Confidence and Performance in Tournaments and Contests*

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Abstract

This paper studies the effect of confidence biases on players' relative efforts in tournaments and contests. We uncover a non-monotonic effect of confidence on equilibrium relative efforts and winning probabilities. A player with either a low or a high confidence exerts less effort than his rival at equilibrium. However, for intermediate confidence levels, the player exerts more effort than his rival. These results show that a less able or a cost disadvantaged player may nevertheless outcompete his rival because of confidence biases.

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1 Introduction

Evidence from psychology and economics shows that humans tend to display biases in confidence. For example, a majority of people believe they are better than others in a wide variety of positive traits and skills (Myers 1996, Santos-Pinto and Sobel 2005).[1] However, when tasks are perceived to be difficult, humans often display underconfidence (Kruger 1999, Moore and Healy 2008). Gender, race, socio-economic status differences in confidence have also been documented and shown to matter for economic decisions in the field and in the lab (e.g. Gneezy et al. 2003, Niederle and Vesterlund 2007, Buser et al. 2014, Guyon and Huillery 2021).

This paper inquires how confidence biases affect behavior and outcomes of competitions that take the form of tournaments and contests. Some examples include promotions in organizations, R&D races, election campaigns, rent-seeking games, competitions for markets, litigation, wars, and sport competitions. The paper addresses the following questions. How do confidence biases affect players' relative efforts? Is a more confident player more or less likely to win a competition? Can overconfidence (underconfidence) make a less (more) able player the most likely winner? We provide answers to these questions in a two player setup where the players can differ in their confidence, abilities, and cost of effort.

In our setup, players compete for a prize by exerting effort. Efforts, abilities, and noise generate outputs that map into winning probabilities. An overconfident (underconfident) player overestimates (underestimates) his ability.[2] We make three main assumptions. First, we impose ability and effort to be complements as is often

[1] Examples include car drivers (Svenson 1981), entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006), currency traders (Oberlechner and Osler 2008), poker and chess players (Park and Santos-Pinto 2010), CFOs (Ben-David et al. 2013), marathon runners (Krawczyk and Wilamowski 2017), freedivers (Lackner and Sonnabend 2020), and truck drivers (Hoffman and Burks 2020).

[2] Biases in confidence can be of three types. Players may have a mistaken assessment of their absolute ability, their relative ability, or the precision of their estimates (Moore and Healy 2008). Our focus is on the first two types of confidence biases.

done in the literature (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). Second, we exclude the possibility for both players to be underconfident to rule out multiple equilibria. Third, we focus on a wide a range of noise distributions encompassing commonly used ones such as the Normal.

Our model uncovers a non-monotonic effect of confidence on the relative effort provision of players in a Lazear-Rosen tournament (Lazear and Rosen 1981), for any given heterogeneity in abilities and/or costs. A player with a low confidence perceives that for a given confidence level of the rival, the marginal contribution of his effort to output is limited. Consequently, at equilibrium a player with a low confidence exerts less effort than his rival. Increasing the confidence level of the focal player while keeping the rival's confidence fixed, the difference in efforts shrinks as a result of the increase in the perceived marginal contribution of his effort to output. If the ability and cost asymmetries between the two players are not too large, then when the focal player is moderately confident, he will exert a higher effort than his rival. Finally, a highly overconfident focal player perceives his winning probability to be high for low efforts, thence leaving limited scope for further increasing it by raising effort. Consequently, since effort is costly, a highly overconfident focal player saves on effort, and exerts less effort than his rival.

Additionally, we show that for equal abilities, the least cost efficient player may choose a higher equilibrium effort in the tournament. This will happen if the rival is either sufficiently underconfident or overconfident. Likewise, if players have the same cost function, the least able player may choose a higher equilibrium effort in the tournament.

We next explore the effect of confidence biases on Tullock contests that have been widely used to study competition. The above results for Lazear-Rosen tournaments are shown to also hold when competition is modelled with a generalized Tullock contest. However, the effect of confidence biases on these two types of competitions can differ. In setups where players have the same abilities and cost functions, in

Lazear-Rosen tournaments perceptional biases can raise both players' equilibrium efforts, while we show that this is never the case in a contest.

2 Related Literature

Our paper contributes to the literature studying the effects of perceptional biases on competition in strategic environments. Santos-Pinto (2010) considers the effect of overconfidence on Lazear-Rosen rank-order tournaments where all players have the same confidence, ability, and cost of effort. Goel and Thakor (2008) study the effect of overconfidence defined as an underestimation of risk on the promotion of managers to CEO positions. Ludwig et al. (2011) analyze a contest where an overconfident player underestimates his cost of effort. Santos-Pinto and Sekeris (2023) explore the effect of confidence gaps on equilibrium efforts in Tullock contests, where all players have the same ability and cost of effort. The present paper differs from these earlier works since we extend the analysis to both contests and tournaments. Importantly, we consider a much more general setup where we allow players to differ in their confidence, abilities, and cost of effort. In contrast to these earlier studies, we uncover a non-monotonicity between overconfidence and relative effort provision. As a consequence, we show that a less able or a cost disadvantaged overconfident player can outcompete a rival.

Some scholars have explored how rationality biases influence behavior in contests. Baharad and Nitzan (2008), and Keskin (2018) focus on Prospect Theory's probability weighing. Fu et al. (2021) and Fu et al. (2022) incorporate Köszegi and Rabin's (2006) reference-dependent preferences. Last, Yang (2020) studies the effect of Rank-Dependent Utility probability weighing. In our paper the perception bias is different since an overconfident (underconfident) player overestimates (underestimates) the winning probability for any effort of the rival. Moreover, we consider a generalized Tullock contest and allow for heterogeneity in players' abilities and costs.

Finally, our paper also contributes to the literature on tournaments and contests where players differ in their abilities and/or costs (Lazear and Rosen 1981, Schotter and Weigelt 1992, Höffler and Sliwka 2003, Kräkel and Sliwka 2004, Garfinkel and Skaperdas 2007, Drugov and Ryvkin 2022). We extend this literature by allowing also for heterogeneity in players' confidence in their abilities. We show that some earlier results on the impact of heterogeneity on equilibrium efforts and winning probabilities can be overturned when players display confidence biases. For example, the result that the most able and/or cost efficient player has a higher winning probability both in tournaments and in contests, may not always hold if players have biased beliefs on their abilities.

3 Set-up

Consider two players, 1 and 2, competing in a tournament. The player who produces the highest output receives the winner's prize y_W and the other receives the loser's prize y_L, with $0 < y_L < y_W$. The players are weakly risk averse and expected utility maximizers and have utility functions that are separable in income (y_i) and effort (a_i):

$$U_i(y_i, a_i) = u(y_i) - c(a_i),$$

for $i = 1, 2$. I assume u and c are twice differentiable with $u' > 0$, $u'' \leq 0$, $c' > 0$, $c'' > 0$, $c(0) = 0$, $c'(0) = 0$, and $c(a_i) = \infty$, for $a_i \to \infty$, where the last two conditions ensure that equilibrium effort is strictly positive but finite. The two players have an outside option which guarantee each $\bar{u} \geq 0$. We assume $\bar{u} = 0$ such that the players' participation is ensured.

When player i exerts effort a_i his output is given by

$$Q_i = h(\theta_i q_i(a_i)) + \varepsilon_i, \tag{1}$$

where both $h(.)$ and $q_i(.)$ are increasing functions, θ_i is ability and ε_i is a unimodal random shock with zero mean. The above specification assumes that effort and ability are complements.

The output function (1) captures situations where effort and ability are complements in generating output since $\partial^2 Q_i/\partial a_i \partial\theta > 0$. This specification for output where noise is additively separable is chosen for its analytical simplicity and is often used in the tournament literature (see Lazear and Rosen 1981, Nalebuff and Stiglitz 1983, Akerlof and Holden 2012). The random variables ε_1 and ε_2 are identically and independently distributed and represent individualistic noise. Throughout, the contract must be signed before ε_1, and ε_2 are known; the players decide on a_1 and a_2, neither of which is observable to the firm. The probability distribution of ε_i is known to both firm and players.

The two players can differ from one another in terms of their ability perceptions. Either player can be overconfident, underconfident, or unbiased. The degree of over/underconvidence of player i is captured by the parameter λ_i, that affects how a players perceives his output as follows:

$$\tilde{Q}_i = h(\lambda_i \theta_i q_i(a_i)) + \varepsilon_i, \tag{2}$$

Accordingly, $\tilde{\theta}_i = \lambda_i \theta$ is player i's believed ability, and for an overconfident player i, $\lambda_i > 1$, for an underconfident player, $\lambda_i < 1$, while for an unbiased player, $\lambda_i = 1$. Under this specification player i perceives his marginal output is increasing with his confidence bias λ_i, that is, $\partial^2 \tilde{Q}_1/\partial a_1 \partial \lambda_i > 0$. This describes situations where effort, ability and confidence are complements in generating output. This way of modeling overconfidence is often used in the literature that analyzes its impact on labor contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Krähmer 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011). This assumption applies to tasks where time (effort) and cognitive skills (ability) determine the output and where a more able employee produces higher output in the same time than a less able one (Sautmann, 2013). Chen and Schilberg-Hörisch (2019) find experimental support for this assumption.

To ensure that the problem has a unique pure strategy equilibrium, we impose the following restriction on confidence parameters:

Assumption 1. $\lambda_1\lambda_2 \geq 1$.

This assumption de facto implies that although both players can be overconfident, it is forbidden by assumption that they are both underconfident.[3]

Hence, player i's perceived probability of winning the tournament is

$$\begin{aligned} P_i(a_i, a_j, \lambda_i) &= \Pr(\tilde{Q}_i \geq Q_j) \\ &= \Pr(h(\lambda_i\theta_i q_i(a_i)) + \varepsilon_i \geq h(\theta_j q_j(a_j)) + \varepsilon_j) \\ &= \Pr(\varepsilon_j - \varepsilon_i \leq h(\lambda_i\theta_i q_i(a_i)) - h(\theta_j q_j(a_j))). \end{aligned}$$

Player i chooses the optimal level of effort that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j, \lambda_i)] = u(y_L) + P_i(a_i, a_j, \lambda_i)\Delta u - c(a_i), \tag{3}$$

where $\Delta u = u(y_W) - u(y_L)$ captures the utility prize spread.

Following Heifetz et al. (2007a,2007b) for games with complete information, and Squintani (2006) for games with incomplete information, we assume: (1) a player who faces a biased rival is aware that the latter's perception of his own ability is mistaken, (2) each player thinks that his own perception of his ability is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their rival's beliefs. Hence, players agree to disagree about their abilities. These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin and Ross, 2002; Pronin and Kugler, 2007).

The firm is risk neutral and correctly assesses the players' abilities. The firm's profits are the difference between expected benefits and compensation costs:

$$E[\pi] = E[Q_1 + Q_2] - (y_L + y_W).$$

[3]This assumption can be relaxed when considering specific functional forms in what follows, but given the degree of generality we wish to preserve we need to restrict the parameter space to ensure the equilibrium is unique and well behaved.

The timing of the events is as follows. The players simultaneously and independently choose their effort levels. The firm observes the players' output realizations and awards the prizes according to the prize schedule.

We shall sequentially consider Lazear-Rosen tournaments in Section 4 and Tullock contests in Section 5, which are both nested in our general setup.

4 Lazear-Rosen tournaments

In this section we analyze the effect of overconfidence on players' effort in the canonical Lazear and Rosen (1981) rank-order tournament. In this case, equations (1) and (2) become, respectively,

$$\tilde{Q}_i = \theta_i a_i + \varepsilon_i, \tag{4}$$

and

$$\tilde{Q}_i = \lambda_i \theta_i a_i + \varepsilon_i. \tag{5}$$

Accordingly, player i's perceived probability of winning the tournament is

$$\begin{aligned} P_i(a_i, a_j, \lambda_1) &= \Pr(\tilde{Q}_i \geq Q_j) \\ &= \Pr(\lambda_i \theta_i a_i + \varepsilon_i \geq \theta_j a_j + \varepsilon_j) \\ &= \Pr(\varepsilon_j - \varepsilon_i \leq \lambda_i \theta_i a_i - \theta_j a_j) \\ &= G(\lambda_i \theta_i a_i - \theta_j a_j). \end{aligned}$$

Since the difference between the random shocks ε_i and ε_j will be crucial, we define the random variable $x = \varepsilon_j - \varepsilon_i$ with cumulative distribution function $G(x)$ and density $g(x)$. Since ε_i is a unimodal random shock with zero mean, it follows that $g(x)$ is unimodal, and symmetric around zero. Moreover, we impose the following additional assumptions on $g(x)$:

Assumption 2.

(a) $g(x)$ is continuously differentiable on $\mathbb{R}$,

(b) $g'(0) = 0$, $g'(x) > 0$ for $x < 0$, and $g'(x) < 0$ for $x > 0$,

(c) over $[0, \hat{x}[$, $g''(x) < 0$, *and over* $[\hat{x}, \infty[$, $g''(x) \geq 0$, *with* $\hat{x} \in [0, \infty[$.

The above assumptions are flexible enough to accommodate a host of density functions, including e.g. the Normal distribution, the logistic distribution, or the Laplace distribution. The shape of the distribution $G(.)$ is intimately related to the assumptions on the random variables ε_i and ε_j that define the noise in the tournament. For example, when the random shocks ε_i and ε_j are i.i.d., a sufficient condition for the unimodality of $g(x)$ is that the pdf of ε_i and ε_j are unimodal (Hodges and Lehmann 1954). Also, $g(x)$ follows a Normal distribution when the noise terms are normally distributed, and $g(x)$ follows a Logistic distribution when the noise terms follow a Gumbel distribution.

Player i chooses the optimal level of effort that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j, \lambda_i)] = u(y_L) + G(\lambda_i \theta_i a_i - \theta_j a_j)\Delta u - c_i(a_i), \tag{6}$$

where $\Delta u = u(y_W) - u(y_L)$.

The first-order condition of player i is

$$\frac{\partial E[U_i(a_i, a_j, \lambda_i)]}{\partial a_i} = \lambda_i \theta_i g(\theta \lambda_i \theta_i a_i - \theta_j a_j)\Delta u - c_i'(a_i) = 0. \tag{7}$$

Hence, the second-order condition of player i is

$$\frac{\partial^2 E[U_i(a_i, a_j, \lambda_i)]}{\partial a_i^2} = \lambda_i^2 \theta_i^2 g'(\lambda_i \theta_i a_i - \theta_j a_j)\Delta u - c_i''(a_i) < 0. \tag{8}$$

A sufficient condition for existence of a pure-strategy Nash equilibrium to exist is that

$$\lambda_i^2 \theta_i^2 g'(\lambda_i \theta_i a_i - \theta_j a_j)\Delta u < c_i''(a_i), \forall a_i, a_j, \lambda_i. \tag{9}$$

As it is known in the tournament literature, a pure-strategy Nash equilibrium will only exist if there is sufficient noise in the tournament and the cost function $c_i(a)$ is sufficiently convex (Lazear and Rosen, 1981). Hence, existence of a pure-strategy Nash equilibrium is assured when the following assumption holds

Assumption 3.

$$\lambda_i^2\theta_i^2\Delta u \sup_x g'(x) < \inf_{a>0} c_i''(a), i = \{1,2\}.$$

Assumption 3 ensures (9) is satisfied. Note that the lower is $\sup_x g'(x)$ the flatter is $g(x)$ and hence the higher is the noise in the tournament. Note also that $0 < c_0 = \inf_{a>0} c''(a)$ defines a class of cost functions with a second derivative bounded away from zero.[4] Let $a_i = R_i(a_j)$ denote player i's best response obtained from (7). Lemma 1 describes the shape of the player i's best response.

Lemma 1. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $\theta_j a_j = \lambda_i\theta_i a_i$.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high effort of the rival, $\theta_j a_j > \lambda_i\theta_i a_i$, player i reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, $\theta_j a_j < \lambda_i\theta_i a_i$, player i reacts to an increase in effort of the rival by increasing effort.

Lemma 2 describes how player i's best response changes with his confidence bias λ_i.

Lemma 2. *An increase in player i's confidence λ_i leads to an expansion of his best response function, $\partial R_i(a_j)/\partial\lambda_i > 0$ for $\partial^2 P_i(a_i, a_j, \lambda_i)/\partial a_i\partial\lambda_i > 0$, and to a contraction of his best response function, $\partial R_i(a_j)/\partial\lambda_i < 0$, for $\partial^2 P_i(a_i, a_j, \lambda_i)/\partial a_i\partial\lambda_i < 0$. Moreover, the maximal effort player i is willing to exert in the tournament, a_i^{max}, increases in player i's confidence λ_i.*

Lemma 2 characterizes how player i's confidence shifts his best response. This is determined by how the bias changes player 1's perceived marginal probability of winning the tournament:

$$\frac{\partial^2 P_i(a_i, a_j, \lambda_i)}{\partial a_i\partial\lambda_i} = \theta_i g(\lambda_i\theta_i a_i - \theta_j a_j) + \theta_i^2\lambda_i a_i g'(\lambda_i\theta_i a_i - \theta_j a_j). \tag{10}$$

[4] Quadratic costs are, obviously, in this class since for $c(a) = c_0 a^2/2$ with $c_0 > 0$ we have $c''(a) = c_0 > 0$.

We see from (10) that player i's perceived marginal probability of winning is composed of two terms. The first term is positive since $g(x)$ is a density function. The second term is positive when $\theta_j a_j > \lambda_i \theta_i a_i$ and negative when $\theta_j a_j < \lambda_i \theta_i a_i$. In sum, overconfidence can shift player i's best response in two ways. First, it can shift it outwards for all effort levels of the rival. Second, it can shift it outwards for high effort of the rival and inwards for low effort of the rival.

A pure-strategy Nash equilibrium (a_1^*, a_2^*) satisfies the first-order conditions of the two players simultaneously and is given by

$$\lambda_1 \theta_1 g(\lambda_1 \theta_1 a_1^* - \theta_2 a_2^*) \Delta u = c_1'(a_1^*), \tag{11}$$

and

$$\lambda_2 \theta_2 g(\theta_1 a_1^* - \lambda_2 \theta_2 a_2^*) \Delta u = c_2'(a_2^*). \tag{12}$$

A third useful lemma establishes the uniqueness of the equilibrium.

Lemma 3. *The tournament has a unique pure-strategy Nash equilibrium*

4.1 Symmetric Abilities and Cost Functions

We begin by considering players endowed with symmetric abilities and cost functions so that $\theta_1 = \theta_2 = \theta$ and $c_1(a) = c_2(a) = c(a)$. Proposition 1 uncovers the effect of player 1's confidence on equilibrium efforts and winning probabilities.

Proposition 1. *For any confidence level of player* 2*, there exist two thresholds for the confidence level of player* 1*,* $\underline{\lambda}_1(\lambda_2)$ *and* $\bar{\lambda}_1(\lambda_2)$*, with* $\underline{\lambda}_1(\lambda_2) < \bar{\lambda}_1(\lambda_2)$*,* $\lambda_2 = \{\underline{\lambda}_1(\lambda_2), \bar{\lambda}_1(\lambda_2)\}$*, and such that*

(i) $a_1^* > a_2^*$ *if* $\underline{\lambda}_1(\lambda_2) < \lambda_1 < \bar{\lambda}_1(\lambda_2)$

(ii) $a_1^* = a_2^*$ *if* $\lambda_1 = \underline{\lambda}_1(\lambda_2)$ *or* $\lambda_1 = \bar{\lambda}_1(\lambda_2)$

(iii) $a_1^* < a_2^*$ *if* $\lambda_1 < \underline{\lambda}_1(\lambda_2)$ *or* $\lambda_1 > \bar{\lambda}_1(\lambda_2)$

Proposition 1 uncovers that for any confidence level of player 2, there exist two confidence levels of player 1, $\underline{\lambda}_1 = \underline{\lambda}_1(\lambda_2)$, and $\bar{\lambda}_1 = \bar{\lambda}_1(\lambda_2)$, such that both players exert the same efforts at equilibrium. Moreover, since players have the same abilities ($\theta_1 = \theta_2$) and cost of effort ($c_1(a) = c_2(a)$), it is necessarily the case that either $\underline{\lambda}_1 = \lambda_2$, or $\bar{\lambda}_1 = \lambda_2$, and either situation may be observed depending on the value of λ_2.

Case (i) tells us that player 1 exerts a higher effort at equilibrium when his confidence level lies between these two thresholds, thence for values of λ_1 that may be higher (when $\underline{\lambda}_1 = \lambda_2$) or lower (when $\bar{\lambda}_1 = \lambda_2$) than λ_2. Hence, for players with the same abilities and cost of effort, the most confident player exerts higher effort at equilibrium when (a) he is moderately more confident than the rival, and (b) the rival is not too overconfident. Case (iii) tells us that if either or both conditions are not satisfied, then the more overconfident player exerts lower effort at equilibrium.

Figure 1 illustrates Proposition 1 when $\theta_1 = \theta_2 = 1$. The x-axis depicts the confidence level of player 1. The bell-shaped curve depicts the density of $G(.)$ when noise is normally distributed and when both players exert the same effort a^*. The downward slopping curve depicts the ratio of the marginal cost of effort at a^* to the product of player 1's confidence level λ_1 and the utility prize spread Δu. Note that when the two curves intersect, the first-order condition of player 1 is satisfied and both players exert the same effort. For this to be an equilibrium, it is necessary that $\lambda_1 = \lambda_2$, hence implying that the only values of the players' confidence parameters compatible with an equilibrium are $\lambda_1 = \lambda_2 = \underline{\lambda}_1$ or $\lambda_1 = \lambda_2 = \bar{\lambda}_1$.

Consider the left panel of Figure 1. Assume the equilibrium compatible with players' efforts $a_1^* = a_2^* = a^*$ is such that $\lambda_1 = \lambda_2 = \underline{\lambda}_1$, so that the upper crossing of the two curves describes that equilibrium. If player 1 is marginally more confident than player 2, the marginal benefit of exerting effort will be larger than its marginal cost for fixed efforts of both players. In this case the bell-shaped curve will lie above the downward slopping curve. Accordingly, player 1's best response to $a_2 = a^*$ is to exert an effort $a_1 > a^*$. Since the best response functions have been shown in Lemma

1 to be quasi-concave, this necessarily implies that when player 1 is marginally more confident than player 2, $a_1^* > a_2^*$.

The intuition behind this result lies in the following trade-off: player 1 aims at exploiting the complementarities between confidence and effort while attempting to save on cost of effort. An increase in player 1's confidence raises his effort because the increase in the perceived probability of winning times the utility prize spread is greater than the associated marginal cost of effort.

Further increases in player 1's confidence will gradually reduce the effectiveness of effort in raising the perceived probability of winning. Graphically, this is represented by the wedge between the two curves shrinking, and eventually flipping. Consequently, there is a second intersection of the two curves that takes place for $\lambda_1 = \bar{\lambda}_1 > \underline{\lambda}_1 = \lambda_2$.

Now assume that the equilibrium compatible with players' efforts $a_1^* = a_2^* = a^*$ is such that $\lambda_1 = \lambda_2 = \bar{\lambda}_1$ so that the lower crossing of the two curves describes that equilibrium. Now both players have a high confidence level, and player 1 therefore has a high perceived probability of winning the tournament. Accordingly, any marginal increase in the confidence of player 1 has a limited scope for further increasing his perceived probability of winning. Hence, increasing the confidence of player 1 will lead his marginal perceived benefit of exerting effort to drop below the marginal cost. Since the best response functions are quasi-concave, this necessarily implies that when both players are highly confident and player 1 is more confident than player 2, then $a_1^* < a_2^*$.

The left panel in Figure 1 depicts Proposition 1 when player 2 is overconfident since $\bar{\lambda}_1 > \underline{\lambda}_1 > 1$. The right panel in Figure 1 shows that the results of Proposition 1 are qualitatively the same when $\underline{\lambda}_1 < 1$.

4.2 Asymmetric Cost Functions

We now consider asymmetries in costs across players for any confidence levels and for equal abilities.

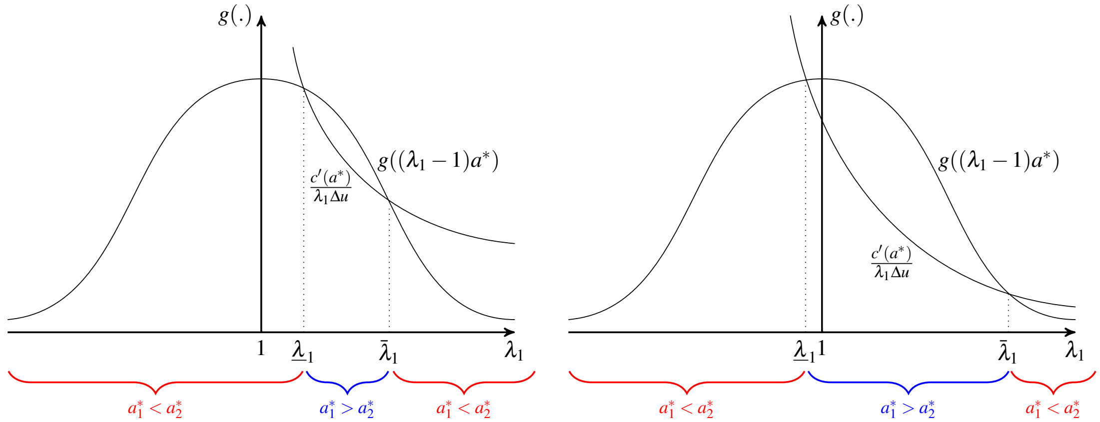

Figure 1: Equilibrium in a Tournament with normally distributed noise

To inquire the effect of cost asymmetries on the game's equilibrium, we build our reasoning starting from the fully symmetric benchmark, and by then gradually modifying the players' cost functions. We therefore take the previous setup, and redefine the cost function of player i as $c_i(a_i) = c(a_i; k_i)$, with k_i capturing the player's cost-efficiency. As such we assume that $c(a_i; k_i) < c(a_i; k_i')$ and that $c'(a_i, k_i) < c'(a_i, k_i')$, for any $k_i < k_i'$. The analysis in the previous section therefore assumed that $k_1 = k_2$, and we now inspect the effect of an increase in k_i on the game's equilibrium efforts. Proposition 2 uncovers the effect of player 1's confidence on equilibrium efforts when players can differ in their cost functions.

Proposition 2. *For any confidence level of player* 2*, if players have the same ability and player* 1 *is more cost efficient* ($k_1 < k_2$)*, there exist two thresholds* $\underline{\lambda}_1'(\lambda_2)$ *and* $\bar{\lambda}_1'(\lambda_2)$*, such that* $\underline{\lambda}_1'(\lambda_2) < \underline{\lambda}_1(\lambda_2)$*,* $\bar{\lambda}_1'(\lambda_2) > \bar{\lambda}_1(\lambda_2)$ *and* $\lambda_2 = \{\underline{\lambda}_1(\lambda_2), \bar{\lambda}_1(\lambda_2)\}$*, and such that*

(i) $a_1^* > a_2^*$ *if* $\underline{\lambda}_1'(\lambda_2) < \lambda_1 < \bar{\lambda}_1'(\lambda_2)$

(ii) $a_1^* = a_2^*$ *if* $\lambda_1 = \underline{\lambda}_1'(\lambda_2)$ *or* $\lambda_1 = \bar{\lambda}_1'(\lambda_2)$

(iii) $a_1^* < a_2^*$ *if* $\lambda_1 < \underline{\lambda}_1'(\lambda_2)$ *or* $\lambda_1 > \bar{\lambda}_1'(\lambda_2)$

If, on the other hand, player 1 *is the least cost efficient* ($k_1 > k_2$)*, these two thresholds are such that* $\underline{\lambda}_1'(\lambda_2) > \underline{\lambda}_1(\lambda_2)$, $\bar{\lambda}_1'(\lambda_2) < \bar{\lambda}_1(\lambda_2)$*. Last, there exists a threshold* $\bar{k}_1$ *such that* $\forall k_1 > \bar{k}_1$ *and for all* λ_1, $a_1^* < a_2^*$.

Case *(i)* tells us that for players with the same confidence level, if player 1 is the most cost efficient, then he chooses a higher equilibrium effort in the tournament. Indeed, having proven that we either have $\underline{\lambda}_1 = \lambda_2$ or $\bar{\lambda}_1 = \lambda_2$, and that $\underline{\lambda}_1' < \underline{\lambda}_1 < \bar{\lambda}_1 < \bar{\lambda}_1'$, if follows that when $\lambda_1 = \lambda_2$, then $\lambda_1 \in]\underline{\lambda}_1', \bar{\lambda}_1'[$, and thus that $a_1^* > a_2^*$. In addition, for players with different confidence levels, the most cost-efficient player 1 still exerts a higher equilibrium effort if his confidence is in the range $[\underline{\lambda}_1', \bar{\lambda}_1']$.

Case (iii) reveals that if player 1 is the most cost efficient, then he may exert a lower equilibrium effort than his rival if his confidence is sufficiently low ($\lambda_1 < \underline{\lambda}_1'$), or high ($\lambda_1 > \bar{\lambda}_1'$).

Proposition 2 equally uncovers that if player 1 is the least cost efficient, he may choose a higher equilibrium effort in the tournament. This may happen if the rival player 2 is less confident than player 1 ($\lambda_2 = \underline{\lambda}_1 < \underline{\lambda}_1' < \lambda_1 < \bar{\lambda}_1'$) but also in cases where player 2 is more confident than player 1 ($\lambda_2 = \bar{\lambda}_1 > \bar{\lambda}_1' > \lambda_1 > \underline{\lambda}_1'$). In the latter case, if player 2 is highly overconfident, there can be instances where player 1 has a cost disadvantage, is underconfident, and yet exerts more effort at equilibrium (if $\underline{\lambda}_1 < \underline{\lambda}_1' < \lambda_1 < 1$). This result is a consequence of the highly overconfident player 2 exerting a low effort at equilibrium.

Figure 2 illustrates Proposition 2 by depicting the effect of a decrease in player 1's marginal cost of effort on the equilibrium relative efforts. Figure 2 reveals that there is a wider range of confidence levels of player 1 for which he exerts more effort than the rival when player 1's marginal cost of effort is lower. The decrease in the marginal cost of effort of player 1 (decrease in k_1) shifts the downward slopping curve to the left, while leaving the bell-shaped curve unaffected. This places the threshold $\underline{\lambda}_1'$ to the left of $\underline{\lambda}_1$ and the threshold $\bar{\lambda}_1'$ to the right of $\bar{\lambda}_1$. This result is highly intuitive and reflecting the fact that following a reduction in a player's marginal cost, he is incentivized to increase his effort for any expected effort of the rival.

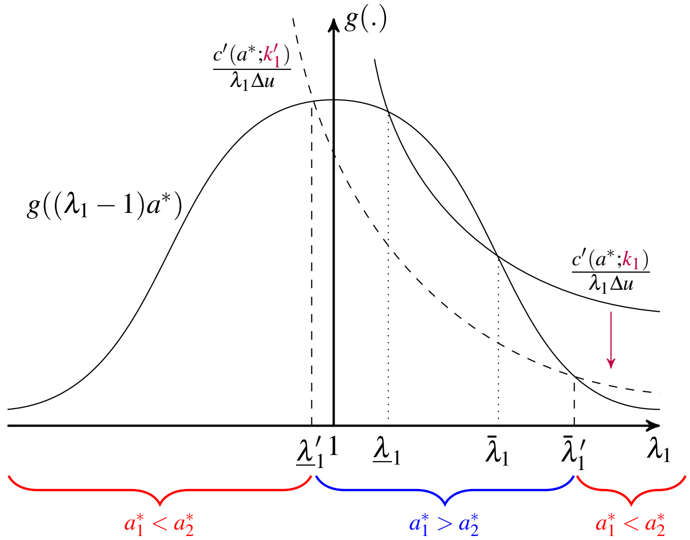

Figure 2: Equilibrium in a Tournament with cost asymmetries

4.3 Asymmetric Abilities and Cost Functions

We now consider asymmetries in abilities across players for any confidence levels and cost functions. Proposition 3 uncovers the effect of player 1's confidence on equilibrium efforts when players can differ in their abilities ($\theta_1 \neq \theta_2$).

Proposition 3. *For any confidence level of player* 2*, and for any potential cost asymmetries among players, there exist two thresholds* $\underline{\lambda}_1''(\lambda_2)$ *and* $\bar{\lambda}_1''(\lambda_2)$*, such that*

(i) $a_1^* > a_2^*$ *if* $\underline{\lambda}_1''(\lambda_2) < \lambda_1 < \bar{\lambda}_1''(\lambda_2)$

(ii) $a_1^* = a_2^*$ *if* $\lambda_1 = \underline{\lambda}_1''(\lambda_2)$ *or* $\lambda_1 = \bar{\lambda}_1''(\lambda_2)$

(iii) $a_1^* < a_2^*$ *if* $\lambda_1 < \underline{\lambda}_1''(\lambda_2)$ *or* $\lambda_1 > \bar{\lambda}_1''(\lambda_2)$

Moreover, if $\theta_1 > \theta_2$*, then* $\underline{\lambda}_1''(\lambda_2) < \underline{\lambda}_1'(\lambda_2)$ *and* $\bar{\lambda}_1''(\lambda_2) > \bar{\lambda}_1'(\lambda_2)$*, whereas if* $\theta_1 < \theta_2$*, then* $\underline{\lambda}_1''(\lambda_2) > \underline{\lambda}_1'(\lambda_2)$ *and* $\bar{\lambda}_1''(\lambda_2) < \bar{\lambda}_1'(\lambda_2)$*. Last, there exists a threshold* $\bar{\theta}_1$ *such that* $\forall \theta_1 < \bar{\theta}_1$ *and for all* λ_1*,* $a_1^* < a_2^*$*.*

Case *(i)* tells us that for players with the same confidence level and cost functions, if player 1 is the most able, then he exerts a higher equilibrium effort in the tournament. In addition, for players with different confidence levels and the same cost function, the most able player 1 still exerts a higher equilibrium effort if his confidence is in the range $[\underline{\lambda}_1''(\lambda_2), \bar{\lambda}_1''(\lambda_2)]$. Case (iii) reveals that if player 1 is the most able, then he may exert a lower equilibrium effort than his rival if his confidence is sufficiently low ($\lambda_1 < \underline{\lambda}_1''$) or high ($\lambda_1 > \bar{\lambda}_1''$).

Proposition 3 equally uncovers that if players have the same cost function and player 1 is the least able, he may choose a higher equilibrium effort in the tournament ($\underline{\lambda}_1'' < \lambda_1 < \bar{\lambda}_1''$). Here too, as in the context of Proposition 2, if player 2 is highly overconfident, there can be instances where player 1 is the least able, is underconfident, and yet exerts more effort at equilibrium (if $\underline{\lambda}_1 < \underline{\lambda}_1'' < \lambda_1 < 1$). This result is a consequence of the highly overconfident player 2 exerting a low effort at equilibrium.

We thus deduce that for players with the same confidence level, if a player is (weakly) more cost efficient ($k_1 \leq k_2$), and has a higher ability ($\theta_1 > \theta_2$), then the most able player produces a higher equilibrium effort in the tournament. Yet, the most able player could always produce a smaller effort than his rival if the confidence gap is large enough.

Figure 3 illustrates Proposition 3 by depicting the effect of an increase in player 1's ability from θ_1 to θ_1', on the equilibrium relative efforts. Figure 3 reveals that there is a wider range of confidence levels of player 1 for which he exerts more effort than the rival when player 1's ability is higher. The increase in player 1's ability shifts both the bell-shaped curve and the downward slopping curve to the left. The combined shift of the curves unambiguously moves the smaller threshold $\underline{\lambda}_1''$ to the left since the upper crossing of the two curves will necessarily occur more leftwards. In the Appendix we demonstrate that the larger threshold $\bar{\lambda}_1''$ moves the right even though its position is determined by the shift of the two curves, each one pushing it in an opposite direction.

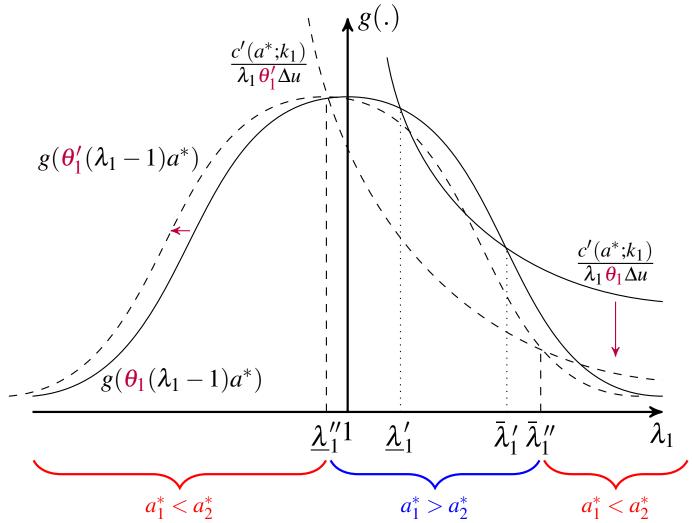

Figure 3: Equilibrium in a Tournament with asymmetries in ability

5 Tullock Contests

We now turn our attention to Tullock contests, which have been shown to be nested in the general tournament introduced in Section 3 (Hirshleifer and Riley 1992, Jia et al. 2013, Ryvkin and Drugov 2020, Santos-Pinto and Sekeris 2023). Indeed, this will be the case when $h(.) = ln(.)$ and ε_i follows a standard Gumbel distribution. Hence, we assume in this section that equations (1) and (2) become, respectively,

$$Q_i = ln(q_i(a_i)) + \varepsilon_i, \tag{13}$$

and,

$$\tilde{Q}_i = ln(\lambda_i q_i(a_i)) + \varepsilon_i, \tag{14}$$

In this section we use $q_i(.)$ to model heterogeneity in abilities in the contest as in Baik (1994), Singh and Wittman (2001), Stein (2002), or Fonseca (2009). Observe that although we assume $\theta_1 = \theta_2 = 1$, this is without any loss of generality since we could instead done the entire reasoning with a function $\tilde{q}_i(.) = \theta_i q_i(.)$.

In this case, player i's perceived probability of winning if at least one player exerts strictly positive effort is given by:

$$P_i(a_i, a_j, \lambda_i) = \begin{cases} \frac{\lambda_i q_i(a_i)}{\lambda_i q_i(a_i)+q_j(a_j)} & \text{if } q_i(a_i) + q_j(a_j) > 0 \\ 1/2 & \text{otherwise} \end{cases}$$

where $q_i(0) \geq 0$, $q_i'(a_i) > 0$ and $q_i''(a_i) \leq 0$.[5]

Any player i, $i = \{1, 2\}$, chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_j; \lambda_i)] = \frac{\lambda_i q_i(a_i)}{\lambda_i q_i(a_i) + q_j(a_j)} \Delta u - c_i(a_i).$$

The first-order condition is

$$\frac{\partial E[U_i(a_i, a_j; \lambda_i)]}{\partial a_i} = \frac{\lambda_i q_i'(a_i) q_j(a_j)}{[\lambda_i q_i(a_i) + q_j(a_j)]^2} \Delta u - c_i'(a_i) = 0. \tag{15}$$

The second-order condition is

$$\frac{\partial^2 E[U_i(a_i, a_j; \lambda_i)]}{\partial a_i^2} = \frac{q_i''(a_i)[\lambda_i q_i(a_i) + q_j(a_j)] - 2\lambda_i [q_i'(a_i)]^2}{[\lambda_i q_i(a_i) + q_j(a_j)]^3} \lambda_i q_j(a_j) v - c_i''(a_i) < 0, \tag{16}$$

and the above inequality is satisfied since $q_i''(a_i) \leq 0$ and $c_i''(a_i) \geq 0$.

Let $a_i = R_i(a_j)$ denote player i's best response obtained from (15). Along player i's best response we have

$$\lambda_i q_i'(a_i) q_j(a_j) v = c_i'(a_i) \left[\lambda_i q_i(a_i) + q_j(a_j)\right]^2.$$

Lemma 4 describes the shapes of the players' best responses.

Lemma 4. *$R_i(a_j)$ is quasi-concave in a_j and reaches a maximum for $q_j(a_j) = \lambda_i q_i(a_i)$.*

[5]The function $q_i(.)$, typically known as the impact function, can capture differences in players' abilities in a contest.

Lemma 4 tells us that the players' best responses are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

A second useful lemma establishes the uniqueness of the equilibrium:

Lemma 5. *The contest has a unique pure-strategy Nash equilibrium.*

Another useful lemma describes how a player's best response changes with his confidence parameter λ_i.

Lemma 6. *An increase in player i's confidence λ_i leads to a contraction of his best response, $\frac{\partial R_i(a_j)}{\partial \lambda_i} < 0$, for $q_j(a_j) < \lambda_i q_i(a_i)$ and to an expansion of his best response, $\frac{\partial R_i(a_j)}{\partial \lambda_i} > 0$, for $q_j(a_j) > \lambda_i q_i(a_i)$. Moreover, the maximal effort player i is willing to exert in the contest, a_i^{max}, is independent of his degree of confidence λ_i.*

Lemma 6 characterizes the best response function of a player who is subject to a confidence bias. For a high effort of the rival, an increase in confidence raises player i's effort, while for low effort of the rival, an increase in confidence lowers player i's effort. Moreover, the maximal value taken by player i's best response is independent of his confidence bias.

5.1 Symmetric Impact and Cost Functions

We next present a proposition that uncovers the effect of confidence on players' equilibrium efforts in a contest where players display identical impact functions ($q_i(a) = q_j(a) = q(a)$) and cost functions ($c_i(a) = c_j(a) = c(a)$).

Proposition 4. *For any confidence level of player 2, there exist two thresholds for the confidence level of player 1, $\underline{\lambda}_1^c(\lambda_2)$ and $\bar{\lambda}_1^c(\lambda_2)$, with $\underline{\lambda}_1^c(\lambda_2) < \bar{\lambda}_1^c(\lambda_2)$, $\lambda_2 = \{\underline{\lambda}_1^c(\lambda_2), \bar{\lambda}_1^c(\lambda_2)\}$, and such that*

(i) $a_1^* > a_2^*$ *if* $\underline{\lambda}_1^c(\lambda_2) < \lambda_1 < \bar{\lambda}_1^c(\lambda_2)$

(ii) $a_1^* = a_2^*$ *if* $\lambda_1 = \underline{\lambda}_1^c(\lambda_2)$ *or* $\lambda_1 = \bar{\lambda}_1^c(\lambda_2)$

(iii) $a_1^* < a_2^*$ *if* $\lambda_1 < \underline{\lambda}_1^c(\lambda_2)$ *or* $\lambda_1 > \bar{\lambda}_1^c(\lambda_2)$

Observe that although the contest game differs from the tournament, the results uncovered in Propositions 1 and 4 are qualitatively similar.

The particular structure of the contest implies that the maximal effort a player will ever exert does not depend on his confidence bias (Lemma 6). This in turn enables us to compare the equilibrium efforts with and without confidence biases.

Corollary 1. *For any confidence levels with* $\lambda_i \neq 1$ *for at least one player, both players exert less effort than if both were rational.*

This result is driven by the fact that in a contest with symmetric impact and cost functions, the maximal effort level of a player, a_i^{max}, is attained at equilibrium when both players are rational. Observe that this result does not hold in a Lazear-Rosen tournament since the maximal effort a player will ever exert in such instances has been shown to depend on his confidence bias (Lemma 2).

5.2 Asymmetric Impact and Cost Functions

In the previous section we assumed that the only source of asymmetry was the degree of overconfidence of players. We now lift this assumption to consider the effect of asymmetries in the players' impact functions, $q_i(a_i)$, and cost functions, $c_i(a_i)$. As such, we are not imposing that $q_1(a_1) = q_2(a_2)$ for $a_1 = a_2$, nor that $c_1(a_1) = c_2(a_2)$.

Proposition 5. *For any confidence level of player* 2*, and for any potential cost asymmetries among players, there exist two thresholds* $\underline{\lambda}_1^{cc}(\lambda_2)$ *and* $\bar{\lambda}_1^{cc}(\lambda_2)$*, such that*

(i) $a_1^* > a_2^*$ *if* $\underline{\lambda}_1^{cc}(\lambda_2) < \lambda_1 < \bar{\lambda}_1^{cc}(\lambda_2)$

(ii) $a_1^* = a_2^*$ *if* $\lambda_1 = \underline{\lambda}_1^{cc}(\lambda_2)$ *or* $\lambda_1 = \bar{\lambda}_1^{cc}(\lambda_2)$

(iii) $a_1^* < a_2^*$ *if* $\lambda_1 < \underline{\lambda}_1^{cc}(\lambda_2)$ *or* $\lambda_1 > \bar{\lambda}_1^{cc}(\lambda_2)$

Last, there always exist cost and impact functions such that for all λ_1, $a_1^* < a_2^*$.

Proposition 5 tells us that for any possible asymmetries between the players' impact and/or cost functions, there always exists confidence parameters such that a player produces higher or lower effort than his rival. As in the Lazear-Rosen tournament, when a player is either very overconfident or very underconfident, for a given confidence level of the rival, the player will exert less effort in equilibrium than the rival.

For instance, suppose player 1 has a cost advantage and both players have identical impact functions. There exist confidence levels of player 1 that will lead him to exert lower effort than player 2 in equilibrium. Alternatively, suppose player 1 has a cost disadvantage and both players have identical impact functions. There exist confidence levels of player 2 that lead player 1 to exert higher effort than player 2.

Observe that the thresholds identified in Proposition 5 may not exist if players have sufficiently asymmetric impact and cost functions. Indeed, if one player has a highly inefficient impact or cost function, then for a fixed confidence of the rival, the player will always exert less effort at equilibrium.

6 Conclusion

In this paper we investigate the role of confidence biases in tournaments and contests where players can differ in their ability and cost functions. We uncover a non-monotonic effect of confidence on the relative effort provision of players in a tournament, for any given heterogeneity in abilities and/or costs. A player with either a low or a high confidence exerts less effort than his rival at equilibrium. However, for intermediate confidence levels, the player exerts more effort than his rival.

We also show that a less able or a higher cost player may nevertheless outcompete his rival because of confidence biases. Indeed, provided the disadvantaged player does not feature a too high ability or cost disadvantage, for a fixed confidence level of the rival, there exist an intermediate range of confidence levels that lead the disadvantaged player to exert more effort than the rival at equilibrium. Moreover, a more able or a lower cost player may, nevertheless, be outcompeted by his rival. For any fixed level of confidence of his rival, the advantaged player will exert a lower effort at equilibrium if his confidence is either low enough, or high enough. Indeed, an advantaged player with a low confidence expects his effort to map into a low winning probability, thus inducing him to restrain effort provision. Moreover, an advantaged player with a high confidence expects his winning probability to be high with low effort thence inducing him to save on effort while securing a high perceived equilibrium probability.

Next, we show that the results extend to a generalized Tullock contest. In addition, the effects of confidence biases on players' equilibrium efforts may differ across these two types of competitive environments. In setups where players have the same abilities and costs, in Lazear-Rosen tournaments confidence biases can raise both players' equilibrium efforts, while this is never the case in a generalized Tullock contest.

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A Appendix

Proof of Lemma 1

The best response of player i is defined implicitly by (7). Hence, the slope of the best response of player i, $R_i'(a_j)$, is given by

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i(a_i,a_j,\lambda_i)]}{\partial a_i \partial a_j}}{\frac{\partial^2 E[U_i(a_i,a_j,\lambda_i)]}{\partial a_i^2}} = \frac{\lambda_i \theta_i^2 g'(\lambda_i\theta_i a_i - \theta_j a_j)\Delta u}{\lambda_i^2\theta_i^2 g'(\lambda_i\theta_i a_i - \theta_j a_j)\Delta u - c_i''(a_i)}.$$

The denominator is the second derivative of player i's perceived expected utility and so it is negative. Therefore, the sign of the slope of player i's best response is only determined by the (inverse of the) sign of the numerator which only depends on $g'(\lambda_i\theta_i a_i - \theta_j a_j)$. Hence, $R_i'(a_j)$ is positive for $\lambda_i\theta_i a_i > \theta_j a_j$, zero for $\lambda_i\theta_i a_i = \theta_j a_j$, and negative for $\lambda_i\theta_i a_i < \theta_j a_j$. This implies that $R_i(a_j)$ increases in a_j for $\lambda_i\theta_i a_i > \theta_j a_j$, reaches the maximum at $\lambda_i\theta_i a_i = \theta_j a_j$, and decreases in a_j for $\lambda_i\theta_i a_i < \theta_j a_j$.

Proof of Lemma 2

player i's best response is defined by (7):

$$\lambda_i\theta_i g(\lambda_i\theta_i a_i - \theta_i a_j)\Delta u - c_i'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{\partial^2 G(\lambda_i\theta_i a_i - \theta_j a_j)}{\partial a_i \partial \lambda_i}\Delta u = \left[g(\lambda_i\theta_i a_i - \theta_j a_j) + \lambda_i\theta_i a_i g'(\lambda_i\theta_i a_i - \theta_j a_j)\right]\theta_i \Delta u.$$

Since $\Delta u > 0$, we see that $\partial R_i(a_j)/\partial \lambda_i \lesseqgtr 0$ for

$$\frac{\partial^2 G(\lambda_i\theta_i a_i - \theta_j a_j)}{\partial a_i \partial \lambda_i} = g(\lambda_i\theta_i a_i - \theta_j a_j) + \lambda_i\theta_i a_i g'(\lambda_i\theta_i a_i - \theta_j a_j) \lesseqgtr 0.$$

Substituting next $\lambda\theta_i a_i = \theta_j a_j$ into the first-order condition of player i and denoting the maximal effort he is willing to exert in the tournament by $a_i^{\max}$ we obtain

$$\lambda_i\theta_i g(0)\Delta u = c'(a_i^{\max}).$$

This implies that $a_i^{\max}$ increases in λ_i.

Proof of Lemma 3

To prove that the equilibrium is unique, we first observe that when the players' best responses cross it is impossible that they are both negatively slopped. Indeed, for the two players' best responses to be negatively slopped at equilibrium, we require that $\lambda_i\theta_i a_i^* < \theta_j a_j^*$ and $\theta_i a_i^* > \lambda_j\theta_j a_j^*$, which combined imply that $\lambda_j < \frac{\theta_i a_i^*}{\theta_j a_j^*} < \frac{1}{\lambda_i}$. Yet, this is impossible since by assumption 1 we require $\lambda_i \geq \frac{1}{\lambda_j}$.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_i) = R_i'(a_j) \circ R_j'(a_i)$ has a slope smaller than 1 for any equilibrium pair (a_i^*, a_j^*), since the function is continuous on $\mathbf{R}$. If $R_i'(a_i^*) < 0$, then since $R_i'(a_j^*) > 0$, the condition is necessarily satisfied. If, on the other hand, $R_j'(a_i^*) > 0$, then we simply need to prove that if $R_i'(a_j^*) > 0$ for both players, then the product of the best responses is smaller than 1. Since $R_i'(a_j)$ is decreasing in $c_i''(a_i)$, it is thus sufficient to establish the result for $c_i''(a_i) = 0$. Rewriting the product of the players' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{\lambda_i\theta_i^2 g'(\lambda_i\theta_i a_i^* - \theta_j a_j^*)\Delta u}{\lambda_i^2\theta_i^2 g'(\lambda_i\theta_i a_i^* - \theta_j a_j^*)\Delta u - c_i''(a_i^*)} \cdot \frac{\lambda_j\theta_j^2 g'(\theta_i a_i^* - \lambda_j\theta_j a_j^*)\Delta u}{\lambda_j^2\theta_j^2 g'(\theta_i a_i^* - \lambda_j\theta_j a_j^*)\Delta u - c_j''(a_j^*)} < 1$$

Since we want to show that the above condition is true when $R_i'(a_j^*) > 0$ and $R_j'(a_i^*) > 0$, if the above condition is true for $c_i''(a_i) = c_j''(a_j) = 0$, then it is also true for any values $c_i''(a_i) > 0$ and $c_j''(a_j) > 0$. Consequently, the above condition is true if $\lambda_i\lambda_j \geq 1$, which is true by Assumption 1.

Proof of Proposition 1

Since the abilities and cost functions of the two players are symmetric, if $\lambda_1 = \lambda_2$, then we necessarily have that at the unique equilibrium $a_1^* = a_2^* = a^*$. The first-order condition for player 1 at any such symmetric equilibrium can be written as:

$$\phi(\lambda_1) = \lambda_1\theta g(\theta(\lambda_1 - 1)a^*)\Delta u - c'(a^*) = 0.$$

To prove the result, we use the fact that $a_1^* = a_2^* = a^*$ when $\lambda_1 = \lambda_2$, and we then explore the effect of a change in λ_1 on the best response of player 1. If, for these effort values the first-order condition of player 1 is not satisfied, then two cases need to be considered. First, if $\phi(\lambda_1) = \partial E[U_1(a_1^*, a_2^*, \lambda_1)]/\partial a_1 < 0$, then for this level of λ_1, $R_1(a_2^*) < a_1^*$. Given the quasi-concavity of $R_2(a_1)$ and the fact that λ_1 does not impact $R_2(a_1)$, this implies that at the equilibrium associated with this value of λ_1, $a_1^* < a_2^*$. Second, if $\phi(\lambda_1) = \partial E[U_1(a_1^*, a_2^*, \lambda_1)]/\partial a_1 > 0$, then at the equilibrium associated with this value of λ_1, $a_1^* > a_2^*$. In what follows, we shall prove that function $\phi(\lambda_1)$ crosses twice the x-axis, and is negatively-valued for $\lambda_1 = 0$ and for λ_1 tending to infinity. We shall denote the two threshold values of λ_1 satisfying $\phi(\lambda_1) = 0$ by $\underline{\lambda}_1(\lambda_2)$ and $\bar{\lambda}_1(\lambda_2)$, with $\underline{\lambda}_1(\lambda_2) < \bar{\lambda}_1(\lambda_2)$.

Consider first $\phi(0)$. Since $g(-\theta a^*) > 0$, it follows that for $\lambda_1 = 0$, $\lambda_1 g(\theta[\lambda_1 - 1]a^*) = 0$, and thus that $\phi(0) < 0$. Second, consider $\lim_{\lambda_1 \to \infty} \phi(\lambda_1)$. To show that $\phi(\lambda_1)$ is negative as λ_1 tends to infinity, it is sufficient to show that $\lambda_1 \theta g(\theta[\lambda_1 - 1]a^*)$ converges to zero as $\lambda_1 \to \infty$. To prove this we proceed in several steps. First, we observe that $\int_{\lambda_1}^{\infty} \theta g(\theta[\lambda_1 - 1]a^*) \leq \theta$ for any λ_1, since $\int_{-\infty}^{+\infty} \theta g(\theta[\lambda_1 - 1]a^*) = \theta$. Moreover, the value of this expression is monotonically decreasing in λ_1 since $g(\theta[\lambda_1 - 1]a^*)$ is monotonically decreasing in λ_1. Assume then that, contrary to what we want to prove, $\lim_{\lambda \to \infty} \lambda_1 \theta g(\theta[\lambda_1 - 1]a^*) > 0$. Accordingly, there must exist some arbitrarily large λ_1 that we designate by λ_L and some value $k \in \mathbb{R}^+$ such that $\lambda_L \theta g(\theta[\lambda_L - 1]a^*) > k$. Moreover, we also have $\int_{\lambda_L}^{+\infty} \theta g(\theta[\lambda_1 - 1]a^*) < d < \theta$. Consider next a value $\lambda_{\hat{L}} > \lambda_L$ that is close enough to λ_L and is such that $\lambda_{\hat{L}} \theta g(\theta[\lambda_{\hat{L}} - 1]a^*) > k$. We know that $g(\theta[\lambda_1 - 1]a^*)$ is a monotone decreasing function, and we thus deduce that:

$$\int_{\lambda_L}^{\lambda_{\bar{L}}} \theta g(\theta[\lambda_1 - 1]a^*) > [\lambda_{\bar{L}} - \lambda_L]\theta g(\theta[\lambda_{\bar{L}} - 1]a^*) > [\lambda_{\bar{L}} - \lambda_L]\frac{k}{\lambda_{\bar{L}}} = \left[1 - \frac{\lambda_L}{\lambda_{\bar{L}}}\right] k.$$

We can then choose a value $\lambda_{\bar{L}} > 2\lambda_L$ so that $[1 - \lambda_L/\lambda_{\bar{L}}] > 1/2$, and then deduce $\int_{\lambda_L}^{\lambda_{\bar{L}}} \theta g(\theta[\lambda_1 - 1]a^*) > \frac{k}{2}$. Since $d > \int_{\lambda_L}^{\infty} \theta g(\theta[\lambda_1 - 1]a^*)$, we deduce that $d > \frac{k}{2}$. But since $g(\theta[\lambda_1 - 1]a^*)$ is monotonically decreasing in λ_1, we can always choose a

λ_1 that is large enough so that $d < k/2$, thence contradicting the initial assertion. Consequently we obtain that $\lim_{\lambda_1 \to \infty} \phi(\lambda_1) = -c'(a^*) < 0$.

Since an equilibrium exists and that $\phi(0) < 0$, it is necessarily the case that there exists a $\underline{\lambda}_1$ such that $\phi(\underline{\lambda}_1) = 0$, with $\phi(\lambda) < 0$, $\forall \lambda < \underline{\lambda}_1$. To complete the proof, we need to demonstrate therefore that there exists a $\bar{\lambda}_1 > \underline{\lambda}_1$, with $\phi(\bar{\lambda}_1) = 0$, and that there is no other confidence value $\hat{\lambda}$ with $\phi(\hat{\lambda}) = 0$. We consider all possible cases.

(i) If $\underline{\lambda}_1 < 1$, so that $g'(\theta[\underline{\lambda}_1 - 1]a^*) > 0$, then $\phi'(\underline{\lambda}_1) = [\theta g(\theta[\underline{\lambda}_1 - 1]a^*) + \theta^2 a^* \underline{\lambda}_1 g'(\theta[\underline{\lambda}_1 - 1]a^*)]\Delta u > 0$. Since we know that $\lim_{\lambda_1 \to \infty} \phi(\lambda_1) < 0$, there must exist a second value $\bar{\lambda}_1$ such that $\phi(\bar{\lambda}_1) = 0$. To reach that value we must have that $\phi'(\lambda_1) < 0$ for values in $]\underline{\lambda}_1, \bar{\lambda}_1[$, since otherwise $\phi(\lambda_1)$ would be monotonically increasing. Take any value in that interval and denote it by $\check{\lambda}$. Thence, it is necessary that $g'(\theta[\check{\lambda} - 1]a^*) < 0$, which implies that $g'(\theta[\bar{\lambda}_1 - 1]a^*) < 0$ since $\bar{\lambda}_1 > \check{\lambda}$. We next evaluate $\phi'(\lambda_1)$ when $\lambda_1 = \bar{\lambda}_1$,

$$\phi'(\bar{\lambda}_1) = [g(\theta[\bar{\lambda}_1 - 1)a^*] + \theta a^* \bar{\lambda}_1 g'(\theta[\bar{\lambda}_1 - 1]a^*)]\theta \Delta u.$$

In $\lambda_1 = \bar{\lambda}_1$ we cannot have $\phi'(\bar{\lambda}_1) > 0$, since the function is smooth and decreasing in the left neigbourhood of $\bar{\lambda}_1$. If $\phi'(\bar{\lambda}_1) = 0$, then since $\lim_{\lambda_1 \to \infty} \phi(\lambda_1) = -c'(a^*)$, then there must exist another value $\hat{\lambda} > \bar{\lambda}_1$ such that $\phi(\hat{\lambda}) = 0$. Yet, to have $\phi'(\bar{\lambda}_1) = 0$, we then need to have:

$$\phi''(\bar{\lambda}_1) = [2g'([\bar{\lambda}_1 - 1]a^*) + \theta a^* g''([\bar{\lambda}_1 - 1]a^*)]\theta^2 a^* \Delta u > 0,$$

which necessitates that $g''(\theta[\bar{\lambda}_1 - 1]a^*) > 0$. Following assumption A.4, this implies that $g''(\theta[\bar{\lambda}_1 - 1]a^*) > 0$ for any $\lambda_1 > \bar{\lambda}_1$. But then we can rewrite $\phi(\lambda_1) = 0$ as:

$$\lambda_1 = \frac{c'(a^*)}{g'(\theta[\lambda_1 - 1]a^*)\theta \Delta u}, \tag{17}$$

and since the LHS is linearly increasing in λ_1 and the RHS is a concave function of λ_1, there can be but a single solution to the above problem, which would then be the value previously identified, $\bar{\lambda}_1$, therefore excluding the existence of a value $\hat{\lambda}$ s.t. $\phi(\hat{\lambda}) = 0$. This in turn would imply that $\lim_{\lambda_1 \to \infty} \phi(\lambda_1) > 0$, which would contradict our earlier finding that $\lim_{\lambda_1 \to \infty} \phi(\lambda_1) < 0$.

Consequently, we must have that $\phi'(\bar{\lambda}_1) < 0$. Then, if $g''(\theta[\bar{\lambda}_1 - 1]a^*) > 0$, there can be no other value $\hat{\lambda} > \bar{\lambda}_1$ such that $\phi(\bar{\lambda}_1) = 0$ because we would reach the same contradiction as above. If on the contrary $g''(\theta[\bar{\lambda}_1 - 1]a^*) \leq 0$, then to have a value $\hat{\lambda} > \bar{\lambda}$ such that $\phi(\hat{\lambda}) = 0$, it must be the case that for some $\lambda \in]\hat{\lambda}, \bar{\lambda}[$, $g''(\theta[\lambda - 1]a^*) > 0$. By Assumption A.4, this implies that for $\lambda = \hat{\lambda}$ we have $g''(\theta[\hat{\lambda} - 1]a^*) > 0$. Yet, if the function $g(\theta[\lambda - 1]a^*)$ is strictly convex on an interval $[\check{\lambda}, \infty[$ with $\hat{\lambda}$ belonging to this interval, then for any value of λ in this interval such that $\lambda < \hat{\lambda}$, we deduce from equation (17) that $\phi(\lambda) > 0$, which is a contradiction since for $\lambda \in]\bar{\lambda}, \hat{\lambda}[$, $\phi(\lambda) < 0$.

(ii) The second scenario is such that $\underline{\lambda}_1 > 1$, so that $g'(\theta[\underline{\lambda}_1 - 1]a^*) < 0$. The rest of the reasoning to prove that there can be but another value of $\bar{\lambda}_1$ such that $\phi(\bar{\lambda}) = 0$ follows the lines above.

Proof of Proposition 2

Since the best response function of player i is described by his first-order condition, and observing that the cost parameter k_i only affects the best response function of player i, we can inspect the effect of a change in k_i on $R_i(a_j)$. Applying the implicit function theorem on player i's best response function, this effect is given by the next expression:

$$\frac{\partial R_i(a_j)}{\partial k_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial k_i}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}}.$$

Since the second-order condition is satisfied, it follows that the sign of $\frac{\partial R_i(a_j)}{\partial k_i}$ is then given by the sign of $\frac{\partial^2 E[U_i]}{\partial a_i \partial k_i}$, or:

$$sign\left\{\frac{\partial^2 E[U_i]}{\partial a_i \partial k_i}\right\} = sign\left\{\frac{\partial c'(a_i, k_i)}{\partial k_i}\right\} < 0.$$

Any increase in k_i then unambiguously leads to contractions of player i's best response, which, given the quasi-concavity of the rival player's best response function, necessarily implies a reduction of a_i^*/a_j^*, and thence a drop in the (actual and perceived) probability that player i wins the tournament. Therefore, the new thresholds on λ_1 guaranteeing that $a_1^* = a_2^*$ are now $\lambda = \{\underline{\lambda}_1', \bar{\lambda}_1'\}$ and are such that $\underline{\lambda}_1' < \underline{\lambda}_1$ and $\bar{\lambda}_1' > \bar{\lambda}_1$.

Proof of Proposition 3

Allowing for confidence, ability, and cost asymmetries, the first-order condition of player 1 can be written as:

$$\frac{\partial E[U_1(a_1, a_2, \lambda_1)]}{\partial a_1} = \lambda_1 \theta_1 g(\lambda_1 \theta_1 a_1 - \theta_2 a_2)\Delta u - c'(a_1; k_1) = 0. \tag{18}$$

If $\theta_1 = \theta_2$, then we know that the equilibrium relative efforts of players is determined by the value of λ_1 as compared to the thresholds $\underline{\lambda}_1'$ and $\bar{\lambda}_1'$. We are therefore interested in the effect of changes in θ_1 on these thresholds, which completely characterize the sign of $(a_1^* - a_2^*)$. To deduce how these thresholds are affected, we consider an initial situation such that the model's parameters induce $a_1^* = a_2^*$ and we inspect the effect of θ_1 on $R_1(a_2)$ at this equal effort equilibrium (i.e. for $a_2^* = a_1^*$) and we then have:

$$\begin{aligned} sign\left\{\frac{\partial R_1(a_2^*)}{\partial \theta_1}\right\} &= sign\left\{\frac{\partial E[U_1(a_1^*, a_2^*, \lambda_1)]}{\partial a_1 \partial \theta_1}\right\} \\ &= sign\left\{g(\lambda_1\theta_1 a_1^* - \theta_2 a_2^*) + \lambda_1\theta_1 a_1^* g'(\lambda_1\theta_1 a_1^* - \theta_2 a_2^*)\right\} \\ &= sign\{\phi'(\lambda_1)\}, \end{aligned}$$

where $\phi(\lambda_1)$ is given by:

$$\phi(\lambda_1) = \lambda_1\theta_1 g([\lambda_1\theta_1 - \theta_2]a^*)\Delta u - c'(a^*) = 0.$$

Note that $\phi'(\lambda_1)$ satisfies the properties derived in the proof of Proposition 1, so that $\phi'(\lambda_1) < 0$ for $\lambda_1 = \underline{\lambda}_1(\lambda_2)$. We can extend the reasoning to deduce that $\phi'(\underline{\lambda}_1') < 0$, thence implying that if we define the new lower threshold below which $a_1^* < a_2^*$ by $\underline{\lambda}_1''$, we necessarily have $\underline{\lambda}_1'' < \underline{\lambda}_1'$. Likewise, having demonstrated that $\phi'(\bar{\lambda}_1) > 0$, we can here too extend the reasoning to deduce that $\phi'(\bar{\lambda}_1') > 0$. If we define the new upper threshold above which $a_1^* < a_2^*$ by $\bar{\lambda}_1''$, we then have $\bar{\lambda}_1'' > \bar{\lambda}_1'$.

Proof of Lemma 4 The best response of player i, $i = \{1, 2\}$, is defined implicitly by (15). Hence, the slope of the best response of player i, $R_i'(a_j)$ is given by

$$-\frac{\partial R_i/\partial a_j}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial a_j}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda_i q_i(a_i) - q_j(a_j)}{[\lambda_i q_i(a_i) + q_j(a_j)]^3} \lambda_i q_i'(a_i) q_j'(a_j) \Delta u}{\frac{q_i''(a_i)[\lambda_i q_i(a_i) + q_j(a_j)] - 2\lambda_i [q_i'(a_i)]^2}{[\lambda_i q_i(a_i) + q_j(a_j)]^3} \lambda_i q_j(a_j) \Delta u - c_i''(a_i)}. \qquad (19)$$

The denominator is negative because player i's second-order condition is satisfied. Therefore, the sign of the slope of player i's best response is only determined by the sign of the numerator which only depends on $\lambda_i q_i(a_i) - q_j(a_j)$. Hence, $R_i'(a_j)$ is positive for $\lambda_i q_i(a_i) > q_j(a_j)$, zero for $\lambda_i q_i(a_i) = q_j(a_j)$, and negative for $\lambda_i q_i(a_i) < q_j(a_j)$. This implies that $R_i(a_j)$ increases in a_j for $\lambda_i q_i(a_i) > q_j(a_j)$, reaches the maximum at $\lambda_i q_i(a_i) = q_j(a_j)$, and decreases in a_j for $\lambda_i q_i(a_i) < q_j(a_j)$.

Proof of Lemma 5 To prove that the equilibrium is unique, we first show that when the players' best responses cross it is impossible that they are both negatively slopped. We proceed by contradiction here too and suppose that there is an equilibrium such that $R_1'(a_2^*) < 0 \Leftrightarrow q_2(a_2^*) > \lambda_1 q_1(a_1^*)$ and $R_2'(a_1^*) < 0 \Leftrightarrow q_1(a_1^*) > \lambda_2 q_2(a_2^*)$. Assume, without loss of generality, $\lambda_1 > \lambda_2$, so that $\lambda_1 > 1$. Then $q_2(a_2^*) > \lambda_1 q_1(a_1^*) \Rightarrow q_2(a_2^*) > q_1(a_1^*)$.

To show that an equilibrium such that $R_1'(a_2^*) < 0$ and $R_2'(a_1^*) < 0$ cannot admit $q_2(a_2^*) > q_1(a_1^*)$, consider any pair $q_1(a_1) = q_2(a_2) = q$. Since $\lambda_1 > \lambda_2$, then $\frac{\partial E[U_1(a_1,a_2;\lambda_1)]}{\partial a_1} > \frac{\partial E[U_2(a_2,a_1;\lambda_2)]}{\partial a_2}$ for $q_1(a_1) = q_2(a_2)$. This in turn would imply that if player 2's first-order condition is satisfied then player 1 has incentives to increase his effort, and if player 1's first-order condition is satisfied, then player 2 has incentives to reduce his effort. Consequently, the best response of player 2 needs to cross the 45-degrees line for efforts a_1 and a_2 such that $q_2(a_2) < q_1(a_1)$. The quasi-concavity of the players' best responses allows us to conclude that $q_1(a_1^*) > q_2(a_2^*)$, thence the contradiction.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(a_1) = R_1'(a_2) \circ R_2'(a_1)$ has a slope smaller than 1 for any equilibrium pair (a_1^*, a_2^*), since the function is continuous on $\mathbf{R}$. Having shown that at equilibrium we cannot have $R_1'(a_2) < 0$ and $R_2'(a_1) < 0$, we simply need to prove that when both best responses are positively slopped at equilibrium, the product of the best responses is smaller than 1. Since $R_1'(a_2)$ is decreasing in $c_1''(a_1)$, it is thus sufficient to establish the result for $c_1''(a_1) = 0$. Rewriting the product of the players' best responses with this restriction, and simplifying expressions, we thus want to show that:

$$\frac{(\lambda_1 q_1(a_1) - q_2(a_2))(\lambda_2 q_2(a_2) - q_1(a_1))\,(q_1'(a_1) q_2'(a_2))^2}{[q_1''(a_1)[\lambda_1 q_1(a_1) + q_2(a_2)] - 2\lambda_1 [q_1'(a_1)]^2]\,[q_2''(a_2)[\lambda_2 q_2(a_2) + q_1(a_1)] - 2\lambda_2 [q_2'(a_2)]^2]\,q_1(a_1) q_2(a_2)} < 1.$$

Since the LHS is decreasing in both $q_1''(a_1)$ and $q_2''(a_2)$ the above expression is *a fortiori* true when setting $q_1''(a_1) = q_2''(a_2) = 0$, thence implying the above inequality is verified if:

$$\frac{(\lambda_1 q_1(a_1) - q_2(a_2))(\lambda_2 q_2(a_2) - q_1(a_1))\,(q_1'(a_1) q_2'(a_2))^2}{4\lambda_1 [q_1'(a_1)]^2 \lambda_2 [q_2'(a_2)]^2 q_1(a_1) q_2(a_2)} < 1,$$

an expression that simplifies to:

$$(\lambda_1 q_1(a_1) - q_2(a_2))(\lambda_2 q_2(a_2) - q_1(a_1)) < 4\lambda_1 \lambda_2 q_1(a_1) q_2(a_2).$$

And this inequality is always satisfied since $\lambda_1 \lambda_2 \geq 1$.

Proof of Lemma 6

(This proof follows Baik 1994) Player i's best response is defined by (15):

$$\frac{\lambda_i q_i'(a_i) q_j(a_j)}{[\lambda_i q_i(a_i) + q_j(a_j)]^2} \Delta u - c_i'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(a_j)}{\partial \lambda_i} = \frac{q_j(a_j) - \lambda_i q_i(a_i)}{[\lambda_i q_i(a_i) + q_j(a_j)]^3} q_i'(a_i) q_j(a_j) \Delta u.$$

We see that $\partial R_i(a_j)/\partial \lambda_i \gtreqless 0$ for $q_j(a_j) \gtreqless \lambda_j q_i(a_i)$. We also know from Lemma 1 that $sign\{R_i'(a_j)\} = -sign\left\{\frac{\partial R_i(a_j)}{\partial \lambda_i}\right\}$.

Substituting next $q_j(a_j) = \lambda_i q_i(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a_i^{max} we obtain

$$\frac{\lambda_i q_i'(a_i^{max})\lambda_i q_i(a_i^{max})}{[\lambda_i q_i(a_i^{max}) + \lambda_i q_i(a_i^{max})]^2}\Delta u = c_i'(a_i^{max}),$$

or

$$\frac{\lambda_i^2 q_i'(a_i^{max}) q_i(a_i^{max})}{4\lambda_i^2 \left[q_i(a_i^{max})\right]^2}\Delta u = c_i'(a_i^{max}),$$

or

$$\frac{q_i'(a_i^{max})}{4q_i(a_i^{max})}\Delta u = c_i'(a_i^{max}).$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a_i^{max}, does not depend on λ_i.

Proof of Proposition 4

To prove this result we follow the same reasoning as in the proof of Proposition 1, and therefore consider the unique equilibrium when $\lambda_1 = \lambda_2$ such that $a_1^* = a_2^* = a^*$. The first-order condition for player 1 at any such symmetric equilibrium can be written as:

$$\xi(\lambda_1) = \frac{\lambda_1 q'(a^*)}{q(a^*)}\Delta u - (1+\lambda_1)^2 c'(a^*) = 0$$

For $\lambda_1 = 0$, $\xi(0) < 0$. Next, $\lim_{\lambda_1 \to \infty} \xi(\lambda_1) < 0$. Last, since the function is a inverted parabola, and given the fact that an equilibrium exists, then there are exactly two values of λ_1 satisfying $\xi(\lambda_1) = 0$. We denote the smaller value by $\underline{\lambda}_1^c$ and the larger value by $\bar{\lambda}_1^c$. The rest of the reasoning replicates the one in the proof of Proposition 1.

Proof of Corollary 1

The first-order condition of player i when $\lambda_i = 1$ is given by:

$$\frac{q_i'(a_i) q_j(a_j)}{[q_i(a_i) + q_j(a_j)]^2}\Delta u - c_i'(a_i) = 0.$$

If players have identical impact and cost functions, and consequently produce the same equilibrium effort a^*, this expression becomes:

$$\frac{q'(a^*)}{4q(a^*)}\Delta u = c'(a^*),$$

and this value coincides with a_i^{max}.

Proof of Proposition 5

Consider any impact functions $q_i(.)$ and $q_j(.)$, as well as any cost functions $c_i(.)$ and $c_j(.)$. We begin by showing that there always exist a pair (λ_i, λ_j) producing equilibrium efforts $a_1^* = a_2^* = a^*$. To see that, take any pair (λ_1, λ_2) such that, without loss of generality, $a_1^* > a_2^*$. observe first that the best response function of any player 2 does not depend on λ_1. Consider then the expected effort of player 1 such that $R_2(a_1) = a_1$, and denote this effort value of player 2 by $\check{a}_2$. Take next the best response of player 1 which is defined by:

$$\frac{\lambda_1 q_1'(a_1) q_2(a_2)}{[\lambda_1 q_1(a_1) + q_2(a_2)]^2}\Delta u - c_1'(a_1) = 0.$$

Recalling the assumption that $c_1'(0) = 0$, we thus have that $\lim_{\lambda_1 \to \infty} R_1(a_2) = 0$. Last, since the best response of player 1 shifts continuously with λ_1, there must exist a value of λ_1 such that $R_1(\check{a}_2) = \check{a}_2$.

Consider then a pair of confidence parameters such that $a_1^* = a_2^*$. The first-order condition for player 1 at this equilibrium can be written as:

$$\xi(\lambda_1) = \frac{\lambda_1 q_1'(a^*)}{q_1(a^*)}\Delta u - (1+\lambda_1)^2 c_1'(a^*) = 0.$$

For $\lambda_1 = 0$, $\xi(0) < 0$. Next, $\lim_{\lambda_1 \to \infty} \xi(\lambda_1) < 0$. Last, since the function is a inverted parabola, and given the fact that an equilibrium exists, then there are exactly two values of λ_1 satisfying $\xi(\lambda_1) = 0$. We denote the smaller value by $\underline{\lambda}_1^{cc}$ and the larger value by $\bar{\lambda}_1^{cc}$. The rest of the reasoning replicates the one in the proof of Proposition 1.