, Santos-Pinto and Sobel 2005). Examples include car drivers (Svenson 1981), entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006),

[1] Moore and Healy (2008) distinguish between three types of overconfidence: (i) overestimation of one's absolute skills or performance, (ii) overestimation of one's relative skills or performance (overplacement or the "better-than-average" effect), and (iii) excessive confidence in the precision of one's private information, estimates, and forecasts (overprecision or miscalibration). In this paper I consider overconfidence of the first two types.

currency traders (Oberlechner and Osler 2008), poker and chess players (Park and Santos-Pinto 2010), CFOs (Ben-David et al. 2013), marathon runners (Krawczyk and Wilamowski 2017) and freedivers (Lackner and Sonnabend 2020)

Laboratory based evidence shows that overconfidence affects the outcome of labor markets (Sautmann 2013, Dargnies et al. 2019, Hoffman and Burks 2020, Santos-Pinto and de la Rosa 2020). The evidence also shows that self-confidence biases matter for entry and performance in tournaments. Men tend to enter tournaments more often than women and this is partly due to their higher self-confidence (Niederle and Vesterlund 2007).[2] Experimental participants are more likely to select into tournaments the more they overplace themselves (Dohmen and Falk 2011). An experimental participant's self-confidence causally increases that participant's propensity to enter a tournament (Möbius et al. 2022). Field evidence shows that managers of a chain of food-and-beverage stores competing in high-stakes tournaments are persistently overconfident (Huffman et al. 2019). Furthermore, field and laboratory evidence shows that gender and socio-economic differences in self-confidence are important determinants of educational and career choices (Kamas and Preston 2012, Buser et al. 2014, Dreber et al. 2014, Wiswall and Zafar 2015, Reuben et al. 2017, Guyon and Huillery 2021). For example, gender differences in self-confidence and competitiveness are correlated with boys choosing more prestigious academic tracks than girls (Buser et al. 2014).[3] Boys are significantly more likely to choose to compete in a mathematical task than girls due to their higher self-confidence (Dreber et al. 2014).[4]

[2]Niederle and Vesterlund (2007) find that, despite there being no gender differences in performance, 73 percent of the men select to enter a tournament but only 35 percent of the women make this choice. The gender gap in tournament entry is driven by two factors. First, men have a stronger preference for competing than women. Second, men are substantially more overconfident about their relative performance than women.

[3]In addition, Buser et al. (2014) find that controlling for performance, girls are about 23 percentage points less likely to enter the tournament. Slightly over 30 percent of this gender gap can be explained by gender differences in confidence.

[4]Balafoutas and Sutter (2012) and Niederle et al. (2013) show experimentally that policy inter-

This paper investigates how heterogeneity in self-confidence in the workforce determines who is more likely to win a bonus or be promoted. More precisely, the paper addresses the following questions. Is an overconfident (underconfident) worker more (less) likely to win a tournament than an unbiased worker? Does this depend on the type of self-confidence and/or its degree? If men display higher self-confidence than women, are men more likely to win tournaments than women? What is the welfare impact of overconfidence and underconfidence for the firm and for the workers?

Section 3 sets-up the model. Two workers compete in a tournament. Each worker chooses an effort level independently and simultaneously. Effort plus random factors determine who produces the highest output or who attains the best performance. One worker is overconfident (underconfident) in the sense that he overestimates (underestimates) his productivity of effort and, as a consequence, his winning probability.[5] The other worker is unbiased, knows about the rival's bias, and optimally reacts to it. Both workers have identical productivity, preferences, outside options, and face identically distributed random shocks. These symmetry assumptions allow me to focus exclusively on the role self-confidence biases play in determining the winner of the tournament. Moreover, they imply that the worker who exerts the highest effort has the highest winning probability. I define as the Nash winner (loser) the worker with the highest (lowest) ex-ante probability of winning at the pure-strategy equilibrium.

Section 4 studies tournaments where an overconfident worker competes against an unbiased worker. Proposition 1 shows that if the overconfident worker perceives his marginal output is increasing with self-confidence, then the identity of the Nash winner depends critically on the size of the overconfident worker's bias. The overconfident worker is the Nash winner when he is slightly overconfident and the Nash

ventions that increase female participation in competitions can be welfare improving.

[5] This way of modeling overconfidence is often used for analyzing the impact of overconfidence on contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). Alternatively, an overconfident player could underestimate his or her cost of effort.

loser when he is significantly overconfident. In other words, too much overconfidence jeopardizes one's chances of winning the tournament but a little overconfidence helps. The intuition behind this result is as follows. If the overconfident worker is slightly overconfident, he *overestimates* his *marginal* probability of winning the tournament. This raises the effort of the overconfident worker and lowers the effort of the unbiased worker. In contrast, if the overconfident worker is significantly overconfident, he *underestimates* his *marginal* probability of winning the tournament. This lowers the effort of both workers but more so that of the overconfident worker.

Proposition 1 has welfare implications. The firm is better off with a slightly overconfident worker since his increase in effort is greater than the decrease in effort of the unbiased worker. The firm is worse off with a significantly overconfident worker since both workers exert less effort than if both were unbiased. Overconfidence can make a slightly overconfident worker better off but always makes a significantly overconfident worker worse off. The unbiased worker is worse off when the rival is slightly overconfident and better off when the rival is significantly overconfident.

The assumption the overconfident worker perceives his marginal output is increasing with self-confidence describes situations where effort and ability are complements in generating output. However, this assumption might not always hold. Alternatively, the overconfident worker might perceive his marginal output is unaffected by self-confidence. Proposition 2 shows that in this case the overconfident worker is the Nash loser and the unbiased worker the Nash winner. Taken together, Propositions 1 and 2 provide conditions under which overconfidence raises or lowers a worker's chances of winning a bonus or being promoted.

Section 5 studies tournaments where an underconfident worker competes against an unbiased worker. Proposition 3 assumes the underconfident worker perceives his marginal output is increasing with self-confidence. Instead, Proposition 4 assumes the underconfident worker perceives his marginal output is unaffected by self-confidence. In both cases the underconfident worker is the Nash loser and the unbiased worker is the Nash winner. This happens because underconfidence leads the biased worker to *underestimate* his *marginal* probability of winning the tournament which leads him to exert less effort than his rival. Taken together, Propositions 3 and 4 show that underconfidence lowers a worker's chances of winning a bonus or being promoted. The firm is unambiguously worse off with an underconfident worker since both workers exert less effort than if both were unbiased.

Section 6 discusses two extensions of the model. first, it shows that in tournaments where the workers' best responses are monotonic, the overconfident worker is the Nash winner (loser) when effort and self-confidence are complements (substitutes). Second, it shows the main results still hold when the unbiased worker is unaware that the rival's bias.

The rest of the paper is organized as follows. Section 2 explains the paper's contribution to the literature. Section 3 sets-up the model. Sections 4 and 5 derive the results for overconfidence and underconfidence, respectively. Section 6 discusses two extensions of the model. Section 7 concludes the paper. All proofs are in the Appendix.

2 Contribution to the Literature

This study relates to three strands of literature. First, it contributes to the fast growing literature on gender and confidence.[6] Laboratory based evidence shows that gender is sometimes associated with differences in performance in tournaments and in self-confidence. Men tend to outperform women in mixed tournaments (Gneezy et al. 2003), men tend to enter tournaments more often than women (Niederle and Vesterlund 2007, Niederle et al. 2013, Buser et al. 2014, Möbius et al. 2022), and men are more confident than women in some tasks (Bengtsson et al. 2005, Dreber et al. 2014, Ring et al. 2016, Bordalo et al. 2019). Knowing whether men and women behave differently in tournaments is of great economic importance because

[6] Waldman (1994) provides an evolutionary explanation for gender differences in self-confidence. Murphy et al. (2015) show that overconfidence plays a role in mate competition and acquisition in the presence of intrasexual competition.

tournaments are ubiquitous, especially for selecting top managers and CEOs. This paper clarifies the conditions under which higher male self-confidence leads men to outperform women in tournaments. Consider a tournament where an overconfident man competes against an unbiased woman. Proposition 1 shows if the man perceives his marginal output is increasing with self-confidence and he is slightly overconfident, then he will exert more effort than the woman and hence he will be the Nash winner. In contrast, if the man perceives his marginal output is increasing with self-confidence and he is significantly overconfident, then he will exert less effort than the woman and hence he will be the Nash loser. Proposition 2 shows that if an overconfident man perceives his marginal output is unaffected by self-confidence, then the man is the Nash loser and the woman is the Nash winner. Taken together, Propositions 1 and 2 imply that when an overconfident man competes against an unbiased woman, the man is not necessarily more likely to be promoted than the woman. Now, consider a tournament where an underconfident woman competes against an unbiased man. Propositions 3 and 4 show that the woman will exert less effort than the man and hence she will be the Nash loser. Thus, when an underconfident woman competes against an unbiased man, the woman's chances of being promoted are always lower than the man's. Overall, these results shows that the type of self-confidence displayed by males and females plays a critical role in determining whether a man has a higher or lower chance of being promoted than a woman. These results also indicate that the most effective intervention to even the playing field is to debias underconfident women.

Second, this study contributes to the literature on tournaments with overconfident players. The most closely related papers are Goel and Thakor (2008) and Santos-Pinto (2010). Goel and Thakor (2008) study tournaments where overconfident and unbiased managers compete against each other to be promoted to CEO. The managers compete by choosing the level of risk of their projects and overconfident managers underestimate the risk of their projects. Goel and Thakor (2008) find that overconfident managers have a higher likelihood of being promoted to CEO than unbiased ones. My results show that the degree of manager overconfidence matters for the likelihood of being promoted to CEO. An overconfident manager has a higher (lower) probability of being promoted to CEO than an unbiased manager when he perceives his marginal output is increasing with self-confidence and when he is slightly (significantly) overconfident. My results differ from those of Goel and Thakor (2008) due to two reasons. First, here managers compete by choosing effort instead of risk. Second, here an overconfident manager overestimates his productivity of effort instead of underestimating the risk of his project. Santos-Pinto (2010) studies tournaments where all workers equally overestimate their productivity of effort. The main finding is that firms can be better off with an overconfident workforce if they wisely structure tournament prizes. Here I consider tournaments where workers display heterogeneity in beliefs and look at their implications for promotions and welfare taking tournament prizes fixed.

Third, this study also contributes to the literature on tournaments with heterogeneous players. This literature finds that greater heterogeneity between players tends to lower aggregate effort. Lazear and Rosen (1981) show that heterogeneity in effort costs leads to inefficient tournament outcomes. Weigelt et al. (1989) find that when one player has an unfair headstart over another, both players exert lower effort than symmetric players. Shotter and Weigelt (1992) study the impact of equal opportunity laws and affirmative actions on effort provision. They find that policies that increase the probability of winning for disadvantaged (high cost) workers reduce the effort they exert when heterogeneity is low but increase the effort exerted by both advantaged and disadvantaged workers when heterogeneity is high. Harbring and Lünser (2008) show that an increase in the prize spread raises effort provision in a tournament with heterogeneous competitors and that, for larger prize spreads, weaker competitors exert higher effort than in a tournament with identical competitors. Gürtler and Kräkel (2010) show that inefficiencies can arise if the firm sets uniform prizes (i.e., prizes that are independent from workers' identity) in a tournament with heterogeneous competitors while efficient effort provision can be induced if the firm sets individualized prizes. In this paper I consider tournaments where one worker is biased and the other one is unbiased. Proposition 1 shows show that heterogeneity in beliefs can raise aggregate effort when the biased worker is slightly overconfident. However, Propositions 2, 3, and 4 show that heterogeneity in beliefs lowers aggregate effort.

3 Set-up

Consider two workers, 1 and 2, competing in a tournament. The worker who produces the highest output receives the winner's prize y_W and the other receives the loser's prize y_L, with $0 < y_L < y_W$. The two workers have an identical productivity of effort, utility function, and outside option. However, they differ from one another in terms of the perception of their own productivity. Worker 1 is biased as he misperceives his productivity of effort. Worker 2 is unbiased since he has an accurate perception of his productivity of effort. Worker 1 is not aware of being biased while worker 2 is aware that worker 1 is biased. Finally, both workers correctly assess their utility functions and their outside options.

The workers are weakly risk averse and expected utility maximizers and have utility functions that are separable in income (y_i) and effort (a_i):

$$U_i(y_i, a_i) = u(y_i) - c(a_i),$$

for $i = 1, 2$. I assume u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' > 0$, $c(0) = 0$, $c'(0) = 0$, and $c(a_i) = \infty$, for $a_i \to \infty$, where the last two conditions ensure that equilibrium effort is strictly positive but finite. The two workers have outside options which guarantee each $\bar{u}$; so unless the perceived expected utility from participation is at least equal to $\bar{u}$, workers will not be willing to participate. Income y_i is equal to y_W if worker i wins the tournament and to y_L if worker i loses the tournament.

The output of worker i is a stochastic function of effort. Each level of effort of worker i induces a distribution over output given by

$$F_i(q_i|e_i(a_i, \omega)),$$

for $i = 1, 2$. Here $e_i(a_i, \omega)$ defines worker i's productivity as a function of effort a_i and the common environmental shock ω (e.g., the weather). Individual productivity strictly increases in effort, i.e., $e_i' > 0$, and marginal productivity is subject to diminishing returns to effort.

Worker 1's perceived productivity of effort is

$$e_1 = e_1(a_1, \omega, \lambda)$$

where λ is a parameter that captures worker 1's bias. Given worker 1's perceived productivity of effort, his perceived distribution over output is

$$F_1(q_1|e_1(a_1, \omega, \lambda)).$$

If worker 1 is overconfident, then $F_1(q_1|e_1(a_1, \omega, \lambda))$ first order stochastically dominates $F_1(q_1|e_1(a_1, \omega))$ for all levels of effort a_1: for each level of effort exerted, worker i believes he is more likely to produce a higher output than he actually does. If worker 1 is underconfident, then $F_1(q_1|e_1(a_1, \omega))$ first order stochastically dominates $F_1(q_1|e_1(a_1, \omega, \lambda))$ for all levels of effort a_1: for each level of effort exerted, worker 1 believes he is less likely to produce a higher output than he actually does.

The unbiased worker 2 has an accurate perception of his own productivity $e_2 = e_2(a_2, \omega)$ and thus his perceived and actual distribution over output coincide at $F_2(q_2|e_2(a_2, \omega))$. This implies that if worker 1 is overconfident and worker 2 is unbiased, then $F_1(q_1|e_1(a_1, \omega, \lambda))$ first order stochastically dominates $F_2(q_2|e_2(a_2, \omega))$ when $a_1 = a_2$. That is, worker 1 believes he is more likely to produce a higher output than worker 2 when both exert the same effort. Similarly, if worker 1 is underconfident and worker 2 is unbiased, then $F_2(q_2|e_2(a_2, \omega))$ first order stochastically dominates $F_1(q_1|e_1(a_1, \omega, \lambda))$ when $a_1 = a_2$. That is, worker 1 believes he is less likely to produce a higher output than worker 2 when both exert the same effort.

Let Q_1 denote worker 1's output and $\tilde{Q}_1$ worker 1's perceived output. Worker 1's objective probability of winning the tournament is

$$\Pr(Q_1 \geq q_2) = 1 - \Pr(Q_1 \leq q_2) = 1 - F_1(q_2|e_1(a_1, \omega)),$$

and his unconditional objective probability of winning the tournament is

$$P_1(a_1, a_2) = \Pr(Q_1 \geq Q_2) = \int [1 - F_1(q_2|e_1(a_1, \omega))] f_2(q_2|e_2(a_2, \omega)) dq_2.$$

Worker 1's perceived probability of winning the tournament is

$$\Pr(\tilde{Q}_1 \geq q_2) = 1 - \Pr(\tilde{Q}_1 \leq q_2) = 1 - F_1(q_2|e_1(a_1, \omega, \lambda)),$$

and his unconditional perceived probability of winning the tournament is

$$P_1(a_1, a_2, \lambda) = \Pr(\tilde{Q}_1 \geq Q_2) = \int [1 - F_1(q_2|e_1(a_1, \omega, \lambda))] f_2(q_2|e_2(a_2, \omega)) dq_2.$$

Worker 1's perceived expected utility is

$$E[U_1(a_1, a_2, \lambda)] = u(y_L) + P_1(a_1, a_2, \lambda)\Delta u - c(a_1),$$

where $\Delta u = u(y_W) - u(y_L)$.

The firm is risk neutral and correctly assesses the workers' productivity. The firm's profits are the difference between expected benefits and compensation costs:

$$E[\pi] = E[Q_1 + Q_2] - (y_L + y_W).$$

The timing of the events is as follows. The firm commits to a prize schedule. The workers decide whether or not to participate. All workers who agree to participate observe the realization of a common shock and then simultaneously and independently choose their effort levels. The firm observes the workers' output realizations and awards the prizes according to the prize schedule.

4 Overconfidence

This section studies tournaments where an overconfident worker competes against an unbiased worker. I specialize the model by assuming output is linearly additive in effort and an idiosyncratic shock (the common environmental shock is set to zero). That is, if worker i exerts effort a_i his output is given by

$$Q_i = a_i + \varepsilon_i, \; i = 1, 2, \tag{1}$$

where ε_i is a random variable with zero mean. The random variables ε_1 and ε_2 are identically and independently distributed and represent individualistic noise. This specification for output is chosen for its analytical simplicity and is often used in the tournament literature (see Lazear and Rosen 1981, Green and Stokey 1983, Akerlof and Holden 2012).[7] Throughout, the contract must be signed before ε_1, and ε_2 are known; the workers decide on a_1 and a_2, neither of which is observable to the firm. The probability distribution of ε_i is known to both firm and workers.

Worker 1 mistakenly perceives his stochastic production function to be equal to

$$\tilde{Q}_1 = \lambda a_1 + \varepsilon_1, \tag{2}$$

with $\lambda > 0$ and $\lambda \neq 1$. Under this specification worker 1 is overconfident when $\lambda > 1$ and underconfident when $\lambda \in (0, 1)$. Furthermore, worker 1 perceives his marginal output is increasing with self-confidence, that is, $\partial^2 \tilde{Q}_1 / \partial a_1 \partial \lambda > 0$. This describes situations where effort and ability are complements in generating output and where an overconfident (underconfident) worker overestimates (underestimates) his ability.[8]

[7] Nalebuff and Stiglitz (1983) consider an alternative output function: $Q_i = a_i \omega + \varepsilon_i$.

[8] Denote ability by $\theta > 0$. Let $Q_i = \theta a_i + \varepsilon_i$. The overconfident (underconfident) worker perceives $\tilde{Q}_1 = \lambda a_1 + \varepsilon_1$, where $\lambda > \theta$ ($0 < \lambda < \theta$). One can set $\theta = 1$ without loss of generality.

Worker 1 chooses effort to maximize his perceived expected utility:

$$\begin{aligned}
E[U_1(a_1, a_2, \lambda)] &= u(y_L) + P_1(a_1, a_2, \lambda)\Delta u - c(a_1) \\
&= u(y_L) + \Pr(\tilde{Q}_1 \geq Q_2)\Delta u - c(a_1) \\
&= u(y_L) + \Pr(\varepsilon_2 - \varepsilon_1 \leq a_1\lambda - a_2)\Delta u - c(a_1) \\
&= u(y_L) + G(\lambda a_1 - a_2)\Delta u - c(a_1). \qquad (3)
\end{aligned}$$

Worker 2 chooses effort to maximize his expected utility:

$$\begin{aligned}
E[U_2(a_1, a_2)] &= u(y_L) + P_2(a_1, a_2)\Delta u - c(a_2) \\
&= u(y_L) + \Pr(Q_2 \geq Q_1)\Delta u - c(a_2) \\
&= u(y_L) + \Pr(\varepsilon_2 - \varepsilon_1 \geq a_1 - a_2)\Delta u - c(a_1) \\
&= u(y_L) + [1 - G(a_1 - a_2)]\Delta u - c(a_2). \qquad (4)
\end{aligned}$$

Since the difference between the random variables ε_1 and ε_2 will be crucial, I define the random variable $x = \varepsilon_2 - \varepsilon_1$ with cumulative distribution function $G(x)$ and density $g(x)$. I assume $G(x)$ is continuous and twice differentiable. Because ε_1 and ε_2 are identically distributed, $g(x)$ is symmetric around zero. Additionally, $g(x)$ satisfies $g'(x) > 0$ for $x < 0$, and $g'(x) < 0$ for $x > 0$.[9]

The pure-strategy Nash equilibrium (a_1^*, a_2^*) satisfies the first-order conditions of the two workers simultaneously and is given by

$$\lambda g(\lambda a_1^* - a_2^*)\Delta u = c'(a_1^*), \qquad (5)$$

and

$$g(a_1^* - a_2^*)\Delta u = c'(a_2^*). \qquad (6)$$

The second-order conditions are satisfied when the cost function is sufficiently convex (see Appendix).

[9] For example, when ε_1 and ε_2 are normally distributed with mean 0 and variance σ^2, then x is normally distributed with mean 0 and variance $2\sigma^2$. When ε_1 and ε_2 are uniformly distributed with mean 0, then x follows a triangular distribution with mean 0. See, e.g., Drago et al. (1996), Hvide (2002), Chen (2003), among others.

Lemma 1: *Consider a tournament where worker i's output is given by (1), worker 1 is overconfident with a bias given by (2), and worker 2 is unbiased. In any pure-strategy Nash equilibrium of this tournament we have* $\lambda a_1^* > a_2^*$.

This result shows that in a pure-strategy Nash equilibrium, the product of the overconfident worker's perceived marginal productivity of effort and own effort is strictly greater than the effort of the unbiased worker. As we shall see, the impact of overconfidence on the pure-strategy equilibrium efforts depends on the size of worker 1's bias. The following definition will prove helpful to characterize the magnitude of worker 1's bias.

Definition 1: When worker 1's bias is given by (2) with $\lambda > 1$, worker 1 is said to be *slightly overconfident* if

$$\lambda < \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)}.$$

Conversely, worker 1 is said to be *significantly overconfident* if

$$\lambda \geq \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)}.$$

I denote the value of the threshold that determines whether worker 1 is either slightly or significantly overconfident by $\bar{\lambda}$. A necessary condition for $\bar{\lambda}$ to be greater than 0 is that $g'(\lambda a_1^* - a_2^*) < 0$, or, equivalently, $\lambda a_1^* > a_2^*$. As we have seen, Lemma 1 tells us that $\lambda a_1^* > a_2^*$ is always satisfied and so $\bar{\lambda} > 0$. A necessary condition for $\bar{\lambda}$ to be greater than 1 is that $g(\lambda a_1^* - a_2^*) + a_1^* g'(\lambda a_1^* - a_2^*) > 0$.

Proposition 1: *Consider a tournament where worker i's output is given by (1), worker 1 is overconfident with a bias given by (2), and worker 2 is unbiased.*
(i) The overconfident worker is the Nash winner when he is slightly overconfident, i.e., $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$ *when* $\lambda \in (1, \bar{\lambda}]$. *In this case, the overconfident worker exerts more effort and the unbiased worker exerts less effort than if both were unbiased. Furthermore, an increase in overconfidence raises the effort of the overconfident worker and lowers that of the unbiased worker, i.e.,* $\partial a_1^*/\partial\lambda > 0 > \partial a_2^*/\partial\lambda$.

(ii) The overconfident worker is the Nash loser when he is significantly overconfident, i.e., $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$ *when* $\lambda > \bar{\lambda}$. *In this case, both workers exert less effort than if both were unbiased, with the overconfident worker exerting the least effort. Furthermore, an increase in overconfidence lowers the efforts of both workers and more so that of the overconfident worker, i.e.,* $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

In the pure-strategy Nash equilibrium, the overconfident worker wins the tournament with probability $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ and the unbiased worker with probability $P_2(a_1^*, a_2^*) = 1 - G(a_1^* - a_2^*)$. When both workers are unbiased ($\lambda = 1$), the tournament is symmetric and the pure-strategy Nash equilibrium is $a_1^* = a_2^* = a^*$ where a^* solves $g(0)\Delta u = c'(a^*)$. Symmetry of $g(x)$ implies $P_1(a_1^*, a_2^*) = P_2(a_1^*, a_2^*) = G(0) = 1/2$. Hence, when both workers are unbiased, each is equally likely to win the tournament (i.e., the winner is purely random).

Proposition 1 shows that the identity of the Nash winner depends critically on the size of overconfident worker's bias. Part (i) tells us that a slightly overconfident worker exerts more effort than the unbiased worker and therefore is the Nash winner. In this case, the overconfident worker believes, mistakenly, he is slightly more productive than the unbiased worker. This raises the overconfident worker's perceived marginal probability of winning and leads him to exert more effort.[10] The unbiased worker, knowing that the overconfident worker will raise his effort, decides to lower his effort in response. Note that a slightly overconfident worker anticipates, correctly, he will be the Nash winner but overestimates his winning probability. In fact, the overconfident worker's perceived probability of winning $P_1(a_1^*, a_2^*, \lambda) = G(\lambda a_1^* - a_2^*)$ is greater than his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ since $\lambda > 1$.

Part (ii) tells us that a significantly overconfident worker exerts less effort than the unbiased worker and therefore is the Nash loser. In this case, the overconfident worker believes, mistakenly, he is significantly more productive than the unbiased

[10] The proof of Proposition 1 shows that whether worker 1's effort raises or falls with self-confidence is determined by the sign of $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)$. When $\lambda < \bar{\lambda}$ and $\lambda a_1^* > a_2^*$ we have $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$, that is, at the Nash equilibrium worker 1's effort is increasing with self-confidence.

worker. This lowers the overconfident worker's perceived marginal probability of winning and leads him to exert less effort.[11] The unbiased worker, knowing that the overconfident worker will lower his effort also decides to lower his effort but not as much as the overconfident worker. Interestingly, even though a significantly overconfident worker anticipates, correctly, he will exert less effort than the unbiased worker, he anticipates, incorrectly, he will be the Nash winner. This happens because the overconfident worker's perceived probability of winning the tournament $P_1(a_1^*, a_2^*, \lambda) = G(\lambda a_1^* - a_2^*)$ is greater than 1/2 (in equilibrium $\lambda a_1^* > a_2^*$) whereas his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ is less than 1/2 (in equilibrium $a_2^* > a_1^*$).

Proposition 1 has welfare implications. The firm is better off with a slightly overconfident worker since his increase in effort is greater than the decrease in effort of the unbiased worker. The firm is always worse off with a significantly overconfident worker since both workers exert less effort than if both were unbiased. Hence, the firm would not want to de-bias a slightly overconfident worker but would prefer to de-bias a significantly overconfident worker.

To evaluate the welfare implications for the overconfident worker I consider how his equilibrium objective expected utility

$$E\left[U_1(a_1^*, a_2^*)\right] = u(y_L) + G(a_1^* - a_2^*)\Delta u - c(a_1^*)$$

changes with λ:

$$\begin{aligned} \frac{\partial E\left[U_1(a_1^*, a_2^*)\right]}{\partial \lambda} &= g(a_1^* - a_2^*)\Delta u \left(\frac{\partial a_1^*}{\partial \lambda} - \frac{\partial a_2^*}{\partial \lambda}\right) - c'(a_1^*)\frac{\partial a_1^*}{\partial \lambda} \\ &= \left[g(a_1^* - a_2^*) - \lambda g(\lambda a_1^* - a_2^*)\right]\Delta u \frac{\partial a_1^*}{\partial \lambda} - g(a_1^* - a_2^*)\Delta u \frac{\partial a_2^*}{\partial \lambda}, \quad (7) \end{aligned}$$

where the second equality follows from the first-order condition of the overconfident worker. The first term on the right-hand side of (7) is the direct effect and the second term is the strategic effect. The direct effect is always negative because the

[11] When $\lambda > \bar{\lambda}$ and $\lambda a_1^* > a_2^*$ we have $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$, that is, at the Nash equilibrium worker 1's effort is decreasing with self-confidence.

overconfident worker fails to play a best response against his rival.[12] The sign of the strategic effect is negative when $\partial a_2^*/\partial\lambda > 0$ (worker 1 is significantly overconfident) and positive when $\partial a_2^*/\partial\lambda < 0$ (worker 1 is slightly overconfident). Hence, when worker 1 is significantly overconfident, the direct and the strategic effects are both negative and an increase in overconfidence always makes a significantly overconfident worker worse off. However, when worker 1 is slightly overconfident, the direct effect is negative and the strategic effect is positive. Therefore, an increase in overconfidence can make a slightly overconfident worker better off. This happens when the strategic effect dominates the direct effect.

To evaluate the welfare implications for the unbiased worker I consider how his equilibrium objective expected utility $E\left[U_2(a_1^*, a_2^*)\right]$ changes with λ:

$$\begin{aligned}\frac{\partial E\left[U_2(a_1^*, a_2^*)\right]}{\partial\lambda} &= -g(a_1^* - a_2^*)\Delta u\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right) - c'(a_2^*)\frac{\partial a_2^*}{\partial\lambda} \\ &= -g(a_1^* - a_2^*)\frac{\partial a_1^*}{\partial\lambda}\Delta u,\end{aligned}$$

where the second equality follows from the first-order condition of the unbiased worker. Hence, an increase in overconfidence makes the unbiased worker worse off when overconfidence raises the effort of the overconfident worker. This is the case when the rival is slightly overconfident. In contrast, an increase in overconfidence makes the unbiased worker better off when overconfidence lowers the effort of the overconfident worker. This is the case when the rival is significantly overconfident: the unbiased worker has a higher probability of winning the tournament and exerts less effort than if both workers were unbiased. The welfare results for the workers are in line with Heifetz et al. (2007) who show, using evolutionary game theory, that slightly overconfident agents will do better than unbiased as well as significantly overconfident agents and will gradually take over the whole population.

[12] When player 1 is slightly overconfident $a_1^* > a_2^*$ and $\partial a_1^*/\partial\lambda > 0$. The first-order conditions and $a_1^* > a_2^*$ imply $g(a_1^* - a_2^*) < \lambda g(\lambda a_1^* - a_2^*)$. Hence, when player 1 is slightly overconfident, the direct effect is negative. When player 1 is significantly overconfident $a_1^* < a_2^*$ and $\partial a_1^*/\partial\lambda < 0$. The first-order conditions and $a_1^* < a_2^*$ imply $g(a_1^* - a_2^*) > \lambda g(\lambda a_1^* - a_2^*)$. Hence, when player 1 is significantly overconfident, the direct effect is also negative.

The assumption the overconfident worker perceives his marginal output is increasing with self-confidence is critical for Proposition 1. To see this consider that if worker i exerts effort a_i his output is given by

$$Q_i = 1 + a_i + \varepsilon_i,\ \ i = 1, 2. \tag{8}$$

Furthermore, assume worker 1 mistakenly perceives his stochastic production function to be equal to

$$\tilde{Q}_1 = 1 + \lambda + a_1 + \varepsilon_1. \tag{9}$$

with $\lambda \neq 0$. Under this specification worker 1 is overconfident when $\lambda > 0$ and underconfident when $\lambda \in (-1, 0)$.[13] Furthermore, worker 1 perceives his marginal output is unaffected by self-confidence, that is, $\partial^2 \tilde{Q}_1 / \partial a_1 \partial \lambda = 0$. This describes situations where effort and ability are neither complements nor substitutes in generating output and where an overconfident (underconfident) worker overestimates (underestimates) his ability.[14]

Worker 1 chooses effort to maximize his perceived expected utility given by

$$E[U_1(a_1, a_2, \lambda)] = u(y_L) + G(\lambda + a_1 - a_2)\Delta u - c(a_1).$$

Worker 2 chooses effort to maximize his expected utility

$$E[U_2(a_1, a_2)] = u(y_L) + [1 - G(a_1 - a_2)]\Delta u - c(a_2).$$

The pure-strategy Nash equilibrium (a_1^*, a_2^*) satisfies the first-order conditions of the two workers simultaneously and is given by

$$g(\lambda + a_1^* - a_2^*)\Delta u = c'(a_1^*), \tag{10}$$

[13] The reason underconfidence is assumed to be bounded below is to rule out situations where the underconfident player expects to obtain a negative output if he exerts zero effort.

[14] Denote ability by $\theta > 0$. Let $Q_i = \theta + a_i + \varepsilon_i$. The overconfident (underconfident) worker perceives $\tilde{Q}_1 = \theta + \lambda + a_1 + \varepsilon_1$, where $\lambda > 0$ $(-\theta < \lambda < 0)$. One can set $\theta = 1$ without loss of generality.

and

$$g(a_1^* - a_2^*)\Delta u = c'(a_2^*). \tag{11}$$

The second-order conditions are satisfied when the cost function is sufficiently convex (see Appendix).

Proposition 2: *Consider a tournament where worker i's output is given by (8), worker 1 is overconfident with a bias given by (9), and worker 2 is unbiased. The overconfident worker is the Nash loser, that is, $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$. Both workers exert less effort than if both were unbiased, with the overconfident worker exerting the least effort. Furthermore, an increase in the bias lowers the efforts of both workers and more so that of the overconfident worker.*

Proposition 2 shows that an overconfident worker who perceives his marginal output is unaffected by self-confidence has a lower probability of winning the tournament than an unbiased worker. The assumption the overconfident worker perceives his marginal output is unaffected by self-confidence implies that an increase in the overconfident worker's bias lowers his perceived marginal probability of winning the tournament. This leads the overconfident worker to exert less effort. The unbiased worker, knowing that the overconfident worker will lower his effort, also decides to lower his effort but not as much. This implies that the overconfident worker exerts less effort than the unbiased worker and therefore is the Nash loser. Interestingly, even though an overconfident worker anticipates, correctly, he will exert less effort than the unbiased worker, he anticipates, incorrectly, he will be the Nash winner. This happens because the overconfident worker's perceived probability of winning the tournament $P_1(a_1^*, a_2^*, \lambda) = G(\lambda + a_1^* - a_2^*)$ is greater than 1/2 (in equilibrium $\lambda + a_1^* > a_2^*$) whereas his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ is less than 1/2 (in equilibrium $a_2^* > a_1^*$).

Proposition 2 also has welfare consequences. The firm is worse off when worker 1 is overconfident since both workers exert less effort than if both were unbiased. An increase in overconfidence can make the overconfident worker better off (worse of) when the positive strategic effect is larger (smaller) than the negative direct effect.

An increase in overconfidence makes the unbiased worker better off: he has a higher probability of winning the tournament and exerts less effort than if both workers were unbiased.

5 Underconfidence

This section studies tournaments where an underconfident worker competes against an unbiased worker. Output is given by (1), worker 1 is underconfident with a bias given by (2), and worker 2 is unbiased.

Proposition 3: *Consider a tournament where worker i's output is given by (1), worker 1 is underconfident with a bias given by (2), and worker 2 is unbiased. The underconfident worker is the Nash loser, i.e.,* $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$*, both workers exert less effort than if both were unbiased, with the underconfident worker exerting the least effort. Furthermore, an increase in underconfidence lowers the efforts of both workers and more so that of the underconfident worker, i.e.,* $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

This results shows that if an underconfident worker perceives his marginal output is increasing with self-confidence, then he is always the Nash loser. The underconfident worker believes, mistakenly, he is less productive than the unbiased worker. This lowers the underconfident worker's perceived marginal probability of winning and leads him to exert less effort. The unbiased worker, knowing the underconfident worker will lower his effort, decides to lower his effort in response. Note that an underconfident worker anticipates, correctly, he will be the Nash loser but underestimates his winning probability. In fact, the underconfident worker's perceived probability of winning $P_1(a_1^*, a_2^*, \lambda) = G(\lambda a_1^* - a_2^*)$ is smaller than his objective probability of winning $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$ since $\lambda < 1$.

Proposition 3 has welfare consequences. The firm is always worse off when worker 1 is underconfident since both workers exert less effort than if both were unbiased. Hence, the firm would want to de-bias an underconfident worker. An increase in underconfidence has an ambiguous effect on the underconfident worker's welfare since the direct effect is negative but the strategic effect is positive. An increase in underconfidence makes the unbiased worker better off: he has a higher probability of winning the tournament and exerts less effort than if both workers were unbiased.

Finally, I study a tournament where output is given by (8), worker 1 is underconfident with a bias given by (9), and worker 2 is unbiased.

Proposition 4: *Consider a tournament where worker i's output is given by (8), worker 1 is underconfident with a bias given by (9), and worker 2 is unbiased. The underconfident worker is the Nash loser, that is,* $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$. *Both workers exert less effort than if both were unbiased, with the underconfident worker exerting the least effort. Furthermore, an increase in the bias lowers the efforts of both workers and more so that of the underconfident worker.*

Proposition 4 shows that an underconfident worker who perceives his marginal output is unaffected by self-confidence has a lower probability of winning the tournament than an unbiased worker. Moreover, the same result also holds when the underconfident worker perceives his marginal output is decreasing with self-confidence.[15] Hence, the result that underconfidence lowers effort provision of both workers and more so that of the underconfident worker holds generally. The reason is straightforward. Underconfidence leads to a downward shift of the best response of the underconfident worker. This implies a movement along the upward sloping part of the best response of the unbiased worker. Hence, it can only be that both workers' efforts go down with an increase in underconfidence. Together, Propositions 3 and 4 show that underconfidence always lowers a worker's chances of being promoted.

[15] An underconfident worker perceives his marginal output is decreasing with self-confidence when $\partial^2 \tilde{Q}_i / \partial a_i \partial \lambda_i < 0$. This can be modeled by assuming $Q_i = 1 + a_i + \varepsilon_i$, $i = 1, 2$, and $\tilde{Q}_1 = (1-\gamma)a_1 + \varepsilon_1$, where $\gamma \in (0, 1)$. This situation is isomorphic to assuming $Q_i = a_i + \varepsilon_i$, $i = 1, 2$, and $\tilde{Q}_1 = \lambda a_1 + \varepsilon_1$, with $\lambda \in (0, 1)$.

6 Extensions

This section discusses two extensions of the model. First, tournaments where the workers' best responses are monotonic. Second, tournaments where the unbiased worker is unaware of the rival's bias.

6.1 Monotonic Best Responses

In the tournaments studied so far workers' best responses are non-monotonic. More precisely, for a given low effort of the rival, a worker reacts to an increase in effort of the rival by increasing effort but, given high effort of the rival, a worker reacts to an increase in effort of the rival by decreasing effort (e.g., Lazear and Rosen 1981, Green and Stokey 1983, Akerlof and Holden 2012). However, there exist tournaments where workers' best responses are monotonic. For example, when efforts are strategic complements (substitutes), each worker best responds in a monotone increasing (decreasing) way to an increase in the rival's effort (e.g., Nalebuff and Stiglitz 1983, Santos-Pinto 2010).

Appendix B studies tournaments where workers' best responses are monotonic and where a biased worker competes against an unbiased worker. Proposition 5 shows that the overconfident worker is the Nash winner when his effort and self-confidence are complements. In this case, the overconfident worker overestimates his *marginal* probability of winning the tournament and hence exerts higher effort than the unbiased worker. In contrast, Proposition 6 shows that the overconfident worker is the Nash loser when his effort and self-confidence are substitutes. In this case, the overconfident worker underestimates his *marginal* probability of winning the tournament and hence exerts lower effort than the unbiased worker. These two results hold regardless of whether workers' efforts are strategic complements or substitutes.

Propositions 5 and 6 also carry welfare implications. Overconfidence makes the firm better off (worse off) when the overconfident worker's effort and self-confidence are complements (substitutes). Overconfidence can make the overconfident worker better off when it lowers the effort of the unbiased worker but always makes the overconfident worker worse off when it raises the effort of the unbiased worker. The unbiased worker is worse off (better off) when the overconfident worker's effort and self-confidence are complements (substitutes).

6.2 Unbiased Worker Unaware of the Rival's Bias

I now discuss tournaments where a biased worker competes against an unbiased worker who, contrary to what has been assumed so far, is unaware of the rival's bias. This is relevant for tournaments where workers' biases are not observable by their rivals. Since the unbiased worker is unaware of the rival's bias, he selects the same effort as he would if both workers were unbiased. As we shall see, all the results obtained in terms of the identity of the Nash winner and loser are left unchanged. However, the winning probabilities of each worker change.

Consider a tournament where an overconfident worker 1 competes against an unbiased worker 2 and where the overconfident worker perceives his marginal output is increasing with self-confidence. We know from Proposition 1 that if the overconfident worker is slightly overconfident, the workers' equilibrium efforts satisfy $a_1^* > a^N > a_2^*$, where a^N denotes the Nash equilibrium effort if both workers are unbiased. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) > 1/2 > P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash winner. However, the winning probability of the overconfident worker 1 is not as high when the unbiased worker 2 is unaware that 1 is overconfident since $a^N > a_2^*$ implies $P_1(a_1^*, a^N) < P_1(a_1^*, a_2^*)$. We also know from Proposition 1 that if the overconfident worker is significantly overconfident, the workers' equilibrium efforts satisfy $a^N > a_2^* > a_1^*$. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) < 1/2 < P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash loser. However, the winning probability of the overconfident worker 1 is even lower when the unbiased worker 2 is unaware that 1 is overconfident since $a^N > a_2^*$ implies $P_1(a_1^*, a^N) < P_1(a_1^*, a_2^*)$.

Consider a tournament where an underconfident worker 1 competes against an unbiased worker 2 and where the underconfident worker perceives his marginal output is increasing with self-confidence. We know from Proposition 3 that the workers' equilibrium efforts satisfy $a^N > a_2^* > a_1^*$. If the unbiased worker 2 is unaware of worker 1's underconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) < 1/2 < P_2(a_1^*, a^N)$. Hence, the underconfident worker 1 is still the Nash loser. However, the winning probability of the underconfident worker 1 is even lower when the unbiased worker 2 is unaware that 1 is underconfident since $a^N > a_2^*$ implies $P_1(a_1^*, a^N) < P_1(a_1^*, a_2^*)$.

Consider a tournament where workers' best responses are monotonic, effort and overconfidence are complements, and efforts are strategic complements. Proposition 5 in Appendix B shows that in this tournament the workers' equilibrium efforts satisfy $a_1^* > a_2^* > a^N$. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) > 1/2 > P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash winner. However, the winning probability of the overconfident worker 1 is even higher when the unbiased worker 2 is unaware that 1 is overconfident since $a^N < a_2^*$ implies $P_1(a_1^*, a^N) > P_1(a_1^*, a_2^*)$. In a tournament where workers' best responses are monotonic, effort and overconfidence are complements, and efforts are strategic substitutes, the workers' equilibrium efforts satisfy $a_1^* > a^N > a_2^*$. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) > 1/2 > P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash winner. However, the winning probability of the overconfident worker 1 is not has high when the unbiased worker 2 is unaware that 1 is overconfident since $a^N > a_2^*$ implies $P_1(a_1^*, a^N) < P_1(a_1^*, a_2^*)$.

Finally, consider a tournament where workers' best responses are monotonic, effort and overconfidence are substitutes, and efforts are strategic complements. Proposition 6 in Appendix B shows that in this tournament the workers' equilibrium efforts satisfy $a_1^N > a_2^* > a_1^*$. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) < 1/2 < P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash loser. However, the winning probability of the overconfident worker 1 is even lower when the unbiased worker 2 is unaware that 1 is overconfident since $a^N > a_2^*$ implies $P_1(a_1^*, a^N) < P_1(a_1^*, a_2^*)$. In a tournament where workers' best responses are monotonic, effort and overconfidence are substitutes, and efforts are strategic substitutes, the workers' equilibrium efforts satisfy $a_2^* > a^N > a_1^*$. If the unbiased worker 2 is unaware of worker 1's overconfidence he will choose a^N. This implies $P_1(a_1^*, a^N) < 1/2 < P_2(a_1^*, a^N)$. Hence, the overconfident worker 1 is still the Nash loser. However, the winning probability of the overconfident worker 1 is not as low when the unbiased worker 2 is unaware that 1 is overconfident since $a^N < a_2^*$ implies $P_1(a_1^*, a^N) > P_1(a_1^*, a_2^*)$.

7 Conclusion

This paper investigates whether overconfident workers are more likely to win bonuses or be promoted. Knowing whether overconfident individuals behave differently in tournaments is of great economic importance because tournaments are ubiquitous, especially for selecting top managers and CEOs. Moreover, laboratory based evidence shows that gender differences in self-confidence play a critical role for entry and performance in tournaments. To address this question I consider a tournament where a biased worker competes against an unbiased worker. The biased worker can be either overconfident or underconfident. An overconfident (underconfident) worker overestimates (underestimates) his productivity of effort and, as a consequence, his winning probability.

The paper finds that if an overconfident worker perceives his marginal output is increasing with self-confidence, then the identity of the Nash winner depends critically on the size of the bias. The overconfident worker is the Nash winner when he is slightly overconfident. In contrast, the overconfident worker is the Nash loser when he is significantly overconfident. This result shows that if an overconfident man competes against an unbiased woman, then the man is not necessarily more likely to be promoted than the woman. In addition, the paper shows that when an underconfident worker competes against an unbiased worker the underconfident worker is always the Nash loser. This result shows that if an underconfident woman competes against an unbiased man, then the woman's chances of being promoted are always lower than the man's. Overall, these results shows that the type of self-confidence displayed by males and females plays a critical role in determining whether a man has a higher or lower chance of being promoted than a woman. These results also suggest that the most effective intervention to even the playing field is to debias underconfident women.

All results were derived under the assumption that tournament prizes are exogenously specified. However, if the firm is aware that one worker is biased it can select the winning and losing prizes optimally taking into account how the bias affects workers' effort provision. Santos-Pinto (2010) shows that in a tournament where all workers are equally overconfident, the firm can counteract any adverse impact of overconfidence on effort by raising the prize spread. Similarly, in a tournament where an overconfident worker competes against an unbiased worker, the firm can raise the prize spread to counteract an unfavorable effect of overconfidence on effort. However, this comes at a cost when workers are risk averse since they need to be compensated for the increase in risk associated with a higher prize spread. Understanding how the firm endogenously selects tournament prizes when one worker is overconfident and the other is unbiased is an interesting avenue for future research.

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Appendix A: Proofs

Second-Order Conditions: The first-order conditions of workers 1 and 2 are

$$\frac{\partial E[U_1(a_1, a_2, \lambda)]}{\partial a_1} = \lambda g(\lambda a_1 - a_2)\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1, a_2)]}{\partial a_2} = g(a_1 - a_2)\Delta u - c'(a_2) = 0,$$

respectively. Hence, the second-order conditions of workers 1 and 2 are

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda)]}{\partial a_1^2} = \lambda^2 g'(\lambda a_1 - a_2)\Delta u - c''(a_1) < 0, \tag{12}$$

and

$$\frac{\partial^2 E[U_2(a_1, a_2)]}{\partial a_2^2} = -g'(a_1 - a_2)\Delta u - c''(a_2) < 0, \tag{13}$$

respectively. A sufficient condition for a pure-strategy Nash equilibrium to exist is that

$$\lambda^2 g'(\lambda a_1 - a_2)\Delta u < c''(a_1), \forall a_1, a_2, \lambda$$

and

$$-g'(a_1 - a_2)\Delta u < c''(a_2), \forall a_1, a_2,$$

that is, the cost functions are sufficiently convex.

Proof of Lemma 1: Assume worker 1 is overconfident and worker 2 is unbiased. Let (a_1^*, a_2^*) denote the pure-strategy equilibrium of the tournament. This implies that (a_1^*, a_2^*) solves $\lambda g(\lambda a_1^* - a_2^*)\Delta u = c'(a_1^*)$ and $g(a_1^* - a_2^*)\Delta u = c'(a_2^*)$. Suppose, by contradiction, that $\lambda a_1^* \leq a_2^*$. Note that $\lambda > 1$ and $\lambda a_1^* \leq a_2^*$ imply $a_1^* < \lambda a_1^* \leq a_2^*$. This, in turn, implies $a_1^* - a_2^* < \lambda a_1^* - a_2^* \leq 0$. Since $g'(x) > 0$ for $x < 0$ this implies $g(a_1^* - a_2^*) < g(\lambda a_1^* - a_2^*)$. However, $\lambda > 1$, $g(a_1^* - a_2^*) < g(\lambda a_1^* - a_2^*)$, and the two first-order conditions imply $c'(a_1^*) > c'(a_2^*)$ or $a_1^* > a_2^*$, a contradiction. Hence, it must be that $\lambda a_1^* > a_2^*$.

Proof of Proposition 1: Assume worker 1 is overconfident and worker 2 is unbiased. The impact of overconfidence on the pure-strategy equilibrium efforts is obtained from total differentiation of the first-order conditions (5) and (6):

$$\partial\lambda g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)(a_1^*\partial\lambda + \lambda\partial a_1^* - \partial a_2^*)\Delta u = c''(a_1^*)\partial a_1^*$$

and

$$g'(a_1^* - a_2^*)(\partial a_1^* - \partial a_2^*)\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial\lambda$ we obtain

$$g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)\left(a_1^* + \lambda\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}, \tag{14}$$

and

$$g'(a_1^* - a_2^*)\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}. \tag{15}$$

Solving (15) for $\partial a_2^*/\partial\lambda$ we have

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}. \tag{16}$$

Substituting (16) into (14) we obtain

$$g(\lambda a_1^* - a_2^*)\Delta u + \lambda g'(\lambda a_1^* - a_2^*)\left[a_1^* + \lambda\frac{\partial a_1^*}{\partial\lambda} - \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}\right]\Delta u$$
$$= c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}.$$

Solving this equation for $\partial a_1^*/\partial\lambda$ we obtain

$$\frac{\partial a_1^*}{\partial\lambda} = \frac{1}{D^*}\left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u, \tag{17}$$

where

$$D^* = \left[\lambda^2 g'(\lambda a_1^* - a_2^*)\Delta u - c''(a_1^*)\right]\left[-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*)\right]$$
$$+ \lambda g'(\lambda a_1^* - a_2^*)g'(a_1^* - a_2^*)(\Delta u)^2.$$

Substituting (17) into (16) we obtain

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{1}{D^*}g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right](\Delta u)^2. \tag{18}$$

Note that the two terms inside square brackets in D^* are the second-order conditions of workers 1 and 2, respectively, and their signs are negative. Hence, the sign of the product of the terms inside square brackets is positive. Assume $a_1^* > a_2^*$. This implies $g'(a_1^* - a_2^*) < 0$. Furthermore, since $\lambda > 1$, $a_1^* > a_2^*$ also implies $\lambda a_1^* > a_2^*$ which, in turn, implies $g'(\lambda a_1^* - a_2^*) < 0$. This, in turn, implies the last term in D^* is positive. Hence, if $a_1^* > a_2^*$, then $D^* > 0$. Now, assume $a_2^* > \lambda a_1^* > a_1^*$. This implies $g'(a_1^* - a_2^*) > 0$ and $g'(\lambda a_1^* - a_2^*) > 0$. This, in turn, implies the last term in D^* is positive. Hence, if $a_2^* > \lambda a_1^* > a_1^*$, then $D^* > 0$. We are left with the case $\lambda a_1^* > a_2^* > a_1^*$. This implies $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$. Hence, the last term in D^* is negative. However, rearranging D^* we obtain

$$\begin{aligned} D^* &= \lambda(1-\lambda)g'(\lambda a_1^* - a_2^*)g'(a_1^* - a_2^*)(\Delta u)^2 - \lambda^2 g'(\lambda a_1^* - a_2^*)c''(a_2^*)\Delta u \\ &\quad + g'(a_1^* - a_2^*)c''(a_1^*)\Delta u + c''(a_1^*)c''(a_2^*). \end{aligned} \tag{19}$$

When $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, the first term in (19) is positive since $\lambda > 1$. The second and third terms in (19) are also positive since $g'(\lambda a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, respectively. Finally, the fourth term in (19) is also positive since $c'' > 0$. Hence, if $\lambda a_1^* > a_2^* > a_1^*$, then $D^* > 0$. Thus, we have shown that $D^* > 0$. I now consider the four possible ways an increase in worker 1's overconfidence can change the pure-strategy Nash equilibrium efforts and show that only two of them are feasible.

(i) Assume $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda > 0$. If $\partial a_1^*/\partial\lambda > 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$. If $\partial a_2^*/\partial\lambda > 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$, then $g'(a_1^* - a_2^*) > 0$. Now, $g'(a_1^* - a_2^*) > 0$ and $g'(x) > 0$ for $x < 0$ implies $a_2^* > a_1^*$. This, in turn, implies $\partial a_2^*/\partial\lambda > \partial a_1^*/\partial\lambda$ or

$$\begin{aligned} & g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right](\Delta u)^2 \\ > \; & \left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u, \end{aligned}$$

or

$$g'(a_1^* - a_2^*)\Delta u > g'(a_1^* - a_2^*)\Delta u + c''(a_2^*),$$

or

$$c''(a_2^*) < 0,$$

which contradicts $c'' > 0$. Hence, $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda > 0$ do not characterize the impact of worker 1's overconfidence on the Nash equilibrium efforts.

(ii) Assume $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$. This implies $a_1^* < a_2^*$, which, in turn, implies $g'(a_1^* - a_2^*) > 0$. If $\partial a_1^*/\partial\lambda < 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$. If $\partial a_2^*/\partial\lambda > 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$, then $g'(a_1^* - a_2^*) < 0$. But $g'(a_1^* - a_2^*) < 0$ contradicts $g'(a_1^* - a_2^*) > 0$. Hence, $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$ do not characterize the impact of worker 1's overconfidence on the Nash equilibrium efforts.

(iii) Assume $\partial a_2^*/\partial\lambda < 0 < \partial a_1^*/\partial\lambda$. This implies $a_1^* > a_2^*$. This, in turn, implies $g'(a_1^* - a_2^*) < 0$. Furthermore, $a_1^* > a_2^*$ and $\lambda > 1$ imply $\lambda a_1^* > a_2^*$. This, in turn, implies $g'(\lambda a_1^* - a_2^*) < 0$. Since $D^* > 0$, for $\partial a_1^*/\partial\lambda > 0$ it must be that

$$g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0,$$

or

$$\lambda < \frac{g(\lambda a_1^* - a_2^*)}{-a_1^* g'(\lambda a_1^* - a_2^*)} = \bar{\lambda}.$$

Note that $g(\lambda a_1^* - a_2^*) > 0$ and $g'(\lambda a_1^* - a_2^*) < 0$ imply that $\bar{\lambda}$ is strictly positive. Furthermore, $g(\lambda a_1^* - a_2^*) + a_1^* g'(\lambda a_1^* - a_2^*) > g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) > 0$ implies $\bar{\lambda} > 1$. Hence, $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda < 0$ characterize the impact of worker 1's overconfidence on the Nash equilibrium efforts when $\lambda \in (1, \bar{\lambda})$.

(iv) Assume $\partial a_1^*/\partial\lambda < 0$ and $\partial a_2^*/\partial\lambda < 0$. If $\partial a_1^*/\partial\lambda < 0$ and $D^* > 0$, then $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$. A necessary (but not sufficient) condition for $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$ is that $g'(\lambda a_1^* - a_2^*) < 0$. If $g'(\lambda a_1^* - a_2^*) < 0$, then $\lambda a_1^* > a_2^*$. If $\partial a_2^*/\partial\lambda < 0$, $D^* > 0$, and $g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*) < 0$, then $g'(a_1^* - a_2^*) > 0$. Furthermore, if $g'(a_1^* - a_2^*) > 0$, then $a_2^* > a_1^*$. Hence, we have $\lambda a_1^* > a_2^* > a_1^*$. This, in turn, implies $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda$ or

$$\begin{aligned} & \left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right]\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right]\Delta u \\ < \ & g'(a_1^* - a_2^*)\left[g(\lambda a_1^* - a_2^*) + \lambda a_1^* g'(\lambda a_1^* - a_2^*)\right](\Delta u)^2, \end{aligned}$$

or

$$g'(a_1^* - a_2^*)\Delta u + c''(a_2^*) > g'(a_1^* - a_2^*)\Delta u,$$

or

$$c''(a_2^*) > 0,$$

which is true. Hence, $\partial a_1^*/\partial\lambda < 0$ and $\partial a_2^*/\partial\lambda < 0$ characterize the impact of worker 1's overconfidence on the Nash equilibrium efforts when $\lambda > \bar{\lambda}$.

I now show that worker 1 has a higher (lower) probability of winning if he is slightly (significantly) overconfident. Worker 1's probability of winning is $P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*)$. Worker 2's probability of winning is $P_2(a_1^*, a_2^*) = 1 - G(a_1^* - a_2^*)$. We have $G(0) = 0$ and $G' > 0$. When $\lambda = 1$ the tournament is symmetric and the pure-strategy Nash equilibrium is $a_1^* = a_2^* = a$. Symmetry of $g(x)$ implies that $P_1(a_1^*, a_2^*) = P_2(a_1^*, a_2^*) = G(0) = 1/2$. If worker 1 is slightly overconfident, then $\partial a_2^*/\partial\lambda < 0 < \partial a_1^*/\partial\lambda$ which implies $a_1^* > a_2^*$. Hence, if worker 1 is slightly overconfident, then

$$P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*) > 1/2 > 1 - G(a_1^* - a_2^*) = P_2(a_1^*, a_2^*).$$

In contrast, if worker 1 is significantly overconfident, then $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$ which implies $a_1^* < a_2^*$. Hence, if worker 1 is significantly overconfident, then

$$P_1(a_1^*, a_2^*) = G(a_1^* - a_2^*) < 1/2 < 1 - G(a_1^* - a_2^*) = P_2(a_1^*, a_2^*).$$

Proof of Proposition 2: The first-order conditions of workers 1 and 2 are

$$\frac{\partial E[U_1(a_1, a_2, \lambda)]}{\partial a_1} = g(\lambda + a_1 - a_2)\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1, a_2)]}{\partial a_2} = g(a_1 - a_2)\Delta u - c'(a_2) = 0,$$

respectively. The second-order conditions of workers 1 and 2 are

$$\frac{\partial^2 E[U_1(a_1, a_2, \lambda)]}{\partial a_1^2} = g'(\lambda + a_1 - a_2)\Delta u - c''(a_1) < 0, \tag{20}$$

and

$$\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2} = -g'(a_1 - a_2)\Delta u - c''(a_2) < 0, \tag{21}$$

respectively. Hence, a sufficient condition for a pure-strategy Nash equilibrium to exist is that

$$g'(\lambda + a_1 - a_2)\Delta u < c''(a_1), \forall a_1, a_2, \lambda$$

and

$$-g'(a_1 - a_2)\Delta u < c''(a_2), \forall a_1, a_2,$$

that is, the cost functions are sufficiently convex. Assume worker 1 is overconfident and worker 2 is unbiased. Let (a_1^*, a_2^*) denote the pure-strategy Nash equilibrium of the tournament, that is, (a_1^*, a_2^*) solves $g(\lambda + a_1^* - a_2^*)\Delta u = c'(a_1^*)$ and $g(a_1^* - a_2^*)\Delta u = c'(a_2^*)$. Suppose, by contradiction, $\lambda + a_1^* - a_2^* \leq 0$. Since $\lambda > 0$ we have $0 < \lambda \leq a_2^* - a_1^*$. This implies $a_2^* > a_1^*$. If $a_2^* > a_1^*$, then $c'(a_2^*) > c'(a_1^*)$. Hence, for the first-order conditions to be satisfied it must be that $g(a_1^* - a_2^*)\Delta u > g(\lambda + a_1^* - a_2^*)\Delta u$ or $g(a_1^* - a_2^*) > g(\lambda + a_1^* - a_2^*)$. This is a contradiction since $a_1^* - a_2^* < \lambda + a_1^* - a_2^* \leq 0$ and $g'(x) > 0$ for $x < 0$ imply $g(a_1^* - a_2^*) < g(\lambda + a_1^* - a_2^*)$. Thus, it must be that $\lambda + a_1^* - a_2^* > 0$. Now, suppose, by contradiction, $a_1^* \geq a_2^*$. If $a_1^* \geq a_2^*$, then $c'(a_1^*) \geq c'(a_2^*)$. Hence, for the first-order conditions to be satisfied it must be that $g(\lambda + a_1^* - a_2^*)\Delta u \geq g(a_1^* - a_2^*)\Delta u$ or $g(\lambda + a_1^* - a_2^*) \geq g(a_1^* - a_2^*)$. This is a contradiction since $\lambda + a_1^* - a_2^* > a_1^* - a_2^* \geq 0$ and $g'(x) < 0$ for $x > 0$ imply $g(\lambda + a_1^* - a_2^*) < g(a_1^* - a_2^*)$. Thus, it must be that $a_2^* > a_1^*$. Finally, suppose, by contradiction, $a_2^* \geq a^N$ where a^N is the solution to $g(0)\Delta u = c'(a^N)$. If $a_2^* \geq a^N$, then $c'(a_2^*) \geq c'(a^N)$. This, in turn, implies $g(a_1^* - a_2^*)\Delta u \geq g(0)\Delta u$ or $g(a_1^* - a_2^*) \geq g(0)$. This is a contradiction: the inequality $g(a_1^* - a_2^*) \geq g(0)$ can never be satisfied since $a_1^* < a_2^*$ and $g'(x) > 0$ for $x < 0$ imply $g(a_1^* - a_2^*) < g(0)$. Thus, it must be that $a_2^* < a^N$. Putting together the last two results we have that if $\lambda > 1$, then (a_1^*, a_2^*) satisfies

$$a^N > a_2^* > a_1^*.$$

Hence, worker 1's overconfidence lowers the efforts of both workers and more so that of the overconfident worker: $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

Proof of Proposition 3: Assume worker 1 is underconfident and worker 2 is unbiased. Let (a_1^*, a_2^*) denote the pure-strategy Nash equilibrium of the tournament, that is, (a_1^*, a_2^*) solves $\lambda g(\lambda a_1^* - a_2^*)\Delta u = c'(a_1^*)$ and $g(a_1^* - a_2^*)\Delta u = c'(a_2^*)$. Suppose, by contradiction, that $a_1^* \geq a^N$ where a^N is the solution to $g(0)\Delta u = c'(a^N)$. If $a_1^* \geq a^N$, then $c'(a_1^*) \geq c'(a^N)$. This, in turn, implies $\lambda g(\lambda a_1^* - a_2^*)\Delta u \geq g(0)\Delta u$ or $\lambda g(\lambda a_1^* - a_2^*) \geq g(0)$. This is a contradiction: the inequality $\lambda g(\lambda a_1^* - a_2^*) \geq g(0)$ can never be satisfied since $\lambda \in (0, 1)$ and $g(\lambda a_1^* - a_2^*) \leq g(0)$. Hence, it must be that $a_1^* < a^N$. Suppose, by contradiction, that $a_2^* \geq a^N$. If $a_2^* \geq a^N$, then $c'(a_2^*) \geq c'(a^N)$. This, in turn, implies $g(a_1^* - a_2^*)\Delta u \geq g(0)\Delta u$ or $g(a_1^* - a_2^*) \geq g(0)$. This is a contradiction: the inequality $g(a_1^* - a_2^*) \geq g(0)$ can never be satisfied since $a_1^* < a^N \leq a_2^*$ and $g'(x) > 0$ for $x < 0$ imply $g(a_1^* - a_2^*) < g(0)$. Hence, it must be that $a_2^* < a^N$. This shows that if worker 1 is underconfident and worker 2 is unbiased, then both workers exert less effort than if both were unbiased. This rules out $\partial a_1^*/\partial\lambda > 0$ and $\partial a_2^*/\partial\lambda > 0$. Therefore, there is only one candidate to characterize the impact of worker 1's underconfidence on the Nash equilibrium efforts for any $\lambda \in (0, 1)$: $\partial a_1^*/\partial\lambda < 0$ and $\partial a_2^*/\partial\lambda < 0$. Note that if $\partial a_1^*/\partial\lambda < 0$, $\partial a_2^*/\partial\lambda < 0$, and (16) imply $g'(a_1^* - a_2^*) > 0$. This, in turn, implies $a_1^* < a_2^*$ and $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$. This shows that an increase in underconfidence lowers the efforts of both workers but more so that of the underconfident worker. Since the underconfident worker always exerts less effort than the unbiased worker, the underconfident worker is the Nash loser and the unbiased worker the Nash winner.

Proof of Proposition 4: Assume worker 1 is underconfident and worker 2 is unbiased. Let (a_1^*, a_2^*) denote the pure-strategy Nash equilibrium of the tournament, that is, (a_1^*, a_2^*) solves $g(-\lambda + a_1^* - a_2^*)\Delta u = c'(a_1^*)$ and $g(a_1^* - a_2^*)\Delta u = c'(a_2^*)$. The impact of underconfidence on the pure-strategy equilibrium efforts is obtained from total differentiation of the first-order conditions:

$$g'(-\lambda + a_1^* - a_2^*)(-\partial\lambda + \partial a_1^* - \partial a_2^*)\Delta u = c''(a_1^*)\partial a_1^*$$

and

$$g'(a_1^* - a_2^*)(\partial a_1^* - \partial a_2^*)\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial\lambda$ we obtain

$$g'(-\lambda + a_1^* - a_2^*)\left(-1 + \frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}, \tag{22}$$

and

$$g'(a_1^* - a_2^*)\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}. \tag{23}$$

Solving (23) for $\partial a_2^*/\partial\lambda$ we have

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}. \tag{24}$$

Substituting (24) into (22) we obtain

$$g'(-\lambda + a_1^* - a_2^*)\left[-1 + \frac{\partial a_1^*}{\partial\lambda} - \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}$$

Solving this equation for $\partial a_1^*/\partial\lambda$ we obtain

$$\frac{\partial a_1^*}{\partial\lambda} = \frac{1}{D^*}\left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right] g'(-\lambda + a_1^* - a_2^*)\Delta u, \tag{25}$$

where

$$D^* = g'(-\lambda + a_1^* - a_2^*)c''(a_2^*)\Delta u - g'(a_1^* - a_2^*)c''(a_1^*)\Delta u - c''(a_1^*)c''(a_2^*). \tag{26}$$

Substituting (25) into (24) we obtain

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{1}{D^*}g'(a_1^* - a_2^*)g'(-\lambda + a_1^* - a_2^*)(\Delta u)^2. \tag{27}$$

I now discuss the sign of D^*. Three cases need to be distinguished. (a) Suppose $a_2^* > a_1^*$. This implies $g'(a_1^* - a_2^*) > 0$ and so $-g'(a_1^* - a_2^*)c''(a_1^*)\Delta u < 0$. Furthermore, the second-order condition for worker 1 implies $g'(-\lambda + a_1^* - a_2^*)\Delta u - c''(a_1^*) < 0$. Since $c''(a_2^*) > 0$ this implies $g'(-\lambda + a_1^* - a_2^*)c''(a_2^*)\Delta u - c''(a_1^*)c''(a_2^*) < 0$. Hence, if $a_2^* > a_1^*$, then $D^* < 0$. (b) Suppose $a_1^* > a_2^*$ with $a_1^* > -\lambda + a_1^* \geq a_2^*$. This implies $g'(-\lambda + a_1^* - a_2^*) \leq 0$ and so $g'(-\lambda + a_1^* - a_2^*)c''(a_2^*)\Delta u \leq 0$. Furthermore, the second-order condition for worker 2 implies $-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*) < 0$. Since

$c''(a_1^*) > 0$ this implies $-g'(a_1^* - a_2^*)c''(a_1^*)\Delta u - c''(a_1^*)c''(a_2^*) < 0$. Hence, if $a_1^* > a_2^*$ with $a_1^* > -\lambda + a_1^* \geq a_2^*$, then $D^* < 0$. (c) Suppose $a_1^* > a_2^*$ with $a_1^* > a_2^* > -\lambda + a_1^*$. In this case the sign of D^* can be either negative or positive and both cases need to be considered.

Suppose, by contradiction, $-\lambda + a_1^* - a_2^* \geq 0$. This implies $a_1^* > a_2^*$ which, in turn, implies $D^* < 0$. If $-\lambda + a_1^* - a_2^* \geq 0$, $D^* < 0$, and (25) hold, then $\partial a_1^*/\partial\lambda > 0$. However, $\partial a_1^*/\partial\lambda > 0$ implies $a_1^* > a^N$ which, in turn, implies $g(-\lambda + a_1^* - a_2^*) > g(0)$. This leads to a contradiction since $-\lambda + a_1^* - a_2^* \geq 0$ and $g'(x) < 0$ for $x > 0$ imply $g(-\lambda + a_1^* - a_2^*) \leq g(0)$. Thus, it must be that $-\lambda + a_1^* - a_2^* < 0$. This rules out case (b) above. Hence, we are left with cases (a) and (c). Consider case (c) first. Suppose $a_1^* > a_2^*$ and $-\lambda + a_1^* - a_2^* < 0$. We know that in this case, the sign of D^* can be either negative or positive. Suppose first that $D^* > 0$. If $a_1^* > a_2^*$, $-\lambda + a_1^* - a_2^* < 0$, $D^* > 0$, and (25) hold, then $\partial a_1^*/\partial\lambda > 0$. However, this leads to a contradiction as shown above. Now, suppose that $D^* < 0$. If $a_1^* > a_2^*$, $-\lambda + a_1^* - a_2^* < 0$, $D^* < 0$, and (25) hold, then $\partial a_1^*/\partial\lambda < 0$. Also, $a_1^* > a_2^*$, $-\lambda + a_1^* - a_2^* < 0$, $D^* < 0$, and (27) hold, then $\partial a_2^*/\partial\lambda > 0$. But $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$ imply $a_1^* < a_2^*$ which contradicts $a_1^* > a_2^*$. Thus, case (c) is ruled out. This means that we are left with case (a) $a_2^* > a_1^*$ and $D^* < 0$. Hence, the pure-strategy Nash equilibrium efforts must satisfy $a_2^* > a_1^*$, $-\lambda + a_1^* - a_2^* < 0$, and $D^* < 0$. If $a_2^* > a_1^*$, $-\lambda + a_1^* - a_2^* < 0$, $D^* < 0$, and (25) hold, then $\partial a_1^*/\partial\lambda < 0$. Also, if $a_2^* > a_1^*$, $-\lambda + a_1^* - a_2^* < 0$, $D^* < 0$, and (27) hold, then $\partial a_2^*/\partial\lambda < 0$. Moreover, $a_2^* > a_1^*$ implies $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$. If not, the alternative case $\partial a_2^*/\partial\lambda < \partial a_1^*/\partial\lambda < 0$ would imply $a_2^* < a_1^*$, leading to a contradiction. Thus, we found that (a_1^*, a_2^*) satisfies

$$a^N > a_2^* > a_1^*.$$

Hence, worker 1's underconfidence lowers the efforts of both workers and more so that of the underconfident worker: $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

Appendix B: Monotonic Best Responses

This section studies tournaments where an overconfident worker competes against an unbiased worker and best responses are monotonic. The pure-strategy equilibrium efforts are the solution to the first-order conditions:

$$\frac{\partial E[U_1(a_1,a_2,\lambda)]}{\partial a_1} = \frac{\partial P_1(a_1,a_2,\lambda)}{\partial a_1}\Delta u - c'(a_1) = 0,$$

and

$$\frac{\partial E[U_2(a_1,a_2)]}{\partial a_2} = \frac{\partial P_2(a_1,a_2)}{\partial a_2}\Delta u - c'(a_2) = 0.$$

The second-order conditions are

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2} < 0 \text{ and } \frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1^2} < 0. \tag{28}$$

I assume the second-order conditions are satisfied. I also assume the tournament has a unique pure-strategy Nash equilibrium. A sufficient condition for this to hold is that the derivatives of the workers' best responses are less than 1 in absolute value over the relevant range.[16] Thus,

$$\left|\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1^2}\right| > \left|\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2}\right| \text{ and } \left|\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_2^2}\right| > \left|\frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2}\right|. \tag{29}$$

is a sufficient condition for uniqueness. Finally, I assume the workers' best responses are monotonic, that is, workers' efforts are either strategic complements or substitutes over all effort levels. The assumption that efforts are strategic complements represents tournaments where a worker's increase in effort makes it more desirable for the rival to increase effort too. This happens when higher effort by a worker raises the rival's *marginal* expected utility. In this case we have

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_1(a_1,a_2,\lambda)}{\partial a_1 \partial a_2} > 0 \text{ and } \frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_2(a_1,a_2)}{\partial a_1 \partial a_2} > 0.$$

[16] A sufficient condition for the pure strategy Nash equilibrium to be unique is that best responses intersect only once.

The assumption that efforts are strategic substitutes represents tournaments where a worker's increase in effort makes it more desirable for the rival to lower effort. This happens when higher effort by a worker lowers the rival's *marginal* expected utility. In this case we have

$$\frac{\partial^2 E[U_1(a_1,a_2,\lambda)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_1(a_1,a_2,\lambda)}{\partial a_1 \partial a_2} < 0 \text{ and } \frac{\partial^2 E[U_2(a_1,a_2)]}{\partial a_1 \partial a_2} = \frac{\partial^2 P_2(a_1,a_2)}{\partial a_1 \partial a_2} < 0.$$

The unique pure-strategy equilibrium (a_1^*, a_2^*) satisfies the first-order conditions simultaneously and is given by:

$$\frac{\partial P_1(a_1^*, a_2^*, \lambda)}{\partial a_1}\Delta u = c'(a_1^*), \tag{30}$$

and

$$\frac{\partial P_2(a_1^*, a_2^*)}{\partial a_2}\Delta u = c'(a_2^*). \tag{31}$$

The impact of overconfidence on the pure-strategy equilibrium efforts is obtained from total differentiation of (30) and (31) which gives us:

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\partial a_1^* + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\partial a_2^* + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\partial \lambda\right]\Delta u = c''(a_1^*)\partial a_1^*,$$

and

$$\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\partial a_1^* + \frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\partial a_2^*\right]\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial \lambda$ we obtain

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\frac{\partial a_2^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial \lambda}, \tag{32}$$

and

$$\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\frac{\partial a_2^*}{\partial \lambda}\right]\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial \lambda}. \tag{33}$$

Solving (33) for $\partial a_2^* / \partial \lambda$ we have

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda}. \tag{34}$$

Substituting (34) into (32) we obtain

$$\left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\frac{\partial a_1^*}{\partial \lambda} - \frac{\frac{\partial^2 P_1(a_1^*,a_2^*,\lambda)}{\partial a_1 \partial a_2}\frac{\partial^2 P_2(a_1^*,a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*,a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\right]\Delta u$$
$$= c''(a_1^*)\frac{\partial a_1^*}{\partial \lambda}.$$

Solving this equation for $\partial a_1^*/\partial \lambda$ we find

$$\frac{\partial a_1^*}{\partial \lambda} = -\frac{1}{D^*}\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)\right]\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}\Delta u. \tag{35}$$

Substituting (35) into (34) we obtain

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{1}{D^*}\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2}\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda}(\Delta u)^2, \tag{36}$$

where

$$D^* = \left[\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1^2}\Delta u - c''(a_1^*)\right]\left[\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)\right]$$
$$-\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial a_2}\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2}(\Delta u)^2. \tag{37}$$

The sign of the two terms inside square brackets in (37) is strictly negative given the second-order conditions. Note that (29) and (37) imply $D^* > 0$. It follows from (28), (29), (35), and (37) that the relation between worker 1's effort and overconfidence only depends on the sign of $\partial^2 P_1(a_1^*, a_2^*, \lambda)/\partial a_1 \partial \lambda$, that is, how overconfidence influences worker 1's perceived marginal probability of winning the tournament. If effort and overconfidence are complements, that is, $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda > 0$, then an increase in overconfidence raises worker 1's perceived marginal probability of winning the tournament and worker 1's effort. If worker 1's effort and overconfidence are substitutes, that is, $\partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda < 0$, then an increase in overconfidence lowers worker 1's perceived marginal probability of winning the tournament and worker 1's effort.

It follows from (28), (29), (36), and (37) that the relation between worker 2's effort and worker 1's overconfidence depends on the signs of $\partial^2 P_1(a_1^*, a_2^*, \lambda)/\partial a_1 \partial \lambda$ and $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$. The sign of $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$ is determined by the nature of the strategic relation between the workers' efforts. When efforts are strategic complements (substitutes), the sign of $\partial^2 P_2(a_1^*, a_2^*)/\partial a_1 \partial a_2$ is positive (negative).

The next result characterizes the impact of overconfidence on the pure-strategy equilibrium efforts and winning probabilities when worker 1's effort and overconfidence are complements. As before, I define as the Nash winner (loser) the worker with the higher (lower) probability of winning the tournament at the pure-strategy equilibrium.

Proposition 5: *The overconfident worker is the Nash winner when his effort and overconfidence are complements, i.e.,*

$$P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*) \text{ when } \partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda > 0.$$

If efforts are strategic complements, then both workers exert more effort than if both were unbiased, with the overconfident worker exerting the greatest effort. Furthermore, an increase in overconfidence raises the effort of both workers and more so that of the overconfident worker, i.e., $\partial a_1^*/\partial \lambda > \partial a_2^*/\partial \lambda > 0$. *If efforts are strategic substitutes, then the overconfident worker exerts more effort and the unbiased worker exerts less effort than if both were unbiased. Furthermore, an increase in overconfidence raises the effort of the overconfident worker and lowers that of the unbiased worker, i.e.,* $\partial a_1^*/\partial \lambda > 0 > \partial a_2^*/\partial \lambda$.

Proof of Proposition 5:
i) When worker 1's overconfidence and effort are complements

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} > 0.$$

In addition, if efforts are strategic complements, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} > 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (35) and (36) imply

$$\frac{\partial a_1^*}{\partial \lambda} > 0 \text{ and } \frac{\partial a_2^*}{\partial \lambda} > 0.$$

We know from (34) that

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*,a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*,a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda}. \tag{38}$$

Assumptions (28), and (29) imply that the first term on the right-hand side of (38) is greater than 0 and less than 1. Hence, it follows that

$$\frac{\partial a_1^*}{\partial \lambda} > \frac{\partial a_2^*}{\partial \lambda} > 0. \tag{39}$$

If (39) holds and workers have identical utility functions, then $a_1^* > a_2^*$. If $a_1^* > a_2^*$ and workers have identical productivity of effort, then $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$.
ii) When worker 1's overconfidence and effort are complements

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} > 0.$$

In addition, if efforts are strategic substitutes, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} < 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (35) and (36) imply

$$\frac{\partial a_1^*}{\partial \lambda} > 0 > \frac{\partial a_2^*}{\partial \lambda}. \tag{40}$$

If (40) holds and workers have identical utility functions, then $a_1^* > a_2^*$. If $a_1^* > a_2^*$ and workers have identical productivity of effort, then $P_1(a_1^*, a_2^*) > 1/2 > P_2(a_1^*, a_2^*)$.

Proposition 5 tells us that an overconfident worker has a higher probability of winning when his effort and overconfidence are complements. The intuition behind this result is as follows. When effort and overconfidence are complements, an increase in overconfidence raises the overconfident worker's perceived marginal probability of winning the tournament. This, in turn, raises the overconfident worker's effort. When efforts are strategic complements, the unbiased worker's optimal response to the higher effort of the overconfident worker is to raise her effort. However, the increase in effort of the unbiased worker is less pronounced than that of the overconfident worker. When efforts are strategic substitutes, the unbiased worker's optimal response to the higher effort of the overconfident worker is to lower her effort. One way or the other, the overconfident worker exerts higher effort than the unbiased worker and therefore has a higher probability of winning.

When effort and overconfidence are complements, individuals with higher beliefs about their abilities work harder. Chen and Schildberg-Hörisch (2019) find experimental support for this assumption using a real effort task. They also show that informing individuals about their true abilities lowers effort provision which further reinforces the idea that overconfidence and effort are complements. However, whether this assumption holds generally is unclear. It might just as well the case that individuals with higher beliefs about their abilities exert less effort, that is, effort and overconfidence are substitutes.

The next result characterizes the impact of overconfidence on the pure-strategy equilibrium effort and winning probabilities when worker 1's effort and overconfidence are substitutes.

Proposition 6: *The overconfident worker is the Nash loser when his effort and overconfidence are substitutes, i.e.,*

$$P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*) \text{ when } \partial^2 P_1(a_1, a_2, \lambda)/\partial a_1 \partial \lambda < 0.$$

If efforts are strategic complements, then both workers exert less effort than if both were unbiased, with the overconfident worker exerting the least effort. Furthermore, an increase in overconfidence lowers the effort of both workers and more so that of the overconfident worker, i.e., $\partial a_1^*/\partial \lambda < \partial a_2^*/\partial \lambda < 0$. *If efforts are strategic substitutes,* *then the overconfident worker exerts less effort and the unbiased worker exerts more effort than if both were unbiased. Furthermore, an increase in overconfidence lowers the effort of the overconfident worker and raises that of the unbiased worker, i.e.,* $\partial a_1^*/\partial\lambda < 0 < \partial a_2^*/\partial\lambda$.

Proof of Proposition 6:

i) When worker 1's overconfidence and effort are substitutes

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} < 0.$$

In addition, if efforts are strategic complements, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} > 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (35) and (36) imply

$$\frac{\partial a_1^*}{\partial \lambda} < 0 \text{ and } \frac{\partial a_2^*}{\partial \lambda} < 0.$$

We know from (34) that

$$\frac{\partial a_2^*}{\partial \lambda} = -\frac{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2 \partial a_1}\Delta u}{\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_2^2}\Delta u - c''(a_2^*)} \frac{\partial a_1^*}{\partial \lambda}.$$

Assumptions (28), and (29) imply that the first term on the right-hand side of (38) is greater than 0 and less than 1. Hence, it follows that

$$\frac{\partial a_1^*}{\partial \lambda} < \frac{\partial a_2^*}{\partial \lambda} < 0. \tag{41}$$

If (41) holds and workers have identical utility functions, then $a_2^* > a_1^*$. If $a_2^* > a_1^*$ and workers have identical productivity of effort, then $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.

ii) When worker 1's overconfidence and effort are substitutes

$$\frac{\partial^2 P_1(a_1^*, a_2^*, \lambda)}{\partial a_1 \partial \lambda} < 0.$$

In addition, if efforts are strategic substitutes, then

$$\frac{\partial^2 P_2(a_1^*, a_2^*)}{\partial a_1 \partial a_2} < 0.$$

Since the second-order conditions are satisfied and $D^* > 0$, these two inequalities and equations (35) and (36) imply

$$\frac{\partial a_2^*}{\partial \lambda} > 0 > \frac{\partial a_1^*}{\partial \lambda}. \tag{42}$$

If (42) holds and workers have identical utility functions, then $a_2^* > a_1^*$. If $a_2^* > a_1^*$ and workers have identical productivity of effort, then $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.

Proposition 6 shows that an overconfident worker has a lower probability of winning when his effort and overconfidence are substitutes. In this case, an increase in overconfidence lowers the overconfident worker's perceived marginal probability of winning the tournament. This, in turn, lowers the overconfident worker's effort. When efforts are strategic complements, the unbiased worker's optimal response to the lower effort of the overconfident worker is to lower her effort. However, the decrease in effort of the unbiased worker is less pronounced than that of the overconfident worker. When efforts are strategic substitutes, the unbiased worker's optimal response to the lower effort of the overconfident worker is to increase her effort. Since the overconfident worker exerts lower effort than the unbiased worker in either case, he has a lower probability of winning.

Propositions 5 and 6 carry welfare implications. Overconfidence makes the firm better off when worker 1's effort and overconfidence are complements. When workers' efforts are strategic complements both workers exert more effort than if both were unbiased. When workers' efforts are strategic substitutes, the increase in effort of the overconfident worker is larger than the decrease in effort of the unbiased worker (an increase in overconfidence leads to a downward movement along the unbiased worker's best response). In either case, the firm is better off than if both workers were unbiased. Hence, the firm would not want to de-bias the overconfident worker when his effort and overconfidence are complements. In contrast, overconfidence makes the firm worse off when worker 1's effort and overconfidence are substitutes. When workers' efforts are strategic complements both workers exert less effort than if both were unbiased. When workers' efforts are strategic substitutes, the decrease in effort of the overconfident worker is larger than the increase in effort of the unbiased worker (an increase in overconfidence leads to an upward movement along the unbiased worker's best response). In either case, the firm is worse off than if both workers were unbiased. Hence, the firm would want to de-bias the overconfident worker when his effort and overconfidence are substitutes.

To evaluate the welfare implications of overconfidence for the overconfident worker I take the perspective of an outside observer who knows the overconfident worker's true productivity (knows that $\lambda = 1$). I consider how the overconfident worker's equilibrium objective expected utility $E\left[U_1(a_1^*, a_2^*)\right] = u(y_L) + P_1(a_1^*, a_2^*)\Delta u - c(a_1^*)$ changes with λ:

$$\begin{aligned} \frac{\partial E\left[U_1(a_1^*, a_2^*)\right]}{\partial \lambda} &= \left[\frac{\partial P_1(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial P_1(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial \lambda}\right]\Delta u - c'(a_1^*)\frac{\partial a_1^*}{\partial \lambda} \\ &= \left[\frac{\partial P_1(a_1^*, a_2^*)}{\partial a_1} - \frac{\partial P_1(a_1^*, a_2^*, \lambda)}{\partial a_1}\right]\frac{\partial a_1^*}{\partial \lambda}\Delta u + \frac{\partial P_1(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial \lambda}\Delta u, \end{aligned} \tag{43}$$

where the second equality follows from the first-order condition of the overconfident worker. The first term on the right-hand side of (43) is the direct effect and the second term is the strategic effect. The direct effect is always negative because the overconfident worker fails to play a best response against his rival.[17] Given that

[17] When effort and overconfidence are complements, player 1's marginal perceived probability of winning the tournament is higher than his actual marginal probability. Hence, the term inside square brakets in (43) is negative. Furthermore, an increase in λ raises the effort of the overconfident player, i.e., $\partial a_1^*/\partial\lambda > 0$. Hence, an increase in λ has an unfavorable direct effect when effort and overconfidence are complements. When effort and overconfidence are substitutes, player 1's marginal perceived probability of winning the tournament is lower than his actual marginal probability. Hence, the term inside square brakets in (43) is positive. Furthermore, an increase in λ lowers the effort of the overconfident player, i.e., $\partial a_1^*/\partial\lambda < 0$. Hence, an increase in λ also has an unfavorable direct effect when effort and overconfidence are substitutes.

$\partial P_1(a_1^*, a_2^*)/\partial a_2 < 0$, the sign of the strategic effect is negative when $\partial a_2^*/\partial \lambda > 0$ and positive when $\partial a_2^*/\partial \lambda < 0$. Hence, an increase in overconfidence always makes the overconfident worker worse off when it raises the effort of the unbiased worker. However, an increase in overconfidence can make the overconfident worker better off when it lowers the effort of the unbiased worker. This happens when the strategic effect dominates the direct effect.

To evaluate the welfare implications of overconfidence for the unbiased worker I consider how his equilibrium expected utility $E\left[U_2(a_1^*, a_2^*)\right]$ changes with λ:

$$\begin{aligned} \frac{\partial E\left[U_2(a_1^*, a_2^*)\right]}{\partial \lambda} &= \left[\frac{\partial P_2(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial \lambda} + \frac{\partial P_2(a_1^*, a_2^*)}{\partial a_2}\frac{\partial a_2^*}{\partial \lambda}\right]\Delta u - c'(a_2^*)\frac{\partial a_2^*}{\partial \lambda} \\ &= \frac{\partial P_2(a_1^*, a_2^*)}{\partial a_1}\frac{\partial a_1^*}{\partial \lambda}\Delta u, \end{aligned}$$

where the second equality follows from the first-order condition of the unbiased worker. The sign of the derivative $\partial P_2(a_1^*, a_2^*)/\partial a_1$ is negative since an increase in the effort of the overconfident worker lowers the winning probability of the unbiased worker. Hence, an increase in overconfidence makes the unbiased worker worse off when the sign of $\partial a_1^*/\partial \lambda$ is positive, i.e., when overconfidence raises the effort of the overconfident worker. In contrast, an increase in overconfidence makes the unbiased worker better off when the sign of $\partial a_1^*/\partial \lambda$ is negative, i.e., when overconfidence lowers the effort of the overconfident worker.