Overconfidence in Tullock Contests

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Abstract

This paper investigates the role of overconfidence on contests. An overconfident player overestimates his probability of wining the contest. In two player contests where players have the same technology and preferences, the more overconfident player is the one who exerts lower effort. In addition, an increase in overconfidence of either player lowers the efforts of both players. In two player contests where players have different technologies and preferences, for any advantage a player may have on his contest technology or cost function, a large enough overconfidence bias can always make that player's winning odds smaller than 1/2. We show that in a symmetric $n > 2$ player contest where all players are equally overconfident, an increase in overconfidence lowers the efforts of all players if the bias is large enough relative to the number of players. These findings provide conditions under which overconfidence lowers the dissipation ratio and explains Tullock's paradox.

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1 Introduction

This paper investigates the role of overconfidence in contests. This question is of relevance since evidence from psychology and economics shows that humans tend to be overconfident. A majority of people believe they are better than others in a wide variety of positive traits and skills (Myers 1996, Santos-Pinto and Sobel 2005). Examples include car drivers (Svenson 1981), entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006), currency traders (Oberlechner and Osler 2008), poker and chess players (Park and Santos-Pinto 2010), CFOs (Ben-David et al. 2013), marathon runners (Krawczyk and Wilamowski 2017), freedivers (Lackner and Sonnabend 2020), and truck drivers (Hoffman and Burks 2020).

Competitions often take the form of contests. For example, an R&D race to be the first to develop or get a patent in new product or technology, election campaigns, rent-seeking games, competitions for monopolies, litigation, and wars, are examples of contests. Overconfidence matters for entry and performance in competitions and for labor markets (Camerer and Lovallo 1999, Niederle and Vesterlund 2007, Moore and Healy 2008, Dohmen and Falk 2011, Malmendier and Taylor 2015, Huffman et al. 2019, Santos-Pinto and de la Rosa 2020). Overconfidence also seems to play a role in mate competition and acquisition (Waldman 1994, Murphy et al. 2015). Interestingly, Lyons et al. (2020) provide evidence that high-status lobbyists working for private interest groups in Washington, DC, USA tend to be overconfident: they overate their achievements and their success. This empirical finding is in line with the experimental findings of Niederle and Vesterlund (2007) and Dohmen and Falk (2011) according to which overconfident subjects tend to self select more into more competitive environments.

Does overconfidence make a player more or less likely to win a contest? What is the effect of players' overconfidence on their effort provision and on rent dissipation? What is the welfare impact of overconfidence for the contest organizer and for the players? These are important questions since although the extant literature has characterized in depth equilibria in contests, behavioral biases have so far received limited attention by scholars (e.g. Baharad and Nitzan 2008). Moreover, some real-world observations have so far been difficult to rationalize with standard frameworks, and overconfidence may help reconciling theory with reality. Indeed, Tullock's paradox is the unresolved puzzle that the resources invested by lobbyists in high stakes rent-seeking contests appear dim compared to the prize that is being contested (Tullock 1989). Our own theoretical predictions demonstrate that overconfidence may push players' equilibrium efforts to a tiny share of the contested prize, thence contributing to explaining Tullock's paradox. Another yet unresolved puzzle is that the experimental literature on contests has repeatedly shown that participants invest more resources in contests than predicted at the Nash equilibrium of setups featuring rational agents (e.g. Sheremeta 2018). With overconfident players, we demonstrate that such predictions may obtain, and that even rent over-dissipation—i.e., the fact that aggregate spending is higher than the value of the contested rent—may equally obtain.

To answer these questions we consider a generalized Tullock contest where players can overestimate their winning probabilities while holding a correct assessment of the monetary value of winning and costs of effort. More precisely, we consider contests where player i's winning probability is $P_i(a_i, a_{-i}) = q_i(a_i)/\sum_j q_j(a_j)$, with $q_j'(a_j) > 0$ and $q_j''(a_j) \leq 0$ for all j. The function $q_j(a_j)$ is often referred to as the impact function (Ewerhart 2015) and models the technology whereby players' efforts or investments translate into probabilities of winning the contest. We assume an overconfident player i thinks, mistakenly, his impact function is $\lambda_i q_i(a_i)$, where $\lambda_i > 1$, but has correct beliefs about his rivals' impact functions. As a consequence, an overconfident player i's perceived winning probability is $P_i(a_i, a_{-i}; \lambda_i) = \lambda_i q_i(a_i)/[\lambda_i q_i(a_i) + \sum_{j \neq i} q_j(a_j)]$.[1]

We start by considering two player contests where technology and preferences are symmetric, that is, the players have identical impact, cost, and utility functions.

[1]Section 2 discusses alternative ways of modelling overconfidence in contests. Section 3 shows that our way of modeling overconfidence in a contest satisfies four desirable properties.

These symmetry assumptions allow us to focus exclusively on the role that the heterogeneity in beliefs plays in determining effort provision and the winner of the contest. Moreover, they imply that the player who exerts the highest effort has the highest objective winning probability. We define as the Nash winner (loser) the player with the highest (lowest) objective probability of winning at the pure-strategy equilibrium. Proposition 1 shows that in such a contest, the more overconfident player is the one who exerts the lowest effort. Hence, the more overconfident player is the Nash loser. Furthermore, as the overconfidence of either player increases, both players' efforts decrease.

Next, we consider two player contests where technology, preferences and beliefs can be asymmetric. Here we show that the more overconfident and the less efficient player always exerts less effort. More importantly, Proposition 2 shows that for any advantage a player may have on his contest technology or cost function, a large enough overconfidence bias can always make that player the Nash loser in the contest. In addition, we show that very large levels of overconfidence lead both players to exert very low effort. Hence, overconfidence reduces rent dissipation and provides a new explanation for Tullock's paradox: the empirical fact that lobbying contributions are small relative to the value of the policies at stake. The rationale uncovered in our study for explaining the Tullock paradox is quite straightforward: overconfident players are (mistakenly) convinced that they are able to optimally bid in a contest (e.g. in lobbying) with lower efforts than what they would need to provide if they were rational. As the competitors fully perceive and integrate in their reasoning this overconfidence bias, they, in turn, are equally incentivized to reduce their own bids for any degree of overconfidence they may themselves be subject to. Eventually, we end up with both players under-investing in the contest as compared to what rational players would have done.

Finally, we consider symmetric n player contests where all players are equally overconfident. Proposition 4 shows that in these contests, individual and aggregate efforts decrease (increase) with overconfidence if λ is greater (smaller) than $n-1$. This finding has two important consequences. First, with a large number of overconfident contestants we will witness over-dissipation of the rent. Second, if contestants are highly overconfident, their equilibrium efforts will be very low, in line with Tullock's paradox. Hence, the degree of overconfidence as well as the number of players in a contest matters in terms of understanding the effects of overconfidence on effort provision and rent dissipation in contests. In contests like these, the contest organizer would prefer to de-bias overconfident players when their bias is large relative to the number of players. In contrast, the contest organizer would prefer not to de-bias overconfident players when their bias is small relative to the number of players.

2 Related Literature

This study relates to four strands of literature. First, it contributes to the literature on behavioral biases in contests. The most closely related studies in the literature on contests are Ando (2004) and Ludwig et al. (2011). Ando (2004) studies a contest between two players who are uncertain about their monetary value of winning the contest. Both players are overconfident and two definitions of overconfidence are considered. An overconfident player can either overestimate his monetary value of winning the contest or, alternatively, underestimate the rival's monetary value of winning it. Ando (2004) finds that an overconfident player who overestimates his monetary value of winning the contest always exerts more effort. In contrast, an overconfident player who underestimates his rival's monetary value of winning the contest might exert less effort. Ludwig et al. (2011) analyze a Tullock contest where an overconfident player competes against a rational player. The overconfident player is assumed to underestimate his cost of effort. Ludwig et al. (2011) find that the overconfident player exerts more effort and the rational player exerts less effort than if both players were rational. They also find that the bias makes the contest organizer better off since the overconfident player's increase in effort more than compensates the rational player's decrease in effort. Our results show that when overconfidence is an overestimation of the probability of winning the contest its effects on effort and welfare are quite different than those in found in Ando (2004) and Ludwig et al. (2011). Our definition of overconfidence is adequate when both the monetary value of winning the contest and the cost of effort are known before entry.

Baharad and Nitzan (2008) and Keskin (2018) amend the standard model of contests by introducing probability weighting in line with Tversky and Kahneman's (1992) Cumulative Prospect Theory. This behavioral bias is modeled with an inverse S-shaped probability weighting function, i.e., a function where the marginal increase in the (perceived) subjective probability is higher for extreme (i.e. low and high) probabilities. Our own approach assumes a constant bias in players' beliefs that they are better than they really are at contesting their opponents. We thus see our approach as complementary to these earlier works since nothing precludes players from both assigning 'weights' to probabilities and be subject to an overconfidence bias. Notice that in terms of contribution to the literature on behavioral biases, our approach has the advantage to be flexible enough to accommodate a very large family of contest success functions while also allowing for any possible heterogeneities among players. Last, whereas Baharad and Nitzan (2008) and Keskin (2018)'s approach applies exclusively to probabilistic setups, our own model is equally suited to describe sharing contests that have gained in importance over the years (e.g. Dickson et al. 2018).[2]

Second, the paper contributes to the literature on under-spending in lobbying contests. While there is a popular perception that there is too much money in politics, empirical evidence shows that lobbying contributions are extremely small with respect to the enormous interests involved in the policymaking process. [3] This

[2]Other scholars have equally focused on the effect of behavioural biases on equilibrium outcomes in the presence of uncertainty. Kelsey and Melkonyan (2018) consider both optimistic and pessimistic attitudes to ambiguity, while Cornes and Hartley (2012) and Fu et al. (2022) introduce loss aversion in probabilistic contests.

[3]For example, the Farm Security and Rural Investment Act of 2002 states that "dairy producers, who since 1996 have had to have subsidies renewed annually, gave 1.3 million in 2000 and received

has been termed the Tullock paradox. Several explanations have been proposed for it, such as risk aversion (Konrad and Schleisinger 1997, Treich 2010), Heterogeneity in valuations (Hillman and Riley 1998), uncertain number of contestants (Kahana and Klunover 2015), loss-aversion of lobbies (Cornes and Hartley 2003), repeated interaction between lobbyists (Polborn 2006), or out of the equilibrium threats to overbid competitors (Chamon and Kaplan 2013).

Third, it also contributes to the experimental literature on behavior in contests. Scholars have also long tried to explain the puzzle that contestants in lab experiments spend significantly higher amounts than the game's Nash equilibrium (Price and Sheremeta 2011, 2015, Mago et al. 2016). The theoretical literature has attempted to explain over-spending, but also extreme manifestations of such phenomena where contestants over-dissipate the rent by expending on aggregate more resources than the value of the prize that is contested. Over-spending has so far been attributed to players' risk attitudes (Jindapon and Whaley 2015), to mixed strategy equilibria where over-spending occurs with some probability but not in expectation (Baye et al. 1999), or to behavioral biases (e.g. Hillman and Long 2019). In this article we demonstrate that with overconfident contestants, over-spending and even over-dissipation can result when the number of players is sufficiently large and the overconfidence bias is relatively mild; overconfident players individually expend more effort than rational players when their odds of winning are low because of the high number of participants. It has also been documented that women tend to bid more than men in experimental contests (Price and Sheremeta 2012, Mago et al. 2013), especially when competing against other women (Mago and Razzolini 2019). If, as evidence suggests (Bengtsson et al. 2005, Niederle and Vesterlund 2007, Buser et al. 2014, Dreber et al. 2014, Bordalo et al. 2019, Möbius et al. 2022), men tend to be more confident than women, then our theory can explain such gender differences

price supports worth almost 1 billion." According to Chamon and Kaplan (2013) "[. . .] the sugar program led to a net gain of over one billion dollars to the sugar industry in 1998. However the sugar industry's total campaign contributions in the two years of that election cycle were a mere $2.8 million."

in bidding. Indeed, our results suggest that increasing a contestant's overconfidence can lower his equilibrium bid, while also inducing the competitor to reduce his own bid, but by a lesser amount.

Last, our paper contributes to the literature on contests with heterogeneous players. Baik (1994) analyzes two player contests where the players differ in their valuation of the prize and in their marginal productivity of effort. Singh and Wittman (2001) show that when players differ in marginal productivity of effort, output increases in ability, and effort provision decreases in effort costs. Stein (2002) determines the equilibrium number of active players when players are heterogeneous. Building on previous findings of Baik (1994), we are able to characterize the game's equilibrium for a very wide array of contest success functions and for any type of heterogeneity, thereby providing useful guidance for scholars of contests.

3 Set-up

In a standard two player Tullock (1980) contest with linear effort costs the players compete for the winner prize V. player i chooses an effort level a_i to maximize $E[U_i] = P_i(a_i, a_j)V - a_i$, where $P_i(a_i, a_j)$ is the probability player i wins the contest– the contest success function (CSF). Tullock (1980) assumes the CSF is:

$$P_i(a_i, a_j) = \begin{cases} a_i^r/(a_i^r + a_j^r) & \text{if } \; a_i + a_j > 0 \\ 1/2 & \text{if } \; a_i + a_j = 0 \end{cases},$$

where $r \geq 0$.[4] Note that the player who exerts the highest effort does not necessarily win the contest. However, a player who exerts zero effort has a zero probability of winning if the other player exerts some positive amount of effort no matter how

[4] The parameter r captures the degree of noise in the Tullock contest. The higher is r, the more sensitive is the success probability to effort. When $r = 0$ effort plays no role and each player always has a success probability of 1/2. The most popular versions of the Tullock contest are the lottery ($r = 1$) and the first-price all-pay auction ($r = \infty$).

small.[5]

To study the role of overconfidence in contests we consider a generalized Tullock contest. The utility of the monetary prize V is $v_i = u_i(V)$ with $u_i' > 0$. The effort cost is $c_i(a_i)$ with $c_i(0) = 0$, $c_i'(a_i) > 0$ and $c_i''(a_i) \geq 0$. Following Baik (1994) we assume the CSF is:

$$P_i(a_i, a_{-i}) = \begin{cases} q_i(a_i)/\sum_j q_j(a_j) & \text{if } \sum_j q_j(a_j) > 0 \\ 1/n & \text{if } \sum_j q_j(a_j) = 0 \end{cases},$$

where $q_i(0) \geq 0$, $q_i^{'}(a_i) > 0$ and $q_i^{''}(a_i) \leq 0$. The overconfident player i mistakenly perceives his impact function to be $\lambda_i q_i(a_i)$, with $\lambda_i > 1$, and correctly perceives the rivals' impact functions. This way of modelling overconfidence in a contest implies that an overconfident player i's perceived winning probability is equal to

$$P_i(a_i, a_{-i}; \lambda_i) = \begin{cases} \lambda_i q_i(a_i)/[\lambda_i q_i(a_i) + \sum_{j \neq i} q_j(a_j)] & \text{if } \lambda_i q_i(a_i) + \sum_{j \neq i} q_j(a_j) > 0 \\ 1/n & \text{if } \lambda_i q_i(a_i) + \sum_{j \neq i} q_j(a_j) = 0 \end{cases}.$$

This specification of overconfidence in a contest satisfies four desirable properties. First, contests where players have heterogeneous productivity of effort are modelled similarly, that is, the players are assumed to have heterogeneous impact functions (Baik 1994, Singh and Wittman 2001, Stein 2002, Fonseca 2009). Second, the overconfident player's perceived winning probability is well defined for any value of $\lambda_i > 1$.[6] Third, the overconfident player's perceived winning probability is increasing in λ_i. Fourth, overestimating one's impact function is equivalent to un-

[5]There are at least three reasons why Tullock contests are widely employed. First, a number of studies have provided axiomatic justification for it (Skaperdas 1996, Clark and Riis 1998). Second, a variety of rent-seeking contests, innovation tournaments, and patent-race games are strategically equivalent to the Tullock contest (Baye and Hoppe 2003). Third, its tractability. The drawback of Tullock contests is that they do not separate the degree to which luck as opposed to effort affects behavior (Amegashie 2006).

[6]This is not the case with alternative specifications. For example, if one assumes an overconfident player's perceived winning probability is $P_i(a_i, a_j, \lambda_i) = \lambda_i q(a_i)/[q(a_i) + q(a_j)]$, with $\lambda_i > 1$, then $P_i(a_i, a_j, \lambda_i)$ is not a well defined probability for any value of $\lambda_i > 1$.

derestimating the rivals' impact functions since $\lambda_i q_i(a_i)/[\lambda_i q_i(a_i) + \sum_{j\neq i} q_j(a_j)] = q_i(a_i)/[q_i(a_i) + \sum_{j\neq i} q_j(a_j)/\lambda_i]$.

Under this specification, overconfidence is equivalent to overestimation of the productivity of effort. This way of modeling overconfidence is often used in studies that analyze the impact of overconfidence on contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011).

4 Contests with Symmetric Technology and Preferences

This section studies a contest where an overconfident player 1 competes against an overconfident player 2. Player i mistakenly perceives his impact function to be $\lambda_i q(a_i)$, with $\lambda_i > 1$ and correctly perceives the rival's impact function to be $q(a_j)$. In this section we assume, without loss of generality, that $\lambda_1 > \lambda_2$. We will later relax this assumption when introducing further asymmetries in the model. Any player i, $i = \{1, 2\}$, chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = P_i(a_i, a_j; \lambda_i)v - c(a_i) = \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)}v - c(a_i).$$

The first-order condition is

$$\frac{\partial E[U_i(a_i, a_j; \lambda_i)]}{\partial a_i} = \frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2}v - c'(a_i) = 0. \quad (1)$$

The second-order condition is

$$\frac{\partial^2 E[U_i(a_i, a_j; \lambda_i)]}{\partial a_i^2} = \frac{q''(a_i)[\lambda_i q(a_i) + q(a_j)] - 2\lambda_i [q'(a_i)]^2}{[\lambda_i q(a_i) + q(a_j)]^3}\lambda_i q(a_j)v - c''(a_i) < 0, \quad (2)$$

and the above inequality is satisfied since $q''(a_i) \leq 0$ and $c''(a_i) \geq 0$.

Let $a_i = R_i(a_j)$ denote player i's best response obtained from (1). Along player i's best response we have

$$\lambda_i q'(a_i) q(a_j) v = c'(a_i) \left[\lambda_i q(a_i) + q(a_j)\right]^2 .$$

Lemma 1 describes the shapes of the players' best responses.

Lemma 1. *$R_i(a_j)$ is concave in a_j and reaches a maximum for $q(a_j) = \lambda_i q(a_i)$.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort. A second useful lemma establishes the uniqueness of the equilibrium:

Lemma 2. *The contest game with overconfident contestants admits a unique equilibrium.*

Lemma 3 describes how the players' best responses changes with their overconfidence parameter λ_i.

Lemma 3. *An increase in player i's overconfidence λ_i leads to a contraction of his best response function, $\frac{\partial R_i(a_j)}{\partial \lambda_i} < 0$, for $q(a_j) < \lambda_i q(a_i)$ and to an expansion of his best response function, $\frac{\partial R_i(a_j)}{\partial \lambda_i} > 0$, for $q(a_j) > \lambda_i q(a_i)$. Moreover, the maximum value of the players' best response is independent of their degree of overconfidence.*

Lemma 3 characterizes the best response function of players who are subject to an overconfidence bias. For a high effort of the rival, an increase in overconfidence raises player i's effort level, while for low effort of the rival, an increase in overconfidence lowers player i's effort level. Moreover, the maximal value taken by player i's best response is independent of his overconfidence bias.

We next present our first proposition that uncovers the effect of overconfidence on equilibrium efforts.

Proposition 1. *In a two player generalized Tullock contest where both players are overconfident the more overconfident player exerts lower effort. Hence, the more overconfident player is the Nash loser since* $P_i(a_i^*, a_j^*) < 1/2 < P_j(a_i^*, a_j^*)$.

Corollary 1. *Both players exert less effort than if both were rational, and as the overconfidence of either player increases, both players' efforts decrease.*

If the overconfidence of player i goes up, then player i's best response shifts inwards for $q(a_j) < \lambda_i q(a_i)$ (as shown in Lemma 3). Corollary 1 follows from the fact that the players' best responses are positively-slopped at the Nash equilibrium.

We illustrate Proposition 1 on Figure 1. On that figure we represent the two players' best response functions given that player 1 is more overconfident than player 2, i.e. given that $\lambda_1 > \lambda_2$. From Lemma 1 we know that the best response functions are concave, while from Lemma 3 we also know that the maximal value player i's best response function takes is given by $q(a_j) = \lambda_i q_(a_i)$, hence the crossing of the dotted lines with the maxima of the best response functions. To better gauge the effect of overconfidence, we have also drawn the best response functions of fully rational players as seen in the two concave dotted curves crossing on the 45^o line at (a_1^{max}, a_2^{max}). The higher is a player's overconfidence, the more the best response function flattens for values of the rival's effort a_j below $q^{-1}(\lambda_i q(a_i))$, and steepens for values above that threshold, while the maximand of the best response function increases with overconfidence. Consequently, and in line with Proposition 1, the more overconfident player 1 will experience a harsher contraction of his best response function below a_2^{max}, and since the best response functions of both players $i = \{1, 2\}$ are strictly increasing in $[0, a_j^{max}]$, the equilibrium E will lie above the 45^o line in the space where $a_2 > a_1$.

Increasing the overconfidence of player 1, implies that the player's best response function shifts inwards for low values of a_2 as represented graphically by the dashed and dotted best response function. Consequently, since $R_2(a_1)$ remains unaffected by this shift in the overconfidence of his rival, at the new equilibrium E' both players will necessarily exert less effort that in E, while the concavity of $R_2(a_1)$ also implies

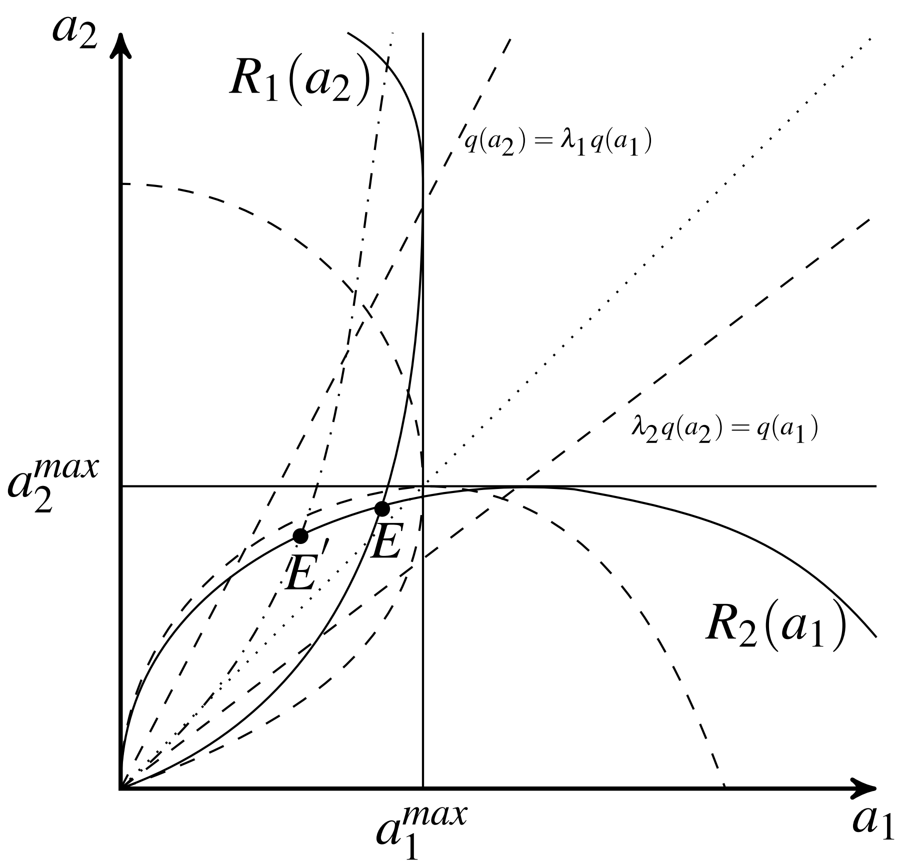

Figure 1: Equilibrium with $\lambda_1 > \lambda_2$.

that the new probability that player 1 wins the contest is now lower.

Upon observing the figure, it is equally obvious that an increase in λ_2 will also result in lower equilibrium efforts of *both* players, while the probability that player 1 wins the contest would then increase instead.

5 Contests with Asymmetric Technology and Preferences

In the previous section we assumed that the only source of asymmetry was the degree of overconfidence of players. We now lift this assumption to consider the effect of asymmetries in the players' impact functions, $q_i(a_i)$, and cost functions, $c_i(a_i)$. As such, we are not imposing that $q_1(a_1) = q_2(a_2)$ for $a_1 = a_2$, nor that $c_1(a_1) = c_2(a_2)$, and we maintain the hypothesis that $\lambda_1, \lambda_2 > 1$.

It is immediate to observe that the first-order condition for player i is given by:

$$\frac{\partial E[U_i(a_i, a_j; \lambda_i)]}{\partial a_i} = \frac{\lambda_i q_i'(a_i) q_j(a_j)}{[\lambda_i q_i(a_i) + q_j(a_j)]^2} v - c_i'(a_i) = 0, \tag{3}$$

while the second-order condition is easily shown to be satisfied and, adopting the same approach as before, the equilibrium can be shown to be unique. Reproducing the reasoning of the proof of Lemma 3 we can also show that the maximal value taken by the best response function of player i, a_i^{max}, is implicitly defined as:

$$\frac{q_i'(a_i^{max})}{4 q_i(a_i^{max})} v = c_i'(a_i^{max}). \tag{4}$$

A first observation allowing us to characterize the equilibrium is contained in the next lemma:

Lemma 4. *If the two players are subject to the same overconfidence bias,* $a_1^{max} > a_2^{max} \Leftrightarrow a_1^* > a_2^*$.

This lemma shows that any competitive edge in the contest technology or in the cost of effort by a player will map in a higher equilibrium effort, and therefore in a higher probability that the most efficient player wins the contest. Building on our earlier results, we immediately deduce the next lemma:

Lemma 5. *If* $a_1^{max} > a_2^{max}$ *and* $\lambda_2 \geq \lambda_1 > 1$*, then* $a_1^* > a_2^*$.

This lemma reinforces the results of the previous section; we showed in Proposition 1 that when players are symmetric along all dimensions but overconfidence, the more overconfident player exerts a lower equilibrium effort. In Lemma 4 we also show that the most efficient player produces a higher equilibrium effort. Lemma 5 shows that the combination of overconfidence and lower efficiency will always result in the more overconfident and less efficient player exerting less effort at equilibrium.

Extending lemma 5, we can state the following result:

Proposition 2. *For any a_1^{max}, a_2^{max} and λ_2, there always exist a value $\tilde{\lambda}_1$ such that if $\lambda_1 > \tilde{\lambda}_1$, then $q_1(a_1^*) < q_2(a_2^*)$.*

This is an important finding since it implies that for any advantage a player may have on his contest technology or cost function, a large enough overconfidence bias can always make that player the Nash loser in a contest.

Corollary 2. *If $\lambda_i \to \infty$, for any $i \in \{1, 2\}$, $a_1^* \to 0$ and $a_2^* \to 0$.*

Very large levels of overconfidence are thus shown to push both players to contain their contest expenditures to infinitesimally small levels at the limit. The intuition of this result is quite straightforward: when a contestant is extremely overconfident, then for any expected effort of his adversary he will be incentivized to exert a very small effort. The adversary then anticipates this and best responds by providing a very small effort as well, yet one that still guarantees him to be the Nash winner, as shown in Proposition 2.

Last, we inspect the effect of overconfidence on the equilibrium effort levels to further explore the extent to which overconfidence can explain under-spending.

Proposition 3. *An increase in player 1's overconfidence implies that*

$$\begin{cases} \text{if } \lambda_1 q_1(a_1^*) > q_2(a_2^*) \text{ and } \lambda_2 q_2(a_2^*) > q_1(a_1^*) \text{ then } \partial a_1^*/\partial\lambda_1 < 0 \text{ and } \partial a_2^*/\partial\lambda_1 < 0 \\ \text{if } \lambda_2 q_2(a_2^*) < q_1(a_1^*) \text{ then } \partial a_1^*/\partial\lambda_1 < 0 \text{ and } \partial a_2^*/\partial\lambda_1 > 0 \\ \text{otherwise, if } \lambda_1 q_1(a_1^*) < q_2(a_2^*) \text{ then } \partial a_1^*/\partial\lambda_1 > 0 \text{ and } \partial a_2^*/\partial\lambda_1 > 0 \end{cases}$$

A driving force underlying our analysis is worth exposing prior to describing the above results. A higher degree of overconfidence—for the player whom we label the focal player—renders, in the focal player's mind, the outcome of the contest less sensitive to one's own effort: if the expected winning probability is higher than 1/2, a higher degree of overconfidence pushes the player to reduce his contest effort for a given effort of the opponent since the victory is more likely than not and can now be achieved at lower cost. On the other hand, if the expected winning probability is lower to 1/2, the player will increase his effort with overconfidence because the higher marginal return to investing effort in the contest allows the player to close the gap with the opponent. This effect, which is known in the contest literature when performing comparative statics exercises in asymmetric contests (see e.g. Malueg and Yates 2005), is therefore shown to be equally at play when players are subject to rationality biases.

Proposition 3 is quite instructive since it uncovers a non-trivial effect of overconfidence on equilibrium efforts in a general contest with asymmetric players. When both players are sufficiently overconfident that they both expect (at equilibrium) to win the contest with a probability larger than 1/2, the best responses of the two players are increasing in their adversary's effort. This general feature of contests has deep implications in the study of overconfidence. Indeed, the same condition defining the slope of the players' best responses equally determines whether overconfidence expands or contracts the players' best responses. Under the stated conditions mapping in upward-sloping best responses, the players' best responses will contract following an increase in their overconfidence. Indeed, since both players believe they have better odds to win the contest, following an increase in their degree of overconfidence, they can afford reducing their efforts while still believing they retain a competitive edge over their opponent. Thence, as the focal player becomes less aggressive following an increase in his overconfidence, since the players' efforts are strategic complements under the stated conditions, the resulting equilibrium will involve lower efforts by both players.

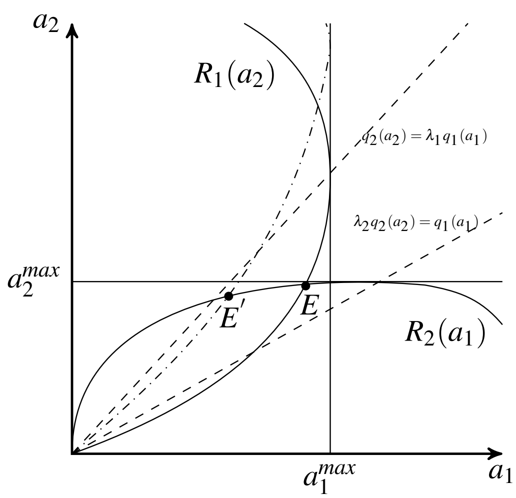

Figure 2a

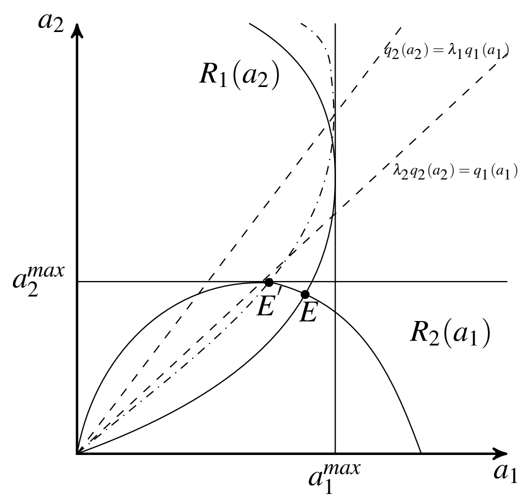

Figure 2b

Figure 2: Asymmetric contest technologies

Consider next the case where player 2 instead (correctly) believes that his winning odds are less than 1/2 so that his best response function is downward slopping. In such an instance, an increase in player 1's overconfidence will make him contract his effort at equilibrium, which will in turn push player 1 to reduce his equilibrium effort, while player 2 who (correctly) believes to be the Nash loser of the contest will increase his equilibrium effort.

Last, if player 1 (correctly) anticipates to be the Nash loser, his best response is negatively slopped (strategic substitutes) and it expands with overconfidence. On the other hand, player 2 (correctly) anticipates to be the Nash winner, and his best response is thus increasing in player 1's effort (strategic complements). As the focal player 1 becomes more aggressive, both players will then increase their equilibrium efforts.

On Figure 2 we depict the comparative statics exercise on player 1's overconfidence parameter when $R_i^{'}(a_j^*) > 0$ for $i = \{1, 2\}$ (left panel) and when $R_1^{'}(a_2^*) > 0$ and $R_2^{'}(a_1^*) < 0$ (right panel). Observe that we are considering a situation where player 1 is endowed with a more efficient contest technology (i.e. a more efficient impact and/or cost function than player 2), so that $a_1^{max} > a_2^{max}$. On Figure 2a we have drawn a situation where both players are subject to some overconfidence, and we consider the effect of increasing the overconfidence of player 1. Since player 1 anticipates to be the Nash winner, the increase in λ_1 will push inwards his reaction function for effort levels of player 2 such that $q_2(a_2) < \lambda_1 q_1(a_1)$. Since, however, the reaction function of player 2 remains unaffected by this shock in his adversary's rationality bias, we observe that the resulting equilibrium E' will feature lower efforts for both players compared to the initial equilibrium E.

On Figure 2b we are considering a situation where at the initial equilibrium player 2 expects to be Nash loser, despite his overconfidence bias. The best response function of player 2 is then downward slopping at equilibrium, while the best response function of player 1 is upward slopping. Further increasing the overconfidence of the Nash winner, λ_1, implies once more that for $q_2(a_2) < \lambda_1 q_1(a_1)$ the best response of player 1 moves inwards. Since the initial equilibrium, E is on the downward slopping part of player 2's best response function, this contraction in player 1's effort will incentivize player 2 to increase his effort thus implying that a_1^* drops while a_2^* increases.

6 Contests Between $n > 2$ Overconfident Players

We now extend the analysis to $n > 2$ players and we begin the analysis by focusing on the fully symmetric case where players have a common overconfidence bias $\lambda > 1$. The first order condition for any player i is then given by:

$$\frac{\lambda q'(a_i) \sum_{j \neq i} q(a_j)}{\left[\lambda q(a_i) + \sum_{j \neq i} q(a_j)\right]^2} v - c'(a_i) = 0, \tag{5}$$

and the second-order condition can here too easily be shown to be satisfied.

The next proposition summarizes our findings on the effect of overconfidence on equilibrium efforts:

Proposition 4. *With $n > 2$ symmetric players Individual and aggregate efforts decrease (increase) with overconfidence if $\lambda > (<) n - 1$.*

The intuition of this result follows the one underlying the finding of Proposition 1 and critically depends on whether players' efforts are strategic complements or strategic substitutes at equilibrium. Consider first a small number of competitors and/or a high degree of overconfidence. In such instances, the players will all expect to be highly likely to win the contest and their best response functions will then be positively-slopped at equilibrium. Indeed a low n or a high λ both imply that (at the symmetric equilibrium) the opponents' sum of impact functions is relatively low, and all players consequently expect to have a high probability of winning the contest. Any expected increase in the opponents' contest effort would then push players to increase their own effort so as to avoid the winning odds from deteriorating too much. In such instances, an increase in overconfidence will incentivize contestants to all reduce their effort for a given expected (equilibrium) effort of their opponent: the high expected winning probability can now be achieved at lower cost as in Proposition 3. The exact opposite mechanism is at play when the number of contestants is high and/or the degree of overconfidence is low. In such instances, the players's best response functions will be downward slopping because (at the symmetric equilibrium) the opponents' sum of impact functions is relatively high, and all players consequently expect to have a small probability of winning the contest. In such instances, an increase in overconfidence incentivizes players to increase effort with overconfidence so as to close the gap with the opponents as a consequence of the higher (expected) marginal returns to investing effort in the contest. This mechanism once more echoes the one in Proposition 3.

From the above observation, we are able to obtain the following corollary:

Corollary 3. *With $n > 2$ symmetric players, the maximal rent dissipation is always attained when $\lambda = n - 1$. There always exists a finite n^D such that over-dissipation can be observed at equilibrium for $n > n^D$.*

It is widely known in the literature on contests that with rational agents over-dissipation can never be observed at equilibrium if the player's valuation of the prize is equal to the actual value of the prize.[7] Although the dissipation ratio, defined as the ratio of total expenditures (or aggregate effort) to the value of the prize, $D = \frac{\sum_i a_i}{v}$, does increase in the number of players, it is bounded by unity because individual equilibrium effort drops as the number of contestants increases. Indeed, a larger number of contestants implies that the competitors' aggregate effort is expected to be higher, thence reducing the marginal return to investing in the contest, which in turn pushes all contestants to individually contract their equilibrium effort. In Proposition 4 we demonstrated, however, that some overconfidence may push players to increase their equilibrium effort compared to a setup with fully rational players. Corollary 2 shows that there always exists a degree of overconfidence such that equilibrium individual efforts of overconfident players will equal the maximal equilibrium individual efforts that can be obtained in the game, i.e., the individual efforts produced in setups with two fully rational players. Consequently, with sufficiently many overconfident players the aggregate effort can be higher than the value of the contested prize.

To visualize the last two results, on Figure 3 we depict the individual equilibrium effort of (symmetric) contestants as a function of their overconfidence parameter in the most simple contest where players' payoffs are given by:

$$E[U_i, a_i, \mathbf{a_{-i}}; \lambda_i] = \frac{\lambda_i a_i}{\lambda_i a_i + \sum_{j \neq i} a_j} - a_i,$$

where $\mathbf{a_{-i}}$ designates the vector of player i's competitors' efforts. With $n = 2$ and $\lambda = 1$, the equilibrium efforts are equal to $1/4$. If we consider games with more players, the individual efforts can be kept equal to $1/4$ if $\lambda = n - 1$. Consequently, under such circumstances, full dissipation can result with $n = 4$ and $\lambda = 3$, and over-dissipation can therefore obtain for any $n > 4$.

[7]See Dickson et al. (2022) for instances where players' valuation of the prize differs from the actual value of the prize.

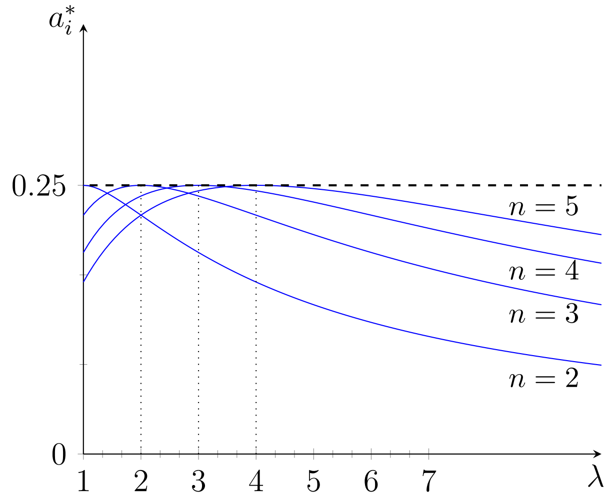

Figure 3: Individual equilibrium efforts as a function of λ with $q(a) = a$, $v = 1$ and $c(a) = a$.

It is important at this stage to underline that although for over-dissipation to be observed it is necessary to have $n > n^D > 2$ players, the required degree of overconfidence may be quite low. Indeed, to visualize this we consider again the previous basic contest setup, and we impose for the sake of the argument the parameter restriction $\lambda < n - 1$, for $n \geq 3$. Since $a^* = \frac{\lambda(n-1)}{(\lambda+n-1)^2}$, this parameter restriction can easily be shown to imply that $\partial na^*/\partial n > 0$, $\partial^2 na^*/\partial n^2 < 0$, and $\partial a^*/\partial \lambda < 0$. We then plot the equilibrium aggregate effort, na^*, against the number of players, n, for various levels of overconfidence on Figure 4. It is well known that as n becomes arbitrarily large, the dissipation ratio converges to unity, without ever reaching total rent dissipation. We know from Corollary 3 that for any number of players $n > n^D > 2$, there always exists a degree of overconfidence conducive to over-dissipation. For example, Figure 4 shows that with $n = 6$ over-dissipation is already observed when $\lambda = 1.5$, which corresponds to a perceived winning probability of 0.231 as opposed to the actual winning probability of 1/6. Increasing the number of players to, say, $n = 8$ implies that over-dissipation can be achieved with an even lower degree of overconfidence (e.g. $\lambda = 1.25$). It is immediate to deduce that as the number of players becomes arbitrarily large in this setup, the required degree of overconfidence for observing over-dissipation will become arbitrarily small (i.e. λ close to 1).

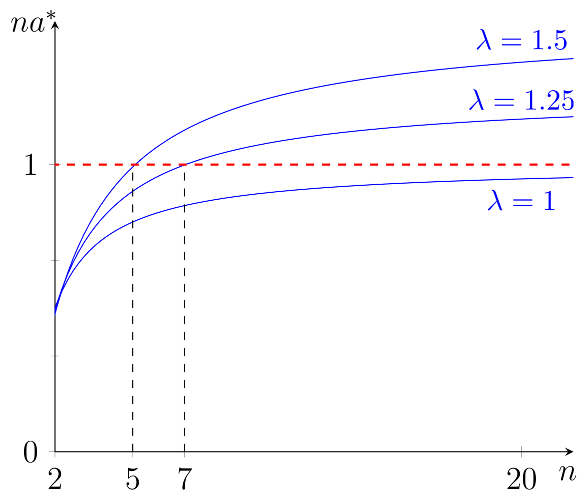

Figure 4: Equilibrium aggregate effort na^* as a function of n.

Last, we extend the analysis to asymmetric players, by allowing both overconfidence and technology (impact and cost functions) to be player-specific. The first-order condition for player i then reads as:

$$\frac{\lambda_i q'(a_i) \sum_{j \neq i} q_j(a_j)}{\left[\lambda_i q_i(a_i) + \sum_{j \neq i} q_j(a_j)\right]^2} v - c_i'(a_i) = 0. \tag{6}$$

Observe first that the second-order condition to this optimization problem will always be verified. Next, by applying the implicit function theorem to the above expression we can once more deduce that the sign of $R_i'(a_j)$, for any $j \neq i$, is given by the sign of $\lambda_i q_i(a_i) - \sum_{j \neq i} q_j(a_j)$. Accordingly, since $R_i(a_j)$ is concave in a_j, the maximal effort player i would be willing to produce is found by replacing the condition that $R_i'(a_j) = 0$ in player i's first-order condition, and this results in a_i^{max} being uniquely defined identically as in (4). This in turn enables us to state the following result:

Proposition 5. *With $n > 2$ asymmetric players, there always exists a finite n^{AD} such that over-dissipation can be observed at equilibrium for $n > n^{AD}$.*

The proof of this statement is straightforward. Since a_i^{max} is uniquely defined by player i's characteristics (except its degree of overconfidence), the vector $\mathbf{a^{max}} = \{a_1^{max}, a_1^{max}, \ldots a_n^{max}\}$ of players' efforts, can always be implemented at equilibrium with a vector of overconfidence parameters $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \ldots \lambda_n\}$ such that $\lambda_i = \frac{\sum_{j\neq i} q_j(a_j)}{q_i(a_i)}$, $\forall i \in N$. Consequently, adding players implies that the aggregate efforts can always be made to equal $\sum_j a_j^{max}$, and this term will necessarily be larger than v for a large enough number of players.[8]

7 Implications of Overconfidence for the Dissipation Ratio

Introducing overconfidence in contests allows us to contribute to the literature on the dissipation ratio. When considering contests between two overconfident players with symmetric technology and preferences, we showed that overconfidence unambiguously reduces the equilibrium efforts of both contestants. With asymmetric players, we show in Proposition 3 that the dissipation ratio could increase following an increase in a player's overconfidence, provided the player is not overly overconfident. Yet, in Corollary 2 we show that extreme overconfidence even for a single player is sufficient to obtain extreme forms of under-spending, i.e., $D \to 0$, even with asymmetric players. It thus follows that with sufficiently symmetric players, or sufficiently overconfident players, overconfidence can indeed explain under-spending.

[8]Observe that unlike setups with rational agents where heterogeneity induces some players to be inactive in the contest, our own result holds true for any degree of heterogeneity since inefficient players (i.e. low impact function, or high cost function) are being compensated with a higher degree of overconfidence, which makes them willing to produce strictly positive efforts at equilibrium.

Our theoretical findings on under-spending complement well the existing literature pioneered by Tullock (1980) who emphasized the disproportionately small lobbying expenditures in relation to the stakes in many contexts of applied interest. In light of some recent evidence which uncovers that lobbyists are overconfident (Lyons et al. 2020), our article therefore connects different bits of empirical evidence and provides a theoretical foundation for observed phenomena of under-spending in lobbying.

The rationale uncovered in our study for explaining under-spending is quite straightforward: overconfident players are (mistakenly) convinced that they are able to optimally bid in a contest (e.g. in lobbying) with lower efforts than what they would need to provide if they were rational. As the competitors fully perceive and integrate in their reasoning this overconfidence bias, they, in turn, are equally incentivized to reduce their own bids for any degree of overconfidence they may themselves be subject to. Eventually, we end up with both players under-spending in the contest as compared to what rational players would have done.

Interestingly, the above results remain true with more than 2 players with symmetric technology, preferences, and beliefs, provided they are sufficiently overconfident. Indeed, if players are highly overconfident, we have shown in Proposition 4 that increases in the degree of overconfidence push all players to contract their equilibrium effort because they anyway each individually expect to win with a higher probability than $1/n$: increasing overconfidence in such instances implies that all players (mistakenly) expect to obtain a very high share of the prize even if they save on effort. Under-spending will therefore equally be observed with many players, provided contestants are highly overconfident.

Corollary 3 also uncovers that overconfidence with $n > 2$ players can lead to over-spending at equilibrium, and that with $n > n^D$ the excess spending can be such that we witness over-dissipation at equilibrium. The rationale of this rather counter-intuitive result rests in a feature of contests according to which increasing the efficiency (perceived or real) of a contestant whose expected equilibrium winning odds are low pushes him to increase his equilibrium effort. In the presence of many contestants, therefore, since all contestants individually expect to have a low probability of winning, some overconfidence will incentivize all contestants to increase their equilibrium efforts, and this may map in over-dissipation.

Our work thus points at overconfidence being an additional element that allows us to understand why under-spending may be observed in contexts of applied interest, while equally providing an potential explanation for observed over-spending and over-dissipation in lab experiments (e.g. Sheremeta 2010, 2011, Mago et al. 2013).

8 Conclusion

This paper studies the impact of overconfidence on contests. We assume an overconfident player overestimates his probability of winning the contest while holding a correct assessment of the winning prize and his cost of effort. We start by showing that in two player contests where players have the same technology and preferences, the most overconfident player exerts less effort and is therefore the Nash loser of the contest. We also show that an increase in overconfidence of either player lowers the efforts of both players. Next, we show that in two player contests where players can have different technology and preferences, for any advantage a player may have on his contest technology or cost function, a large enough overconfidence bias can always make that player the Nash loser in the contest. Finally, we show that in symmetric $n > 2$ player contests where all players are equally overconfident, an increase in overconfidence increases the efforts of all players provided that the bias is small relative to the number of players. With sufficiently high levels of overconfidence, on the other hand, an increase in overconfidence will lead to lower equilibrium efforts. Our findings provide conditions under which overconfidence lowers the dissipation ratio and is a potential explanation for Tullock's paradox: the overconfidence of lobbyists can lower their contributions to a policymaker. Finally, our paper also provides conditions under which over-spending and even over-dissipation can result from overconfidence. This may explain the observed over-spending in contest experiments. Moreover, it can also explain why women tend to bid more than men in experimental contests.

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9 Appendix

Proof of Lemma 1: The best response of player i, $i = \{1, 2\}$, is defined implicitly by (1). Hence, the slope of the best response of player i, $R_i'(a_j)$ is given by

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial a_j}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda_i q(a_i)-q(a_j)}{[\lambda_i q(a_i)+q(a_j)]^3}\lambda_i q'(a_i)q'(a_j)v}{\frac{q''(a_i)[\lambda_i q(a_i)+q(a_j)]-2\lambda_i[q'(a_i)]^2}{[\lambda_i q(a_i)+q(a_j)]^3}\lambda_i q(a_j)v - c''(a_i)}. \tag{7}$$

The denominator is player 1's second-order condition and so it is negative. Therefore, the sign of the slope of player 1's best response is only determined by the sign of the numerator which only depends on $\lambda_i q(a_i) - q(a_j)$. Hence, $R_i'(a_j)$ is positive for $\lambda_i q(a_i) > q(a_j)$, zero for $\lambda_i q(a_i) = q(a_j)$, and negative for $\lambda_i q(a_i) < q(a_j)$. This implies that $R_i(a_j)$ increases in a_j for $\lambda_i q(a_i) > q(a_j)$, reaches the maximum at $\lambda_i q(a_i) = q(a_j)$, and decreases in a_j for $\lambda_i q(a_i) < q(a_j)$.

Proof of Lemma 2: To prove that the equilibrium is unique, observe first that when the contestants' best responses cross it is impossible that they are both negatively slopped. Indeed, from 7 we know that if at equilibrium $R_i^{'}(a_j^*) < 0$, then it must be the case that $\lambda_i q(a_i^*) < q(a_j^*)$. This implies that $\lambda_j q(a_j^*) > q(a_i^*)$, so that $R_j^{'}(a_i^*) > 0$. To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_i) = R_i^{'}(a_j) \circ R_j^{'}(a_i)$ has a slope smaller than 1 for any equilibrium pair (a_i^*, a_j^*), since the function is continuous on $\mathbf{R}$. Having excluded that $R_i^{'}(a_j^*) < 0$ for both contestants, we simply need to prove that if $R_i^{'}(a_j^*) > 0$ for both players, then the product of the best response functions is smaller than 1. Since $R_i^{'}(a_j)$ is decreasing in $c^{''}(a_i)$, it is thus sufficient to establish the result for $c^{''}(a_i) = 0$. Rewriting the product of the contestants' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1))\left(q^{'}(a_1)q^{'}(a_2)\right)^2}{[q^{''}(a_1)[\lambda_1 q(a_1) + q(a_2)] - 2\lambda_1[q^{'}(a_1)]^2]\,[q^{''}(a_2)[\lambda_2 q(a_2) + q(a_1)] - 2\lambda_2[q^{'}(a_2)]^2]\,q(a_1)q(a_2)} < 1.$$

Since the LHS is decreasing in $q^{''}(a_i)$, $i = \{1, 2\}$, the above expression is *a fortiori* true if:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1)) \left(q^{'}(a_1) q^{'}(a_2)\right)^2}{4\lambda_1 [q^{'}(a_1)]^2 \lambda_2 [q^{'}(a_2)]^2 q(a_1) q(a_2)} < 1,$$

an expression that simplifies to:

$$(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1)) < 4\lambda_1 \lambda_2 q(a_1) q(a_2).$$

And this inequality is always satisfied.

Proof of Lemma 3: (This proof follows Baik 1994) Player i's best response is defined by (1):

$$\frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2} v - c'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{q(a_j) - \lambda_i q(a_i)}{[\lambda_i q(a_i) + q(a_j)]^3} q'(a_i) q(a_j) v.$$

We see that $\partial R_i(a_j)/\partial \lambda_i \gtreqless 0$ for $q(a_j) \gtreqless \lambda_j q(a_i)$. We also know from Lemma 1 that $sign\{R_i'(a_j)\} = -sign\left\{\frac{\partial R_i(a_j)}{\partial \lambda_i}\right\}$.

Substituting next $q(a_j) = \lambda q(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a_i^{max} we obtain

$$\frac{\lambda_i q'(a_i^{max}) \lambda q(a_i^{max})}{[\lambda_i q(a_i^{max}) + \lambda q(a_i^{max})]^2} v = c'(a_i^{max}),$$

or

$$\frac{\lambda_i^2 q'(a_i^{max}) q(a_i^{max})}{4\lambda_i^2 \left[q(a_i^{max})\right]^2} v = c'(a_i^{max}),$$

or

$$\frac{q'(a_i^{max})}{4q(a_i^{max})} v = c'(a_i^{max}).$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a_i^{max}, does not depend on λ_i.

Proof of Proposition 1: To prove this result we show that the best response of the more overconfident player crosses the 45 degree line at a lower value of effort than the best response of the less overconfident player. If player 1 is the more overconfident player, then $\lambda_i > \lambda_j > 1$. At the 45 degree line the best response of player 1 takes the value a_L given by

$$\frac{\lambda_i q'(a_L)}{(1+\lambda_i)^2 q(a_L)} v - c'(a_L) = 0. \tag{8}$$

At 45 degree line the best response of player 2 takes the value a_H given by

$$\frac{\lambda_j q'(a_H)}{(1+\lambda_j)^2 q(a_H)} v - c'(a_H) = 0. \tag{9}$$

Note that $\lambda_i > \lambda_j$ implies

$$\frac{\lambda_i}{(1+\lambda_i)^2} < \frac{\lambda_j}{(1+\lambda_j)^2}. \tag{10}$$

Therefore, (8), (9), and (10) imply

$$\frac{q'(a_H)}{q(a_H)c'(a_H)} < \frac{q'(a_L)}{q(a_L)c'(a_L)}.$$

Given that $q(.)$ is (weakly) concave and that $c(.)$ is (weakly) convex, this inequality can only be satisfied provided $a_L < a_H$.

Proof of Lemma 4: Assume first that $\lambda_1 = \lambda_2 > 1$ and $a_1^{max} = a_2^{max}$, so that $a_1^* = a_2^*$ and the best response functions cross on the 45^o line. Consider then any change leading to an increase in a_1^{max}, so that $a_1^{max} > a_2^{max}$. This will be the case if $c_i^{'}(a_i)$ gets lower, if $q_i^{'}(a_i)$ gets lower, or if $q_i(a_i)$ gets higher. Upon observing the first-order condition (3) we see that any such change leads to an increase of $R_i(a_j)$ for any effort level of the rival. Consequently, since the two best response functions both start at the origin of the graph and are strictly concave in the rival's effort, it is necessarily the case that after such a change we have $a_2^* < a_1^*$.

Proof of Lemma 5: By reproducing the steps in the proof of Lemma 3 for the present case with asymmetric players, we deduce that $\partial R_i(a_j)/\partial \lambda_i \lesseqgtr 0 \Leftrightarrow q_j(a_j) \lesseqgtr \lambda_i q_i(a_i)$. Fix λ_1. By Lemma 4 we know that when $\lambda_1 = \lambda_2 > 1$ and $a_1^{max} > a_2^{max}$ then $a_1^* > a_2^*$, which *de facto* implies that if $q_1(a_1^*) < \lambda_2 q_2(a_2^*)$, then the best response function of player 2 shifts down with the overconfidence level of player 2 and we necessarily have that $\partial a_1^*/\partial \lambda_2 < 0$, $\partial a_2^*/\partial \lambda_2 < 0$, and, by the concavity of $R_1(a_2)$ we also have that $\partial(a_1^*/a_2^*)/\partial\lambda_2 > 0$ and thus for $\lambda_2 > \lambda_1$, we necessarily have $a_1^* > a_2^*$. Consider next the case where $q_1(a_1^*) \geq \lambda_2 q_2(a_2^*)$ so that $R_2^{'}(a_1^*) < 0$. In such a case, observe that for $\lambda_1 = \lambda_2 = 1$, the best response function of player 2 hits first the 45^o line, and for any $\lambda_2 > 1$, the value of a_1 for which $R_2(a_1)$ is maximized is larger since $\partial R_2(a_1)/\partial\lambda_2 < 0$ for $q_1(a_1) < \lambda_2 q(a_2)$. It thus follows that the crossing between the two best response functions must occur for values $a_1^* > a_2^*$.

Proof of Proposition 2: Observe that the best response function of any player i does not depend on λ_j. To establish the result, consider the equation $q_1(a_1) = q_2(a_2)$, or $a_2 = q_2^{-1}(q_1(a_1))$. Define next by $\tilde{a}_1$ the effort of player 1 such that the best response of player 2 commands him to exert an effort such that $q_1(a_1) = q_2(a_2)$. Replacing for this equality in $R_2(a_1)$, this condition reads as:

$$\frac{\lambda_2 q_2^{-1'}(q_1(\tilde{a}_1))}{[1+\lambda_2]^2 q_1(\tilde{a}_1)}v - c^{'}(q_2^{-1}(q_1(\tilde{a}_1)) = 0.$$

Observe that for any finite values λ_2, the above expression is satisfied for a strictly positive value $\tilde{a}_1$.

To establish our result, we then demonstrate that the value of α_1 that commands player 1 exert an effort such that $q_1(a_1) = q_2(a_2)$, a value we shall denote by $\check{a}_1$, is such that $\check{a}_1 < \tilde{a}_1$ for high enough values of λ_1. Replacing for $q_1(a_1) = q_2(a_2)$ in $R_1(a_2)$, we obtain:

$$\frac{\lambda_1 q_1^{'}(\check{a}_1)}{[1+\lambda_1]^2 q_1(\tilde{a}_1)}v - c_1^{'}(\tilde{a}_1) = 0.$$

Since the limit of the first term when λ_1 goes to infinity is zero, it is immediate to deduce that $\lim_{\lambda_1\to\infty} \check{a}_1 = 0$. Consequently, because of the reaction functions' concavity, it is necessarily the case that as $\lambda_1 \to \infty$, then $q_1(a_1^*) < q_2(a_2^*)$.

Proof of Corollary 2 Recall that $R_1(a_2)$ is defined by:

$$\frac{\lambda_1 q_1(a_1) q_2(a_2)}{[\lambda_1 q_1(a_1) + q_2(a_2)]^2}v - c_1^{'}(a_1) = 0.$$

It follow that for any $a_2 \geq 0$, $\lim_{\lambda_1 \to \infty} a_1(a_2) = 0$. Consider next the reaction function of player 2:

$$\frac{\lambda_2 q_2(a_2) q_1(a_1)}{\left[\lambda_2 q_2(a_2) + q_1(a_1)\right]^2} v - c_2^{'}(a_2) = 0.$$

For any finite value of λ_2, if $a_1 \to 0$, the above expression tends to $-c_2^{'}(a_2)$, and since $c_2(.)$ is convex, this implies that $a_2^* \to 0$.

Proof of Proposition 3: Observe first that there can be only three cases, since the fact that $\lambda_1 > 1$ and $\lambda_2 > 1$ precludes the possibility to have $\lambda_i q(a_i^*) < q(a_j^*)$, $i \neq j \in \{1, 2\}$.

If $\lambda_i q(a_i^*) > q(a_j^*)$, $i \neq j \in \{1, 2\}$, then $R_i^{'}(a_j^*) > 0$ for both players at equilibrium, and $\partial R_1(a_2)/\partial \lambda_1 < 0$. It then follows that $\partial a_1^*/\partial \lambda_1 < 0$ and $\partial a_2^*/\partial \lambda_1 < 0$.

If $\lambda_1 q(a_1^*) > q(a_2^*)$ and $\lambda_2 q(a_2^*) < q(a_1^*)$, then $R_1^{'}(a^*2) > 0$, $R_2^{'}(a^*1) < 0$, and $\partial R_1(a_2)/\partial \lambda_1 < 0$. It then follows that $\partial a_1^*/\partial \lambda_1 < 0$ and $\partial a_2^*/\partial \lambda_1 > 0$, since $R_1(a_2)$ will contract along the decreasing part of $R_2(a_1)$.

Last if $\lambda_1 q(a_1^*) < q(a_2^*)$ and $\lambda_2 q(a_2^*) > q(a_1^*)$, then $R_1^{'}(a^*2) < 0$, $R_2^{'}(a^*1) > 0$, and $\partial R_1(a_2)/\partial \lambda_1 > 0$. It then follows that $\partial a_1^*/\partial \lambda_1 > 0$ and $\partial a_2^*/\partial \lambda_1 > 0$.

Proof of Proposition 4: We begin by imposing symmetry so that $a_i = a_j = a^*$, $\forall i, j \in N$. Consequently, at equilibrium the first-order condition (5) reads as:

$$\frac{\lambda q'(a^*)(n-1)q(a^*)}{\left[\lambda q(a^*) + (n-1)q(a^*)\right]^2} v - c'(a^*) = 0,$$

or

$$\frac{\lambda(n-1)q'(a^*)}{(\lambda + n - 1)^2 q(a^*)} v - c'(a^*) = 0.$$

To inspect the sign of $\partial a^*/\partial \lambda$ we apply the implicit function theorem to the above expression to obtain:

$$\begin{aligned} \frac{\partial a^*}{\partial \lambda} &= -\frac{\frac{(n-1)(\lambda+n-1)^2 - 2(\lambda+n-1)\lambda(n-1)}{(\lambda+n-1)^4} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)} \\ &= -\frac{\frac{(n-1)(n-1-\lambda)}{(\lambda+n-1)^3} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)}. \end{aligned}$$

Since the denominator of this expression is unambiguously negative, the sign of the expression is therefore given by the sign of $(n - 1 - \lambda)$.

Proof of Corollary 2: Consider n symmetric rational players. Their equilibrium effort is given by:

$$\frac{(n-1)q'(a^*)}{(n-1)^2 \, q(a^*)} v - c'(a^*) = 0.$$

Since it is immediate to show that $da^*/dn < 0$, it follows that the optimal *individual* effort is maximal when $n = 2$. Now, from Proposition 4 we know that for any given n the maximal individual effort obtains when $n = \lambda + 1$. Observe that the equilibrium effort when $n = \lambda + 1$ is the same as the maximal individual effort that the game admits, i.e. it is the same as when $n = 2$ and $\lambda = 1$. Indeed, this will is true since the maximal individual effort is given by:

$$\frac{q'(a^*)}{4q(a^*)} v - c'(a^*) = 0,$$

and for any $n > 2$, the players' equilibrium individual efforts will equal this value if $\frac{\lambda(n-1)}{(\lambda+n-1)^2} = 1/4$, an equality that is true if $n = \lambda + 1$.

Since $a^* > 0$, and since respecting $n = \lambda + 1$ implies that the dissipation ratio is given by $D = \frac{na^*}{v}$, there always exists a finite value of n above which over-dissipation can be observed at equilibrium.