Confidence and Gender Gaps in Competitive Environments

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Abstract

This paper analyzes theoretically how confidence gaps affect behavior in tournaments as well as in contests. An overconfident player overestimates his ability and hence his probability of winning. Our results help organizing experimental evidence on gender gaps in outcomes and behavior in tournaments and contests. Namely, the fact that in experiments men often tend to exert more effort than women in a tournament, whereas the opposite holds in a contest. Lastly, in contests with $n > 2$ players, overconfidence can raise the equilibrium efforts above the Nash prediction with rational players (overspending), and even lead to aggregate efforts larger than the prize (over-dissipation).

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1 Introduction

Many economic studies document systematic gender gaps in outcomes and behavior. There is a gap on how much women earn. For example, in 2022 women in the US earn 82% of their male counterparts (Kochhar 2023). There is also a gap in the number of women in top business positions. For instance, women represent 6% of top business executives in the US (Keller et al. 2022). The wage gender gap is larger in high skilled work, and much of it seems to be caused by gaps in promotions (Blau and DeVaro 2007, Blau and Kahn 2017, Bronson and Thoursie 2020).

Experimental evidence from economics also documents gender gaps. The influential article by Gneezy et al. (2003) shows that competing in a tournament causes males to increase their performance by more than females. The seminal article by Niederle and Vesterlund (2007) finds that men are more likely to enter a tournament than women. The importance of both studies has stimulated much experimental research on this topic, and the gender gap in tournament entry has been replicated in many other studies (Markowsky and Beblo 2022).[1]

Intriguingly, experimental evidence on contests produces quite different gender gaps in behavior as compared to the experimental literature on tournaments. Indeed, some scholars find that females invest higher effort than males (Anderson and Stafford 2003, Mago et al. 2013, Price and Sheremeta 2015, Mago and Razzolini 2019). For example, Mago and Razzolini (2019) document a gender gap whereby females spend more effort than males. Females also tend to bid more than males in experimental all-pay auction contests (Ham and Kagel 2006, Charness and Levin 2009, Hyndman et al. 2012, Breaban et al. 2020).

Such gender gaps in outcomes and behavior documented in experimental studies are due to gender differences in confidence and preferences (Croson and Gneezy 2009). Most of the literature on gender and tournament entry finds that men are

[1] There is also a gender gap in performance in highly competitive academic exams (Jurajda and Münich 2011, Ors et al. 2013), although this gap is not directly comparable with the more controlled environment of the laboratory.

more confident than women (Buser et al. 2021b). In fact, three recent studies argue that the gender gap in tournament entry is mainly driven by gender differences in confidence and risk attitudes (Gillen et al. 2019, Price 2020, van Veldhuizen 2022).[2]

Even though the experimental literature has identified gender gaps in confidence as one of the drivers of gender gaps in behavior in both tournaments and contests, to date no theoretical explanation has been proposed to rationalize these findings, and the apparent differences in behavior across the two types of competition. Understanding the mechanisms driving these gender gaps is important in order to optimally design affirmative action policies (e.g. Calsamiglia et al. 2013, Niederle et al. 2013).

In this paper we analyze theoretically how confidence gaps affect behavior in tournaments as well as in contests. These two types of models both describe competitive environments where a player's probability of winning is an increasing function of his effort. However, they also differ since in a tournament, unlike in a contest, noise can play an important role and a player may win the prize with zero effort.[3]

We set-up a two player tournament model where players are identical in all respects except their confidence. An overconfident player overestimates his ability and ability and effort are complements.[4] One player is overconfident, whereas the other player can be either overconfident, rational, or underconfident. We assume noise follows a standard Gumbel distribution and fully characterize the equilibrium of a Lazear-Rosen rank-order tournament. Following earlier literature, we show that a generalized Tullock (1980) contest is a special case of the tournament (Hirshleifer and Riley 1992, Jia et al. 2013, Ryvkin and Drugov 2020), and we fully characterize the

[2]Gender and socio-economic differences in confidence are also important determinants of educational and career choices (Buser et al. 2014, Wiswall and Zafar 2015, Reuben et al. 2017, Guyon and Huillery 2021).

[3]Promotions in firms, sports competitions, election campaigns, rent-seeking games, R&D races, competition for monopolies, litigation, wars are examples of tournaments and contests.

[4]Moore and Healy (2008) distinguish between three types of overconfidence: overestimation of one's skill (absolute overconfidence), overplacement (relative oveconfidence), and excessive precision in one's beliefs (miscalibration or overprecision). Our own study conceptualizes overconfidence as an overestimation of one's skill or ability, thence precluding the third type of overconfidence.

equilibrium of such a contest. We also extend the analysis to n-player tournaments and contests where all players are equally overconfident.

Our theoretical results uncover that confidence gaps may have different implications for behavior in tournaments than in contests. In a two player tournament where the confidence gap is small and neither player is too overconfident, the more overconfident player exerts higher effort at equilibrium. However, for a sufficiently large confidence gap, the more overconfident player exerts lower effort at equilibrium. The intuition behind these results lies in the following trade-off. Players aim at exploiting the complementarities between confidence and effort while attempting to save on cost of effort. When neither player is too overconfident, an increase in the confidence of the most confident player raises his effort because the increase in the perceived probability of winning times the utility prize spread is greater than the associated increase in cost of effort. For high levels of confidence, on the other hand, there is limited scope for further increasing one's perceived probability of winning, thence implying that the latter effect dominates and the more overconfident player exerts lower effort.

In contrast, in a contest opposing two overconfident players the more overconfident player always exerts lower effort at equilibrium. Indeed, an increase in overconfidence leads to a drop in the perceived marginal probability of winning in a contest. Consequently, the scope for further improving one's winning probability will be reduced, thereby incentivizing the overconfident player to reduce effort and save on costs of effort. Observe, however, that in a contest opposing an overconfident to an underconfident player, the more confident player may exert a higher effort when the confidence gap is not too large.

Our theoretical results on the effect of the confidence gap on equilibrium efforts in tournaments and contests are able to organize the experimental evidence. In a tournament, the complementarity between confidence and effort implies that the most overconfident player exerts more effort when neither player is too overconfident. This is consistent with the experimental findings in Gneezy et al. (2003), Gneezy and

Rustichini (2004), Niederle et al. (2013), Buser et al. (2021b), and van Veldhuizen (2022), who show that men perform better than women in tournaments. In contrast, in a two player contest featuring no underconfident player, the most overconfident player always exerts lower effort. This is consistent with the experimental evidence in Anderson and Stafford (2003), Mago et al. (2013), Price and Sheremeta (2015), and Mago and Razzolini (2019) who show that women invest more effort than men in contests.

We also study theoretically the effect of overconfidence on single-prize tournaments and contests with $n > 2$ players. To keep the analysis tractable, we restrict our attention to the case where players are equally confident. In both tournaments and contests, we show that, for a given number of players, the equilibrium efforts are higher when players are mildly overconfident than if they were rational. However, for high levels of overconfidence, the equilibrium efforts are lower than if the players were rational. If there is a high number of players and/or a low degree of overconfidence, then the players have a small perceived probability of winning. In such instances, an increase in overconfidence leads to a large perceived marginal probability of winning and therefore it is beneficial to raise effort. On the contrary, if the number of players is low and/or there is a high degree of overconfidence, the players have a large perceived probability of winning. In such instances, an increase in overconfidence leads to a small perceived marginal probability of winning and therefore it is beneficial to lower effort. In addition, we show that in $n > 2$ player contests, moderate overconfidence can not only raise the equilibrium efforts above the Nash prediction with rational players (overspending), but even lead to aggregate efforts larger than the prize (over-dissipation).

Our theoretical results in n player contests address the puzzle that contestants in lab experiments spend significantly higher amounts than the game's Nash equilibrium (Price and Sheremeta 2015, Mago et al. 2016), and that even over-dissipation can occur (Sheremeta 2011, Lim et al. 2014).

The paper is organized as follows. Section 2 discusses related literature. Section

3 sets-up the general model. Sections 4 and 5 derive, respectively, the results for two player and $n > 2$ player tournaments. Sections 6 and 7 derive, respectively, the results for two player and $n > 2$ player contests. Section 8 concludes the paper. All proofs are in the Appendix.

2 Related Literature

This study relates to four strands of literature. First, it contributes to the large literature on gender and competition. This literature documents gender gaps in outcomes and behavior and tries to identify the drivers behind these gaps (Bertrand and Hallock 2001). Several explanations have been given for the gender pay gap, including discrimination (Becker 1971), differences in human capital (Mincer and Polachek 1974), in ability in non-market activities (Lazear and Rosen 1990), in risk attitudes (Eckel and Grossman 2003), or in mobility (Goldin et al. 2017).

Beyond the aforementioned factors, economic experiments find gender differences in confidence and preferences—risk, social, and competitiveness—(Croson and Gneezy 2009). In particular, gender gaps in tournament entry and behavior have been experimentally shown to be driven by gender differences in confidence and risk attitudes (Niederle and Vesterlund 2007, Gillen et al. 2019, Price 2020, van Veldhuizen 2022), alongside competitiveness (Niederle and Vesterlund 2007).[5] Our study focuses on the effect of gender differences in confidence on behavior and outcomes in tournaments and contests. We provide theoretical results that help organizing experimental findings on gender gaps in tournaments and contests.

The experimental evidence on gender and tournaments points to three main regularities. First, men tend to perform better than women, especially in mixed gender tournaments where gender is observable (Gneezy et al. 2003, Gneezy and Rustichini 2004, Niederle et al. 2013, Buser et al. 2021b, van Veldhuizen 2022). Second,

[5] There is an ongoing debate as to the relative importance of these different explanations (Niederle 2017, Gillen et al. 2019, Buser et al. 2021a, Lozano and Reuben 2022, van Veldhuizen 2022).

men enter more in tournaments than women (Niederle and Vesterlund 2007, Niederle 2016, Markowsky and Beblo 2022). Third, the gender difference in entry seems to be mainly driven by two factors: (1) men are more confident about their performance than women, and (2) men are less risk averse than women (Gillen et al. 2019, Price 2020, van Veldhuizen 2022). Our theoretical results confirm that a confidence gap between two players in a tournament can lead the most confident player to outperform the other, thereby providing a theoretical explanation to the findings of Gneezy et al. (2003), Gneezy and Rustichini (2004), Niederle et al. (2013), Buser et al. (2021b), and van Veldhuizen (2022). We also highlight the conditions under which gender gaps in confidence will lead men to be overly represented at CEO positions. Even though we do not study the decision to enter a tournament, our results also suggest that a gender gap in confidence can lead to a significant increase in the perceived expected utility of the more confident player, especially in situations where the confidence gap leads to a decrease in the equilibrium effort of the least confident player.

The experimental evidence on gender differences in contests, which is less abundant, shows that women tend to invest higher effort than men (Anderson and Stafford 2003, Mago et al. 2013, Price and Sheremeta 2015, Mago and Razzolini 2019). Our theoretical results confirm that a confidence gap between two players in a contest can lead the most confident player to exert lower. In particular, this is always the case if the less confident player is not underconfident.

Second, our study contributes to the theoretical literature on behavioral biases in tournaments. Goel and Thakor (2008) study tournaments where overconfident and unbiased managers compete for promotion to become CEO by choosing the level of risk of their projects. They find that overconfident managers, those who underestimate project risk, have a higher likelihood of being promoted to CEO than unbiased ones. Santos-Pinto (2010) studies a two player tournament where both players are equally overconfident and shows that the tournament organizer can exploit the players' perceptual bias. We broaden the study of overconfidence in tournaments (i) by allowing players to differ in their confidence levels, and (ii) by considering tournaments with more than 2 players. Moreover, we clarify the conditions under which managers who are overconfident about their abilities are more likely to be promoted or not to a CEO position.

Third, it contributes to the theoretical literature on behavioral biases in contests. The most closely related studies are Ando (2004) and Ludwig et al. (2011). Ando (2004) studies a two player contest where overconfidence is an overestimation of the monetary value of winning the contest. Ludwig et al. (2011) analyze a contest where an overconfident player competes against a rational player and overconfidence is an underestimation of the cost of effort. Our results show that when overconfidence is an overestimation of own ability and consequently of the winning probability, its effects on effort provision are quite different than those in found in Ando (2004) and Ludwig et al. (2011). The differences in the results are driven by the fact that overconfidence in our setup raises the marginal perceived utility from winning for low values of effort whereas it lowers it for high value of effort. As a consequence, and in contrast to Ando (2004) and Ludwig et al. (2011), in our study, overconfidence shifts a player's best response in a non-monotonic way.

Baharad and Nitzan (2008) and Keskin (2018) amend the standard model of contests by introducing probability weighting in line with Tversky and Kahneman's (1992) Cumulative Prospect Theory. This behavioral bias is modeled with an inverse S-shaped probability weighting function, i.e., a function where the marginal increase in the (perceived) subjective probability is higher for extreme (i.e. low and high) probabilities. Our own approach assumes a constant bias in players' beliefs that they are better than they really are at contesting their opponents. We thus see our approach as complementary to these earlier works since nothing precludes players from both assigning 'weights' to probabilities and be subject to an overconfidence bias. Notice that in terms of contribution to the literature on behavioral biases in contests, our approach has the advantage to be flexible enough to accommodate a very large family of contest success functions.

Finally, our study also contributes to the literature on overdissipation in contests. Scholars have long tried to explain the puzzle that contestants in lab experiments sometimes spend significantly higher amounts than the game's Nash equilibrium (Price and Sheremeta 2015, Mago et al. 2016), and even over-dissipation can occur (Sheremeta 2011). Overspending has so far been attributed to players' risk attitudes (Jindapon and Whaley 2015) or to mixed strategy equilibria where overspending occurs with some probability but not in expectation (Baye et al. 1999). Our paper demonstrates that with overconfident contestants, overspending and even over-dissipation can result when the number of players is sufficiently large and the overconfidence bias is relatively mild; overconfident players individually expend more effort than rational players when their odds of winning are low because of the high number of participants.

3 Set-up

Consider two players, i and j, competing in a tournament. The player who produces the highest output receives the winner's prize y_W and the other receives the loser's prize y_L, with $0 < y_L < y_W$. The players are expected utility maximizers and have utility functions that are separable in income (y_k) and effort ($a_k \in \mathbb{R}^+$), $k = i, j$. Player i's utility function (likewise for j) is given by:

$$U_i(y_i, a_i) = u(y_i) - c(a_i).$$

We assume u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' \geq 0$, $c(0) = 0$, $c'(0) = 0$, and $c(a_i) = \infty$, for $a_i \to \infty$, where the last two conditions ensure that equilibrium effort is strictly positive but finite. The two players have an outside option which guarantee each $\bar{u} \geq 0$. We assume $\bar{u} = 0$.

When player i exerts effort a_i his output is given by

$$Q_i = h(q(a_i)) + \varepsilon_i, \tag{1}$$

where both $h(.)$ and $q(.)$ are increasing functions. We assume that ε_i follows a standard Gumbel distribution, that is, its density function is $f(\varepsilon_i) = e^{-\varepsilon_i - e^{-\varepsilon_i}}$, and its cumulative distribution function is $F(\varepsilon_i) = e^{-e^{-\varepsilon_i}}$. This noise distribution enables us to fully characterize the equilibrium in Lazear and Rosen (1981) rank-order tournaments with overconfident players. Moreover, the difference between two Gumbel random variables with the same variance follows a logistic distribution, which has a similar shape to the Normal distribution. Lastly, as shown below, this assumption also enables us to characterize generalized Tullock contests as a particular form of tournament with the same definition of overconfidence.

The two players can differ from one another in terms of their beliefs about their productivity of effort while holding a correct assessment of the winning prize and their cost of effort. An overconfident player i overestimates his productivity of effort, that is, he thinks his output function is

$$\tilde{Q}_i = h(\lambda_i q(a_i)) + \varepsilon_i,$$

where $\lambda_i > 1$. Under this specification player i perceives his marginal output is increasing with his overconfidence bias λ_i, that is, $\partial^2 \tilde{Q}_i / \partial a_i \partial \lambda_i > 0$. This describes situations where effort and ability are complements in generating output and where an overconfident worker overestimates his ability. This way of modeling overconfidence is often used in the literature that analyzes its impact on labor contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). An overconfident player i ($\lambda_i > 1$) competes against a player j that can be either overconfident ($\lambda_j > 1$), rational ($\lambda_j = 1$), or underconfident ($0 < \lambda_j < 1$).

Hence, player i's perceived probability of winning the tournament is

$$\begin{aligned} P_i(a_i, a_j, \lambda_i) &= \Pr(\tilde{Q}_i \geq Q_j) \\ &= \Pr(h(\lambda_i q(a_i)) + \varepsilon_i \geq h(q(a_j)) + \varepsilon_j) \\ &= \Pr(\varepsilon_j - \varepsilon_i \leq h(\lambda_i q(a_i)) - h(q(a_j))) \\ &= \frac{1}{1 + e^{-(h(\lambda_i q(a_i)) - h(q(a_j)))}}. \end{aligned}$$

Player i chooses the optimal level of effort that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j, \lambda_i)] = u(y_L) + P_i(a_i, a_j, \lambda_i)\Delta u - c(a_i), \tag{2}$$

where $\Delta u = u(y_W) - u(y_L)$.

To be able to derive equilibria when players hold mistaken beliefs we adopt Squintani's (2006) definition of what he terms a naïve equilibrium, namely equilibria where "despite their possibly mistaken self-perceptions, the players are rational (i.e. utility maximizing) and correctly anticipate each other's strategies, without attempting to make sense of them." (Squintani 2006: 617). As such we assume that: (1) a player who faces a biased opponent is aware that the latter's ability perception (and probability of winning) is mistaken, (2) each player thinks that his own ability perception (and probability of winning) is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players' agree to disagree about their ability perceptions (and winning probabilities).

These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007). As stated by Pronin et al. (2002: 369) "people recognize the existence, and the impact, of most of the biases that social and cognitive psychologists have described over the past few decades. What they *lack* recognition of, we argue, is the role that those same biases play in governing their *own* judgments and inferences." For example, Libby and Rennekamp (2012) conduct a survey which shows that experienced financial managers believe that other managers are likely to be overconfident while failing to recognize their own overconfidence. Hoffman (2016) runs a field experiment which finds that internet businesspeople recognize others tend to be overconfident while being unaware of their own overconfidence.[6]

[6]Ludwig and Nafziger (2011) conduct a lab experiment that elicits participants' beliefs about own and others' overconfidence and abilities. On the one hand they find that the largest group of

Note that throughout the paper we assume players are fully identical except for their beliefs. This assumption allows us to isolate the effect of confidence gaps on behavior. In addition, most of the experimental evidence we are going to relate our theoretical results to assumes players have identical abilities/productivities and outside options.

In Sections 4 and 5 we analyze the effect of overconfidence in the canonical Lazear-Rosen rank-order tournament where $\tilde{Q}_i = \lambda_i a_i + \varepsilon_i$. In this case player i's perceived probability of winning the tournament is:

$$P_i(a_i, a_j, \lambda_i) = \frac{1}{1 + e^{-(\lambda_i a_i - a_j)}}.$$

In Sections 6 and 7 we analyze the effect of overconfidence in a generalized Tullock contest. When $h(.) = \ln(.)$, the tournament collapses into a Tullock contest since player i's perceived probability of winning becomes:

$$P_i(a_i, a_j, \lambda_i) = \frac{1}{1 + e^{-\ln(\lambda_i q(a_i)/q(a_j))}} = \frac{1}{1 + \frac{q(a_j)}{\lambda_i q(a_i)}} = \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)},$$

where function $q(a)$ is often referred to as the impact function (Ewerhart 2015) and models the technology whereby players' efforts or investments translate into probabilities of winning the contest.

Note that this specification of overconfidence satisfies three desirable properties. First, the overconfident player's perceived winning probability is well defined for any value of λ_i. Second, the overconfident player's perceived winning probability is increasing in λ_i. Third, in a contest, overestimating one's ability (or impact function) is equivalent to underestimating the rivals' ability.

4 Overconfidence in Tournaments

In this section we analyze the effect of the confidence gap on a two player Lazear and Rosen (1981) rank-order tournament. Player i chooses the optimal effort level

participants thinks that they are themselves better at judging their ability correctly than others. On the other hand, they find that with a few exceptions, most people believe that others are unbiased.

that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = u(y_L) + \frac{1}{1 + e^{-(\lambda_i a_i - a_j)}} \Delta u - c(a_i).$$

The first order-conditions for players i and j are as follows:

$$\begin{aligned} \lambda_i \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1 + e^{-(\lambda_i a_i - a_j)}\right]^2} \Delta u &= c'(a_i) \\ \lambda_j \frac{e^{-(\lambda_j a_j - a_i)}}{\left[1 + e^{-(\lambda_j a_j - a_i)}\right]^2} \Delta u &= c'(a_j) \end{aligned}$$

The second-order conditions are then given by:

$$-\lambda_i^2 \frac{1 - e^{-(\lambda_i a_i - a_j)}}{\left[1 + e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \Delta u - c''(a_i) < 0,$$
$$-\lambda_j^2 \frac{1 - e^{-(\lambda_j a_j - a_i)}}{\left[1 + e^{-(\lambda_j a_j - a_i)}\right]^3} e^{-(\lambda_j a_j - a_i)} \Delta u - c''(a_j) < 0.$$

We assume in what follows that these second-order conditions are satisfied.

Lemma 1. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $a_j = \lambda_i a_i$.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

A second useful lemma describes how player i's best response changes with his confidence λ_i.

Lemma 2. *An increase in player i's confidence λ_i leads to an expansion of his best response function, $\partial R_i(a_j)/\partial \lambda_i > 0$, for $e^{-(\lambda_i a_i - a_j)} > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$ and to a contraction of his best response, $\partial R_i(a_j)/\partial \lambda_i < 0$, for $e^{-(\lambda_i a_i - a_j)} < \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$. Moreover, the maximum value of player i's best response increases in player i's confidence λ_i.*

Lemma 2 characterizes the best response of players who are subject to a confidence bias. For a high effort of the rival, an increase in confidence raises player

i's effort level. For low effort of the rival, however, depending on the size of the bias, an increase in confidence can either expand or contract player i's best response. Moreover, the maximal value taken by player i's best response is increasing in his confidence bias. Making use of these results, we can establish equilibrium uniqueness in the following lemma:

Lemma 3. *A two player tournament featuring an overconfident player* 1 *admits a unique equilibrium if* $\lambda_1 \lambda_2 \geq 1$.

We next present our main result on the effect of the confidence gap on the tournament equilibrium efforts.

Proposition 1. *Consider a two player tournament where player* 1 *is overconfident and* $\lambda_1 \lambda_2 \geq 1$. *For any level of confidence* λ_2 *of player* 2*, there is a threshold value of player* 1*'s overconfidence bias* $\bar{\lambda}_1(\lambda_2) \geq \lambda_2$ *such that* $a_1^* > a_2^*$ *if* $\lambda_1 < \bar{\lambda}_1(\lambda_2)$ *and* $a_1^* < a_2^*$ *otherwise. There exists a threshold value of* λ_2 *that we denote by* $\bar{\lambda}_2$ *such that if* $\lambda_2 < \bar{\lambda}_2$*, then* $\bar{\lambda}_1(\lambda_2) > \lambda_2$*, otherwise* $\bar{\lambda}_1(\lambda_2) = \lambda_2$.

This proposition uncovers that the effects of the confidence gap on a player's equilibrium effort depend on the size of the gap as well as on the confidence level of the least confident player. If the confidence gap is small and neither player is too overconfident, then the more overconfident player 1 exerts more effort at equilibrium. Hence, the more confident player 1 will be the Nash winner since $P_1(a_1^*, a_2^*) > 1/2$. In contrast, if the confidence gap is large, then the more overconfident player 1 exerts less effort at equilibrium and is therefore the Nash loser.

The intuition behind this result is as follows. Note that if the players are equally confident, they exert the same equilibrium effort, $a_1^* = a_2^*$. First, if the confidence gap is small and neither player is too overconfident, an increase in player 1's overconfidence expands his best response at equilibrium. Given the quasi-concavity of the players' best responses, this in turn implies that a_1^*/a_2^* will increase.

We illustrate Proposition 1 in Figures 1 and 2. On Figure 1, we represent the best responses when both players are overconfident and the confidence gap is small. From

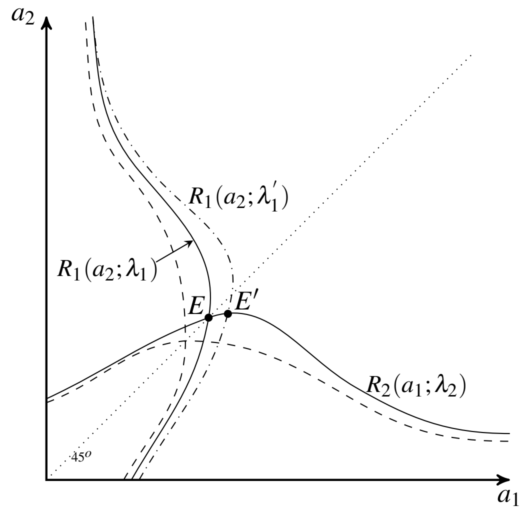

Figure 1: Equilibrium in a Tournament with a Small Confidence Gap

Lemma 1, we know that the best responses are quasi-concave. To better gauge the effect of overconfidence, we have also drawn the best responses of rational players as depicted by the two dashed curves crossing on the 45^o line of the left panel of Figure 1. The equilibrium when both players are equally overconfident is depicted by point E and involves higher equilibrium efforts than if both players were rational. Increasing the overconfidence of player 1 implies that his best response shifts outwards (see Lemma 2) as represented by the dashed and dotted best response. Consequently, since efforts are strategic complements in this range, the new equilibrium, E', has the more confident player 1 exerting higher effort. On the right panel of Figure 1 we depict a situation where the confidence gap is larger, leading the less confident player 2 to reduce his equilibrium effort, which corresponds to the shift from equilibrium E' to E''.

Second, if the confidence gap is large (player 1 is highly overconfident and player 2 is not), then an increase in player 1's overconfidence leads to a contraction of his best response at equilibrium. This in turn leads to a reduction of a_1^*/a_2^*, and if player 1 is sufficiently overconfident, then player 1 will exert a lower effort than player 2 at equilibrium. Third, if both players are highly overconfident, an increase in player 1's overconfidence always leads to a contraction of his best response at equilibrium.

Figure 2 depicts equilibrium efforts in the case where the confidence gap is large. In this case, the highly overconfident player 1 exerts less effort than the less confident player 2.

Even though we do not model the choice between a piece-rate and the tournament, as in Niederle and Vesterlund (2007), the above analysis uncovers instances where the more overconfident player will be overly attracted to enter the tournament. In the case depicted in Figure 1 where the confidence gap is small and neither player is too overconfident, the equilibrium perceived winning probability of the more overconfident player 1, $P_1(a_1^*, a_2^*, \lambda_1)$, will be larger than the true winning probability for two reasons. First, a higher confidence for given efforts raises the perceived winning probability. Second, player 1 equally anticipates he will exert a higher equilibrium effort that player 2. This will increase the perceived equilibrium expected utility of player 1 if his effort provision does not increase too much, as is the case when player 2 is not too overconfident. Even in the case depicted in Figure 2 where the confidence gap is large, and the more confident player exerts lower effort at equilibrium, the equilibrium perceived probability of the more overconfident player 1 could be larger than the rival's. This in turn would lead to a higher perceived expected utility. Interestingly, even if the equilibrium perceived probability of the more overconfident player 1 is lower than the rival's, the drop in effort by player 1 would still make the tournament more attractive than to the less confident player 2.

Proposition 1 helps organizing the evidence on gender gaps in performance in tournaments. Several studies find a gender gap in performance in tournaments, whereby men perform better than women (Gneezy et al. 2003, Gneezy and Rustichini 2004, Niederle et al. 2013, Buser et al. 2021b, van Veldhuizen 2022). For example, Gneezy et al. (2003) set up a six player single-prize tournament featuring three female and three male participants who compete by solving mazes online. In their setup where participants observe each other's gender, the authors find a significant

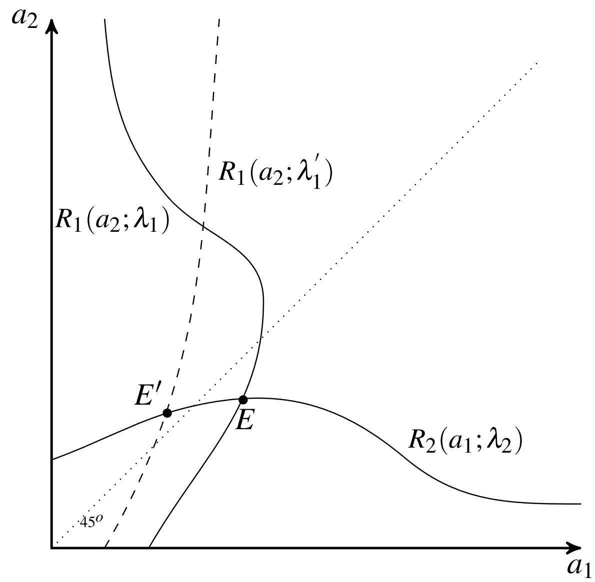

Figure 2: Equilibrium in a Tournament with a Large Confidence Gap

gender gap in performance: on average, men solve 15 mazes whereas women solve 10.8 mazes. Gneezy and Rustichini (2004) measure the running speed of boys and girls from 9 to 10 years old. The children first run alone and then paired with another child. They find that boys and girls run at the same speed when alone. However, boys run faster whereas girls run slower in the second round. In addition, boys only races led to a better performance whereas girls only races led to a worse performance in the second round.[7]

In our model, the players' equilibrium outputs $(Q_1^*, Q_2^*, \ldots)$ determine their performance, and output is a function of ability, effort, and luck. In experimental tournaments performance is a function of these same variables. If men and women participating to the experiments are equally able in expectation and have the same

[7]Not all studies find evidence of a gender gap in performance in tournaments (e.g. Niederle and Vesterlund 2007, Gillen et al. 2019). The existence of gender differences in performance is sensitive to the task used, and, for instance, is more pronounced in a math task and when gender is observable (Niederle and Vesterlund 2010, Iriberri and Rey-Biel 2017).

preferences and cost of effort, then systematic differences in performance can only be due to differences in effort provision. Proposition 1 shows that the confidence gap can lead to differences in effort which, in turn, lead to differences in performance. In particular, if the confidence gap is small, and neither player is too overconfident, overconfident males will exert more effort than less confident females. Consequently, the experimental findings of Gneezy et al (2003) and the other four studies are consistent with our theoretical predictions, provided men are more confident than women, and neither gender is too overconfident. Interestingly, Gneezy et al. (2003) do find that women feel less competent than men, thus further pointing at the salience of our theory.

5 Tournaments With $n > 2$ Overconfident Players

We now consider an n-player single-prize tournament with symmetric overconfident players. From player i's perspective, his perceived probability of winning is

$$\begin{aligned} P_i(a_i, \mathbf{a}_{-i}, \lambda_i) &= \Pi_{j\neq i} \Pr(\tilde{Q}_i \geq q) \\ &= \Pi_{j\neq i} \Pr(\lambda_i a_i + \varepsilon_i \geq a_j + \varepsilon_j) \\ &= \Pi_{j\neq i} \Pr(\varepsilon_j - \varepsilon_i \leq \lambda_i a_i - a_j) \\ &= \Pi_{j\neq i} \frac{1}{1+e^{-(\lambda_i a_i - a_j)}}. \end{aligned}$$

The optimization problem of player i is given by:

$$max_{a_i} \left\{ u(y_L) + \Pi_{j\neq i} \frac{1}{1+e^{-(\lambda_i a_i - a_j)}} \Delta u - c(a_i) \right\}.$$

The first-order condition for player i is then given by:

$$\frac{\lambda_i e^{-(\lambda_i a_i - a_1)}(1+e^{-(\lambda_i a_i - a_1)})^{-1}}{\Pi_{j\neq i}(1+e^{-(\lambda_i a_i - a_j)})}\Delta u + \frac{\lambda_i e^{-(\lambda_i a_i - a_2)}(1+e^{-(\lambda_i a_i - a_2)})^{-1}}{\Pi_{j\neq i}(1+e^{-(\lambda_i a_i - a_j)})}\Delta u$$
$$+ \frac{\lambda_i e^{-(\lambda_i a_i - a_3)}(1+e^{-(\lambda_i a_i - a_3)})^{-1}}{\Pi_{j\neq i}(1+e^{-(\lambda_i a_i - a_j)})}\Delta u + \ldots - c'(a_i) = 0.$$

Imposing symmetry in beliefs and simplifying terms we then have:

$$\frac{\lambda(n-1)e^{-a(\lambda-1)}}{\left[1+e^{-a(\lambda-1)}\right]^n}\Delta u - c'(a) = 0. \tag{3}$$

We assume that the second-order conditions are satisfied.

Proposition 2. *Consider an n-player tournament where all players are equally overconfident,* $\lambda_1 = ... = \lambda_n = \lambda > 1$. *Let* $\bar{\lambda} > 1$ *be given by*

$$\frac{[1+e^{-\hat{a}(\bar{\lambda}-1)}]^n}{e^{-\hat{a}(\bar{\lambda}-1)}} - 2^n\bar{\lambda} = 0,$$

where $\hat{a}$ *is given by* $c'(\hat{a}) = \Delta u/2^n$. *If the overconfidence bias is moderate,* $\lambda \in (1, \bar{\lambda})$, *then* $a_1^* = ... = a_n^* > \hat{a}$. *If the overconfidence bias is large,* $\lambda > \bar{\lambda}$, *then* $a_1^* = ... = a_n^* < \hat{a}$.

This proposition echoes the findings of Proposition 1 since it uncovers players' incentives to increase their equilibrium effort in tournaments when the overconfidence bias is small (i.e. lower than $\bar{\lambda}$). The mechanism underlying this result is also similar to the two player case. When players are not very confident, overconfidence and effort are complements, so that an increase in their confidence expands their best responses. Since efforts are strategic complements, all players are incentivized to increase their equilibrium efforts. On the contrary, if players are highly overconfident, overconfidence and effort are substitutes, and an increase in their confidence therefore results in a contraction of their best responses. Indeed, since overconfidence multiplies a player's perceived expected effort, if a player is highly overconfident then following an increase in his overconfidence bias he can afford reducing costly effort while still expecting a higher perceived probability of winning. Eventually this maps in lower equilibrium efforts for all players.

Proposition 2 is consistent with findings in Gneezy et al. (2003) on 6 player tournaments featuring either only males, or only females. They find that the performance of males is higher than the one of females. This is in accordance with our own findings provided men are more confident than women, yet not too much.

6 Overconfidence in Contests

In this section we analyze the effect of the confidence gap on a two player generalized Tullock contest. The perceived probability of winning of player i is as follows:

$$P_i(a_i, a_j; \lambda_i) = \begin{cases} \lambda_i q(a_i)/[\lambda_i q(a_i) + q(a_j)] & \text{if } \lambda_i q(a_i) + q(a_j) > 0 \\ 1/2 & \text{if } \lambda_i q(a_i) + q(a_j) = 0 \end{cases},$$

where $q(0) \geq 0$, $q'(a_i) > 0$ and $q''(a_i) \leq 0$.

Player i chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = u(y_L) + \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)} \Delta u - c(a_i).$$

The first-order condition is

$$\frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2} \Delta u - c'(a_i) = 0. \tag{4}$$

The second-order condition is

$$\frac{q''(a_i)[\lambda_i q(a_i) + q(a_j)] - 2\lambda_i [q'(a_i)]^2}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q(a_j) \Delta u - c''(a_i) < 0, \tag{5}$$

and the above inequality is satisfied since $q''(a_i) \leq 0$ and $c''(a_i) \geq 0$.

Let $a_i = R_i(a_j)$ denote player i's best response obtained from (4). Along player i's best response we have

$$\lambda_i q'(a_i) q(a_j) \Delta u = c'(a_i) \left[\lambda_i q(a_i) + q(a_j)\right]^2.$$

Lemma 4 describes the shapes of the players' best responses.

Lemma 4. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $q(a_j) = \lambda_i q(a_i)$.*

Lemma 4 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

A second useful lemma describes how the players' best responses changes with their overconfidence parameter λ_i.

Lemma 5. *An increase in player i's overconfidence λ_i leads to a contraction of his best response, $\frac{\partial R_i(a_j)}{\partial \lambda_i} < 0$, for $q(a_j) < \lambda_i q(a_i)$ and to an expansion of his best response, $\frac{\partial R_i(a_j)}{\partial \lambda_i} > 0$, for $q(a_j) > \lambda_i q(a_i)$. Moreover, the maximum value of the players' best response is independent of their degree of overconfidence.*

Lemma 5 characterizes the best response of players who are subject to an overconfidence bias in a contest. For a high effort of the rival, an increase in overconfidence raises player i's effort level, while for low effort of the rival, an increase in overconfidence lowers player i's effort level. Moreover, unlike in the tournament, the maximal value taken by player i's best response is independent of his overconfidence bias.

Making use of these results, we can establish equilibrium uniqueness in the following lemma:

Lemma 6. *A two player contest featuring one overconfident player* 1 *admits a unique equilibrium if* $\lambda_1 \lambda_2 \geq 1/3$.

We next present a proposition that uncovers the effect of the confidence gap on equilibrium efforts in a two player contest.

Proposition 3. *In a contest with two overconfident players where $\lambda_1 > \lambda_2$, the more overconfident player* 1 *exerts lower effort. Hence, the more overconfident player* 1 *is the Nash loser since $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.*

Proposition 3 contrasts with Proposition 1 since it shows that in a contest between two overconfident players, unlike in a tournament, the confidence gap does not matter to determine which player exerts more effort in equilibrium.

Corollary 1. *In a contest with two overconfident players, the players exert less effort than if both were rational, and as the overconfidence of either player increases, both players' efforts decrease.*

If the confidence of player i goes up, then player i's best response shifts inwards for $q(a_j) < \lambda_i q(a_i)$ (as shown in Lemma 5). Corollary 1 follows from the fact that the players' best responses are positively-slopped at the Nash equilibrium.

We illustrate Proposition 3 in Figure 3. On that figure we represent the two players' best responses given that player 1 is more overconfident than player 2, i.e. given that $\lambda_1 > \lambda_2$. From Lemma 4 we know that the best responses are quasi-concave, while from Lemma 5 we also know that the maximal value player i's best response takes is given by $q(a_j) = \lambda_i q(a_i)$, hence the crossing of the dotted lines with the maxima of the best responses. To better gauge the effect of overconfidence, we have also drawn the best responses of rational players as seen in the two concave dashed curves crossing on the 45^o line at (a_1^{max}, a_2^{max}). The higher is a player's overconfidence, the more the best response flattens for values of the rival's effort a_j below $q^{-1}(\lambda_i q(a_i))$, and steepens for values above that threshold, while the maximand of the best response increases with overconfidence. Consequently, and in line with Proposition 3, the more overconfident player 1 will experience a harsher contraction of his best response below a_2^{max}, and since the best response functions of both players are strictly increasing in $[0, a_j^{max}]$, the equilibrium E will lie above the 45^o line in the space where $a_2 > a_1$.

Increasing the overconfidence of player 1, implies that the player's best response shifts inwards for low values of a_2 as represented graphically by the dashed and dotted best response. Consequently, since $R_2(a_1)$ remains unaffected by this shift in the overconfidence of his rival, at the new equilibrium E' both players will necessarily exert less effort that in E, while the concavity of $R_2(a_1)$ also implies that the new probability that player 1 wins the contest is now lower. Upon observing the figure, it is equally obvious that an increase in λ_2 will also result in lower equilibrium efforts of *both* players, while the probability that player 1 wins the contest would then increase instead.

We now consider a contest where an overconfident player i competes against an underconfident player j. Lemma 7 describes how the underconfident player's best

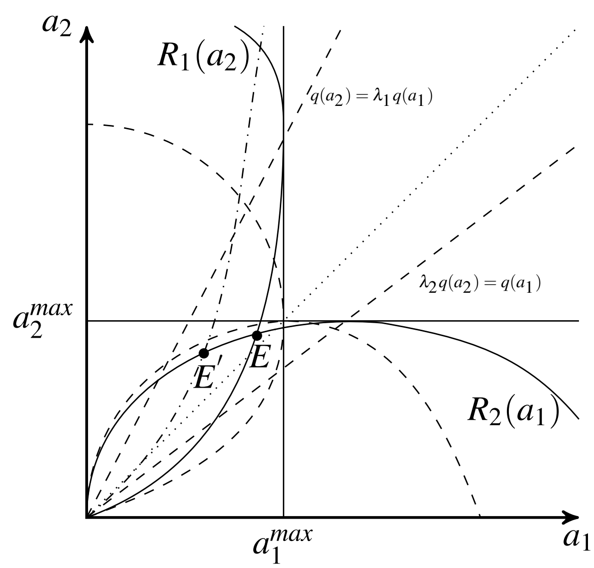

Figure 3: Equilibrium in a Contest with Overconfident Players

response shifts with his bias λ_j.

Lemma 7. *An increase in player j's underconfidence (λ_j goes down) leads to a contraction of player j's best response, $\frac{\partial R_j(a_i)}{\partial \lambda_j} < 0$, for $q(a_i) > \lambda_j q(a_j)$ and to an expansion of player j's best response, $\frac{\partial R_j(a_i)}{\partial \lambda_j} > 0$, for $q(a_i) < \lambda_j q(a_j)$. Moreover, the maximum value of player j's best response is independent of the player's degree of underconfidence.*

In Proposition 3 we show that the confidence gap is not relevant to determine which player exerts more effort at equilibrium in a contest between two overconfident players. Our next result shows that this is no longer the case in a contest where one player is overconfident and the other one is underconfident.

Proposition 4. *In a two player contest where player 1 is overconfident and player 2 is underconfident, $\lambda_1 > 1 > \lambda_2$, the overconfident player exerts less effort than the underconfident player if and only if $\lambda_1 \lambda_2 > 1$.*

If the underconfidence of player 2 goes up (λ_2 goes down), then player 2's best response shifts inwards for $q(a_1) > \lambda_2 q(a_2)$. If player 1 is rational ($\lambda_1 = 1$), then at equilibrium the underconfident player will exert lower effort. This result follows directly from the complementarity between confidence and effort. If the confidence of player 1 increases (λ_1 goes up), then player 1's best response shifts inwards for $q(a_2) < \lambda_1 q(a_1)$. Indeed, player 1 perceives a higher winning probability and thence expects effort not to increase much this probability, so he lowers effort to save on cost of effort. For a sufficiently high confidence gap, $\lambda_1 > 1/\lambda_2$, the overconfident player 1 will exert less effort than the underconfident player 2 at equilibrium.

Corollary 2. *In a two player contest where player* 1 *is overconfident and player* 2 *is underconfident,* $\lambda_1 > 1 > \lambda_2$*, the players exert less effort than if they were both rational.*

Propositions 3 and 4 both show that the more overconfident player exerts lower effort at equilibrium as long as the less confident player is not overly underconfident. This result helps organizing the experimental evidence on gender gaps in contests. These predictions are consistent with Mago and Razzolini's (2019) lab experiment. In their study they elicit the participants' confidence about their relative performance in an IQ test, after which they compete in a two player best-of-five Tullock contest. Note that here, in contrast to the experimental studies on tournaments, a participant's effort is the amount invested in the contest. Hence, effort is observable by the experimenter, and the cost of effort does not differ across participants. Although the best-of-five setup is not directly comparable to our one shot contest, we nevertheless find some parallels. In line with Propositions 3 and 4, and in contrast to the predictions of Ando (2004) and Ludwig et al. (2011), Mago and Razzolini (2019) find that more confident participants exert lower effort. Furthermore, total effort in female only contests is significantly higher than total effort in mixed or male only contests, which, combined with their finding that men are more confident than women, is also in line with our theoretical results.

Observe that our results are also in line with the findings of Anderson and Stafford

(2003), Mago et al. (2013), and Price and Sheremeta (2015) who find that women invest higher amounts than men in contests. Note that in these two experiments, unlike the study of Mago and Razzolini (2019), the participants were unaware of the gender of their rivals.

7 Contests Between $n > 2$ Overconfident Players

We now extend the analysis to single-prize contests with $n > 2$ players and focus on the fully symmetric case where players have a common overconfidence bias $\lambda > 1$.

The perceived probability of winning of player i is as follows:

$$P_i(a_i, a_{-i}; \lambda_i) = \begin{cases} \lambda_i q(a_i)/[\lambda_i q(a_i) + \sum_{j \neq i} q(a_j)] & \text{if } \lambda_i q(a_i) + \sum_{j \neq i} q(a_j) > 0 \\ 1/n & \text{if } \lambda_i q(a_i) + \sum_{j \neq i} q(a_j) = 0 \end{cases},$$

The optimization problem of player i is given by:

$$max_{a_i} \left\{ u(y_L) + \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + \sum_{j \neq i} q(a_j)} \Delta u - c(a_i) \right\}.$$

The first-order condition for any player i is then given by:

$$\frac{\lambda q'(a_i) \sum_{j \neq i} q(a_j)}{\left[\lambda q(a_i) + \sum_{j \neq i} q(a_j)\right]^2} \Delta u - c'(a_i) = 0, \tag{6}$$

and the second-order condition can here too easily be shown to be satisfied.

The next proposition summarizes our findings on the effect of overconfidence on equilibrium efforts:

Proposition 5. *In a contest with $n > 2$ symmetric players, individual and aggregate efforts increase (decrease) with overconfidence if $\lambda < (>) n - 1$.*

The intuition of this result follows the one underlying the finding of Proposition 3 and critically depends on whether players' efforts are strategic complements or strategic substitutes at equilibrium. Consider first a small number of competitors and/or a high degree of overconfidence. In such instances, the players will all expect to be highly likely to win the contest and their best responses will then be positively-slopped at equilibrium. Indeed a low n or a high λ both imply that (at the symmetric equilibrium) the opponents' sum of impact functions is relatively low, and all players consequently expect to have a high probability of winning the contest. Any expected increase in the opponents' contest effort would then push players to increase their own effort so as to avoid the winning odds from deteriorating too much. In such instances, an increase in overconfidence will incentivize contestants to all reduce their effort for a given expected (equilibrium) effort of their opponent: the high expected winning probability can now be achieved at lower cost as in Proposition 3. The exact opposite mechanism is at play when the number of contestants is high and/or the degree of overconfidence is low. In such instances, the players' best responses will be downward slopping because (at the symmetric equilibrium) the opponents' sum of impact functions is relatively high, and all players consequently expect to have a small probability of winning the contest. In such instances, an increase in overconfidence incentivizes players to increase effort with overconfidence so as to close the gap with the opponents as a consequence of the higher (expected) marginal returns to investing effort in the contest. This mechanism once more echoes the one in Proposition 3.

From the above observation, we are able to obtain the following corollary:

Corollary 3. *With $n > 2$ symmetric players, the maximal rent dissipation is always attained when $\lambda = n - 1$. There always exists a finite n^D such that over-dissipation (i.e. the sum of players' effort costs is greater than the value of the prize) can be observed at equilibrium for $n > n^D$.*

It is widely known in the literature on contests that with rational agents over-dissipation can never be observed at equilibrium if the player's valuation of the prize is equal to the actual value of the prize.[8] Although the dissipation ratio, defined as

[8] See Dickson et al. (2022) for instances where players' valuation of the prize differs from the actual value of the prize.

the ratio of total expenditures (or sum of players' effort costs) to the value of the prize, $D = \frac{\sum_i c_i(a_i)}{v}$, does increase in the number of players, it is bounded by unity because individual equilibrium effort drops as the number of contestants increases. Indeed, a larger number of contestants implies that the competitors' aggregate effort is expected to be higher, thence reducing the marginal return to investing in the contest, which in turn pushes all contestants to individually contract their equilibrium effort. In Proposition 5 we demonstrated, however, that some overconfidence may push players to increase their equilibrium effort compared to a setup with fully rational players. Corollary 3 shows that there always exists a degree of overconfidence such that equilibrium individual efforts of overconfident players will equal the maximal equilibrium individual efforts that can be obtained in the game, i.e., the individual efforts produced in setups with two fully rational players. Consequently, with sufficiently many overconfident players the aggregate effort can be higher than the value of the contested prize.

To visualize the last two results, in Figure 4 we depict the individual equilibrium effort of (symmetric) contestants as a function of their overconfidence parameter in the most simple contest where players' payoffs are given by:

$$E[U_i, a_i, \mathbf{a_{-i}}; \lambda_i] = \frac{\lambda_i a_i}{\lambda_i a_i + \sum_{j \neq i} a_j} - a_i,$$

where $\mathbf{a_{-i}}$ designates the vector of player i's competitors' efforts. Accordingly, we are assuming that $u(y_L) = 0$, $u(y_H) = 1$, $q(a) = a$, and $c(a) = a$. With $n = 2$ and $\lambda = 1$, the equilibrium efforts are equal to $1/4$. If we consider games with more players, the individual efforts can be kept equal to $1/4$ if $\lambda = n - 1$. Consequently, under such circumstances, full dissipation can result with $n = 4$ and $\lambda = 3$, and over-dissipation can therefore obtain for any $n > 4$.

It is important at this stage to underline that although for over-dissipation to be observed it is necessary to have $n > n^D > 2$ players, the required degree of overconfidence may be quite low. Indeed, to visualize this we consider again the previous basic contest setup, and we impose for the sake of the argument the parameter restriction

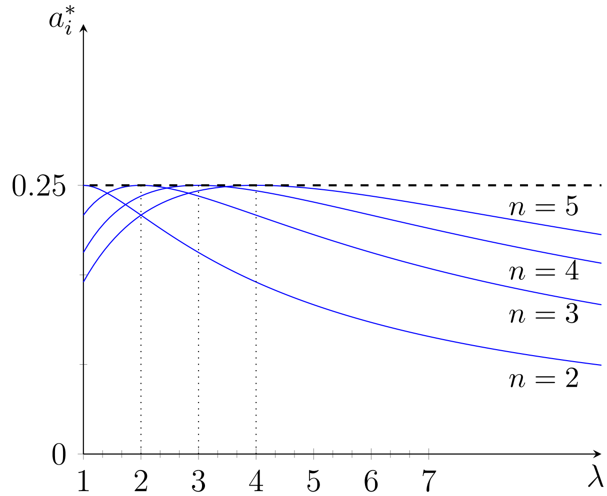

Figure 4: Individual Equilibrium Efforts and Overconfidence in n Player Contests.

$\lambda < n - 1$, for $n \geq 3$. Since $a^* = \frac{\lambda(n-1)}{(\lambda+n-1)^2}$, this parameter restriction can easily be shown to imply that $\partial na^*/\partial n > 0$, $\partial^2 na^*/\partial n^2 < 0$, and $\partial a^*/\partial \lambda < 0$. We then plot the equilibrium aggregate effort, na^*, against the number of players, n, for various levels of overconfidence in Figure 5. It is well known that as n becomes arbitrarily large, the dissipation ratio converges to unity, without ever reaching total rent dissipation. We know from Corollary 3 that for any number of players $n > n^D > 2$, there always exists a degree of overconfidence conducive to over-dissipation. For example, Figure 5 shows that with $n = 6$ over-dissipation is already observed when $\lambda = 1.5$, which corresponds to a perceived winning probability of 0.231 as opposed to the actual winning probability of 1/6. Increasing the number of players to, say, $n = 8$ implies that over-dissipation can be achieved with an even lower degree of overconfidence (e.g. $\lambda = 1.25$). It is immediate to deduce that as the number of players becomes arbitrarily large in this setup, the required degree of overconfidence for observing over-dissipation will become arbitrarily small (i.e. λ close to 1).

Introducing overconfidence in contests allows us to contribute to the literature on the dissipation ratio. When considering contests between two overconfident players,

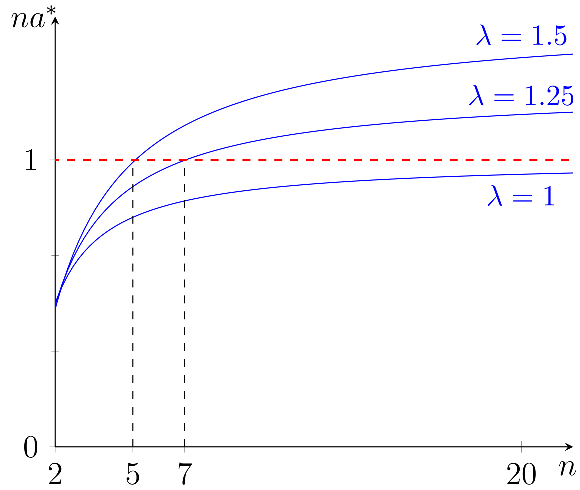

Figure 5: Equilibrium Aggregate Effort as a Function of n.

we showed that overconfidence unambiguously reduces the equilibrium efforts of both contestants. Proposition 4 and Corollary 3 also uncovers that overconfidence with $n > 2$ players can lead to overspending at equilibrium, and that with $n > n^D$ the excess spending can be such that we witness over-dissipation at equilibrium. The rationale of these rather counter-intuitive results rests in a feature of contests according to which increasing the efficiency (perceived or real) of a contestant whose expected equilibrium winning odds are low pushes him to increase his equilibrium effort. In the presence of many contestants, therefore, since all contestants individually expect to have a low probability of winning, some overconfidence will incentivize all contestants to increase their equilibrium efforts, and this may map in over-dissipation.

Corollary 3 is consistent with findings in lab experiments featuring $n > 2$ contestants. These experiments find that participants spend significantly higher amounts than the game's Nash equilibrium (Sheremeta 2011, Price and Sheremeta 2015, Mago et al. 2016), and that even over-dissipation can occur (Anderson and Stafford 2003, Lim et al. 2014). While other behavioral biases could account for these results, like noisy best responses, level-k thinking, or even joy of winning, we show that overconfidence is another potential explanation.

8 Conclusion

This paper studies the impact of overconfidence on tournaments and contests. In a two player tournament we show that the more overconfident player will be the Nash winner if the confidence gap is small and neither player is too overconfident. In this case, the more overconfident player exerts higher effort at equilibrium. However, the more overconfident player is the Nash loser when the confidence gap is sufficiently large because he will exert lower effort in equilibrium. An interesting implication of this result is that an underconfident player can be the Nash winner when the rival is excessively overconfident. These results clarify the conditions under which overconfidence about one's own ability can help a manager being promoted to a CEO position. They also highlight the conditions under which gender gaps in confidence will lead men to be overly represented at CEO positions.

In a contest opposing two overconfident players, the more overconfident player will be the Nash loser because he exerts lower effort at equilibrium. Observe, however, that in a contest opposing an overconfident to an underconfident player, the more confident player may exert a higher effort and be the Nash winner when the confidence gap is not too large. One implication of our findings calls for attention when hiring lawyers or lobbyists since excessive overconfidence may lead to worse outcomes. Moreover, it can also explain why women tend to spend more than men in experimental contests.

We also provide new results on the effect of overconfidence on single-prize tournaments and contests with $n > 2$ equally confident players. For any given number of players, we uncover a non-monotonic effect of overconfidence on individual, and thus aggregate equilibrium efforts. For low (high) levels of overconfidence, players exert more (less) effort than if they were rational. This finding allows us to provide conditions under which overspending and even over-dissipation in contests can result from overconfidence. This may explain the observed overspending in contest experiments.

Our paper also provides new testable implications of confidence gaps in tournaments and contests. In tournaments we uncovered a non-monotonic relationship between a player's overconfidence and his equilibrium effort, keeping the rival's degree of confidence fixed. Likewise, in a contest the same result obtains provided one's rival is underconfident. This study carries important public policy implications. Given the non-monotonic effect of confidence gaps on effort provision, one should be careful when designing public policies (e.g. affirmative action) that affect incentives to enter and perform in competitions. Another implication of our study is that future experimental research should carefully account for the size of players' confidence biases.

An avenue for future theoretical research would be to study tournaments and contests where the players can differ not only in terms of confidence, but also in terms of their preferences towards either risk or competitiveness since the experimental evidence shows that all these aspects seem to matter.

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9 Appendix

Proof of Lemma 1 Using player i's first-order condition, we have

$$\begin{aligned}
\frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i \partial a_j} &= \frac{\partial}{\partial a_j}\left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\right) \\
&= \lambda_i \frac{e^{-(\lambda_i a_i - a_j)}\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2 - 2\left[1+e^{-(\lambda_i a_i - a_j)}\right] e^{-(\lambda_i a_i - a_j)} e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^4} \\
&= \lambda_i \frac{1+e^{-(\lambda_i a_i - a_j)} - 2e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \\
&= \lambda_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)}.
\end{aligned}$$

and

$$\frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i^2} = \frac{\partial}{\partial a_i}\left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\right) = -\lambda_i^2 \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)}.$$

Therefore, the slope of the best response of player i is

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E(U_i)}{\partial a_i \partial a_j}}{\frac{\partial^2 E(U_i)}{\partial a_i^2}} = -\frac{\lambda_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \Delta u}{-\lambda_i^2 \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \Delta u - c''(a_i)}.$$

This is equal to zero when

$$1 - e^{-(\lambda_i a_i - a_j)} = 0,$$

or

$$\lambda_i a_i - a_j = 0.$$

Moreover, for $\lambda_i a_i > a_j$, the slope of player i's best response is positive, while otherwise, if $\lambda_i a_i < a_j$, the slope of player i's best response is negative.

Proof of Lemma 2 Player i's best response is defined as

$$\lambda_i \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} \Delta u = c'(a_i).$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i \partial \lambda_i} \Delta u,$$

where

$$\begin{aligned}
\frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i \partial \lambda_i} &= \frac{\partial}{\partial \lambda_i} \left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} \right) \\
&= \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} - \lambda_i a_i \frac{1+e^{-(\lambda_i a_i - a_j)} - 2e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \\
&= \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} \left[1 - \lambda_i a_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{1+e^{-(\lambda_i a_i - a_j)}} \right].
\end{aligned}$$

Hence, $\partial R_i(a_j)/\partial \lambda_i$ is positive if and only if

$$1 - \lambda_i a_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{1+e^{-(\lambda_i a_i - a_j)}} > 0,$$

or

$$1 + e^{-(\lambda_i a_i - a_j)} - \lambda_i a_i \left[1 - e^{-(\lambda_i a_i - a_j)}\right] > 0,$$

or

$$1 - \lambda_i a_i + e^{-(\lambda_i a_i - a_j)} + \lambda_i a_i e^{-(\lambda_i a_i - a_j)} > 0,$$

or

$$(1+\lambda_i a_i) e^{-(\lambda_i a_i - a_j)} > \lambda_i a_i - 1,$$

or

$$e^{-(\lambda_i a_i - a_j)} > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}.$$

Note that this inequality is always satisfied when $a_j \geq \lambda_i a_i$ since $e^{-(\lambda_i a_i - a_j)} \geq 1 > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$. Substituting next $a_j = \lambda_i a_i$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the tournament by $a_i^{\max}$ we have

$$\lambda_i \frac{e^{-(\lambda_i a_i - \lambda_i a_i)}}{\left[1+e^{-(\lambda_i a_i - \lambda_i a_i)}\right]^2} \Delta u = c'(a_i),$$

or

$$\lambda_i \frac{\Delta u}{4} = c'(a_i^{\max}).$$

Proof of Lemma 3 To prove that the equilibrium is unique, we first show that when the players' best responses cross it is impossible that they are both negatively slopped. We proceed by contradiction and suppose that there is an equilibrium such that $R_1'(a_2^*) < 0 \Leftrightarrow a_2^* > \lambda_1 a_1^*$ and $R_2'(a_1^*) < 0 \Leftrightarrow a_1^* > \lambda_2 a_2^*$. Since $\lambda_1 > 1$, $a_2^* > \lambda_1 a_1^* \Rightarrow a_2^* > a_1^*$.

To show that an equilibrium such that $R_1'(a_2^*) < 0$ and $R_2'(a_1^*) < 0$ cannot admit $a_2^* > a_1^*$, consider any pair $a_1 = a_2 = a$. Since $\lambda_1 > \lambda_2$, then $e^{-a(\lambda_1-1)} < e^{-a(\lambda_2-1)}$. Consequently, using the players' first-order conditions we deduce that for any such pair we would have:

$$\frac{\lambda_1 e^{-a(\lambda_1-1)}}{\left(1+e^{-a(\lambda_1-1)}\right)^2} > \frac{\lambda_2 e^{-a(\lambda_2-1)}}{\left(1+e^{-a(\lambda_2-1)}\right)^2},$$

which in turn would imply that if player 2's first-order condition is satisfied then player 1 has incentives to increase his effort, and if player 1's first-order condition is satisfied, then player 2 has incentives to reduce his effort. Consequently, the best response of player 2 needs to cross the 45-degrees line for lower efforts a_2 than the best response of player 1. The quasi-concavity of the players' best responses allows us to conclude that $a_1^* > a_2^*$, thence the contradiction.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_1) = R_1'(a_2) \circ R_2'(a_1)$ has a slope smaller than 1 for any equilibrium pair (a_1^*, a_j^*), since the function is continuous on $\mathbf{R}$. If $R_1'(a_1^*) < 0$, then since $R_1'(a_2^*) > 0$, the condition is necessarily satisfied. If, on the other hand, $R_2'(a_1^*) > 0$, then we simply need to prove that if $R_1'(a_2^*) > 0$ for both players, then the product of the best responses is smaller than 1. Since $R_1'(a_2)$ is decreasing in $c''(a_1)$, it is thus sufficient to establish the result for $c''(a_1) = 0$. Rewriting the product of the players' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{\lambda_1 \frac{1-e^{-(\lambda_1 a_1 - a_2)}}{\left[1+e^{-(\lambda_1 a_1 - a_2)}\right]^3} e^{-(\lambda_1 a_1 - a_2)} \Delta u}{\lambda_1^2 \frac{1-e^{-(\lambda_1 a_1 - a_2)}}{\left[1+e^{-(\lambda_1 a_1 - a_2)}\right]^3} e^{-(\lambda_1 a_1 - a_2)} \Delta u + c''(a_1)} \frac{\lambda_2 \frac{1-e^{-(\lambda_2 a_2 - a_1)}}{\left[1+e^{-(\lambda_2 a_2 - a_1)}\right]^3} e^{-(\lambda_2 a_2 - a_1)} \Delta u}{\lambda_2^2 \frac{1-e^{-(\lambda_2 a_2 - a_1)}}{\left[1+e^{-(\lambda_2 a_2 - a_1)}\right]^3} e^{-(\lambda_2 a_2 - a_1)} \Delta u + c''(a_2)} < 1.$$

Since we want to show that the above condition is true when $R_1'(a_2) > 0$ and $R_2'(a_1) > 0$, if the above condition is true for $c''(a_1) = c''(a_2) = 0$, then it is also true for any values $c''(a_1) > 0$ and $c''(a_2) > 0$. Consequently, the above condition is true if $\lambda_1\lambda_2 \geq 1$.

Proof of Proposition 1 We proceed in several steps. First, we show that for any λ_2, there are two values of λ_1 such that $a_1^* = a_2^*$. We define those two thresholds as $\underline{\lambda}_1(\lambda_2)$ and $\bar{\lambda}_1(\lambda_2)$, with $\underline{\lambda}_1(\lambda_2) < \bar{\lambda}_1(\lambda_2)$ and $\lambda_2 = \underline{\lambda}_1(\lambda_2)$ or $\lambda_2 = \bar{\lambda}_1(\lambda_2)$. We show that for $\lambda_1 \in]\underline{\lambda}_1(\lambda_2), \bar{\lambda}_1(\lambda_2)[$, $a_1^* > a_2^*$, whereas for $\lambda_1 > \bar{\lambda}_1(\lambda_2)$, $a_1^* < a_2^*$. Consequently, if $\lambda_2 = \underline{\lambda}_1(\lambda_2)$, then $a_1^* > a_2^*$ for $\lambda_1 < \bar{\lambda}_1(\lambda_2)$ and $a_1^* < a_2^*$ otherwise. Moreover, if $\lambda_2 = \bar{\lambda}_1(\lambda_2)$, then $a_1^* < a_2^*$ for any $\lambda_1 > \lambda_2$.

In a 2 player setup, the first-order condition of player 1 is:

$$\lambda_1 \frac{e^{-[\lambda_1 a_1 - a_2]}}{(1+e^{-[\lambda_1 a_1 - a_2]})^2} \Delta u - c'(a_1) = 0$$

Consider then any value λ_2. If $\lambda_1 = \lambda_2$, then $a_1 = a_2 = a^*(\lambda_2)$ and the above first-order condition can be written as:

$$\lambda_2 \frac{e^{-a^*(\lambda_2)[\lambda_2 - 1]}}{(1+e^{-a^*(\lambda_2)[\lambda_2 - 1]})^2} \Delta u - c'(a^*(\lambda_2)) = 0$$

We can then define function $\phi(\lambda_1)$ the function where we maintain $a^*(\lambda_2)$ fixed, and is defined as:

$$\phi(\lambda_1) = \lambda_1 \frac{e^{-a^*(\lambda_2)[\lambda_1 - 1]}}{(1+e^{-a^*(\lambda_2)[\lambda_1 - 1]})^2} \Delta u - c'(a^*).$$

For $\phi(\lambda_1) = 0$, the players' efforts are mutual best responses when $a_1 = a_2 = a^*(\lambda_2)$. Otherwise, if $\phi(\lambda_1) < 0$, the best response of player 1 to $a_2 = a^*(\lambda_2)$ commands player 1 to produce a smaller effort than $a^*(\lambda_2)$. Combining this with the facts that (i) player 2's best response is not affected by λ_1, and (ii) the best response of player 2 is quasi-concave, implies that at equilibrium $a_1^* < a_2^*$. By a similar reasoning, if $\phi(\lambda_1) > 0$, then $a_1^* > a_2^*$.

We next show that function $\phi(\lambda_1)$ is quasi-concave, it crosses twice the x-axis at values $\underline{\lambda}_1(\lambda_2)$ and $\bar{\lambda}_1(\lambda_2)$, and that $\phi(\lambda_1) = 0$ for either $\lambda_2 = \underline{\lambda}_1(\lambda_2)$, or $\lambda_2 = \bar{\lambda}_1(\lambda_2)$

We begin by showing that the function $\phi(\lambda_1)$ is quasi-concave. Using the short notation $e = e^{-\hat{a}(\lambda-1)}$, we first have that:

$$\phi'(\lambda_1) = \frac{e}{[1+e]^3}\left[1 + e + a^*(\lambda_2)\lambda_1[e-1]\right].$$

To show quasi-concavity, we first observe from the above expression that $\phi'(\lambda_1) > 0$ for any $\lambda_1 \leq 1$. Second, we show that for $\lambda > 1$, that $\phi''(\lambda_1) < 0$ for $\phi'(\lambda_1) = 0$. Indeed,

$$\phi''(\lambda_1) = \frac{a^* e}{[1+e]^4}\left[2e-1\right]\left[1 + e + a^*\lambda_1(e-1)\right] - \frac{a^* e}{[1+e]^4}[1+e][a^*\lambda_1 e + 1],$$

and this expression is negative if the following inequality is true:

$$[1+e][2[e-1] - a^*\lambda_1 e] + a^*(\lambda_2)\lambda_1[e-1][2e-1] < 0,$$

or,

$$2[e-1][1+e] + a^*\lambda_1[e^2 - 4e + 1] < 0.$$

If we evaluate this expression at $\phi'(\lambda_1) = 0 \Leftrightarrow a^*(\lambda_2)\lambda_1 = \frac{1+e}{1-e}$, we then obtain:

$$2[e-1][1+e] + \frac{1+e}{1-e}[e^2 - 4e + 1] < 0.$$

And since $e < 1$ for $\lambda_1 > 1$, this inequality can be re-written as:

$$2[e-1]^2 > e^2 - 4e + 1,$$

which is always true.

Having shown that $\phi(\lambda_1)$ is quasi-concave, we next show that $\phi(\lambda_1) < 0$ for $\lambda_1 \to 0$ and also that for $\lambda_1 \to \infty$, thence implying that, since an equilibrium exists, there exists at least one value λ_1 satisfying $\phi(\lambda_1) = 0$. If $\underline{\lambda_1} = \bar{\lambda}_1$, then there is but one value of λ_1 such that $\phi(\lambda_1) = 0$. Otherwise there are exactly 2 such values.

Focusing first on the case where $\lambda_1 \to 0$, we have:

$$\lim_{\lambda_1 \to 0} \phi(\lambda_1) = 0 - c'(a^*(\lambda_2)),$$

with the limit of the first component of $\phi(\lambda_1)$ tending to 0 since $\lim_{\lambda_1 \to 0} e^{-a^*(\lambda_2)[\lambda_1 - 1]} = e^{a^*(\lambda_2)}$, which is finite.

Turning next to the case where $\lim_{\lambda_1 \to \infty}$ we have:

$$\lim_{\lambda \to \infty} \phi(\lambda) = \frac{\infty e^{-\infty}}{[1+e^{-\infty}]^2} \Delta u - c'(a)$$

The denominator of the first component of the expression tends to 1 but the limit of the numerator is undetermined. We apply l'Hospital's rule to the numerator so that:

$$\lim_{\lambda \to \infty} (\lambda e^{-\lambda}) = \lim_{\lambda \to \infty} \frac{\lambda}{e^{\lambda}} = \lim_{\lambda \to \infty} \frac{1}{e^{\lambda}} = 0$$

with the next to last equality following from the application of l'Hospital's rule. Consequently $\lim_{\lambda \to \infty} \phi(\lambda) = -c'(a) < 0$.

Proof of Proposition 2 The equilibrium effort is the solution to

$$(n-1)\lambda \frac{e^{-a^*(\lambda-1)}}{[1+e^{-a^*(\lambda-1)}]^n} \Delta u = c'(a^*).$$

Let $\hat{a}$ describe the players' symmetric equilibrium effort when $\lambda = 1$. This value is then implicitly defined by:

$$c'(\hat{a}) = \frac{(n-1)\Delta u}{2^n}.$$

When the bias λ satisfies

$$(n-1)\lambda\frac{e^{-a^*(\lambda-1)}}{[1+e^{-a^*(\lambda-1)}]^n}\Delta u=(n-1)\frac{\Delta u}{2^n}, \tag{7}$$

then $a^*=\hat{a}$.

After simplifying terms in (7) we can define $\phi(\lambda;\hat{a},n)$ as:

$$\phi(\lambda;\hat{a},n)=\frac{[1+e^{-\hat{a}(\lambda-1)}]^n}{e^{-\hat{a}(\lambda-1)}}-2^n\lambda.$$

We know that $\phi(1;\hat{a},n)=0$. We then show that there exists only one value $\lambda>1$, that we denote by $\bar{\lambda}$, such that $\phi(\bar{\lambda};\hat{a},n)=0$. To show that, we demonstrate that $\phi'(1;\hat{a},n)<0$, that $\lim_{\lambda\to\infty}\phi(\lambda;\hat{a},n)>0$, and that the function $\phi(\lambda;\hat{a},n)$ is convex.

We begin by computing $\phi'(\lambda;\hat{a},n)$ which is given by:

$$\phi'(\lambda;\hat{a},n)=a^*\frac{1-(n-1)e^{-\hat{a}(\lambda-1)}}{e^{-\hat{a}(\lambda-1)}}[1+e^{-\hat{a}(\lambda-1)}]^{n-1}-2^n.$$

If $\lambda=1$, $e^{-\hat{a}(\lambda-1)}=1$, and so $\phi'(1;\hat{a},n)=a^*(2-n)2^{n-1}-2^n<0$. Next, if $\lambda\to\infty$, $\lim_{\lambda\to\infty}e^{-\hat{a}(\lambda-1)}=0$, and we thus conclude that $\lim_{\lambda\to\infty}\phi(\lambda;\hat{a},n)=+\infty$.

The last step to complete the proof is to show that the function $\phi(\lambda;\hat{a},n)$ is convex. Using the short notation $e=e^{-\hat{a}(\lambda-1)}$, we have:

$$\phi''(\lambda;\hat{a},n)=\hat{a}\left[\frac{\hat{a}\left[(n-1)e(1+e)^{n-1}-(n-1)\hat{a}e(1+e)^{n-2}(1-(n-1)e)\right]e}{e^2}\right]+\hat{a}\left[\frac{\hat{a}e(1-(n-1)e)(1+e)^{n-1}}{e^2}\right]$$

Factoring out $(\hat{a})^2e(1+e)^{n-2}$, the sign of which is positive, we have that the sign of $\phi''(\lambda;\hat{a},n)$ is then given by the sign of the next expression:

$$sign\{\phi''(\lambda;\hat{a},n)\}=sign\left\{e\left[(n-1)(1+e)-(n-1)(1-(n-1)e)\right]+(1-(n-1)e)(1+e)\right\},$$

and hence,

$$sign\{\phi''(\lambda;\hat{a},n)\} = \{-e(n-1)(1-(n-1)e) + (1+e)\}$$

or,

$$sign\{\phi''(\lambda;\hat{a},n)\} = \{e^2(n-1)^2 - e(n-1) + 1 + e\}. \quad (8)$$

As a last step we show that the sign of the above expression is positive for any $n \geq 2$. If $n = 2$, this expression is positive since:

$$e^2 - e + 1 + e > 0.$$

For $n \to \infty$ expression (8) is also positive since:

$$\lim_{n\to\infty}[e^2(n-1)^2 - e(n-1) + 1 + e] = \lim_{n\to\infty} e(n-1)[(n-1)e - 1] + 1 + e > 0.$$

Since $\hat{a}$ is monotonically decreasing in n, and given the fact that the argument of the exponential is $-\hat{a}(\lambda - 1)$, $\lim_{n\to\infty} e = K$, where $K \in \mathbf{R}^+$. Consequently, $\lim_{n\to\infty}\{(n-1)e - 1\} = +\infty$, so that the limit of the above entire expression (8) is infinity as well. Last, we show that this expression is convex in n, reaching a minimum for some $n < \infty$, thence implying that if the expression is positive for that minimal value, then it is positive for any $n \geq 2$. To establish this last step we differentiate expression (8), and obtain

$$\frac{\partial\{e^2(n-1)^2 - e(n-1) + 1 + e\}}{\partial n} = 2(n-1)e^2 - e = e(2(n-1)e - 1),$$

and,

$$\frac{\partial^2\{e^2(n-1)^2 - e(n-1) + 1 + e\}}{(\partial n)^2} = 2e > 0.$$

A local extremum is then reached for:

$$n = 1/2e + 1.$$

We therefore have that expression (8) is decreasing in n below that threshold and increasing for values of n above that threshold. As a last step we show that for that threshold value of n the expression is positive. And this is true since after replacing in the condition we have,

$$\frac{1}{4} - \frac{1}{2} + 1 + e > 0.$$

Proof of Lemma 4 The best response of player i, $i = \{1, 2\}$, is defined implicitly by (4). Hence, the slope of the best response of player i, $R_i'(a_j)$ is given by

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial a_j}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda_i q(a_i) - q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q'(a_i) q'(a_j) \Delta u}{\frac{q''(a_i)[\lambda_i q(a_i) + q(a_j)] - 2\lambda_i [q'(a_i)]^2}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q(a_j) \Delta u - c''(a_i)}. \tag{9}$$

The denominator is negative because player i's second-order condition is satisfied. Therefore, the sign of the slope of player i's best response is only determined by the sign of the numerator which only depends on $\lambda_i q(a_i) - q(a_j)$. Hence, $R_i'(a_j)$ is positive for $\lambda_i q(a_i) > q(a_j)$, zero for $\lambda_i q(a_i) = q(a_j)$, and negative for $\lambda_i q(a_i) < q(a_j)$. This implies that $R_i(a_j)$ increases in a_j for $\lambda_i q(a_i) > q(a_j)$, reaches the maximum at $\lambda_i q(a_i) = q(a_j)$, and decreases in a_j for $\lambda_i q(a_i) < q(a_j)$.

Proof of Lemma 5 (This proof follows Baik 1994) Player i's best response is defined by (4):

$$\frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2} \Delta u - c'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{q(a_j) - \lambda_i q(a_i)}{[\lambda_i q(a_i) + q(a_j)]^3} q'(a_i) q(a_j) \Delta u.$$

We see that $\partial R_i(a_j)/\partial \lambda_i \gtreqless 0$ for $q(a_j) \gtreqless \lambda_j q(a_i)$. We also know from Lemma 4 that $sign\{R_i'(a_j)\} = -sign\left\{\frac{\partial R_i(a_j)}{\partial \lambda_i}\right\}$.

Substituting next $q(a_j) = \lambda_i q(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a_i^{max} we obtain

$$\frac{\lambda_i q'(a_i^{max}) \lambda_i q(a_i^{max})}{[\lambda_i q(a_i^{max}) + \lambda_i q(a_i^{max})]^2} \Delta u = c'(a_i^{max}),$$

or

$$\frac{\lambda_i^2 q'(a_i^{max}) q(a_i^{max})}{4\lambda_i^2 \left[q(a_i^{max})\right]^2} \Delta u = c'(a_i^{max}),$$

or

$$\frac{q'(a_i^{max})}{4q(a_i^{max})} \Delta u = c'(a_i^{max}).$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a_i^{max}, does not depend on λ_i.

Proof of Lemma 6 To prove that the equilibrium is unique, we reproduce the steps of the proof of Lemma 3, and we first show that when the players' best responses cross it is impossible that they are both negatively slopped. We proceed by contradiction here too and suppose that there is an equilibrium such that $R_1'(a_2^*) < 0 \Leftrightarrow q(a_2^*) > \lambda_1 q(a_1^*)$ and $R_2'(a_1^*) < 0 \Leftrightarrow q(a_1^*) > \lambda_2 q(a_2^*)$. Since $\lambda_1 > 1$, $q(a_2^*) > \lambda_1 q(a_1^*) \Rightarrow q(a_2^*) > q(a_1^*) \Rightarrow a_2^* > a_1^*$.

To show that an equilibrium such that $R_1'(a_2^*) < 0$ and $R_2'(a_1^*) < 0$ cannot admit $a_2^* > a_1^*$, consider any pair $a_1 = a_2 = a$. Since $\lambda_1 > \lambda_2$, then $\frac{\partial E[U_1(a_1,a_2;\lambda_1)]}{\partial a_1} > \frac{\partial E[U_2(a_2,a_1;\lambda_2)]}{\partial a_2}$ for $a_1 = a_2$. This in turn would imply that if player 2's first-order condition is satisfied then player 1 has incentives to increase his effort, and if player 1's first-order condition is satisfied, then player 2 has incentives to reduce his effort. Consequently, the best response of player 2 needs to cross the 45-degrees line for lower efforts a_2 than the best response of player 1. The quasi-concavity of the players' best responses allows us to conclude that $a_1^* > a_2^*$, thence the contradiction.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_1) = R_1'(a_2) \circ R_2'(a_1)$ has a slope smaller than 1 for any equilibrium pair (a_1^*, a_2^*), since the function is continuous on $\mathbf{R}$. Having shown that at equilibrium we cannot have $R_1'(a_2) < 0$ and $R_2'(a_1) < 0$, we simply need to prove that when both best responses are positively slopped at equilibrium, the product of the best responses is smaller than 1. Since $R_1'(a_2)$ is decreasing in $c''(a_1)$, it is thus sufficient to establish the result for $c''(a_1) = 0$. Rewriting the product of the contestants' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1))\,(q'(a_1)q'(a_2))^2}{[q''(a_1)[\lambda_1 q(a_1) + q(a_2)] - 2\lambda_1[q'(a_1)]^2]\,[q''(a_2)[\lambda_2 q(a_2) + q(a_1)] - 2\lambda_2[q'(a_2)]^2]\,q(a_1)q(a_2)} < 1.$$

Since the LHS is decreasing in both $q''(a_1)$ and $q''(a_2)$ the above expression is *a fortiori* true if:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1))\,(q'(a_1)q'(a_2))^2}{4\lambda_1[q'(a_1)]^2\lambda_2[q'(a_2)]^2 q(a_1)q(a_2)} < 1,$$

an expression that simplifies to:

$$(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1)) < 4\lambda_1\lambda_2 q(a_1)q(a_2).$$

And this inequality is always satisfied if $\lambda_1\lambda_2 \geq 1/3$.

Proof of Proposition 3 To prove this result we show that the best response of the more overconfident player crosses the 45 degree line at a lower value of effort than the best response of the less overconfident player. If player 1 is the more overconfident player, then $\lambda_1 > \lambda_2 > 1$. At the 45 degree line the best response of player 1 takes the value a_L given by

$$\frac{\lambda_1 q'(a_L)}{(1+\lambda_1)^2 q(a_L)}\Delta u - c'(a_L) = 0. \tag{10}$$

At 45 degree line the best response of player 2 takes the value a_H given by

$$\frac{\lambda_2 q'(a_H)}{(1+\lambda_2)^2 q(a_H)}\Delta u - c'(a_H) = 0. \tag{11}$$

Note that $\lambda_1 > \lambda_2$ implies

$$\frac{\lambda_1}{(1+\lambda_1)^2} < \frac{\lambda_2}{(1+\lambda_2)^2}. \tag{12}$$

Therefore, (10), (11), and (12) imply

$$\frac{q'(a_H)}{q(a_H)c'(a_H)} < \frac{q'(a_L)}{q(a_L)c'(a_L)}.$$

Given that $q(.)$ is (weakly) concave and that $c(.)$ is (weakly) convex, this inequality can only be satisfied provided $a_L < a_H$.

Proof of Lemma 7 See the proof of Lemma 5.

Proof of Proposition 4 To prove this result we show that if $\lambda_1\lambda_2 < 1$, then the best response of the overconfident player 1 crosses the 45 degree line at a higher value of effort than the best response of the underconfident player 2.

At the 45 degree line the best response of player 1 takes the value $\bar{a}_1$ given by

$$\frac{\lambda_1 q'(\bar{a}_1)}{(1+\lambda_1)^2 q(\bar{a}_1)}\Delta u - c'(\bar{a}_1) = 0. \tag{13}$$

At 45 degree line the best response of player 2 takes the value $\bar{a}_2$ given by

$$\frac{\lambda_2 q'(\bar{a}_2)}{(1+\lambda_2)^2 q(\bar{a}_2)}\Delta u - c'(\bar{a}_2) = 0. \tag{14}$$

Observe that

$$\frac{\lambda_1}{(1+\lambda_1)^2} > \frac{\lambda_2}{(1+\lambda_2)^2},$$

is equivalent to:

$$\lambda_1\lambda_2^2 + \lambda_1 > \lambda_2\lambda_1^2 + \lambda_2,$$

which is true when $\lambda_1\lambda_2 < 1$. This implies

$$\frac{q'(\bar{a}_1)}{q(\bar{a}_1)c'(\bar{a}_1)} > \frac{q'(\bar{a}_2)}{q(\bar{a}_2)c'(\bar{a}_2)}.$$

Given that $q(.)$ is (weakly) concave and that $c(.)$ is (weakly) convex, this inequality can only be satisfied provided $\bar{a}_1 > \bar{a}_2$.

Likewise, if $\lambda_1\lambda_2 > 1$, then $\bar{a}_1 < \bar{a}_2$.

Proof of Proposition 5 We begin by imposing symmetry so that $a_i = a_j = a^*$, $\forall i, j \in N$. Consequently, at equilibrium the first-order condition (6) reads as:

$$\frac{\lambda q'(a^*)(n-1)q(a^*)}{[\lambda q(a^*) + (n-1)q(a^*)]^2}\Delta u - c'(a^*) = 0,$$

or

$$\frac{\lambda(n-1)q'(a^*)}{(\lambda+n-1)^2\, q(a^*)}\Delta u - c'(a^*) = 0.$$

To inspect the sign of $\partial a^*/\partial\lambda$ we apply the implicit function theorem to the above expression to obtain:

$$\begin{aligned}\frac{\partial a^*}{\partial \lambda} &= -\frac{\frac{(n-1)(\lambda+n-1)^2-2(\lambda+n-1)\lambda(n-1)}{(\lambda+n-1)^4}\Delta u\frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2}\Delta u\frac{q''(a^*)q(a^*)-[q'(a^*)]^2}{q^2(a^*)}-c''(a^*)} \\ &= -\frac{\frac{(n-1)(n-1-\lambda)}{(\lambda+n-1)^3}v\frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2}v\frac{q''(a^*)q(a^*)-[q'(a^*)]^2}{q^2(a^*)}-c''(a^*)}.\end{aligned}$$

Since the denominator of this expression is unambiguously negative, the sign of the expression is therefore given by the sign of $(n-1-\lambda)$.

Proof of Corollary 3 Consider n symmetric rational players. Their equilibrium effort is given by:

$$\frac{(n-1)q'(a^*)}{(n-1)^2\, q(a^*)}\Delta u - c'(a^*) = 0.$$

Since it is immediate to show that $da^*/dn < 0$, it follows that the optimal *individual* effort is maximal when $n = 2$. Now, from Proposition 5 we know that for any given n the maximal individual effort obtains when $n = \lambda + 1$. Observe that the equilibrium effort when $n = \lambda + 1$ is the same as the maximal individual effort that the game admits, i.e. it is the same as when $n = 2$ and $\lambda = 1$. Indeed, this will is true since the maximal individual effort is given by:

$$\frac{q'(a^*)}{4q(a^*)}\Delta u - c'(a^*) = 0,$$

and for any $n > 2$, the players' equilibrium individual efforts will equal this value if $\frac{\lambda(n-1)}{(\lambda+n-1)^2} = 1/4$, an equality that is true if $n = \lambda + 1$.

Since $a^* > 0$, and since respecting $n = \lambda + 1$ implies that the dissipation ratio is given by $D = \frac{na^*}{v}$, there always exists a finite value of n above which over-dissipation can be observed at equilibrium.