Confidence and Gender Gaps in Competitive Environments

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Abstract

This paper analyzes theoretically how confidence gaps affect behavior in tournaments as well as in contests. An overconfident player overestimates his ability and hence his probability of winning. Our results help organizing experimental evidence on gender gaps in outcomes and behavior in tournaments and contests. Namely, the fact that women are less likely to enter tournaments, women often do not perform as well in tournaments, and women tend to bid more in contests.

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1 Introduction

Many economic studies document systematic gender gaps in outcomes and behavior. There is a gap on how much women earn. For example, in 2022 women in the US earn 82% of their male counterparts (Kochhar 2023). There is also a gap in the number of women in top business positions. For instance, women represent 6% of top business executives in the US (Keller et al. 2022). The wage gender gap is larger in high skilled work, and much of it seems to be caused by gaps in promotions (Blau and DeVaro 2007, Blau and Kahn 2017, Bronson and Thoursie 2020).

Experimental evidence from economics also documents gender gaps. The influential article by Gneezy et al. (2003) shows that competing in a tournament causes males to increase their performance by more than females. The seminal article by Niederle and Vesterlund (2007) finds that men are more likely to enter a tournament than women. The importance of both studies has stimulated much experimental research on this topic, and the gender gap in tournament entry has been replicated in many other studies (Markowsky and Beblo 2022).[1]

Intriguingly, experimental evidence on contests produces quite different gender gaps in behavior as compared to the experimental literature on tournaments. Indeed, some scholars find that females invest higher effort than males (Anderson and Stafford 2003, Mago et al. 2013, Price and Sheremeta 2015, Mago and Razzolini 2019). For example, Mago and Razzolini (2019) document a gender gap whereby females spend more effort than males. Females also tend to bid more than males in experimental all-pay auction contests (Ham and Kagel 2006, Charness and Levin 2009, Hyndman et al. 2012, Breaban et al. 2020).

Such gender gaps in outcomes and behavior documented in experimental studies are due to gender differences in confidence and preferences (Croson and Gneezy 2009). Most of the literature on gender and tournament entry finds that men are

[1]There is also a gender gap in performance in highly competitive academic exams (Jurajda and Münich 2011, Ors et al. 2013), although this gap is not directly comparable with the more controlled environment of the laboratory.

more confident than women (Buser et al. 2021b). In fact, four recent studies argue that the gender gap in tournament entry is mainly driven by gender differences in confidence and risk attitudes (Kamas and Preston 2012, Gillen et al. 2019, Price 2020, van Veldhuizen 2022).[2]

Even though the experimental literature has identified gender gaps in confidence as one of the drivers of gender gaps in behavior in both tournaments and contests, to date no theoretical explanation has been proposed to rationalize these findings, and the apparent differences in behavior across the two types of competition. Understanding the mechanisms driving these gender gaps is important in order to optimally design affirmative action policies (e.g. Calsamiglia et al. 2013, Niederle et al. 2013).

In this paper we analyze theoretically how confidence gaps affect behavior in tournaments as well as in contests. These two types of models both describe competitive environments where a player's probability of winning is an increasing function of his effort. However, they also differ since in a tournament, unlike in a contest, noise can play an important role and a player may win the prize with zero effort.[3]

We set-up a two player tournament model where players are identical in all respects except their confidence. An overconfident player overestimates his ability, and ability and effort are complements.[4] One player is overconfident, whereas the other player can be either overconfident, rational, or underconfident. We assume noise follows a standard Gumbel distribution and fully characterize the equilibrium of a Lazear-Rosen rank-order tournament. Following earlier literature, we show that a generalized Tullock (1980) contest is a special case of the tournament (Hirshleifer

[2]Gender and socio-economic differences in confidence are also important determinants of educational and career choices (Buser et al. 2014, Wiswall and Zafar 2015, Reuben et al. 2017, Guyon and Huillery 2021).

[3]Promotions in firms, sports competitions, election campaigns, rent-seeking games, R&D races, competition for monopolies, litigation, wars are examples of tournaments and contests.

[4]Moore and Healy (2008) distinguish between three types of overconfidence: overestimation of one's skill (absolute overconfidence), overplacement (relative overconfidence), and excessive precision in one's beliefs (miscalibration or overprecision). Our own study conceptualizes overconfidence as an overestimation of one's skill or ability, thence precluding the third type of overconfidence.

and Riley 1992, Jia et al. 2013, Ryvkin and Drugov 2020), and we fully characterize the equilibrium of such a contest.

Our theoretical results uncover that confidence gaps may have different implications for behavior in tournaments than in contests. In a two player tournament where the confidence gap is small and neither player is too overconfident, the more overconfident player exerts higher effort at equilibrium. However, when either the confidence gap is large or both players are too overconfident, the more overconfident player exerts lower effort at equilibrium. The intuition behind these results lies in the following trade-off. Players aim at exploiting the complementarities between confidence and effort while attempting to save on cost of effort. When neither player is too overconfident, an increase in the confidence of the most confident player raises his effort because the increase in the perceived probability of winning times the utility prize spread is greater than the associated increase in cost of effort. For high levels of confidence, on the other hand, there is limited scope for further increasing one's perceived probability of winning, thence implying that the latter effect dominates and the more overconfident player exerts lower effort.

Our model also sheds lights on how confidence gaps affect players' incentives to enter a tournament. We predict that in a two player tournament where a man is more confident than a woman, and where the confidence gap is not too large, the man will be relatively more attracted by the tournament. This result is driven by a more confident man overestimating his winning probability, and thus exerting disproportionately high levels of effort, which in turn leads the less confident woman to reduce her own effort.

We then show that in a contest opposing two overconfident players the more overconfident player always exerts lower effort at equilibrium. Indeed, an increase in overconfidence leads to a drop in the perceived marginal probability of winning in a contest. Consequently, the scope for further improving one's winning probability will be reduced, thereby incentivizing the overconfident player to reduce effort and save on costs of effort. Observe, however, that in a contest opposing an overconfident to an underconfident player, the more confident player may exert a higher effort when the confidence gap is not too large.

Our theoretical results on the effect of the confidence gap on equilibrium efforts in tournaments and contests are able to organize the experimental evidence. In a tournament, the complementarity between confidence and effort implies that the most overconfident player exerts more effort when neither player is too overconfident. This is consistent with the experimental findings in Gneezy et al. (2003), Gneezy and Rustichini (2004), Niederle et al. (2013), Buser et al. (2021b), and van Veldhuizen (2022), who show that women do not perform as well as men in tournaments.[5] In contrast, in a two player contest featuring no underconfident player, the most overconfident player always exerts lower effort. This is consistent with the experimental evidence in Anderson and Stafford (2003), Mago et al. (2013), Price and Sheremeta (2015), and Mago and Razzolini (2019) who show that women invest more effort than men in contests.

The paper is organized as follows. Section 2 discusses related literature. Section 3 sets-up the general model. Sections 4 derives the results for tournaments and Section 5 for contests. Section 6 concludes the paper. All proofs are in the Appendix.

2 Related Literature

This study relates to four strands of literature. First, it contributes to the large literature on gender and competition. This literature documents gender gaps in outcomes and behavior and tries to identify the drivers behind these gaps (Bertrand and Hallock 2001). Several explanations have been given for the gender pay gap, including discrimination (Becker 1971), differences in human capital (Mincer and Polachek 1974), in ability in non-market activities (Lazear and Rosen 1990), in risk

[5] Not all studies find evidence of a gender gap in performance in tournaments (e.g. Niederle and Vesterlund 2007, Gillen et al. 2019). The existence of gender differences in performance in tournaments is sensitive to the task used, and, for instance, is more pronounced in a math task and when gender is observable (Niederle and Vesterlund 2010, Price 2012, Iriberri and Rey-Biel 2017).

attitudes (Eckel and Grossman 2003), or in mobility (Goldin et al. 2017).

Beyond the aforementioned factors, the experimental evidence on tournaments shows that gender differences in entry and performance are driven by differences in beliefs and preferences (Croson and Gneezy 2009, Niederle and Vesterlund 2011, Dechenaux et al. 2015). In particular, women are less likely to enter tournaments than men are (Niederle and Vesterlund 2007, Niederle 2016, Markowsky and Beblo 2022). Second, women often do not perform as well as men, especially in mixed gender tournaments where gender is observable (Gneezy et al. 2003, Gneezy and Rustichini 2004, Niederle et al. 2013, Buser et al. 2021b, van Veldhuizen 2022).[6] Third, gender differences in confidence and risk attitudes can explain a significant part of these differences in behavior: (1) women are less confident about their performance than men, and (2) women are more risk averse than men (Niederle and Vesterlund 2007, Kamas and Preston 2012, Gillen et al. 2019, Price 2020, van Veldhuizen 2022).[7,8]

Our study focuses on the effect of gender differences in confidence on behavior and outcomes in tournaments and contests. We provide theoretical results that help

[6] In a summary of the related experimental literature, Niederle (2016, p.502) writes: "*Gender differences in performance increase when moving from a competitive to a non-competitive incentive scheme, a result that has been replicated several times. This implies that a woman of an ability and performance in a non-competitive piece rate scheme comparable to that of a man will have an inferior performance to that man if the performance is measured in a competitive environment where women and men compete against each other.*"

[7] These gaps were already suggested by Dechenaux et al. (2015, p.650): "*Several experimental studies do indeed find robust evidence that men and women behave differently in tournaments (Croson and Gneezy 2009; Niederle and Vesterlund 2011). In fact, this evidence is corroborated both by laboratory and field experiments. Generally speaking, women are less likely to enter tournaments than men are (women "shy away from competition"). Second, women do not perform as well as men under tournament incentives. Third, possible differences in attitudes toward risk can only partially explain these differences.*"

[8] There is an ongoing debate as to the relative importance of confidence, risk attitudes, and gender differences in attitudes towards competition (Niederle 2017, Gillen et al. 2019, Buser et al. 2021a, Lozano and Reuben 2022, van Veldhuizen 2022).

organizing experimental findings on gender gaps in tournaments and contests. Our theoretical results confirm that a confidence gap between two players in a tournament can lead the most confident player to outperform the other, thereby providing a theoretical explanation to the findings of Gneezy et al. (2003), Gneezy and Rustichini (2004), Niederle et al. (2013), Buser et al. (2021b), and van Veldhuizen (2022). Regarding the decision to enter a tournament, our results suggest that a gender gap in confidence can lead to an increase in the perceived expected utility of the more confident player, especially in situations where the confidence gap leads to a decrease in the equilibrium effort of the least confident player.

The experimental evidence on gender differences in contests, which is less abundant, shows that women tend to invest higher effort than men (Anderson and Stafford 2003, Mago et al. 2013, Price and Sheremeta 2015, Mago and Razzolini 2019). Our theoretical results confirm that a confidence gap between two players in a contest can lead the most confident player to exert lower effort. In particular, this is always the case if the less confident player is not underconfident.

Second, our study contributes to the theoretical literature on behavioral biases in tournaments. Goel and Thakor (2008) study tournaments where overconfident and unbiased managers compete for promotion to become CEO by choosing the level of risk of their projects. They find that overconfident managers, those who underestimate project risk, have a higher likelihood of being promoted to CEO than unbiased ones. Santos-Pinto (2010) studies a two player tournament where both players are equally overconfident and shows that the tournament organizer can exploit the players' perceptual bias. Unlike Santos-Pinto (2010), we consider an asymmetric setup where players can hold different confidence levels. Moreover, we clarify the conditions under which managers who are overconfident about their abilities are more likely to be promoted or not to a CEO position.

Third, it contributes to the theoretical literature on behavioral biases in contests. The most closely related studies are Ando (2004) and Ludwig et al. (2011). Ando (2004) studies a two player contest where overconfidence is an overestimation of the monetary value of winning the contest. Ludwig et al. (2011) analyze a contest where an overconfident player competes against a rational player and overconfidence is an underestimation of the cost of effort. Our results show that when overconfidence is an overestimation of own ability and consequently of the winning probability, its effects on effort provision are quite different than those in found in Ando (2004) and Ludwig et al. (2011). The differences in the results are driven by the fact that overconfidence in our setup raises the marginal perceived utility from winning for low values of effort whereas it lowers it for high value of effort. As a consequence, and in contrast to Ando (2004) and Ludwig et al. (2011), in our study, overconfidence shifts a player's best response in a contest in a non-monotonic way.

Baharad and Nitzan (2008) and Keskin (2018) amend the standard model of contests by introducing probability weighting in line with Tversky and Kahneman's (1992) Cumulative Prospect Theory. This behavioral bias is modeled with an inverse S-shaped probability weighting function, i.e., a function where the marginal increase in the (perceived) subjective probability is higher for extreme (i.e. low and high) probabilities. Our own approach assumes a constant bias in players' beliefs that they are better than they really are at contesting their opponents. We thus see our approach as complementary to these earlier works since nothing precludes players from both assigning 'weights' to probabilities and be subject to an overconfidence bias. Notice that in terms of contribution to the literature on behavioral biases in contests, our approach has the advantage to be flexible enough to accommodate a very large family of contest success functions.

3 Set-up

Consider two players, i and j, competing in a tournament. The player who produces the highest output receives the winner's prize y_W and the other receives the loser's prize y_L, with $0 < y_L < y_W$. The players are expected utility maximizers and have utility functions that are separable in income (y_k) and effort ($a_k \in \mathbb{R}^+$), $k = i, j$.

Player i's utility function (likewise for j) is given by:

$$U_i(y_i, a_i) = u(y_i) - c(a_i).$$

We assume u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' \geq 0$, $c(0) = 0$, $c'(0) = 0$, and $c(a_i) = \infty$, for $a_i \to \infty$, where the last two conditions ensure that equilibrium effort is strictly positive but finite. The two players have an outside option which guarantee each $\bar{u} \geq 0$. We assume $\bar{u} = 0$.

When player i exerts effort a_i his output is given by

$$Q_i = h(q(a_i)) + \varepsilon_i, \tag{1}$$

where both $h(.)$ and $q(.)$ are increasing functions. We assume that ε_i follows a standard Gumbel distribution, that is, its density function is $f(\varepsilon_i) = e^{-\varepsilon_i - e^{-\varepsilon_i}}$, and its cumulative distribution function is $F(\varepsilon_i) = e^{-e^{-\varepsilon_i}}$. This noise distribution enables us to fully characterize the equilibrium in Lazear and Rosen (1981) rank-order tournaments with overconfident players. Moreover, the difference between two Gumbel random variables with the same variance follows a logistic distribution, which has a similar shape to the Normal distribution. Lastly, as shown below, this assumption also enables us to characterize generalized Tullock contests as a particular form of tournament with the same definition of overconfidence.

The two players can differ from one another in terms of their beliefs about their productivity of effort while holding a correct assessment of the winning prize and their cost of effort. An overconfident player i overestimates his productivity of effort, that is, he thinks his output function is

$$\tilde{Q}_i = h(\lambda_i q(a_i)) + \varepsilon_i,$$

where $\lambda_i > 1$. Under this specification player i perceives his marginal output is increasing with his overconfidence bias λ_i, that is, $\partial^2 \tilde{Q}_i / \partial a_i \partial \lambda_i > 0$. This describes situations where effort and ability are complements in generating output and where an overconfident player overestimates his ability. This way of modeling overconfidence is often used in the literature that analyzes its impact on labor contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). An overconfident player i ($\lambda_i > 1$) competes against a player j that can be either overconfident ($\lambda_j > 1$), rational ($\lambda_j = 1$), or underconfident ($0 < \lambda_j < 1$).

Hence, player i's perceived probability of winning the tournament is

$$\begin{aligned} P_i(a_i, a_j, \lambda_i) &= \Pr(\tilde{Q}_i \geq Q_j) \\ &= \Pr(h(\lambda_i q(a_i)) + \varepsilon_i \geq h(q(a_j)) + \varepsilon_j) \\ &= \Pr(\varepsilon_j - \varepsilon_i \leq h(\lambda_i q(a_i)) - h(q(a_j))) \\ &= \frac{1}{1 + e^{-(h(\lambda_i q(a_i)) - h(q(a_j)))}}. \end{aligned}$$

Player i chooses the optimal level of effort that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j, \lambda_i)] = u(y_L) + P_i(a_i, a_j, \lambda_i)\Delta u - c(a_i), \tag{2}$$

where $\Delta u = u(y_W) - u(y_L)$.

To be able to compute equilibria when workers' hold mistaken beliefs we assume that: (1) a player who faces a biased opponent is aware that the latter's ability perception (and probability of winning) is mistaken, (2) each player thinks that his own ability perception (and probability of winning) is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players agree to disagree about their ability perceptions (and winning probabilities). This approach follows Heifetz et al. (2007a,2007b) for games with complete information, and Squintani (2006) for games with incomplete information.

These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007). As stated by Pronin et al. (2002: 369) "people recognize the existence, and the impact, of most of the biases that social and cognitive psychologists have described over the past few decades. What they *lack* recognition of, we argue, is the role that those same biases play in governing their *own* judgments and inferences." For example, Libby and Rennekamp (2012) conduct a survey which shows that experienced financial managers believe that other managers are likely to be overconfident while failing to recognize their own overconfidence. Hoffman (2016) runs a field experiment which finds that internet businesspeople recognize others tend to be overconfident while being unaware of their own overconfidence.[9]

Note that throughout the paper we assume players are fully identical except for their beliefs. This assumption allows us to isolate the effect of confidence gaps on behavior. In addition, most of the experimental evidence we are going to relate our theoretical results to assumes players have identical abilities/productivities and outside options.

In Section 4 we analyze the effect of overconfidence in the canonical Lazear-Rosen rank-order tournament where $\tilde{Q}_i = \lambda_i a_i + \varepsilon_i$. In this case player i's perceived probability of winning the tournament is:

$$P_i(a_i, a_j, \lambda_i) = \frac{1}{1 + e^{-(\lambda_i a_i - a_j)}}.$$

In Section 5 we analyze the effect of overconfidence in a generalized Tullock contest. When $h(.) = \ln(.)$, the tournament collapses into a Tullock contest since player i's perceived probability of winning becomes:

$$P_i(a_i, a_j, \lambda_i) = \frac{1}{1 + e^{-\ln(\lambda_i q(a_i)/q(a_j))}} = \frac{1}{1 + \frac{q(a_j)}{\lambda_i q(a_i)}} = \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)},$$

where function $q(a)$ is often referred to as the impact function (Ewerhart 2015) and models the technology whereby players' efforts or investments translate into probabilities of winning the contest.

[9] Ludwig and Nafziger (2011) conduct a lab experiment that elicits participants' beliefs about own and others' overconfidence and abilities. On the one hand they find that the largest group of participants thinks that they are themselves better at judging their ability correctly than others. On the other hand, they find that with a few exceptions, most people believe that others are unbiased.

Note that this specification of overconfidence satisfies three desirable properties. First, the overconfident player's perceived winning probability is well defined for any value of λ_i. Second, the overconfident player's perceived winning probability is increasing in λ_i. Third, in a contest, overestimating one's ability (or impact function) is equivalent to underestimating the rivals' ability.

4 Confidence Gaps in Tournaments

In this section we analyze the effect of confidence gaps on a two player Lazear and Rosen (1981) rank-order tournament. First, we study the effect of confidence gaps on effort provision, and then discuss the players' incentives to participate to the tournament.

4.1 Confidence and effort in tournaments

Player i chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = u(y_L) + \frac{1}{1 + e^{-(\lambda_i a_i - a_j)}}\Delta u - c(a_i).$$

The first order-conditions for players i and j are as follows:

$$\begin{aligned} \lambda_i \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1 + e^{-(\lambda_i a_i - a_j)}\right]^2}\Delta u &= c'(a_i), \\ \lambda_j \frac{e^{-(\lambda_j a_j - a_i)}}{\left[1 + e^{-(\lambda_j a_j - a_i)}\right]^2}\Delta u &= c'(a_j). \end{aligned}$$

The second-order conditions are then given by:

$$\begin{aligned} -\lambda_i^2 \frac{1 - e^{-(\lambda_i a_i - a_j)}}{\left[1 + e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)}\Delta u - c''(a_i) < 0, \\ -\lambda_j^2 \frac{1 - e^{-(\lambda_j a_j - a_i)}}{\left[1 + e^{-(\lambda_j a_j - a_i)}\right]^3} e^{-(\lambda_j a_j - a_i)}\Delta u - c''(a_j) < 0. \end{aligned}$$

We assume in what follows that these second-order conditions are satisfied.

Denoting by $R_i(a_j)$ player i's best response, we show the following result:

Lemma 1. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $a_j = \lambda_i a_i$.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

A second useful lemma describes how player i's best response changes with his confidence λ_i.

Lemma 2. *An increase in player i's confidence λ_i leads to an expansion of his best response function, $\partial R_i(a_j)/\partial \lambda_i > 0$, for $e^{-(\lambda_i a_i - a_j)} > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$, and to a contraction of his best response, $\partial R_i(a_j)/\partial \lambda_i < 0$, for $e^{-(\lambda_i a_i - a_j)} < \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$. Moreover, the maximum value of player i's best response increases in player i's confidence λ_i.*

Lemma 2 characterizes the best responses of players who are subject to a confidence bias. For a high effort of the rival, an increase in confidence raises player i's effort level. For low effort of the rival, however, depending on the size of the bias, an increase in confidence can either expand or contract player i's best response. Moreover, the maximal value taken by player i's best response is increasing in his confidence bias. Making use of these results, we can establish equilibrium uniqueness in the following lemma:

Lemma 3. *A two player tournament featuring an overconfident player 1 admits a unique equilibrium if $\lambda_1 \lambda_2 \geq 1$.*

We next present our main result on the effect of the confidence gap on the tournament equilibrium efforts.

Proposition 1. *Consider a two player tournament where player 1 is overconfident and $\lambda_1 \lambda_2 \geq 1$. For any level of confidence λ_2 of player 2, there is a threshold value of player 1's overconfidence bias $\bar{\lambda}_1(\lambda_2) \geq \lambda_2$ such that $a_1^* > a_2^*$ if $\lambda_1 < \bar{\lambda}_1(\lambda_2)$ and $a_1^* < a_2^*$ otherwise. There exists a threshold value of λ_2 that we denote by $\bar{\lambda}_2$ such that if $\lambda_2 < \bar{\lambda}_2$, then $\bar{\lambda}_1(\lambda_2) > \lambda_2$, otherwise $\bar{\lambda}_1(\lambda_2) = \lambda_2$.*

This proposition uncovers that the effects of the confidence gap on a player's equilibrium effort depend on the size of the gap as well as on the confidence level of the least confident player. If the confidence gap is small and neither player is too overconfident, then the more overconfident player 1 exerts more effort at equilibrium. Hence, the more confident player 1 will be the Nash winner since $P_1(a_1^*, a_2^*) > 1/2$. In contrast, if either the confidence gap is large or both players are too overconfident, then the more overconfident player 1 exerts less effort at equilibrium and is therefore the Nash loser.

We illustrate Proposition 1 in Figures 1 and 2. On Figure 1, we represent the best responses and equilibrium efforts when both players are overconfident, the confidence gap is small, and neither player is too overconfident. To better gauge the effect of overconfidence, we have also drawn the best responses of rational players as depicted by the two dashed curves crossing on the 45^o line of the left panel of Figure 1. The equilibrium when both players are equally overconfident is depicted by point E and involves higher equilibrium efforts than if both players were rational. Increasing the overconfidence of player 1 implies that his best response shifts outwards (see Lemma 2) as represented by the dashed and dotted best response. Consequently, since efforts are strategic complements in this range, the new equilibrium, E', has the more confident player 1 exerting higher effort. On the right panel of Figure 1 we depict a situation where the confidence gap is larger than on the left panel, leading the less confident player 2 to reduce his equilibrium effort, which corresponds to the shift from equilibrium E' to E''. The intuition behind this result is as follows. If the players are equally confident, they exert the same equilibrium effort, $a_1^* = a_2^*$. If the confidence gap is small and neither player is too overconfident, an increase in player 1's overconfidence expands his best response at equilibrium. Given the quasi-concavity of the players' best responses, this in turn implies that a_1^*/a_2^* will increase.

Figure 2 depicts the best responses and equilibrium efforts in the case where the confidence gap is large (player 1 is highly overconfident and player 2 is not). In this

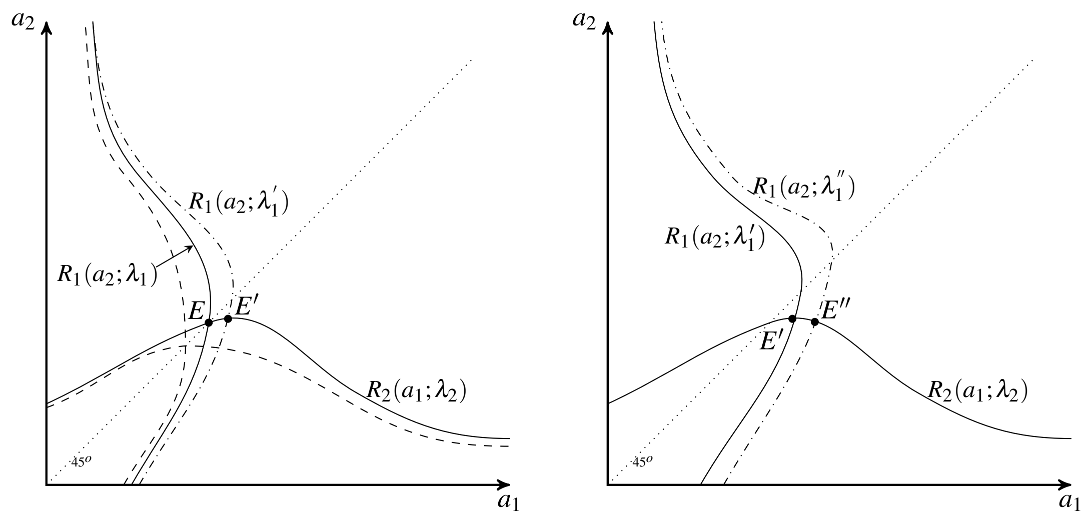

Figure 1: Tournament with a Small Confidence Gap ($1 < \lambda_1 < \lambda_1' < \lambda_1''$)

case, the highly overconfident player 1 exerts less effort than the less confident player 2. Indeed, if the confidence gap is large, then an increase in player 1's overconfidence leads to a contraction of his best response at equilibrium. This in turn leads to a reduction of a_1^*/a_2^*, and if player 1 is sufficiently overconfident, then player 1 will exert a lower effort than player 2 at equilibrium. Observe that if both players are highly overconfident, an increase in player 1's overconfidence always leads to a contraction of his best response at equilibrium.

Proposition 1 helps organizing the evidence on gender gaps in performance in tournaments. Several studies find a gender gap in performance in tournaments, whereby women do not perform as well as men, especially on mixed-gender tournaments where gender is observable (Gneezy et al. 2003, Gneezy and Rustichini 2004, Niederle et al. 2013, Buser et al. 2021b, van Veldhuizen 2022). For example, Gneezy et al. (2003) set up a six player single-prize tournament featuring three female and three male participants who compete by solving mazes online. In their setup where participants observe each other's gender, the authors find a significant gender gap in performance: on average, men solve 15 mazes whereas women solve 10.8 mazes.

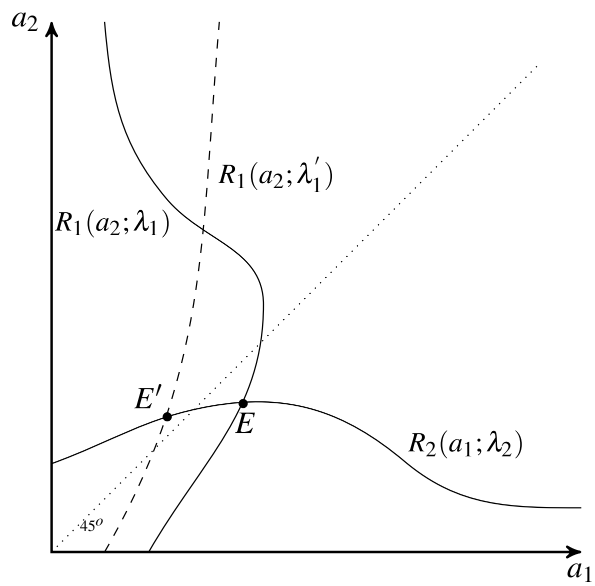

Figure 2: Tournament with a Large Confidence Gap ($1 < \lambda_1 < \lambda_1'$)

Gneezy and Rustichini (2004) measure the running speed of boys and girls from 9 to 10 years old. The children first run alone and then paired with another child. They find that boys and girls run at the same speed when alone. However, boys run faster whereas girls run slower in the second round. In addition, boys only races led to a better performance whereas girls only races led to a worse performance in the second round.

In our model, the players' equilibrium outputs, Q_1^* and Q_2^*, determine their performance, and output is a function of ability, effort, and luck. In experimental tournaments performance is a function of these same variables. If men and women participating to the experiments are equally able in expectation and have the same preferences and cost of effort, then systematic differences in performance can only be due to differences in effort provision. Proposition 1 shows that the confidence gap can lead to differences in effort which, in turn, lead to differences in performance. In particular, if the confidence gap is small, and neither player is too overconfident, overconfident males will exert more effort than less confident females. Consequently, the experimental findings of Gneezy et al (2003) and the other studies cited above are consistent with our theoretical predictions, provided women are less confident than men, and neither gender is too overconfident. Interestingly, Gneezy et al. (2003) do find that women feel less competent than men, thus further pointing at the salience of our theory.

There are also experimental studies that do not find gender differences in performance in tournaments (e.g. Niederle and Vesterlund 2007, Gillen et al. 2019). Our theoretical results do not preclude this possibility since the confidence gap has a non-monotonic effect on effort provision. As Proposition 1 shows, if the confidence gap is large, the more confident player may exert the same or even less effort than the less confident player at equilibrium. In view of our theoretical predictions, it is therefore important to measure the size of the confidence gap in experimental studies on gender differences in behavior in tournaments.

4.2 Confidence and entry in tournaments

We now uncover instances where confidence gaps make entry in the tournament more attractive. The analysis so far assumes that the players are better off by participating in the tournament than staying out. This is the case because even when exerting zero effort a player has a strictly positive probability of winning the tournament, and the outside option is normalized to zero. However, if the outside option is high enough, it is possible that the perceived expected utility of participating to the tournament is too low to make the tournament attractive. To analyze how confidence affects entry, we assume that player 1 is more confident than player 2, and study the effects of an increase in player 1's confidence on both players' perceived equilibrium utilities.

We begin by focusing on player 1 whose perceived equilibrium utility is

$$E[U_1(a_1^*, a_2^*, \lambda_1)] = u(y_L) + \frac{1}{1 + e^{-(\lambda_1 a_1^* - a_2^*)}} \Delta u - c(a_1^*).$$

The impact of player 1's confidence on his perceived equilibrium utility is

$$\begin{aligned}\frac{\partial E\left[U_1(a_1^*, a_2^*, \lambda_1)\right]}{\partial \lambda_1} &= \frac{e^{-(\lambda_1 a_1^* - a_2^*)}}{\left[1+e^{-(\lambda_1 a_1^* - a_2^*)}\right]^2}\left[a_1^* + \lambda_1 \frac{\partial a_1^*}{\partial \lambda_1} - \frac{\partial a_2^*}{\partial \lambda_1}\right]\Delta u - c'(a_1^*)\frac{\partial a_1^*}{\partial \lambda_1} \\ &= \frac{e^{-(\lambda_1 a_1^* - a_2^*)}}{\left[1+e^{-(\lambda_1 a_1^* - a_2^*)}\right]^2}\left[a_1^* - R_2'(a_1^*)\frac{\partial a_1^*}{\partial \lambda_1}\right]\Delta u, \qquad (3)\end{aligned}$$

where the second equality follows from the Envelope Theorem.

Equation (3) shows that the effect of player 1's overconfidence on his perceived equilibrium utility depends on the sign of the term inside the square brackets. The first term in the square brackets is positive and captures the direct effect of player 1's confidence on his perceived winning probability weighted by the prize spread. The second term describes a strategic effect of player 1's confidence on player 2's effort. The sign of this second term is generally ambiguous, and is positive under two scenarios.

First, if efforts are strategic substitutes for player 2 and an increase in player 1's confidence incentivizes the latter to increase effort, this will lead player 2 to reduce effort. This case is depicted in the right panel of Figure 1 where the confidence gap between players 1 and 2 is small and an increase in player 1's confidence raises his equilibrium effort while lowering player 2's equilibrium effort. Second, if efforts are strategic complements for player 2 and an increase in player 1's confidence incentivizes the latter to reduce effort, then player 2 will also reduce effort. This case is depicted in Figure 2 where the confidence gap is large, and further increases in player 1's confidence incentivize both players to reduce their equilibrium efforts. In any other situation, the sign of the second term is ambiguous, and thus the effect of an increase in player 1's confidence on his perceived equilibrium utility is equally ambiguous.

We now focus on player 2 whose perceived equilibrium utility is only indirectly impacted by a change in player 1's confidence. Consequently, player 2's perceived equilibrium utility will decrease if an increase in player 1's confidence leads player 1 to raise his effort. This case is depicted in both panels of Figure 1. On the other hand, if player 1 reduces his effort following an increase in his confidence, then player 2' perceived expected utility will increase. This case can be visualized in Figure 2.

The above analysis reveals that for not too large confidence gaps, the more confident player may have higher incentives to participate to the tournament, while the less confident one could be disincentivized to participate (right panel of Figure 1). Surprisingly, the model also predicts that when a player is highly overconfident then both players' perceived expected utility of participating to the tournament will be higher than if that same player was less confident (Figure 2).

These predictions shed light on the mechanisms whereby confidence gaps can lead to the gender gap in tournament entry which has first been reported in Niederle and Vesterlund (2007) and later replicated in many studies (Markowsky and Beblo 2022). We predict that in a two player tournament where a man is more confident than a woman, and where the confidence gap is not too large and neither player is too overconfident, the man will be relatively more incentivized to enter the tournament. This is driven by a more confident man overestimating his winning probability, and thus exerting disproportionately high levels of effort, which in turn leads the less confident woman to reduce her own effort. Our model describes how behavior in competitive setups is affected by confidence gaps, provided players are aware of each other's biases. Consequently, for establishing a connection between our theoretical findings and experimental results, it is crucial that the participants in a laboratory experiment are aware of each other's confidence biases, which may be gauged upon observing each other's gender, as in Niederle and Vesterlund (2007).

5 Confidence Gaps in Contests

In this section we analyze the effect of confidence gaps on a two player generalized Tullock contest. First, we study the effect of confidence gaps on effort provision, and then discuss the players' incentives to participate to the contest.

5.1 Confidence and effort in contests

The perceived probability of winning of player i is as follows:

$$P_i(a_i, a_j; \lambda_i) = \begin{cases} \lambda_i q(a_i)/[\lambda_i q(a_i) + q(a_j)] & \text{if } \lambda_i q(a_i) + q(a_j) > 0 \\ 1/2 & \text{if } \lambda_i q(a_i) + q(a_j) = 0 \end{cases},$$

where $q(0) \geq 0$, $q'(a_i) > 0$ and $q''(a_i) \leq 0$.

Player i chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = u(y_L) + \frac{\lambda_i q(a_i)}{\lambda_i q(a_i) + q(a_j)} \Delta u - c(a_i).$$

The first-order condition is

$$\frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2} \Delta u - c'(a_i) = 0. \tag{4}$$

The second-order condition is

$$\frac{q''(a_i)[\lambda_i q(a_i) + q(a_j)] - 2\lambda_i [q'(a_i)]^2}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q(a_j) \Delta u - c''(a_i) < 0, \tag{5}$$

and the above inequality is satisfied since $q''(a_i) \leq 0$ and $c''(a_i) \geq 0$.

Let $a_i = R_i(a_j)$ denote player i's best response obtained from (4). Along player i's best response we have

$$\lambda_i q'(a_i) q(a_j) \Delta u = c'(a_i) \left[\lambda_i q(a_i) + q(a_j)\right]^2.$$

Lemma 4 describes the shape of the player i's best response.

Lemma 4. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $q(a_j) = \lambda_i q(a_i)$.*

Lemma 4 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

A second useful lemma describes how player i's best response changes with the confidence parameter λ_i.

Lemma 5. *An increase in player i's confidence* λ_i *leads to a contraction of his best response,* $\frac{\partial R_i(a_j)}{\partial \lambda_i} < 0$*, for* $q(a_j) < \lambda_i q(a_i)$*, and to an expansion of his best response,* $\frac{\partial R_i(a_j)}{\partial \lambda_i} > 0$*, for* $q(a_j) > \lambda_i q(a_i)$*. Moreover, the maximum value of player i's best response is independent of player i's confidence.*

Lemma 5 characterizes the best responses of players who are subject to a confidence bias in a contest. For a high effort of the rival, an increase in confidence raises player i's effort level, while for low effort of the rival, an increase in confidence lowers player i's effort level. Moreover, unlike in the tournament, the maximal value taken by player i's best response is independent of his confidence bias.

Making use of these results, we can establish equilibrium uniqueness in the following lemma:

Lemma 6. *A two player contest featuring one overconfident player* 1 *admits a unique equilibrium if* $\lambda_1 \lambda_2 \geq 1/3$.

We next present a proposition that uncovers the effect of the confidence gap on equilibrium efforts in a two player contest.

Proposition 2. *In a contest with two overconfident players where* $\lambda_1 > \lambda_2 > 1$*, the more overconfident player* 1 *exerts lower effort. Hence, the more overconfident player* 1 *is the Nash loser since* $P_1(a_1^*, a_2^*) < 1/2 < P_2(a_1^*, a_2^*)$.

Proposition 2 contrasts with Proposition 1 since it shows that in a contest between two overconfident players, unlike in a tournament, the confidence gap does not matter to determine which player exerts more effort in equilibrium.

Corollary 1. *In a contest with two overconfident players, the players exert less effort than if both were rational, and as the overconfidence of either player increases, both players' efforts decrease.*

If the confidence of player i goes up, then player i's best response shifts inwards for $q(a_j) < \lambda_i q(a_i)$ (as shown in Lemma 5). Corollary 1 follows from the fact that the players' best responses are positively-slopped at the Nash equilibrium.

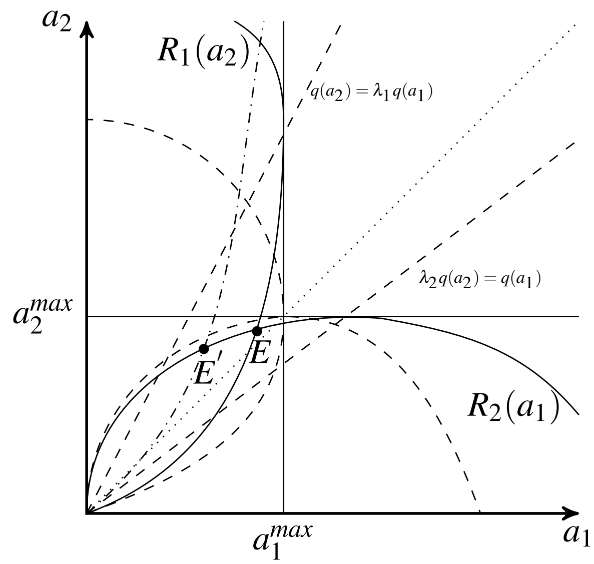

Figure 3: Contest with Overconfident Players

We illustrate Proposition 2 in Figure 3. On that figure we represent the players' best responses and equilibrium efforts given that player 1 is more overconfident than player 2, i.e. $\lambda_1 > \lambda_2$. From Lemma 4 we know that the best responses are quasi-concave, while from Lemma 5 we also know that the maximal value player i's best response takes is given by $q(a_j) = \lambda_i q(a_i)$, hence the crossing of the dotted lines with the maxima of the best responses. To better gauge the effect of overconfidence, we have also drawn the best responses of rational players as seen in the two concave dashed curves crossing on the 45^o line at (a_1^{max}, a_2^{max}). The higher is a player's overconfidence, the more the best response flattens for values of the rival's effort a_j below $q^{-1}(\lambda_i q(a_i))$, and steepens for values above that threshold, while the maximand of the best response increases with overconfidence. Consequently, and in line with Proposition 2, the more overconfident player 1 will experience a harsher contraction of his best response below a_2^{max}, and since the best response functions of both players are strictly increasing in $[0, a_j^{max}]$, the equilibrium E will lie above the 45^o line in the space where $a_2 > a_1$.

Increasing the overconfidence of player 1, implies that the player's best response shifts inwards for low values of a_2 as represented graphically by the dashed and dotted best response. Consequently, since $R_2(a_1)$ remains unaffected by this shift in the overconfidence of his rival, at the new equilibrium E' both players will necessarily exert less effort that in E, while the concavity of $R_2(a_1)$ also implies that the new probability that player 1 wins the contest is now lower. Upon observing the figure, it is equally obvious that an increase in λ_2 will also result in lower equilibrium efforts of *both* players, while the probability that player 1 wins the contest would then increase instead.

We now consider a contest where an overconfident player i competes against an underconfident player j. Lemma 7 describes how the underconfident player's best response shifts with his bias λ_j.

Lemma 7. *An increase in player j's underconfidence (λ_j goes down) leads to a contraction of player j's best response, $\frac{\partial R_j(a_i)}{\partial \lambda_j} < 0$, for $q(a_i) > \lambda_j q(a_j)$ and to an expansion of player j's best response, $\frac{\partial R_j(a_i)}{\partial \lambda_j} > 0$, for $q(a_i) < \lambda_j q(a_j)$. Moreover, the maximum value of player j's best response is independent of player j's confidence.*

In Proposition 2 we show that the confidence gap is not relevant to determine which player exerts more effort at equilibrium in a contest between two overconfident players. Our next result shows that this is no longer the case in a contest where one player is overconfident and the other one is underconfident.

Proposition 3. *In a two player contest where player 1 is overconfident and player 2 is underconfident, $\lambda_1 > 1 > \lambda_2$, and where $\lambda_1\lambda_2 \geq 1/3$, the overconfident player exerts less effort than the underconfident player if and only if $\lambda_1\lambda_2 > 1$.*

If the underconfidence of player 2 goes up (λ_2 goes down), then player 2's best response shifts inwards for $q(a_1) > \lambda_2 q(a_2)$. If player 1 is rational ($\lambda_1 = 1$), then at equilibrium the underconfident player will exert lower effort. This result follows directly from the complementarity between confidence and effort. If the confidence of player 1 increases (λ_1 goes up), then player 1's best response shifts inwards for

$q(a_2) < \lambda_1 q(a_1)$. Indeed, player 1 perceives a higher winning probability and thence expects effort not to increase much this probability, so he lowers effort to save on cost of effort. For a sufficiently high confidence gap, $\lambda_1 > 1/\lambda_2$, the overconfident player 1 will exert less effort than the underconfident player 2 at equilibrium.

Corollary 2. *In a two player contest where player* 1 *is overconfident and player* 2 *is underconfident,* $\lambda_1 > 1 > \lambda_2$*, and where* $\lambda_1\lambda_2 \geq 1/3$*, the players exert less effort than if they were both rational.*

Propositions 2 and 3 both show that the more overconfident player exerts lower effort at equilibrium as long as the less confident player is not overly underconfident. This result helps organizing the experimental evidence on gender gaps in contests. These predictions are consistent with Mago and Razzolini's (2019) lab experiment. In their study they elicit the participants' confidence about their relative performance in an IQ test, after which they compete in a two player best-of-five Tullock contest. Note that here, in contrast to the experimental studies on tournaments, a participant's effort is the amount invested in the contest. Hence, effort is observable by the experimenter, and the cost of effort does not differ across participants. Although the best-of-five setup is not directly comparable to our one shot contest, we nevertheless find some parallels. In line with Propositions 2 and 3, and in contrast to the predictions of Ando (2004) and Ludwig et al. (2011), Mago and Razzolini (2019) find that more confident participants exert lower effort. Furthermore, total effort in female only contests is significantly higher than total effort in mixed or male only contests, which, combined with their finding that women are less confident than men, is also in line with our theoretical results.

Observe that our results are also in line with the findings of Anderson and Stafford (2003), Mago et al. (2013), and Price and Sheremeta (2015) who find that women invest higher amounts than men in contests. Note that in these three experiments, unlike the study of Mago and Razzolini (2019), the participants were unaware of the gender of their rivals.

5.2 Confidence and entry in contests

Proceeding as in Section 4.2, we deduce that the effect of a change in player 1's confidence on his perceived equilibrium utility in a contest is given by the following expression

$$\frac{\partial E[U_1(a_1^*, a_2^*, \lambda_1)}{\partial \lambda_1} = \frac{q(a_1^*)}{[\lambda_1 q(a_1^*) + q(a_2^*)]^2} \Delta u \left[q(a_2^*) - \lambda_1 q'(a_2^*) R_2'(a_1^*) \frac{\partial a_1^*}{\partial \lambda_1} \right]. \quad (6)$$

Equation (6) shows that the effect of player 1's overconfidence on his perceived equilibrium utility depends on the sign of the term inside square brackets which captures the same effects as in Section 4.2 on tournaments. As before, the first term is positive while the sign of the second term is ambiguous. Observe that $\partial a_1^* / \partial \lambda_1$ is always negative. Indeed, the more confident player always perceives his equilibrium probability of winning the contest to be larger than 1/2. As a consequence, his perceived marginal probability of winning the contest is decreasing with confidence, which implies that an increase in his confidence will incentivize him to save on cost of effort. Hence, when efforts are strategic complements, as depicted in Figure 3, for the less confident player 2, the term in squared brackets is positive, and an increase in player 1's confidence makes entry in the contest more attractive for player 1. On the other hand, when efforts are strategic substitutes for the less confident player, the overall effect of confidence on the attractiveness of entry will be ambiguous. Player 2's perceived equilibrium utility will unambiguously increase with an increase in the confidence of player 1 since this leads player 1 to lower his effort.

There is scarce experimental evidence on gender and entry in contests. The two exceptions we are aware of both find that women enter as much as men in Tullock contests (Cason et al. 2010, Morgan et al. 2012), and less than men in the particular case of proportional prize contests (Cason et al. 2010). Our results above show that having a more confident man as a rival in a contest does not necessarily make entry in the contest less attractive to a less confident woman.

6 Conclusion

This paper studies the impact of confidence gaps on tournaments and contests. In a two player tournament we show that the more confident player will be the Nash winner if the confidence gap is small and neither player is too overconfident. In this case, the more confident player exerts higher effort at equilibrium. However, the more confident player is the Nash loser when either the confidence gap is large or both players are too overconfident, because he will exert lower effort in equilibrium. An interesting implication of this result is that an underconfident player can be the Nash winner when the rival is excessively overconfident. These results clarify the conditions under which overconfidence about one's own ability can help a manager being promoted to a CEO position. They also highlight the conditions under which gender gaps in confidence will lead men to be overly represented at CEO positions.

Our model also sheds light on the mechanisms linking gender gaps in confidence to differences in entry in tournaments. We find that a more confident player may be relatively more attracted to enter a tournament than a less confident rival. These results help understanding experimental evidence on gender differences in entry and performance in tournaments.

In a contest opposing two overconfident players, the more overconfident player will be the Nash loser because he exerts lower effort at equilibrium. Observe, however, that in a contest opposing an overconfident to an underconfident player, the more confident player may exert a higher effort and be the Nash winner when the confidence gap is not too large. One implication of our findings calls for attention when hiring lawyers or lobbyists since excessive overconfidence may lead to worse outcomes. Moreover, it can also explain why women tend to spend more than men in experimental contests.

Our paper also provides new testable implications of confidence gaps in tournaments and contests. In tournaments we uncovered a non-monotonic relationship between a player's overconfidence and his equilibrium effort, keeping the rival's degree of confidence fixed. Likewise, in a contest the same result obtains provided one's rival is underconfident. This study carries important public policy implications. Given the non-monotonic effect of confidence gaps on effort provision, one should be careful when designing public policies (e.g. affirmative action) that affect incentives to enter and perform in competitions. Another implication of our study is that future experimental research should carefully account for the size of players' confidence biases.

An avenue for future theoretical research would be to study tournaments and contests where the players can differ not only in terms of confidence, but also in terms of their preferences towards either risk or competitiveness since the experimental evidence shows that all these aspects seem to matter. Last, it would be interesting to consider heterogeneity in skills and cost of effort, since that could result in more able players performing worse at equilibrium due to their confidence bias.

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7 Appendix

Proof of Lemma 1 Using player i's first-order condition, we have

$$\begin{aligned}
\frac{\partial^2 P_i(a_i,a_j,\lambda_i)}{\partial a_i \partial a_j} &= \frac{\partial}{\partial a_j}\left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\right) \\
&= \lambda_i \frac{e^{-(\lambda_i a_i - a_j)}\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2 - 2\left[1+e^{-(\lambda_i a_i - a_j)}\right]e^{-(\lambda_i a_i - a_j)}e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^4} \\
&= \lambda_i \frac{1+e^{-(\lambda_i a_i - a_j)} - 2e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \\
&= \lambda_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)}.
\end{aligned}$$

and

$$\frac{\partial^2 P_i(a_i,a_j,\lambda_i)}{\partial a_i^2} = \frac{\partial}{\partial a_i}\left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\right) = -\lambda_i^2 \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)}.$$

Therefore, the slope of the best response of player i is

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E(U_i)}{\partial a_i \partial a_j}}{\frac{\partial^2 E(U_i)}{\partial a_i^2}} = -\frac{\lambda_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \Delta u}{-\lambda_i^2 \frac{1-e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \Delta u - c''(a_i)}.$$

This is equal to zero when

$$1 - e^{-(\lambda_i a_i - a_j)} = 0,$$

or

$$\lambda_i a_i - a_j = 0.$$

Moreover, for $\lambda_i a_i > a_j$, the slope of player i's best response is positive, while otherwise, if $\lambda_i a_i < a_j$, the slope of player i's best response is negative.

Proof of Lemma 2 Player i's best response is defined as

$$\lambda_i \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} \Delta u = c'(a_i).$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i \partial \lambda_i} \Delta u,$$

where

$$\begin{aligned}
\frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i \partial \lambda_i} &= \frac{\partial}{\partial \lambda_i}\left(\frac{\lambda_i e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\right) \\
&= \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2} - \lambda_i a_i \frac{1+e^{-(\lambda_i a_i - a_j)} - 2e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^3} e^{-(\lambda_i a_i - a_j)} \\
&= \frac{e^{-(\lambda_i a_i - a_j)}}{\left[1+e^{-(\lambda_i a_i - a_j)}\right]^2}\left[1 - \lambda_i a_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{1+e^{-(\lambda_i a_i - a_j)}}\right].
\end{aligned}$$

Hence, $\partial R_i(a_j)/\partial \lambda_i$ is positive if and only if

$$1 - \lambda_i a_i \frac{1-e^{-(\lambda_i a_i - a_j)}}{1+e^{-(\lambda_i a_i - a_j)}} > 0,$$

or

$$1+e^{-(\lambda_i a_i - a_j)} - \lambda_i a_i \left[1-e^{-(\lambda_i a_i - a_j)}\right] > 0,$$

or

$$1 - \lambda_i a_i + e^{-(\lambda_i a_i - a_j)} + \lambda_i a_i e^{-(\lambda_i a_i - a_j)} > 0,$$

or

$$(1+\lambda_i a_i) e^{-(\lambda_i a_i - a_j)} > \lambda_i a_i - 1,$$

or

$$e^{-(\lambda_i a_i - a_j)} > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}.$$

Note that this inequality is always satisfied when $a_j \geq \lambda_i a_i$ since $e^{-(\lambda_i a_i - a_j)} \geq 1 > \frac{\lambda_i a_i - 1}{\lambda_i a_i + 1}$. Substituting next $a_j = \lambda_i a_i$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the tournament by $a_i^{\max}$ we have

$$\lambda_i \frac{e^{-(\lambda_i a_i - \lambda_i a_i)}}{\left[1+e^{-(\lambda_i a_i - \lambda_i a_i)}\right]^2} \Delta u = c'(a_i),$$

or

$$\lambda_i \frac{\Delta u}{4} = c'(a_i^{\max}).$$

Proof of Lemma 3 To prove that the equilibrium is unique, we first show that when the players' best responses cross it is impossible that they are both negatively slopped. We proceed by contradiction and suppose that there is an equilibrium such that $R_1'(a_2^*) < 0 \Leftrightarrow a_2^* > \lambda_1 a_1^*$ and $R_2'(a_1^*) < 0 \Leftrightarrow a_1^* > \lambda_2 a_2^*$. Since $\lambda_1 > 1$, $a_2^* > \lambda_1 a_1^* \Rightarrow a_2^* > a_1^*$.

To show that an equilibrium such that $R_1'(a_2^*) < 0$ and $R_2'(a_1^*) < 0$ cannot admit $a_2^* > a_1^*$, consider any pair $a_1 = a_2 = a$. Since $\lambda_1 > \lambda_2$, then $e^{-a(\lambda_1 - 1)} < e^{-a(\lambda_2 - 1)}$. Consequently, using the players' first-order conditions we deduce that for any such pair we would have:

$$\frac{\lambda_1 e^{-a(\lambda_1-1)}}{\left(1+e^{-a(\lambda_1-1)}\right)^2} > \frac{\lambda_2 e^{-a(\lambda_2-1)}}{\left(1+e^{-a(\lambda_2-1)}\right)^2},$$

which in turn would imply that if player 2's first-order condition is satisfied then player 1 has incentives to increase his effort, and if player 1's first-order condition is satisfied, then player 2 has incentives to reduce his effort. Consequently, the best response of player 2 needs to cross the 45-degrees line for lower efforts a_2 than the best response of player 1. The quasi-concavity of the players' best responses allows us to conclude that $a_1^* > a_2^*$, thence the contradiction.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_1) = R_1'(a_2) \circ R_2'(a_1)$ has a slope smaller than 1 for any equilibrium pair (a_1^*, a_j^*), since the function is continuous on $\mathbf{R}$. If $R_1'(a_1^*) < 0$, then since $R_1'(a_2^*) > 0$, the condition is necessarily satisfied. If, on the other hand, $R_2'(a_1^*) > 0$, then we simply need to prove that if $R_1'(a_2^*) > 0$ for both players, then the product of the best responses is smaller than 1. Since $R_1'(a_2)$ is decreasing in $c''(a_1)$, it is thus sufficient to establish the result for $c''(a_1) = 0$. Rewriting the product of the players' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{\lambda_1 \frac{1-e^{-(\lambda_1 a_1 - a_2)}}{\left[1+e^{-(\lambda_1 a_1 - a_2)}\right]^3} e^{-(\lambda_1 a_1 - a_2)} \Delta u}{\lambda_1^2 \frac{1-e^{-(\lambda_1 a_1 - a_2)}}{\left[1+e^{-(\lambda_1 a_1 - a_2)}\right]^3} e^{-(\lambda_1 a_1 - a_2)} \Delta u + c''(a_1)} \frac{\lambda_2 \frac{1-e^{-(\lambda_2 a_2 - a_1)}}{\left[1+e^{-(\lambda_2 a_2 - a_1)}\right]^3} e^{-(\lambda_2 a_2 - a_1)} \Delta u}{\lambda_2^2 \frac{1-e^{-(\lambda_2 a_2 - a_1)}}{\left[1+e^{-(\lambda_2 a_2 - a_1)}\right]^3} e^{-(\lambda_2 a_2 - a_1)} \Delta u + c''(a_2)} < 1.$$

Since we want to show that the above condition is true when $R_1'(a_2) > 0$ and $R_2'(a_1) > 0$, if the above condition is true for $c''(a_1) = c''(a_2) = 0$, then it is also true for any values $c''(a_1) > 0$ and $c''(a_2) > 0$. Consequently, the above condition is true if $\lambda_1\lambda_2 \geq 1$.

Proof of Proposition 1 We proceed in several steps. First, we show that for any λ_2, there are two values of λ_1 such that $a_1^* = a_2^*$. We define those two thresholds as $\underline{\lambda}_1(\lambda_2)$ and $\bar{\lambda}_1(\lambda_2)$, with $\underline{\lambda}_1(\lambda_2) < \bar{\lambda}_1(\lambda_2)$ and $\lambda_2 = \underline{\lambda}_1(\lambda_2)$ or $\lambda_2 = \bar{\lambda}_1(\lambda_2)$. We show that for $\lambda_1 \in]\underline{\lambda}_1(\lambda_2), \bar{\lambda}_1(\lambda_2)[$, $a_1^* > a_2^*$, whereas for $\lambda_1 > \bar{\lambda}_1(\lambda_2)$, $a_1^* < a_2^*$. Consequently, if $\lambda_2 = \underline{\lambda}_1(\lambda_2)$, then $a_1^* > a_2^*$ for $\lambda_1 < \bar{\lambda}_1(\lambda_2)$ and $a_1^* < a_2^*$ otherwise. Moreover, if $\lambda_2 = \bar{\lambda}_1(\lambda_2)$, then $a_1^* < a_2^*$ for any $\lambda_1 > \lambda_2$.

In a 2 player setup, the first-order condition of player 1 is:

$$\lambda_1 \frac{e^{-[\lambda_1 a_1 - a_2]}}{(1 + e^{-[\lambda_1 a_1 - a_2]})^2} \Delta u - c'(a_1) = 0$$

Consider then any value λ_2. If $\lambda_1 = \lambda_2$, then $a_1 = a_2 = a^*(\lambda_2)$ and the above first-order condition can be written as:

$$\lambda_2 \frac{e^{-a^*(\lambda_2)[\lambda_2 - 1]}}{(1 + e^{-a^*(\lambda_2)[\lambda_2 - 1]})^2} \Delta u - c'(a^*(\lambda_2)) = 0$$

We can then define function $\phi(\lambda_1)$ the function where we maintain $a^*(\lambda_2)$ fixed, and is defined as:

$$\phi(\lambda_1) = \lambda_1 \frac{e^{-a^*(\lambda_2)[\lambda_1 - 1]}}{(1 + e^{-a^*(\lambda_2)[\lambda_1 - 1]})^2} \Delta u - c'(a^*).$$

For $\phi(\lambda_1) = 0$, the players' efforts are mutual best responses when $a_1 = a_2 = a^*(\lambda_2)$. Otherwise, if $\phi(\lambda_1) < 0$, the best response of player 1 to $a_2 = a^*(\lambda_2)$ commands player 1 to produce a smaller effort than $a^*(\lambda_2)$. Combining this with the facts that (i) player 2's best response is not affected by λ_1, and (ii) the best response of player 2 is quasi-concave, implies that at equilibrium $a_1^* < a_2^*$. By a similar reasoning, if $\phi(\lambda_1) > 0$, then $a_1^* > a_2^*$.

We next show that function $\phi(\lambda_1)$ is quasi-concave, it crosses twice the x-axis at values $\underline{\lambda}_1(\lambda_2)$ and $\bar{\lambda}_1(\lambda_2)$, and that $\phi(\lambda_1) = 0$ for either $\lambda_2 = \underline{\lambda}_1(\lambda_2)$, or $\lambda_2 = \bar{\lambda}_1(\lambda_2)$

We begin by showing that the function $\phi(\lambda_1)$ is quasi-concave. Using the short notation $e = e^{-a^*(\lambda_2)[\lambda-1]}$, we first have that:

$$\phi'(\lambda_1) = \frac{e}{[1+e]^3}\left[1 + e + a^*(\lambda_2)\lambda_1[e-1]\right].$$

To show quasi-concavity, we first observe from the above expression that $\phi'(\lambda_1) > 0$ for any $\lambda_1 \leq 1$. Second, we show that for $\lambda > 1$, that $\phi''(\lambda_1) < 0$ for $\phi'(\lambda_1) = 0$. Indeed,

$$\phi''(\lambda_1) = \frac{a^*e}{[1+e]^4}\left[2e-1\right]\left[1 + e + a^*\lambda_1(e-1)\right] - \frac{a^*e}{[1+e]^4}[1+e][a^*\lambda_1 e + 1],$$

and this expression is negative if the following inequality is true:

$$[1+e][2[e-1] - a^*\lambda_1 e] + a^*(\lambda_2)\lambda_1[e-1][2e-1] < 0,$$

or,

$$2[e-1][1+e] + a^*\lambda_1[e^2 - 4e + 1] < 0.$$

If we evaluate this expression at $\phi'(\lambda_1) = 0 \Leftrightarrow a^*(\lambda_2)\lambda_1 = \frac{1+e}{1-e}$, we then obtain:

$$2[e-1][1+e] + \frac{1+e}{1-e}[e^2 - 4e + 1] < 0.$$

And since $e < 1$ for $\lambda_1 > 1$, this inequality can be re-written as:

$$2[e-1]^2 > e^2 - 4e + 1,$$

which is always true.

Having shown that $\phi(\lambda_1)$ is quasi-concave, we next show that $\phi(\lambda_1) < 0$ for $\lambda_1 \to 0$ and also that for $\lambda_1 \to \infty$, thence implying that, since an equilibrium exists, there exists at least one value λ_1 satisfying $\phi(\lambda_1) = 0$. If $\underline{\lambda_1} = \bar{\lambda_1}$, then there is but one value of λ_1 such that $\phi(\lambda_1) = 0$. Otherwise there are exactly 2 such values.

Focusing first on the case where $\lambda_1 \to 0$, we have:

$$\lim_{\lambda_1 \to 0} \phi(\lambda_1) = 0 - c'(a^*(\lambda_2)),$$

with the limit of the first component of $\phi(\lambda_1)$ tending to 0 since $\lim_{\lambda_1 \to 0} e^{-a^*(\lambda_2)[\lambda_1 - 1]} = e^{a^*(\lambda_2)}$, which is finite.

Turning next to the case where $\lim_{\lambda_1 \to \infty}$ we have:

$$\lim_{\lambda \to \infty} \phi(\lambda) = \frac{\infty e^{-\infty}}{[1 + e^{-\infty}]^2} \Delta u - c'(a)$$

The denominator of the first component of the expression tends to 1 but the limit of the numerator is undetermined. We apply l'Hospital's rule to the numerator so that:

$$\lim_{\lambda \to \infty} (\lambda e^{-\lambda}) = \lim_{\lambda \to \infty} \frac{\lambda}{e^{\lambda}} = \lim_{\lambda \to \infty} \frac{1}{e^{\lambda}} = 0$$

with the next to last equality following from the application of l'Hospital's rule. Consequently $\lim_{\lambda \to \infty} \phi(\lambda) = -c'(a) < 0$.

Proof of Lemma 4 The best response of player i, $i = \{1, 2\}$, is defined implicitly by (4). Hence, the slope of the best response of player i, $R_i'(a_j)$ is given by

$$-\frac{\partial R_i / \partial a_j}{\partial R_i / \partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial a_j}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda_i q(a_i) - q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q'(a_i) q'(a_j) \Delta u}{\frac{q''(a_i)[\lambda_i q(a_i) + q(a_j)] - 2\lambda_i [q'(a_i)]^2}{[\lambda_i q(a_i) + q(a_j)]^3} \lambda_i q(a_j) \Delta u - c''(a_i)}. \quad (7)$$

The denominator is negative because player i's second-order condition is satisfied. Therefore, the sign of the slope of player i's best response is only determined by the sign of the numerator which only depends on $\lambda_i q(a_i) - q(a_j)$. Hence, $R_i'(a_j)$ is positive for $\lambda_i q(a_i) > q(a_j)$, zero for $\lambda_i q(a_i) = q(a_j)$, and negative for $\lambda_i q(a_i) < q(a_j)$. This implies that $R_i(a_j)$ increases in a_j for $\lambda_i q(a_i) > q(a_j)$, reaches the maximum at $\lambda_i q(a_i) = q(a_j)$, and decreases in a_j for $\lambda_i q(a_i) < q(a_j)$.

Proof of Lemma 5 (This proof follows Baik 1994) Player i's best response is defined by (4):

$$\frac{\lambda_i q'(a_i) q(a_j)}{[\lambda_i q(a_i) + q(a_j)]^2} \Delta u - c'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{q(a_j) - \lambda_i q(a_i)}{[\lambda_i q(a_i) + q(a_j)]^3} q'(a_i) q(a_j) \Delta u.$$

We see that $\partial R_i(a_j)/\partial \lambda_i \gtreqless 0$ for $q(a_j) \gtreqless \lambda_j q(a_i)$. We also know from Lemma 4 that $sign\{R_i'(a_j)\} = -sign\left\{\frac{\partial R_i(a_j)}{\partial \lambda_i}\right\}$.

Substituting next $q(a_j) = \lambda_i q(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a_i^{max} we obtain

$$\frac{\lambda_i q'(a_i^{max}) \lambda_i q(a_i^{max})}{[\lambda_i q(a_i^{max}) + \lambda_i q(a_i^{max})]^2} \Delta u = c'(a_i^{max}),$$

or

$$\frac{\lambda_i^2 q'(a_i^{max}) q(a_i^{max})}{4\lambda_i^2 [q(a_i^{max})]^2} \Delta u = c'(a_i^{max}),$$

or

$$\frac{q'(a_i^{max})}{4q(a_i^{max})} \Delta u = c'(a_i^{max}).$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a_i^{max}, does not depend on λ_i.

Proof of Lemma 6 To prove that the equilibrium is unique, we reproduce the steps of the proof of Lemma 3, and we first show that when the players' best responses cross it is impossible that they are both negatively slopped. We proceed by contradiction here too and suppose that there is an equilibrium such that $R_1'(a_2^*) < 0 \Leftrightarrow q(a_2^*) >$

$\lambda_1 q(a_1^*)$ and $R_2'(a_1^*) < 0 \Leftrightarrow q(a_1^*) > \lambda_2 q(a_2^*)$. Since $\lambda_1 > 1$, $q(a_2^*) > \lambda_1 q(a_1^*) \Rightarrow q(a_2^*) > q(a_1^*) \Rightarrow a_2^* > a_1^*$.

To show that an equilibrium such that $R_1'(a_2^*) < 0$ and $R_2'(a_1^*) < 0$ cannot admit $a_2^* > a_1^*$, consider any pair $a_1 = a_2 = a$. Since $\lambda_1 > \lambda_2$, then $\frac{\partial E[U_1(a_1,a_2;\lambda_1)]}{\partial a_1} > \frac{\partial E[U_2(a_2,a_1;\lambda_2)]}{\partial a_2}$ for $a_1 = a_2$. This in turn would imply that if player 2's first-order condition is satisfied then player 1 has incentives to increase his effort, and if player 1's first-order condition is satisfied, then player 2 has incentives to reduce his effort. Consequently, the best response of player 2 needs to cross the 45-degrees line for lower efforts a_2 than the best response of player 1. The quasi-concavity of the players' best responses allows us to conclude that $a_1^* > a_2^*$, thence the contradiction.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_1) = R_1'(a_2) \circ R_2'(a_1)$ has a slope smaller than 1 for any equilibrium pair (a_1^*, a_2^*), since the function is continuous on $\mathbf{R}$. Having shown that at equilibrium we cannot have $R_1'(a_2) < 0$ and $R_2'(a_1) < 0$, we simply need to prove that when both best responses are positively slopped at equilibrium, the product of the best responses is smaller than 1. Since $R_1'(a_2)$ is decreasing in $c''(a_1)$, it is thus sufficient to establish the result for $c''(a_1) = 0$. Rewriting the product of the contestants' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1))\left(q'(a_1)q'(a_2)\right)^2}{[q''(a_1)[\lambda_1 q(a_1) + q(a_2)] - 2\lambda_1[q'(a_1)]^2]\,[q''(a_2)[\lambda_2 q(a_2) + q(a_1)] - 2\lambda_2[q'(a_2)]^2]\,q(a_1)q(a_2)} < 1.$$

Since the LHS is decreasing in both $q''(a_1)$ and $q''(a_2)$ the above expression is *a fortiori* true if:

$$\frac{(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1))\left(q'(a_1)q'(a_2)\right)^2}{4\lambda_1[q'(a_1)]^2\lambda_2[q'(a_2)]^2 q(a_1)q(a_2)} < 1,$$

an expression that simplifies to:

$$(\lambda_1 q(a_1) - q(a_2))(\lambda_2 q(a_2) - q(a_1)) < 4\lambda_1\lambda_2 q(a_1)q(a_2).$$

And this inequality is always satisfied if $\lambda_1\lambda_2 \geq 1/3$.

Proof of Proposition 2 To prove this result we show that the best response of the more overconfident player crosses the 45 degree line at a lower value of effort than the best response of the less overconfident player. If player 1 is the more overconfident player, then $\lambda_1 > \lambda_2 > 1$. At the 45 degree line the best response of player 1 takes the value a_L given by

$$\frac{\lambda_1 q'(a_L)}{(1+\lambda_1)^2 q(a_L)}\Delta u - c'(a_L) = 0. \tag{8}$$

At 45 degree line the best response of player 2 takes the value a_H given by

$$\frac{\lambda_2 q'(a_H)}{(1+\lambda_2)^2 q(a_H)}\Delta u - c'(a_H) = 0. \tag{9}$$

Note that $\lambda_1 > \lambda_2$ implies

$$\frac{\lambda_1}{(1+\lambda_1)^2} < \frac{\lambda_2}{(1+\lambda_2)^2}. \tag{10}$$

Therefore, (8), (9), and (10) imply

$$\frac{q'(a_H)}{q(a_H)c'(a_H)} < \frac{q'(a_L)}{q(a_L)c'(a_L)}.$$

Given that $q(.)$ is (weakly) concave and that $c(.)$ is (weakly) convex, this inequality can only be satisfied provided $a_L < a_H$.

Proof of Lemma 7 See the proof of Lemma 5.

Proof of Proposition 3 To prove this result we show that if $\lambda_1\lambda_2 < 1$, then the best response of the overconfident player 1 crosses the 45 degree line at a higher value of effort than the best response of the underconfident player 2.

At the 45 degree line the best response of player 1 takes the value $\bar{a}_1$ given by

$$\frac{\lambda_1 q'(\bar{a}_1)}{(1+\lambda_1)^2 q(\bar{a}_1)}\Delta u - c'(\bar{a}_1) = 0. \tag{11}$$

At 45 degree line the best response of player 2 takes the value $\bar{a}_2$ given by

$$\frac{\lambda_2 q'(\bar{a}_2)}{(1+\lambda_2)^2 q(\bar{a}_2)}\Delta u - c'(\bar{a}_2) = 0. \tag{12}$$

Observe that

$$\frac{\lambda_1}{(1+\lambda_1)^2} > \frac{\lambda_2}{(1+\lambda_2)^2},$$

is equivalent to:

$$\lambda_1\lambda_2^2 + \lambda_1 > \lambda_2\lambda_1^2 + \lambda_2,$$

which is true when $\lambda_1\lambda_2 < 1$. This implies

$$\frac{q'(\bar{a}_1)}{q(\bar{a}_1)c'(\bar{a}_1)} > \frac{q'(\bar{a}_2)}{q(\bar{a}_2)c'(\bar{a}_2)}.$$

Given that $q(.)$ is (weakly) concave and that $c(.)$ is (weakly) convex, this inequality can only be satisfied provided $\bar{a}_1 > \bar{a}_2$.

Likewise, if $\lambda_1\lambda_2 > 1$, then $\bar{a}_1 < \bar{a}_2$.