Subjective Evaluation Contracts for Overconfident Workers

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--- Online Appendix ---

A. Large Outside Option

This Appendix generalizes the model by assuming the agent has a large outside option. We start by assuming the agent is either rational and his outside option satisfies $\bar{u} > \bar{u}_1$ or displays overprecision and his outside option satisfies $\bar{u} \in (\bar{u}_1, \bar{u}_3]$. Next, we assume the agent displays overestimation and his outside option satisfies $\bar{u} > \bar{u}_2$. Finally, we assume the agent displays overprecision and his outside option satisfies $\bar{u} > \bar{u}_3$. Note that the thresholds $\bar{u}_2$ and $\bar{u}_3$ are not independent of the bias, and they are therefore, agent-specific Our first result generalizes the BPE contract.

Lemma 23. *When $\bar{u} \in (\bar{u}_1, \bar{u}_3]$, the APE satisfies the (PC) while the BPE violates it.*

Proof. To see that the BPE violates the (PC) simply compute:

$$
\begin{aligned}
&c^*_{aa}\tilde{\gamma}^H_{aa} + c^*_{au}\tilde{\gamma}^H_{au} + c^*_{ua}\tilde{\gamma}^H_{ua} + c^*_{uu}\tilde{\gamma}^H_{u} - V(\lambda^H) \geq \bar{u}_1 \\
&\frac{\Delta V}{\Delta \Gamma_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) - V(\lambda^H) \geq \bar{u}_1 \\
&\frac{V(\lambda^H)\Gamma^L_a - V(\lambda^L)\Gamma^H_a}{\Delta \Gamma_a} \geq \bar{u}_1 \\
&\bar{u}_1 = \bar{u}_1
\end{aligned}
$$

Hence any value above $\bar{u}_1$ violates the (PC). As for the APE, notice that:

$$
\begin{aligned}
& c^{\dagger}_{aa}\tilde{\gamma}^H_{aa} + c^{\dagger}_{au}\tilde{\gamma}^H_{au} + c^{\dagger}_{ua}\tilde{\gamma}^H_{ua} + c^{\dagger}_{uu}\tilde{\gamma}^H_{u} - V(\lambda^H) \geq \bar{u}_1 \\
& c^{\dagger}_{au}\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu} + \tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{aa}\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\right) - V(\lambda^H) \geq \bar{u}_1 \\
& c^{\dagger}_{au}\left(\frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{P}_{au}}\right) - V(\lambda^H) \geq \bar{u}_1 \\
& \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a \frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - V(\lambda^H) \geq \bar{u}_1 \\
& \bar{u}_2 \geq \bar{u}_1
\end{aligned}
$$

To conclude the proof we need to show that it is indeed the case that $\bar{u}_2 \geq \bar{u}_1$.

$$
\begin{aligned}
& \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_a \frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - V(\lambda^H) \geq \frac{\Delta V}{\Delta \Gamma_a}(\tilde{\gamma}^H_{aa} + \tilde{\gamma}^H_{au}) - V(\lambda^H) \\
& \Leftrightarrow \frac{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}\Gamma^H_u}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - (\tilde{P}_{aa} + \tilde{P}_{au}) \geq 0 \\
& \Leftrightarrow 2\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{P}_{aa} + \tilde{\gamma}^H_{aa}\tilde{P}_{uu} - \tilde{\gamma}^H_{au}\tilde{P}_{au} + \tilde{P}_{aa}\tilde{P}_{uu} \geq 0 \\
& \Leftrightarrow \tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}(1 - \tilde{P}_{aa}) \geq 0
\end{aligned}
$$

which is trivially true.[29] ∎

Proposition 6. *When $\bar{u} > \bar{u}_1$, then the principal offers a rational agent a Generalized BPE (GBPE) contract $\tilde{w}^*_{ts}, \tilde{c}^*_{ts}$ given by:*

$$
\begin{array}{llll}
\tilde{w}^*_{aa} = \tilde{c}^*_{aa} & \tilde{w}^*_{au} = \tilde{c}^*_{aa} & \tilde{w}^*_{uu} = \tilde{c}^*_{uu} & \tilde{w}^*_{ua} = \tilde{c}^*_{aa} + \frac{\Delta V P_{au}}{\Delta \Gamma_a P_{aa}} \\
\tilde{c}^*_{aa} = \frac{\Delta V}{\Delta \Gamma_a} + \tilde{c}^*_{uu} & \tilde{c}^*_{au} = \tilde{c}^*_{aa} & \tilde{c}^*_{uu} = \bar{u} - \bar{u}_1 & \tilde{c}^*_{ua} = \tilde{c}^*_{uu}.
\end{array}
$$

When $\bar{u} \in (\bar{u}_1, \bar{u}_3]$, then the principal offers an agent who displays overprecision

[29]Notice that denominator $\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)$ is also weakly positive. To see this, notice that it is decreasing in Γ^H_a. At the maximum value of Γ^H_a it equals $\tilde{P}_{aa}\tilde{P}_{uu} + \tilde{P}_{au} - \tilde{P}_{uu}$. Finally, notice that $\min\left(\tilde{P}_{aa}\tilde{P}_{uu} + \tilde{P}_{au} - \tilde{P}_{uu}\right) = \tilde{P}_{aa} + \tilde{P}_{au} - 1 = 0$.

either an APE contract or the GBPE contract. The GBPE contract has the same properties of the BPE contract and converges to the latter as $\bar{u} \to \bar{u}_1$.

Proof. To prove this proposition, we are going to show first that the GBPE is the only optimal contract when case (i) of Lemma 6 is assumed. From the analysis in the paper we know that the APE is the only optimal contract under cases (i) of Lemma 5 and (ii) of Lemma 6. We then prove that no feasible contract is possible under case (ii) of both Lemmas when $\bar{u} \in (\bar{u}_1, \bar{u}_2]$. This concludes the set of possible cases and leaves only the APE and the GBPE as potentially optimal contracts.

Part 1 Case (i) of Lemma 6.
Under case (i) of Lemma 6, it must be that:

$$c_{aa} = c_{au} \quad \text{and} \quad c_{uu} = c_{ua}.$$

First, we rearrange the (PC):

$$\begin{aligned}
&\sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{H} - V(\lambda^H) \geq \bar{u} \\
&c_{aa}(\tilde{\gamma}_{aa}^{H} + \tilde{\gamma}_{au}^{H}) + c_{uu}(\tilde{\gamma}_{uu}^{H} + \tilde{\gamma}_{ua}^{H}) - V(\lambda^H) \geq \bar{u} \\
&c_{aa}\Gamma_a^H + c_{uu}\Gamma_u^H - V(\lambda^H) \geq \bar{u} \\
&c_{aa} \geq \frac{\bar{u} + V(\lambda^H)}{\Gamma_a^H} - c_{uu}\frac{\Gamma_u^H}{\Gamma_a^H}.
\end{aligned}$$

Similarly, we can rearrange the (IC) as:

$$\begin{aligned}
&\sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{H} - V(\lambda^{H}) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{L} - V(\lambda^{L}) \\
&\sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0 \\
&c_{aa}(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au}) + c_{uu}(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}) - \Delta V \geq 0 \\
&c_{aa}\Delta\Gamma_{a} + c_{uu}\Delta\Gamma_{u} - \Delta V \geq 0 \\
&c_{aa} \geq \frac{\Delta V}{\Delta\Gamma_{a}} + c_{uu}.
\end{aligned}$$

We then draw these new versions of the (IC) and (PC) in (c_{uu}, c_{aa}) space to study which one is tighter. To do so, notice the following:

1. the (PC) is negatively sloped;
2. the intercept of the (PC) with the c_{aa}-axis is given by $\frac{\bar{u}+V(\lambda^{H})}{\Gamma_{a}^{H}}$;
3. contracts that lie on and above the locus of points satisfying the (PC) with equality satisfy the constraint;
4. the (IC) is positively sloped with slope 1;
5. the intercept of the (IC) with the c_{aa}-axis is given by $\frac{\Delta V}{\Delta\Gamma_{a}}$;
6. contracts that lie on and above the locus of points satisfying the (IC) with equality satisfy the constraint;
7. Since $\bar{u} \in (\bar{u}_1, \bar{u}_2]$ the intercept of the (IC) with the c_{aa}-axis is lower than the

one of the (PC). To see this, calculate

$$\begin{aligned}
\frac{\Delta V}{\Delta \Gamma_a} &< \frac{\bar{u} + V(\lambda^H)}{\Gamma_a^H} \\
\frac{\Delta V \Gamma_a^H}{\Delta \Gamma_a} - V(\lambda^H) &< \bar{u} \\
\bar{u} &> \frac{V(\lambda^H)\Gamma_a^L - V(\lambda^L)\Gamma_a^H}{\Delta \Gamma_a},
\end{aligned}$$

which corresponds to $\bar{u} > \bar{u}_1$.

With these in mind, we can produce Figure 5.

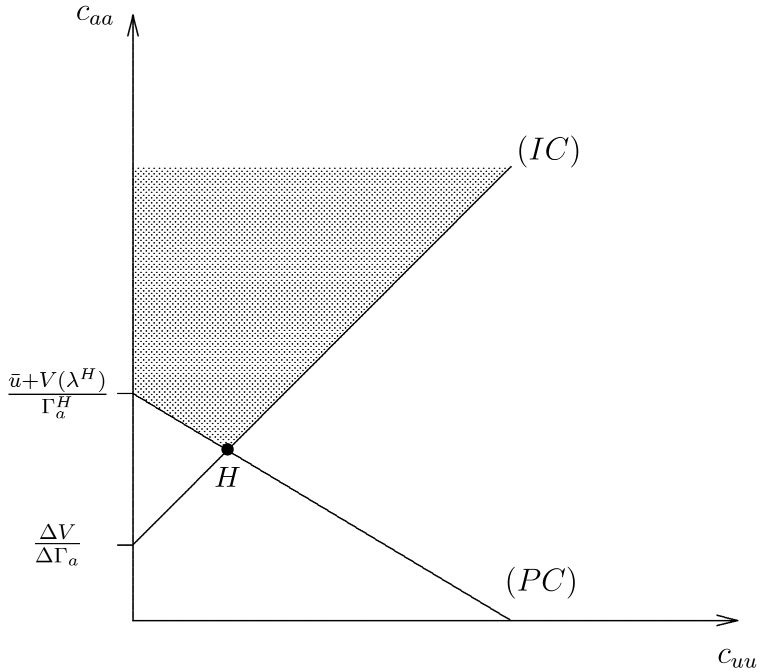

Figure 5: Contracts lying in the shaded area satisfy both the (PC) and the (IC) under case (i) of Lemma 6.

In the figure, all contracts lying in the shaded area satisfy both the (PC) and the (IC). Recall that the (TR_A) constraints are trivially solved, since we are in case (i) of Lemma 6. This immediately calls for an observation.

Lemma 24. *When $c_{aa} = c_{au}$ and $c_{uu} = c_{ua}$, the optimal contract features $c_{aa} > c_{uu}$.*

Proof. In Figure 5, the shaded area always lies above the 45 degree line. This is because the (IC) has slope 1 but an intercept larger than the 45 degree line (unless $\Delta V \leq 0$, which we rule out). ∎

As for the w_{ts}, of course, we can either be in case (i) or (ii) of Lemma 5. That is, either:

$$\text{(case (i))} \quad w_{ua} = w_{aa} \quad \text{and} \quad w_{au} = w_{uu}$$

or

$$\text{(case (ii))} \quad w_{ua} > w_{aa} \quad \text{and} \quad w_{au} > w_{uu}.$$

Assume we are in case (i). Under $w_{ua} = w_{aa}$ and $w_{au} = w_{uu}$, the principal is left with minimizing the following cost function

$$\min_{w_{aa}, w_{uu}} w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{uu}(\gamma_{au}^H + \gamma_{uu}^H),$$

subject to the (LL_{ts}) constraints. Given the cases we are studying, these constraints imply

$$w_{aa} \geq c_{aa}, \; w_{aa} = w_{ua} \geq c_{uu}, \; w_{uu} \geq c_{uu}, \; w_{uu} = w_{au} \geq c_{aa}$$

and therefore they yield

$$w_{aa} \geq \max\{c_{aa}, c_{uu}\} = c_{aa} \; w_{uu} \geq \max\{c_{aa}, c_{uu}\} = c_{aa}.$$

All this means that, under case (i) of both Lemmas, the principal solves

$$\min_{w_{aa}, w_{uu}} w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{uu}(\gamma_{au}^H + \gamma_{uu}^H)$$
$$\text{s.t.} \quad w_{aa} \geq c_{aa} \quad \text{and} \quad w_{uu} \geq c_{aa},$$

which is trivially solved by setting $w_{aa} = w_{uu} = c_{aa}$ and setting the contract that minimizes c_{aa} among the ones that satisfy the (PC) and the (IC). That is, point H

in Figure 3. To derive the final values of the optimal contract under case (i) of both Lemmas, we set the RHS of the (PC) and (IC) equal and solve for c_{uu}.

$$\begin{aligned}\frac{\bar{u}+V(\lambda^H)}{\Gamma_a^H} - c_{uu}\frac{\Gamma_u^H}{\Gamma_a^H} &= \frac{\Delta V}{\Delta \Gamma_a} + c_{uu} \\ c_{uu} &= \bar{u} + \frac{V(\lambda^L)\Gamma_a^H - V(\lambda^H)\Gamma_a^L}{\Delta\Gamma_a},\end{aligned}$$

which consistently with our findings converges to 0 as $\bar{u} \to \bar{u}_1$.

We can then substitute the above in the (PC) to get c_{aa} and finalize the contract values. This leads to the following contract:

$$\begin{array}{llll} w_{aa} = c_{aa} & w_{au} = c_{aa} & w_{uu} = c_{aa} & w_{ua} = c_{aa} \\ c_{aa} = \frac{\Delta V}{\Delta \Gamma_a} + c_{uu} & c_{au} = c_{aa} & c_{uu} = \bar{u} + \frac{V(\lambda^L)\Gamma_a^H - V(\lambda^H)\Gamma_a^L}{\Delta\Gamma_a} & c_{ua} = c_{uu}. \end{array}$$

where the principal pays a fixed wage.

Now we move to case (ii) of Lemma 5. First of all, notice that, because of the Lemma above, we can disregard (LL_{ua}) since we have $w_{ua} > w_{aa} \geq c_{aa} > c_{uu} = c_{ua}$. On the other end, (LL_{au}) cannot be disregarded just yet, since we haven't derived the result of $\max\{w_{uu}, c_{aa}\}$. When setting wages that satisfy case (ii) of Lemma 5, the principal solves:

$$\begin{aligned} \min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} \quad & w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \\ w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H &\leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H \\ w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H &\leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H \\ w_{aa} &\geq c_{aa} \\ w_{uu} &\geq c_{uu} \\ w_{au} &\geq c_{aa} \end{aligned}$$

Lemma 25. *Given case (i) of Lemma 6 and case (ii) of Lemma 5, at optimum the*

(LL_{aa}) *binds and* $w_{aa} = c_{aa}$.

Proof. Suppose not, then the principal can decrease both w_{aa} and w_{ua} by the same amount, this has no effect on the (TR_P) constraints and it decreases the objective function. Hence, (LL_{aa}) cannot be slack at optimum. ∎

Rewriting the two (TR_P) constraints, the principal now solves:

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H$$

$$(w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H} \leq (w_{ua} - c_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma_{uu}^H}{\gamma_{ua}^H}$$

$$w_{uu} \geq c_{uu}$$

$$w_{au} \geq c_{aa}$$

Lemma 26. *Given case (i) of Lemma 6 and case (ii) of Lemma 5, constraints* (LL_{uu}) *and* (LL_{au}) *bind at optimum.*

Proof. Suppose not, and both constraints hold with inequality at optimum, then the principal can decrease both w_{uu} and w_{au}, keeping their difference constant so that (TR_P) is not affected, reducing the objective function. This, however, is not enough to prove the lemma. We need to show that even if only one of the two binds the other cannot be slack.

Suppose only (LL_{uu}) binds, while (LL_{au}) does not, then the principal can decrease both w_{au} and w_{ua} in such a way that (TR_P) still holds, decreasing the objective function. This is always possible, since both wages have positive signs in (TR_P).

Suppose now only (LL_{au}) binds, while (LL_{uu}) does not, then the proof is a bit trickier since decreasing both w_{uu} and w_{ua} tightens the (TR_P^a). Suppose the latter binds, the principal can decrease w_{uu} by ϵ and increase w_{ua} by $\epsilon_1 = \epsilon\frac{\gamma_{au}^H}{\gamma_{aa}^H}$. This has no effect on the (TR_P^a), but it affects the objective function. To see that the overall effect on costs is negative, let it be denoted by ΔC and calculate

$$\Delta C = -\epsilon\gamma_{uu}^H + \epsilon_1\gamma_{au}^H = -\epsilon\frac{(\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{ua}^H)}{\gamma_{aa}^H} < 0$$

This concludes the proof. ∎

Given the above, the principal is left with

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}(\gamma_{aa}^H + \gamma_{au}^H) + w_{ua}\gamma_{ua}^H + c_{uu}\gamma_{uu}^H$$
$$(c_{aa} - c_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H} \leq (w_{ua} - c_{aa}) \leq (c_{aa} - c_{uu})\frac{\gamma_{uu}^H}{\gamma_{ua}^H}$$

where it is easy to see that, since the first and the last element of (TR_P) are different from 0, we need $w_{ua} > c_{aa}$ for (TR_P) to hold. Now notice that the first inequality of (TR_P) corresponds to (TR_P^a).

Lemma 27. *Given case (i) of Lemma 6 and case (ii) of Lemma 5, at optimum* (TR_P^a) *binds. Hence*

$$w_{ua} = c_{aa} + (c_{aa} - c_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$$

Proof. Suppose not, by decreasing w_{ua} the principal decreases the objective function and relaxes (TR_P^u). ∎

The final form of the problem that the principal solves is therefore

$$\min_{c_{aa},c_{uu}} c_{aa}\left(\gamma_{aa}^H + \gamma_{au}^H + \gamma_{ua}^H + \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right) + c_{uu}\left(\gamma_{uu}^H - \frac{\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H}\right)$$
$$= \min_{c_{aa},c_{uu}} c_{aa}(\gamma_{aa}^H\gamma_{aa}^H + \gamma_{aa}^H\gamma_{au}^H + \gamma_{aa}^H\gamma_{ua}^H + \gamma_{au}^H\gamma_{ua}^H) + c_{uu}(\gamma_{uu}^H\gamma_{aa}^H - \gamma_{au}^H\gamma_{ua}^H)$$

with c_{aa} and c_{uu} that have to lie inside the shaded area of Figure 5.

Since both c_{aa} and c_{uu} enter negatively in the objective function, the cost of implementing high effort decreases towards the origin in Figure 5. This implies that the optimal contract for this case is either at point H or it corresponds to the intercept of the (PC) with the c_{aa}-axis. The next Lemma solves this dilemma and shows how the optimal contract for this case, the GBPE one, also dominates the contract derived under case (i) of both Lemmas.

Lemma 28. *Given case (i) of Lemma 6, the optimal contract is the GBPE contract.*

Proof. We start from showing that the optimal contract under case (ii) of Lemma 5 lies at H. To see this, notice that iso-costs decrease in value towards the origin in Figure 3. Hence, if they are flatter than the (PC), the contract minimising costs lies at H. The absolute value of the slope of the iso-costs is given by:

$$\begin{aligned}
&\frac{\gamma_{uu}^H\gamma_{aa}^H-\gamma_{au}^H\gamma_{ua}^H}{\gamma_{aa}^H\gamma_{aa}^H+\gamma_{aa}^H\gamma_{au}^H+\gamma_{aa}^H\gamma_{ua}^H+\gamma_{au}^H\gamma_{ua}^H}\\
&=\frac{(P_{uu}P_{aa}-P_{au}P_{ua})\Gamma_a^H\Gamma_u^H}{P_{aa}P_{aa}(\Gamma_a^H)^2+P_{aa}P_{au}(\Gamma_a^H)^2+P_{aa}P_{ua}\Gamma_a^H\Gamma_u^H+P_{au}P_{ua}\Gamma_a^H\Gamma_u^H}\\
&=\frac{(P_{aa}-P_{ua})\Gamma_u^H}{P_{aa}\Gamma_a^H(P_{aa}+P_{au})+P_{ua}\Gamma_u^H(P_{aa}+P_{au})}=\frac{(P_{aa}-P_{ua})(1-\Gamma_a^H)}{P_{aa}\Gamma_a^H+P_{ua}(1-\Gamma_a^H)}.
\end{aligned}$$

The absolute value of the slope of the (PC), instead, is given by $\frac{1-\Gamma_a^H}{\Gamma_a^H}$. To see that the latter is always larger than the former, calculate

$$\begin{aligned}
&\frac{(P_{aa}-P_{ua})(1-\Gamma_a^H)}{P_{aa}\Gamma_a^H+P_{ua}(1-\Gamma_a^H)}<\frac{1-\Gamma_a^H}{\Gamma_a^H}\\
&\frac{P_{aa}\Gamma_a^H-P_{ua}\Gamma_a^H}{P_{aa}\Gamma_a^H+P_{ua}(1-\Gamma_a^H)}<1\\
&P_{aa}\Gamma_a^H-P_{ua}\Gamma_a^H<P_{aa}\Gamma_a^H+P_{ua}(1-\Gamma_a^H),
\end{aligned}$$

which is trivially true. Hence, the optimal contract for this case always lies at point H. Following the same calculations of case (i) of Lemma 5, we then get the values of c_{aa} and c_{uu}. To obtain the value of w_{ua}, substitute for $c_{uu}=c_{aa}-\frac{\Delta V}{\Delta\Gamma_a}$ in $w_{ua}=c_{aa}+(c_{aa}-c_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H}$:

$$w_{ua}=c_{aa}+\left(c_{aa}-c_{aa}-\frac{\Delta V}{\Delta\Gamma_a}\right)\frac{\gamma_{au}^H}{\gamma_{aa}^H}=c_{aa}+\frac{\Delta V}{\Delta\Gamma_a}\frac{P_{au}\Gamma_a^H}{P_{aa}\Gamma_a^H}=c_{aa}+\frac{\Delta V}{\Delta\Gamma_a}\frac{P_{au}}{P_{aa}}.$$

To conclude the proof, we need to show that the cost of implementing high effort, i.e. $E(w_{ts})$, is lower under the GBPE contract compared to the one of case (i) of Lemma 5. First of all, notice that the latter is equal to c_{aa}, since the wage is constant, and that c_{aa} takes the same value in both contracts. We then check for

$$c_{aa}\gamma_{aa}^{H} + c_{aa}\gamma_{au}^{H} + c_{uu}\gamma_{uu}^{H} + \left(c_{aa} + \frac{\Delta V}{\Delta \Gamma_a}\frac{P_{au}}{P_{aa}}\right)\gamma_{ua}^{H} < c_{aa}$$

$$c_{aa}\underbrace{\left(\gamma_{aa}^{H} + \gamma_{au}^{H} + \gamma_{ua}^{H} + \gamma_{uu}^{H}\right)}_{1} + \frac{\Delta V}{\Delta \Gamma_a}\left(\frac{P_{au}}{P_{aa}}\gamma_{ua}^{H} - \gamma_{uu}^{H}\right) < c_{aa}$$

$$\frac{\Delta V}{\Delta \Gamma_a P_{aa}}\left(P_{au}P_{ua} - P_{aa}P_{uu}\right)\Gamma_u^{H} < 0,$$

which is true by positive correlation. ∎

Part 2 Case (ii) of both Lemmas.

We are now left to show that no feasible contract emerges from case (ii) of both Lemmas. The reason for doing so is that the GBPE is optimal for a smaller set of parameter values compared to the BPE, since it is more expensive. Call P the difference between this set and the one under which the BPE is optimal when $\bar{u} < \bar{u}_1$. We are looking for any potentially optimal contract that differs structurally from the APE and could be optimal in P. If we were to find none, we would be sure that the APE is optimal in P.[30]

Given the Lemmas, we have

$$c_{uu} > c_{ua},\, c_{aa} > c_{au} \quad \text{and} \quad w_{ua} > w_{aa},\, w_{au} > w_{uu}.$$

[30]This is due to the different approach taken to derive the optimal contracts under the assumption that $\bar{u} < \bar{u}_1$. Under the latter, we haven't studied the problem assuming specific cases of Lemmas 6 and 5, we have not proven yet that the APE is the only feasible contract under case (ii) of Lemma 6, but only that it is optimal w.r.t. the BPE under a certain parameter restriction.

Hence, the problem solved by the principal is the starting one.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^{H} + w_{au}\gamma_{au}^{H} + w_{ua}\gamma_{ua}^{H} + w_{uu}\gamma_{uu}^{H}$$

$$\text{s.t.} \quad \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{H} - V(\lambda^{H}) \geq \bar{u} \qquad (PC)$$

$$\sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{H} - V(\lambda^{H}) \geq \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^{L} - V(\lambda^{L}) \qquad (IC)$$

$$w_{aa}\gamma_{aa}^{H} + w_{au}\gamma_{au}^{H} \leq w_{ua}\gamma_{aa}^{H} + w_{uu}\gamma_{au}^{H} \qquad (TR_P^a)$$

$$w_{ua}\gamma_{ua}^{H} + w_{uu}\gamma_{uu}^{H} \leq w_{aa}\gamma_{ua}^{H} + w_{au}\gamma_{uu}^{H} \qquad (TR_P^u)$$

$$c_{aa}\tilde{\gamma}_{aa}^{H} + c_{ua}\tilde{\gamma}_{ua}^{H} \geq c_{au}\tilde{\gamma}_{aa}^{H} + c_{uu}\tilde{\gamma}_{ua}^{H} \qquad (TR_A^a)$$

$$c_{au}\tilde{\gamma}_{au}^{H} + c_{uu}\tilde{\gamma}_{uu}^{H} \geq c_{aa}\tilde{\gamma}_{au}^{H} + c_{ua}\tilde{\gamma}_{uu}^{H} \qquad (TR_A^u)$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t,s \in\{a,u\}. \qquad (LL_{ts})$$

First of all, notice that at least one of the (TR_P) constraints must bind. If this were not the case, then the (LL_{ts}) would be the only constraints on the w_{ts}. Clearly these would then be set binding. This violates Lemma 7 and the resulting contract would never implement high effort under truthful reporting. We, therefore, divide the analysis in two sections depending on which one of the (TR_P) binds.

Let the (TR_P^a) bind. We can then solve the (TR_P^a) for $w_{au}\gamma_{au}^{H}$ to get

$$w_{au}\gamma_{au}^{H} = w_{uu}\gamma_{au}^{H} + (w_{ua} - w_{aa})\gamma_{aa}^{H}.$$

When we plug this back into the objective function we get

$$w_{aa}\gamma_{aa}^{H} + w_{au}\gamma_{au}^{H} + w_{ua}\gamma_{ua}^{H} + w_{uu}\gamma_{uu}^{H} = w_{uu}(\gamma_{au}^{H} + \gamma_{uu}^{H}) + w_{ua}(\gamma_{ua}^{H} + \gamma_{aa}^{H}).$$

From the (PC) binding, we know that

$$c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^{H}}\left(\bar{u} + V(\lambda^{H}) - c_{au}\tilde{\gamma}_{au}^{H} - c_{ua}\tilde{\gamma}_{ua}^{H} - c_{uu}\tilde{\gamma}_{uu}^{H}\right),$$

which plugged back into the (IC) yields

$$\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \geq c_{ua}\tilde{P}_{ua} + c_{uu}\tilde{P}_{uu}.$$

The resulting maximization problem is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{uu}(\gamma_{au}^H + \gamma_{uu}^H) + w_{ua}(\gamma_{ua}^H + \gamma_{aa}^H) \tag{31}$$

$$\text{s.t.} \quad \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \geq c_{ua}\tilde{P}_{ua} + c_{uu}\tilde{P}_{uu} \tag{IC}$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \tag{TR_A^u}$$

$$w_{uu} \geq c_{uu} \geq 0 \tag{LL_{uu}}$$

$$w_{ua} \geq c_{ua} \geq 0. \tag{LL_{ua}}$$

By case (ii) of Lemma 6, we know that the (TR_A) will not bind together. Suppose one of them binds, we can plug c_{aa} into it and solve for c_{au}, removing the (TR_A) constraints from the problem. At this point, in the problem above, (LL_{uu}) and (LL_{ua}) are set binding since they are the only constraints on the relevant wages. This implies that the objective function is minimized at $c_{uu} = c_{ua} = 0$. This is incompatible with case (ii) of Lemma 6. Suppose no (TR_A) constraint binds, we have the same solution, since the (TR_A) are completely disregarded, and therefore the same contradiction. Hence, there exists no optimal contract for this case when the (TR_P^a) binds.

Let the (TR_P^u) bind. The proof is a little more tricky. As above, solve the (TR_P^u) for $w_{uu}\gamma_{uu}^H$ to get

$$w_{uu}\gamma_{uu}^H = w_{au}\gamma_{uu}^H + (w_{aa} - w_{ua})\gamma_{ua}^H.$$

When we plug this back into the objective function we get

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H = w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}(\gamma_{au}^H + \gamma_{uu}^H).$$

From the (PC), we know that

$$c_{uu} = \frac{1}{\tilde{\gamma}^H_{uu}} \left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}^H_{au} - c_{ua}\tilde{\gamma}^H_{ua} - c_{aa}\tilde{\gamma}^H_{aa} \right).$$

When we plug this into the (IC) we obtain

$$\bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_u \leq c_{aa}\tilde{P}_{aa} + c_{au}\tilde{P}_{au}.$$

Differently from above, we first are going to assume that a (TR_A) binds. Suppose it is the (TR^u_A), and solve it for

$$c_{ua}\tilde{\gamma}^H_{uu} = c_{uu}\tilde{\gamma}^H_{uu} - (c_{aa} - c_{au})\tilde{\gamma}^H_{au}.$$

Substituting for c_{uu} from the (PC), we obtain

$$c_{ua} = \frac{\bar{u} + V(\lambda^H) - c_{aa}\Gamma^H_a}{\Gamma^H_u}.$$

If instead of the (TR^u_A) we set the (TR^a_A) binding, and still solve for c_{uu} and c_{ua} from the (TR^a_A) and the (PC) constraint, we get

$$\begin{aligned} c_{uu} &= \frac{\bar{u} + V(\lambda^H) - c_{au}\Gamma^H_a}{\Gamma^H_u}, \\ c_{ua} &= \frac{1}{\tilde{\gamma}^H_{ua}} \left[\left(\bar{u} + V(\lambda^H) \right) \tilde{P}_{ua} + c_{au}(\tilde{P}_{aa} - \tilde{P}_{ua})\Gamma^H_a - c_{aa}\tilde{\gamma}^H_{aa} \right], \end{aligned}$$

which leads to the symmetric problem. Hence, regardless of the (TR_A) set binding, the principal faces

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad w_{aa}(\gamma_{aa}^H+\gamma_{ua}^H)+w_{au}(\gamma_{au}^H+\gamma_{uu}^H) \tag{32}$$

$$\text{s.t.} \quad \bar{u}+V(\lambda^H)+\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \leq c_{aa}\tilde{P}_{aa}+c_{au}\tilde{P}_{au} \tag{IC}$$

$$w_{aa}\geq c_{aa}\geq 0 \tag{LL_{aa}}$$

$$w_{au}\geq c_{au}\geq 0 \tag{LL_{au}}$$

where it is immediate to see that the (LL_{aa}) and (LL_{au}) bind. Following this, also the (IC) must bind. We, therefore, solve it for c_{aa} to get

$$c_{aa}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{aa}}+\frac{\bar{u}+V(\lambda^H)}{\tilde{P}_{aa}}-c_{au}\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}.$$

We can then plug this into the objective function to obtain

$$\text{(a fixed positive term)}-c_{au}\left[\left(\gamma_{aa}^H+\gamma_{ua}^H\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\right].$$

To see that the coefficient of c_{au} is always positive, notice that the bracket is always negative:

$$\begin{aligned}
&\left(\gamma_{aa}^H+\gamma_{ua^H}\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}-\left(\gamma_{au}^H+\gamma_{uu}^H\right)\\
&\quad=P_{au}P_{aa}\Gamma_a^H+P_{au}P_{ua}\Gamma_u^H-P_{au}P_{aa}\Gamma_a^H-P_{uu}P_{aa}\Gamma_u^H-b_a(\gamma_{aa}^H+\gamma_{ua}^H+\gamma_{au}^H+\gamma_{uu}^H)\\
&\quad=P_{au}P_{ua}\Gamma_u^H-P_{uu}P_{aa}\Gamma_u^H-b_a=(P_{ua}-P_{aa})\Gamma_u^H-b_a<0.
\end{aligned}$$

Hence, the principal sets $c_{au}=0$ to minimize the objective function. This is enough to prove our contradiction. Notice, in fact, that given c_{au} and $c_{aa}=w_{aa}$, from the (TR_P^u) we have that

$$w_{uu}=(w_{aa}-w_{ua})\frac{P_{ua}}{P_{uu}},$$

which is ≥ 0 if and only if $w_{ua} \leq w_{aa}$. This violate case (ii) of Lemma 5, and shows that there cannot be an optimal contract where one of the (TR_A) binds.

To conclude the proof, we need to show that no optimal contract exists when both the (TR_A) are slack. Problem (32) above, however, still holds when the (TR_A) are disregarded. The solution follows the same steps above and leads, therefore, to $c_{au} = 0$ and to the same contradiction.
This concludes the proof of part 2 and of Proposition 6. ∎

When the principal finds the BPE contract optimal and the GBPE contract is available, it must be that the expected wage of the GBPE contract is larger. On the other hand, its competitor for optimality, the APE, hasn't changed in structure, and therefore in expected wage cost. This allows us to state the following Corollary.

Lemma 29. *When $\bar{u} \in (\bar{u}_1, \bar{u}_3]$, the APE is optimal for at least all parameter values under which it is optimal when $\bar{u} < \bar{u}_1$.*

Hence for intermediate outside option values, the APE contract becomes relatively more attractive than the GBPE contract. This is due to the fact that the APE contract relaxes the participation constraint, which holds under the GBPE one.

Proof. The proof follows from the text. The expected wage cost of the APE contract hasn't changed from the case of $\bar{u} < \bar{u}_1$, while the expected wage cost of the GBPE one is given by

$$E(\tilde{w}_{ts}^{*}) = \bar{u} + V(\lambda^{H}) + \frac{\Delta V}{\Delta \Gamma_{a}} \frac{\gamma_{ua}^{H}}{P_{aa}}$$

and it is larger than the expected wage cost of the BPE contract when $\bar{u} > \bar{u}_1$. Hence, the comparison between the APE contract and the GBPE one ends in favor of the APE contract for a larger — at least weakly — set of parameters compared to the case of the BPE vs. the APE contract. ∎

Proposition 3 showed that there are situations where the APE contract does in fact, taking the perspective of an outside observer, raise the agent's expected compensation leading to a Pareto improvement compared to the BPE contract. Given Lemma 29, we know not only that Proposition 3 still holds in the case of the APE vs. the GBPE contract, but that it may even hold for a larger portion of the parameter space. Hence, an outside option utility such that $\bar{u} \in (\bar{u}_1, \bar{u}_3]$ reinforces (at least weakly) the welfare result on the Pareto dominance of the APE.

Proposition 7. *When $\bar{u} > \bar{u}_2$, then the principal offers an agent who displays over-estimation a contract $\tilde{w}^{\diamond}_{ts}, \tilde{c}^{\diamond}_{ts}$ given by:*

$$\begin{array}{llll} \tilde{w}^{\diamond}_{aa} = \tilde{c}^{\diamond}_{aa} & \tilde{w}^{\diamond}_{au} = \tilde{c}^{\diamond}_{aa} & \tilde{w}^{\diamond}_{uu} = \tilde{c}^{\diamond}_{uu} & \tilde{w}^{\diamond}_{ua} = \tilde{c}^{\diamond}_{aa} + \frac{\Delta V P_{au}}{\Delta \tilde{\Gamma}_a P_{aa}} \\ \tilde{c}^{\diamond}_{aa} = \frac{\Delta V}{\Delta \tilde{\Gamma}_a} + \tilde{c}^{\diamond}_{uu} & \tilde{c}^{\diamond}_{au} = \tilde{c}^{\diamond}_{aa} & \tilde{c}^{\diamond}_{uu} = \bar{u} - \left[\frac{\Delta V}{\Delta \tilde{\Gamma}_a} \tilde{\Gamma}^H_a - V(\lambda^H) \right] & \tilde{c}^{\diamond}_{ua} = \tilde{c}^{\diamond}_{uu}. \end{array}$$

This contract converges to contract in Proposition 2 as $\bar{u} \to \bar{u}_2$.

Proof. The proof is similar to that of Proposition 6.

We now let the outside option of an agent who displays overprecision be larger than $\bar{u}_3$. First of all, notice that the GBPE is feasible when $\bar{u} > \bar{u}_3$, while the APE is no longer feasible, since it violates the (PC).

Proposition 8. *Let $\bar{u} > \bar{u}_3$.*

1) If the agent displays overprecision and his beliefs violate condition (40) *and satisfy condition* (42)*, then the principal offers the agent a Generalized APE contract, GAPE $_1$, given by:*

$$\begin{array}{llll} \tilde{w}^{\dagger}_{aa} = \tilde{c}^{\dagger}_{aa} & \tilde{w}^{\dagger}_{au} = \tilde{c}^{\dagger}_{au} & \tilde{w}^{\dagger}_{uu} = \tilde{c}^{\dagger}_{au} & \tilde{w}^{\dagger}_{ua} = \tilde{c}^{\dagger}_{aa} \\ \tilde{c}^{\dagger}_{aa} = \tilde{c}^{\dagger}_{ua} + \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & & \tilde{c}^{\dagger}_{au} = \tilde{c}^{\dagger}_{ua} + \frac{\Delta V}{\Delta \Gamma_a} \frac{\tilde{\gamma}^H_{au}}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} & \\ \tilde{c}^{\dagger}_{uu} = \tilde{c}^{\dagger}_{au} & & \tilde{c}^{\dagger}_{ua} = \bar{u} - \bar{u}_3. & \end{array}$$

where the (PC) *and* (IC) *bind together. Contract* $GAPE_1$ *has the same properties as an APE and converges to the latter as* $\bar{u} \to \bar{u}_3$.

2) If the agent's bias satisfies conditions (40) *and* (41)*, then the principal offers the agent a* $GAPE_2$ *contract given by:*

$$\begin{array}{llll} \tilde{w}^{\dagger'}_{aa} = \tilde{c}^{\dagger'}_{aa} & \tilde{w}^{\dagger'}_{au} = \tilde{c}^{\dagger'}_{au} & \tilde{w}^{\dagger'}_{uu} = \tilde{c}^{\dagger'}_{au} & \tilde{w}^{\dagger'}_{ua} = \tilde{c}^{\dagger'}_{aa} \\ \tilde{c}^{\dagger'}_{aa} = c^{\dagger'}_{au}\left(1 + \frac{\tilde{\gamma}^{H}_{uu}}{\tilde{\gamma}^{\dagger'}_{au}}\right) & \tilde{c}^{\dagger'}_{au} = \frac{\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au}}{\left(\tilde{\gamma}^{H}_{au}+\tilde{\gamma}^{H}_{uu}\right)} & \tilde{c}^{\dagger'}_{uu} = \tilde{c}^{\dagger'}_{au} & \tilde{c}^{\dagger'}_{ua} = 0. \end{array}$$

where the (PC) *binds but the* (IC) *is slack.*

Proof. First, note that the derivation of the optimality of the GBPE contract under case (i) of Lemma 6 does not depend on $\bar{u}$ being larger or smaller than $\bar{u}_2$. Hence, once again we focus our attention on case (ii) of Lemma 6. Second, our analysis of cases (ii) of both Lemmas in the proof of Proposition 5 also does not depend on the value of $\bar{u}$ being larger or smaller than $\bar{u}_2$. Hence we are left simply with case (i) of Lemma 5 and (ii) of Lemma 6. Any optimal contract besides the GBPE must respect these features. This proves that any GAPE contract uses information from SPE reports in the same fashion of the APE contract.

The rest of this appendix is the proof of Proposition 6. As this is quite long, it is split into several parts. First, focusing on case (i) of Lemma 5 we solve for the optimal w_{ts} (Lemma 30). Second, we use that to rewrite the problem and prove that at optimum, $c_{aa} \geq c_{ua}$ (Lemma 31). Third, we use a different rewriting of the problem to show that, at optimum, $c_{au} = c_{uu}$ (Lemma 32). The proof of this second result is itself divided in two parts: part 1, where we show that $c_{uu} > c_{au}$ leads to a contradiction, and part 2, where we show that also $c_{uu} < c_{au}$ leads to unfeasible contracts. Fourth, we state the final form of the problem in (38) and show graphically the potential solutions to it (Figure 8 to 10). Fifth, we select the contracts that are indeed optimal under certain parameter conditions (Lemma 33). Finally, we derive the values for GAPE_1 and GAPE_2 together with their feasibility and optimality conditions (Lemmas 34 and 35).

Under case (i) of Lemma 5 and (ii) of Lemma 6, the principal faces the following problem (where the (TR_P) are trivially solved and we reduced the (LL_{ts}) to only two constraints)

$$\begin{aligned} \min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} & w_{aa}(\gamma_{aa}^H+\gamma_{ua}^H)+w_{uu}(\gamma_{au}^H+\gamma_{uu}^H) \\ \text{s.t.} \quad & \sum_{ts} c_{ts}\tilde{\gamma}_{ts}^H - V(\lambda^H) \geq \bar{u} \\ & \sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0 \\ & c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H + c_{uu}\tilde{\gamma}_{ua}^H \\ & c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \\ & w_{aa} \geq \max\{c_{aa}, c_{ua}\} \\ & w_{uu} \geq \max\{c_{au}, c_{uu}.\} \end{aligned}$$

Given that $\bar{u} > \bar{u}_2$, and that the BPE and the APE contracts are the only optimal one resulting from the (PC) being slack, we set the (PC) binding and use it to rewrite the (TR_A) constraints. First, solve for

$$c_{aa}\tilde{\gamma}_{aa}^H = \bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H$$

and plug this into the (TR_A^a) to obtain

$$\bar{u} + V(\lambda^H) \geq c_{au}\Gamma_a^H + c_{uu}\Gamma_u^H.$$

Similarly, solve the (PC) for

$$c_{au}\tilde{\gamma}_{au}^H = \bar{u} + V(\lambda^H) - c_{aa}\tilde{\gamma}_{aa}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H$$

and plug this into the (TR_A^u) to obtain

$$\bar{u} + V(\lambda^H) \geq c_{aa}\Gamma_a^H + c_{ua}\Gamma_u^H.$$

This allows us to rewrite the problem as

$$\begin{aligned}
\min_{\{w_{ts}, c_{ts}\}_{t,s\in\{u,a\}}} \quad & w_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{uu}(\gamma_{au}^H + \gamma_{uu}^H) \\
\text{s.t.} \quad & c_{aa}\tilde{\gamma}_{aa}^H + c_{au}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{ua}^H + c_{uu}\tilde{\gamma}_{uu}^H = \bar{u} + V(\lambda^H) \\
& \sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0 \\
& \bar{u} + V(\lambda^H) \geq c_{au}\Gamma_a^H + c_{uu}\Gamma_u^H \\
& \bar{u} + V(\lambda^H) \geq c_{aa}\Gamma_a^H + c_{ua}\Gamma_u^H \\
& w_{aa} \geq \max\{c_{aa}, c_{ua}\} \\
& w_{uu} \geq \max\{c_{au}, c_{uu}\}
\end{aligned}$$

Lemma 30. *When $\bar{u} > \bar{u}_2$, given case (ii) of Lemma 6 and case (i) of Lemma 5*

$$w_{aa} = \max\{c_{aa}, c_{ua}\} \quad w_{uu} = \max\{c_{au}, c_{uu}\}$$

Proof. Given case (i) of Lemma 5, the (TR_P) are trivially satisfied. Both w_{aa} and w_{uu} affect the cost negatively and therefore the principal has the incentive to decrease them as much as she can. Since the (LL_{ts}) are the only constraints on wages, they are set binding. ∎

Let us rearrange the problem using the optimal w_{ts} derived.

$$\begin{aligned}\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} \quad & \max\{c_{aa},c_{ua}\}(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\\ \text{s.t.} \quad & c_{aa}\tilde{\gamma}^H_{aa}+c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}=\bar{u}+V(\lambda^H)\\ & \sum_{ts} c_{ts}\Delta\tilde{\gamma}_{ts}-\Delta V\geq 0\\ & \bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u\\ & \bar{u}+V(\lambda^H)\geq c_{aa}\Gamma^H_a+c_{ua}\Gamma^H_u\end{aligned}$$

Lemma 31. *When $\bar{u}>\bar{u}_2$, given case (ii) of Lemma 6 and case (i) of Lemma 5, $c_{aa}\geq c_{ua}$.*

Proof. Suppose not, then $c_{ua}-c_{aa}=X>0$. The principal can then increase c_{aa} by

$$\epsilon\equiv X\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{ua}+\tilde{\gamma}^H_{aa}}$$

and decrease c_{ua} by $(X-\epsilon)$. This strictly increases profits, since by assumption c_{aa} does not enter the cost function.

First of all, notice that ϵ is picked in such a way that it leaves the (PC) binding. The change to the LHS of the (PC) is in fact:

$$\epsilon\tilde{\gamma}^H_{aa}-(X-\epsilon)\tilde{\gamma}^H_{ua}=\epsilon(\tilde{\gamma}^H_{aa}+\tilde{\gamma}^H_{ua})-X\tilde{\gamma}^H_{ua}=X\tilde{\gamma}^H_{ua}-X\tilde{\gamma}^H_{ua}=0.$$

Now let's check that this satisfies all other constraints. Start by the (IC). Since $\Delta\tilde{\gamma}^H_{ua}<0$ and $\Delta\tilde{\gamma}_{aa}>0$, the (IC) is relaxed by this deviation.

Now consider the (TR^u_A). The change to its RHS has to be weakly negative. This happens when

$$\epsilon\Gamma_a^H - (X-\epsilon)\Gamma_u^H \leq 0$$
$$\epsilon - X\Gamma_u^H \leq 0 \;\Rightarrow\; \epsilon \leq X\Gamma_u^H.$$

To see that this holds, notice that

$$\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H} \leq \Gamma_u^H$$
$$\tilde{P}_{ua} \leq \tilde{\gamma}_{ua}^H + \tilde{\gamma}_{aa}^H$$
$$\tilde{P}_{ua} - \tilde{P}_{ua}\Gamma_u^H - \tilde{P}_{aa}\Gamma_a^H \leq 0$$
$$\tilde{P}_{ua}\Gamma_a^H - \tilde{P}_{aa}\Gamma_a^H \leq 0,$$

which is true by positive perceived correlation. This concludes the proof since neither c_{aa} nor c_{ua} enter (TR_A^a). ∎

Before going ahead we are going to rewrite the problem in a more convenient way. From the (PC), we substitute for

$$c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right)$$

in all the constraints and the objective function. The (TR_A^a) does not change from the above. The (IC) becomes

$$c_{ts}\Delta\tilde{\gamma}_{ts} - \Delta V \geq 0$$
$$\tfrac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right) + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} + c_{uu}\Delta\tilde{\gamma}_{uu} - \Delta V \geq 0$$
$$c_{au}\left(\Delta\tilde{\gamma}_{au} - \tilde{\gamma}_{au}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) + c_{ua}\left(\Delta\tilde{\gamma}_{ua} - \tilde{\gamma}_{ua}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right)$$
$$+ c_{uu}\left(\Delta\tilde{\gamma}_{uu} - \tilde{\gamma}_{uu}^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) + \frac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V,$$

where the coefficients of each c_{ts} can be rearranged as

$$\Delta\tilde{\gamma}_{ts} - \tilde{\gamma}_{ts}^{H}\frac{\Delta\Gamma_a}{\Gamma_a^H} = \tilde{P}_{ts}\left(\Delta\Gamma_t - \Gamma_t^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right)$$

which is equal to 0 if $t = a$ and to

$$\tilde{P}_{us}\left(\Delta\Gamma_u - \Gamma_u^H\frac{\Delta\Gamma_a}{\Gamma_a^H}\right) = \tilde{P}_{us}\left(\Delta\Gamma_u + \Gamma_u^H\frac{\Delta\Gamma_u}{\Gamma_a^H}\right) = \tilde{P}_{us}\frac{\Delta\Gamma_u}{\Gamma_a^H}\left(\Gamma_a^H + \Gamma_u^H\right) = \tilde{P}_{us}\frac{\Delta\Gamma_u}{\Gamma_a^H}$$

otherwise. This gives us

$$\begin{aligned}
&c_{ua}\tilde{P}_{ua}\frac{\Delta\Gamma_u}{\Gamma_a^H} + c_{uu}\tilde{P}_{uu}\frac{\Delta\Gamma_u}{\Gamma_a^H} + \frac{\Delta\Gamma_a}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V \\
&-c_{ua}\tilde{P}_{ua}\Delta\Gamma_a - c_{uu}\tilde{P}_{uu}\Delta\Gamma_a + \Delta\Gamma_a\left(\bar{u} + V(\lambda^H)\right) \geq \Delta V\Gamma_a^H \\
&\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - c_{uu}\tilde{P}_{uu} \geq \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H.
\end{aligned}$$

Consider the (TR_A^u) in its standard form and substitute for c_{aa} from the (PC) to obtain

$$\begin{aligned}
&c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \\
&c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H \geq \left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right)\frac{\tilde{P}_{au}}{\tilde{P}_{aa}} + c_{ua}\tilde{\gamma}_{uu}^H \\
&c_{au}\tilde{\gamma}_{au}^H\left(1 + \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) + c_{uu}\tilde{\gamma}_{uu}^H\left(1 + \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) - c_{ua}\left(\tilde{\gamma}_{uu}^H - \tilde{\gamma}_{ua}^H\frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\right) \geq \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\left(\bar{u} + V(\lambda^H)\right) \\
&c_{au}\frac{\tilde{\gamma}_{au}^H}{\tilde{P}_{aa}} + c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{aa}} - c_{ua}\frac{\Gamma_u^H}{\tilde{P}_{aa}}\left(\tilde{P}_{uu}\tilde{P}_{aa} - \tilde{P}_{ua}\tilde{P}_{au}\right) \geq \frac{\tilde{P}_{au}}{\tilde{P}_{aa}}\left(\bar{u} + V(\lambda^H)\right) \\
&c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au}.
\end{aligned}$$

Finally, we need to rearrange the objective function.

$$
\begin{aligned}
&c_{aa}(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\\
&\quad=\frac{1}{\tilde{\gamma}^H_{aa}}\left(\bar{u}+V(\lambda^H)-c_{au}\tilde{\gamma}^H_{au}-c_{ua}\tilde{\gamma}^H_{ua}-c_{uu}\tilde{\gamma}^H_{uu}\right)(\gamma^H_{aa}+\gamma^H_{ua})+\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\\
&\quad\propto\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}\right)
\end{aligned}
$$

We can now rewrite the problem.

$$
\min_{\{c_{ts}\}_{t,s\in\{u,a\}}}\max\{c_{au},c_{uu}\}(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{au}\tilde{\gamma}^H_{au}+c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}\right) \tag{33}
$$

$$
\begin{aligned}
\text{s.t.}\quad &\bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a && (IC)\\
&\bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u && (TR^a_A)\\
c_{au}\tilde{\gamma}^H_{au}+c_{uu}\tilde{\gamma}^H_{uu}-c_{ua}&\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} && (TR^u_A)
\end{aligned}
$$

Lemma 32. *When $\bar{u}>\bar{u}_2$, given case (ii) of Lemma 6 and case (i) of Lemma 5, $c_{au}=c_{uu}$.*

Proof. We first prove that assuming $c_{uu}>c_{au}$ leads to a contradiction. Then we prove the same for the case of $c_{au}>c_{uu}$.

PART 1: suppose $\boldsymbol{c_{uu}>c_{au}}$ We are going to show that all possible optimal contracts emerging from this case contradict this assumption. The new problem would be

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}\right]-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{ua}\tilde{\gamma}^H_{ua}+c_{au}\tilde{\gamma}^H_{au}\right)$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a \qquad (IC)$$

$$\bar{u}+V(\lambda^H)\geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u \qquad (TR^a_A)$$

$$c_{au}\tilde{\gamma}^H_{au}+c_{uu}\tilde{\gamma}^H_{uu}-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \qquad (TR^u_A)$$

where the coefficient of c_{uu} in the objective function

$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}$$

is of no clear sign. We start by assuming it is positive.

It is obvious to see that the (TR^a_A) binds. Suppose it did not, then the principal could increase c_{au} enough for it to do so. As a consequence of this change, the objective function would be decreased, the (TR^u_A) relaxed and the (IC) unaffected. Hence we can solve (TR^a_A) for c_{au}

$$c_{au}=\frac{1}{\Gamma^H_a}\left(\bar{u}+V(\lambda^H)-c_{uu}\Gamma^H_u\right).$$

Plugging this into the objective function yields

$$\begin{aligned}
&c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}\right]-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{ua}\tilde{\gamma}^H_{ua}+c_{au}\tilde{\gamma}^H_{au}\right)\\
=&c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}\right]\\
&\qquad-(\gamma^H_{aa}+\gamma^H_{ua})\left[c_{ua}\tilde{\gamma}^H_{ua}+\frac{\tilde{\gamma}^H_{au}}{\Gamma^H_a}\left(\bar{u}+V(\lambda^H)-c_{uu}\Gamma^H_u\right)\right]\\
\propto&c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{uu}\right]-(\gamma^H_{aa}+\gamma^H_{ua})\left[c_{ua}\tilde{\gamma}^H_{ua}-\tilde{P}_{au}c_{uu}\Gamma^H_u\right]\\
=&c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(\tilde{\gamma}^H_{uu}-\tilde{P}_{au}\Gamma^H_u\right)\right]-(\gamma^H_{aa}+\gamma^H_{ua})c_{ua}\tilde{\gamma}^H_{ua}\\
=&c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)\right]-(\gamma^H_{aa}+\gamma^H_{ua})c_{ua}\tilde{\gamma}^H_{ua}. \qquad (34)
\end{aligned}$$

Since we assumed

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H > 0,$$

it is immediate to see how

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) > 0,$$

given that $\tilde{\gamma}_{uu}^H \leq \tilde{\gamma}_{uu}^H - \Gamma_u^H \tilde{P}_{au} = \Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)$. Hence, the principal's objective is still to decrease c_{uu} and increase c_{ua}.

We now plug the value for c_{au} into the (TR_A^u).

$$\begin{aligned}
&\frac{\tilde{\gamma}_{au}^H}{\Gamma_a^H}\left(\bar{u} + V(\lambda^H) - c_{uu}\Gamma_u^H\right) + c_{uu}\tilde{\gamma}_{uu}^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au} \\
&\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{au} - \tilde{P}_{au}\right) + c_{uu}(\tilde{\gamma}_{uu}^H - \tilde{P}_{au}\Gamma_u^H) - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq 0 \\
&c_{uu}(\tilde{P}_{uu} - \tilde{P}_{au})\Gamma_u^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq 0 \\
&c_{uu} \geq c_{ua},
\end{aligned}$$

where we used the fact that $(\tilde{P}_{uu} - \tilde{P}_{au}) = (1 - \tilde{P}_{ua} - 1 + \tilde{P}_{aa}) = (\tilde{P}_{aa} - \tilde{P}_{ua})$.

The new (TR_A^u) and the (IC) can then be plotted in Figure 6 below. Contracts that satisfy all constraints must lie above the (TR_A^u) line and below the (IC) one.

By the analysis above, isocosts are positively sloped and decrease in value towards the bottom right of Figure 6. This implies that, depending on the relative slope of the isocosts and the (TR_A^u), the optimal contract for this case lies either at point A or B.[31] Both of these points, however, contradict the assumption that $c_{uu} > c_{au}$.

To see why A contradicts the assumption, simply notice that, at A, $c_{uu} = 0$ and therefore it cannot be $c_{uu} > c_{au} \geq 0$.

To see why also point B contradicts our assumption, we can solve for the optimal

[31] Or any linear combination fo the two when the slopes exactly coincide.

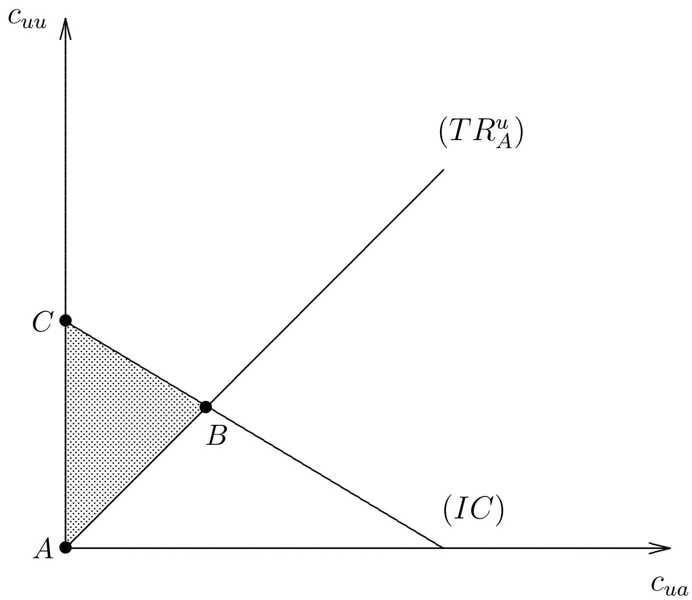

Figure 6: Contracts lying in the shaded area satisfy all constraints when $c_{uu} > c_{au}$.

c_{uu} and c_{au}. At B, we have that $c_{ua} = c_{uu}$. Hence we solve for c_{uu} in the (IC) to get

$$c_{uu} = \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a.$$

While we can prove that the above is always positive, notice that plugging this value in the formula for c_{au} yields

$$\begin{aligned} c_{au} =& \frac{1}{\Gamma^H_a}\left\{\bar{u} + V(\lambda^H) - \left[\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\right]\Gamma^H_u\right\} \\ =& \frac{1}{\Gamma^H_a}\left[\left(\bar{u} + V(\lambda^H)\right)\underbrace{(1 - \Gamma^H_u}_{\Gamma^H_a} + \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\Gamma^H_u\right] \\ =& \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u \;\geq\; \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a = c_{uu}. \end{aligned}$$

This proves that, when

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H > 0,$$

c_{au} must be larger or equal than c_{uu}. We are now going to prove the same for the opposite case.

Start again from the problem

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{uu}\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\left(c_{ua}\tilde{\gamma}_{ua}^H + c_{au}\tilde{\gamma}_{au}^H\right)$$

$$\text{s.t.} \quad \bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - c_{uu}\tilde{P}_{uu} \geq \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \qquad (IC)$$

$$\bar{u} + V(\lambda^H) \geq c_{au}\Gamma_a^H + c_{uu}\Gamma_u^H \qquad (TR_A^a)$$

$$c_{au}\tilde{\gamma}_{au}^H + c_{uu}\tilde{\gamma}_{uu}^H - c_{ua}\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H \geq \left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au} \qquad (TR_A^u)$$

and suppose now that the coefficient of c_{uu} is negative. That is,

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H < 0.$$

First of all, notice that the (TR_A^a) still binds, since the discussion above still holds. This implies that we can solve for c_{au} again and substitute it in the objective function and other constraints, exactly as we did above. Now, however, the new coefficient of c_{uu} featured in the new objective function may itself be positive or negative. Suppose it is positive, then the problem is exactly the same as above and the proof for the case of $(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{uu}^H > 0$ holds.

Things change when the new coefficient is still negative. That is, when

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H\left(\tilde{P}_{uu} - \tilde{P}_{au}\right) < 0.$$

In this case, while the contracts that satisfy all constraints are still the ones in the shaded area of Figure 6, the isocosts now decrease in value towards the top right corner. Hence, depending on the relative slope of isocosts and (IC), the optimal contract lies either at point B or C of the Figure. We have already shown how point B contradicts $c_{uu} > c_{au}$. Instead of checking that the same is true for point C, we are going to show that this point is never an equilibrium and that B is the sole optimal contract for this case. To do so, we show that the isocosts are always strictly steeper than the (IC) when

$$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right) < 0.$$

The slope of the (IC) is given by $\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}}$. The one of the isocosts, according to the new objective function (34), is given by

$$\frac{(\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H}{(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)}.$$

The optimal contract lies at point C iff

$$\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}} \geq \frac{(\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H}{(\gamma_{au}^H + \gamma_{uu}^H)\tilde{\gamma}_{aa}^H - (\gamma_{aa}^H + \gamma_{ua}^H)\Gamma_u^H \left(\tilde{P}_{uu} - \tilde{P}_{au}\right)}.$$

To see that the above never holds calculate

$$\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}} \geq \frac{(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{ua}}{(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)}$$
$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}\tilde{P}_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)\tilde{P}_{ua} \geq (\gamma^H_{aa}+\gamma^H_{ua})\tilde{P}_{ua}\Gamma^H_u\tilde{P}_{uu}$$
$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}\tilde{P}_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})\left[\tilde{P}_{ua}\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)+\tilde{P}_{uu}\tilde{P}_{ua}\Gamma^H_u\right] \geq 0$$
$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left[\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)+\tilde{P}_{uu}\Gamma^H_u\right] \geq 0$$
$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u\left(2\tilde{P}_{uu}-\tilde{P}_{au}\right) \geq 0,$$

which is never true since we are in the case of

$$(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right) < 0,$$

and $\Gamma^H_u\left(2\tilde{P}_{uu}-\tilde{P}_{au}\right) > \Gamma^H_u\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)$.

This concludes the proof of Part 1 by showing that all potentially optimal contracts resulting from the assumption that $c_{uu} > c_{au}$ in (33) actually feature $c_{au} \geq c_{uu}$.

PART 2: suppose $c_{au} > c_{uu}$ We start by re-stating the problem

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{au}\Gamma^H_a\left[(P_{aa}-P_{ua})\Gamma^H_u+b_a\right]-(\gamma^H_{aa}+\gamma^H_{ua})\left(c_{ua}\tilde{\gamma}^H_{ua}+c_{uu}\tilde{\gamma}^H_{uu}\right) \tag{35}$$
$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu} \geq \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a \tag{IC}$$
$$\bar{u}+V(\lambda^H) \geq c_{au}\Gamma^H_a+c_{uu}\Gamma^H_u \tag{TR^a_A}$$
$$c_{au}\tilde{\gamma}^H_{au}+c_{uu}\tilde{\gamma}^H_{uu}-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u \geq \left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \tag{TR^u_A.}$$

We then can see that the (TR^u_A) binds. Suppose not. The principal can decrease c_{au} by an ϵ such that $c_{au} > c_{uu}$ is preserved. This does not affect the (IC), it relaxes the (TR^a_A) and decreases the objective function.

We can now solve for c_{au} from the (TR_A^u) to get

$$\begin{aligned} c_{au} =& \frac{1}{\tilde{\gamma}_{au}^H} \left[\left(\bar{u} + V(\lambda^H) \right) \tilde{P}_{au} - c_{uu} \tilde{\gamma}_{uu}^H + c_{ua} \left(\tilde{P}_{aa} - \tilde{P}_{ua} \right) \Gamma_u^H \right] \\ =& \frac{\left(\bar{u} + V(\lambda^H) \right)}{\Gamma_a^H} - c_{uu} \frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H} + c_{ua} \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua} \right) \Gamma_u^H}{\tilde{\gamma}_{au}^H} \end{aligned}$$

and plug it in the objective function to obtain the following:

$$\left(\left(\bar{u} + V(\lambda^H) \right) - c_{uu} \frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}} + c_{ua} \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua} \right) \Gamma_u^H}{\tilde{P}_{au}} \right) \left[\left(P_{aa} - P_{ua} \right) \Gamma_u^H + b_a \right] + \\ - \left(\gamma_{aa}^H + \gamma_{ua}^H \right) \left(c_{ua} \tilde{\gamma}_{ua}^H + c_{uu} \tilde{\gamma}_{uu}^H \right),$$

which is equivalent to minimizing

$$\left(-c_{uu} \frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}} + c_{ua} \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua} \right) \Gamma_u^H}{\tilde{P}_{au}} \right) \left[\left(P_{aa} - P_{ua} \right) \Gamma_u^H + b_a \right] - \left(\gamma_{aa}^H + \gamma_{ua}^H \right) \left(c_{ua} \tilde{\gamma}_{ua}^H + c_{uu} \tilde{\gamma}_{uu}^H \right).$$

Let us study the coefficients of c_{uu} and c_{ua} separately. Start from the coefficient of c_{ua} and recall that we obtained $\left(P_{aa} - P_{ua} \right) \Gamma_u^H + b_a$ from the simplification of

$(\gamma_{au}^H + \gamma_{uu}^H)\tilde{P}_{aa} - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au}.$

$$\begin{aligned}
&\frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}} \left[(P_{aa} - P_{ua})\,\Gamma_u^H + b_a\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H \\
&= \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}} \left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{P}_{aa} - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au}\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{\gamma}_{ua}^H \\
&\propto \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\left[(\gamma_{au}^H + \gamma_{uu}^H)\tilde{P}_{aa} - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au}\right] - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{ua}\tilde{P}_{au} \\
&= \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma_{au}^H + \gamma_{uu}^H)\tilde{P}_{aa} - (\gamma_{aa}^H + \gamma_{ua}^H)\left[\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{au} + \tilde{P}_{ua}\tilde{P}_{au}\right] \\
&= \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma_{au}^H + \gamma_{uu}^H)\tilde{P}_{aa} - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au}\tilde{P}_{aa} \\
&\propto \left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)(\gamma_{au}^H + \gamma_{uu}^H) - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au} \\
&= \left(1 - \tilde{P}_{au} - \tilde{P}_{ua}\right)(\gamma_{au}^H + \gamma_{uu}^H) - (\gamma_{aa}^H + \gamma_{ua}^H)\tilde{P}_{au} \\
&= \left(1 - \tilde{P}_{ua}\right)(\gamma_{au}^H + \gamma_{uu}^H) - \tilde{P}_{au} \\
&= \tilde{P}_{uu}(\gamma_{au}^H + \gamma_{uu}^H) - \tilde{P}_{au}.
\end{aligned}$$

This implies that the coefficient of c_{ua} in the new objective function has no clear sign. It is positive only if

$$b_a \geq P_{au} - (\gamma_{au}^H + \gamma_{uu}^H)(P_{uu} - b_u). \tag{36}$$

The coefficient of c_{uu}, instead, is always negative and it simplifies to

$$
\begin{aligned}
&-\frac{(P_{aa}-P_{ua})\,\Gamma_u^H+b_a}{\tilde{P}_{au}}\tilde{\gamma}_{uu}^H-(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{\gamma}_{uu}^H\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[(P_{aa}-P_{ua})\,\Gamma_u^H+b_a+(\gamma_{aa}^H+\gamma_{ua}^H)\tilde{P}_{au}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H+b_a+\gamma_{aa}^H\tilde{P}_{au}+\gamma_{ua}^H\tilde{P}_{au}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H+b_a+P_{aa}\Gamma_a^H P_{au}+P_{ua}\Gamma_u^H P_{au}-b_a(P_{aa}\Gamma_a^H+P_{ua}\Gamma_u^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H(1-P_{au})+P_{aa}\Gamma_a^H P_{au}+b_a\underbrace{(1-\gamma_{aa}^H-\gamma_{ua}^H)}_{\gamma_{au}^H+\gamma_{uu}^H}\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H-P_{ua}\Gamma_u^H P_{aa}+P_{aa}\Gamma_a^H P_{au}+b_a(\gamma_{au}^H+\gamma_{uu}^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}\left[P_{aa}\Gamma_u^H P_{uu}+P_{aa}\Gamma_a^H P_{au}+b_a(\gamma_{au}^H+\gamma_{uu}^H)\right]\\
&=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\,(P_{aa}+b_a)=-\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\tilde{P}_{aa}.
\end{aligned}
$$

Before re-writing the problem fully, let us calculate the new (TR_A^u).

$$
\begin{aligned}
&\bar{u}+V(\lambda^H)\geq\left(\bar{u}+V(\lambda^H)\right)-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}+c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}+c_{uu}\Gamma_u^H\\
&c_{uu}\left(\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}-\Gamma_u^H\right)\geq c_{ua}\frac{\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H}{\tilde{P}_{au}}\\
&c_{uu}\Gamma_u^H\left(\tilde{P}_{uu}-\tilde{P}_{au}\right)\geq c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma_u^H\\
&c_{uu}\geq c_{ua}.
\end{aligned}
$$

Hence, the new problem for the case of $c_{au} > c_{uu}$ is given by

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{ua}\frac{\Gamma_u^H}{\tilde{P}_{au}}\left(\tilde{P}_{uu}(\gamma_{au}^H+\gamma_{uu}^H)-\tilde{P}_{au}\right)-c_{uu}\frac{\tilde{\gamma}_{uu}^H}{\tilde{P}_{au}}(\gamma_{au}^H+\gamma_{uu}^H)\,(P_{aa}+b_a) \tag{37}$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-c_{uu}\tilde{P}_{uu}\geq\frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H \tag{IC}$$

$$c_{uu}\geq c_{ua}. \tag{TR_A^a}$$

We now represent the constraints and feasible contracts in Figure 7 below. In the graph, contracts above the (TR_A^a) and below the (IC) satisfy all constraints (are feasible). Recall, however, that we are in case (ii) of Lemma 6. Hence, all contracts lying on the locus of points that sets the (TR_A^a) binding, although the lie in the feasible set, if optimal, would contraddict $c_{uu} > c_{ua}$.

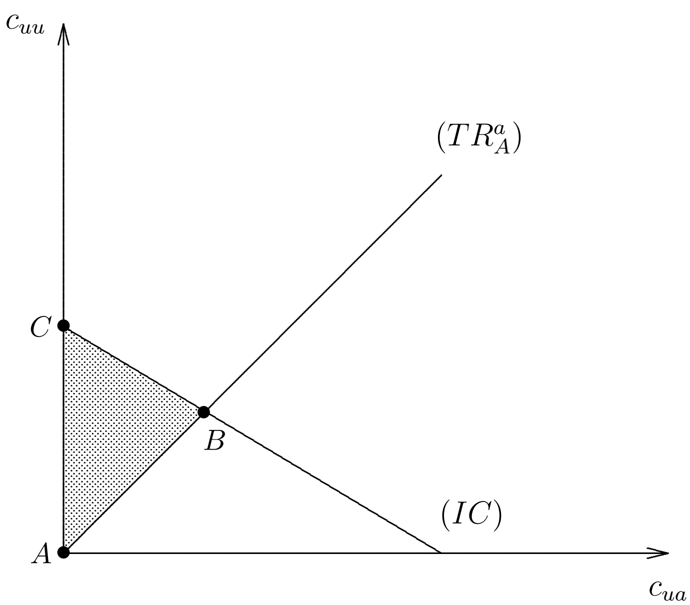

Figure 7: Contracts lying in the shaded area satisfy all constraints when $c_{au} > c_{uu}$. The ones lying on the (TR_A^a), however, violate case (ii) of Lemma 6.

Suppose (36) holds, then the objective function is minimized towards the top left corner of the Figure. This, inevitably, yields as an optimal contract the one lying in point C.

Suppose (36) fails, then the objective function is minimized towards the top right corner of the Figure. If the isocosts are flatter than the (IC) then point C lies on an isocosts that features a lower cost than the one where B lies. The reverse is true when the isocost is steeper. We are now going to show that at point C the assumption $c_{au} > c_{uu}$ is violated as well, leading to the desired contradiction.

At point C, $c_{ua} = 0$ is trivial. c_{uu} comes from the (IC) binding and c_{au} is derived from

$$
\begin{aligned}
c_{au} =& \frac{1}{\tilde{\gamma}_{au}^{H}} \left[\left(\bar{u} + V(\lambda^{H}) \right) \tilde{P}_{au} - c_{uu} \tilde{\gamma}_{uu}^{H} + c_{ua} \left(\tilde{P}_{aa} - \tilde{P}_{ua} \right) \Gamma_{u}^{H} \right] \\
=& \frac{\left(\bar{u} + V(\lambda^{H}) \right)}{\Gamma_{a}^{H}} - \frac{\bar{u} + V(\lambda^{H}) - \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H}}{\tilde{P}_{uu}} \cdot \frac{\tilde{\gamma}_{uu}^{H}}{\tilde{\gamma}_{au}^{H}} \\
=& \frac{1}{\tilde{P}_{au} \Gamma_{a}^{H}} \left\{ \left(\bar{u} + V(\lambda^{H}) \right) \tilde{P}_{au} - \left(\bar{u} + V(\lambda^{H}) \right) \Gamma_{u}^{H} + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \Gamma_{u}^{H} \right\} \\
=& \frac{1}{\tilde{P}_{au} \Gamma_{a}^{H}} \left\{ \left(\bar{u} + V(\lambda^{H}) \right) \left(\tilde{P}_{au} - \Gamma_{u}^{H} \right) + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \Gamma_{u}^{H} \right\}.
\end{aligned}
$$

Finally, to get c_{aa} simply plug the values in

$$
c_{aa} = \frac{1}{\tilde{\gamma}_{aa}^{H}} \left(\bar{u} + V(\lambda^{H}) - c_{au} \tilde{\gamma}_{au}^{H} - c_{ua} \tilde{\gamma}_{ua}^{H} - c_{uu} \tilde{\gamma}_{uu}^{H} \right).
$$

We now derive the contradiction. Start by the following calculations

$$
\begin{aligned}
& c_{au} > c_{uu} \\
& \frac{\left(\bar{u} + V(\lambda^{H}) \right) \left(\tilde{P}_{au} - \Gamma_{u}^{H} \right) + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \Gamma_{u}^{H}}{\tilde{P}_{au} \Gamma_{a}^{H}} > \frac{\bar{u} + V(\lambda^{H}) - \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H}}{\tilde{P}_{uu}} \\
& \left(\bar{u} + V(\lambda^{H}) \right) \left[\tilde{P}_{uu} (\tilde{P}_{au} - \Gamma_{u}^{H}) - \tilde{P}_{au} \Gamma_{a}^{H} \right] + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \left(\tilde{P}_{uu} \Gamma_{u}^{H} + \tilde{P}_{au} \Gamma_{a}^{H} \right) > 0 \\
& \left(\bar{u} + V(\lambda^{H}) \right) \left[\tilde{P}_{uu} (1 - \tilde{P}_{aa} - 1 + \Gamma_{a}^{H}) - \tilde{P}_{au} \Gamma_{a}^{H} \right] + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \left(\tilde{P}_{uu} \Gamma_{u}^{H} + \tilde{P}_{au} \Gamma_{a}^{H} \right) > 0 \\
& - \left(\bar{u} + V(\lambda^{H}) \right) \left[\tilde{P}_{au} \Gamma_{a}^{H} + \tilde{P}_{uu} (\tilde{P}_{aa} - \Gamma_{a}^{H}) \right] + \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{a}^{H} \left(\tilde{P}_{uu} \Gamma_{u}^{H} + \tilde{P}_{au} \Gamma_{a}^{H} \right) > 0.
\end{aligned}
$$

and notice that $\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)$ corresponds to the denominator of the first component of $\bar{u}_2$, which is always positive.[32] Hence, the sign of the inequality is not trivial.

Let's now substitute $\bar{u}$ for $\bar{u}_2$. We are going to show that, when this holds, the inequality above fails. Hence, since the LHS is decreasing in $\bar{u}$, for values larger than $\bar{u}_2$ the inequality fails too. To see this, calculate

$$- \left(\bar{u} + V(\lambda^H)\right) \left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)\right] + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \left(\tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\right) > 0$$

$$- \left(\frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} - V(\lambda^H) + V(\lambda^H)\right) \left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)\right] + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \left(\tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\right) > 0$$

$$- \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \left(\tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\right) + \tfrac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \left(\tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H\right) > 0$$

which obviously fails. Hence, point C violates our assumption and is ruled out.[33] This concludes the proof of this part and of the Lemma. ∎

[32] To see this calculate:

$$\begin{aligned}
\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H) &= \tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(1 - \Gamma_a^H - 1 + \tilde{P}_{aa}) \\
&= \tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H - \tilde{P}_{au}\tilde{P}_{uu} = (1 - \tilde{P}_{au})\tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{au}\Gamma_a^H(1 - \tilde{P}_{uu}) \\
&= \tilde{P}_{aa}\tilde{P}_{uu}\Gamma_u^H + \tilde{\gamma}_{au}^H\tilde{P}_{ua} > 0.
\end{aligned}$$

[33] Technically when the slopes of isocosts and (IC) are identical, any point lying on segment $\bar{CB}$ is optimal. There could very well be a contract satisfying the assumption that $c_{au} > c_{uu}$ there. However, given that our parameters are continuous values, the probability of such an event happening is zero. We therefore rule this case out.

Given Lemma 32, we can derive the final version of the problem:

$$\min_{\{c_{ts}\}_{t,s\in\{u,a\}}} c_{uu}\left[(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)\right]-(\gamma^H_{aa}+\gamma^H_{ua})c_{ua}\tilde{\gamma}^H_{ua} \tag{38}$$

$$\text{s.t.}\quad \bar{u}+V(\lambda^H)-c_{ua}\tilde{P}_{ua}-\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\geq c_{uu}\tilde{P}_{uu} \tag{IC}$$

$$\bar{u}+V(\lambda^H)\geq c_{uu} \tag{TR^a_A}$$

$$c_{uu}(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu})-c_{ua}\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\Gamma^H_u\geq\left(\bar{u}+V(\lambda^H)\right)\tilde{P}_{au} \tag{TR^u_A}$$

where the coefficient of c_{uu} is simplified to

$$\begin{aligned}
&(\gamma^H_{au}+\gamma^H_{uu})\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)\\
&\quad=\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(\Gamma^H_a+\tilde{\gamma}^H_{uu}\right)\\
&\quad=\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(\Gamma^H_a+\Gamma^H_u-\tilde{\gamma}^H_{ua}\right)=\tilde{\gamma}^H_{aa}-(\gamma^H_{aa}+\gamma^H_{ua})\left(1-\tilde{\gamma}^H_{ua}\right)\\
&\quad=\gamma^H_{aa}+b_a\Gamma^H_a-\gamma^H_{aa}-\gamma^H_{ua}+(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{ua}
\end{aligned}$$

and is positive iff

$$b_a\geq\frac{\gamma^H_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})\tilde{\gamma}^H_{ua}}{\Gamma^H_a}$$

$$b_a\geq\frac{(\gamma^H_{au}+\gamma^H_{uu})\gamma^H_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})b_u\Gamma^H_u}{\Gamma^H_a}. \tag{39}$$

Now, consider the constraints in (c_{ua},c_{uu}) space. Notice that the (IC) is negatively sloped with intercept

$$\frac{1}{\tilde{P}_{uu}}\left[\bar{u}+V(\lambda^H)-\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\right].$$

On the contrary, the (TR^u_A) is positively sloped with intercept

$$\left(\bar{u}+V(\lambda^H)\right)\frac{\tilde{P}_{au}}{\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}}.$$

Hence, we need the intercept of (TR_A^u) to be smaller than the one of the (IC), or the set of feasible contracts would be empty. To see that this is always the case for $\bar{u} > \bar{u}_2$, calculate

$$\begin{aligned}
&\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}\tilde{P}_{au} - \tilde{\gamma}_{au}^H - \tilde{\gamma}_{uu}^H\right) + \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) < 0. \\
&\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}(1 - \tilde{P}_{aa}) - \tilde{P}_{au}\Gamma_a^H - \tilde{P}_{uu}\Gamma_u^H\right) + \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) < 0. \\
&-\left(\bar{u} + V(\lambda^H)\right)\left(\tilde{P}_{uu}\tilde{P}_{aa} + \tilde{P}_{au}\Gamma_a^H - \tilde{P}_{uu}\Gamma_a^H\right) + \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) < 0. \\
&\bar{u} > \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H\frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} - V(\lambda^H) \equiv \bar{u}_2.
\end{aligned}$$

We plot in the next set of Figures the (IC), (TR_A^a) and the (TR_A^u) and study, once again, the area of feasible contracts. Notice that we have three different cases. This is because, while we know that the intercept of the (TR_A^u) is lower than the one of the (IC), and we can prove that the intercept of the (TR_A^u) is lower than the one of the (TR_A^a), it is not straightforward to prove whether the (TR_A^a) lies above or below the intercept of the (IC), or above or below the intersection between the (TR_A^u) and the (IC).

In Figure 8, we assume the (TR_A^a) is always slack, since it is looser than the (IC).

In Figure 9, we assume the (TR_A^a) crosses the (IC) and it does so above the point where (IC) and (TR_A^u) cross each other.

In Figure 10 , we assume the (TR_A^a) crosses the (IC) and it does so below the point where (IC) and (TR_A^u) cross each other.

Point A, present in all figures, is contract GAPE$_2$ in the Proposition and it features the interesting aspect of setting the (IC) slack. Point B, in Figure 8 and 9, is the closest to the APE derived for the case of $\bar{u}_2$, since it features exactly the same binding constraints with the addition of the (PC). This corresponds to our GAPE$_1$. Point C, in Figure 8, and point D, in Figure 9, are potentially new optimal contracts where both (TR_A) are slack. Finally, notice that point E, in Figure 10, is never feasible since both (TR_A) bind. This can only happen in case (i) of Lemma 6

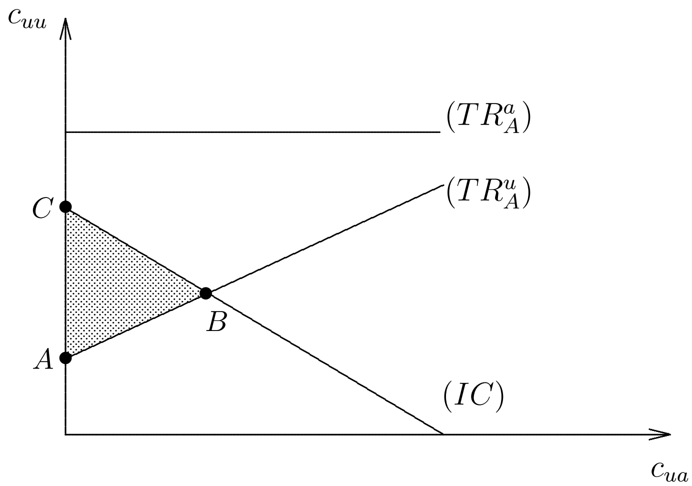

Figure 8: Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR^a_A) is looser than the (IC).

and it is therefore, ruled out from the analysis.

We now show why C and D are never optimal and then present the conditions for GAPE$_1$ and GAPE$_2$ to be optimal and feasible, and derive their equilibrium values.

Lemma 33. *When $\bar{u} > \bar{u}_2$ and the GBPE is not optimal, the principal assigns the worker a Generalized APE contract that lies either at point A of Figures 8-10 or at point B of Figures 8 and 9.*

Proof. We have already shown how E can never be an equilibrium. For either C or D to be optimal, it must be that (39) fails, so that costs decrease towards the top right corner of each Figure. Second, it must be that isocosts in the reduced problem (38) are flatter than the (IC), otherwise the optimal contract would lie at point B. To prove the Lemma, we show how this second condition is never feasible.

The slope of the (IC) in (38) is given by $\frac{\tilde{P}_{ua}}{\tilde{P}_{uu}}$, while the one of isocosts — when (39) fails — is given by

$$\frac{\left(\gamma^H_{aa} + \gamma^H_{ua}\right) \tilde{P}_{ua}}{\left(\gamma^H_{aa} + \gamma^H_{ua}\right) \left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right) - \left(\gamma^H_{au} + \gamma^H_{uu}\right) \tilde{\gamma}^H_{aa}}$$

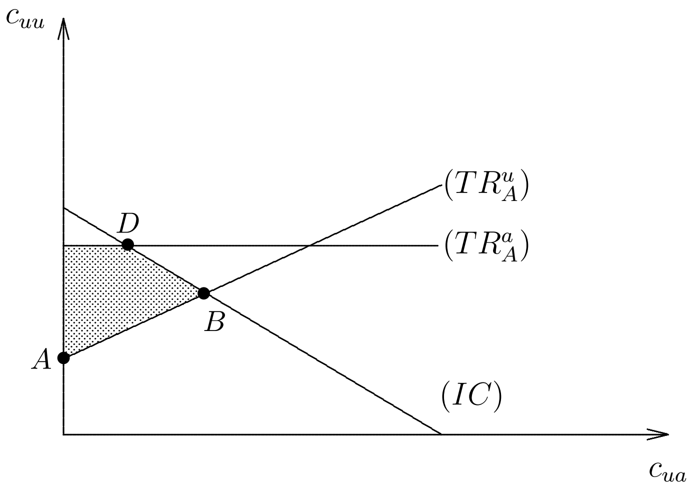

Figure 9: Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR_A^a) crosses the (IC) after (in terms of the value of c_{uu}) the (TR_A^u).

Hence, for C or D to be optimal, we would need:

$$\begin{aligned}
&\frac{\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{ua}}{\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H} < \frac{\tilde{P}_{ua}}{\tilde{P}_{uu}} \\
&\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{uu} < \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H \\
&\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{uu}\Gamma_a^H - \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{\gamma}_{au}^H + \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{\gamma}_{aa}^H < 0 \\
&\left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{uu}\Gamma_a^H - \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{\gamma}_{au}^H + \left(\gamma_{au}^H + \gamma_{uu}^H\right)\Gamma_a^H - \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{\gamma}_{au}^H < 0 \\
&\left(\gamma_{aa}^H + \gamma_{ua}^H\right)(1 - \tilde{P}_{ua}) - \tilde{P}_{au} + \left(\gamma_{au}^H + \gamma_{uu}^H\right) < 0 \\
&1 - \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\tilde{P}_{ua} - \tilde{P}_{au} < 0 \\
&1 - \left(1 - \gamma_{au}^H - \gamma_{uu}^H\right)\tilde{P}_{ua} - \tilde{P}_{au} < 0 \\
&\tilde{P}_{aa} - \tilde{P}_{ua} + \left(\gamma_{au}^H + \gamma_{uu}^H\right)\tilde{P}_{ua} < 0,
\end{aligned}$$

which is never true by positive perceived correlation ($\tilde{P}_{aa} - \tilde{P}_{ua} > 0$). Conditions for feasibility and optimality of A and B are derived in the Lemmas below. ∎

Now that we know the features of the GAPE contracts, we derive their equilibrium

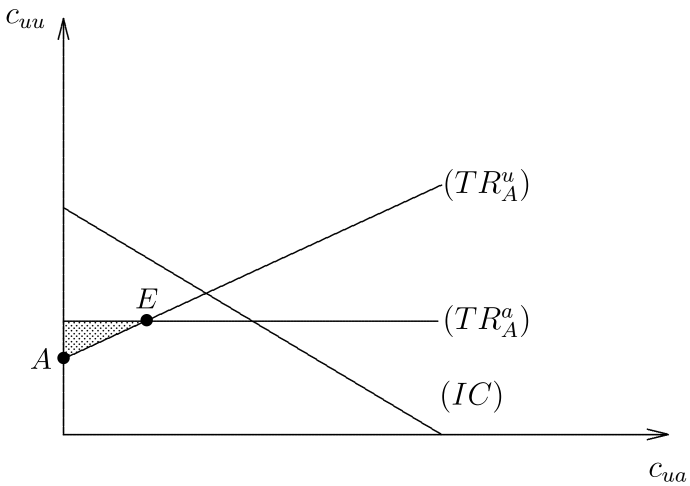

Figure 10: Contracts lying in the shaded area satisfy all constraints when $c_{au} = c_{uu}$. This graph represents the case where the (TR_A^a) crosses the (IC) before (in terms of the value of c_{uu}) the (TR_A^u).

values and the conditions for their feasibility and optimality. In order to simplify the algebraic derivations, we start with GAPE$_2$ (point A).

Lemma 34. *When the bias of the worker is large, that is*

$$b_a \geq \frac{\left(\gamma_{au}^H + \gamma_{uu}^H\right) P_{ua} + b_u \left(\gamma_{au}^H - \gamma_{uu}^H\right)}{\Gamma_a^H} \tag{40}$$

and

$$b_a \geq P_{au} - (P_{uu} - b_u) \frac{\left[\bar{u} + V(\lambda^H)\right]\left(\gamma_{au}^H - \gamma_{ua}^H\right) + \frac{\Delta V}{\Delta \Gamma_a} \frac{\gamma_{ua}^H}{P_{aa}} \Gamma_a^H}{\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a} \frac{P_{ua}}{P_{aa}} \Gamma_a^H}. \tag{41}$$

the optimal contract set by the principal is the GAPE$_2$ of Proposition 6.

Proof. First of all, at A we have $c_{ua} = 0$ and $c_{uu} = c_{au}$ equal the intercept of the (TR_A^u). That is,

$$c_{uu} = c_{au} = \frac{(\bar{u} + V(\lambda^H))\tilde{P}_{au}}{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H}.$$

Plugging these into the (PC) constraint to obtain c_{aa}, we get

$$\begin{aligned} c_{aa} &= \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right) \\ &= \frac{\bar{u} + V(\lambda^H)}{\Gamma_a^H}. \end{aligned}$$

It is immediate to see how this contract satisfies $c_{uu} > c_{ua} = 0$. It is also easy to see that $c_{aa} > c_{au}$, since

$$\frac{1}{\Gamma_a^H} > \frac{\tilde{P}_{au}}{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H} \iff 1 > \frac{\tilde{\gamma}_{au}^H}{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H},$$

which is obvious. This ensures that the contract satisfies case (ii) of Lemma 6. Finally, to derive the wages, simply recall that we are in case (i) of Lemma 5 and that Lemma 30 above holds. Now we move to the derivations of its feasibility and optimality.

For the GAPE$_2$ contract to be optimal, quite a few conditions have to hold. First of all, point A is feasible only when the agent believes signals to be positively correlated. We know that this happens when (3) holds

$$\tilde{P}_{aa} - \tilde{P}_{ua} \geq 0 \qquad \iff \qquad b_a \geq P_{au} - (P_{uu} - b_u).$$

Second, for GAPE$_2$ to be optimal in the restricted problem of case (ii) of Lemma 6 and (i) of Lemma 5, condition (39) must hold, so that costs decrease towards the bottom right corner:

$$b_a \geq \frac{(\gamma_{au}^H + \gamma_{uu}^H)\gamma_{ua}^H - (\gamma_{aa}^H + \gamma_{ua}^H)b_u\Gamma_u^H}{\Gamma_a^H}$$

Third, also for GAPE$_2$ to be optimal in the restricted problem of case (ii) of

Lemma 6 and (i) of Lemma 5, the isocosts of the restricted problem have to be flatter than the (TR^u_A), otherwise the optimal contract would either be at point B (Figure 8 and 9) or non-existent (Figure 10 where E yields to a contradiction). Looking at problem (38), this happens when:

$$\frac{\tilde{P}_{aa}-\tilde{P}_{ua}}{\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}} \geq \frac{\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\tilde{P}_{ua}}{\left(\gamma^H_{au}+\gamma^H_{uu}\right)\tilde{\gamma}^H_{aa}-\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)}$$
$$\left(\tilde{P}_{aa}-\tilde{P}_{ua}\right)\left(\left(\gamma^H_{au}+\gamma^H_{uu}\right)\tilde{\gamma}^H_{aa}-\left(\gamma^H_{aa}+\gamma^H_{ua}\right)\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)\right) \geq \left(\gamma^H_{aa}+\gamma^H_{ua}\right)\tilde{P}_{ua}\left(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}\right)$$
$$\tilde{P}_{uu}\left[\Gamma^H_a\left(\gamma^H_{au}+\gamma^H_{uu}\right)-\Gamma^H_u\left(\tilde{\gamma}^H_{aa}+\tilde{\gamma}^H_{ua}\right)\right]-\tilde{\gamma}^H_{au}\geq 0$$
$$\tilde{P}_{uu}\left(P_{au}\Gamma^H_a-P_{ua}\Gamma^H_u\right)-\tilde{\gamma}^H_{au}\geq 0$$
$$b_a \geq \frac{\left(\gamma^H_{au}+\gamma^H_{uu}\right)P_{ua}+b_u\left(\gamma^H_{au}-\gamma^H_{ua}\right)}{\Gamma^H_a},$$

which generates (40). Notice that the above is true only if (39) holds, so that the denominator of the slope of the isocost is positive. However, we are now going to show that (40) is a stronger requirement compared to (39), which can therefore be disregarded. To see this, simply compare the numerators of the two and notice that

$$(\gamma^H_{au}+\gamma^H_{uu})\gamma^H_{ua}-(\gamma^H_{aa}+\gamma^H_{ua})b_u\Gamma^H_u \leq \left(\gamma^H_{au}+\gamma^H_{uu}\right)P_{ua}+b_u\left(\gamma^H_{au}-\gamma^H_{ua}\right)$$
$$\iff (\gamma^H_{au}+\gamma^H_{uu})P_{ua}(\Gamma^H_u-1)-b_u\left[(\gamma^H_{aa}+\gamma^H_{ua})\Gamma^H_u+\gamma^H_{au}-\gamma^H_{ua}\right]\leq 0.$$

The first component of the LHS is clearly negative. The second one is as well, since the bracket is always positive. To see this, notice that it is decreasing linearly in P_{ua}, so if it is positive at the maximum value of P_{ua} then it always is. By positive correlation of s an t, P_{ua} is strictly lower than P_{aa}. As $P_{ua}\to P_{aa}$, the bracket converges to

$$P_{aa}\Gamma^H_u(\Gamma^H_a+\Gamma^H_u)+P_{au}\Gamma^H_a-P_{aa}\Gamma^H_u=P_{au}\Gamma^H_a>0.$$

Hence, we can disregard (39) in the rest of the analysis.

The tightness of (40) allows us to remove a further condition, the one of positive correlation. To show this, we calculate

$$
\begin{aligned}
&P_{au} - P_{uu} + b_u < \frac{\left(\gamma^H_{au} + \gamma^H_{uu}\right) P_{ua} + b_u \left(\gamma^H_{au} - \gamma^H_{uu}\right)}{\Gamma^H_a} \\
&b_u(-\gamma^H_{au} + \gamma^H_{uu} + \Gamma^H_a) < P_{au}P_{ua}\Gamma^H_a + P_{uu}P_{ua}\Gamma^H_u + P_{uu}\Gamma^H_a - P_{au}\Gamma^H_a \\
&b_u(\gamma^H_{aa} + \gamma^H_{uu}) < -P_{au}P_{uu}\Gamma^H_a + P_{uu}P_{ua}\Gamma^H_u + P_{uu}\Gamma^H_a \\
&b_u(\gamma^H_{aa} + \gamma^H_{uu}) < P_{uu}P_{ua}\Gamma^H_u + P_{uu}P_{aa}\Gamma^H_a \\
&b_u < P_{uu},
\end{aligned}
$$

which is always true by definition of b_u. Hence (40) is a necessary condition. Condition (41) comes, instead, from the comparison of the expected wage costs of the GBPE and GAPE$_2$ contracts. Hence, we have to calculate the two.

$$
\begin{aligned}
E(\tilde{w}^*_{ts}) &= \tilde{w}^*_{aa}\gamma^H_{aa} + \tilde{w}^*_{au}\gamma^H_{au} + \tilde{w}^*_{ua}\gamma^H_{ua} + \tilde{w}^*_{uu}\gamma^H_{uu} \\
&= \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma^H_{aa} + \gamma^H_{ua}}{P_{aa}} + \tilde{c}^*_{uu} = \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\frac{\gamma^H_{ua}}{P_{aa}},
\end{aligned}
$$

$$
\begin{aligned}
E(\tilde{w}^{\dagger\prime}_{ts}) &= \tilde{w}^{\dagger\prime}_{aa}\gamma^H_{aa} + \tilde{w}^{\dagger\prime}_{au}\gamma^H_{au} + \tilde{w}^{\dagger\prime}_{ua}\gamma^H_{ua} + \tilde{w}^{\dagger\prime}_{uu}\gamma^H_{uu} \\
&= \left[\bar{u} + V(\lambda^H)\right] \frac{\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right) - \tilde{\gamma}^H_{uu}\left(\gamma^H_{au} + \gamma^H_{uu}\right)}{\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}\right)\Gamma^H_a}.
\end{aligned}
$$

The GAPE_2 is set when

$$E(\tilde{w}_{ts}^{*}) > E(\tilde{w}_{ts}^{\dagger'})$$

$$\bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}} > \left[\bar{u} + V(\lambda^H)\right] \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) - \tilde{\gamma}_{uu}^H\left(\gamma_{au}^H + \gamma_{uu}^H\right)}{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\Gamma_a^H}$$

$$\left(\bar{u} + V(\lambda^H)\right)\Gamma_u^H\left[\tilde{P}_{uu}\left(\gamma_{au}^H - \gamma_{ua}^H\right) - \tilde{\gamma}_{au}^H\right] + \frac{\Delta V}{\Delta \Gamma_a^H}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) \geq 0$$

$$\tilde{\gamma}_{uu}^H\left[\left[\bar{u} + V(\lambda^H)\right]\left(\gamma_{au}^H - \gamma_{ua}^H\right) + \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H\right] + \tilde{\gamma}_{au}^H\Gamma_u^H\left[\frac{\Delta V}{\Delta \Gamma_a}\frac{P_{ua}}{P_{aa}}\Gamma_a^H - \bar{u} - V(\lambda^H)\right] \geq 0$$

$$b_a \geq P_{au} - (P_{uu} - b_u)\frac{\left[\bar{u} + V(\lambda^H)\right]\left(\gamma_{au}^H - \gamma_{ua}^H\right) + \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma_{ua}^H}{P_{aa}}\Gamma_a^H}{\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\frac{P_{ua}}{P_{aa}}\Gamma_a^H}.$$

This concludes the proof. ∎

Now we move the derivation of the GAPE_1.

Lemma 35. *When the worker's bias is moderate (but such that he still perceives signals as positively correlated), that is, condition* (40) *fails and*

$$b_a\left((P_{uu} - b_u + \Gamma_a^H)P_{ua} - \gamma_{aa}^H\right) \leq \gamma_{au}^H(P_{ua} + b_u)(P_{aa} - P_{ua}) \tag{42}$$

then the principal assigns contract $GAPE_1$ of Proposition 5.

Proof. Before deriving contract B, notice two things. First, there are values for b_u such that (42) holds for all b_a. Second, formally the requirement on b_a deriving from the failing of (40), should be stated as

$$b_a \in \left[P_{au} - P_{uu} + b_u, \frac{\left(\gamma_{au}^H + \gamma_{uu}^H\right)P_{ua} + b_u\left(\gamma_{au}^H - \gamma_{uu}^H\right)}{\Gamma_a^H}\right].$$

At B, we need to study the intersection between the (IC) and the (TR_A^u). From the (IC)

$$c_{uu} = \tfrac{1}{\tilde{P}_{uu}} \left(\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \right).$$

Recall that we need $c_{uu} > c_{ua}$. This implies that

$$c_{ua} < \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H.$$

We check for this later on. Plug the above in the (TR_A^u) binding to get

$$c_{ua} = \frac{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H} \left(\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \right) - \frac{\left(\bar{u} + V(\lambda^H)\right)\tilde{P}_{au}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H},$$

which is equivalent to

$$c_{ua}\left[1 + \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}\right] = \frac{\left(\bar{u} + V(\lambda^H)\right)}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\Gamma_u^H}\left[\frac{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H}{\tilde{P}_{uu}} - \tilde{P}_{au}\right] - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \frac{\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}.$$

Let's work one coefficient at a time. The one of c_{ua} is

$$\begin{aligned}
\left[1 + \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}\right] &= \frac{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H + \left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)\tilde{P}_{ua}}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H} \\
&= \frac{1}{(...)}\left[\tilde{P}_{aa}\tilde{P}_{uu}\Gamma_u^H + \tilde{\gamma}_{au}^H\tilde{P}_{ua}\right] = \frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H - \tilde{P}_{au}\tilde{P}_{uu}\right] \\
&= \frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)\right].
\end{aligned}$$

Now we work on the one of $\bar{u} + V(\lambda^H)$ and show that it is identical.

$$\begin{aligned}
&\frac{1}{\left(\tilde{P}_{aa} - \tilde{P}_{ua}\right)\tilde{P}_{uu}\Gamma_u^H}\left[\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H - \tilde{P}_{au}\tilde{P}_{uu}\right] \\
&= \frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H - \tilde{P}_{au}\tilde{P}_{uu}\right] \\
&= \frac{1}{(...)}\left[\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)\right]
\end{aligned}$$

This implies that

$$c_{ua} = \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} = \bar{u} - \bar{u}_2,$$

which is always positive when $\bar{u} > \bar{u}_2$. As promised, we now check that $c_{ua} < \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H$.

$$\begin{aligned}
&c_{ua} < \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \\
&\bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \frac{\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right)}{\tilde{P}_{aa}\Gamma_u^H - \tilde{P}_{ua}(\tilde{P}_{aa} - \Gamma_a^H)} < \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta \Gamma_a}\Gamma_a^H \\
&\left(\tilde{\gamma}_{au}^H + \tilde{\gamma}_{uu}^H\right) > \tilde{P}_{aa}\Gamma_u^H - \tilde{P}_{ua}(\tilde{P}_{aa} - \Gamma_a^H) \\
&\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H - \tilde{P}_{aa} + \tilde{P}_{aa}\Gamma_a^H + \tilde{P}_{ua}(\tilde{P}_{aa} - \Gamma_a^H) > 0 \\
&-\tilde{P}_{aa} + \Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H + \tilde{P}_{ua}(\tilde{P}_{aa} - \Gamma_a^H) > 0 \\
&\tilde{P}_{uu}\Gamma_u^H + (\tilde{P}_{ua} - 1)(\tilde{P}_{aa} - \Gamma_a^H) > 0 \\
&\Gamma_u^H - \tilde{P}_{aa} + \Gamma_a^H) > 0 \\
&\tilde{P}_{au} > 0,
\end{aligned}$$

which is always true. Hence we can derive c_{uu}

$$
\begin{aligned}
c_{uu} &= \tfrac{1}{\tilde{P}_{uu}}\left(\bar{u} + V(\lambda^H) - c_{ua}\tilde{P}_{ua} - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right) \\
&= \tfrac{1}{\tilde{P}_{uu}}\left(\bar{u} + V(\lambda^H) - (\bar{u} - \bar{u}_2)\tilde{P}_{ua} - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\right) \\
&= \bar{u} + V(\lambda^H) - \frac{\Delta V}{\Delta\Gamma_a}\Gamma_a^H\Gamma_u^H \frac{(\tilde{P}_{aa} - \tilde{P}_{ua})}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)} = c_{au}
\end{aligned}
$$

We now derive c_{aa} with

$$
\begin{aligned}
c_{aa} &= \frac{1}{\tilde{\gamma}_{aa}^H}\left(\bar{u} + V(\lambda^H) - c_{au}\tilde{\gamma}_{au}^H - c_{ua}\tilde{\gamma}_{ua}^H - c_{uu}\tilde{\gamma}_{uu}^H\right) \\
&= \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \frac{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}\Gamma_u^H}{\tilde{P}_{au}\Gamma_a^H + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma_a^H)},
\end{aligned}
$$

where it is immediate to see that this is larger than c_{au}. This ensures that the GAPE_1 contract satisfies case (ii) of Lemma 6. To derive wage levels, simply recall that we are in case (i) of Lemma 5 and that Lemma 30 above holds.

For the GAPE_1 contract to be optimal, we need the following conditions. First of all, we need positive perceived correlation of signals ensured by (3). Second, we need isocosts in the reduced problem (38) to be steeper than (TR_A^u) when (39) holds. That is (40) failing.[34] Third, we need for the GAPE_1 contract to be less costly than

[34]To see that no further condition is needed for optimality in the reduced problem, notice that when (39) fails, Lemma 33 ensures that the GAPE_1 contract is the only optimal one.

the GBPE. This latter condition is the last calculation needed to prove the Lemma.

$$E(\tilde{w}^{*}_{ts}) = \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma^H_{aa}+\gamma^H_{ua}}{P_{aa}} + \tilde{c}^{*}_{uu} = \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta \Gamma_a}\frac{\gamma^H_{ua}}{P_{aa}},$$

$$\begin{aligned} E(\tilde{w}^{\dagger}_{ts}) &= \tilde{w}^{\dagger}_{aa}\gamma^H_{aa} + \tilde{w}^{\dagger}_{au}\gamma^H_{au} + \tilde{w}^{\dagger}_{ua}\gamma^H_{ua} + \tilde{w}^{\dagger}_{uu}\gamma^H_{uu} \\ &= \bar{u} + V(\lambda^H) + \frac{\Delta V}{\Delta \Gamma_a}\frac{\left[(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu})(\gamma^H_{aa}+\gamma^H_{ua}) - (\gamma^H_{au}+\gamma^H_{uu})(\tilde{\gamma}^H_{aa} - \tilde{P}_{ua}\Gamma^H_a)\right]\Gamma^H_u}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}. \end{aligned}$$

The GAPE_1 is therefore set when

$$\begin{aligned} & E(\tilde{w}^{\dagger}_{ts}) < E(\tilde{w}^{*}_{ts}) \\ & \frac{\Delta V}{\Delta \Gamma_a}\Gamma^H_u\left[\frac{(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu})(\gamma^H_{aa}+\gamma^H_{ua}) - (\gamma^H_{au}+\gamma^H_{uu})(\tilde{\gamma}^H_{aa} - \tilde{P}_{ua}\Gamma^H_a)}{\tilde{\gamma}^H_{au} + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)} - \frac{P_{ua}}{P_{uu}}\right] < 0 \end{aligned}$$

which, after some algebraical manipulation, yields (42). ∎

This Lemma concludes the proof of Proposition 6. ∎

B. Pareto Improving GAPE Contract

This Appendix derives conditions for the GAPE contract to Pareto improve over the GBPE contract. As in the case of $\bar{u} < \bar{u}_1$, we know that, if a GAPE contract is optimal for the firm, it is lowering the expected wage cost. Hence, we only look for true expected compensations. We start from GAPE_1.

First of all, notice that the true expected compensation from the GBPE is given by:

$$E(\tilde{c}^{*}_{ts}) = \tilde{c}^{*}_{aa}\gamma^H_{aa} + \tilde{c}^{*}_{au}\gamma^H_{au} + \tilde{c}^{*}_{ua}\gamma^H_{ua} + \tilde{c}^{*}_{uu}\gamma^H_{uu} = \bar{u} + V(\lambda^H).$$

Similarly, we can calculate the true expected compensation from the GAPE_1

contract

$$
\begin{aligned}
E(\tilde{c}^{\dagger}_{ts}) &= \tilde{c}^{\dagger}_{aa}\gamma^H_{aa} + \tilde{c}^{\dagger}_{au}\gamma^H_{au} + \tilde{c}^{\dagger}_{ua}\gamma^H_{ua} + \tilde{c}^{\dagger}_{uu}\gamma^H_{uu} \\
&= \bar{u} + V(\lambda^H) + \gamma^H_{aa}\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u \frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \\
&\qquad - (\gamma^H_{au} + \gamma^H_{uu})\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a\Gamma^H_u \frac{(\tilde{P}_{aa} - \tilde{P}_{ua})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right) \\
&\qquad\qquad - \gamma^H_{ua}\left(\frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_a \frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})}{\tilde{P}_{au}\Gamma^H_a + \tilde{P}_{uu}(\tilde{P}_{aa} - \Gamma^H_a)}\right).
\end{aligned}
$$

Hence, for the GAPE_1 to be a Pareto Improvement over the GBPE, the extra terms in $E(\tilde{c}^{\dagger}_{ts})$ beyond $\bar{u}$ and $V(\lambda^H)$ must be positive. Given the common components, it is possible to show that this happens when

$$
\gamma^H_{aa}(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) - (\gamma^H_{au} + \gamma^H_{uu})(\tilde{P}_{aa} - \tilde{P}_{ua})\Gamma^H_a - P_{ua}\Gamma^H_a(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}) > 0,
$$

which can be rearranged to yield

$$
b_a < b_u \frac{P_{au}}{P_{uu}\Gamma^H_A}. \tag{43}
$$

A Pareto Improving GAPE_1 can be derived, for example, setting $P_{aa} = 0.55$, $P_{uu} = 0.8$, $\Gamma^H_a = 0.65$, $b_a = 0.1$ and $b_u = 0.2$ (other parameters are irrelevant).

To prove the same for the GAPE_2, we calculate

$$
\begin{aligned}
E(\tilde{c}^{\dagger'}_{ts}) &= \tilde{c}^{\dagger'}_{aa}\gamma^H_{aa} + \tilde{c}^{\dagger'}_{au}\gamma^H_{au} + \tilde{c}^{\dagger'}_{ua}\gamma^H_{ua} + \tilde{c}^{\dagger'}_{uu}\gamma^H_{uu} \\
&= \left(\bar{u} + V(\lambda^H)\right)\left[\frac{(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu})P_{aa} + \tilde{P}_{au}(\gamma^H_{au} + \gamma^H_{uu})}{\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{uu}}\right]
\end{aligned}
$$

Hence, for the GAPE$_2$ to be a Pareto Improvement over the GBPE, we solve

$$\left(\bar{u}+V(\lambda^H)\right)\left[\frac{(\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu})P_{aa}+\tilde{P}_{au}(\gamma^H_{au}+\gamma^H_{uu})}{\tilde{\gamma}^H_{au}+\tilde{\gamma}^H_{uu}}\right] > \bar{u}+V(\lambda^H)$$

which can be rearranged to yield

$$b_a < b_u \frac{P_{au}}{P_{uu}}. \tag{44}$$

A Pareto Improving GAPE$_2$ can be derived setting, for example, $P_{aa} = 0.3$, $P_{uu} = 0.9$, $\Gamma^H_a = 0.9$, $\Gamma^H_u = 0.4$, $V(\lambda^L) = 0.5$, $V(\lambda^H) = 6$, $\bar{u} = 3$, $b_a = 0.5$ and $b_u = 0.66$.

C. Perceived Negative Correlation

This Appendix derives the optimal contracts for an agent who displays underprecision and who perceives a negative correlation between the subjective performance evaluations. We conclude with a welfare analysis of the contracts derived.

Consider an agent who displays underprecision and who believes signals are negatively correlated, that is, he holds beliefs such that

$$b_a < 0 \text{ and } -\frac{\gamma^G_a}{\gamma^G_u} b_a < b_u < -\frac{\gamma^B_a}{\gamma^B_u} b_a \text{ and } b_u - b_a > P_{aa} - P_{ua}.$$

In this case, the compensation scheme of the contract has to subdue to different properties in order to ensure truthful reporting.

Lemma 36. *If the agent believes signals are negatively correlated, that is,* (3) *fails to hold, then any optimal contract implementing high effort features either (i)* $c_{aa} = c_{au}$ *and* $c_{uu} = c_{ua}$ *or (ii)* $c_{aa} < c_{au}$ *and* $c_{uu} < c_{ua}$.

Proof. When the agent believes signals are negatively correlated, that is, $\tilde{\gamma}^H_{aa}\tilde{\gamma}^H_{uu} - \tilde{\gamma}^H_{au}\tilde{\gamma}^H_{ua} < 0$, we have:

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} > \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}.$$

That is, the (TR_A) becomes

$$(c_{ua}-c_{uu})\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq (c_{au}-c_{aa}) \leq (c_{ua}-c_{uu})\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}.$$

Where either all brackets are 0 or $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$. ∎

Lemma is the equivalent of 6 for the case of perceived negative correlation between signals. Lemma 6 showed that, if the agent believes signals are positively correlated, then the principal might opt for offering an optimal contract where, taking as given her performance evaluation report, the compensation is higher in the agreement cases than in the disagreement cases, that is, a contract with $c_{aa} > c_{au}$ and $c_{uu} > c_{ua}$. Lemma 7 shows that the opposite happens when the agent believes signals are negatively correlated. In this case the principal might opt for offering a contract where, taking as given her performance evaluation report, the compensation is higher in the disagreement cases than in the agreement cases, that is, a contract with $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$. This follows the exact opposite intuition of Lemma 6.

In order to solve problem (10), when the agent believes signals are negatively correlated, we present a set of Lemmas that select the binding constraints for this case and reduce the problem to the following one.

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \quad c_{aa}\gamma^H_{aa}+w_{au}\gamma^H_{au}+w_{ua}\gamma^H_{ua} \tag{45}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa}+c_{au}\left(\tilde{P}_{au}-\Gamma^H_a\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma^H_u \tag{IC}$$

$$w_{au}\frac{P_{au}}{P_{aa}} \leq (w_{ua}-c_{aa}) \tag{(TR^a_P)}$$

$$(w_{ua}-c_{aa}) \leq w_{au}\frac{P_{uu}}{P_{ua}} \tag{(TR^u_P)}$$

$$c_{au} \geq c_{aa} \tag{(TR^u_A)}$$

$$w_{ua} \geq c_{ua} \geq 0 \tag{LL_{ua}}$$

$$w_{au} \geq c_{au} \geq 0. \tag{LL_{au}}$$

Reducing the problem to (45). Lemma 37 below studies the effect of underprecision on the (TR_A^s) in this case. Intuitively, given Lemma 7, the agent now underestimates the chances of obtaining premium $c_{ua} - c_{uu}$, and overestimates that of obtaining $c_{au} - c_{aa}$. Hence, his incentive to exert λ^L is higher, since $T = u$ is more probable under low effort, and the (IC) tightens.

Lemma 37. *If the optimal contract implementing high effort features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then underprecision relaxes the (TR_A^s).*

Proof. Consider the proof to Lemma 14, which is the equivalent version of 37 for the case of positive correlation and overconfidence. Notice that if the optimal contract features $c_{aa} < c_{au}$ and $c_{uu} < c_{ua}$, then the third term in the LHS of (19) is positive and the third term in the RHS of (19) is negative. Furthermore, the third term in the LHS of (20) is positive and the third term in the RHS of (20) is negative. Hence, underconfidence relaxes the (TR_A^s). ∎

Lemma 38. *If the agent believes signals are negatively correlated, that is* (3) *fails, then the optimal contract implementing high effort features $c_{au} > c_{uu} = 0$.*

Proof. Recall that any optimal contract with truthful reporting for an agent who believes signals are negatively correlated satisfies either case (i) or case (ii) of Lemma 7. Assume case (i) of Lemma 7 holds, then the (IC) becomes:

$$c_{au} \underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{>0} + c_{uu} \underbrace{(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})}_{<0} \geq \Delta V.$$

Because of the negative sign of the second bracket, and since $\Delta V > 0$ and $c_{uu} \geq 0$, the above requires $c_{au} > 0$ to always hold. Assume now case (ii) of Lemma 7 holds, for a similar argument, we need at least one between c_{aa} and c_{au} to be positive. If $c_{au} > 0$, the Lemma is trivially proven. If $c_{aa} \geq 0$, case (ii) implies $c_{au} > c_{aa} \geq 0$. This proves the first part of Lemma 38, i.e. that $c_{au} > 0$.

To prove the second part of Lemma 38, i.e. that $c_{uu} = 0$ and therefore smaller that c_{au}, we suppose it is false, that is, at optimum the contract features $c_{uu} > 0$, and we prove that there exists a profitable deviation from it, which contradicts its optimality. From Lemma 7, we know that $c_{ua} \geq c_{uu}$ and also $c_{au} \geq c_{aa}$. The proof now depends on whether $c_{aa} > 0$ or $c_{aa} = 0$.

Let $c_{aa} > 0$. Let the principal decrease both c_{uu} and c_{ua} by ϵ so that their difference remains constant (so not to affect the (TR_A) constraints). From the rearrangement of the constraint above, we see that both c_{uu} and c_{ua} enter negatively in the LHS of the (IC). Hence, decreasing them, would relax the (IC) rather than tightening it. In particular, the LHS of the (IC) constraint has increased by $-\epsilon(\Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu})$. Since we are in the case where $c_{aa} > 0$, the principal can also decrease both c_{aa} and c_{au} by ϵ. In this way, the overall change in the LHS of the (IC) is given by:

$$\begin{aligned}
&-\epsilon\left(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au} + \Delta\tilde{\gamma}_{ua} + \Delta\tilde{\gamma}_{uu}\right) \\
&\quad = -\epsilon\left(\tilde{P}_{aa}\Delta\Gamma_a + \tilde{P}_{au}\Delta\Gamma_a + \tilde{P}_{ua}\Delta\Gamma_u + \tilde{P}_{uu}\Delta\Gamma_u\right) \\
&\quad = -\epsilon\left(\Delta\Gamma_a + \Delta\Gamma_u\right) = -\epsilon\left(\Delta\Gamma_a - \Delta\Gamma_a\right) = 0,
\end{aligned}$$

and therefore the (IC) binds again.

Finally, since both c_{ua} and c_{aa} have been decreased by ϵ, then the principal can decrease also w_{ua} and w_{aa} by the same amount. This does not violate the relevant (LL_{ts}) and holds their difference constant. Hence, it does not violate any of the (TR_P) constraints. This new contract $\{w_{ts}, c_{ts}\}_{t,s}$ implements high effort at a lower cost. Hence, a contract where $c_{uu} > 0$ and $c_{aa} > 0$ cannot be the solution to the problem.

Let now, instead, the optimal contract feature $c_{aa} = 0$ and define $\Delta c_u = c_{ua} - c_{uu}$. We divide the proof for this case in three steps.

Step 1. When $c_{aa} = 0$, the (TR_A) imply:

$$\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} \leq c_{au} \leq \Delta c_u \frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}}, \tag{46}$$

where, since we are in case (ii) of Lemma 7, either only one of the two inequalities holds as equality, or none. Suppose none of the two does, or only the second one, the principal can decrease both c_{ua} and c_{uu} by ϵ keeping Δc_u constant, relaxing the (IC) constraint. In particular, the LHS of the (IC) has decreased by $\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})<0$. He can then decrease c_{au} by $\delta \equiv \frac{\epsilon(\Delta\tilde{\gamma}_{ua}+\Delta\tilde{\gamma}_{uu})}{\Delta\tilde{\gamma}_{au}}$ bringing the LHS of the (IC) back to its original value. Clearly for some ϵ, this deviation can be done until the first inequality in (46) binds. Finally, to see that this is optimal for the principal, notice that according to the (LL) constraints, she can now decrease w_{uu} up to ϵ and w_{au} up to δ. By decreasing both by $\min\{\epsilon,\delta\}$, their difference does not change. Hence, (TR_P) constraints are not affected while the objective function decreases. This implies that at optimum if $c_{aa}=0$, the first inequality of (46) binds.

Step 2. Given that $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$ must hold at optimum if $c_{aa}=0$, we now show that the principal has at her disposal the following optimal deviation from a contract with $c_{aa}=0$ and $c_{uu}>0$. Let her decrease c_{ua} by ϵ and c_{uu} by $\epsilon_0<\epsilon$. Then Δc_u has decreased by $(\epsilon-\epsilon_0)$. In order to keep $\Delta c_u \frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} = c_{au}$, the principal decreases c_{au} by $(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}$. It remains to check if this deviation can be made in such a way that it does not violate the (IC). The change in the (IC) is:

$$
\begin{aligned}
& -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}}\Delta\tilde{\gamma}_{au} - \epsilon\Delta\tilde{\gamma}_{ua} - \epsilon_0\Delta\tilde{\gamma}_{uu} \\
= \;& -(\epsilon-\epsilon_0)\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\Delta\Gamma_a + \epsilon\tilde{P}_{ua}\Delta\Gamma_a + \epsilon_0\tilde{P}_{uu}\Delta\Gamma_a \\
= \;& \Delta\Gamma_a\left[\epsilon\left(\tilde{P}_{ua} - \frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a}\right) + \epsilon_0\left(\frac{\tilde{\gamma}^H_{uu}}{\Gamma^H_a} + \tilde{P}_{uu}\right)\right] \\
= \;& \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon\left(\tilde{P}_{ua}\Gamma^H_a - \tilde{P}_{uu}\Gamma^H_u\right) + \epsilon_0\left(\tilde{P}_{uu}\Gamma^H_u + \tilde{P}_{uu}\Gamma^H_a\right)\right] \\
= \;& \frac{\Delta\Gamma_a}{\Gamma^H_a}\left[\epsilon(\Gamma^H_a - \tilde{P}_{uu}) + \epsilon_0\tilde{P}_{uu}\right],
\end{aligned}
$$

which is positive when:

$$\epsilon\left(\Gamma^H_a - \tilde{P}_{uu}\right) + \epsilon_0\tilde{P}_{uu} > 0.$$

If $\Gamma^H_a > \tilde{P}_{uu}$, the above is always true. If, instead, $\Gamma^H_a < \tilde{P}_{uu}$ then the principal has to choose $\epsilon \in \left\{\epsilon_0, \epsilon_0 \frac{\tilde{P}_{uu}}{\tilde{P}_{uu}-\Gamma_a^H}\right\}$.

Step 3. To conclude, given the decreases in the c_{ts}, the principal can now decrease w_{uu} up to ϵ_0 and w_{au} up to $(\epsilon - \epsilon_0)\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H}$. By an argument similar to the one of Step 1, she can decrease both by the smallest of the two limits, decreasing the objective function. This provides the desired contradiction.

Finally, since a contract where $c_{uu} > 0$ and $c_{aa} \geq 0$ cannot be a solution to the problem it follows that $c_{uu} = 0$. This concludes the proof of the Lemma. ∎

Lemma 39. *If the agent believes signals are negatively correlated, that is* (3) *fails, then constraint* (TR_A^a) *always binds in any optimal contract implementing high effort. Therefore:*

$$c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}(c_{au} - c_{aa}).$$

Proof. Suppose not. Given the Lemmas proven till now and that the (TR_A^a) is slack, the problem that the principal faces is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H + w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \tag{47}$$

$$\text{s.t.} \quad c_{aa}\Delta\tilde{\gamma}_{aa} + c_{au}\Delta\tilde{\gamma}_{au} + c_{ua}\Delta\tilde{\gamma}_{ua} - \Delta V \geq 0 \tag{IC}$$

$$w_{aa}\gamma_{aa}^H + w_{au}\gamma_{au}^H \leq w_{ua}\gamma_{aa}^H + w_{uu}\gamma_{au}^H \tag{TR_P^a}$$

$$w_{ua}\gamma_{ua}^H + w_{uu}\gamma_{uu}^H \leq w_{aa}\gamma_{ua}^H + w_{au}\gamma_{uu}^H \tag{TR_P^u}$$

$$c_{aa}\tilde{\gamma}_{aa}^H + c_{ua}\tilde{\gamma}_{ua}^H \geq c_{au}\tilde{\gamma}_{aa}^H \tag{TR_A^a}$$

$$c_{au}\tilde{\gamma}_{au}^H \geq c_{aa}\tilde{\gamma}_{au}^H + c_{ua}\tilde{\gamma}_{uu}^H \tag{TR_A^u}$$

$$w_{ts} \geq c_{ts} \geq 0 \qquad \forall t, s \in \{a, u\}. \tag{LL_{ts}}$$

and we can rewrite the (TR_P) and (TR_A) constraints as

$$(w_{au} - w_{uu})\frac{\gamma_{au}^H}{\gamma_{aa}^H} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{\gamma_{uu}^H}{\gamma_{ua}^H} \tag{(TR_P)}$$

$$c_{ua}\frac{\tilde{\gamma}_{uu}^H}{\tilde{\gamma}_{au}^H} \leq (c_{au} - c_{aa}) \leq c_{ua}\frac{\tilde{\gamma}_{ua}^H}{\tilde{\gamma}_{aa}^H}. \tag{(TR_A)}$$

The principal can then decrease c_{ua} by ϵ, such that the (TR_A^a) still holds, and c_{au} and c_{aa} by $\epsilon\tilde{P}_{ua}$. Since the difference $c_{au} - c_{aa}$ is constant, the (TR_A) still hold. The (IC) is invariant, since its LHS has changed by

$$-\epsilon\tilde{P}_{ua}\underbrace{(\Delta\tilde{\gamma}_{aa} + \Delta\tilde{\gamma}_{au})}_{\Delta\Gamma_a} - \epsilon\Delta\tilde{\gamma}_{ua} = \Delta\Gamma_a(\epsilon\tilde{P}_{ua} - \epsilon\tilde{P}_{ua}) = 0.$$

We are now left to show that this is optimal for the principal. Notice that both c_{ua} and c_{aa} have decreased. Hence, the principal can decrease both w_{ua} and w_{aa} by $\epsilon\tilde{P}_{ua}$. This does not violate (TR_P) and decreases the objective function, providing the desired contradiction. ∎

Given this, we can rewrite the (IC) as

$$\begin{aligned}
&c_{aa}\left(\Delta\tilde{\gamma}_{aa} - \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\Delta\tilde{\gamma}_{ua}\right) + c_{au}\left[\Delta\tilde{\gamma}_{au} + \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\Delta\tilde{\gamma}_{ua}\right] - \Delta V > 0 \\
&c_{aa}\left(\tilde{P}_{aa}\Gamma_u^H + \tilde{P}_{aa}\Gamma_a^H\right) + c_{au}\left(\tilde{P}_{au}\Gamma_u^H - \tilde{P}_{aa}\Gamma_a^H\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \\
&c_{aa}\tilde{P}_{aa}(\Gamma_a^H + \Gamma_u^H) + c_{au}\left[\tilde{P}_{au}(1 - \Gamma_a^H) - \tilde{P}_{aa}\Gamma_a^H\right] > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \\
&c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H
\end{aligned}$$

Lemma 40. *If the agent believes signals are negatively correlated, that is* (3) *fails, then* (LL_{aa}) *and* (LL_{uu}) *bind in any optimal contract implementing high effort, that is,* $w_{aa} = c_{aa}$ *and* $w_{uu} = 0$.

Proof. Consider the (TR_P)

$$\underbrace{(w_{au} - w_{uu})\frac{P_{au}}{P_{aa}}}_{LHS} \leq \underbrace{(w_{ua} - w_{aa})}_{\text{middle term}} \leq \underbrace{(w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}}}_{RHS}$$

Start from (LL_{aa}). Suppose it does not bind. Then the principal can increase w_{au} by ϵ and decrease w_{aa} by $\epsilon_1 \equiv \epsilon\frac{P_{uu}}{P_{ua}}$. The values in (TR_P) change. The RHS increases by ϵ_1. The middle term also increases by ϵ_1. The LHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. To see that the LHS stays lower than the middle term, notice that

$$\epsilon\frac{P_{au}}{P_{aa}} \leq \epsilon\frac{P_{uu}}{P_{ua}} \quad \text{since} \quad P_{au}P_{ua} < P_{aa}P_{uu},$$

by Assumption 2. This creates an overall effect on the objective function given by

$$\epsilon\gamma^H_{au} - \epsilon\frac{P_{uu}}{P_{ua}}\gamma^H_{aa} = \epsilon\frac{1}{P_{ua}}\left(\gamma^H_{au}P_{ua} - \gamma^H_{aa}P_{uu}\right) = \epsilon\frac{1}{P_{ua}}(P_{au}P_{ua} - P_{aa}P_{uu})\Gamma^H_a < 0.$$

Hence, this deviation contradicts the optimality of $w_{aa} > c_{aa}$.

For the (LL_{uu}) we follow the same logic. Suppose it does not bind. The principal can decrease w_{uu} by ϵ and increase w_{ua} by $\epsilon_1 \equiv \epsilon\frac{P_{au}}{P_{aa}}$. The values in (TR_P) change. The RHS increases by $\epsilon\frac{P_{au}}{P_{aa}}$. The middle term increases by ϵ_1. The LHS increases also by ϵ_1. To see that the RHS stays larger than the middle term, notice that

$$\epsilon\frac{P_{au}}{P_{aa}} \leq \epsilon\frac{P_{uu}}{P_{ua}}$$

as above. This creates an overall effect on the objective function given by

$$-\epsilon\gamma^H_{uu} + \epsilon\frac{P_{au}}{P_{aa}}\gamma^H_{ua} = \epsilon\frac{1}{P_{aa}}\left(\gamma^H_{ua}P_{au} - \gamma^H_{uu}P_{aa}\right) = \epsilon\frac{1}{P_{aa}}(P_{au}P_{ua} - P_{aa}P_{uu})\Gamma^H_u < 0.$$

Hence this deviation contradicts the optimality of $w_{uu} > c_{uu}$. ∎

This concludes the set of Lemmas that yield us problem (45). The study of its solution is longer and more complicated than the solutions to (11). First of all, notice that this time we have $c_{uu} = 0$ (Lemma 38) instead of $c_{ua} = 0$ (as in Lemma 11). When the agent believes signals to be negatively correlated the agreement payoffs (that is the ones following $T = S$) are now surprising for an agent. In particular if the agent observes $S = u$, he believes that the principal has observed $T = a$, and is not easily convinced that $T = u$ instead.

The first implication of the above, is that the Lemma 7 does no longer hold. That is, there exist values for the bias of an agent who displays underprecision and who believes signals are negatively correlated such that the existence of a deadweight loss is no longer a necessary condition for the implementation of high effort. Further, as we show later, there exists a portion of the parameter space where the optimal contract does not, in fact, feature any deadweight loss.

In terms of contract structure, we show how the optimal contract when the agent displays underprecision and believes signals are negatively correlated can take two new forms: the Disagreement Performance Evaluation Deadweight Loss (DPE-DL) contract and the Disagreement Performance Evaluation No Deadweight Loss (DPE-NDL) contract. We are going to present each new contract in a proposition. While when the agent believes that signals are negatively correlated the existence of a deadweight loss is no longer a necessary condition for the implementation of high effort, this can still be optimal as the next result shows.

Proposition 9. *If the agent displays underprecision, believes signals are negatively correlated, and has beliefs that satisfy*

$$b_u P_{au} \Gamma_u^H - b_a P_{aa} \Gamma_a^H \geq P_{aa}^2 \Gamma_a^H - P_{au} P_{ua} \Gamma_u^H, \tag{48}$$

and

$$b_a < -P_{aa} \Gamma_A^H, \tag{49}$$

then he is offered a DPE-DL contract $\{\hat{w}_{ts}, \hat{c}_{ts}\}$ *given by:*

$$\hat{w}_{aa} = 0 \quad \hat{w}_{au} = \hat{c}_{au} \qquad \hat{w}_{uu} = 0 \quad \hat{w}_{ua} = \frac{P_{au}}{P_{aa}}\hat{c}_{au}$$
$$\hat{c}_{aa} = 0 \quad \hat{c}_{au} = \frac{\Delta V}{\Delta \Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} \quad \hat{c}_{uu} = 0 \quad \hat{c}_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\hat{c}_{au}.$$

The DPE-DL contract features:

(i) a wage and compensation that depend on both parties' performance evaluation reports;

(ii) a deadweight loss when the principal reports an unacceptable performance and the agent reports an acceptable performance (unless (48) *holds with equality);*

(iii) a wage and a compensation only when the parties report misaligned performance evaluations.

Proof. See the proof of Proposition 10. ∎

Proposition 7 shows that if an agent who displays underprecision, believes signals are negatively correlated, and has a "large" bias, then the optimal contract is very different from the BPE contract described in Proposition 1. First, the principal's wage cost and the agent's compensation depend on both parties' performance evaluations. Second, wage and compensation are positive only when the parties disagree on their performance evaluations. Next, we provide the economic intuition behind the DPE-DL contract.

When the agent believes that signals are negatively correlated, two very similar and connected effects take place: (i) he believes states (a, u) and (u, a) more probable than (a, a) and (u, u) (at least jointly); (ii) his believes the most probable events are the symmetric opposite of the ones believed by the principal. Hence, it is straightforward to see why an agent who displays underprecision and believes signals are negatively correlated never accepts a APE contract. First, he rarely expects to obtain $c_{aa}^{\dagger}$. Second, he is not willing any longer to accept a contract that features $c_{ua} = 0$. Similarly to the APE contract, the principal can take advantage of this in order to decrease the expected wage paid. She can wager on the misalignment of beliefs by increasing (decreasing) the compensation of the agent in states he wrongly deems more (less) probable than she does. Because of (ii), the principal is therefore happy to offer the agent a positive c_{ua} and a larger c_{au} (compared to the BPE case) in exchange for a lower c_{aa} and/or c_{uu}. The fact that they no longer disagree only on the extent of the correlation, but also on the direction of it, opens up to a stronger manipulation of the standard contract, compared to the switch from BPE to APE. This is behind the result that $c_{aa} = c_{uu} = 0$.

Obviously, the above has to be feasible, that is, the agent has to be biased enough to accept such a manipulation compared to the BPE contract, and optimal, that is, the DPE contract has to implement high effort at a lower cost. These are precisely the meanings of condition (48) and (49). Condition (48) requires the agent to be biased enough to accept a DPE contract while (49) requires the agent to be biased enough for the principal to find it optimal to offer a DPE contract. To better understand the meaning of, and intuitions behind the conditions, let us represent them in (b_a, b_u) space. Figure 11 represents the two conditions for an agent who displays underprecision when $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.[35]

When (49) holds, the bias of the agent is such that the principal has the incentive to wager as described above. In other words, the agent perceives a negative enough correlation for the principal to be able to offer a new contract with a lower c_{aa} and higher c_{au} which is accepted by the agent.[36]

Before going ahead, note that a key aspect of this contract lies in the full implementation of both performance evaluation reports. The disagreement about the correlation between the signals allows the principal to design contracts that take ad-

[35]If condition (49) holds with equality, the slopes of the (IC) and isocosts are identical and the problem has many solutions. At this point of indifference, we assume the principal sets up a DPE-DL contract. In any case, given that all our parameters are continuous, the probability of (49) holding with equality is zero.

[36]To see why this is reflected in the Figure, consider the area where the DPE-DL is set. Notice that it is at its largest when b_u is large and b_a is low. This is precisely when the disagreement on correlation is at its maximum, since $\tilde{P}_{aa}$ is close to zero and $\tilde{P}_{uu}$ is at its minimum.

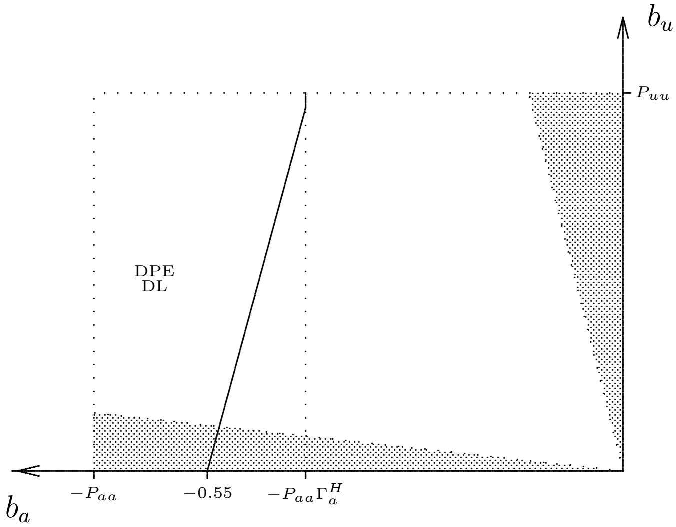

Figure 11: In the Figure, we represent conditions (48) and (49). Together they define the area where an agent who displays underprecision is assigned a DPE-DL contract. The shaded area rule out biases that fall outside our definition of underprecision. In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

vantage of both information sources. This is confirmed in the next type of contract as well. From Holmström's (1979) informativeness principle and its extension to risk neutrality and limited liability due to Chaigneau et al. (2019) we know that, since T is a sufficient statistic for the pair T, S with respect to effort, the fact that the contract fully implements both sources does not add any informational value to the transaction. If anything it may even decrease the correlation between the actual performance of the project and the resulting w and c.

As we mentioned already, the result of Lemma 7 does not necessarily hold in the presence of an agent who (wrongly) believes signals to be negatively correlated. This is originated by the disagreement on the direction of the correlation.[37] To see

[37] It would, in fact, still hold if the true correlation were negative and the agent and principal agreed on it.

this, suppose the agent observes $S = a$ and that he believes signals to be negatively correlated. Clearly the agent would be very happy to hear the principal reporting $T = a$, but what happens if the principal reports $T = u$? On the one hand, the agent is upset because the principal deems his performance unacceptable, and therefore would like to punish her in general. On the other hand, however, the agent expects $T = u$ because she believes signals to be negatively correlated! So he is less prone to punish the principal because he is more convinced that $T = u$ is indeed the true realization. As already explained above, the principal takes advantage of this by setting $c_{aa} = c_{uu} = 0$. When the agent reports $S = a$, he knows that if the principal reports $T = a$, he will get no compensation at all. This makes the agent (i) willing to report $S = a$ only when it is indeed true, (ii) less prone to punish the principal compared to the positive correlation case. Under some particular levels of bias, this effect is so strong that the presence of a deadweight loss case in the contract is no longer a necessary condition for its implementation. This is highlighted in the following proposition.

Proposition 10. *If the agent displays underprecision, believes signals are negatively correlated, and has beliefs that violate* (48) *but satisfy*

$$b_a \leq -P_{aa}\left(\frac{P_{ua} + b_u\Gamma_a^H}{P_{ua}(1+\Gamma_u^H) + b_u}\right) \tag{50}$$

then he is offered a DPE-NDL contract $\hat{w}'_{ts}, \hat{c}'_{ts}$ given by:

$$\begin{array}{llll} \hat{w}'_{aa} = 0 & \hat{w}'_{au} = \hat{c}'_{au} & \hat{w}'_{uu} = 0 & \hat{w}'_{ua} = \hat{c}'_{ua} \\ \hat{c}'_{aa} = 0 & \hat{c}'_{au} = \frac{\Delta V}{\Delta \Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & \hat{c}'_{uu} = 0 & \hat{c}'_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\hat{c}'_{au}. \end{array}$$

The DPE-NDL contract features:

(i) a wage and compensation that depend on both parties' performance evaluation reports;

(ii) no deadweight loss;

(iii) a wage and a compensation only when the parties report misaligned performance evaluations.

Proof. To prove both Proposition 7 and 8, we are going to start from problem (45). At optimum, it is of course true that either (TR_P^a) or (TR_P^u) bind, or both, or neither. Since, however, $P_{aa}P_{uu} - P_{au}P_{ua} > 0$ and $w_{uu} = 0$, the only way to have both binding would be for $w_{ua} = w_{aa}$ and $w_{au} = 0$. From Lemma 38, we know that $c_{au} > 0$. Hence, at least one for the two constraint has to be slack. The proof is quite long and therefore we divide it in several parts. First, we assume only the (TR_P^a) binds, which is further split into two cases on the basis of the sign of the slope of the (IC). Second, we assume no (TR_P) binds. Finally, we assume only the (TR_P^u) binds.

Constraint (TR_P^a) binding Suppose the optimal contract sets the (TR_P^a) binding. We then have

$$w_{ua} = w_{au}\frac{P_{au}}{P_{aa}} + c_{aa},$$

which results in the following objective function

$$c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right).$$

Since w_{au} has a clear positive effect on it and the only constraint left on w_{ua} is (LL_{au}), we have that $w_{au} = c_{au}$. We can further simplify the objective function

$$\begin{aligned}
&c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + w_{au}\left(\gamma_{au}^H + \frac{P_{au}}{P_{aa}}\gamma_{ua}^H\right) \\
&= c_{aa}\left(\gamma_{aa}^H + \gamma_{ua}^H\right) + c_{au}\frac{P_{au}}{P_{aa}}\left(P_{aa}\Gamma_a^H + P_{ua}\Gamma_u^H\right) \\
&= \left(\gamma_{aa}^H + \gamma_{ua}^H\right)\left[c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}\right],
\end{aligned}$$

which is equivalent to minimizing:

$$c_{aa} + c_{au}\frac{P_{au}}{P_{aa}}.$$

This implies that iso-costs have slope $-P_{au}/P_{aa} < 0$.

On the other hand, the (IC) is not necessarily negatively sloped. Its slope is given by

$$-\frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}},$$

which is negative only if $b_a < P_{au} - \Gamma_a^H$.

The reduced problem for this case is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} c_{aa}+c_{au}\frac{P_{au}}{P_{aa}} \tag{51}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa}+c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa} \tag{(TR_A^u)}$$

Positively Sloped (IC). Suppose $b_a > P_{au} - \Gamma_a^H$, the slope of the (IC) is positive and smaller than 1.[38] To see this, notice that

$$\Gamma_a^H - \tilde{P}_{au} < \tilde{P}_{aa} \Rightarrow \Gamma_a^H - 1 < 0,$$

which is always true. Hence, the binding constraints can be represented in (c_{au}, c_{aa}) space as in Figure 12.

In the Figure, costs decrease towards the origin of the graph. The shaded area represents the set of contracts satisfying all constraints and the optimal contract is therefore at point Y. At Y, $c_{au} = c_{aa} > 0 = c_{ua}$. We derive the full contract below in Lemma 41 and show that it is equivalent to the BPE contract.

[38]Note that since $b_A < 0$ for an underconfident agent, this may never be the case when $P_{au} > \Gamma_a^H$.

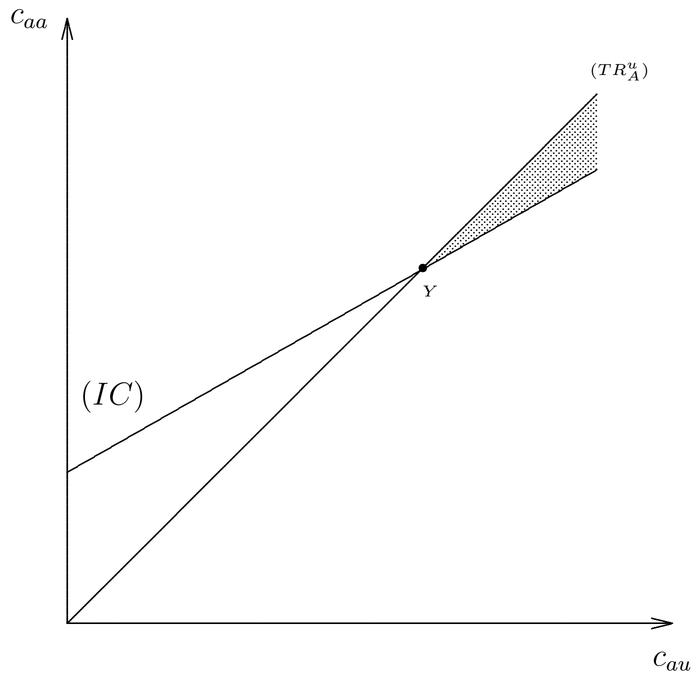

Figure 12: The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (45), when the (IC) is positively sloped and the agent believes signals are negatively correlated.

Negatively Sloped (IC). Now suppose that $b_a < P_{au} - \Gamma^H_a$.[39] The problem can be represented as in Figure 7 below.

Once again, costs decrease towards the origin, but whether the minimum point lies at Y or X depends on the comparison between the slope of the (IC) and the one of the iso-costs. In particular, the minimum lies at X if iso-costs are flatter than the

[39]Notice that this restriction may always hold for an under confident agent if $P_{au} > \Gamma^H_a$ since $b_A > 0$.

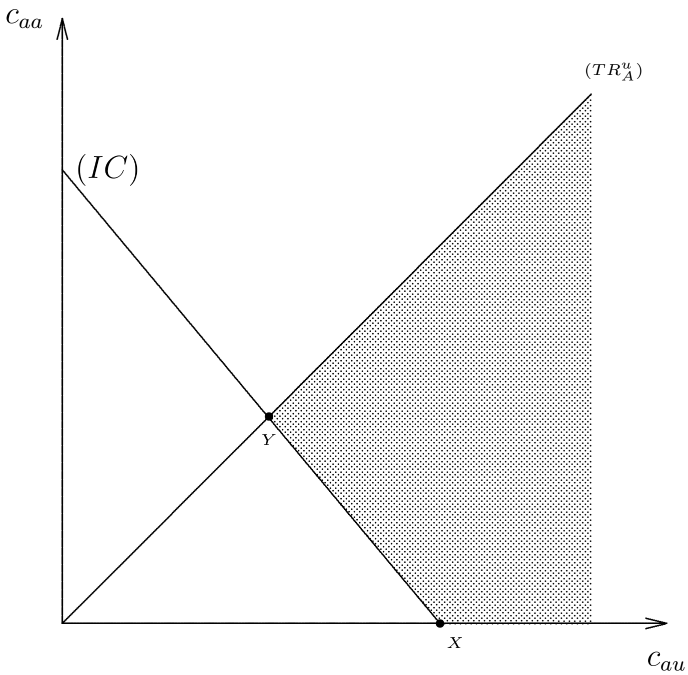

Figure 13: The shaded area represents the set of contracts satisfying all the constraints of the minimization problem (45), when the (IC) is negatively sloped and the agent believes signals are negatively correlated.

(IC). This happens when

$$\begin{aligned}
&\frac{P_{au}}{P_{aa}} \leq \frac{\tilde{P}_{au} - \Gamma_a^H}{\tilde{P}_{aa}} \\
&P_{au}\tilde{P}_{aa} \leq \tilde{P}_{au}P_{aa} - \Gamma_a^H P_{aa} \\
&b_a P_{au} + b_a P_{aa} \leq P_{au}P_{aa} - \Gamma_a^H P_{aa} - P_{au}P_{aa} \\
&b_a(P_{au} + P_{aa}) \leq -\Gamma_a^H P_{aa} \\
&b_a \leq -P_{aa}\Gamma_a^H.
\end{aligned} \tag{52}$$

Notice that (i) $-P_{aa}\Gamma_a^H < P_{au} - \Gamma_A^H$, and hence that (52) implies the negative slope of the (IC), but also that for an overconfident agent b_a will always be larger than $-P_{aa}\Gamma_a^H$ and therefore the optimal contract will never lie at X in figure 7. This implies that when (TR_P^a) binds, so do the IC and the (TR_A^u). This allows us to derive the optimal contract for this case.

Lemma 41. *If the agent believes signals are negatively correlated, i.e.* (3) *fails, the* (TR_P^a) *binds, and* (49) *holds, then the optimal contract implementing high effort is given by:*

$$\begin{array}{llll} \hat{w}_{aa}=0 & \hat{w}_{au}=\hat{c}_{au} & \hat{w}_{uu}=0 & \hat{w}_{ua}=\frac{P_{au}}{P_{aa}}\hat{c}_{au} \\ \hat{c}_{aa}=0 & \hat{c}_{au}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H} & \hat{c}_{uu}=0 & \hat{c}_{ua}=\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\hat{c}_{au}. \end{array}$$

If (49) *fails, the optimal contract is a BPE contract*

$$\begin{array}{llll} w_{aa}=c_{au} & w_{au}=c_{au} & w_{uu}=0 & w_{ua}=\frac{c_{au}}{P_{aa}} \\ c_{aa}=c_{au} & c_{au}=\frac{\Delta V}{\Delta\Gamma_a} & c_{uu}=0 & c_{ua}=0. \end{array}$$

Proof. Let (49) fail and substitute $c_{aa}=c_{au}$ from the binding (TR_A^u), as per the graphical analysis above, into the, again, binding (IC) and notice that

$$c_{au}(\tilde{P}_{aa}+\tilde{P}_{au}-\Gamma_a^H)=\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u$$

implies

$$c_{au}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u}{(1-\Gamma_a^H)}=\frac{\Delta V}{\Delta\Gamma_a}.$$

For w_{ua}, notice that

$$w_{ua}=c_{au}\frac{P_{au}}{P_{aa}}+c_{aa}=c_{au}\left(\frac{P_{au}}{P_{aa}}+1\right)=c_{au}\left(\frac{P_{au}+P_{aa}}{P_{aa}}\right)=\frac{c_{au}}{P_{aa}}$$

Now let 49 hold and notice that from Figure $c_{aa}=0$, c_{ua} follows from Lemma 39 and c_{au} comes from

$$c_{au}(\tilde{P}_{au}-\Gamma_a^H)=\frac{\Delta V}{\Delta\Gamma_a}\Gamma_u \Rightarrow c_{au}=\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u}{\tilde{P}_{au}-\Gamma_a^H}.$$

Notice that given the value of w_{ua}, it is not straightforward whether (LL_{ua}) holds or not. Hence, we have

$$\begin{aligned}
& w_{ua} \geq c_{ua} \\
& \frac{P_{au}}{P_{aa}} c_{au} \geq \frac{\tilde{\gamma}_{aa}^{H}}{\tilde{\gamma}_{ua}^{H}} c_{au} \\
& P_{au} \tilde{P}_{ua} \Gamma_{u}^{H} \geq P_{aa} \tilde{P}_{aa} \Gamma_{a}^{H} \\
& P_{au} P_{ua} \Gamma_{u}^{H} - P_{aa}^{2} \Gamma_{a}^{H} + b_{u} P_{au} \Gamma_{u}^{H} - b_{a} P_{aa} \Gamma_{a}^{H} \geq 0,
\end{aligned}$$

which generates (48). ∎

No (TR_P) **constraint binding (no deadweight loss contract)** Suppose now all (TR_P) are slack. Then clearly all (LL_{ts}) constraint bind, since the principal wants to decrease the expected wage paid as much as she can, and they are the only constraints preventing her to set the $w_{ts} = 0$. We have

$$w_{ts} = c_{ts}, \quad c_{uu} = 0, \quad c_{ua} = \frac{\tilde{\gamma}_{aa}^{H}}{\tilde{\gamma}_{ua}^{H}} (c_{au} - c_{aa}).$$

Then the principal solves

$$\min_{\{w_{ts}, c_{ts}\}_{t,s \in \{u,a\}}} c_{aa} \gamma_{aa}^{H} + c_{au} \gamma_{au}^{H} + \frac{\tilde{\gamma}_{aa}^{H}}{\tilde{\gamma}_{ua}^{H}} (c_{au} - c_{aa}) \gamma_{ua}^{H} \tag{53}$$

$$\text{s.t.} \quad c_{aa} \tilde{P}_{aa} + c_{au} \left(\tilde{P}_{au} - \Gamma_{a}^{H} \right) > \frac{\Delta V}{\Delta \Gamma_{a}} \Gamma_{u}^{H} \tag{IC}$$

$$c_{au} \geq c_{aa}. \tag{TR_A^u}$$

To derive the sign of the slope of iso-costs note that the objective function can be rearranged to obtain

$$\frac{1}{\tilde{\gamma}^H_{ua}} c_{aa}(\gamma^H_{aa}\tilde{\gamma}^H_{ua} - \tilde{\gamma}^H_{aa}\gamma^H_{ua}) + c_{au}(\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua})$$
$$\propto c_{aa}(P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua}) + c_{au}(P_{au}\tilde{P}_{ua} + \tilde{P}_{aa}P_{ua})$$

which is clearly negatively sloped since $P_{aa}\tilde{P}_{ua} - \tilde{P}_{aa}P_{ua} = b_u P_{aa} - b_a P_{ua} > 0$ for an underconfident agent. Now notice two things

1. If the (IC) is positively sloped, the optimal contract would feature $c_{au} = c_{aa}$. This, however, yields an unfeasible contract, since $c_{au} = c_{aa} \Rightarrow c_{ua} = 0 = w_{ua}$, which violates the (TR^a_P) constraint as

$$c_{au}\frac{P_{au}}{P_{aa}} > -c_{au}.$$

2. If the (IC) is negatively sloped, the constraint of the problem are the same as the ones represented already in Figure 7. When iso-costs are negatively sloped but steeper than the (IC), the optimal point would be at $c_{au} = c_{aa}(\Rightarrow c_{ua} = 0 = w_{ua})$ again and therefore not be feasible, again.

These two observations imply that the only possible feasible contract for this case is one where the negatively sloped iso-costs are flatter than the (IC).[40] This happens

[40]Notice that, since we assumed that the (TR_P) are slack, they cannot be considered as restrictions to the problem. On the contrary, when we assumed the (TR^a_P) binding in the previous case, we made no assumption about the (LL_{ts}) and, therefore, they were considered as potentially binding.

when

$$\begin{aligned}
&\frac{P_{au}\tilde{P}_{ua}+\tilde{P}_{aa}P_{ua}}{P_{aa}\tilde{P}_{ua}-\tilde{P}_{aa}P_{ua}} \leq \frac{P_{au}-b_a-\Gamma_a^H}{P_{aa}+b_a}\\
&\frac{(P_{au}+P_{aa})P_{ua}+b_uP_{au}+b_aP_{ua}}{P_{aa}b_u-b_aP_{ua}} \leq \frac{P_{au}-b_a-\Gamma_a^H}{P_{aa}+b_a}\\
&(P_{ua}+b_uP_{au}+b_aP_{ua})(P_{aa}+b_a) \leq (P_{aa}b_u-b_aP_{ua})(P_{au}-b_a-\Gamma_a^H)\\
&P_{aa}P_{ua}+b_uP_{aa}P_{au}+b_aP_{ua}P_{aa}+b_aP_{ua}+b_ab_uP_{au}+b_a^2P_{ua}\\
&\qquad \leq b_uP_{aa}P_{au}-b_ab_uP_{aa}-b_u\Gamma_a^HP_{aa}-b_aP_{ua}P_{au}+b_a^2P_{ua}+b_a\Gamma_a^HP_{ua}\\
&P_{aa}P_{ua}+b_aP_{ua}+b_aP_{ua}\Gamma_u^H+b_ab_u+b_u\Gamma_a^HP_{aa}\geq 0\\
&b_a(P_{ua}(1+\Gamma_u^H)+b_u) \leq -P_{aa}\left(P_{ua}+b_u\Gamma_a^H\right),
\end{aligned}$$

which generates

$$b_a \leq -P_{aa}\frac{\left(P_{ua}+b_u\Gamma_a^H\right)}{(P_{ua}(1+\Gamma_u^H)+b_u)} \tag{54}$$

When (54) holds, the optimal contract is similar to the DPE-DL derived above with the difference that now w_{ua} derives from the (LL_{ua}) instead of the (TR_P^a) and therefore

$$w_{ua}=c_{ua}=\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}c_{au}.$$

Further checks have to be carried out to be sure that the contract satisfies the (TR_P) constraints. We start from the (TR_P^a) and see that it holds as long as

$$\begin{aligned}
&w_{ua} > \frac{P_{au}}{P_{aa}}w_{au} \;\Rightarrow\; c_{ua} > \frac{P_{au}}{P_{aa}}c_{au}\\
&\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}c_{au} > \frac{P_{au}}{P_{aa}}c_{au} \;\Rightarrow\; \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\frac{P_{aa}}{P_{au}}c_{au} > c_{au}\\
&\frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\frac{P_{aa}}{P_{au}} \geq 1 \;\Rightarrow\; P_{aa}\tilde{\gamma}_{aa}^H - P_{au}\tilde{\gamma}_{ua}^H \geq 0\\
&P_{aa}\tilde{P}_{aa}\Gamma_a^H - P_{au}\tilde{P}_{ua}\Gamma_u^H > 0,
\end{aligned}$$

which yields the opposite of (48). This perfectly separates the parameter space where the DPE contracts are feasible and therefore relieves us from the burden of having to check for optimality between the two.

Now we check for (TR_P^u) to hold:

$$
\begin{aligned}
& w_{ua} \leq \frac{P_{uu}}{P_{ua}} w_{au} \;\Rightarrow\; c_{ua} < \frac{P_{uu}}{P_{ua}} c_{au} \\
& \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au} < \frac{P_{uu}}{P_{ua}} c_{au} \;\Rightarrow\; \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \frac{P_{uu}}{P_{ua}} c_{au} < c_{au} \\
& \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} \frac{P_{uu}}{P_{ua}} \leq 1 \;\Rightarrow\; P_{uu} \tilde{\gamma}_{ua}^H - P_{ua} \tilde{\gamma}_{aa}^H > 0 \\
& P_{uu} P_{ua} \Gamma_u^H - P_{ua} P_{aa} \Gamma_a^H + b_u P_{uu} \Gamma_u^H - b_a P_{ua} \Gamma_a^H > 0 \\
& b_u P_{uu} \Gamma_u^H - b_a P_{ua} \Gamma_a^H > P_{aa} P_{ua} \Gamma_a^H - P_{uu} P_{ua} \Gamma_u^H \\
& b_a < b_u P_{uu} \Gamma_u^H + P_{ua} P_{uu} \Gamma_u^H - P_{ua} P_{aa} \Gamma_a^H,
\end{aligned}
$$

which however always holds when (54) holds. To see this notice that comparing right hand sides one obtains the following

$$
-P_{aa} \frac{\left(P_{ua} + b_u \Gamma_a^H\right)}{\left(P_{ua}(1 + \Gamma_u^H) + b_u\right)} < b_u P_{uu} \Gamma_u^H + P_{ua} P_{uu} \Gamma_u^H - P_{ua} P_{aa} \Gamma_a^H
$$

$$
\begin{aligned}
& b_u^2 P_{uu} \Gamma_u^H + b_u P_{ua} P_{uu} \Gamma_u^H + b_u P_{uu} P_{ua} \Gamma_u^H + b_u P_{uu} P_{ua} (\Gamma_u^H)^2 + P_{uu} P_{ua}^2 \Gamma_u^H \\
& \quad + P_{uu} P_{ua}^2 (\Gamma_u^H)^2 + b_u P_{aa} \Gamma_a^H + P_{aa} P_{ua} \geq b_u P_{aa} P_{ua} \Gamma_a^H + P_{aa} P_{ua}^2 \Gamma_a^H + P_{aa} P_{ua}^2 \Gamma_a^H \Gamma_u^H
\end{aligned}
$$

which is clearly tighter the lower is b_u (since $P_{aa} P_{ua} \Gamma_a^H$ on the RHS is lower than $P_{aa} \Gamma_a^H$ on the LHS). Hence we test whether the inequality holds at $b_u = 0$.[41] This

[41] Notice that this is a very strong assumption as given a value for b_a the floor value of b_u is larger than zero, but it simplifies the calculations a lot.

implies:

$$
\begin{aligned}
&P_{uu}P_{ua}^2\Gamma_u^H + P_{uu}P_{ua}^2(\Gamma_u^H)^2 + P_{aa}P_{ua} - P_{aa}P_{ua}^2\Gamma_a^H(1+\Gamma_u^H) \geq 0\\
&P_{uu}P_{ua}\Gamma_u^H + P_{uu}P_{ua}(\Gamma_u^H)^2 + P_{aa} - P_{aa}P_{ua}(1-\Gamma_u^H)(1+\Gamma_u^H) \geq 0\\
&P_{uu}P_{ua}\Gamma_u^H + P_{uu}P_{ua}(\Gamma_u^H)^2 + P_{aa} - P_{aa}P_{ua} + P_{aa}P_{ua}(\Gamma_u^H)^2 \geq 0\\
&P_{uu}P_{ua}\Gamma_u^H + P_{uu}P_{ua}(\Gamma_u^H)^2 + P_{aa}P_{uu} + P_{aa}P_{ua}(\Gamma_u^H)^2 \geq 0
\end{aligned}
$$

which is clearly true.

Notice that the above contracts are feasible in completely distinct areas (since (48) separates them) and that if all the conditions derived hold, they also “dominate” the BPE, that is, they are optimal.

Constraint (TR_P^u) **binding** Suppose finally that (TR_P^u) binds. We have

$$w_{ua} = w_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}.$$

In this case, the objective function is given by

$$
\begin{aligned}
&c_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}\left(\gamma_{au}^H + \frac{P_{uu}}{P_{ua}}P_{ua}\Gamma_u^H\right)\\
&= c_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + w_{au}\left(\gamma_{au}^H + \gamma_{uu}^H\right).
\end{aligned}
$$

Since w_{au} has a clear positive effect on it, and the only constraint left on w_{au} is (LL_{au}), we have that $w_{au} = c_{au}$. The reduced problem for this case is given by

$$\min_{\{w_{ts},c_{ts}\}_{t,s\in\{u,a\}}} \; c_{aa}(\gamma_{aa}^H + \gamma_{ua}^H) + c_{au}\left(\gamma_{au}^H + \gamma_{uu}^H\right) \tag{55}$$

$$\text{s.t.} \quad c_{aa}\tilde{P}_{aa} + c_{au}\left(\tilde{P}_{au} - \Gamma_a^H\right) > \frac{\Delta V}{\Delta\Gamma_a}\Gamma_u^H \tag{IC}$$

$$c_{au} \geq c_{aa}. \tag{TR_A^u}$$

We can immediately see that iso-costs are always negatively sloped. Suppose first that $b_a > P_{au} - \Gamma_a^H$, than the IC is positively sloped and Figure 12 represents the constraints of the problem. The optimal contract would feature $c_{ua} = 0$ and the contract resemble the one of Lemma 41 with the only difference that

$$w_{ua} = c_{au}\frac{P_{uu}}{P_{ua}} + c_{au} = \frac{c_{au}}{P_{ua}}.$$

However, since $P_{ua} < P_{aa}$ (from Assumption 3) this contract is clearly dominated by the BPE contract in Lemma 41.

If instead the (IC) is negatively sloped, we are, once again, in Figure 7, where a new contract may arise if iso-costs are flatter than the (IC) (if instead they are steeper we have the BPE contract again, for the reasons just explained).

However, we now show that (i) the resulting contract with $c_{aa} = 0$ and (TR_P^u) binding is always dominated by the contract without a deadweight loss derived above when the latter is feasible, (ii) it is always dominated by the contract with (TR_P^a) binding derived above when the latter is feasible. Hence, this new contract, even if optimal given the assumption of (TR_P^u) binding, is never generally optimal and can be ignored.

Let's start from deriving the contract. Notice that the contract lying at point X of Figure 7 for this case is given by

$$\begin{array}{llll} w_{aa} = 0 & w_{au} = c_{au} & w_{uu} = 0 & w_{ua} = \frac{P_{uu}}{P_{ua}} c_{au} \\ c_{aa} = 0 & c_{au} = \frac{\Delta V}{\Delta \Gamma_a} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_a^H} & c_{uu} = 0 & c_{ua} = \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au} \end{array}$$

The calculations follow the same identical derivations of the case of (TR_P^a) binding, save for w_{ua} that follows from $w_{ua} = c_{au}\frac{P_{uu}}{P_{ua}} + c_{aa}$, given the (TR_P^u). Given this, constraint (LL_{ua}) holds if

$$\frac{P_{uu}}{P_{ua}} c_{au} \geq \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H} c_{au},$$

For (i), we compare the average wage cost in both contracts. Without the need of any algebra, we note that the no deadweight loss contract features $w_{ts} = c_{ts}$ for all t and s and the compensations and wages offered by the two contracts are identical, save for w_{ua}. Hence, the only way for the contract with (TR_P^u) binding to grant a lower expected wage cost than the no deadweight loss contract is for it to feature $w_{ts} < c_{ts}$ for some ts, which is unfeasible. Finally, for (ii), notice that the two contracts, again, feature identical c_{ts} and w_{ts}, save for w_{ua}. The contract with (TR_P^a) binding grants a lower average wage cost if

$$\frac{P_{au}}{P_{aa}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right) \leq \frac{P_{uu}}{P_{ua}}\left(\frac{\Delta V}{\Delta\Gamma_a}\frac{\Gamma_u^H}{\tilde{P}_{au}-\Gamma_a^H}\right),$$

which boils down to simply

$$P_{au}P_{ua} - P_{uu}P_{aa} \leq 0,$$

which is always true. This concludes the proof of Propositions 9 and 10. ∎

Proposition 8 shows that there exist DPE contracts without deadweight loss. Identifying the set of parameter values under which a DPE-NDL contract is feasible and optimal is no easy task. In fact, none of the conditions behind it implies any of the others for all parameter values. In Figure 14 below, we plot the area where a DPE-NDL contract is feasible and optimal for an agent who displays underprecision when $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

While condition (50) play the exact same role for a DPE-NDL contract as (48) and (49) do for a DPE-DL contract, the requirement of (48) to fail needs attention. In the proof of Proposition 8, we show how condition (48) determines whether the (LL_{ua}) or the (TR_P^a) is more stringent in problem (45). When (48) holds, the (TR_P^a) is more stringent and the contract must feature a deadweight loss. When it fails, the contract features no deadweight loss. In other words, the failing of condition (48) identifies an area where an agent who disagrees with the principal on the direction of the correlation of signals has a bias such that he is willing to sign a contract without deadweight loss. It is possible to show that the area where a DPE-NDL contract is set may not exist, under some parameter conditions. This implies that the presence

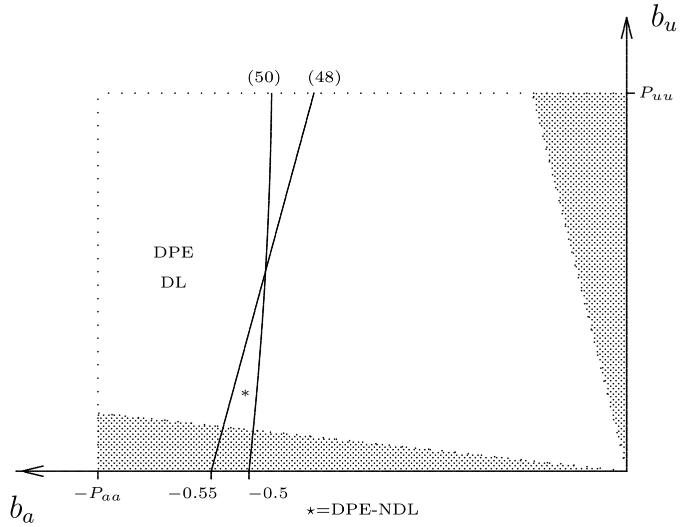

Figure 14: In the Figure we represent conditions (48) and (50). Together they define the area where an agent who displays underprecision is assigned an DPE-NDL contract - marked with a star. In the Figure, we assume $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

of a (particularly) biased agent may not be enough for the principal to be able to set up a contract without a deadweight loss.

D. Social Welfare Analysis for Perceived Negative Correlation

This Appendix performs a social welfare comparison between the DPE contracts and the BPE contracts.

Proposition 9: *Let $\tilde{E}(\cdot)$ denote the biased expectations of the agent. Given the BPE contract $\{w^*_{ts}, c^*_{ts}\}$, the DPE-DL $\{\hat{w}_{ts}, \hat{c}_{ts}\}$ and the DPE-NDL $\{\hat{w}'_{ts}, \hat{c}'_{ts}\}$ contracts, the following are true:*

i $E(\hat{w}'_{ts}) \leq E(\hat{w}_{ts}) < E(w^*_{ts})$ *whenever the DPE contracts are optimal;*

ii $\tilde{E}(\hat{c}_{ts}) = \tilde{E}(\hat{c}'_{ts}) > \tilde{E}(c^*_{ts})$ *always;*

iii $E(\hat{c}_{ts}) > E(c^*_{ts})$ *whenever (50) fails.*

Proof. Point (i) follows from the fact that the DPE contracts feature the same wage but for the $(t,s) = (u,a)$ case and the optimality of the DPE contracts (as in Proposition 4).

Point (ii)'s equality is straightforward. To see why the inequality is true we calculate

$$\begin{aligned}\tilde{E}(\hat{c}_{ts}) =& \hat{c}_{aa}\tilde{\gamma}^H_{aa} + \hat{c}_{au}\tilde{\gamma}^H_{au} + \hat{c}_{ua}\tilde{\gamma}^H_{ua} + \hat{c}_{uu}\tilde{\gamma}^H_{uu}\hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\tilde{\gamma}^H_{ua}\right)\\ =& \hat{c}_{au}\left(\tilde{\gamma}^H_{au} + \tilde{\gamma}^H_{aa}\right) = \hat{c}_{au}\Gamma^H_a = \frac{\Delta V}{\Delta\Gamma^H_a}\frac{\Gamma^H_a\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a}.\end{aligned}$$

Hence, to prove point (ii), we simply need

$$\frac{\Gamma^H_u}{\tilde{P}_{au} - \Gamma^H_a} > 1 \quad \Rightarrow \quad \underbrace{\Gamma^H_a + \Gamma^H_u}_{1} - \tilde{P}_{au} > 0,$$

which always holds.

Finally, to prove point (iii) we calculate

$$\begin{aligned}E(\hat{c}_{ts}) =& \hat{c}_{aa}\gamma^H_{aa} + \hat{c}_{au}\gamma^H_{au} + \hat{c}_{ua}\gamma^H_{ua} + \hat{c}_{uu}\gamma^H_{uu} = \hat{c}_{au}\left(\gamma^H_{au} + \frac{\tilde{\gamma}^H_{aa}}{\tilde{\gamma}^H_{ua}}\gamma^H_{ua}\right)\\ =& \frac{\Delta V}{\Delta\Gamma^H_a}\frac{\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua}}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}}.\end{aligned}$$

Hence, we need

$$\begin{aligned}&\frac{\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua}}{\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua}} > \Gamma^H_a\\ &\gamma^H_{au}\tilde{\gamma}^H_{ua} + \tilde{\gamma}^H_{aa}\gamma^H_{ua} > \Gamma^H_a(\tilde{P}_{au}\tilde{P}_{ua} - \Gamma^H_a\tilde{P}_{ua})\\ &P_{au}\tilde{P}_{ua}\Gamma^H_u + \tilde{P}_{aa}P_{ua}\Gamma^H_u - \tilde{P}_{au}\tilde{P}_{ua} + \Gamma^H_a\tilde{P}_{ua} > 0,\end{aligned}$$

which, using our usual tools, further simplifies to

$$\begin{aligned}
&\underbrace{P_{au}P_{ua}\Gamma_u^H + P_{aa}P_{ua}\Gamma_u^H}_{P_{ua}\Gamma_u^H} - P_{au}P_{ua} + \Gamma_a^H P_{ua} \\
&\qquad + b_u P_{au}\Gamma_u^H + b_a P_{ua}\Gamma_u^H + b_a P_{ua} - b_u P_{au} + b_a b_u + b_u \Gamma_a^H > 0 \\
&P_{ua}(\Gamma_u^H - P_{au} + \Gamma_a^H) + b_u(P_{au}\Gamma_u^H - P_{au} + \Gamma_a^H) + b_a(P_{ua}\Gamma_u^H + P_{ua} + b_u) > 0 \\
&P_{ua}(1 - P_{au}) + b_u(P_{au}\underbrace{(\Gamma_u^H - 1)}_{\Gamma_a^H} + \Gamma_a^H) + b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > 0 \\
&P_{ua}P_{aa} + b_u\Gamma_a^H(1 - P_{au}) + b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > 0 \\
&b_a\left(P_{ua}(1 + \Gamma_u^H) + b_u\right) > -P_{ua}P_{aa} - b_u\Gamma_a^H P_{aa}
\end{aligned}$$

which generates the opposite of (50). ∎

Point (i) states, once again, that whenever DPE contracts are assigned, they must be optimal. There is a difference, however, compared to the case of the APE. It is trivial to observe that $E(\hat{w}'_{ts}) \leq E(\hat{w}_{ts})$ whenever (48) strictly holds, since the DPE contracts feature the same wage costs but for $\hat{w}'_{ua} \leq \hat{w}_{ua}$. As a matter of fact, the principal would always like to set up the DPE-NDL contract rather than the DPE-DL one. The former however, may not be feasible under some parameter conditions. Point (ii) is due to the wrong believed direction of correlation between signals by the agent. We know that the DPE contracts feature the largest possible compensations among the contracts set up for an agent who displays underprecision, while featuring zero compensation in the agreement states. The bias of the agent is, however, enough for him to believe that his expected compensation is higher under a DPE contract than under the baseline one. This, once again, connects to the idea of exploitation, where the principal takes advantage of agent's bias. Point (iii) follows the same intuition behind point (iv) of Lemma 4, but provides even more interesting insights summarized in the following proposition.

Proposition 10: *A DPE-NDL contract is never socially desirable. A DPE-DL contract is socially desirable whenever it is optimal, feasible and (50) fails.*

Proof. The first statement is trivial since the DPE-NDL contract is optimal only if (50) and it would be socially desirable only when (50) fails. Hence, the DPE-NDL contract never Pareto improves over the BPE contract when it is assigned.

The second statement follows from point (iii) of Proposition 9. ∎

Proposition 10 states a very controversial result on the DPE-NDL contract. On the one hand, the DPE-NDL contract features no deadweight loss. On the other hand, with a DPE-NDL contract the principal takes so much advantage of the agent's biased beliefs that the agent never gains from switching from a BPE to a DPE-NDL contract. The wagering motive is so strong that the agent is, in fact, always exploited by a DPE-NDL contract.

On the positive side, social desirability may take place when the DPE-DL contract is assigned. When the principal is not capable of eliminating the deadweight loss, her taking advantage of the agent's bias may put the latter in a better position compared to the baseline contract. To see that this is possible, consider Figure 15 Figure 16 below where we assume show the area where a DPE-DL is socially desirable under two different parameter conditions.

We conclude with a result on the magnitude of deadweight losses.

Proposition 11: *The BPE contract always features the highest deadweight loss also compared to the DPE contracts. That is*

$$\sum_{ts}(\hat{w}_{ts} - \hat{c}_{ts})\gamma_{ts}^{H} < \sum_{ts}(w_{ts}^{*} - c_{ts}^{*})\gamma_{ts}^{H}.$$

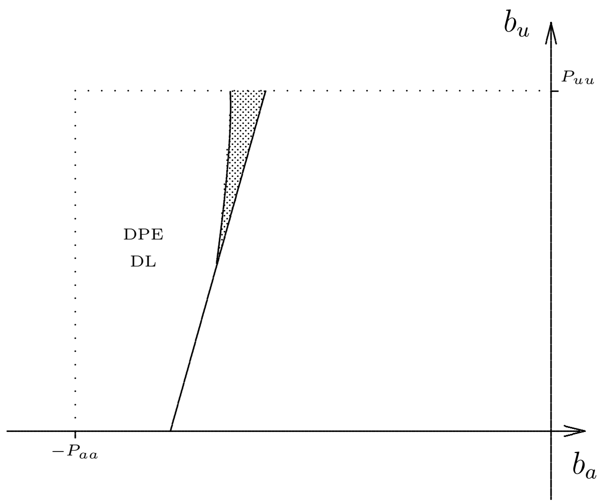

Figure 15: The shaded area between the two curves features a DPE-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of an agent who displays underprecision is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.7, 0.5, 0.6)$.

Proof. The deadweight loss under the DPE-DL contract is given by

$$\begin{aligned}\sum_{ts} \left(\hat{w}_{ts} - \hat{c}_{ts}\right) \gamma_{ts}^H &= (\hat{w}_{ua} - \hat{c}_{ua})\gamma_{ua}^H = \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\right) \hat{c}_{au} \\ &= \left(\frac{P_{au}}{P_{aa}} - \frac{\tilde{\gamma}_{aa}^H}{\tilde{\gamma}_{ua}^H}\right) \frac{\Delta V}{\Delta \Gamma_A} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_A^H} = \left(\frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{P_{aa}\tilde{P}_{ua}\Gamma_u^H}\right) \frac{\Delta V}{\Delta \Gamma_A} \frac{\Gamma_u^H}{\tilde{P}_{au} - \Gamma_A^H} \\ &= \frac{\Delta V}{\Delta \Gamma_A} \frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_A^H\right)}.\end{aligned}$$

To see that the deadweight loss in a DPE-DL contract is always lower than that in

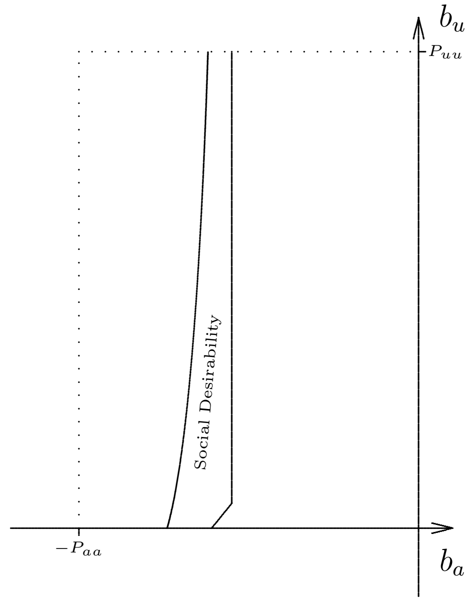

Figure 16: The area between the curve and the straight lines on the left features a DPE-DL contract with a higher expected compensation and a lower expected wage compared to the benchmark contract. Inside this area, the presence of an agent who displays underprecision is socially optimal. The Figure assumes $\left(P_{aa}, P_{uu}, \Gamma_a^H\right) = (0.55, 0.7, 0.5)$.

a BPE contract when the DPE-DL one is optimal, we calculate

$$\frac{\Delta V}{\Delta \Gamma_a}\frac{1}{P_{aa}} > \frac{\Delta V}{\Delta \Gamma_A}\frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{P_{aa}\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_A^H\right)}$$

$$1 > \frac{P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H}{\tilde{P}_{ua}\left(\tilde{P}_{au} - \Gamma_A^H\right)}$$

$$P_{au}\tilde{\gamma}_{ua}^H - P_{aa}\tilde{\gamma}_{aa}^H - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma_A^H < 0$$

$$P_{au}\tilde{P}_{ua}\Gamma_u^H - P_{aa}\tilde{P}_{aa}\Gamma_a^H - \tilde{P}_{ua}\tilde{P}_{au} + \tilde{P}_{ua}\Gamma_A^H < 0$$

$$P_{au}P_{ua}\underbrace{(\Gamma_u^H - 1)}_{-\Gamma_a^H} - P_{aa}P_{aa}\Gamma_a^H + P_{ua}\Gamma_A^H + b_u(P_{au}\Gamma_u^H - P_{au} + \Gamma_a^H) + b_a(P_{ua} + b_u - P_{aa}\Gamma_a^H) < 0$$

$$P_{ua}\Gamma_a^H\underbrace{(1 - P_{au})}_{P_{aa}} - P_{aa}P_{aa}\Gamma_a^H + b_u(P_{au}(\Gamma_u^H - 1) + \Gamma_a^H) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma_a^H) < 0$$

$$P_{aa}\Gamma_a^H(P_{ua} - P_{aa}) + b_u\Gamma_a^H(1 - P_{au}) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma_a^H) < 0$$

$$P_{aa}\Gamma_a^H(P_{ua} - P_{aa} + b_u) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma_a^H) < 0$$

$$P_{aa}\Gamma_a^H(\tilde{P}_{ua} - P_{aa}) + b_a(\tilde{P}_{ua} - P_{aa}\Gamma_a^H) < 0.$$

Recall that for the DPE-DL contract to be optimal $b_a \in \left[-P_{aa}, -P_{aa}\Gamma_a^H\right]$. Since the above inequality is linear in b_a, but its effect on the LHS is not straightforward, we can check that it holds at the extremes of the interval. At $b_a = -P_{aa}$, we have

$$-P_{aa}\tilde{P}_{ua}+P_{aa}^2\Gamma_a^H+P_{aa}\tilde{P}_{ua}\Gamma_a^H-P_{aa}^2\Gamma_a^H = -P_{aa}\tilde{P}_{ua}+P_{aa}\tilde{P}_{ua}\Gamma_a^H = P_{aa}\tilde{P}_{ua}(\Gamma_a^H-1) < 0.$$

At $b_a = -P_{aa}\Gamma_a^H$, we have

$$-P_{aa}\tilde{P}_{ua}\Gamma_a^H+P_{aa}^2(\Gamma_a^H)^2+P_{aa}\tilde{P}_{ua}\Gamma_a^H-P_{aa}^2\Gamma_a^H = P_{aa}^2(\Gamma_a^H)^2-P_{aa}^2\Gamma_a^H = P_{aa}^2\Gamma_a^H(\Gamma_a^H-1) < 0.$$

This proves that the DPE-DL contract always features a smaller deadweight loss than the BPE contract. ∎

E. Guile-Free vs. Good-Faith Contracts

In order to solve for the guile-free Perfect Bayesian Equilibrium contract, rename the (TR_P^i) and (TR_A^i) constraints in (4) as $(TR_{P,H}^i)$ and $(TR_{A,H}^i)$ for $i = a, u$. We can add to (4) the following constraints

$$\begin{aligned}
w_{aa}\gamma_{aa}^L + w_{au}\gamma_{au}^L &\leq w_{ua}\gamma_{aa}^L + w_{uu}\gamma_{au}^L \\
w_{ua}\gamma_{ua}^L + w_{uu}\gamma_{uu}^L &\leq w_{aa}\gamma_{ua}^L + w_{au}\gamma_{uu}^L \\
c_{aa}\tilde{\gamma}_{aa}^L + c_{ua}\tilde{\gamma}_{ua}^L &\geq c_{au}\tilde{\gamma}_{aa}^L + c_{uu}\tilde{\gamma}_{ua}^L \\
c_{au}\tilde{\gamma}_{au}^L + c_{uu}\tilde{\gamma}_{uu}^L &\geq c_{aa}\tilde{\gamma}_{au}^L + c_{ua}\tilde{\gamma}_{uu}^L.
\end{aligned}$$

These ensure that both the principal and the agent are incentivize to truthfully report also when the agent exerts low effort. First of all, notice that $(TR_{P,L}^i)$ and $(TR_{P,H}^i)$ coincide for all $i = a, u$. To see this, notice that they both imply

$$(w_{au} - w_{uu})\frac{P_{au}}{P_{aa}} \leq (w_{ua} - w_{aa}) \leq (w_{au} - w_{uu})\frac{P_{uu}}{P_{ua}}.$$

On the other hand, this is not true for the (TR_A) constraints. Among the four, we can show that $(TR^a_{A,L})$ implies $(TR^a_{A,H})$ while $(TR^u_{A,H})$ implies $(TR^u_{A,L})$. Recall that, by Assumption 1 and the definition of λ, we have $\Gamma^H_a > \Gamma^L_a$ and $\Gamma^H_u < \Gamma^L_u$.

Constraint $(TR^a_{A,j})$ implies

$$(c_{uu} - c_{ua})\frac{\tilde{\gamma}^j_{ua}}{\tilde{\gamma}^j_{aa}} \leq (c_{aa} - c_{au}) \quad j = H, L$$

where we have

$$\frac{\tilde{\gamma}^H_{ua}}{\tilde{\gamma}^H_{aa}} < \frac{\tilde{\gamma}^L_{ua}}{\tilde{\gamma}^L_{aa}} \quad \text{since} \quad \frac{\Gamma^H_u}{\Gamma^H_a} < \frac{\Gamma^L_u}{\Gamma^L_a}$$

proving that the $(TR^a_{A,L})$ is tighter. Similarly, constraint $(TR^u_{A,j})$ implies

$$(c_{aa} - c_{au}) \leq (c_{uu} - c_{ua})\frac{\tilde{\gamma}^j_{uu}}{\tilde{\gamma}^j_{au}} \quad j = H, L$$

where we have

$$\frac{\tilde{\gamma}^H_{uu}}{\tilde{\gamma}^H_{au}} < \frac{\tilde{\gamma}^L_{uu}}{\tilde{\gamma}^L_{au}} \quad \text{since} \quad \frac{\Gamma^H_u}{\Gamma^H_a} < \frac{\Gamma^L_u}{\Gamma^L_a}$$

proving that the $(TR^u_{A,H})$ is tighter. Hence, in a problem with both the $(TR^u_{A,H})$ and the $(TR^u_{A,L})$, the latter would be slack. To obtain a guile-free PBE, we can therefore solve problem (4) substituting $(TR^a_{A,L})$ for $(TR^a_{A,H})$. To see that this would not change the key tools we use to solve the model, notice that i) Lemma 7 does not depend on which of the (TR^a_A) constraint we use and ii) Lemma 6 still holds. Hence, solving, for example, the problem for the case of an overconfident worker using the $(TR^a_{A,L})$ leads to an equivalent of condition (5) and the APE contracts that depend on the $\tilde{\gamma}^L_{ts}$. While this does not affect our results qualitatively, it makes the study of the problem, and the parameter space where the APE contract is set up, needlessly complicated. For example, as discussed in the paper, a good-faith APE contract depends on many conditions, all of which are implied by (5). The equivalent condition for a guile-free APE contract, instead, can be shown to be necessary but not sufficient. That is, an area where the equivalent of the APE contract is set up exists, but it is harder to identify and graphically describe it.