Subjective Evaluation Contracts for Overconfident Workers

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This version: March 22, 2021

--- Main Appendix ---

Proof of Lemma 1: To prove (i), note that:

$$\gamma_{ts}^j = \lambda^j \gamma_{ts}^G + (1-\lambda^j)\gamma_{ts}^B = \lambda^j P_{ts}\gamma_t^G + (1-\lambda^j)P_{ts}\gamma_t^B = P_{ts}\left[\lambda^j \gamma_t^G + (1-\lambda^j)\gamma_t^B\right] = P_{ts}\Gamma_t^j.$$

To prove (ii), start from Assumption 2 and use (i) to obtain:

$$\begin{aligned} &P_{aa}P_{uu} - P_{au}P_{ua} > 0 \\ &P_{aa}P_{uu}\Gamma_a^j\Gamma_u^j - P_{au}P_{ua}\Gamma_a^j\Gamma_u^j > 0 \\ &\gamma_{aa}^j\gamma_{uu}^j - \gamma_{au}^j\gamma_{ua}^j > 0 \end{aligned} \tag{7}$$

by positivity of $\Gamma_a^j\Gamma_u^j$. To prove (iii), note that $P_{aa} = 1 - P_{au}$ and $P_{ua} = 1 - P_{uu}$. Substitute for the latter in Assumption 2 to obtain:

$$\begin{aligned} &(1 - P_{au})P_{uu} - P_{au}(1 - P_{uu}) > 0 \\ &P_{uu} - P_{au} > 0 \end{aligned}$$

Similarly, substitute for $P_{au} = 1 - P_{aa}$ and $P_{uu} = 1 - P_{ua}$ to obtain that $P_{aa} - P_{ua} > 0$. Finally, to prove (iv), note that

$$\Delta\Gamma_t = \Gamma_t^H - \Gamma_t^L = \lambda^H\gamma_t^G + (1-\lambda^H)\gamma_t^B - \left[\lambda^L\gamma_t^G + (1-\lambda^L)\gamma_t^B\right] = \left(\lambda^H - \lambda^L\right)\left(\gamma_t^G - \gamma_t^B\right).$$

Therefore

$$\begin{aligned}
\Delta\Gamma_a + \Delta\Gamma_u &= \left(\lambda^H - \lambda^L\right)\left(\gamma_a^G - \gamma_a^B\right) - \left(\lambda^H - \lambda^L\right)\left(\gamma_u^G - \gamma_u^B\right) \\
&= \left(\lambda^H - \lambda^L\right)\left[(\gamma_a^G - \gamma_u^G) - (\gamma_u^B - \gamma_a^B)\right] \\
&= \left(\lambda^H - \lambda^L\right)\left[(\gamma_a^G - \gamma_u^G) - (\gamma_a^G - \gamma_u^G)\right] = 0.
\end{aligned}$$

This proves Lemma 1.

Proof of Lemma 2: Simple checking yields:

$$\tilde{\gamma}_{aa}^j \tilde{\gamma}_{uu}^j - \tilde{\gamma}_{au}^j \tilde{\gamma}_{ua}^j = (\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua})\Gamma_a^j \Gamma_u^j$$

which is positive when

$$\begin{aligned}
&\tilde{P}_{aa}\tilde{P}_{uu} - \tilde{P}_{au}\tilde{P}_{ua} = \tilde{P}_{aa}(1 - \tilde{P}_{ua}) - (1 - \tilde{P}_{aa})\tilde{P}_{ua} \\
&= \tilde{P}_{aa} - \tilde{P}_{aa}\tilde{P}_{ua} - \tilde{P}_{ua} + \tilde{P}_{aa}\tilde{P}_{ua} = P_{aa} - P_{ua} + b_a - b_u > 0.
\end{aligned}$$

Since, by Lemma 1, $P_{aa} > P_{ua}$, the latter inequality always holds for $b_a \geq b_u$. For values of $b_u > b_a$, it yields condition (3). This proves Lemma 2.

Basic Features of Contracts: We now state and prove three results on problem (4) which are valid for biased as well as for rational agents. Note that in order for the truthful reporting constraints to hold, it cannot be optimal for either party to always report the same performance evaluation regardless of that party's performance evaluation realization. Hence, truthful reporting imposes constraints on the equilibrium wages and compensation levels. For example, suppose we had $w_{aa} \geq w_{ua}$ and $w_{au} \geq w_{uu}$, with at least one inequality holding strictly. In this case, it would be optimal for the principal to always report an unacceptable performance, regardless of her performance evaluation realization, and this would violate the principal's truthful reporting constraints. Similarly, suppose we had $c_{aa} \geq c_{au}$ and $c_{ua} \geq c_{uu}$, with at least one inequality holding strictly. In this case, it would be optimal for the agent to always report an acceptable performance, regardless of his performance evaluation realization, and this would violate the agent's truthful reporting constraints. Since the principal wants to pay the lowest possible wage, the direction of the inequalities must be such that the wages are the lowest when the performance evaluation reports are identical, that is, $t = s$, the most probable outcome (under truthful reporting) given that signals are positively correlated. Similarly, since the agent wants to obtain the highest possible compensation, if he believes signals are positively correlated, then the direction of the inequalities must be such that the compensations are the highest when $t = s$, the most probable believed outcome. This produces the following two Lemmas.

Lemma 5. *Given Assumption 2, any optimal contract implementing high effort features either (i) $w_{ua} = w_{aa}$ and $w_{au} = w_{uu}$ or (ii) $w_{ua} > w_{aa}$ and $w_{au} > w_{uu}$.*

Proof. Rearranging the two (TR_P) constraints:

$$(w_{ua} - w_{aa}) \geq (w_{au} - w_{uu}) \frac{\gamma_{au}^H}{\gamma_{aa}^H}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu}) \frac{\gamma_{uu}^H}{\gamma_{ua}^H}$$

$$\Rightarrow (w_{au} - w_{uu}) \frac{\gamma_{au}^H}{\gamma_{aa}^H} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu}) \frac{\gamma_{uu}^H}{\gamma_{ua}^H}. \quad (8)$$

Given Assumption 2, either all the brackets in (8) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 5. ∎

Lemma 6. *If the agent believes signals are positively correlated, i.e. (3) holds, then any optimal contract implementing high effort features either (i) $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$ or (ii) $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$.*

Proof. For Lemma 6 follow the same steps with the (TR_A) constraints to obtain:

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} \leq (c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}. \tag{9}$$

When the agent believes signals are positively correlated, that is, $\tilde{\gamma}^H_{aa}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{\gamma}^H_{ua} > 0$, we have:

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} < \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}.$$

Given this last inequality, either all the brackets in (9) are 0 (case (i)), or they have positive signs (case (ii)). This proves Lemma 6. ∎

Lemmas 5 and 6 allow us to state a Lemma which confirms one of the main results of MacLeod (2003) for an agent who believes signals are positively correlated. That is, unless the optimal contract features a deadweight loss, it is impossible to implement high effort under truthful reporting. This proves Lemma 6.

Lemma 7. *If the principal wishes to implement high effort under truthful reporting and the agent believes signals are positively correlated, then there ought to exist at least one combination of realizations of t and s where $w_{ts} > c_{ts}$.*

Proof. Suppose not, then $w_{ts} = c_{ts}$ for all t and s. Given Lemma 5 and 6, we have:

$$c_{uu} \geq c_{ua} \geq c_{aa} \geq c_{au} \geq c_{uu},$$

where the first and third inequalities follow from Lemma 6 and the second and fourth follow from Lemma 5. Obviously, for all inequalities to hold together we need

$$c_{uu} = c_{ua} = c_{aa} = c_{au}.$$

This implies that $\tilde{E}(c_{ts}|\lambda^H) = \tilde{E}(c_{ts}|\lambda^L)$, since the agent compensation is completely independent from the realization of t and s. This, of course, violates the (IC)

constraint since

$$\tilde{E}(c_{ts}|\lambda^H) - V(\lambda^H) < \tilde{E}(c_{ts}|\lambda^L) - V(\lambda^L).$$

This proves Lemma 7. ∎

Simplifying the Effort Implementation Problem: When the (PC) is slack problem (4) becomes:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \tag{10}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^L - V(\lambda^L) \tag{IC}$$

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H \leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H \tag{TR_P^a}$$

$$w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H \tag{TR_P^u}$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \tag{TR_A^u}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t,s \in \{a,u\}. \tag{LL_{ts}}$$

We now show that problem (10) can be simplified to:

$$\min_{c_{aa},c_{au}} c_{aa}[(\gamma_{aa}^H)^2\tilde{\gamma}_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H + \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H]$$

$$+ c_{au}\left(\gamma_{aa}^H\gamma_{au}^H\tilde{\gamma}_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H\right) \tag{11}$$

$$\text{s.t.} \quad c_{aa}\left(\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) + c_{au}\left(\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right) \geq \Delta V \tag{IC}$$

$$c_{aa} \leq \left(1 + \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right) c_{au} \tag{TR_P^u}$$

$$c_{aa} \geq c_{au}, \tag{TR_A^a}$$

where $\Delta V = V(\lambda^H) - V(\lambda^L)$.

Lemma 8 below states that an agent believing that signals are positively correlated ought to be compensated in the "most positive" case, that is, when both principal and agent report an acceptable performance. It also states that the agent obtains no compensation when the principal deems the performance unacceptable and the agent disagrees. Together with Lemma 10 below, Lemma 8 proves that a deadweight loss happens only when the principal deems unacceptable a performance deemed acceptable by the agent.

Lemma 8. *If the agent believes signals are positively correlated, i.e.* (3) *holds, then any optimal contract implementing high effort features* $c_{aa} > c_{ua} = 0$.

Proof. Define $\Delta\gamma_{ts} = \gamma_{ts}^H - \gamma_{ts}^L$ and $\Delta\tilde{\gamma}_{ts} = \tilde{\gamma}_{ts}^H - \tilde{\gamma}_{ts}^L$. First, we prove that $\Delta\tilde{\gamma}_{as} > 0$ and $\Delta\tilde{\gamma}_{us} < 0$ for any $s \in \{a, u\}$ (it is easy to see that the same holds for $\Delta\gamma_{as}$ and $\Delta\gamma_{us}$). Notice that Assumption 1 is independent from Assumption 3. Therefore:

$$\begin{aligned}
\Delta\tilde{\gamma}_{ts} &= \tilde{\gamma}_{ts}^H - \tilde{\gamma}_{ts}^L \\
&= \lambda^H \tilde{\gamma}_{ts}^G + (1-\lambda^H)\tilde{\gamma}_{ts}^B - \lambda^L \tilde{\gamma}_{ts}^G - (1-\lambda^L)\tilde{\gamma}_{ts}^B \\
&= \lambda^H \tilde{P}_{ts}\gamma_t^G + (1-\lambda^H)\tilde{P}_{ts}\gamma_t^B - \lambda^L \tilde{P}_{ts}\gamma_t^G - (1-\lambda^L)\tilde{P}_{ts}\gamma_t^B \\
&= (\lambda^H - \lambda^L)\tilde{P}_{ts}(\gamma_t^G - \gamma_t^B),
\end{aligned}$$

which is positive at $t = a$ and negative otherwise.[23] Now we rewrite the (IC) in the following way:

$$c_{aa}\Delta\tilde{\gamma}_{aa} + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} + c_{uu}\Delta\tilde{\gamma}_{uu} \geq \Delta V, \tag{12}$$

Recall that any optimal contract with truthful reporting for an agent who believes signals are positively correlated satisfies either case (i) or case (ii) of Lemma 6. Assume case (i) of Lemma 6 holds, then (12) becomes:

$$c_{aa}\underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{>0} + c_{uu}\underbrace{(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})}_{<0} \geq \Delta V.$$

[23]For future reference, this also proves that, as long as b_a and b_u are both positive, $\Delta\tilde{\gamma}_{ta} > \Delta\gamma_{ta}$ and $\Delta\tilde{\gamma}_{tu} < \Delta\gamma_{tu}$ for any t.

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{aa} > 0$ to always hold. Assume now case (ii) of Lemma 6 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. Since $c_{au} \geq 0$, case (ii) implies $c_{aa} > c_{au} \geq 0$. This proves the first part of Lemma 8.

To prove the second part of Lemma 8, we suppose it is false, i.e., at optimum $c_{ua} > 0$, and prove that there exists a profitable deviation from such a contract, which contradicts its optimality. First of all, from Lemma 6 we know that $c_{uu} \geq c_{ua}$ and also $c_{aa} \geq c_{au}$. The proof now depends on whether $c_{au} > 0$ or $c_{au} = 0$.

Suppose $c_{au} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From (12) above, we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence, decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{au} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way, the overall change in the LHS of the (IC) is given by

$$\begin{aligned}
&-\epsilon\left(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au} + \Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}\right) \\
&\quad = -\epsilon\left(\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Delta\Gamma_a + \tilde{P}_{ua}\Delta\Gamma_u + \tilde{P}_{uu}\Delta\Gamma_u\right) \\
&\quad = -\epsilon\left(\Delta\Gamma_a + \Delta\Gamma_u\right) = -\epsilon\left(\Delta\Gamma_a - \Delta\Gamma_a\right) = 0
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, the principal can decrease also w_{ua} and w_{aa} by the same amount. This holds their difference constant and does not violate any of the relevant (LL_{ts}). Hence, it does not violate any of the (TR_P) constraints either. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{ua} > 0$ and $c_{au} > 0$ cannot be the solution to the problem.

Suppose now, instead, that the optimal contract features $c_{au} = 0$ and define $\Delta c_u = c_{uu} - c_{ua}$. Notice that this implies $c_{aa} > c_{au}$ and that we are in case (ii) of

Lemma 6. We divide the proof for this case in three steps.

Step 1

When $c_{au} = 0$, the (TR_A) constraints imply

$$\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} \leq c_{aa} \leq \Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}, \tag{13}$$

where, since we are in case (ii) of Lemma 6 either only one of the two inequalities holds as equality, or none. Suppose none of the two does, or just the second one, the principal can decrease both c_{uu} and c_{ua} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular, the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. He can then decrease c_{aa} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{aa}}$ bringing the LHS of the (IC) back to its original value. Clearly, for some ϵ, this deviation can lead to the first inequality in (13) binding. Finally, to see that this is optimal for the principal, notice that according to the (LL_{ts}) constraints, she can decrease w_{ua} up to ϵ and w_{aa} up to δ. By decreasing both by $\min\{\epsilon, \delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected, while the objective function decreases. This implies that, at optimum, if $c_{au} = 0$ the first inequality of (13) must bind and $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$.

Step 2

Given that $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$ when $c_{au} = 0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{au} = 0$. Let her decrease c_{uu} by ϵ and c_{ua} by $\epsilon_0 < \epsilon$. Then Δc_u has decreased by $(\epsilon - \epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} = c_{aa}$, the principal decreases c_{aa} by $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$\begin{aligned}
&-(\epsilon-\epsilon_0)\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{aa}^H}\Delta\tilde{\gamma}_{aa}-\epsilon_0\Delta\tilde{\gamma}_{ua}-\epsilon\Delta\tilde{\gamma}_{uu}\\
&\quad=-(\epsilon-\epsilon_0)\frac{\tilde{P}_{ua}\Gamma_u^H}{\Gamma_a^H}\Delta\Gamma_a+\epsilon_0\tilde{P}_{ua}\Delta\Gamma_a+\epsilon\tilde{P}_{uu}\Delta\Gamma_a\\
&\quad=\Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{uu}-\tilde{P}_{ua}\frac{\Gamma_u^H}{\Gamma_a^H}\right)+\epsilon_0\tilde{P}_{ua}\left(\frac{\Gamma_u^H}{\Gamma_a^H}+1\right)\right]\\
&\quad=\frac{\Delta\Gamma_a}{\Gamma_a^H}\left[\epsilon\left(\tilde{P}_{uu}\Gamma_a^H-\tilde{P}_{ua}+\tilde{P}_{ua}\Gamma_a^H\right)+\epsilon_0\tilde{P}_{ua}\right]\\
&\quad=\frac{\Delta\Gamma_a}{\Gamma_a^H}\left[\epsilon\left(\Gamma_a^H-\tilde{P}_{ua}\right)+\epsilon_0\tilde{P}_{ua}\right],
\end{aligned}$$

which is positive when:

$$\epsilon\left(\Gamma_a^H-\tilde{P}_{ua}\right)+\epsilon_0\tilde{P}_{ua}>0.$$

If $\Gamma_a^H>\tilde{P}_{ua}$, the above is always true. If instead $\Gamma_a^H<\tilde{P}_{ua}$ then the principal has to choose $\epsilon\in\left\{\epsilon_0,\epsilon_0\frac{\tilde{P}_{ua}}{\tilde{P}_{ua}-\Gamma_a^H}\right\}$.

Step 3

To conclude, we show that the above deviation is optimal. Given the decreases in the c_{ts}, the principal can now decrease w_{ua} up to ϵ_0 and w_{aa} up to $(\epsilon-\epsilon_0)\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{aa}^H}$. By an argument similar to the one in Step 1, she can decrease both wages by the smallest of the two limits, decreasing the objective function. This proves that a contract with $c_{ua}>0$ and $c_{au}=0$ cannot be optimal, since the principal can deviate optimally from it.

Hence, since a contract where $c_{ua}>0$ and $c_{au}\geq 0$ cannot be a solution to the problem it follows that $c_{ua}=0$. This concludes the proof of the Lemma. ∎

We now study the principal's incentives to report her performance evaluation truthfully.

Lemma 9. *If the agent believes signals are positively correlated, i.e.* (3) *holds, then constraint* (TR_P^a) *always binds in any optimal contract implementing high effort.*

Proof. Of course, in case (i) of Lemma 5 this is trivially proven. Assume now case (ii) of Lemma 5 holds and suppose (TR_P^a) is slack. Then $w_{ua} > 0$ must hold. From Lemma 8, then $c_{ua} = 0$, and the principal can simply decrease w_{ua} until (TR_P^a) binds. This would relax (TR_P^u), not affect (LL_{ua}) and decrease the objective function. ∎

We now solve for all w_{ts} as functions of the compensation c_{ts}.

Lemma 10. *If the agent believes signals are positively correlated, i.e.* (3) *holds, then any optimal contract implementing high effort features:*

(i) $w_{aa} = c_{aa}$;

(ii) $w_{uu} = c_{uu}$;

(iii) $w_{au} = \max\{c_{au}, c_{uu}\}$;

(iv) $w_{ua} = c_{aa} + (\max\{c_{au}, c_{uu}\} - c_{uu})\,\frac{\gamma_{au}^H}{\gamma_{aa}^H}$.

Proof. First of all, notice that, by Lemma 9, $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$. Hence, the principal's objective function in (4) can be rearranged as:

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + \left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}\right]\gamma_{ua}^H + w_{uu}\gamma_{uu}^H,$$

and further as:

$$w_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right) + w_{uu}\left(\gamma_{uu}^H - \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right),$$

where the last bracket is positive by Assumption 2. Furthermore, setting $w_{ua} = w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$ in (TR_P^u) we have

$$\left[w_{aa} + (w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}\right]\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H,$$

which is equivalent to

$$w_{uu} \leq w_{au}.$$

Hence, given the Lemmas so far, w_{aa}, w_{au}, and w_{uu} are only bound by $w_{uu} \leq w_{au}$ and the three corresponding (LL_{ts}). This implies that w_{aa}, w_{au}, and w_{uu} will be set to the lowest possible value. By Lemma 5, and in order to minimize the objective function, $w_{aa} = c_{aa}$, $w_{uu} = c_{uu}$ and $w_{au} = \max\{c_{au}, w_{uu}\}$, implying points (i), (ii) and (iii) of Lemma 10. Point (iv) follows by substitution. ∎

The next Lemma completes case (ii) of Lemma 6 by ranking c_{au} and c_{uu}. As expected, when the principal deems the performance acceptable, the agent may obtain a compensation premium even when he observes $S = u$.

Lemma 11. *If the agent believes signals are positively correlated, i.e.* (3) *holds, then any optimal contract implementing high effort features* $c_{au} \geq c_{uu}$.

Proof. Suppose not. Then $c_{uu} > c_{au} \geq 0$. By Lemma 8, $c_{ua} = 0$. Hence $c_{uu} > c_{ua}$, implying we are in case (ii) of Lemma 6 and $c_{aa} > c_{au}$. By Lemma 10, we have $w_{uu} = w_{au} = c_{uu}$ and $w_{ua} = c_{aa} = w_{aa}$. This implies that c_{au} disappears from the objective function and from constraints. The principal can, therefore, increase c_{au} and decrease other compensation (and therefore wage payments) in such a way that the rest of the constraints are still satisfied. This operation can be repeated until $c_{au} = c_{uu}$. Hence, the contradiction. ∎

Given this, we can further decrease the amount of binding constraints by proving the following:

Lemma 12. *If the agent believes signals are positively correlated, i.e.* (3) *holds, then constraint* (TR_A^u) *always binds in any optimal contract implementing high effort. Therefore:*

$$c_{uu} = \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}(c_{aa} - c_{au}).$$

Proof. Let $c_{uu} = 0$. Then we are in case (i) of Lemma 6 and (TR_A^u) is trivially binding. Suppose now that $c_{uu} > 0$ and (TR_A^u) is not binding. The principal can reduce c_{uu} until it binds. Given the proven Lemmas, the (TR_P) still hold, while

(TR_A^a) and (IC) are relaxed by this change. To complete the proof, we need to check whether a decrease in c_{uu} would decrease the objective function as well. By Lemmas 10 and 11, we can substitute for all wages in the objective function and find that the coefficient of c_{uu} becomes $\left(\gamma_{uu}^H - \frac{\gamma_{au}^H \gamma_{ua}^H}{\gamma_{aa}^H}\right)$, which is positive by Assumption 2. Hence, decreasing c_{uu} also decreases cost and it is therefore optimal for the principal to do so. This provides the desired contradiction and proves that (TR_A^u) always binds at optimum. ∎

This concludes the set of Lemmas yielding problem (11). Notice that, when plugging in the values from Lemma 10, the objective function in (10), simplifies to (11) divided by $\gamma_{aa}^H \tilde{\gamma}_{uu}^H$. This is however irrelevant for the minimization problem and therefore omitted.

The next Lemma presents a condition on the agent's overprecision that leads to a result original to our model. That is, as we show later, the existence of a new contract where the principal's wage cost is determined only by the agent's performance evaluation report and the agent's compensation is determined by both parties' performance evaluation reports. This stands in contrast to the baseline subjective evaluation contract in the literature where the principal's wage cost is determined by both parties' performance evaluation reports and the agent's compensation is tied only to the principal's performance evaluation report.

Lemma 13. *If the agent is overconfident in the sense of overprecision and his beliefs satisfy:*

$$b_a \geq P_{au} \frac{\Gamma_u^H \Gamma_a^H \left(P_{aa} - P_{ua}\right) + \left(P_{uu} - b_u\right) \left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) \Gamma_a^H}{\Gamma_u^H \Gamma_a^H \left(P_{aa} - P_{ua}\right) + \left(P_{uu} - b_u\right) \left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)}$$

then the optimal contract implementing high effort features $c_{aa} > c_{au}$*,* (TR_A^a) *slack and* (TR_P^u) *binding. If the agent is overconfident in the sense of overprecision and his beliefs violate* (5)*, then the optimal contract implementing high effort features* $c_{aa} = c_{au}$*,* (TR_A^a) *binding and* (TR_P^u) *slack.*

Proof. The inequality in Lemma 13 follows from the comparisons of the slope of the (IC) with the slope of the iso-costs. This produces the following condition

$$\frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}} \leq \frac{\gamma_{aa}^H\gamma_{au}^H\tilde{\gamma}_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H}{(\gamma_{aa}^H)^2\tilde{\gamma}_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H + \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H}.$$

We start from simplifying the slope of the (IC)

$$LHS = \frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}{\Delta\tilde{\gamma}_{aa} + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}} = \frac{\tilde{\gamma}_{au}^H - \tilde{\gamma}_{au}^L - \tilde{\gamma}_{au}^H + \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\tilde{\gamma}_{uu}^L}{\tilde{\gamma}_{aa}^H - \tilde{\gamma}_{aa}^L + \tilde{\gamma}_{au}^H - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\tilde{\gamma}_{uu}^L} = \frac{\frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\tilde{\gamma}_{uu}^L - \tilde{\gamma}_{au}^L}{\tilde{\gamma}_{aa}^H - \tilde{\gamma}_{aa}^L + \tilde{\gamma}_{au}^H - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\tilde{\gamma}_{uu}^L}$$
$$= \frac{\frac{\tilde{P}_{au}\tilde{P}_{uu}\Gamma_a^H\Gamma_u^L}{\tilde{P}_{uu}\Gamma_u^H} - \tilde{P}_{au}\Gamma_a^L}{\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Gamma_a^H\left(1 - \frac{\Gamma_u^L}{\Gamma_u^H}\right)} = \frac{\tilde{P}_{au}\left(\Gamma_a^H\Gamma_u^L - \Gamma_a^L\Gamma_u^H\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\Delta\Gamma_u}.$$

Notice that, since $\Gamma_a^J + \Gamma_u^J = 1$ for any $j = H, L$, we can substitute for $\Gamma_u^H = 1 - \Gamma_a^H$ and $\Gamma_u^L = 1 - \Gamma_a^L$. Also, from Lemma 1, $\Delta\Gamma_a = -\Delta\Gamma_u$. Hence we can further simplify the LHS:

$$\frac{\tilde{P}_{au}\left(\Gamma_a^H\Gamma_u^L - \Gamma_a^L\Gamma_u^H\right)}{\tilde{P}_{aa}\Delta\Gamma_a\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\Delta\Gamma_u} = \frac{\tilde{P}_{au}\left(\Gamma_a^H(1 - \Gamma_a^L) - \Gamma_a^L(1 - \Gamma_a^H)\right)}{\tilde{P}_{aa}\Delta\Gamma_a(1 - \Gamma_a^H) + \tilde{P}_{au}\Gamma_a^H(-\Delta\Gamma_a)}$$
$$= \frac{\tilde{P}_{au}\Delta\Gamma_a}{\Delta\Gamma_a\left[\tilde{P}_{aa}(1 - \Gamma_a^H) - \tilde{P}_{au}\Gamma_a^H\right]} = \frac{\tilde{P}_{au}}{\tilde{P}_{aa} - \Gamma_a^H} = \frac{P_{au} - b_a}{P_{aa} - \Gamma_a^H + b_a}.$$

The slope of the iso-costs, instead, is given by

$$\frac{\gamma_{aa}^H\gamma_{au}^H\tilde{\gamma}_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H}{(\gamma_{aa}^H)^2\tilde{\gamma}_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\tilde{\gamma}_{uu}^H + \tilde{\gamma}_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \tilde{\gamma}_{au}^H\gamma_{au}^H\gamma_{ua}^H}$$
$$= \frac{(P_{uu} - b_u)\left(P_{aa}P_{au}\Gamma_a^H + P_{au}P_{ua}\Gamma_u^H\right) - (P_{au} - b_a)\Gamma_a^H\left(P_{aa}P_{uu} - P_{au}P_{ua}\right)}{(P_{uu} - b_u)\left(P_{aa}P_{aa}\Gamma_a^H + P_{aa}P_{ua}\Gamma_u^H\right) + (P_{au} - b_a)\Gamma_a^H\left(P_{aa}P_{uu} - P_{au}P_{ua}\right)}$$
$$= \frac{(P_{uu} - b_u)P_{au}Z - (P_{au} - b_a)W}{(P_{uu} - b_u)P_{aa}Z + (P_{au} - b_a)W},$$

where $Z = \left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right)$ and $W = \Gamma_a^H \left(P_{aa}P_{uu} - P_{au}P_{ua}\right) = \Gamma_a^H(P_{aa} - P_{ua})$. Hence the inequality in Lemma 13 is equivalent to

$$\frac{P_{au} - b_a}{P_{aa} - \Gamma_a^H + b_a} \leq \frac{(P_{uu} - b_u)P_{au}Z - (P_{au} - b_a)W}{(P_{uu} - b_u)P_{aa}Z + (P_{au} - b_a)W},$$

or

$$\begin{aligned} & \left(P_{au} - b_a\right)(P_{uu} - b_u)P_{aa}Z + \left(P_{au} - b_a\right)^2 W \\ \leq \; & \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{uu} - b_u)P_{au}Z - \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{au} - b_a)W, \end{aligned}$$

or

$$\begin{aligned} & \left(P_{au} - b_a\right)^2 W + \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{au} - b_a)W \\ \leq \; & \left(P_{aa} - \Gamma_a^H + b_a\right)(P_{uu} - b_u)P_{au}Z - \left(P_{au} - b_a\right)(P_{uu} - b_u)P_{aa}Z, \end{aligned}$$

or

$$\left(P_{au} - b_a\right)\left[\left(P_{au} + P_{aa}\right) - \Gamma_a^H\right] W \leq \left(P_{uu} - b_u\right)\left[b_a\left(P_{aa} + P_{au}\right) - P_{au}\Gamma_a^H\right] Z,$$

or

$$b_u \leq P_{uu} - \frac{\left(P_{au} - b_a\right)\left(1 - \Gamma_a^H\right) W}{\left(b_a - P_{au}\Gamma_a^H\right) Z}. \tag{14}$$

Solving (14) for b_a we obtain (5). To conclude the proof of Lemma 13 consider the graphical analysis of (11). Figure 3 below shows the three constraints binding in (c_{au}, c_{aa}) space and highlights the set of contracts satisfying all constraints of (11) — and therefore of (4).

In order to understand whether at optimum it is the (TR_P^u) or the (TR_A^a) that binds, and therefore where the optimal contract lies in Figure 3, we compare the sign and magnitude of the slope of the iso-costs and the (IC). Hence, Lemma 13 shows that the optimal contract lies either at point X or Y of Figure 3, depending on how the slope of the (IC) and of the iso-costs compare. ∎

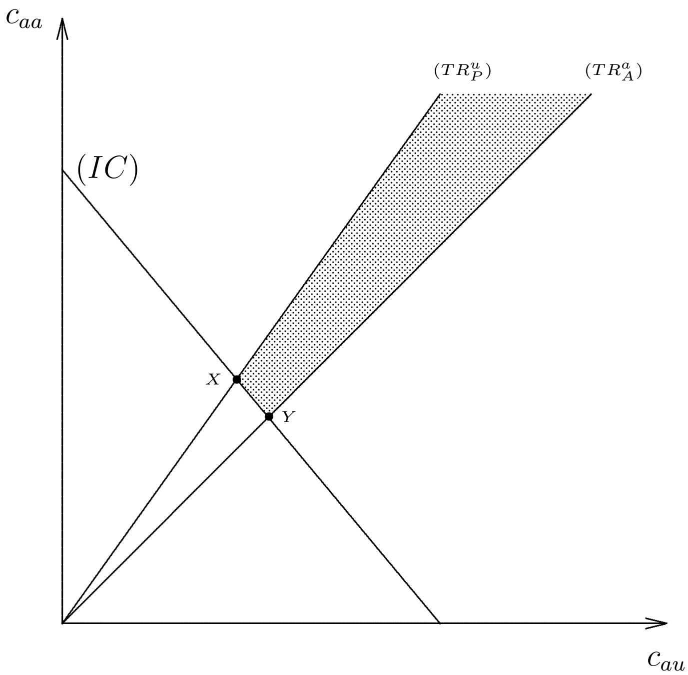

Figure 3: The shaded area represents the set of contracts satisfying all the constraints in the minimization problem (11) — and therefore (4).

Proof of Proposition 1: The proof is divided into three parts. First, we show that when $b_a = b_u = 0$, the slope of the (IC) is never lower than the slope of the iso-costs. Seccond, we derive the optimal contract for a rational agent. Third, we show that the optimal contract satisfies the (PC) when

$$\bar{u} \leq \frac{\Delta V}{\Delta \Gamma_a} \Gamma_a^H - V(\lambda^H) \equiv \bar{u}_1. \tag{15}$$

Using the algebra presented in the proof of Lemma 13, consider the slope of the (IC) when the agent is unbiased:

$$\frac{P_{au}}{P_{aa} - \Gamma_a^H}$$

This implies that the (IC) is negatively sloped if and only if $\Gamma_a^H < P_{aa}$. First we assume $\Gamma_a^H < P_{aa}$ and show that the (16) always holds. Then we move to the case of $\Gamma_a^H > P_{aa}$.

Let $\Gamma_a^H < P_{aa}$. The comparison between slopes then becomes:

$$\frac{P_{au}}{P_{aa} - \Gamma_a^H} > \frac{\gamma_{aa}^H\gamma_{au}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H - \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \tag{16}$$

We now rearrange the RHS, which is less nicely simplified.

$$\begin{aligned} RHS &= \frac{\gamma_{aa}^H\gamma_{au}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H - \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \\ &= \frac{\gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \end{aligned}$$

Before going ahead, notice that this proves that in the case of an unbiased agent isocosts are always negatively sloped. Carrying on we obtain

$$\begin{aligned} &\frac{\gamma_{au}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H\gamma_{uu}^H + \gamma_{aa}^H\gamma_{ua}^H\gamma_{uu}^H + \gamma_{au}^H\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{au}^H\gamma_{ua}^H} \\ &= \frac{P_{au}P_{ua}P_{uu}\Gamma_u^H + P_{au}P_{au}P_{ua}\Gamma_a^H}{P_{aa}P_{aa}P_{uu}\Gamma_a^H + P_{aa}P_{ua}P_{uu}\Gamma_u^H + P_{au}P_{uu}P_{aa}\Gamma_a^H - P_{au}P_{au}P_{ua}\Gamma_a^H} \\ &= \frac{P_{au}P_{ua}(P_{uu}(1-\Gamma_a^H) + P_{au}\Gamma_a^H)}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \\ &= \frac{P_{au}P_{ua}(P_{uu} - \Gamma_a^H(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \end{aligned}$$

This implies that comparing the slopes boils down to:

$$\begin{aligned} \frac{P_{au}}{P_{aa} - \Gamma_a^H} &> \frac{P_{au}P_{ua}(P_{uu} - \Gamma_a^H(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \\ \frac{1}{P_{aa} - \Gamma_a^H} &> \frac{P_{ua}(P_{uu} - \Gamma_a^H(P_{uu} - P_{ua}))}{P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H)} \end{aligned}$$

$$P_{aa}P_{uu}\Gamma_a^H + P_{ua}(P_{aa}P_{uu}(1-\Gamma_a^H) - P_{au}P_{au}\Gamma_a^H) > (P_{aa} - \Gamma_a^H)P_{ua}(P_{uu} - \Gamma_a^H(P_{uu} - P_{ua})).$$

Recall that Lemma 1 showed $P_{aa} > P_{ua}$ and $P_{uu} > P_{au}$.

$$\begin{aligned}
&P_{aa}P_{uu}\Gamma_a^H + P_{ua}P_{aa}P_{uu} - P_{ua}P_{aa}P_{uu}\Gamma_a^H - P_{ua}P_{au}P_{au}\Gamma_a^H \\
&> P_{ua}P_{aa}P_{uu} - P_{ua}P_{uu}\Gamma_a^H - P_{aa}P_{ua}\Gamma_a^H(P_{uu} - P_{ua}) + P_{ua}(\Gamma_A^H)^2(P_{uu} - P_{ua}),
\end{aligned}$$

which, by simplifying and dividing by Γ_a^H on both sides, is equivalent to:

$$\begin{aligned}
&P_{aa}P_{uu} - P_{ua}P_{au}P_{au} > -P_{ua}P_{uu} + P_{aa}P_{ua}P_{ua} + P_{ua}P_{uu}\Gamma_A^H - P_{ua}P_{ua}\Gamma_A^H \\
&P_{aa}P_{uu} - P_{ua}P_{au}^2 > -P_{ua}P_{uu} + P_{aa}P_{ua}^2 + P_{uu}P_{ua}\Gamma_a^H - P_{ua}^2\Gamma_a^H \\
&P_{uu}(P_{aa} + P_{ua}) - P_{ua}\Gamma_a^H(P_{uu} - P_{ua}) - P_{ua}P_{au}^2 - P_{aa}P_{ua}^2 > 0.
\end{aligned}$$

Now we substitute for $P_{uu} = 1 - P_{ua}$ and $P_{au} = 1 - P_{aa}$ and we get:

$$\begin{aligned}
&(1 - P_{ua})\,(P_{aa} + P_{ua}) - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua}(1 - P_{aa})^2 - P_{aa}P_{ua}^2 > 0 \\
&P_{aa} + P_{ua} - P_{aa}P_{ua} - P_{ua}^2 - P_{ua}\Gamma_a^H(1 - 2P_{ua}) - P_{ua} + 2P_{aa}P_{ua} - P_{ua}P_{aa}^2 - P_{aa}P_{ua}^2 > 0 \\
&P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + \underbrace{P_{ua}\Gamma_a^H(2P_{ua} - 1)}_{\boldsymbol{\Gamma}} > 0.
\end{aligned}$$

Suppose first that $P_{ua} < \frac{1}{2}$, then $\Gamma < 0$ and the LHS gets smaller the greater is Γ_a^H. Hence, to be sure the condition holds, we set $\Gamma_a^H \to P_{aa}$, the highest possible value it can get. This yields $\Gamma \to 2P_{aa}P_{ua}^2 - P_{aa}P_{ua}$. Hence the condition converges to

$$\begin{aligned}
&P_{aa} + P_{aa}P_{ua}(1 - P_{ua} - P_{aa}) - P_{ua}^2 + 2P_{aa}P_{ua}^2 - P_{aa}P_{ua} > 0 \\
&P_{aa} + P_{aa}P_{ua}^2 - P_{aa}^2P_{ua} - P_{ua}^2 > 0. \qquad (17)
\end{aligned}$$

Notice that if this holds for all $P_{aa} > P_{ua}$ then so will the condition for the case of $P_{ua} > \frac{1}{2}$. In that case, in fact, $\Gamma > 0$, which means that the LHS would increase with Γ_a^H. Hence, to check it holds we set it to 0. This would set $\Gamma = 0$ and yield a condition looser than (17).

To see that (17) always holds, notice that the derivative of the LHS with respect to P_{ua} is given by:

$$\frac{\partial LHS}{\partial P_{ua}} = 2P_{aa}P_{ua} - P_{aa}^2 - 2P_{ua} = 2P_{ua}(P_{aa} - 1) - P_{aa}^2$$

which is negative for all $P_{aa} < 1$. Hence, the condition is monotonically decreasing in P_{ua}. We therefore check for the maximum value of P_{ua}, which in this case is $\frac{1}{2}$. At this value, condition (17) becomes simply

$$-2P_{aa}^2 + 5P_{aa} - 1 > 0$$

By Lemma 1, P_{aa} must be strictly larger than P_{ua}. The second order equation above always holds for $P_{aa} \in [\frac{1}{2}, 1]$. Hence when the (IC) is negatively sloped and $b_a = b_u = 0$, (5) always holds.

We are now left to show that the same holds when the (IC) is positively sloped. Suppose that the (IC) is positively sloped. This implies that it requires c_{aa} to be smaller than c_{au} times a positive number. First of all, notice from the (IC) that when it is positively sloped, its intercept for $c_{au} = 0$ is negative. Further, its slope is now given by

$$\frac{\Delta\tilde{\gamma}_{au} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}{-\Delta\tilde{\gamma}_{aa} - \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}$$

which is obviously larger than 1. Hence, the set of constraint compatible contracts becomes the one highlighted in Figure 4.

Regardless of whether the iso-costs are positively or negatively sloped, the optimal contract lies at point Y in the graph. Hence, the optimal contract has (IC) binding

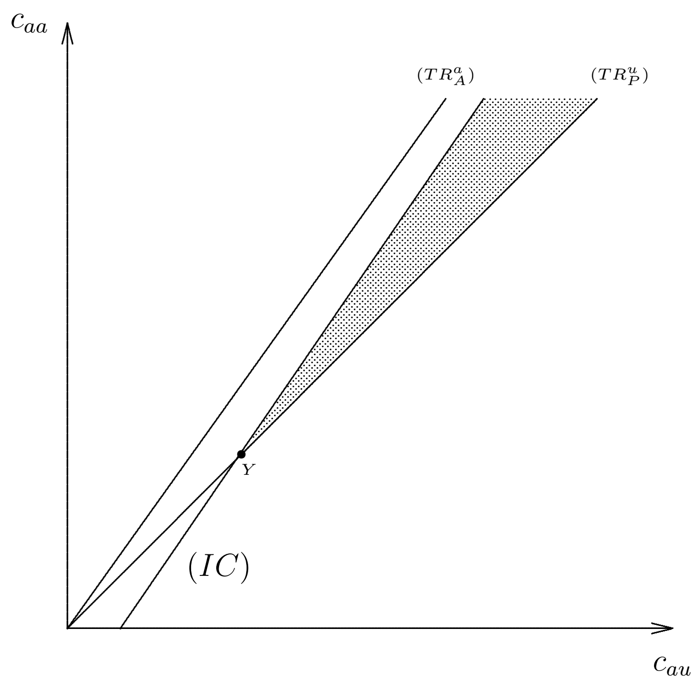

Figure 4: The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (11) when the agent believes that signals are positively correlated and the (IC) is positively sloped.

and (TR_A^a) binding. Therefore it is the solution to:

$$c_{ua}^* = 0$$
$$c_{uu}^* = 0$$
$$c_{aa}^* = c_{au}^*$$
$$c_{aa}^* \frac{1}{\Gamma_u^H}\left(P_{aa} - \Gamma_a^H\right)\Delta\Gamma_a + c_{au}^* \frac{P_{au}}{\Gamma_u^H}\Delta\Gamma_a = \Delta V$$

Setting $c_{au}^* = c_{aa}^*$ and solving the last equation for c_{aa}^* gives us:

$$c_{aa}^* = \frac{\Delta V}{P_{aa} - \Gamma_a^H + P_{au}}\frac{\Gamma_u^H}{\Delta\Gamma_a} = \frac{\Delta V}{1 - \Gamma_a^H}\frac{\Gamma_u^H}{\Delta\Gamma_a} = \frac{\Delta V}{\Gamma_u^H}\frac{\Gamma_u^H}{\Delta\tilde{\Gamma}_a} = \frac{\Delta V}{\Delta\Gamma_a}.$$

Hence, the optimal contract is given by:

$$w^*_{aa} = c^*_{aa} \quad w^*_{au} = c^*_{aa} \quad w^*_{uu} = 0 \quad w^*_{ua} = \tfrac{c^*_{aa}}{P_{aa}}$$
$$c^*_{aa} = \tfrac{\Delta V}{\Delta \Gamma_a} \quad c^*_{au} = c^*_{aa} \quad c^*_{uu} = 0 \quad c^*_{ua} = 0.$$

To complete the proof we show that the optimal contract satisfies the (PC). The LHS of the (PC) is:

$$\begin{aligned}
\sum_{ts} c^*_{ts}\gamma^H_{ts} - V(\lambda^H) &= \frac{\Delta V}{\Delta \Gamma_a}\gamma^H_{aa} + \frac{\Delta V}{\Delta \Gamma_a}\gamma^H_{au} - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta \Gamma_a}(\gamma^H_{aa} + \gamma^H_{au}) - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta \Gamma_a}(P_{aa} + P_{au})\Gamma^H_a - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a - V(\lambda^H) = \bar{u}_1 \geq \bar{u}
\end{aligned}$$

Hence, the optimal contract for a rational agent satisfies the (PC).

Impact of Overestimation on (PC) and (IC) for fixed compensation. The following two results characterize the impact of overestimation on (PC) and (IC) when the agent's compensation is held fixed.

(i) *If the optimal contract implementing high effort features $c_{aa} = c_{au} > c_{uu} = c_{ua}$, then overestimation relaxes the (PC) for fixed compensation.* To see this note that the (PC) is

$$\sum_{ts} c_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) \geq \bar{u},$$

or

$$c_{aa}\tilde{\gamma}^H_{aa} + c_{au}\tilde{\gamma}^H_{au} + c_{ua}\tilde{\gamma}^H_{ua} + c_{uu}\tilde{\gamma}^H_{uu} - V(\lambda^H) \geq \bar{u},$$

or

$$c_{aa}P_{aa}\tilde{\Gamma}^H_a + c_{au}P_{au}\tilde{\Gamma}^H_a + c_{ua}P_{ua}\tilde{\Gamma}^H_u + c_{uu}P_{uu}\tilde{\Gamma}^H_u - V(\lambda^H) \geq \bar{u},$$

or

$$(c_{aa}P_{aa} + c_{au}P_{au})\tilde{\Gamma}^H_a + (c_{ua}P_{ua} + c_{uu}P_{uu})\tilde{\Gamma}^H_u - V(\lambda^H) \geq \bar{u},$$

or

$$c_{aa}(P_{aa}+P_{au})\tilde{\Gamma}_a^H + c_{uu}(P_{ua}+P_{uu})(1-\tilde{\Gamma}_a^H) - V(\lambda^H) \geq \bar{u},$$

or

$$c_{uu} + (c_{aa}-c_{uu})\tilde{\Gamma}_a^H - V(\lambda^H) \geq \bar{u}.$$

Since $\tilde{\Gamma}_a^H > \Gamma_a^H$ it follows that overestimation relaxes the (PC) for fixed compensation.

(ii) *If the optimal contract implementing high effort features* $c_{aa} = c_{au} > c_{uu} = c_{ua}$, *then overestimation relaxes the* (IC) *for fixed compensation if and only if* $\Delta\tilde{\lambda} > \Delta\lambda$. To see this note that the (IC) is

$$\sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^L - V(\lambda^L),$$

or

$$c_{aa}\triangle\tilde{\gamma}_{aa} + c_{au}\triangle\tilde{\gamma}_{au} + c_{ua}\triangle\tilde{\gamma}_{ua} + c_{uu}\triangle\tilde{\gamma}_{uu} \geq \triangle V,$$

or

$$c_{aa}P_{aa}\triangle\tilde{\Gamma}_a + c_{au}P_{au}\triangle\tilde{\Gamma}_a + c_{ua}P_{ua}\triangle\tilde{\Gamma}_u + c_{uu}P_{uu}\triangle\tilde{\Gamma}_u \geq \triangle V,$$

or

$$c_{aa}(P_{aa}+P_{au})\triangle\tilde{\Gamma}_a + c_{uu}(P_{ua}+P_{uu})\triangle\tilde{\Gamma}_u \geq \triangle V,$$

or

$$(c_{aa}-c_{uu})\triangle\tilde{\Gamma}_a \geq \triangle V,$$

Hence, if the optimal contract implementing high effort features $c_{aa} = c_{au} > c_{uu} = c_{ua}$, then overestimation relaxes the (IC) for a fixed compensation as long as

$$\triangle\tilde{\Gamma}_a > \triangle\Gamma_a,$$

or

$$\Delta\tilde{\lambda}\gamma_a^G + (1-\Delta\tilde{\lambda})\gamma_a^B > \Delta\lambda\gamma_a^G + (1-\Delta\lambda)\gamma_a^B,$$

or

$$\Delta\tilde{\lambda} > \Delta\lambda.$$

Similarly, if the optimal contract implementing high effort features $c_{aa} = c_{au} > c_{uu} = c_{ua}$, then overestimation tightens the (IC) for a fixed compensation if and only if $\Delta\tilde{\lambda} < \Delta\lambda$.

Proof of Proposition 2: The proof is divided into three parts. First, we show that the slope of the (IC) is never lower than the slope of the iso-costs. Second, we derive the optimal contract for an agent who displays overestimation. Third, we show that the optimal contract satisfies the (PC) when

$$\bar{u} \leq \frac{\Delta V}{\Delta\tilde{\Gamma}_a}\tilde{\Gamma}_a^H - V(\lambda^H) \equiv \bar{u}_2. \tag{18}$$

The definition of overestimation implies

$$\begin{aligned}
\tilde{\gamma}_{aa}^H &= P_{aa}\tilde{\Gamma}_a^H \text{ and } \tilde{\gamma}_{aa}^L = P_{aa}\tilde{\Gamma}_a^L \\
\tilde{\gamma}_{au}^H &= P_{au}\tilde{\Gamma}_a^H \text{ and } \tilde{\gamma}_{au}^L = P_{au}\tilde{\Gamma}_a^L \\
\tilde{\gamma}_{ua}^H &= P_{ua}\tilde{\Gamma}_u^H \text{ and } \tilde{\gamma}_{ua}^L = P_{ua}\tilde{\Gamma}_u^L \\
\tilde{\gamma}_{uu}^H &= P_{uu}\tilde{\Gamma}_u^H \text{ and } \tilde{\gamma}_{uu}^L = P_{uu}\tilde{\Gamma}_u^L \\
\Delta\tilde{\gamma}_{aa} &= \tilde{\gamma}_{aa}^H - \tilde{\gamma}_{aa}^L = P_{aa}\tilde{\Gamma}_a^H - P_{aa}\tilde{\Gamma}_a^L = P_{aa}\Delta\tilde{\Gamma}_a \\
\Delta\tilde{\gamma}_{au} &= \tilde{\gamma}_{au}^H - \tilde{\gamma}_{au}^L = P_{au}\tilde{\Gamma}_a^H - P_{au}\tilde{\Gamma}_a^L = P_{au}\Delta\tilde{\Gamma}_a \\
\Delta\tilde{\gamma}_{ua} &= \tilde{\gamma}_{ua}^H - \tilde{\gamma}_{ua}^L = P_{ua}\tilde{\Gamma}_u^H - P_{ua}\tilde{\Gamma}_u^L = P_{ua}\Delta\tilde{\Gamma}_u \\
\Delta\tilde{\gamma}_{uu} &= \tilde{\gamma}_{uu}^H - \tilde{\gamma}_{uu}^L = P_{uu}\tilde{\Gamma}_u^H - P_{uu}\tilde{\Gamma}_u^L = P_{uu}\Delta\tilde{\Gamma}_u
\end{aligned}$$

The reduced effort implementation problem (11) becomes

$$
\begin{aligned}
\min_{c_{aa},c_{au}} & \; c_{aa}[((\gamma_{aa}^H)^2+\gamma_{aa}^H\gamma_{ua}^H)P_{uu}\tilde{\Gamma}_u^H+P_{au}\tilde{\Gamma}_a^H(\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{ua}^H)] \\
& + c_{au}\left((\gamma_{aa}^H\gamma_{au}^H+\gamma_{au}^H\gamma_{ua}^H)P_{uu}\tilde{\Gamma}_u^H-P_{au}\tilde{\Gamma}_a^H(\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{ua}^H)\right) \\
\text{s.t.} \quad & c_{aa}\left(P_{aa}\Delta\tilde{\Gamma}_a+\frac{P_{au}\tilde{\Gamma}_a^H}{P_{uu}\tilde{\Gamma}_u^H}P_{uu}\Delta\tilde{\Gamma}_u\right)+c_{au}\left(P_{au}\Delta\tilde{\Gamma}_a-\frac{P_{au}\tilde{\Gamma}_a^H}{P_{uu}\tilde{\Gamma}_u^H}P_{uu}\Delta\tilde{\Gamma}_u\right)\geq\Delta V \\
& c_{aa}\leq\left(1+\frac{P_{uu}\tilde{\Gamma}_u^H}{P_{au}\tilde{\Gamma}_a^H}\right)c_{au} \\
& c_{aa}\geq c_{au},
\end{aligned}
$$

or

$$
\begin{aligned}
\min_{c_{aa},c_{au}} & \; c_{aa}[((\gamma_{aa}^H)^2+\gamma_{aa}^H\gamma_{ua}^H)P_{uu}\tilde{\Gamma}_u^H+P_{au}\tilde{\Gamma}_a^H(\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{ua}^H)] \\
& + c_{au}\left((\gamma_{aa}^H\gamma_{au}^H+\gamma_{au}^H\gamma_{ua}^H)P_{uu}\tilde{\Gamma}_u^H-P_{au}\tilde{\Gamma}_a^H(\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{ua}^H)\right) \\
\text{s.t.} \quad & c_{aa}\left(P_{aa}\Delta\tilde{\Gamma}_a+\frac{P_{au}\tilde{\Gamma}_a^H}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_u\right)+c_{au}\left(P_{au}\Delta\tilde{\Gamma}_a-\frac{P_{au}\tilde{\Gamma}_a^H}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_u\right)\geq\Delta V \\
& c_{aa}\leq\left(1+\frac{P_{uu}\tilde{\Gamma}_u^H}{P_{au}\tilde{\Gamma}_a^H}\right)c_{au} \\
& c_{aa}\geq c_{au},
\end{aligned}
$$

Let's simplify the (IC):

$$
\begin{aligned}
P_{aa}\Delta\tilde{\Gamma}_a + \frac{P_{au}\tilde{\Gamma}_a^H}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_u &= \frac{1}{\tilde{\Gamma}_u^H}\left(P_{aa}\tilde{\Gamma}_u^H\Delta\tilde{\Gamma}_a + P_{au}\tilde{\Gamma}_a^H\Delta\tilde{\Gamma}_u\right) \\
&= \frac{1}{\tilde{\Gamma}_u^H}\left[P_{aa}(1-\tilde{\Gamma}_a^H) - (1-P_{aa})\tilde{\Gamma}_a^H\right]\Delta\tilde{\Gamma}_a \\
&= \frac{1}{\tilde{\Gamma}_u^H}\left(P_{aa} - P_{aa}\tilde{\Gamma}_a^H - \tilde{\Gamma}_a^H + P_{aa}\tilde{\Gamma}_a^H\right)\Delta\tilde{\Gamma}_a \\
&= \frac{1}{\tilde{\Gamma}_u^H}\left(P_{aa} - \tilde{\Gamma}_a^H\right)\Delta\tilde{\Gamma}_a
\end{aligned}
$$

and

$$
\begin{aligned}
P_{au}\Delta\tilde{\Gamma}_a - \frac{P_{au}\tilde{\Gamma}_a^H}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_u &= \frac{P_{au}}{\tilde{\Gamma}_u^H}\left(\tilde{\Gamma}_u^H\Delta\tilde{\Gamma}_a - \tilde{\Gamma}_a^H\Delta\tilde{\Gamma}_u\right) \\
&= \frac{P_{au}}{\tilde{\Gamma}_u^H}\left(\tilde{\Gamma}_u^H + \tilde{\Gamma}_a^H\right)\Delta\tilde{\Gamma}_a \\
&= \frac{P_{au}}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_a
\end{aligned}
$$

The simplified (IC) becomes

$$
c_{aa}\frac{1}{\tilde{\Gamma}_u^H}\left(P_{aa} - \tilde{\Gamma}_a^H\right)\Delta\tilde{\Gamma}_a + c_{au}\frac{P_{au}}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_a \geq \Delta V.
$$

Hence, the reduced effort implementation problem (11) becomes

$$\begin{aligned}
\min_{c_{aa},c_{au}} & c_{aa}[((\gamma^H_{aa})^2 + \gamma^H_{aa}\gamma^H_{ua})P_{uu}\tilde{\Gamma}^H_u + P_{au}\tilde{\Gamma}^H_a(\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua})] \\
& + c_{au}\left((\gamma^H_{aa}\gamma^H_{au} + \gamma^H_{au}\gamma^H_{ua})P_{uu}\tilde{\Gamma}^H_u - P_{au}\tilde{\Gamma}^H_a(\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua})\right) \\
\text{s.t.} \quad & c_{aa}\frac{1}{\tilde{\Gamma}^H_u}\left(P_{aa} - \tilde{\Gamma}^H_a\right)\Delta\tilde{\Gamma}_a + c_{au}\frac{P_{au}}{\tilde{\Gamma}^H_u}\Delta\tilde{\Gamma}_a \geq \Delta V \\
& c_{aa} \leq \left(1 + \frac{P_{uu}\tilde{\Gamma}^H_u}{P_{au}\tilde{\Gamma}^H_a}\right)c_{au} \\
& c_{aa} \geq c_{au},
\end{aligned}$$

The slope of the (IC) is:

$$LHS = \frac{\frac{P_{au}}{\tilde{\Gamma}^H_u}\Delta\tilde{\Gamma}_a}{\frac{1}{\tilde{\Gamma}^H_u}\left(P_{aa} - \tilde{\Gamma}^H_a\right)\Delta\tilde{\Gamma}_a} = \frac{P_{au}}{P_{aa} - \tilde{\Gamma}^H_a}$$

The slope of the iso-cost is

$$\begin{aligned}
& \frac{(\gamma^H_{aa}\gamma^H_{au} + \gamma^H_{au}\gamma^H_{ua})P_{uu}\tilde{\Gamma}^H_u - P_{au}\tilde{\Gamma}^H_a(\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua})}{((\gamma^H_{aa})^2 + \gamma^H_{aa}\gamma^H_{ua})P_{uu}\tilde{\Gamma}^H_u + P_{au}\tilde{\Gamma}^H_a(\gamma^H_{uu}\gamma^H_{aa} - \gamma^H_{au}\gamma^H_{ua})} \\
= \; & \frac{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{au}P_{uu}\Gamma^H_a\tilde{\Gamma}^H_u - P_{au}(P_{aa}P_{uu} - P_{au}P_{ua})\Gamma^H_u\Gamma^H_a\tilde{\Gamma}^H_a}{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{aa}P_{uu}\Gamma^H_a\tilde{\Gamma}^H_u + P_{au}(P_{aa}P_{uu} - P_{au}P_{ua})\Gamma^H_u\Gamma^H_a\tilde{\Gamma}^H_a} \\
= \; & \frac{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{au}P_{uu}\tilde{\Gamma}^H_u - P_{au}(P_{aa} - P_{ua})\Gamma^H_u\tilde{\Gamma}^H_a}{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{aa}P_{uu}\tilde{\Gamma}^H_u + P_{au}(P_{aa} - P_{ua})\Gamma^H_u\tilde{\Gamma}^H_a}.
\end{aligned}$$

Let us start by assumig the (IC) is negatively sloped, that is, $\tilde{\Gamma}^H_a < P_{aa}$. In this case, the slope of the (IC) is never lower than the slope of iso-cost since:

$$\frac{P_{au}}{P_{aa} - \tilde{\Gamma}^H_a} > \frac{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{au}P_{uu}\tilde{\Gamma}^H_u - P_{au}(P_{aa} - P_{ua})\Gamma^H_u\tilde{\Gamma}^H_a}{(P_{aa}\Gamma^H_a + P_{ua}\Gamma^H_u)P_{aa}P_{uu}\tilde{\Gamma}^H_u + P_{au}(P_{aa} - P_{ua})\Gamma^H_u\tilde{\Gamma}^H_a},$$

or

$$\frac{1}{P_{aa}-\tilde{\Gamma}_a^H} > \frac{(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{uu}\tilde{\Gamma}_u^H-(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H}{(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{aa}P_{uu}\tilde{\Gamma}_u^H+P_{au}(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H},$$

or

$$\begin{aligned} &(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{aa}P_{uu}\tilde{\Gamma}_u^H+P_{au}(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H \\ > \;& \left(P_{aa}-\tilde{\Gamma}_a^H\right)\left[(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{uu}\tilde{\Gamma}_u^H-(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H\right], \end{aligned}$$

or

$$\begin{aligned} &(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{aa}P_{uu}\tilde{\Gamma}_u^H+P_{au}(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H \\ > \;& \left(P_{aa}-\tilde{\Gamma}_a^H\right)(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{uu}\tilde{\Gamma}_u^H-\left(P_{aa}-\tilde{\Gamma}_a^H\right)(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H, \end{aligned}$$

or

$$\begin{aligned} &P_{au}(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H+\left(P_{aa}-\tilde{\Gamma}_a^H\right)(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H \\ > \;& \left(P_{aa}-\tilde{\Gamma}_a^H\right)(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{uu}\tilde{\Gamma}_u^H-(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)P_{aa}P_{uu}\tilde{\Gamma}_u^H, \end{aligned}$$

or

$$\begin{aligned} &\left(P_{au}+P_{aa}-\tilde{\Gamma}_a^H\right)(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H \\ > \;& \left[\left(P_{aa}-\tilde{\Gamma}_a^H\right)\tilde{\Gamma}_u^H-P_{aa}\tilde{\Gamma}_u^H\right]P_{uu}(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H), \end{aligned}$$

or

$$\begin{aligned} &\left(1-\tilde{\Gamma}_a^H\right)(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_a^H \\ > \;& \left(P_{aa}\tilde{\Gamma}_u^H-\tilde{\Gamma}_a^H\tilde{\Gamma}_u^H-P_{aa}\tilde{\Gamma}_u^H\right)P_{uu}(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H), \end{aligned}$$

or

$$(P_{aa}-P_{ua})\Gamma_u^H\tilde{\Gamma}_u^H\tilde{\Gamma}_a^H+\tilde{\Gamma}_a^H\tilde{\Gamma}_u^H P_{uu}(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)>0,$$

or

$$(P_{aa} - P_{ua})\Gamma_u^H + P_{uu}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H) > 0,$$

which holds. This implies that, like in Proposition 1, the optimal contract has the (IC) binding and the (TR_A^a) binding when $\tilde{\Gamma}_a^H < P_{aa}$. Let us now assume the (IC) is positively sloped, that is, $\tilde{\Gamma}_a^H > P_{aa}$. In this case, the slope of the (IC) is

$$\frac{P_{au}}{\tilde{\Gamma}_a^H - P_{aa}}.$$

Hence, the slope of the (IC) is larger than 1 since

$$\frac{P_{au}}{\tilde{\Gamma}_a^H - P_{aa}} > 1,$$

or

$$P_{au} > \tilde{\Gamma}_a^H - P_{aa},$$

or

$$1 > \tilde{\Gamma}_a^H.$$

The intercept of the (IC) for $c_{au} = 0$ is negative since $\tilde{\Gamma}_a^H > P_{aa}$. This implies that, just as in Proposition 1, the optimal contract has the (IC) binding and the (TR_A^a) binding when $\tilde{\Gamma}_a^H > P_{aa}$. Hence, the optimal contract has (IC) binding and (TR_A^a) binding. Therefore it is the solution to:

$$c_{ua}^{\diamond} = 0$$
$$c_{uu}^{\diamond} = 0$$
$$c_{aa}^{\diamond} = c_{au}^{\diamond}$$
$$c_{aa}^{\diamond}\frac{1}{\tilde{\Gamma}_u^H}\left(P_{aa} - \tilde{\Gamma}_a^H\right)\Delta\tilde{\Gamma}_a + c_{au}^{\diamond}\frac{P_{au}}{\tilde{\Gamma}_u^H}\Delta\tilde{\Gamma}_a = \Delta V$$

Setting $c^\diamond_{au} = c^\diamond_{aa}$ and solving the last equation for $c^\diamond_{aa}$ gives us:

$$c^\diamond_{aa} = \frac{\Delta V}{P_{aa} - \tilde{\Gamma}^H_a + P_{au}} \frac{\tilde{\Gamma}^H_u}{\Delta\tilde{\Gamma}_a} = \frac{\Delta V}{1 - \tilde{\Gamma}^H_a} \frac{\tilde{\Gamma}^H_u}{\Delta\tilde{\Gamma}_a} = \frac{\Delta V}{\tilde{\Gamma}^H_u} \frac{\tilde{\Gamma}^H_u}{\Delta\tilde{\Gamma}_a} = \frac{\Delta V}{\Delta\tilde{\Gamma}_a}$$

Hence, the optimal contract is given by:

$w^\diamond_{aa} = c^\diamond_{aa} \quad w^\diamond_{au} = c^\diamond_{aa} \quad w^\diamond_{uu} = 0 \quad w^\diamond_{ua} = \frac{c^\diamond_{aa}}{P_{aa}}$

$c^\diamond_{aa} = \frac{\Delta V}{\Delta\tilde{\Gamma}_a} \quad c^\diamond_{au} = c^\diamond_{aa} \quad c^\diamond_{uu} = 0 \quad c^\diamond_{ua} = 0.$

To complete the proof we need to show that the optimal contract satisfies the (PC). The LHS of the (PC) is:

$$\begin{aligned}
\sum_{ts} c^\diamond_{ts}\tilde{\gamma}^H_{ts} - V(\lambda^H) &= \frac{\Delta V}{\Delta\tilde{\Gamma}_a}\tilde{\gamma}^H_{aa} + \frac{\Delta V}{\Delta\tilde{\Gamma}_a}\tilde{\gamma}^H_{au} - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta\tilde{\Gamma}_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta\tilde{\Gamma}_a}(P_{aa} + P_{au})\tilde{\Gamma}^H_a - V(\lambda^H) \\
&= \frac{\Delta V}{\Delta\tilde{\Gamma}_a}\tilde{\Gamma}^H_a - V(\lambda^H) \geq \bar{u}
\end{aligned}$$

Hence, the optimal contract for an agent who displays overestimation satisfies the (PC).

Impact of Overprecision on (IC) and (TR^s_A) for fixed compensation. The following two Lemmas characterize the impact of overprecision on (IC) and (TR^s_A) when the agent's compensation is held fixed.

Lemma 14. *If the optimal contract implementing high effort features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, then overprecision has no impact on (IC) and (TR^s_A).*

Proof. see the proof for Lemma 15. ∎

Lemma 14 states that if the agent's compensation is independent of his own performance evaluation report, then his overprecision has no effect on (IC) and (TR^s_A) and therefore on implementability of any level of effort.

Lemma 15. *If the optimal contract implementing high effort features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$, then overprecision has an ambiguous effect on (IC) but relaxes (TR_A^s).*

Proof. The (IC)

$$\sum_{ts} c_{ts}(\tilde{\gamma}_{ts}^H - \tilde{\gamma}_{ts}^L) \geq \Delta V,$$

can be rewritten as

$$\sum_{ts} c_{ts}(\gamma_{ts}^H - \gamma_{ts}^L) + (c_{aa} - c_{au})(\Gamma_a^H - \Gamma_a^L)b_a + (c_{uu} - c_{ua})(\Gamma_u^L - \Gamma_u^H)b_u \geq \Delta V. \quad (19)$$

Note that $\Gamma_a^H > \Gamma_a^L$, $\Gamma_u^L > \Gamma_u^H$, $b_a > 0$, and $b_u \in \left(-\frac{\gamma_a^G}{\gamma_u^G}b_a, -\frac{\gamma_a^B}{\gamma_u^B}b_a\right)$. It follows from (19) that overconfidence has no impact on (IC) when the optimal contract features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$. It also follows from (19) that overconfidence has an ambiguous impact on (IC) when the optimal contract features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$ since the second term in the LHS of (19) is positive whereas the third term is negative.

The (TR_A^a)

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H,$$

can be rewritten as

$$c_{aa}\gamma_{aa}^H + c_{ua}\gamma_{ua}^H + (c_{aa} - c_{au})\Gamma_a^H b_a \geq c_{au}^H\gamma_{aa}^H + c_{uu}\gamma_{ua}^H + (c_{uu} - c_{ua})\Gamma_u^H b_u. \quad (20)$$

It follows from (20) that overconfidence has no impact on (TR_A^a) when the optimal contract features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$. It also follows from (20) that overconfidence relaxes (TR_A^a) when the optimal contract features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$ since the third term in the LHS of (20) is positive and the third term in the RHS of (20) is negative.

The (TR_A^u)

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H,$$

can be rewritten as

$$c_{au}\gamma_{au}^H + c_{uu}\gamma_{uu}^H + (c_{aa} - c_{au})\Gamma_a^H b_a \geq c_{aa}\gamma_{au}^H + c_{ua}\gamma_{uu}^H + (c_{uu} - c_{ua})\Gamma_u^H b_u \tag{21}$$

It follows from (21) that overconfidence has no impact on (TR_A^u) when the optimal contract features $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$. It also follows from (21) that overconfidence relaxes (TR_A^u) when the optimal contract features $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$ since the third term in the LHS of (21) is positive and the third term in the RHS of (21) is negative. This proves Lemmas 14 and 15. ∎

By Lemma 6, the agent knows that given what the principal observes, he obtains a premium when he reports $T = S$. A positive b_a and a negative b_u increase the agent's belief of both signals showing either a or u. This means that, given effort, an agent who displays overprecision with beliefs satisfying (3) overestimates the chances of obtaining premium $c_{aa} - c_{au}$ more than he overestimates the chances of obtaining premium $c_{uu} - c_{ua}$. Since $T = a$ is most probable when he exerts high effort, an agent who displays overprecision with beliefs satisfying (3) requires a lower incentive to exert λ^H. That is to say, exerting high effort is part of his "strategy" to increase the chance of reports $(t, s) = (a, a)$.

Proof of Proposition 3: The proof is divided into six steps. First, we show that when either the (IC) or the iso-costs (or both) are positively sloped, the optimal contract is the standard one. Then, we derive conditions for this case not to happen. Third, we prove that condition (5) implies all the conditions derived as well as (3) — and hence it is sufficient and necessary to our result — and we identify the shape of the area where the APE contract is set up (that is we provide an explanation to the shape of Figure 1). Fourth, we derive the values of wages and compensations of the APE contract. Fifth, we show that the optimal contract satisfies the (PC) when

$$\bar{u} \leq \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} - V(\lambda^H) \equiv \bar{u}_3. \tag{22}$$

Sixth, we prove how the deadweight loss of the APE contract is lower than the one of the BPE contract.

Step 1

First of all, note from (11) that an increase of c_{aa} always increases the expected cost of implementing high effort. The effect of an increase of c_{au}, however, is not straightforward when $b_a = b_u = 0$. If it is positive, then iso-costs are negatively sloped in (c_{au}, c_{aa}) space and costs decrease towards the origin. If it is negative, then iso-costs are positively sloped and costs decrease towards the bottom right of the graph.

Suppose the latter is true. Since iso-costs are positively sloped in (c_{au}, c_{aa}) space, optimal contracts lie at point Y of Figure 3. Notice, however, that a further check is needed here. Suppose the iso-costs are positively sloped. If their slope is larger than 1, then they are steeper than the locus of points where $c_{aa} = c_{au}$. Hence, for any given $c_{aa} = c_{au} = c$, there would always exists a $c' > c$ lying on an iso-costs further to the right of Figure 3 satisfying all constraints and lowering costs. Hence, an optimal contract would feature $c_{aa} = c_{au} = c \to \infty$. In order to check that this cannot happen, we study the value of the slope of the iso-costs when the latter is positive. From the algebra in the proof of Lemma 13, we can get this value as:

$$\frac{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) - (P_{uu} - b_u)P_{au}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}{(P_{au} - b_a)\Gamma_a^H(P_{aa} - P_{ua}) + (P_{uu} - b_u)P_{aa}(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H)}$$

which is trivially never larger than 1. Hence in equilibrium the baseline contract is set up.

The case of a positively sloped (IC) has already been discussed in the proof of Proposition 1.

Step 2

The slope of the (IC) is negative as long as $b_a \geq \Gamma_a^H - P_{aa}$. The condition for the slope of the iso-cost to be negative, instead, can be derived as follows.

Consider the slope derived in the proof of Lemma 13 again, this time without looking at its absolute value

$$-\frac{(P_{uu}-b_u)P_{au}\left(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H\right)-(P_{au}-b_a)\Gamma_a^H\left(P_{aa}-P_{ua}\right)}{(P_{uu}-b_u)P_{aa}\left(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H\right)+(P_{au}-b_a)\Gamma_a^H\left(P_{aa}-P_{ua}\right)}.$$

The iso-costs are negatively sloped when the numerator of the above is positive. This happens when:

$$(P_{uu}-b_u)P_{au}\underbrace{\left(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H\right)}_{Z}-(P_{au}-b_a)\underbrace{\Gamma_a^H\left(P_{aa}-P_{ua}\right)}_{W}>0$$

which yields condition:

$$b_u<P_{uu}-\frac{(P_{au}-b_a)W}{P_{au}Z}. \tag{23}$$

Step 3

In this part of the proof we show how, for $b_a\in\left[P_{au}\Gamma_a^H,P_{au}\right]$, condition (5) implies the negativity of the slope of the (IC) and condition (23). We also show how the area it delimits has a concave shape in (b_a,b_u) space and how it always lies in the interval $(P_{au}\Gamma_a,P_{au})$ on b_a. In order to study this we use version (14) of condition (5).

First, note that the (IC) is negatively sloped if

$$b_a\geq\Gamma_a^H-P_{aa}=\Gamma_a^H-1+P_{au}$$

and that

$$\Gamma_a^H-1+P_{au}<P_{au}\Gamma_a^H\ \Rightarrow\ P_{au}(1-\Gamma_a^H)<1-\Gamma_a^H.$$

Hence, when $b_a>P_{au}\Gamma_a^H$ (which is necessary for (14) to matter) the (IC) is negatively sloped.

Second, for (14) to imply (23) it is enough for the RHS of (23) to be larger than (14). This comparison corresponds to comparing the second terms of the RHS of each inequality. Condition (23) is looser if

$$\frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a-P_{au}\Gamma_a^H\right)Z} \geq \frac{(P_{au}-b_a)W}{P_{au}Z}$$

which corresponds to

$$P_{au}(1-\Gamma_a^H) \geq b_a - P_{au}\Gamma_a^H \;\Rightarrow\; P_{au} \geq b_a$$

which is always true.

To conclude this part of the proof we show that the RHS of (14) is concave in b_a. To see this, consider the first derivative of the RHS of (14) with respect to b_a

$$\left[\frac{P_{au}(1-\Gamma_a^H)^2 WZ}{\left[\left(b_a-P_{au}\Gamma_a^H\right)Z\right]^2}\right],$$

and note that it is decreasing in b_a. Hence, (14) identifies a concave area.[24] To see that its lower bound is always larger than $P_{au}\Gamma_a^H$, substitute $b_u = 0$ in the condition to obtain

$$0 \leq P_{uu} - \frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{\left(b_a-P_{au}\Gamma_a^H\right)Z}$$

which is equivalent to

$$b_a \geq P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}.$$

To prove our claim we then show that

$$\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z} > \Gamma_a^H.$$

With simple algebra, it is easy to see that this condition boils down to $\Gamma_a^H \leq 1$, which is always true.

[24] Recall that the derivative is of the entire RHS not only of the second term.

This concludes this part and proves that the area identified by the feasible values of b_a and condition (5) always features the APE contract. Its shape, furthermore, always resembles the representation in Figure 1.

Step 4

Given all the above and Lemma 13 we finally solve problem (11) by setting the (TR_P^u) binding together with the (IC). This yields the following system in two equations:

$$\begin{aligned}
&c_{aa}\left(\Delta\tilde{\gamma}_{aa}+\frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right)+c_{au}\left(\Delta\tilde{\gamma}_{au}-\frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right)=\Delta V\\
&c_{aa}=\left(1+\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)c_{au}
\end{aligned}$$

from which we obtain:

$$\begin{aligned}
c_{au}&=\frac{\Delta V}{\left(1+\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)\left(\Delta\tilde{\gamma}_{aa}+\frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}\right)+\Delta\tilde{\gamma}_{au}-\frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{uu}^H}\Delta\tilde{\gamma}_{uu}}\\
&=\frac{\Delta V}{\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{au}+\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\Delta\tilde{\gamma}_{aa}+\Delta\tilde{\gamma}_{uu}}\\
&=\frac{\Delta V}{\Delta\Gamma_a\tilde{P}_{aa}+\Delta\Gamma_a\tilde{P}_{au}+\frac{\tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H}\Delta\Gamma_a\tilde{P}_{aa}-\Delta\Gamma_a\tilde{P}_{uu}}\\
&=\frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(1-\Gamma_a^H)\tilde{P}_{aa}-\tilde{P}_{uu}\tilde{P}_{au}\Gamma_a^H}\\
&=c_{aa}^*\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)}.
\end{aligned}$$

To conclude the proof, we obtain $c_{aa}=\left(1+\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}\right)c_{au}$ from the above discussion, and $c_{au}=c_{uu}$ from Lemma 12.

Hence, the optimal contract is given by

$$
\begin{array}{llll}
w_{aa}^{\dagger}=c_{aa}^{\dagger} & w_{au}^{\dagger}=c_{au}^{\dagger} & w_{uu}^{\dagger}=c_{au}^{\dagger} & w_{ua}^{\dagger}=c_{aa}^{\dagger} \\
c_{aa}^{\dagger}=c_{au}^{\dagger}\left(1+\frac{\tilde{\gamma}_{uu}^{H}}{\tilde{\gamma}_{au}^{H}}\right) & c_{au}^{\dagger}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)} & c_{uu}^{\dagger}=c_{au}^{\dagger} & c_{ua}^{\dagger}=0.
\end{array}
.
$$

Step 5

Now we need to show that the optimal contract satisfies the (PC). The LHS of the (PC) is:

$$
\begin{aligned}
\sum_{ts} c_{ts}^{\dagger}\tilde{\gamma}_{ts}^{H}-V(\lambda^H) &= c_{au}^{\dagger}\left(\tilde{\gamma}_{aa}^{H}+\frac{\tilde{\gamma}_{uu}^{H}}{\tilde{\gamma}_{au}^{H}}\tilde{\gamma}_{aa}^{H}+\tilde{\gamma}_{au}^{H}+\tilde{\gamma}_{uu}^{H}\right)-V(\lambda^H) \\
&= c_{au}^{\dagger}\left(\tilde{P}_{aa}\Gamma_a^H+\frac{\tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H}\tilde{P}_{aa}\Gamma_a^H+\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}\Gamma_u^H\right)-V(\lambda^H) \\
&= c_{au}^{\dagger}\left(\Gamma_a^H+\frac{\tilde{P}_{uu}}{\tilde{P}_{au}}\tilde{P}_{aa}\Gamma_u^H+\tilde{P}_{uu}\Gamma_u^H\right)-V(\lambda^H) \\
&= c_{au}^{\dagger}\left(\Gamma_a^H+\frac{\tilde{P}_{uu}}{\tilde{P}_{au}}\Gamma_u^H\right)-V(\lambda^H) \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma_a^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)}\left(\Gamma_a^H+\frac{\tilde{P}_{uu}}{\tilde{P}_{au}}\Gamma_u^H\right)-V(\lambda^H) \\
&= \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\frac{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H+\tilde{P}_{uu}(\tilde{P}_{aa}-\Gamma_a^H)}-V(\lambda^H)\geq\bar{u}.
\end{aligned}
$$

Step 6

To see that the APE contract features a lower deadweight loss, notice that this is equal to $\sum_{ts}(w_{ts}^{*}-c_{ts}^{*})\gamma_{ts}^{H}=(w_{ua}^{*}-c_{ua}^{*})\gamma_{ua}^{H}$ in a BPE contract and to $\sum_{ts}(w_{ts}^{\dagger}-c_{ts}^{\dagger})\gamma_{ts}^{H}=(w_{ua}^{\dagger}-c_{ua}^{\dagger})\gamma_{ua}^{H}$ in the APE contract. Since $c_{ua}^{*}=c_{ua}^{\dagger}=0$, the deadweight loss is smaller under the APE contract if

$$\begin{aligned}
&w^*_{ua} > w^\dagger_{ua} \quad \iff \quad \frac{\Delta V}{\Delta\Gamma_a}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right)\\
&1 > P_{aa}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a}\\
&\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a) > P_{aa}\tilde{P}_{au}\Gamma^H_a + P_{aa}\tilde{P}_{uu}\Gamma^H_u\\
&(1 - P_{aa})\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}\Gamma^H_a - P_{aa}\tilde{P}_{uu}\Gamma^H_u > 0\\
&P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(P_{aa} + b_a - \Gamma^H_a - P_{aa} + P_{aa}\Gamma^H_a) > 0\\
&P_{au}\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\left[b_a - (1 - P_{aa})\Gamma^H_a\right] > 0,
\end{aligned}$$

which is always true since in the APE contract we have $b_a \in (P_{au}\Gamma^H_a, P_{au}]$.

Proof of Lemma 3: While $c^\dagger_{au} < c^*_{aa}$ we also have $c^\dagger_{aa} > c^\dagger_{au}$. Therefore the check for $c^\dagger_{aa} > c^*_{aa}$ is given by:

$$\left(1 + \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right)\left(\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \geq 1$$

which is equivalent to

$$\left(\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a}\right)\left(\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \geq 1$$

and to

$$\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(1 - \Gamma^H_A)}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} \geq 1,$$

which is always true since $\tilde{P}_{aa} \leq 1$.

Proof of Lemma 4:

Point (i) is trivial. Condition (5) comes from the study of how to minimize cost and it selects the optimal contract precisely on the basis of the lowest possible expected wage. Since both contracts are available at the moment of minimization none of the two can minimize costs when the other is optimal.

To prove point (ii), notice that

$$\begin{aligned} E(c^*_{ts}) &= c^*_{aa}\gamma^H_{aa} + c^*_{au}\gamma^H_{au} + c^*_{ua}\gamma^H_{ua} + c^*_{uu}\gamma^H_{uu} \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\gamma^H_{aa} + \gamma^H_{au}) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a, \end{aligned}$$

and

$$\begin{aligned} \tilde{E}(c^*_{ts}) &= c^*_{aa}\tilde{\gamma}^H_{aa} + c^*_{au}\tilde{\gamma}^H_{au} + c^*_{ua}\tilde{\gamma}^H_{ua} + c^*_{uu}\tilde{\gamma}^H_{uu} \\ &= \frac{\Delta V}{\Delta\Gamma_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) = \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a, \end{aligned}$$

where we used the fact that $\gamma^H_{ta} + \gamma^H_{tu} = \tilde{\gamma}^H_{ta} + \tilde{\gamma}^H_{tu} = \Gamma^H_t$ (which is easily proven from Lemma 1).

Point (iii) requires us to calculate $\tilde{E}(c^\dagger_{ts})$.

$$\begin{aligned} \tilde{E}(c^\dagger_{ts}) &= c^\dagger_{aa}\tilde{\gamma}^H_{aa} + c^\dagger_{au}\tilde{\gamma}^H_{au} + c^\dagger_{ua}\tilde{\gamma}^H_{ua} + c^\dagger_{uu}\tilde{\gamma}^H_{uu} = c^\dagger_{au}\left[\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right] \\ &= \frac{c^\dagger_{au}}{\tilde{\gamma}^H_{au}}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au})(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) = \frac{c^\dagger_{au}}{\tilde{\gamma}^H_{au}}\Gamma^H_a(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) \\ &= \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} = \tilde{E}(c^*_{ts})\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}. \end{aligned}$$

Since $\Gamma^H_u = 1 - \Gamma^H_a$, it is clear that the numerator is at least as large as the denominator. This proves point (iii).

Finally, for point (iv), we need to calculate $E(c^\dagger_{ts})$.

$$\begin{aligned} E(c^\dagger_{ts}) &= c^\dagger_{aa}\gamma^H_{aa} + c^\dagger_{au}\gamma^H_{au} + c^\dagger_{ua}\gamma^H_{ua} + c^\dagger_{uu}\gamma^H_{uu} = c^\dagger_{au}\left[\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\gamma^H_{aa} + \gamma^H_{au} + \gamma^H_{uu}\right] \\ &= \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma^H_{aa}\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\gamma^H_{aa} + \gamma^H_{au}\tilde{\gamma}^H_{au} + \gamma^H_{uu}\tilde{\gamma}^H_{au}}{\tilde{P}_{au}\Gamma^H_a}\frac{\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} \\ &= \frac{\Delta V}{\Delta\Gamma_a}\frac{\tilde{\gamma}_{au}\Gamma^H_a + \tilde{\gamma}_{uu}P_{aa}\Gamma^H_a + \gamma^H_{uu}\tilde{P}_{au}\Gamma^H_a}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} = E(c^*_{ts})\frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}P_{aa}\Gamma^H_u + P_{uu}\tilde{P}_{au}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}. \end{aligned}$$

Hence, to prove our result we are left to show that

$$\frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}P_{aa}\Gamma_u^H + P_{uu}\tilde{P}_{au}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} > 1$$

which is equivalent to

$$\tilde{P}_{uu}P_{aa}\Gamma_u^H + P_{uu}\tilde{P}_{au}\Gamma_u^H \geq \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H).$$

This requires some algebra.

$$\begin{aligned}
\tilde{P}_{uu}P_{aa}(1-\Gamma_a^H) + P_{uu}(1-\tilde{P}_{aa})(1-\Gamma_a^H) - \tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{uu}\Gamma_a^H \geq 0 \\
\tilde{P}_{uu}P_{aa} - \tilde{P}_{uu}P_{aa}\Gamma_a^H + P_{uu} - P_{uu}\tilde{P}_{aa} - P_{uu}\Gamma_a^H + P_{uu}\tilde{P}_{aa}\Gamma_a^H - \tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{uu}\Gamma_a^H \geq 0 \\
\tilde{P}_{uu}P_{aa} - P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}\tilde{P}_{aa} - P_{uu}\Gamma_a^H + \tilde{P}_{uu}\Gamma_a^H - \tilde{P}_{uu}P_{aa}\Gamma_a^H + P_{uu}\tilde{P}_{aa}\Gamma_a^H + P_{uu} \geq 0
\end{aligned}$$

From here, we substitute for some of the $\tilde{P}_{ts}$ to get

$$\begin{gathered}
(\tilde{P}_{uu}P_{aa} - P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}P_{aa} - \tilde{P}_{uu}b_a) + (-P_{uu}\Gamma_a^H + P_{uu}\Gamma_a^H - b_u\Gamma_a^H) + \\
+(-P_{uu}P_{aa}\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu}P_{aa}\Gamma_a^H + P_{uu}b_a\Gamma_a^H) + P_{uu} \geq 0
\end{gathered}$$

and finally

$$\begin{aligned}
& - P_{uu}\tilde{P}_{aa} - \tilde{P}_{uu}b_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
& - P_{uu}P_{aa} - P_{uu}b_a - P_{uu}b_a + b_ub_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
& - P_{uu}\underbrace{(P_{aa} + b_a)}_{1-\tilde{P}_{au}} - P_{uu}b_a + b_ub_a - b_u\Gamma_a^H + P_{uu}b_a\Gamma_a^H + b_uP_{aa}\Gamma_a^H + P_{uu} \geq 0 \\
& b_u\left(b_a - \Gamma_a + P_{aa}\Gamma_a^H\right) + P_{uu}\left[1 - b_a(1-\Gamma_a^H) - (1-\tilde{P}_{au})\right] \geq 0 \\
& b_u\left(b_a - P_{au}\Gamma_a^H\right) \geq P_{uu}\left[b_a(1-\Gamma_a^H) - P_{au} + b_a\right] \\
& b_u\left(b_a - P_{au}\Gamma_a^H\right) \geq P_{uu}\left(b_a(1+\Gamma_u^H) - P_{au}\right).
\end{aligned}$$

Note that the APE requires $b_a > P_{au}\Gamma_a$ as described in the proof of Proposition 2. This means that the LHS is always positive and we can therefore derive the condition presented in the proposition.

Proof of Proposition 4: First we study condition (6).

$$b_u \geq P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}$$

At $b_u = 0$, condition (6) corresponds to $b_a < P_{au}/(1+\Gamma_u^H)$. Hence, $P_{au}/(1+\Gamma_u^H)$ is the intercept of the RHS of the condition with the x-axis. Let

$$P_{au}/(1+\Gamma_u^H) \equiv \underline{b}_a.$$

To show that this condition is compatible with (5), we once again refer to (14), that is (5) solved for b_u, and therefore that an area where overconfidence is socially desirable always exists. We need to show that $\underline{b}_a$ is larger than the intercept of condition (14) (holding with equality) with the x-axis. We start from the latter, which we already calculated in Part 3 of the proof to Proposition 3.

$$b_a = P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z}.$$

We then need to show that

$$P_{au}\frac{(1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z}{(1-\Gamma_a^H)W + P_{uu}Z} < \frac{P_{au}}{(1+\Gamma_u^H)}.$$

To do this, we get

$$
\begin{aligned}
&(1-\Gamma_a^H)W + P_{uu}Z > (1-\Gamma_a^H)W + P_{uu}\Gamma_a^H Z + (1-\Gamma_a^H)W\Gamma_u^H + P_{uu}\Gamma_a^H Z\Gamma_u^H \\
&P_{uu}(1-\Gamma_a^H)Z - (1-\Gamma_a^H)W\Gamma_u^H - P_{uu}\Gamma_a^H\Gamma_u^H Z > 0 \\
&P_{uu}\Gamma_u^H Z - W(\Gamma_u^H)^2 - P_{uu}\Gamma_a^H\Gamma_u^H Z > 0 \\
&P_{uu}\Gamma_u^H Z \underbrace{(1-\Gamma_a^H)}_{\Gamma_u^H} - W(\Gamma_u^H)^2 > 0 \;\Rightarrow\; P_{uu}Z - W > 0
\end{aligned}
$$

We can now expand Z and W to get

$$
\begin{aligned}
&P_{uu}Z - W > 0 \\
&P_{uu}P_{aa}\Gamma_a^H + P_{uu}P_{ua}\Gamma_u^H - \Gamma_a^H P_{aa} + \Gamma_a^H P_{ua} > 0 \\
&-P_{aa}\Gamma_a^H + P_{uu}\Gamma_u^H + \Gamma_a^H > 0 \\
&P_{uu}\Gamma_u^H + \Gamma_a^H(1-P_{aa}) > 0,
\end{aligned}
$$

which is obviously always true. This proves that an area where overconfidence is socially desirable always exists, at least for $b_u = 0$. We now show that this area also exists for positive values of b_u. Since we know that the curve representing condition (5) intercepts the x-axis before (6), it is enough to show that the loci of points where the two conditions hold cross only once in (b_a, b_u) space and that they do so at $(b_a, b_u) = (P_{au}, P_{uu})$. To formally prove the shape of Figure 2, we are also going to show that the locus where (6) binds is concave in (b_a, b_u) space.

Take the two conditions binding and equate the two RHSs to get:

$$
\begin{aligned}
&P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}=P_{uu}-\frac{(P_{au}-b_a)\left(1-\Gamma_a^H\right)W}{(b_a-P_{au}\Gamma_a^H)\,Z}\\
&P_{uu}\left[(b_a(1+\Gamma_u^H)-P_{au})Z-(b_a-P_{au}\Gamma_a^H)Z\right]=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
&P_{uu}(b_a-P_{au})\Gamma_u^H Z=-\left(P_{au}-b_a\right)\left(1-\Gamma_a^H\right)W\\
&(P_{uu}Z-W)(b_a-P_{au})=0\\
&(P_{uu}P_{aa}\Gamma_a^H+P_{uu}P_{ua}\Gamma_u^H-P_{aa}\Gamma_a^H+P_{ua}\Gamma_a^H)(b_a-P_{au})=0\\
&P_{ua}(P_{au}\Gamma_a^H+P_{uu}\Gamma_u^H)(b_a-P_{au})=0,
\end{aligned}
$$

which holds only if $b_a = P_{au}$.When plugged into any of the two conditions we get that the corresponding value is $b_u = P_{uu}$. Hence the two curves cross only at that point. This concludes the proof of the Proposition. To show that the RHS of (6) is concave simply calculate the first derivative and obtain:

$$
\begin{aligned}
&\frac{\partial}{\partial b_a}\left[P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}}{b_a-P_{au}\Gamma_a^H}\right]P_{uu}\frac{b_a(1+\Gamma_u^H)-P_{au}\Gamma_a^H(1+\Gamma_u^H)-b_a(1+\Gamma_u^H)+P_{au}}{(b_a-P_{au}\Gamma_a^H)^2}\\
&=P_{uu}P_{au}\frac{1-\Gamma_a^H(1+\Gamma_u^H)}{(b_a-P_{au}\Gamma_a^H)^2}P_{uu}=P_{au}\frac{1-2\Gamma_a^H+(\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}=P_{uu}P_{au}\frac{(1-\Gamma_a^H)^2}{(b_a-P_{au}\Gamma_a^H)^2}>0.
\end{aligned}
$$

The second derivative is obviously negative since b_a only appears at the denominator.

Proof of Proposition 5: To prove this result, we present the derivations for the optimal contract implementing λ^L for a rational and an agent who displays overprecision. First of all, recall that SPEs are positively correlated regardless of the effort exerted (Assumption 2).

The problem the principal faces is the same as (4) with a reversed (IC) and γ_{ts}^L

instead of γ^H_{ts} for all t and s.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \tag{24}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}^L_{ts} - V(\lambda^L) \geq \bar{u} \tag{PC}$$

$$\sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} \leq \Delta V \tag{IC}$$

$$w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} \leq w_{ua}\gamma^L_{aa} + w_{uu}\gamma^L_{au} \tag{TR^a_P}$$

$$w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \leq w_{aa}\gamma^L_{ua} + w_{au}\gamma^L_{uu} \tag{TR^u_P}$$

$$c_{aa}\tilde{\gamma}^L_{aa} + c_{ua}\tilde{\gamma}^L_{ua} \geq c_{au}\tilde{\gamma}^L_{aa} + c_{uu}\tilde{\gamma}^L_{ua} \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} \geq c_{aa}\tilde{\gamma}^L_{au} + c_{ua}\tilde{\gamma}^L_{uu} \tag{TR^u_A}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

Lemma 16. *For all b_A and b_u, low effort can be implemented by the principal with a truth-telling, budget-balancing contract $w^\ell_{ts} = c^\ell_{ts} = V(\lambda^L) + \bar{u}$. Contract $\left\{w^\ell_{ts}, c^\ell_{ts}\right\}_{t,s}$ is optimal among all budget-balancing contracts. It is also guile-free.*

Proof. First, notice that Lemma 5 and 6 hold also for the case of low effort implementation, given Assumption 2 and $\frac{\tilde{\gamma}^L_{ua}}{\tilde{\gamma}^L_{aa}} > \frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}}$. Hence, following the same logic behind the proof of Lemma 7, any budget balancing contract must feature $w_{ts} = c_{ts} = c$ for all t and s. Contrary to the case of high effort implementation, however, a contract like this always satisfies the (IC) since

$$c\left(\sum_{ts}\Delta\tilde{\gamma}_{ts}\right) = 0 < \Delta V.$$

All truthful reporting constraints also hold, as well as the (LL) ones. The participation constraint becomes

$$c\left(\sum_{ts}\tilde{\gamma}^L_{ts}\right) \geq V(\lambda^L) + \bar{u} \quad \Rightarrow \quad c \geq V(\lambda^L) + \bar{u}.$$

Since the objective function is now simply $\min_{ts} c$ then the restricted problem is solved by $c = V(\lambda^L) + \bar{u}$. ∎

The Lemma above yields the only solution for a rational agent.[25] However, since our agent can display overprecision, we have to check whether the principal can find a way to manipulate the contract taking advantage of the agent's biased beliefs. In other words, a contract that grants $\tilde{E}(c_{ts}) = \bar{u} + V(\lambda^L)$ but that in fact yields (and costs) less, that is $E(c_{ts}) < \bar{u} + V(\lambda^L) = \tilde{E}(c_{ts})$. Lemma 16 states that, if such a contract exists, it must feature some deadweight loss. This is because Lemma 7 does not apply here and Lemma 16 shows that $\left\{w^\ell_{ts}, c^\ell_{ts}\right\}_{t,s}$ is the only optimal contract featuring no deadweight loss. This implies that, if there were to exist another contract implementing optimally low effort, this ought to feature some deadweight loss. This result is key for this analysis.

A second key feature of this analysis is that we are going to assume again that the agent's outside option is small enough . What this does it to allow us to ignore the (PC) and solve the problem without it.[26]

We are now going to present a series of Lemmas, from 17 to 22, to prove that, under (15), $\{w^\ell_{ts}, c^\ell_{ts}\}_{t,s}$ is the only optimal contract to implement low effort.

Lemma 17. *Given* (15)*, for any value of the bias, when the principal implements low effort, the* (PC) *always binds.*

Proof. Suppose not. In that case, the principal can decrease all w_{ts} and all c_{ts} by $\epsilon > 0$. All the other constraints are unchanged and cost of implementation decreases. This does not fully prove the statement, however. It could be that $w_{ts} = c_{ts} = 0$

[25] To see that a rational agent is always assigned $\left\{w^\ell_{ts}, c^\ell_{ts}\right\}_{t,s}$, notice that the (PC) always binds, as we argue below. Hence, a rational agent must be granted at least $\bar{u} + V(\lambda^L)$. Since principal and agent have the same beliefs, there is no room for the principal to manipulate the contract trying to decrease $E(w_{ts})$ below $\bar{u} + V(\lambda^L)$.

[26] Numerical simulations show that for a very large $\bar{u}$, or a very low ΔV, low effort is implemented by a BPE-like contract, the values of which are independent of b_a and b_u as in the case of high effort implementation.

for some t and s, in which case, the principal cannot decrease them all. We need to prove that a deviation is possible in these cases as well.

Recall that the (IC) is assumed slack when (15) holds and let us rewrite the (TR) constraints as in the proofs of Lemmas 5 and 6.

$$(w_{au} - w_{uu})\frac{\gamma^L_{au}}{\gamma^L_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^L_{uu}}{\gamma^L_{ua}}$$
$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}^L_{ua}}{\tilde{\gamma}^L_{aa}} \leq (c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}}.$$

By Lemma 16, we know that at least one $w_{ts} > c_{ts}$ must hold, otherwise any contract derived would be dominated by $\{w^\ell_{ts}, c^\ell_{ts}\}_{t,s}$. Also, from From (TR_A), if only one of the c_{ts} is strictly positive then at least one of the two inequalities will fail. Hence, at least two c_{ts} must be positive for the (TR_A) to hold. Further, notice that they must be c_{aa} and c_{au} or c_{uu} and c_{ua} or c_{uu} and c_{aa}. Suppose c_{aa} and c_{au} are positive, with $c_{ua} = c_{uu} = 0$, then given Lemma 5 at least $w_{ua} > 0$ must hold. The principal can decrease c_{aa}, c_{au}, w_{aa} and w_{au} by $\epsilon \in [0, w_{ua} - c_{ua})$, while also decreasing w_{ua}, to adjust for the (TR_P) to hold. This does not violate any constraints and decreases costs. The symmetric logic holds for c_{uu} and c_{ua} positive and $c_{aa} = c_{au} = 0$. For the case of c_{uu} and c_{aa} positive and $c_{ua} = c_{au} = 0$, instead, notice that, from (TR_P), it must be that w_{ua} and w_{au} are greater then zero. The principal can then decrease c_{uu} and c_{aa} in a way that (TR_A) is not violated. Further, she decreases by the same amount w_{uu} and w_{aa}, and uses a decrease in w_{ua} and w_{au} to adjust the (TR_P) to the new values of w_{uu} and w_{aa}.

Now suppose only one of the c_{ts} is 0. Notice that, from (TR_A), it can only be either the c_{au} or the c_{ua}. Suppose it is c_{au} then, by Lemma 5, $w_{au} > 0$. The principal can then decrease c_{uu}, c_{ua}, w_{uu}, w_{ua} by $\epsilon \in [0, w_{au} - c_{au})$ and use the w_{au} to adjust for the (TR_P) to hold. This does not violate any constraints and decreases costs. The symmetric logic holds for c_{ua}. This concludes the proof. ∎

Using the symmetric versions of the algebra used in all other derivations, we solve the (PC) for c_{aa} and rewrite the (TR_A) constraints.

$$\begin{aligned}
c_{aa} &= \frac{1}{\tilde{\gamma}^L_{aa}}\left(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\tilde{\gamma}^L_{ua}\right) \\
\Rightarrow &\quad \bar{u} + V(\lambda^L) \geq c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u \\
\Rightarrow &\quad c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u \geq \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au}
\end{aligned}$$

The new problem is therefore

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \tag{25}$$

$$\text{s.t.} \quad (w_{au} - w_{uu})\frac{\gamma^L_{au}}{\gamma^L_{aa}} \leq (w_{ua} - w_{aa}) \tag{TR^a_P}$$

$$(w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma^L_{uu}}{\gamma^L_{ua}} \tag{TR^u_P}$$

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u \tag{TR^a_A}$$

$$c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u \geq \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au} \tag{TR^u_A}$$

$$w_{aa} \geq c_{aa} = \frac{1}{\tilde{\gamma}^L_{aa}}(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu} - c_{ua}\tilde{\gamma}^L_{ua}) \geq 0 \tag{LL_{aa}}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

Lemma 18. *Given* (15)*, for any value of the bias, when the principal implements low effort with a contract featuring a deadweight loss, the* (TR^u_A) *always binds.*

Proof. Suppose not. The proof changes depending on which one among the (TR_P) binds at optimum.

Suppose the (TR^a_P) binds, while the (TR^u_P) is slack, then the principal can decrease c_{au} and w_{au} by ϵ. This relaxes the (TR^a_A). At the same time, (LL_{aa}) implies that c_{aa} increases by $\epsilon\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}$ and w_{aa} by an amount at most as large. This affects the

(TR_P). The (TR_P^a) changes to the LHS by $-\epsilon\frac{P_{au}}{P_{aa}}$ and to the RHS by $-\epsilon\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}$. The RHS changes less since, for an overconfident agent, $b_a > 0$ implying $\tilde{P}_{au} < P_{au}$ and $\tilde{P}_{aa} > P_{aa}$. The (TR_P^u) is tightened by the change but, since it is assumed slack, there always exists an ϵ small enough for it to still hold. Finally, to see that this deviation is optimal, notice that the change in the objective function is given by

$$-\epsilon\gamma_{au}^L + \epsilon\frac{\gamma_{aa}^L\tilde{P}_{au}}{\tilde{P}_{aa}} = \epsilon\frac{\Gamma_a^L}{\tilde{P}_{aa}}(P_{aa}\tilde{P}_{au} - \tilde{P}_{aa}P_{au}),$$

which is negative since $\tilde{P}_{aa} > P_{aa}$ and $\tilde{P}_{au} < P_{au}$.

Now suppose it is the (TR_P^u) that binds while the (TR_P^a) is slack, we will show that this can never be the case or otherwise the (TR_A^u) would be violated.

First of all, since the (TR_P^u) binds, we can re-state the objective function as

$$w_{aa}(\gamma_{aa}^L + \gamma_{ua}^L) + w_{au}(\gamma_{uu}^L + \gamma_{au}^L),$$

and set it subject to:

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \qquad (TR_A^a)$$

$$w_{aa} \geq c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^L}(\bar{u} + V(\lambda^L) - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L - c_{ua}\tilde{\gamma}_{ua}^L) \geq 0 \qquad (LL_{aa})$$

$$w_{au} \geq c_{au} \geq 0 \qquad (LL_{au})$$

$$w_{uu} \geq c_{uu} \geq 0 \qquad (LL_{uu})$$

$$w_{ua} \geq c_{ua} \geq 0 \qquad (LL_{ua})$$

Given the above, it is obvious that $w_{aa} = c_{aa}$ and $w_{au} = c_{au}$. Substituting them into the problem and re-ordering constraints for simplicity one obtains

$$c_{aa}(\gamma_{aa}^L + \gamma_{ua}^L) + c_{au}(\gamma_{uu}^L + \gamma_{au}^L),$$

subject to:

$$c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \leq \bar{u} + V(\lambda^L) \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L < \bar{u} + V(\lambda^L) - c_{ua}\tilde{\gamma}_{ua}^L \tag{LL_{aa}}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

Notice that we have rewritten (LL_{aa}) as slack. This is because if it were to bind then, first, c_{aa} would be zero, second, by Lemma (4) also c_{au} would be zero, and third, by the (TR_P^u) binding we would have

$$w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L = 0$$

which by the limited liability constraints implies that $w_{ua} = w_{uu} = 0$ and therefore $c_{ua} = c_{uu} = 0$ which violates the (TR_A^u).To see this, note that the LHS of the constraint will be zero while the RHS will be positive

Given this and that c_{aa} cannot equal zero, to minimize $c_{aa}(\gamma_{aa}^L + \gamma_{ua}^L) + c_{au}(\gamma_{uu}^L + \gamma_{au}^L)$ the principal can only set $c_{au} = 0$ instead – since both (TR_A^a) and (LL_{aa}) are relaxed by this. The problem is therefore to minimize

$$c_{aa}(\gamma_{aa}^L + \gamma_{ua}^L),$$

subject to:

$$c_{uu}\Gamma_u^L \leq \bar{u} + V(\lambda^L) \tag{TR_A^a}$$

$$c_{uu}\tilde{\gamma_{uu}^L} < \bar{u} + V(\lambda^L) - c_{ua}\tilde{\gamma_{ua}^L} \tag{LL_{aa}}$$

$$w_{au} \geq c_{au} \geq 0 \tag{LL_{au}}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

We now use again the binding (TR_P^u) to get $c_{aa} = c_{ua} + c_{uu}\frac{\gamma_{uu}^L}{\gamma_{ua}^L}$ and rewrite the objective function to get

$$\min \left[c_{ua} + c_{uu}\frac{\gamma_{uu}^L}{\gamma_{ua}^L}\right](\gamma_{aa}^L + \gamma_{ua}^L) \tag{26}$$

subject to the same constraints above. However, once again the principal finds itself unconstrained by (TR_A^a) and (LL_{aa}). He will set $c_{ua} = c_{uu} = 0$ and offer a contract that violates the (TR_A^u). As above, note that the LHS of the constraint will be zero while the RHS will be positive Hence, when (TR_A^u) is slack, the optimal contract cannot feature (TR_P^u) binding and the (TR_P^a) being slack.

Finally suppose that both (TR_P) bind, that is $w_{aa} = w_{ua}$ and $w_{au} = w_{uu}$. Given Lemma 16, it must be that either $w_{aa} > c_{aa}$ or $w_{ua} > c_{ua}$ (or both). When $w_{aa} > c_{aa}$, the principal can decrease c_{au}, c_{uu}, w_{au} and w_{uu} keeping w_{aa} constant. There always exists an ϵ small enough for this to be possible. This is obviously optimal and it relaxes all other constraints. When $w_{ua} > c_{ua}$, the principal can increase c_{ua} by ϵ. This will decrease c_{aa} and w_{aa} by $\epsilon\frac{\tilde{\gamma}_{ua}^L}{\tilde{\gamma}_{aa}^L}$ which, by the assumption on the (TR_P) forces the w_{ua} down by the same amount. Regardless of whether $\epsilon\frac{\tilde{\gamma}_{ua}^L}{\tilde{\gamma}_{aa}^L}$ is larger or smaller than ϵ, there always exists an ϵ small enough for $w_{ua} > c_{ua}$ to be preserved.

Of course, if no (TR_P) binds, the principal can decrease c_{au}, c_{uu}, w_{au} and w_{uu} and the increasing effect on w_{aa} is not enough to offset the gain, in the same fashion as above. This proves Lemma 18. ∎

By Lemma 18 we can solve for c_{ua} from the (TR_A^u),

$$c_{ua} = \frac{c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L - \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L},$$

and plug it into the function for c_{aa} from the (PC) to obtain

$$c_{aa} = \frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}_{au}^L - c_{uu}\tilde{\gamma}_{uu}^L}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_a^L}.$$

The above poses the following additional restrictions on c_{au} and c_{uu} respectively (via the (LL_{ts}) constraints):

$$(\bar{u} + V(\lambda^L))\tilde{P}_{au} \leq c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L,$$
$$(\bar{u} + V(\lambda^L))\tilde{P}_{uu} \geq c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L.$$

Rearranging the second equation we have

$$\bar{u} + V(\lambda^L) \geq c_{au}\frac{\tilde{P}_{au}}{\tilde{P}_{uu}}\Gamma_a^L + c_{uu}\Gamma_u^L.$$

Since $\tilde{P}_{au} < \tilde{P}_{uu}$ by positive correlation, this restriction is implied by the (TR_A^a) and can be, therefore, disregarded. It also proves that c_{aa} is always at least weakly positive and that it is strictly positive as long as $c_{au} > 0$. Let us decompose each (LL_{ts}) into (LL_{ts}^1), which requires $c_{ts} > 0$, and (LL_{ts}^2), which requires $w_{ts} > c_{ts}$. The above implies that (LL_{aa}^1) is implied by the other constraints. The new problem of the principal is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \tag{27}$$

$$\text{s.t.} \quad (w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} \le (w_{ua} - w_{aa}) \tag{TR^a_P}$$

$$(w_{ua} - w_{aa}) \le (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}} \tag{TR^u_P}$$

$$\bar{u} + V(\lambda^L) \ge c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u \tag{TR^a_A}$$

$$\bar{u} + V(\lambda^L) \le c_{au}\Gamma^L_a + c_{uu}\frac{\tilde{P}_{uu}}{\tilde{P}_{au}}\Gamma^L_u \tag{LL^1_{ua}}$$

$$w_{au} \ge c_{au} \ge 0 \tag{LL_{au}}$$

$$w_{uu} \ge c_{uu} \ge 0 \tag{LL_{uu}}$$

$$w_{ua} \ge c_{ua} \tag{LL^2_{ua}}$$

$$w_{aa} \ge c_{aa} \tag{LL^2_{aa}}$$

Lemma 19. *Given* (15)*, for any value of the bias, if there exists a contract featuring a deadweight loss that the principal optimally sets to implement low effort, it features the* (TR^a_P) *binding.*

Proof. First of all, notice that the Lemma does not rule out the case that suboptimal contracts can implement low effort with the (TR^a_P) slack. Rather, it states that there exist no generally optimal way to implement low effort with a contract featuring a deadweight loss and the (TR^a_P) slack.

Suppose this is not true and let the (TR^a_P) be slack. Recall that constraint (LL^1_{ua}) ensures that $c_{ua} \ge 0$. When it binds, $c_{ua} = 0$. Suppose the (LL^1_{ua}) does bind and $c_{ua} = 0$. This also implies that $w_{ua} > 0$. To see why, notice that, if $w_{ua} = 0$, then also $w_{aa} = 0$, by Lemma 5. Hence, c_{au} and c_{uu} have to be such that $c_{aa} = 0$ from the

(PC) binding. This then implies

$$c_{ua} = \frac{\left(\bar{u} + V(\lambda^L)\right)(\tilde{P}_{uu} - \tilde{P}_{au})}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^L} > 0.$$

Hence, when (LL_{ua}^1) binds, it must be that $w_{ua} > 0$. It is immediate to see how the (TR_P^a) cannot be slack then, since the principal could simply decrease w_{ua} and decrease the objective function tightening the (TR_P^a).

Now suppose the (LL_{ua}^1) is slack. This implies that $c_{ua} > 0$ and, by Lemma 6, also $c_{uu} > 0$. The proof further divides depending on whether $c_{au} = 0$ or not.

Suppose $c_{au} = 0$. We are going to show that any solution either sets the (LL_{ua}^1) binding or is suboptimal to the no deadweight loss contract. First of all, it must be that $w_{au} > 0$. Otherwise $w_{uu} = 0$ would be implied by Lemma 5. Then $c_{uu} = 0$ would make $c_{ua} < 0$. Hence if $c_{au} = 0$, $w_{au} > 0$. Given this, the principal can decrease w_{au} until (TR_P^u) binds. Hence,

$$w_{au} = (w_{ua} - w_{aa})\frac{P_{ua}}{P_{uu}} + w_{uu}.$$

Further, all other w_{ts} are set equal to their respective c_{ts}. In fact, suppose this was not the case. If $w_{uu} > c_{uu}$ or $w_{ua} > c_{ua}$, the principal can simply decrease them, without violating any constraint. If $w_{aa} > c_{aa}$, instead, the principal can decrease it by ϵ while increasing w_{au} by $\epsilon\frac{P_{ua}}{P_{uu}}$. This does not violate the (TR_P^u) and it is optimal since the change in the objective function is given by

$$-\epsilon\left(\gamma_{aa}^L - \frac{P_{au}P_{ua}}{P_{uu}}\Gamma_a^L\right) = -\epsilon\frac{\Gamma_a^L(P_{aa} - P_{ua})}{P_{uu}} < 0.$$

To conclude this part of the proof, notice that given $w_{uu} = c_{uu}$, $w_{aa} = c_{aa}$, $w_{ua} = c_{ua}$, $w_{au} = (w_{ua} - w_{aa})\frac{P_{ua}}{P_{uu}} + w_{uu}$ and the fact that all c_{ts} can be written as a function of

c_{uu}, the objective function depends only on c_{uu} and is subject to

$$c_{uu} \leq \frac{\bar{u} + V(\lambda^L)}{\Gamma_u^L}.$$

Depending on the sign of the coefficient of c_{uu} in the objective function, if the latter is minimized by minimizing c_{uu}, the assumption that the (LL_{ua}^1) is slack would be violated, yielding a contradiction. If it is minimized by maximizing c_{uu}, then (TR_A^a) binds and $c_{uu} = \frac{\bar{u}+V(\lambda^L)}{\Gamma_u^L}$. At this value, the rest of compensations and wages are given by

$$\begin{array}{llll} w_{aa} = 0 & w_{au} = \frac{c_{uu}}{P_{uu}} & w_{uu} = c_{uu} & w_{ua} = c_{ua} \\ c_{aa} = 0 & c_{au} = 0 & c_{uu} = \frac{\bar{u}+V(\lambda^L)}{\Gamma_u^L} & c_{ua} = c_{uu} \end{array}$$

This implies a contract with $E(w_{ts}) = w_{au}\gamma_{au}^L + \bar{u} + V(\lambda^L)$ which is clearly larger than $E(w_{ts}^\ell)$.[27] Hence, even if this contract implements λ^L, it is never optimal.

To conclude the proof, we derive a similar contradiction for the case of $c_{au} > 0$. In this case, when both (LL_{ua}^1) and (TR_P^a) are slack, the principal faces

$$\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L + w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L \tag{28}$$

$$\text{s.t.} \quad (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}} \qquad (TR_P^u)$$

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma_a^L + c_{uu}\Gamma_u^L \qquad (TR_A^a)$$

$$w_{au} \geq c_{au} \qquad (LL_{au}^2)$$

$$w_{uu} \geq c_{uu} \qquad (LL_{uu}^2)$$

$$w_{ua} \geq c_{ua} \qquad (LL_{ua}^2)$$

$$w_{aa} \geq c_{aa} \qquad (LL_{aa}^2)$$

[27] Technically, to be sure that this is indeed a potential solution, we need to check that it satisfies the (IC). It is easy to see that it does so even regardless of (15), since

$$\sum_{ts} \Delta\tilde{\gamma}_{ts}c_{ts} = \frac{(\bar{u} + V(\lambda))}{\Gamma_u^L}(\Delta\tilde{\gamma}_{uu} + \Delta\tilde{\gamma}_{ua}) = \frac{(\bar{u} + V(\lambda))}{\Gamma_u^L}\Delta\Gamma_u < 0 \leq \Delta V.$$

where all $c_{ts} > 0$ following Lemma 6. It is immediate to see how the (LL^2_{uu}) and (LL^2_{ua}) bind in this case. If they do not, decreasing the relevant w_{ts} decreases costs and does not affect any constraint. At this point, from Lemma 16, we know that only one between (LL^2_{aa}) and (LL^2_{au}) binds. Suppose it is the latter, then $w_{aa} > c_{aa}$. In this case, the the principal can decrease c_{uu} until (LL^1_{ua}) binds, violating the assumption. Suppose instead, that $w_{aa} = c_{aa}$ while $w_{au} > c_{au}$. The (TR^u_P) becomes

$$\frac{c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u\Gamma^L_a} + c_{uu}\frac{P_{uu}}{P_{ua}} - \frac{\bar{u} + V(\lambda^L)}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u\Gamma^L_a}\left(\tilde{\gamma}^L_{au} + \tilde{\gamma}^L_{uu}\right) \leq w_{au}.$$

Since the (LL^2_{au}) is slack, the principal can decrease w_{au} until the (TR^u_P) binds. We can now calculate the new objective function where $w_{ts} = c_{ts}$ with the exception of the w_{au} (which, instead, comes from the (TR^u_P)):

$$\begin{aligned}
&w_{aa}\gamma^L_{aa} + w_{au}\gamma^L_{au} + w_{ua}\gamma^L_{ua} + w_{uu}\gamma^L_{uu} \\
&= \left[\frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_a}\right]\gamma^L_{aa} + w_{au}\gamma^L_{au} \\
&\qquad + \left[\frac{c_{au}\tilde{\gamma}^L_{au} + c_{uu}\tilde{\gamma}^L_{uu} - \left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_u}\right]\gamma^L_{ua} + c_{uu}\gamma^L_{uu} \\
&\propto w_{au}\gamma^L_{au}(\tilde{P}_{aa} - \tilde{P}_{ua}) + c_{au}\tilde{\gamma}^L_{au}(P_{ua} - P_{aa}) + c_{uu}\left(\tilde{\gamma}^L_{uu}(P_{ua} - P_{aa}) + \gamma^L_{uu}\right) \\
&\propto c_{au}\tilde{\gamma}^L_{au}\left[(P_{ua} - P_{aa}) + \frac{P_{au}}{\Gamma^L_u}\right] + c_{uu}\left[\tilde{\gamma}^L_{uu}(P_{ua} - P_{aa}) + \gamma^L_{uu} + P_{au}\tilde{P}_{uu} + \frac{\gamma^L_{au}P_{uu}(\tilde{P}_{aa} - \tilde{P}_{ua})}{P_{ua}}\right].
\end{aligned}$$

In the reduced problem, the objective function is subject only to

$$\bar{u} + V(\lambda^L) \geq c_{au}\Gamma^L_a + c_{uu}\Gamma^L_u.$$

Since the sign of the coefficients of c_{au} and c_{uu} is not trivial, we study all possible cases and show that all of them lead to a contradiction. First, suppose the case where both c_{uu} and c_{au} increase the objective functions, then the principal wants to decrease them both. This violates the assumption that (LL^1_{ua}) is slack. Now suppose c_{uu} increases the objective function while c_{au} decreases it. Then the principal sets $c_{uu} = 0$ and c_{au} such that the (TR^a_A) binds. However, when $c_{uu} = 0$, the (TR^a_A) coincides with the (LL^1_{ua}) and therefore the latter binds, providing a contradiction again. Now suppose c_{uu} decreases the objective function while c_{au} increases it. Then $c_{au} = 0$ which violates the assumption that $c_{au} > 0$. Finally, suppose both c_{uu} and c_{au} decrease the objective function. Then the principal sets the (TR^a_A) binding and solves for c_{au} to get

$$c_{au} = \frac{\bar{u} + V(\lambda^L)}{\Gamma^L_a} - c_{uu}\frac{\Gamma^L_A}{\Gamma^L_u}. \tag{29}$$

Substituting this into the objective function, we obtain the final form of the reduced problem:

$$\min_{c_{uu}} \; c_{uu}\left[(P_{ua} - P_{aa})\frac{(\tilde{\gamma}^L_{au}\Gamma^L_a - \tilde{\gamma}^L_{uu}\Gamma^L_u)}{\Gamma^L_u} + \gamma^L_{uu} + P_{au}\tilde{P}_{uu} + \frac{\gamma^L_{au}P_{uu}(\tilde{P}_{aa} - \tilde{P}_{ua})}{P_{ua}} - \frac{\tilde{\gamma}^L_{au}P_{au}}{\Gamma^L_u}\right]$$

If the coefficient of c_{uu} is positive, then the solution to the problem is $c_{uu} = 0$ and $c_{au} = \frac{\bar{u}+V(\lambda^L)}{\Gamma^L_a}$ yielding, once again, to the (LL^1_{ua}) binding since it coincides with the (TR^a_A). If the coefficient is negative, instead, the problem is solved by the maximum possible c_{uu}. That is, the value that sets $c_{au} = 0$ from (29). This violates the assumption that $c_{au} > 0$. This concludes the proof for the case of a positive c_{au} and (LL^1_{ua}) slack.

This concludes the proof of the Lemma showing that, if there exists an optimal contract that implements low effort with a deadweight loss, it must be that it sets the (TR^a_P) binding. ∎

Now that we know that the (TR^a_P) binds, we are going to re-write the problem in two different ways. With the first one, we are going to prove that $c_{ua} = 0$. With the second, we are going to select a value for each w_{ts} as a function of c_{ts}.

First, solve the (TR^a_P) for $w_{au}\gamma^L_{au} = w_{ua}\gamma^L_{aa} + w_{uu}\gamma^L_{au} - w_{aa}\gamma^L_{aa}$ and substitute it into the objective function. This makes the w_{aa} disappear from the objective function, which is now given by $w_{ua}(\gamma^L_{aa} + \gamma^L_{ua}) + w_{uu}(\gamma^L_{au} + \gamma^L_{uu})$.

Lemma 20. *Given* (15)*, for any value of the bias, if there exists a contract featuring a deadweight loss that implements low effort, it features* $c_{ua} = 0$*.*

Proof. Suppose not, and consider the objective function $w_{ua}(\gamma^L_{aa} + \gamma^L_{ua}) + w_{uu}(\gamma^L_{au} + \gamma^L_{uu})$. Notice that $c_{ua} > 0$ corresponds to (LL^1_{ua}) slack. Hence, the only constraint on c_{au} and c_{uu} is the (TR^a_A). Decreasing c_{au} and c_{uu} by ϵ also decreases c_{ua} and increases c_{aa}. The latter produces no effect on the objective function while the decrease in c_{au}, c_{uu} and c_{ua} allows the principal to decrease the objective function via either w_{uu} or w_{ua}. This provides a contradiction to (LL^1_{ua}) being slack. ∎

Lemma 21. *Given* (15)*, for any value of the bias, if there exists a contract, featuring a deadweight loss that implements low effort, it must feature*

$$\begin{aligned}
w_{uu} &= c_{uu} \\
w_{aa} &= c_{aa} \\
w_{au} &= \max\{c_{au}, c_{uu}\} \\
w_{ua} &= (\max\{c_{au}, c_{uu}\} - c_{uu})\frac{P_{au}}{P_{aa}} + c_{aa}
\end{aligned}$$

Proof. First of all, from Lemma 18, we can solve the (TR^a_P) for

$$w_{ua} = (w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} + w_{aa}.$$

When plugged into the objective function, it yields

$$\begin{aligned}
&w_{aa}(\gamma^L_{aa} + \gamma^L_{ua}) + w_{au}\left(\gamma^L_{au} + \frac{P_{au}}{P_{aa}}\gamma^L_{ua}\right) + w_{uu}\left(\gamma^L_{uu} - \frac{P_{au}}{P_{aa}}\gamma^L_{ua}\right) \\
&\propto w_{aa}(\gamma^L_{aa} + \gamma^L_{ua})P_{aa} + w_{au}(\gamma^L_{au} + \gamma^L_{ua})P_{au} + w_{uu}\Gamma^L_u(P_{aa} - P_{ua}).
\end{aligned}$$

The above is subject only to the (LL_{ua}) and to $w_{au} \geq w_{uu}$, by Lemma 5. Hence, the principal can decrease the wage levels and set $w_{uu} = c_{uu}$, $w_{aa} = c_{aa}$ and $w_{au} = \max\{c_{au}, w_{uu}\} = \max\{c_{au}, c_{uu}\}$. The proof is concluded by plugging these into the function for w_{ua}. ∎

Given Lemmas 20 and 21, we have the new objective function

$$c_{aa}(\gamma^L_{aa} + \gamma^L_{ua})P_{aa} + \max\{c_{au}, c_{uu}\}(\gamma^L_{au} + \gamma^L_{ua}) + c_{uu}\Gamma^L_u(P_{aa} - P_{ua}).$$

Before plugging in the value for c_{aa}, notice that we can solve the (LL^1_{ua}) to get

$$c_{au} = \frac{\bar{u} + V(\lambda^L)}{\Gamma^L_a} - c_{uu}\frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}}.$$

We plug this into the value for c_{aa} to obtain

$$c_{aa} = \frac{\left(\bar{u} + V(\lambda^L)\right)\tilde{P}_{uu} - c_{au}\tilde{\gamma}^L_{au} - c_{uu}\tilde{\gamma}^L_{uu}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma^L_a} = \frac{\bar{u} + V(\lambda^L)}{\Gamma^L_a},$$

which is, therefore, irrelevant for the objective function of the reduced problem. This latter is given by

$$\min_{c_{au}, c_{uu}} \max\{c_{au}, c_{uu}\}(\gamma^L_{au} + \gamma^L_{ua})P_{au} + c_{uu}\Gamma^L_u(P_{aa} - P_{ua}). \tag{30}$$

This allows us to state the final Lemma, that shows how there exist no optimal contract implementing low effort with deadweight loss.

Lemma 22. *When* (15) *holds, the principal implements low effort with* $\{w^\ell_{ts}, c^\ell_{ts}\}_{t,s}$.

Proof. First of all, we show that $c_{au} \geq c_{uu}$ in (30). Suppose not, and $c_{uu} > c_{au}$, then the objective function only depends (positively) on c_{uu}. The problem is then solved by $c_{uu} = 0$, which contradicts $c_{uu} > c_{au}$. Given that $c_{au} \geq c_{uu}$, the problem becomes (disregarding any constant term)

$$\min_{c_{uu}} \left[-c_{uu}\frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}}(\gamma^L_{au} + \gamma^L_{ua}) + c_{uu}\Gamma^L_u(P_{aa} - P_{ua})\right].$$

Suppose the coefficient of c_{uu} is negative, then the solution would imply c_{uu} such that $c_{au} = 0$. This would violate $c_{au} \geq c_{uu}$. Hence, a solution only exists when the coefficient of c_{uu} is negative. Regardless of whether this is the case or not, notice that the solution to the problem would be

$$\begin{array}{llll} w_{aa} = c_{aa} & w_{au} = c_{aa} & w_{uu} = 0 & w_{ua} = \frac{c_{aa}}{P_{aa}} \\ c_{aa} = \frac{\bar{u}+V(\lambda^L)}{\Gamma_a^L} & c_{au} = c_{aa} & c_{uu} = 0 & c_{ua} = 0 \end{array}$$

which yields an expected wage payment of

$$E(w_{ts}) = w_{aa}\left(\Gamma_a^L + \frac{\gamma_{ua}^L}{P_{aa}}\right) = (\bar{u} + V(\lambda^L))\left(1 + \frac{\gamma_{ua}^L}{\gamma_{aa}^L}\right) > (\bar{u} + V(\lambda^L)) = E(w_{ts}^{\ell}).$$

Hence, even when a solution does exist, it is more expensive than the constant wage one.[28] This implies that when (15) holds, low effort is implemented with a constant wage contract. ∎

Given Lemma 22, it is immediate to see that the magnitude, or presence, of the bias does not affect the expected cost of implementing low effort. From Proposition 2 and Lemma 4, we know that the expected cost of implementing high effort, instead, is at least weakly decreasing in the bias. This concludes the proof.

[28]It is possible to show that the contract with deadweight loss above satisfies the (IC) under (15).