Overconfidence and Strategic Behavior in Elimination Contests: Implications for CEO Selection*

Yuxi Chen† Luís Santos-Pinto‡

October 4, 2024

Abstract

We analyze how overconfidence affects behavior in multistage elimination contests. Our findings reveal a nuanced interplay between overconfidence and effort exertion. An overconfident player exerts less effort in the final stage than a rational rival. However, this pattern can be inverted in the semifinals stage, where an overconfident player can exert more effort than a rational rival. We also uncover that an overconfident player can have the highest probability of winning an elimination contest. Our results offer a novel perspective on CEO overconfidence and highlight that high executive compensation renders the pursuit of CEO positions exceptionally appealing to overconfident managers.

Keywords: Overconfidence, Elimination Contest, Encouragement Effect, Complacency Effect.

*We thank José Alcalde, Carmen Beviá, Adrian Bruhin, Kai Konrad, Igor Letina, Ludmila Matyskova, Petros Sekeris, and participants at The Lisbon Meetings in Game Theory and Applications 2024, Conference on Mechanism and Institution Design 2024, 51th Conference of the European Association for Research in Industrial Economics, European Meeting on Game Theory 2024, and North America Summer Meeting of the Econometric Society 2024.

†Department of Economics, University of Lausanne, Quartier UNIL-Dorigny, 1015 Lausanne, Switzerland. Email: Yuxi.Chen@unil.ch.

‡Corresponding author: Luís Santos-Pinto Department of Economics, University of Lausanne, Quartier UNIL-Dorigny, 1015 Lausanne, Switzerland. Email: luispedro.santospinto@unil.ch.

1 Introduction

Elimination contests are a common feature in organizations, politics, sports, and academia. In companies, managers compete for promotion to senior executive positions, and senior executives compete for a chief executive officer (CEO) position. In politics, politicians compete for top positions in the party, and those who reach prominent positions in the party compete to become high-level government officials. In academia, PhDs compete to be hired as assistant professors, and assistant professors compete for tenure. In tennis and many other sports, players compete in elimination contests.

Overconfidence is one of the most widely documented biases in judgment and has been detected both in the laboratory and in the field.[1] Overconfidence has consequences for economic behavior in labor markets (Spinnewijn 2013, Spinnewijn 2015, Köszegi 2014, Santos-Pinto and de la Rosa 2020). A large proportion of CEOs is overconfident and CEO overconfidence affects corporate decisions (Malmendier and Tate, 2005, 2008, 2015). It remains an open research question why do these overconfident CEOs obtain their jobs in the first place.

In this paper we analyze how overconfidence, conceptualized as overestimation of the impact of one's effort, affects behavior in multistage elimination contests. We are interested in finding answers to the following questions. How does overconfidence affect effort provision in the different stages of an elimination contest? Is an overconfident player more or less likely to win an elimination contest than a rational player? What are the welfare implications of overconfidence for the players and for the contest designer?

To address these questions we consider a two-stage elimination contest with four players. In the first stage, the players are matched pairwise, and each pair competes in one semifinal. The first-stage winners go on to the second stage of the contest and compete against each other in the final. The winner receives prize w_1, the runner-up prize w_2, and the first-stage losers receive nothing, with $w_1 > w_2 \geqslant 0$.

In each pairwise interaction the players choose their efforts simultaneously and their winning probabilities are determined by a contest success function. Players are homogeneous, except for their confidence levels. This allow us to zero in on the impact of overconfidence on players' incentives to exert effort and winning probabilities. An overconfident player overestimates the impact of his effort on his chances of winning at each

[1] Moore and Healy (2008) distinguish between three types of overconfidence: overestimation of one's skill (absolute overconfidence), overplacement (relative overconfidence), and excessive precision in one's beliefs (miscalibration or overprecision). Our paper focuses on the first type of overconfidence.

stage but has a correct perception of the prizes and cost of effort. Furthermore, an overconfident player's bias is observable by his rivals.

We solve the elimination contest using backward induction, beginning with the final stage. In the final, each player selects their effort level based on the condition that the product of the perceived marginal probability of winning and the utility prize spread equals the marginal cost of effort. In the semifinal, a player similarly chooses an effort level where the perceived marginal benefit matches the marginal cost. Here, the perceived marginal benefit is calculated as the product of the perceived marginal probability of winning the semifinal and the perceived expected utility of reaching the final.

We begin by demonstrating that, as Rosen (1986) first suggested, overconfidence affects a player's incentives to exert effort in a semifinal in two distinct ways. First, overconfidence increases a player's perceived expected utility of reaching the final, encouraging greater effort. We refer to this as the encouragement effect of overconfidence. Conversely, overconfidence lowers a player's perceived marginal probability of winning the semifinal, which discourages effort. We call this the complacency effect of overconfidence. Hence, the equilibrium effort exerted by an overconfident player in the semifinal depends on the dominance of one effect over the other: if the encouragement effect prevails, the player exerts more effort; if the complacency effect dominates, the player exerts less.

Next, we assume the players' winning probabilities are determined by the most commonly used contest success function (CSF) introduced by Tullock (1980). We assume one player is overconfident while the other three are rational. This setup reflects situations where a minority of players exhibit overconfidence, enabling us to analyze the effects of overconfidence in the most straightforward manner. We identify two key findings. First, in the final, the overconfident player exerts less effort than his rational rival at equilibrium. Intuitively, the (mis)perceived advantage of the overconfident player leads him to lower his effort. The rational player, anticipating the overconfident player will lower his effort, also lowers her effort but not as much. Second, in the semifinal, the overconfident player exerts more effort than his rational rival at equilibrium when the bias is relatively small, as this allows the encouragement effect to outweigh the complacency effect. These two key findings open the possibility that a moderately overconfident player might have a higher chance of winning an elimination contest than a rational player. However, deriving closed-form solutions for equilibrium efforts and winning probabilities under Tullock's CSF is not feasible, which limits our ability to address this question.

We then assume the players' winning probabilities are determined by the contest success function proposed by Alcalde and Dahm (2007). Unlike the Tullock CSF, this allows us to derive closed-form solutions for the players' equilibrium efforts, winning probabilities, and expected utilities.[2] From this, we identify three key findings. First, in the semifinal, the overconfident player exerts more effort than his rational rival at equilibrium when the prize spread is large and the bias is not too extreme. Second, an overconfident player can emerge with the highest equilibrium probability of winning an elimination contest. This occurs when effort significantly influences winning probabilities, the prize spread is large, and the overconfidence bias is relatively small. Third, the bias can improve the overconfident player's welfare, benefits the rational players seeded in the same semifinal, but has an ambiguous effect on the welfare of the rational player seeded in the same semifinal as the overconfident player and on the welfare of the contest designer.

Finally, we discuss four extensions of the model. The first extension involves an elimination contest with two overconfident players and two rational players. The second examines a scenario where the overconfident player's bias is unobservable by the rational players. The third allows for an elimination contest with three stages instead of two. The fourth extension considers an elimination contest with one underconfident player and three rational players. The first three extensions confirm that our main results hold with multiple overconfident players, when overconfidence is unobservable, and when the contest has three stages. The fourth extension demonstrates that an underconfident player exerts less effort than his rational rivals in both the final and semifinal stages, resulting in a lower equilibrium probability of winning an elimination contest. Intuitively, the underconfident player doesn't try hard enough because opponents appear relatively stronger, and also because he underestimates the expected utility of reaching the final.

The paper is organized as follows: Section 2 reviews the related literature, while Section 3 introduces the general model. Section 4 outlines the encouragement and complacency effects. Section 5 specializes the model with Tullock's CSF, and Section 6 with Alcalde and Dahm's (2007) CSF. Section 7 explores four extensions, and Section 8 concludes the paper. Proofs for Tullock's CSF are provided in the Appendix, while those for Alcalde and Dahm's CSF are available in the Online Appendix.

[2]Alcalde and Dahm's CSF yields a tractable model for multi-stage games as the equilibrium efforts and payoffs of the subgames can be easily computed and plugged into earlier stages of the game. Section 6 explains Alcalde and Dahm's CSF in detail.

2 Related Literature

Our study relates to four strands of literature. First, it contributes to the literature on CEO overconfidence. Empirical evidence documents that a substantial share of CEOs are overconfident (for a review see Malmendier and Tate, 2015). The seminal contribution to this literature is Malmendier and Tate (2005, 2008) who measure CEO overconfidence as the tendency to hold stock options longer before exercise. Malmendier and Tate (2015) use this measure together with additional controls and find that approximately 40 percent of CEOs of companies listed in the Standard & Poor's 1500 index are overconfident.

Several theories on the selection of managers into CEO positions have been proposed to explain CEO overconfidence. According to Van den Steen (2005), CEO overconfidence serves as a commitment device that helps attract and retain employees that share the same values as the CEO. For Hackbarth (2008), CEO overconfidence leads to high debt levels which prevent CEOs from diverting funds, which, in turn, increases firm value and reduces conflicts between CEOs and shareholders. Goel and Thakor (2008) study elimination tournaments where risk-averse managers compete to become CEO by choosing the level of risk of their projects. Some managers are rational while others are overconfident. An overconfident manager underestimates project risk which increases the propensity to take risky projects (e.g. R&D activities). Some of the more risky projects will be successful and hence, the higher risk taking of overconfident managers will improve their chances of promotion to CEO. Finally, Gervais et al. (2011), show that firms can find overconfident managers more attractive because they exert higher effort to learn about their projects.

Our findings offer a new perspective on why overconfident managers are promoted to CEO positions. In a seminal contribution, Rosen (1986), models the competition for promotion to a top executive role as a multistage elimination contest where in each stage fewer managers are selected for the next step of the career ladder. Our model suggests that when the prize differential across the corporate ladder is substantial, moderately overconfident managers are more likely to be promoted to CEO than their rational counterparts. This occurs because moderately overconfident managers tend to exert greater effort, such as working longer hours, early in their careers due to the encouragement effect provided by their overconfidence. The larger the disparity between the compensation of a lower-level manager and that of a CEO, the stronger this encouragement effect becomes. Consequently, our results emphasize the significant influence that large increases in executive compensation (Murphy 2013) can have in making the pursuit of a CEO position particularly appealing to overconfident individuals.

Second, our study contributes to the large literature on gender gaps in the labor market. Empirical evidence documents gender gaps in wages and in top business positions. For instance, in 2022 women in the US earn 82 percent of their male counterparts (Kochhar 2023) and women represent only 6 percent of top business executives in the US (Keller et al. 2022). The wage gender gap is larger in high skilled work, and much of it seems to be caused by gaps in promotions (Blau and DeVaro 2007, Blau and Kahn 2017, Bronson and Thoursie 2019). Laboratory experiments show that gender differences in confidence and risk attitudes can account for gender gaps in behavior in tournaments and contests (Niederle and Vesterlund 2007, Kamas and Preston 2012, Gillen et al. 2019, Price 2020, Buser et al. 2021, van Veldhuizen 2022).

Our findings show that the large executive compensation spreads coupled with higher male confidence can make competing for a top business position much more attractive to male candidates. We also predict that much of the gender gap in promotions will take place early in workers' careers. This could place women at a further disadvantage besides the negative effects of childbirth and child-rearing (Bertrand et al. 2010, Goldin and Katz 2011, Goldin 2014).[3]

Third, our study also contributes to the literature on overconfidence, tournaments, and contests. Santos-Pinto (2010) shows how firms can optimally set tournament prizes to exploit workers' overconfidence, defined as overestimation of productivity of effort. Ludwig et al. (2011) show that an overconfident player, defined as someone who underestimates the cost of effort, exerts more effort than a rational player in a Tullock contest. Santos-Pinto and Sekeris (2023) study how heterogeneity in confidence biases affects effort provision in Tullock contests. They find, among other things, that the more confident player exerts lower effort in a Tullock contest with two players. All of these studies focus on one-shot tournaments and contests. To the best of our knowledge, ours is the first study on overconfidence in elimination contests.

Finally, our study contributes to the literature on elimination contests. The seminal work is Rosen (1986) who studies how prizes affect performance in a multistage elimination contest. There are many studies on elimination contests regarding different aspects,

[3]Many studies suggest that gender gap varies with culture (Gneezy et al, 2003 and 2009; Booth and Nolen, 2009 and 2014). In societies where gender equality is more promoted, gender gaps become less significant in many areas, including entry and performance in a competitive environments. Differences in work environment, characteristics of professions, and education also affect the magnitude of gender gaps.

such as the discouragement effects in multi-stage contests (Konrad, 2012), optimal prize setting (Mago et al, 2013; Cheng et al, 2019; Coehn et al, 2018; Moldovanu and Sela, 2006), optimal contest structure (Gradstein and Konrad, 1999; Moldovanu and Sela, 2006; Fu and Lu, 2018; Hou and Zhang, 2021), heterogeneity in abilities (Rosen, 1986; Brown and Minor, 2014) and seeding (Groh et al, 2012). Our paper expands this strand of literature by considering a new dimension: heterogeneity in confidence levels.

3 Set-up

Consider a two-stage elimination contest with four players. In the first stage, the players are matched pairwise, and each pair competes in one semifinal. The first-stage winners go on to the second stage of the contest and compete against each other in the final. The winner receives prize w_1, the runner-up receives prize w_2, and the first-stage losers receive nothing, with $w_1 > w_2 \geqslant 0$.

The players choose their efforts simultaneously to maximize their expected utilities in each pairwise interaction. The effort of player i in a pairwise interaction is denoted by e_i. Player i derives utility $u(w)$ from prize $w \geqslant 0$, where $u'(w) > 0$, $u''(w) \leqslant 0$, and $u(0) = 0$, and has cost of effort $c(e_i) = ce_i$, with $c \geqslant 1$, and $e_i \geqslant 0$. Player i's actual probability of winning when paired with j at stage $t \in \{s, f\}$ is modeled via a contest success function and is denoted by $p^t_{ij}(e_i, e_j)$ with $mgp^t_{ij}(e_i, e_j) = \partial p^t_{ij}/\partial e_i > 0$, $\partial p^t_{ij}/\partial e_j < 0$, and $\partial^2 p^t_{ij}/\partial e_i^2 < 0$.

We assume an overconfident player overestimates the impact of his effort on his probability of winning each pairwise interaction and has a correct perception of the prizes and cost of effort. This definition of overconfidence is in line with Santos-Pinto (2008, 2010) and Santos-Pinto and Sekeris (2023). Accordingly, player i's perceived probability of winning when paired with j at stage $t \in \{s, f\}$ is denoted by $\widetilde{p}^t_{ij}(e_i, e_j, \lambda_i)$, where λ_i represents player i's confidence level, with $mg\widetilde{p}^t_{ij}(e_i, e_j, \lambda_i) = \partial\widetilde{p}^t_{ij}/\partial e_i > 0$, $\partial\widetilde{p}^t_{ij}/\partial e_j < 0$, $\partial\widetilde{p}^t_{ij}/\partial\lambda_i > 0$, and $\partial^2\widetilde{p}^t_{ij}/\partial e_i^2 < 0$.

The solution concept is Subgame Perfect Nash Equilibrium. We solve the elimination contest via backwards induction and determine the equilibrium of the second-stage (the final) before we determine equilibrium in the first-stage (the semifinals). To be able to compute the equilibrium taking into account that players can hold mistaken beliefs we assume: (i) a player who faces a biased opponent is aware that the latter's perception (and probability of winning) is mistaken, (ii) each player thinks that his own perception (and probability of winning) is correct, and (iii) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players agree to disagree about their perceptions (and probabilities of winning). This approach follows Heifetz et al. (2007a, 2007b) for games with complete information, and Squintani (2006) for games with incomplete information.[4] Finally, we assume that each player not only knows the confidence level of his direct rival in the semifinal but also the confidence levels of the other two potential rivals in the other semifinal.[5]

In a final between i and j, player i chooses the level of effort e_i that maximizes his perceived expected utility:

$$\widetilde{E}^f[U_{ij}(e_i, e_j, \lambda_i)] = \widetilde{p}^f_{ij} u(w_1) + (1 - \widetilde{p}^f_{ij}) u(w_2) - ce_i = \widetilde{p}^f_{ij} \Delta u - ce_i + u(w_2),$$

where $\Delta u = u(w_1) - u(w_2)$ represents the utility prize spread. In a final between i and j, the first-order condition of player i is:

$$mg\widetilde{p}^f_{ij}(e_i, e_j, \lambda_i)\Delta u = c.$$

This equation tells us that in the final, the overconfident player chooses the level of effort at which the perceived marginal benefit of effort equals the marginal cost. Since

$$\frac{\partial mg\widetilde{p}^f_{ij}}{\partial e_i} = \frac{\partial^2 \widetilde{p}^f_{ij}}{\partial e_i^2} < 0,$$

we have

$$\frac{\partial^2 \widetilde{E}^f[U_{ij}(e_i, e_j, \lambda_i)]}{\partial e_i^2} < 0,$$

and the second-order condition is satisfied. The Nash equilibrium efforts in a final between players i and j, (e^f_i, e^f_j), are the solution to

$$mg\widetilde{p}^f_{ij}(e^f_i, e^f_j, \lambda_i)\Delta u = c,$$

and

$$mg\widetilde{p}^f_{ji}(e^f_i, e^f_j, \lambda_j)\Delta u = c.$$

Now consider the semifinals stage. Let players i and h be seeded in one semifinal and players j and k be seeded in the other semifinal. If i wins his semifinal, then i faces j in the final with probability p^s_{jk} and k with probability $p^s_{kj} = 1 - p^s_{jk}$. Hence, player i's perceived expected utility of reaching the final (or perceived continuation value), $\widetilde{v}_i$, is:

$$\widetilde{v}_i = p^s_{jk}\widetilde{E}^f[U_{ij}(e^f_i, e^f_j, \lambda_i)] + p^s_{kj}\widetilde{E}^f[U_{ik}(e^f_i, e^f_k, \lambda_i)].$$

[4] These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007).

[5] In Section 7 we discuss what happens when overconfidence is unobservable.

In the semifinal between i and h, player i chooses the level of effort e_i that maximizes his perceived expected utility:

$$\widetilde{E}^s[U_{ih}(e_i, e_h, \lambda_i)] = \widetilde{p}_{ih}^s \widetilde{v}_i - ce_i.$$

Hence, the first-order condition of player i in a semifinal against h is:

$$mg\widetilde{p}_{ih}^s(e_i, e_h, \lambda_i)\widetilde{v}_i = c.$$

Since

$$\frac{\partial mg\widetilde{p}_{ih}^s}{\partial e_i} = \frac{\partial^2 \widetilde{p}_{ih}^s}{\partial e_i^2} < 0,$$

we have

$$\frac{\partial^2 \widetilde{E}^s[U_{ih}(e_i, e_h, \lambda_i)]}{\partial e_i^2} < 0,$$

and the second-order condition is satisfied. The Nash equilibrium efforts in the semifinal between players i and h, (e_i^s, e_h^s), are the solution to

$$mg\widetilde{p}_{ih}^s(e_i^s, e_h^s, \lambda_i)\widetilde{v}_i = c, \tag{1}$$

and

$$mg\widetilde{p}_{hi}^s(e_i^s, e_h^s, \lambda_h)\widetilde{v}_h = c. \tag{2}$$

To ensure there exists a unique equilibrium in each stage stage $t \in \{s, f\}$ of the elimination contest we make the following additional assumption

$$\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i \partial e_j} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_i \partial e_j} < \frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i^2} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_j^2}.$$

4 Encouragement and Complacency Effects

This section reveals the two key effects that overconfidence has on an overconfident player's effort in the semifinal of an elimination contest. Differentiating (1) and (2) and solving for $\partial e_i^s / \partial \lambda_i$ we find how the player i's equilibrium effort changes with his bias

$$\frac{de_i^s}{d\lambda_i} = \frac{mg\widetilde{p}_{ih}^s}{\lambda_i} \frac{\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i} + \frac{\partial mg\widetilde{p}_{ih}^s}{\partial \lambda_i}\frac{\lambda_i}{mg\widetilde{p}_{ih}^s}}{-\frac{\partial mg\widetilde{p}_{ih}^s}{\partial e_i} + \frac{\partial mg\widetilde{p}_{ih}^s}{\partial e_h}\frac{\frac{\partial mg\widetilde{p}_{hi}^s}{\partial e_i}}{\frac{\partial mg\widetilde{p}_{hi}^s}{\partial e_h}}}, \tag{3}$$

where the sign of the denominator in equation (3) is positive.[6] Thus, whether overconfidence raises or lowers the equilibrium effort of the overconfident player in the semifinal

[6]In the Appendix we derive equation (3) and show that its denominator is positive.

depends on the signs and magnitudes of the two terms in the numerator in equation (3). The first term

$$\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i},$$

represents the elasticity of the perceived expected utility of reaching the final with respect to the bias, $\varepsilon_{\widetilde{v}_i,\lambda_i}$. If an increase in the bias raises the overconfident player's perceived expected utility of reaching the final, the first term is positive.

Meanwhile, the second term

$$\frac{\partial mg\widetilde{p}^s_{ih}}{\partial \lambda_i}\frac{\lambda_i}{mg\widetilde{p}^s_{ih}},$$

represents the elasticity of the perceived marginal probability of winning the semifinal with respect to the bias, $\varepsilon_{mg\widetilde{p}^s_{ih},\lambda_i}$. If an increase in the bias lowers the overconfident player's perceived marginal probability of winning the semifinal, the second term is negative.

Hence, we see from equation (3) that the bias can have two effects on the overconfident player's incentives to exert effort in a semifinal. On the one hand, an increase in the bias can increase the overconfident player's perceived expected utility of reaching the final which motivates him to raise effort. We label this the encouragement effect of overconfidence. On the other hand, an increase in the bias can reduce the overconfident player's perceived marginal probability of winning the semifinal which motivates him to lower effort. We label this the complacency effect of overconfidence.

Note that sign of the encouragement effect is positive when

$$\frac{\partial \widetilde{v}_i}{\partial \lambda_i} = p^s_{jk}\frac{\partial \widetilde{E}^f[U_{ij}(e^f_i, e^f_j, \lambda_i)]}{\partial \lambda_i} + (1 - p^s_{jk})\frac{\partial \widetilde{E}^f[U_{ik}(e^f_i, e^f_k, \lambda_i)]}{\partial \lambda_i} > 0,$$

where

$$\frac{\partial \widetilde{E}^f[U_{ij}(e^f_i, e^f_j, \lambda_i)]}{\partial \lambda_i} = \left(\frac{\partial \widetilde{p}^f_{ij}}{\partial \lambda_i} + \frac{\partial \widetilde{p}^f_{ij}}{\partial e_j}\frac{\partial e^f_j}{\partial \lambda_i}\right)\Delta u. \tag{4}$$

Since $\frac{\partial \widetilde{p}^f_{ij}}{\partial \lambda_i} > 0$ and $\frac{\partial \widetilde{p}^f_{ij}}{\partial e_j} < 0$, it follows from (4) that overconfidence generates an encouragement effect when an increase in the bias lowers a rival's equilibrium effort in the final. As we shall see, this will be the case when the players' winning probabilities are determined by either Tullock's CSF or Alcalde and Dahm's CSF.

5 Elimination Contest with Tullock's CSF

In this section, we solve the model using Tullock's CSF, a widely used framework that provides a natural starting point for examining the impact of overconfidence on an elimination contest. This setup also allows us to assess both the encouragement and complacency effects and identify the conditions under which each effect prevails.

Player i's actual winning probability when paired with j at stage $t \in \{s, f\}$ is

$$p_{ij}^t(e_i, e_j) = \begin{cases} \frac{q(e_i)}{q(e_i)+q(e_j)} & \text{if} \quad q(e_i) + q(e_j) > 0 \\ \frac{1}{2} & \text{if} \quad q(e_i) + q(e_j) = 0 \end{cases}$$

Following Santos-Pinto and Sekeris (2023), an overconfident player i's perceived probability of winning a rival j at stage $t \in \{s, f\}$ is

$$\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i) = \begin{cases} \frac{\lambda_i q(e_i)}{\lambda_i q(e_i)+q(e_j)} & \text{if} \quad \lambda_i q(e_i) + q(e_j) > 0 \\ \frac{1}{2} & \text{if} \quad \lambda_i q(e_i) + q(e_j) = 0 \end{cases}$$

where $\lambda_i \geqslant 1$, and $q(e)$ satisfies $q(0) \geqslant 0$, $q'(e) > 0$, and $q''(e) \leqslant 0$. The assumption $q''(e) \leqslant 0$ implies that each stage of the elimination contest has a unique pure strategy Nash equilibrium.

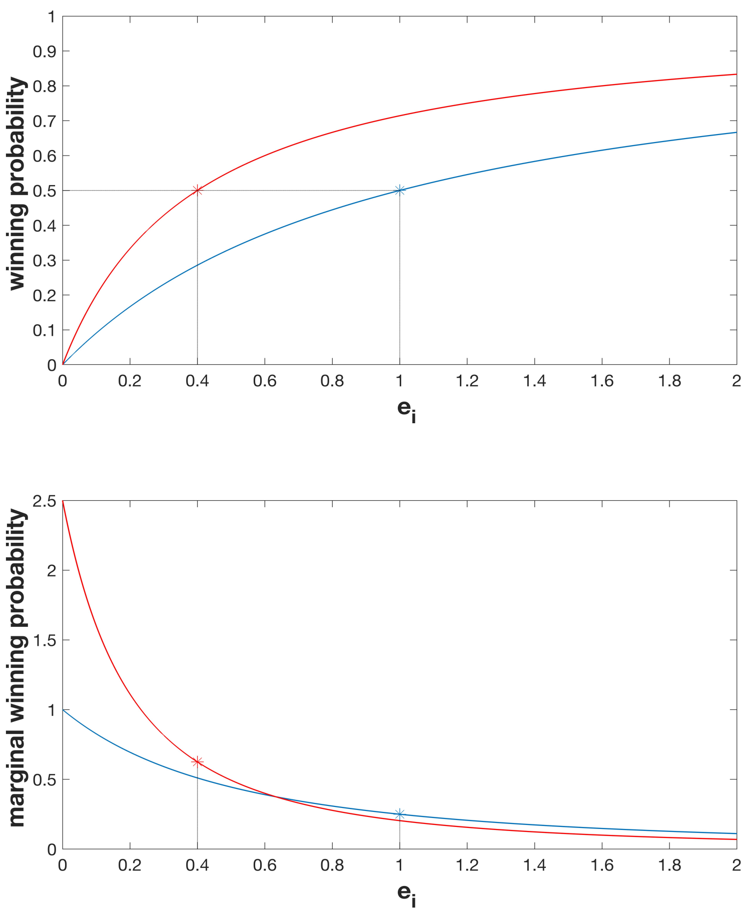

Figure 1: Actual and Perceived Winning Probabilities with Tullock's CSF

The top panel of Figure 1 depicts an overconfident player's actual (solid blue curve) and perceived (solid red curve) probabilities of winning when $\lambda_i = 2.5$, $q(e) = e$, and $e_j = 1$. Note that when the overconfident player's effort equals that of his rival, i.e., $e_i = e_j = 1$, his actual winning probability is $1/2$, but his perceived winning probability is greater than $1/2$. Additionally, we observe that the overconfident player mistakenly believes that if he chooses $e_i = 0.4$ while his rival chooses $e_j = 1$, his winning probability remains $1/2$. The bottom panel of Figure 1 depicts the overconfident player's actual (solid blue curve) and perceived (solid red curve) marginal probabilities of winning. We see that an overconfident player's perceived marginal probability of winning is higher (lower) than his actual marginal probability of winning when his effort is low (high).

We focus on an elimination contest involving one overconfident player and three rational players. The four players are labeled 1, 2, 3, and 4. Players 1 and 2 compete in one semifinal, while players 3 and 4 face off in the other. Throughout we assume player 1 is overconfident with $\lambda_1 > 1$ and players 2, 3, and 4 are rational with $\lambda_2 = \lambda_3 = \lambda_4 = 1$. We start by analyzing the impact of overconfidence on the final. Since players 3 and 4 are identical, we consider a final with between an overconfident player 1 and a rational player 3 without loss of generality. In the final, the overconfident player 1 chooses effort to maximize his perceived expected utility:

$$\widetilde{E}^f[U_{13}(e_1, e_3, \lambda_1)] = \frac{\lambda_1 q(e_1)}{\lambda_1 q(e_1) + q(e_3)} \Delta u - ce_1 + u(w_2).$$

The first-order condition to player 1's problem implicitly defines his best response in the final $e_1 = R_1^f(e_3)$:

$$\frac{\lambda_1 q'(e_1) q(e_3)}{[\lambda_1 q(e_1) + q(e_3)]^2} \Delta u = c, \tag{5}$$

The rational player 3 chooses effort to maximize her expected utility in the final:

$$E^f[U_{31}(e_1, e_3)] = \frac{q(e_3)}{q(e_1) + q(e_3)} \Delta u - ce_3 + u(w_2).$$

The first-order condition to player 3's problem implicitly defines her best response in the final $e_3 = R_3^f(e_1)$:

$$\frac{q'(e_3) q(e_1)}{[q(e_1) + q(e_3)]^2} \Delta u = c. \tag{6}$$

In a final between the overconfident player 1 and the rational player 3, the equilibrium efforts, (e_1^f, e_3^f), simultaneously satisfy equations (5) and (6):

$$\frac{\lambda_1 q'(e_1^f) q(e_3^f)}{[\lambda_1 q(e_1^f) + q(e_3^f)]^2} \Delta u = c, \tag{7}$$

and

$$\frac{q'(e_3^f)q(e_1^f)}{[q(e_1^f)+q(e_3^f)]^2}\Delta u = c. \tag{8}$$

It is not possible to solve equations (7) and (8) explicitly for the equilibrium efforts in the final. Nevertheless, Santos-Pinto and Sekeris (2023) show that in a Tullock contest between two overconfident players, the more overconfident player is the one who exerts lower effort. Here, we show that their results also apply to a Tullock contest between an overconfident player 1 and a rational player 3. Namely, the overconfident player 1 exerts less effort than the rational player 3.

To prove this result we follow the approach in Santos-Pinto and Sekeris (2023). We know from Lemma 1 in Santos-Pinto and Sekeris (2023) that player 1's best response in the final, $R_1^f(e_3)$, is concave in e_3 and reaches a maximum at $q(e_3) = \lambda_1 q(e_1)$. Moreover, the slope of the best response of player 1 is positive for $\lambda_1 q(e_1) > q(e_3)$, zero for $\lambda_1 q(e_1) = q(e_3)$, and negative for $\lambda_1 q(e_1) < q(e_3)$. This implies that $R_1^f(e_3)$ increases in e_3 for $\lambda_1 q(e_1) > q(e_3)$, reaches the maximum at $\lambda_1 q(e_1) = q(e_3)$, and decreases in e_3 for $\lambda_1 q(e_1) < q(e_3)$. We now show, using the same approach as Proposition 1 in Santos-Pinto and Sekeris (2023), that player 1's best response in the final crosses the 45 degree line at a lower value of effort than player 3's best response in the final. At the 45 degree line, player 1's best response in the final takes the value e_L given by

$$\frac{\lambda_1 q'(e_L)}{(1+\lambda_1)^2 q(e_L)}\Delta u = c. \tag{9}$$

At 45 degree line, player 3's best response in the final takes the value e_H given by

$$\frac{q'(e_H)}{4q(e_H)}\Delta u = c. \tag{10}$$

Note that $\lambda_1 > 1$ implies

$$\frac{\lambda_1}{(1+\lambda_1)^2} < \frac{1}{4}, \tag{11}$$

Equations (9), (10), and inequality (11) imply

$$\frac{q'(e_H)}{q(e_H)} < \frac{q'(e_L)}{q(e_L)}.$$

Given that $q(.)$ is (weakly) concave, this inequality can only be satisfied provided $e_L < e_H$. This and the shape of the players' best responses imply the equilibrium in the final lies above the 45 degree line. Hence, the overconfident player 1 exerts less effort than the rational player 3 in the final. Intuitively, the overconfident player, given his (mis)perceived advantage, thinks, mistakenly, he can reduce his effort without endangering his prospects of success.[7]

The rational player, anticipating the overconfident player will lower his effort, also lowers her effort but not as much. Hence, both players exert lower effort than if both were rational. At equilibrium, the overconfident player's perceived probability of winning the final is greater than 1/2 whereas his actual winning probability is less than 1/2 given the lower equilibrium effort:

$$\widetilde{p}_{13}^{f} = \frac{\lambda_1 q(e_1^f)}{\lambda_1 q(e_1^f) + q(e_3^f)} > \frac{1}{2} > \frac{q(e_1^f)}{q(e_1^f) + q(e_3^f)} = p_{13}^{f},$$

where the first inequality follows from the slope of the overconfident player 1's best response in the final being positive at equilibrium, i.e., $\lambda_1 q(e_1^f) > q(e_3^f)$.

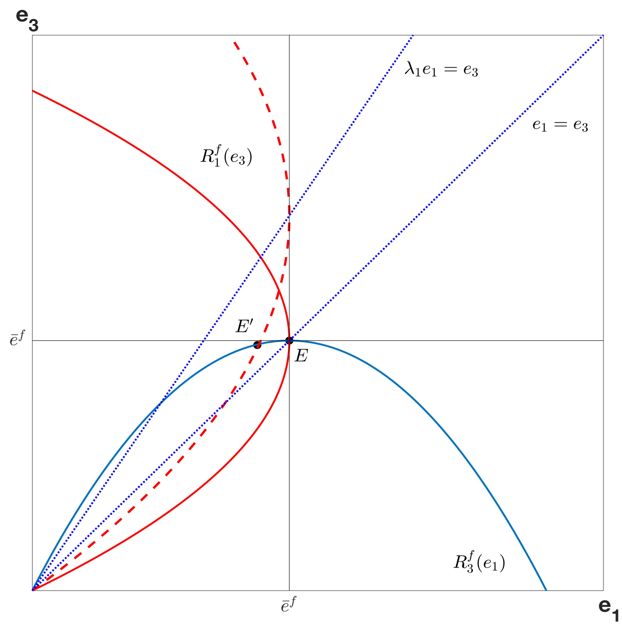

Figure 2: Equilibrium Efforts in the Final of an Elimination Contest with Tullock's CSF

Figure 2 depicts the best responses and equilibrium efforts in the final of an elimination contest with Tullock's CSF where $q(e) = e$, $\Delta u/c = 9$, and $\lambda_1 = 1.5$. The best response of a rational player 1 is depicted in solid red and that of a rational player 3 in solid blue. Point E at the 45 degree line depicts the equilibrium when players 1 and 3 are rational. The best response of an overconfident player 1 is depicted in dashed red. Point E' above

[7]Note that as player 1's overconfidence increases, his best response, $R_1^f(e_3)$, shifts inwards for low values of e_3. Since player 3's best response, $R_3^f(e_1)$, is positively sloped for low values of e_3 and is unaffected by changes in player 1's bias, both players' equilibrium efforts in the final decrease in the bias of player 1.

the 45 degree line depicts the equilibrium when player 1 is overconfident and player 3 is rational.

We now turn our attention to the semifinals. Consider first the semifinal between rational players 3 and 4. Since they are identical in every respect, they will exert the same effort at equilibrium, $e_3^s = e_4^s$, resulting in equal winning probabilities, $p_{34}^s = p_{43}^s = 1/2$. Moreover, both players are incentivized to exert more effort than if all contestants were rational. This is because the winner of the semifinal has a chance of facing the overconfident player 1 in the final, which increases the expected utility of reaching the final for players 3 and 4. Thus, the presence of an overconfident player in the elimination contest creates a spillover effect, increasing the equilibrium efforts of the rational players seeded in the same semifinal.

Consider now the semifinal between the overconfident player 1 and the rational player 2. The overconfident player 1 chooses effort to maximize his perceived expected utility in the semifinal:

$$\widetilde{E}^s[U_{12}(e_1, e_2, \lambda_1)] = \frac{\lambda_1 q(e_1)}{\lambda_1 q(e_1) + q(e_2)} \widetilde{v}_1 - ce_1,$$

where

$$\widetilde{v}_1 = \widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)] = \frac{\lambda_1 q(e_1^f)}{\lambda_1 q(e_1^f) + q(e_3^f)} \Delta u - ce_1^f + u(w_2). \tag{12}$$

Note that the first equality in (12) results from the symmetric equilibrium in the semifinal between rational players 3 and 4. From (12), we have

$$\begin{aligned}\frac{\partial \widetilde{v}_1}{\partial \lambda_1} &= \left[\frac{q(e_1^f) q(e_3^f)}{[\lambda_1 q(e_1^f) + q(e_3^f)]^2} + \frac{\lambda_1 q'(e_1^f) q(e_3^f)}{[\lambda_1 q(e_1^f) + q(e_3^f)]^2} \frac{\partial e_1^f}{\partial \lambda_1} - \frac{\lambda_1 q(e_1^f) q'(e_3^f)}{[\lambda_1 q(e_1^f) + q(e_3^f)]^2} \frac{\partial e_3^f}{\partial \lambda_1} \right] \Delta u - c \frac{\partial e_1^f}{\partial \lambda_1} \\ &= \frac{q(e_1^f)}{[\lambda_1 q(e_1^f) + q(e_3^f)]^2} \left[q(e_3^f) - \lambda_1 q'(e_3^f) \frac{\partial R_3^f(e_1^f)}{\partial e_1} \frac{\partial e_1^f}{\partial \lambda_1} \right] \Delta u > 0\end{aligned}$$

where the second equality follows from the Envelope Theorem and $e_3^f = R_3^f(e_1^f)$. Observe that $\partial R_3^f(e_1^f)/\partial e_1$ is positive and $\partial e_1^f/\partial \lambda_1$ is negative. Hence, overconfidence generates an encouragement effect in an elimination contest with Tullock's CSF.

The rational player 2 chooses effort to maximize her expected utility in the semifinal:

$$E^s[U_{21}(e_1, e_2)] = \frac{q(e_2)}{q(e_1) + q(e_2)} v_2 - ce_2,$$

where

$$v_2 = \frac{q(e_2^f)}{q(e_2^f) + q(e_3^f)} \Delta u - ce_2^f + u(w_2) = \frac{\Delta u}{2} - ce_2^f + u(w_2). \tag{13}$$

The first equality in (13) arises from the symmetric equilibrium in the semifinal between rational players 3 and 4, while the second equality reflects the symmetric equilibrium of a final between rational player 2 and 3.

The first-order condition to player 1's problem implicitly defines his best response in the semifinal $e_1 = R_1^s(e_2)$:

$$\frac{\lambda_1 q'(e_1)q(e_2)}{[\lambda_1 q(e_1) + q(e_2)]^2}\widetilde{v}_1 = c, \tag{14}$$

Similarly, the first-order condition to player 2's problem implicitly defines her best response in the semifinal $e_2 = R_2^s(e_1)$:

$$\frac{q'(e_2)q(e_1)}{[q(e_1) + q(e_2)]^2}v_2 = c, \tag{15}$$

Note that the shape of the overconfident player's best response in the semifinal closely mirrors the shape of his best in the final as can be seen by comparing (5) to (14). However, a change in overconfidence shifts the overconfident player's best response in the semifinal differently than it does in the final. To see this we introduce the following lemma.

Lemma 1 *Consider an elimination contest with Tullock's CSF. An increase in player i's overconfidence leads to a contraction of his best response in the semifinal,* $\partial R_i^s(e_h)/\partial\lambda_i < 0$*, for* $q(e_h) < \lambda_i q(e_i)$ *and* $\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i} < \frac{\lambda_i q(e_i) - q(e_h)}{\lambda_i q(e_i) + q(e_h)}$*, otherwise, it leads to an expansion of his best response in the semifinal,* $\partial R_i^s(e_h)/\partial\lambda_i > 0$*. Moreover, the maximum value of player i's best response in the semifinal increases in player i's overconfidence.*

Lemma 1 characterizes how overconfidence shifts a player's best response in the semifinal. An increase in confidence contracts player i's best response in the semifinal when the rival exerts low effort and the encouragement effect is small, otherwise, an increase in confidence expands player i's best response in the semifinal. Moreover, the maximal value taken by player i's best response in the semifinal increases in his overconfidence bias. This result stands in contrast to Lemma 2 in Santos-Pinto and Sekeris (2023) which shows that the maximal value taken by player i's best response in the final does not depend on his overconfidence bias.

Lemma 2 *The semifinal between an overconfident player 1 and a rational player 2 of an elimination contest with Tullock's CSF, one overconfident player, and four rational players, admits a unique equilibrium.*

Having established uniqueness, we now turn to the equilibrium efforts. In the semifinal between the overconfident player 1 and the rational player 2, the equilibrium efforts, (e_1^s, e_2^s), simultaneously satisfy equations (14) and (15):

$$\frac{\lambda_1 q'(e_1^s)q(e_2^s)}{[\lambda_1 q(e_1^s) + q(e_2^s)]^2}\widetilde{v}_1 = c,$$

and

$$\frac{q'(e_2^s)q(e_1^s)}{[q(e_1^s) + q(e_2^s)]^2}v_2 = c.$$

Proposition 1 *Consider the semifinal between an overconfident player 1 and a rational player 2 of an elimination contest with Tullock's CSF, one overconfident player, and three rational players. If*

$$\frac{\widetilde{v}_1}{v_2} > \frac{(1+\lambda_1)^2}{4\lambda_1}, \tag{16}$$

then the equilibrium efforts and winning probabilities in the semifinal satisfy $e_1^s > e_2^s$ *and* $\widetilde{p}_{12}^s > p_{12}^s > 1/2 > p_{21}^s$.

Proposition 1 shows that an overconfident player can exert higher effort than his rational rival in the semifinal of an elimination contest with Tullock's CSF. As we have seen, overconfidence generates an encouragement effect which incentivizes an overconfident player to increase effort in the semifinal. However, overconfidence also generates a complacency effect which leads an overconfident player to lower effort in the semifinal. The complacency effect is given by

$$\frac{\partial mg\widetilde{p}_{12}^s}{\partial \lambda_1}\frac{\lambda_1}{mg\widetilde{p}_{12}^s} = -q(e_2^s)q'(e_1^s)\frac{\lambda_1 q(e_1^s) - q(e_2^s)}{(\lambda_1 q(e_1^s) + q(e_2^s))^3}\frac{\lambda_1}{\frac{\lambda_1 q(e_2^s)q'(e_1^s)}{(\lambda_1 q(e_1^s)+q(e_2^s))^2}} = -\frac{\lambda_1 q(e_1^s) - q(e_2^s)}{\lambda_1 q(e_1^s) + q(e_2^s)}. \tag{17}$$

In Lemma 2, we demonstrate that at equilibrium, the slope of the overconfident player's best response in the semifinal is positive, which is equivalent to $\lambda_1 q(e_1^s) > q(e_2^s)$. From (17), this implies that overconfidence generates a complacency effect in an elimination contest with Tullock's CSF. Hence, the net effect of overconfidence on the equilibrium effort of the overconfident player in the semifinal depends on the sizes of the encouragement and complacency effects. When inequality (16) holds, the encouragement effect dominates and the overconfident player exerts more effort in the semifinal than his rational rival.[8] Note that inequality (16) is satisfied for values of λ_1 close to 1, i.e., when the

[8] Conversely, when inequality (16) is violated, the overconfident player exerts less effort in the semifinal than his rational rival.

overconfident player's bias is relatively small.[9]

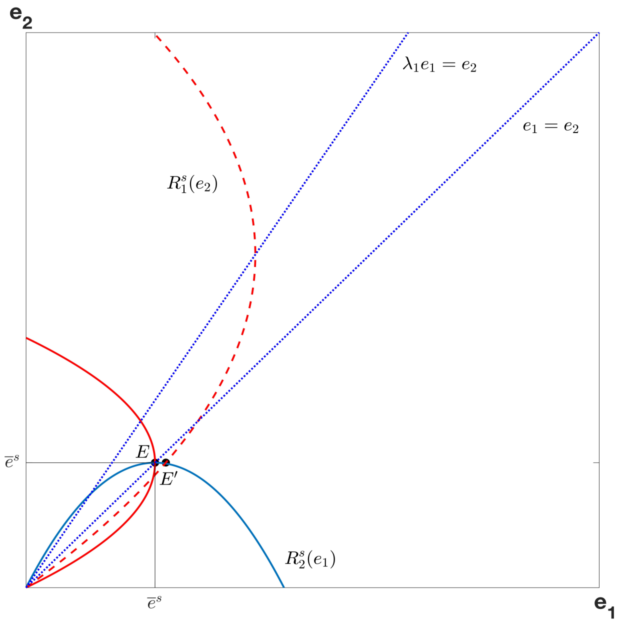

Figure 3: Equilibrium Efforts in Semifinal of an Elimination Contest with Tullock's CSF and $\frac{\widetilde{v}_1}{v_2} > \frac{(1+\lambda_1)^2}{4\lambda_1}$

Figure 3 illustrates Proposition 1. It depicts the best responses and equilibrium efforts in a semifinal of an elimination contest with Tullock's CSF where $q(e) = e$, $\Delta u = 9$, $u(w_2) = 9/4$, $c = 1$, and $\lambda_1 = 1.5$. These parameters imply $\widetilde{v}_1 = 5.4$ and $v_2 = 4.5$ and satisfy inequality (16). The best response of a rational player 1 is depicted in solid red and that of a rational player 2 in solid blue. Point E at the 45 degree line depicts the equilibrium when players 1 and 2 are rational. The best response of an overconfident player 1 is depicted in dashed red. Point E' below the 45 degree line depicts the equilibrium when player 1 is overconfident and player 2 is rational.

When the overconfident player exerts more effort than his rational rival in the semifinal, his probability of winning the semifinal is greater than $1/2$. If the increase in his probability of winning the semifinal is substantial enough, it may offset the reduction in his chances of winning the final. As a result, it is possible for the overconfident player to have the highest overall chances of winning the elimination contest. However, deriving closed-form solutions for equilibrium efforts and winning probabilities under Tullock's CSF is not feasible, limiting our ability to fully understand how overconfidence influences the players' equilibrium probabilities of winning the elimination contest.

[9]The derivative of the left-hand side of (16) is positive at $\lambda = 1$ since $\partial\widetilde{v}_1/\partial\lambda_1 > 0$ whereas the derivative of the right-hand side of (16) is zero at $\lambda_1 = 1$. Moreover, the right-hand side of (16) attains a minimum at $\lambda_1 = 1$.

6 Elimination Contest with Alcalde and Dahm's (2007) CSF

In this section, we solve the elimination contest using Alcalde and Dahm's (2007) CSF. This enables us to derive closed-form solutions for the equilibrium efforts and winning probabilities, allowing us not only to assess how overconfidence affects the players' chances of winning the elimination contest but also its impact on welfare.

Player i's actual winning probability when paired with j at stage $t \in \{s, f\}$ is

$$p_{ij}^t(e_i, e_j) = \begin{cases} 1 - \frac{1}{2}(\frac{e_j}{e_i})^\alpha & \text{if} \quad e_i \geqslant e_j \\ \frac{1}{2}(\frac{e_i}{e_j})^\alpha & \text{if} \quad e_i \leqslant e_j \end{cases}$$

and player i's perceived winning probability when paired with j at stage $t \in \{s, f\}$ is

$$\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i) = \begin{cases} 1 - \frac{1}{2}\frac{e_j^\alpha}{\lambda_i e_i^\alpha} & \text{if} \quad \lambda_i e_i^\alpha \geqslant e_j^\alpha \\ \frac{1}{2}\frac{\lambda_i e_i^\alpha}{e_j^\alpha} & \text{if} \quad \lambda_i e_i^\alpha \leqslant e_j^\alpha \end{cases} \tag{18}$$

Note that the parameter α determines how sensitive Alcalde and Dahm's CSF is to effort. When $\alpha = 0$ the CSF is completely insensitive to effort and we obtain the extreme case of a (fair) lottery. As α increases, the CSF becomes more sensitive to effort, and the contest becomes more deterministic until the extreme case of an all-pay auction is reached when $\alpha \to \infty$. We assume $0 < \alpha \leq 1$ which implies that each stage of the elimination contest has a unique pure strategy Nash equilibrium.

The top panel of Figure 4 depicts the actual (solid blue curve) and the perceived (solid red curve) probabilities of winning of an overconfident player with $\lambda_i = 1.5$ when $\alpha = 0.9$ and $e_j = 1$. The bottom panel of Figure 4 represents the overconfident player's actual (solid blue curve) and perceived (solid red curve) marginal probabilities of winning. Figures 1 and 4 demonstrate that the qualitative effects of overconfidence on a player's perceived probability of winning as well as on his perceived marginal probability of winning are similar under both Tullock's CSF and Alcalde and Dahm's CSF.

We now introduce two lemmas that will be useful to understand the impact of overconfidence on the final stage of an elimination contest with Alcalde and Dahm's (2007) CSF. Lemma 3 describes the shape of the players' best responses in the final, and Lemma 4 describes how an overconfident player's best response in the final changes with his bias.

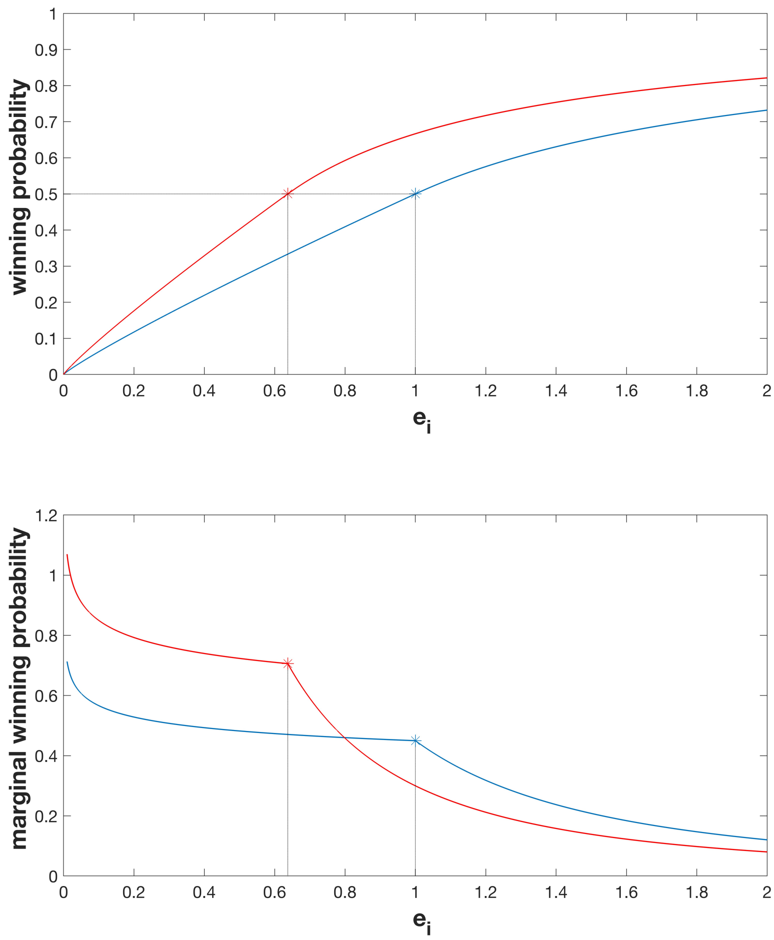

Figure 4: Actual and Perceived Winning Probabilities with Alcalde and Dahm's CSF

Lemma 3 *$R_i^f(e_j)$ is quasi-concave in e_j and reaches a maximum at $\lambda_i e_i^\alpha = e_j^\alpha$.*

Lemma 3 tells us that the players' best responses in the final are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

Lemma 4 *An increase in player i's overconfidence λ_i leads to a contraction of his best response in the final, $\partial R_i^f(e_j)/\partial \lambda_i \leqslant 0$, for $e_j^\alpha \leqslant \lambda_i e_i^\alpha$, and to an expansion of his best response in the final, $\partial R_i^f(e_j)/\partial \lambda_i \geqslant 0$, for $e_j^\alpha \geqslant \lambda_i e_i^\alpha$. Moreover, the maximum value of player i's best response in the final is independent of player i's overconfidence.*

Lemma 4 describes how overconfidence shifts a player's best response in the final. For a high effort of the rival, an increase in confidence raises player i's effort level, while for low effort of the rival, an increase in confidence lowers player i's effort level. Moreover, the maximal value taken by player i's best response in the final is independent of his overconfidence bias.

6.1 Benchmark

This section characterizes the subgame perfect equilibrium of an elimination contest with Alcalde and Dahm's (2007) CSF and four rational players. This serves as a benchmark to which we compare our subsequent results.

Proposition 2 *Consider an elimination contest with Alcalde and Dahm's CSF and four rational players. The equilibrium effort in the final is*

$$\overline{e}^f = \frac{\alpha}{2c}\Delta u,$$

the equilibrium winning probability is

$$\overline{p}^f = \frac{1}{2},$$

and the equilibrium expected utility is

$$\overline{E}^f[U(\overline{e}^f)] = \frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2).$$

The equilibrium effort in the semifinals is

$$\overline{e}^s = \frac{\alpha}{2c}\left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right],$$

the equilibrium winning probability is

$$\overline{p}^s = \frac{1}{2},$$

and the equilibrium expected utility is

$$\overline{E}^s[U(\overline{e}^s, \overline{e}^f)] = \frac{1-\alpha}{2}\left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right].$$

In a final featuring two rational players, equilibrium is reached when both exert the same effort, resulting in an equal winning probabilities. The equilibrium effort in the final increases in the utility prize spread, Δu, in the role that effort plays in determining the winner of the final, α, and decreases in the marginal cost of effort, c. Similarly, in a semifinal featuring two rational players, equilibrium is reached when both exert the same effort, resulting in an equal winning probabilities. The equilibrium effort in the semifinal is smaller than the equilibrium effort in the final for all $\alpha \in (0, 1]$ when $u(w_1) > 3u(w_2)$. Finally, we have $\overline{P} = \overline{p}^s \overline{p}^f = 1/4$. When all players are rational the elimination contest is symmetric and hence each has $1/4$ probability of being the winner.

6.2 Equilibrium Efforts

This section characterizes the subgame perfect equilibrium of an elimination contest with Alcalde and Dahm's (2007) CSF, one overconfident player, and three rational players.

Proposition 3 *Consider an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players. In a final between an overconfident player 1 and a rational player 3, the equilibrium effort of the overconfident player 1 is*

$$e_1^f = \frac{\alpha}{2c} \lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \Delta u,$$

and the equilibrium effort of the rational player 3 is

$$e_3^f = \frac{\alpha}{2c} \lambda_1^{-\frac{\alpha}{2\alpha+1}} \Delta u$$

with $e_1^f < e_3^f < \overline{e}^f$. *The perceived equilibrium winning probability of the overconfident player 1 is*

$$\widetilde{p}_{13}^f = 1 - \frac{1}{2} \lambda_1^{-\frac{\alpha+1}{2\alpha+1}},$$

and the actual equilibrium winning probabilities are

$$p_{13}^f = \frac{1}{2} \lambda_1^{-\frac{\alpha}{2\alpha+1}}$$

$$p_{31}^f = 1 - \frac{1}{2} \lambda_1^{-\frac{\alpha}{2\alpha+1}}$$

with $\widetilde{p}_{13}^f > p_{31}^f > 1/2 > p_{13}^f$. *The perceived equilibrium expected utility of the overconfident player 1 is*

$$\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)] = u(w_1) - \frac{1+\alpha}{2} \lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \Delta u$$

and the equilibrium expected utility of the rational player 3 is

$$E^f[U_{31}(e_1^f, e_3^f)] = u(w_1) - \frac{1+\alpha}{2} \lambda_1^{-\frac{\alpha}{2\alpha+1}} \Delta u,$$

with $\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)] > E^f[U_{31}(e_1^f, e_3^f)] > \overline{E}^f[U(\overline{e}^f)]$.

Proposition 3 demonstrates that overconfidence impacts equilibrium efforts in the final of an elimination contest similarly under both Alcalde and Dahm's CSF and Tullock's CSF. However, the tractability of Alcalde and Dahm's CSF allows to obtain two new results. First, the overconfident player's equilibrium perceived expected utility of the final, $\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)]$, increases in his bias. An increase in the bias raises the overconfident player's perceived probability of winning the final and lowers his cost of effort. Second, the rational player's equilibrium expected utility of the final, $E^f[U_{31}(e_1^f, e_3^f)]$, increases in the overconfident player's bias. An increase in the bias raises the rational player's probability of winning the final and lowers her cost of effort. Hence, the bias makes reaching the final more attractive not only to the overconfident player but also to a rational player. Note also that, at equilibrium, the overconfident player's perceived expected utility of reaching the final is greater than that of the rational player.[10]

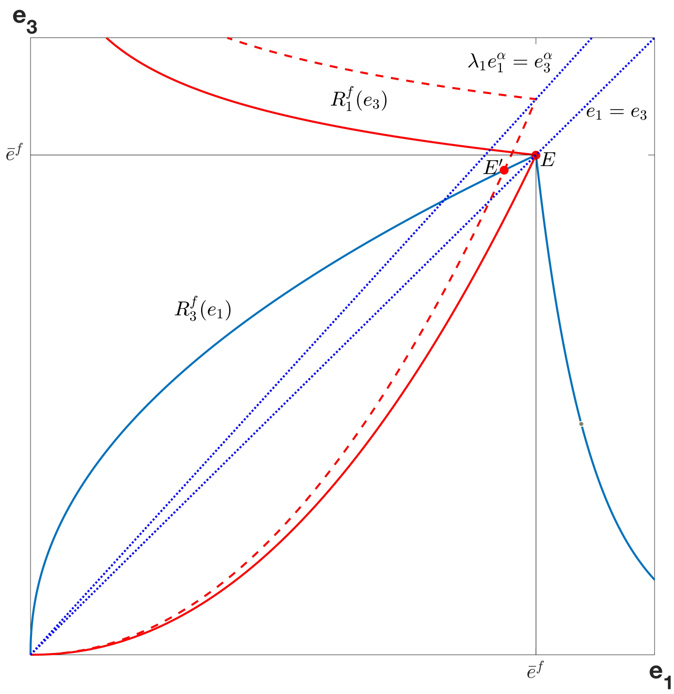

Figure 5: Equilibrium Efforts in the Final of an Elimination Contest with Alcalde and Dahm's CSF

Figure 5 illustrates Proposition 3. It depicts the best responses and equilibrium efforts in a final where $\alpha = 0.9$, $\Delta u/c = 9$, and $\lambda_1 = 1.1$. The equilibrium when players 1 and 3

[10]As the overconfident player's bias converges to infinity, the efforts of both players converge to zero, the overconfident player's perceived probability of winning the final converges to 1, his actual probability of winning the final converges to zero, his perceived expected utility of reaching the final converges to the utility of the winner's prize $u(w_1)$ and so does the rational player's expected utility of the final.

are rational is depicted by point E at the 45 degree line. The equilibrium when player 1 is overconfident and player 3 is rational is depicted by point E' above the 45 degree line.

We now move on to the semifinals. Note that, since players 3 and 4 are identical, player 1's perceived expected utility of reaching the final is:

$$\widetilde{v}_1 = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u. \tag{19}$$

It follows from (19) that

$$\frac{\partial \widetilde{v}_1}{\partial \lambda_1} = \frac{\alpha(1+\alpha)}{2(2\alpha+1)}\lambda_1^{-\frac{3\alpha+1}{2\alpha+1}}\Delta u > 0 \tag{20}$$

Hence, overconfidence also generates an encouragement effect with Alcalde and Dahm's (2007) CSF. Moreover, the encouragement effect is given by:

$$\frac{\partial \widetilde{v}_1}{\partial \lambda_1}\frac{\lambda_1}{\widetilde{v}_1} = \frac{(\alpha+1)^2}{2(2\alpha+1)}\frac{1}{\frac{u(w_1)}{u(w_1)-u(w_2)}\lambda_1^{\frac{\alpha+1}{2\alpha+1}} - \frac{1+\alpha}{2}}. \tag{21}$$

Equation (21) shows that the size of the encouragement effect decreases in the bias and converges to zero when the bias converges to infinity (as $\lambda_1 \to \infty$ we have $\widetilde{v}_1 \to u(w_1)$). We now introduce a lemma that characterizes how overconfidence shifts a player's best response in the semifinal.

Lemma 5 *An increase in player i's overconfidence leads to a contraction of his best response in the semifinal, $\partial R_i^s(e_h)/\partial \lambda_i < 0$, for $e_h^\alpha < \lambda_i e_i^\alpha$ and $\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i} < 1$, otherwise, it leads to an expansion of his best response in the semifinal, $\partial R_i^s(e_h)/\partial \lambda_i > 0$. Moreover, the maximum value of player i's best response in the semifinal increases in player i's overconfidence.*

Lemma 5 shows that an increase in confidence contracts player i's best response in the semifinal when the rival exerts low effort and the encouragement effect is less than 1, otherwise, an increase in confidence expands player i's best response in the semifinal. Moreover, the maximal value taken by player i's best response in the semifinal is increasing in his overconfidence bias.

We now characterize the equilibrium of the semifinal where the overconfident player 1 competes against the rational player 2.

Proposition 4 *Consider the semifinal between an overconfident player 1 and a rational player 2 of an elimination contest with Alcalde and Dahm's CSF, one overconfident*

player, and three rational players.
(i) If $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ *and* $\lambda_1 < \hat{\lambda}$ *where* $\hat{\lambda}$ *is given by* $\frac{1+\alpha}{2}\frac{u(w_1)-u(w_2)}{u(w_1)} = \frac{\hat{\lambda}-1}{\hat{\lambda}-\hat{\lambda}^{-\frac{\alpha+1}{2\alpha+1}}}$, *then the equilibrium efforts and winning probabilities satisfy* $e_1^s > \overline{e}^s > e_2^s$ *and* $\widetilde{p}_{12}^s > p_{12}^s > 1/2 > p_{21}^s$.
(ii) If either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ *or* $\lambda_1 \geqslant \hat{\lambda}$, *then the equilibrium efforts and winning probabilities satisfy* $e_1^s \leqslant e_2^s \leqslant \overline{e}^s$ *and* $\widetilde{p}_{12}^s > p_{21}^s \geqslant \frac{1}{2} \geqslant p_{12}^s$.

Proposition 4 shows that an overconfident player exerts higher effort in a semifinal than a rational rival when the prize spread is large enough, $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$, and overconfidence is not too extreme, $\lambda_1 < \hat{\lambda}$. When either of these two conditions is not met, the overconfident player exerts less effort in the semifinal than a rational rival.

From (18), player i's perceived marginal probability of winning is

$$mg\widetilde{p}_{ij}(e_i, e_j, \lambda_i) = \begin{cases} \frac{\alpha}{2\lambda_i}\frac{e_j^\alpha}{e_i^{\alpha+1}} & \text{if} \quad \lambda_i e_i^\alpha \geqslant e_j^\alpha \\ \frac{\alpha}{2}\lambda_i \frac{e_i^{\alpha-1}}{e_j^\alpha} & \text{if} \quad \lambda_i e_i^\alpha \leqslant e_j^\alpha \end{cases} \tag{22}$$

The proof of Proposition 4 shows that at equilibrium $\lambda_1 e_1^\alpha \geqslant e_3^\alpha$ which, together with (22), implies that the complacency effect is

$$\frac{\partial mg\widetilde{p}_{12}^s}{\partial \lambda_1}\frac{\lambda_1}{mg\widetilde{p}_{12}^s} = -\frac{\alpha}{2\lambda_1^2}\frac{e_2^\alpha}{e_1^{\alpha+1}}\frac{\lambda_1}{\frac{\alpha}{2\lambda_1}\frac{e_2^\alpha}{e_1^{\alpha+1}}} = -1. \tag{23}$$

It follows from equations (21) and (23) that when the bias is relatively small, i.e., λ_1 is close to 1, a necessary condition for the encouragement effect to dominate is that it is greater than 1 when $\lambda_1 = 1$. This is equivalent to $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$. Thus, the encouragement effect dominates when the prize spread is large enough and the bias is close to 1. As the bias increases, the size of the encouragement effect decreases and converges to zero while the size of the complacency effect is fixed at -1. Hence, there exists an upper bound for the bias above which the complacency effect dominates.

Figure 6 illustrates the first part of Proposition 4. It depicts the best responses and equilibrium efforts in a semifinal of an elimination contest where $\alpha = 0.9$, $\Delta u = 9$, $u(w_2) = 9/4$, $c = 1$, and $\lambda_1 = 1.1$. These parameters satisfy $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda} = 2.436$. Point E at the 45 degree line depicts the equilibrium when players 1 and 2 are rational. Point E' below the 45 degree line depicts the equilibrium when player 1 is overconfident with and player 2 is rational. As the encouragement effect dominates, the overconfidence player exerts higher effort than his rational rival in the semifinal.

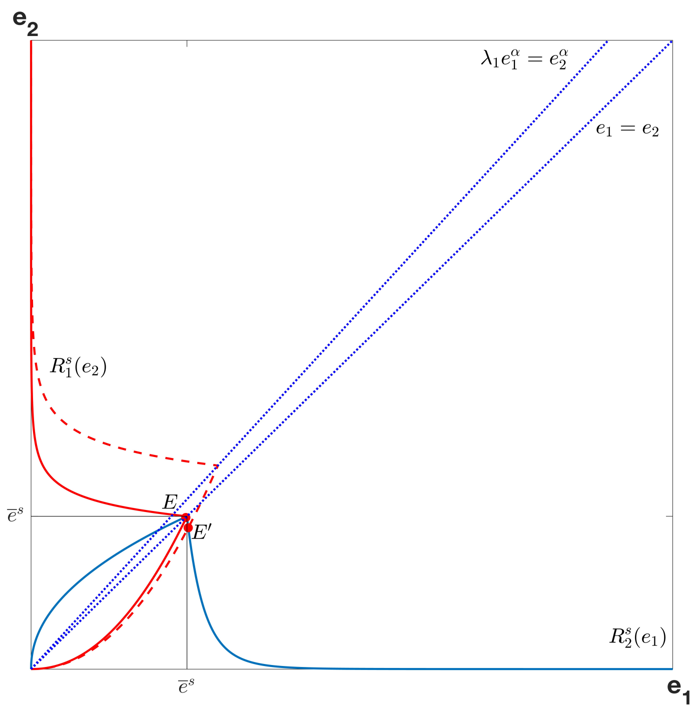

Figure 6: Equilibrium Efforts in Semifinal of an Elimination Contest with Alcalde and Dahm's CSF where $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

Next, we characterize the equilibrium of the semifinal with rational players 3 and 4.

Proposition 5 *In the semifinal between rational players 3 and 4 of an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players, the equilibrium efforts and winning probabilities satisfy* $e_3^s = e_4^s > \overline{e}^s$ *and* $p_{34}^s = p_{43}^s = 1/2$.

Proposition 5 demonstrates that the presence of an overconfident player in an elimination contest has a spillover effect on the equilibrium efforts in the semifinal with two rational players. In that semifinal, both players have an incentive to exert higher effort when player 1 is overconfident than if player 1 were rational. This occurs because the winner of the semifinal with two rational players will face the overconfident player 1 in the final with probability $p_{12}^s > 0$ and this leads to a higher expected utility of reaching the final than if player 1 were rational.

6.3 Equilibrium Winning Probabilities

This section focuses on how overconfidence affects the players' chances of winning the elimination contest. We are particularly interested in knowing if an overconfident player can be the one with the highest chances of winning an elimination contest.

We denote by P_i player i's equilibrium probability of winning the elimination contest. P_i is the product of player i's equilibrium probability of winning his semifinal and his equilibrium probability of winning the final:

$$P_i = p^s_{ih}(p^s_{jk}p^f_{ij} + p^s_{kj}p^f_{ik}).$$

Hence, player 1's equilibrium probability of winning the elimination contest is

$$P_1 = p^s_{12}(p^s_{34}p^f_{13} + p^s_{43}p^f_{14}) = p^s_{12}p^f_{13},$$

where the second equality follows from $p^f_{13} = p^f_{14}$. Player 2's equilibrium probability of winning the elimination contest is

$$P_2 = p^s_{21}(p^s_{34}p^f_{23} + p^s_{43}p^f_{24}) = p^s_{21}\frac{1}{2},$$

where the second equality follow from $p^s_{34} + p^s_{43} = 1$ and $p^f_{23} = p^f_{24} = 1/2$. Players 3 and 4 equilibrium probability of winning the elimination contest is

$$P_3 = P_4 = p^s_{34}(p^s_{12}p^f_{31} + p^s_{21}p^f_{32}) = \frac{1}{2}\left[p^s_{12}(1 - p^f_{13}) + (1 - p^s_{12})\frac{1}{2}\right].$$

Since the overconfident player 1 has an equilibrium probability of winning the final p^f_{13} that is less than 1/2, a necessary condition for him to have the highest equilibrium probability of winning the elimination contest is that his equilibrium probability of winning the semifinal p^s_{12} is greater than 1/2. In other words, the overconfident player must exert higher effort in his semifinal than the rational player 2. When this is the case, rational player 2 has an equilibrium probability of winning which is less than 1/4 since $p^s_{21} = 1 - p^s_{12} < 1/2$. Rational players 3 and 4 have an equilibrium winning probability greater than 1/4 since they have a positive probability of facing the overconfident player 1 in the final. Hence, in equilibrium, the overconfident player 1 has the highest winning probability when $P_1 > P_3 = P_4$ which is equivalent to $6p^s_{12}p^f_{13} - p^s_{12} - 1 > 0$.

Proposition 6 *In an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players, if* $\alpha > \frac{\sqrt{97}-5}{12}$ *and* $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$, *then* *there exist* $\lambda_1 \in (1, \hat{\lambda})$ *for which the overconfident player has the highest equilibrium probability of winning the elimination contest, i.e.,* $P_1 > P_3 = P_4 > 1/4 > P_2$.

Proposition 6 shows that an overconfident player can have the highest equilibrium probability of winning an elimination contest. Moreover, for this to be the case three conditions need to be met. First, the CSF's effort sensitivity parameter α must be greater than $\frac{\sqrt{97}-5}{12} \approx 0.404$. Hence, the role that effort plays in determining the winning probabilities must be sufficiently high. Second, the utility prize spread needs to be large enough, $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$. Third, the overconfidence bias must be relatively low.

6.4 Welfare

This section analyzes the effects of overconfidence on the welfare of the players and of the contest designer. To evaluate how overconfidence affects player i's welfare we study the impact overconfidence has on player i's equilibrium expected utility in the elimination contest:

$$E^s[U_{ih}(e^s, e^f)] = p^s_{ih}\left[p^s_{jk}E^f[U_{ij}(e^f_i, e^f_j)] + p^s_{kj}E^f[U_{ik}(e^f_i, e^f_k)]\right] - ce^s_i.$$

where $e^s = (e^s_i, e^s_h, e^s_j, e^s_k)$, $e^f = (e^f_i, e^f_j, e^f_k)$, p^s_{ih} is player i's equilibrium winning probability in the semifinal with h, p^s_{jk} is player j's equilibrium winning probability in the semifinal with k, and the term inside parenthesis is player i's equilibrium perceived expected utility of reaching the final.

Proposition 7 *Consider an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players.*
(i) If $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(2\alpha+1)}{\alpha(3\alpha+1)}$*, then there exist* $\lambda_1 \in (1, \hat{\lambda})$ *for which* $E^s[U_{12}(e^s, e^f)] > \overline{E}^s[U(\overline{e}^s, \overline{e}^f)]$.
(ii) If $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(2\alpha+1)}{\alpha(3\alpha+1)}$ *and* $\lambda_1 < \hat{\lambda}$*, then* $E^s[U_{21}(e^s, e^f)] < \overline{E}^s[U(\overline{e}^s, \overline{e}^f)]$*, otherwise,* $E^s[U_{21}(e^s, e^f)] \geqslant \overline{E}^s[U(\overline{e}^s, \overline{e}^f)]$.
(iii) $E^s[U_{34}(e^s, e^f)] = E^s[U_{43}(e^s, e^f)] > \overline{E}^s[U(\overline{e}^s, \overline{e}^f)]$.

Part (i) demonstrates that if the prize spread is large enough, there are levels of (moderate) overconfidence where the equilibrium expected utility of the overconfident player 1 exceeds what it would be if all players were rational. Part (ii) shows that when the prize spread is large enough and overconfidence is not too extreme, the equilibrium expected utility of the rational player 2 is lower than if all players were rational, otherwise it is higher. Finally, part (iii) reveals that the equilibrium expected utility of rational players 3 and 4, who are seeded in the same semifinal, is higher than if all players were rational. This is not an obvious result since the presence of an overconfident player has two opposite effects on the equilibrium expected utility of rational players 3 and 4. On the one hand, it leads them to raise their equilibrium efforts in the semifinal stage which raises their cost of effort. On the other hand, the rational player who reaches the final has a positive probability of facing an overconfident opponent which makes the final stage more attractive to players 3 and 4.

Now, we turn to the impact of overconfidence on the contest designer's welfare. We assume the welfare of the contest designer is increasing in aggregate effort, the sum of the players' efforts in the two stages of the elimination contest.

Proposition 8 *Consider an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players.*
(i) The equilibrium aggregate effort in a final with players i and j satisfies $e_i^f + e_j^f \leqslant 2\bar{e}^f$.
(ii) If $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(2\alpha+1)}{\alpha(3\alpha+1)}$ *and* $\alpha < \frac{1}{2}$, *then there exist* $\lambda_1 \in (1, \hat{\lambda})$ *such that the equilibrium aggregate effort in the semifinals stage satisfies* $\sum_{i=1}^{4} e_i^s > 4\bar{e}^s$.

Part (i) shows that the equilibrium aggregate effort in the final is less than or equal to that in a final with two rational players. This result follows directly from Propositions 2 and 3. The impact of overconfidence on aggregate effort in the semifinals is harder to characterize. We know from Proposition 4 that in the semifinal with one overconfident and one rational player two situations can emerge. If the prize spread is large enough and overconfidence is not too extreme, the overconfident player's equilibrium effort is higher than if he were rational and the rational player's equilibrium effort is lower than if she were facing a rational rival. In all other cases, both players' equilibrium efforts are less than if all players were rational. We also know from Proposition 5 that in the semifinal with two rational players equilibrium efforts go up since both players have a higher expected utility of reaching the final. Still, part (ii) shows that overconfidence raises aggregate effort in the semifinals stage when the prize spread is large enough, $\alpha < 1/2$, and the overconfident player's bias is small. This result implies that it is unclear whether overconfidence lowers or raises players' aggregate efforts in the two stages. Hence, we are unable to tell whether overconfidence lowers or raises the contest designer's welfare.

7 Extensions

This section discusses four extensions of the elimination contest with Alcalde and Dahm's CSF. First, allowing for more than one overconfident player. Second, assuming the rational players cannot observe the bias of the overconfident player. Third, allowing the elimination contest to have three stages instead of two. Fourth, assuming the biased player is underconfident instead of overconfident.

7.1 Two Overconfident and Two Rational Players

In the Online Appendix, we analyze an elimination contest featuring two overconfident and two rational players. We consider two seeding scenarios: (i) overconfident players in the same semifinal, and (ii) in separate semifinals. We show that in a final between two overconfident players, the more confident player always exerts lower effort, and both exert less effort than if they were rational. Additionally, when the two overconfident players are seeded in the same semifinal, both can exert higher efforts than if they were rational. Finally, when the two overconfident players are seeded in separate semifinals, both can exert higher efforts than their rational rivals. These results collectively underscore that the findings derived for an elimination contest with one overconfident and three rational players extend to an elimination contest with two overconfident and two rational players, irrespective of the seeding.

7.2 Unobservable Overconfidence

The Online Appendix demonstrates that our results extend to an elimination contest where the rational players cannot observe the overconfident player's bias. In a final between an overconfident player and a rational player unaware of this bias, the rational player exerts the benchmark equilibrium effort, $\bar{e}^f$, while the overconfident player, guided by mistaken beliefs, chooses a best response to $\bar{e}^f$. In equilibrium, the overconfident player exerts less effort than the rational one. Since the rational player doesn't adjust her effort, the overconfident player reduces his effort by less than he would if the bias were known. An increase in the bias raises the overconfident player's perceived expected utility of reaching the final. We also examine the equilibrium in a semifinal between an overconfident player and a rational player unaware of this bias. If the prize spread is large enough and the bias moderate, the overconfident player exerts more effort than the rational player. In the semifinal between two rational players, overconfidence no longer boosts equilibrium effort due to the unobservability of bias. Lastly, we identify conditions under which the overconfident player has the highest probability of winning the elimination contest.

7.3 Three-Stage Elimination Contest

Our results also extend to a three-stage elimination contest featuring one overconfident and seven rational players. In the first-stage, the eight players are seeded pairwise and each pair competes in one of the four quarterfinals. The first-stage winners move on to compete in the second-stage and the second-stage winners move on to the final. The winner of the contest receives prize w_1, the runner-up prize w_2, the second-stage losers receive prize w_3, and the first-stage losers receive nothing, with $w_1 > w_2 > w_3 \geqslant 0$. In the Online Appendix we characterize the equilibrium of the quarterfinal between an overconfident and a rational player. We find that, regardless of the overconfident player exerting more or less effort than a rational rival in the semifinal, overconfidence generates an encouragement effect in the quarterfinal. Moreover, depending on the parameters of the model, in the quarterfinals stage, as in the semifinals stage, the encouragement effect can dominate the complacency effect.

7.4 Underconfidence

In the Online Appendix, we analyze an elimination contest featuring one underconfident and three rational players. The underconfident player underestimates the impact of his effort on his probability of winning each pairwise interaction, i.e., he has a bias $\lambda_i \in (0, 1)$. We show that an underconfident player exerts less effort in a final than his rational rival since the bias leads to a drop in the underconfident player's perceived marginal probability of winning the final. The rational player also lowers her effort but not as much. Hence, the overconfident player's perceived probability of winning the final is less than $1/2$. Furthermore, the underconfident player's perceived expected utility of reaching the final decreases in his bias. This happens since the drop in utility due to the decrease in the perceived marginal probability of winning is greater than the the drop in his cost of effort. We also show that an underconfident player always exerts less effort in a semifinal than a rational rival. In the semifinal, an increase in underconfidence makes reaching the final less attractive due to the fall in the perceived expected utility of reaching the final and also lowers the underconfident player's perceived marginal probability of winning the semifinal. Hence, the underconfident player has the lowest chance of winning the elimination contest.

8 Conclusion

We analyze how overconfidence affects behavior in multistage elimination contests. Our findings reveal a nuanced interplay between overconfidence and effort exertion in a two-stage elimination contest.

In the final stage, an overconfident player always exerts lower effort at equilibrium than a rational player. The (mis)perceived advantage of the overconfident player leads him to think, mistakenly, he can reduce his effort without endangering his prospects of success. The rational player, aware of the rival's bias, also lowers her effort but not as much. Hence, the bias lowers the overconfident player's probability of winning the final.

In the semifinals stage, the bias has two opposite effects on an overconfident player's incentives to exert effort. On the one hand, it raises the overconfident player's perceived expected utility of reaching the final. On the other hand, it leads to a decrease in the overconfident player's perceived marginal probability of winning the semifinal. The first effect encourages an overconfident player to raise effort whereas the second effect makes him complacent and leads him to lower effort. If the encouragement effect dominates, an overconfident player exert higher effort at equilibrium in a semifinal than a rational rival.

We show that for the encouragement effect to dominate two conditions have to be met. First, the prize spread needs to be large enough. Second, the overconfident player's bias cannot be too extreme. The intuition behind these two conditions is as follows. The higher is the prize spread, the higher is the perceived expected utility of reaching the final, and the higher is the encouragement effect. As the overconfident player's bias increases, the increase in the encouragement effect gets smaller whereas the increase in the complacency effect gets larger. Hence, there exists an upper bound for the bias above which the complacency effect dominates.

We also find that an overconfident player can be the one with the highest probability of winning an elimination contest. For this to be the case three conditions have to be met. First, the role that effort plays in determining the winning probabilities must be sufficiently high. Second, the prize spread needs to be large enough. Third, the overconfident player's bias needs to be small.

Our study contributes to the literature on CEO overconfidence by providing a novel explanation for why overconfident managers are selected to CEO positions. In addition, our results highlight the role that increases in executive compensation (interpreted as increases in the prize spread) can have in making elimination contests more attractive to overconfident managers. Our study also contributes to the literature on gender gaps in the labor market by showing that large executive compensation coupled with higher male confidence can make competing for top business positions more attractive to males.

Future research could explore elimination contests where players differ not only in confidence levels but also in abilities. If an overconfident player has lower ability than a rational player, the bias may increase both his perceived expected utility of reaching the final as well as his marginal probability of winning. Additionally, future studies could examine how contest designers should optimally structure prizes in the presence of overconfident players.

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Appendix

Derivation of equation (3)

The first-order conditions in the semifinal are

$$\frac{\partial \widetilde{p}^s_{ih}(e_i, e_h, \lambda_i)}{\partial e_i}\widetilde{v}_i = c$$

$$\frac{\partial \widetilde{p}^s_{hi}(e_i, e_h, \lambda_h)}{\partial e_h}\widetilde{v}_h = c$$

Total differentiation with respect to e_i, e_j, and λ_i gives us

$$\left(\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2}\frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial e_h}\frac{de_h}{d\lambda_i} + \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial \lambda_i}\right)\widetilde{v}_i + \frac{\partial \widetilde{p}^s_{ih}}{\partial e_i}\frac{\partial \widetilde{v}_i}{\partial \lambda_i} = 0$$

$$\left(\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h \partial e_i}\frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h^2}\frac{de_h}{d\lambda_i}\right)\widetilde{v}_h = 0$$

Solving the second equation for $\frac{de_h}{d\lambda_i}$ we obtain

$$\frac{de_h}{d\lambda_i} = -\frac{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h \partial e_i}}{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h^2}} \frac{de_i}{d\lambda_i}$$

Replacing in the first equation we obtain

$$\left(\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2} \frac{de_i}{d\lambda_i} - \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial e_h} \frac{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h \partial e_i}}{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h^2}} \frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial \lambda_i} \right) \widetilde{v}_i + \frac{\partial \widetilde{p}^s_{ih}}{\partial e_i} \frac{\partial \widetilde{v}_1}{\partial \lambda_1} = 0$$

Solving this equation for $\frac{de_i}{d\lambda_i}$ we have

$$\frac{de_i}{d\lambda_i} = -\frac{\frac{\partial \widetilde{p}^s_{ih}}{\partial e_i} \frac{\partial \widetilde{v}_i}{\partial \lambda_i} + \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial \lambda_i} \widetilde{v}_i}{\left(\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2} - \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial e_h} \frac{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h \partial e_i}}{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h^2}} \right) \widetilde{v}_i}$$

or

$$\frac{de_i}{d\lambda_i} = -\frac{1}{\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2}} \frac{\frac{\partial \widetilde{p}^s_{ih}}{\partial e_i} \frac{\partial \widetilde{v}_i}{\partial \lambda_i} + \frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial \lambda_i} \widetilde{v}_i}{\left(1 - \frac{\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i \partial e_h}}{\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2}} \frac{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h \partial e_i}}{\frac{\partial^2 \widetilde{p}^s_{hi}}{\partial e_h^2}} \right) \widetilde{v}_i} = -\frac{\frac{mg\widetilde{p}^s_{ih}}{\lambda_i}}{\frac{\partial mg\widetilde{p}^s_{ih}}{\partial e_i}} \frac{\frac{\partial \widetilde{v}_i}{\partial \lambda_i} \frac{\lambda_i}{\widetilde{v}_i} + \frac{\partial mg\widetilde{p}^s_{ih}}{\partial \lambda_i} \frac{\lambda_i}{mg\widetilde{p}^s_{ih}}}{\left(1 - \frac{\frac{\partial mg\widetilde{p}^s_{ih}}{\partial e_h}}{\frac{\partial mg\widetilde{p}^s_{ih}}{\partial e_i}} \frac{\frac{\partial mg\widetilde{p}^s_{hi}}{\partial e_i}}{\frac{\partial mg\widetilde{p}^s_{hi}}{\partial e_h}} \right)} \tag{24}$$

Note that in equation (24) the sign of the first term is positive due to the assumption $\frac{\partial^2 \widetilde{p}^s_{ih}}{\partial e_i^2} < 0$ and the sign of the denominator of the second term is positive, based on the assumption that ensures the uniqueness of the equilibrium.

Proof of Lemma 1

The best response of player i in the semifinal with h, $R_i^s(e_h)$, is defined by

$$\frac{\lambda_i q'(e_i) q(e_h)}{[\lambda_i q(e_i) + q(e_h)]^2} \widetilde{v}_i = c. \tag{25}$$

From (25) we obtain

$$\begin{aligned} \frac{\partial R_i^s(e_h)}{\partial \lambda_i} &= \frac{q'(e_i) q(e_h) [q(e_h) - \lambda_i q(e_i)]}{[\lambda_i q(e_i) + q(e_h)]^3} \widetilde{v}_i + \frac{\lambda_i q'(e_i) q(e_h)}{[\lambda_i q(e_i) + q(e_h)]^2} \frac{\partial \widetilde{v}_i}{\partial \lambda_i} \\ &= \frac{q'(e_i) q(e_h)}{[\lambda_i q(e_i) + q(e_h)]^2} \left[-\frac{\lambda_i q(e_i) - q(e_h)}{\lambda_i q(e_i) + q(e_h)} \widetilde{v}_i + \lambda_i \frac{\partial \widetilde{v}_i}{\partial \lambda_i} \right]. \end{aligned} \tag{26}$$

It follows from (26) that an increase in overconfidence shifts player i's best response in the semifinal inwards when

$$\frac{\partial \widetilde{v}_i}{\partial \lambda_i} \frac{\lambda_i}{\widetilde{v}_i} < \frac{\lambda_i q(e_i) - q(e_h)}{\lambda_i q(e_i) + q(e_h)},$$

otherwise, an increase in overconfidence shifts player i's best response in the semifinal outwards.

From (25) the slope of player i's best response in the semifinal, $\partial R_i^s(e_h)/\partial e_h$, is equal to zero when $\lambda_i q(e_i) = q(e_h)$. Substituting $q(e_h) = \lambda_i q(e_i)$ into player i's best response in the semifinal and denoting the maximal effort that i is willing to invest in the semifinal by e_i^{smax} we obtain

$$\frac{q(e_i^{smax})}{q'(e_i^{smax})} = \frac{\widetilde{v}_i}{4c}.$$

Since $\widetilde{v}_i$ increases with λ_i, it follows from the last equality that e_i^{smax} increases with λ_i.

Proof of Lemma 2

The proof of this result is similar to Lemma 3 in Santos-Pinto and Sekeris (2023).

To prove that the equilibrium is unique, observe first that when the players' best responses cross it is impossible that they are both negatively slopped, since the best response of the overconfident player is necessarily positively slopped. Indeed, if the two players were unbiased ($\lambda_1 = 1$), then player 1's best response function would be positively slopped for any $e_2 < \overline{e}_2^{smax}$, reach a max at $e_2 = \overline{e}_2^{smax}$, and be negatively slopped for $e_2 > \overline{e}_2^{smax}$.

From Lemma 1 we deduce that increasing the value of the overconfidence parameter λ_1 leads player 1's best response function to be positively sloped for any $e_2 < \hat{e}_2^{smax}$, reach a max at $e_2 = \hat{e}_2^{smax}$, and be negatively slopped for $e_2 > \hat{e}_2^{smax}$, where $q(\hat{e}_2^{smax}) = \lambda_1 q(\hat{e}_1^{smax})$. Note that since e_1^{smax} increases with λ_1 we have $\hat{e}_1^{smax} > \overline{e}_1^{smax}$. This and $\overline{e}_1^{smax} = \overline{e}_2^{smax}$, in turn, implies $\hat{e}_2^{smax} > \overline{e}_2^{smax}$. Hence, $R_1^s(e_2)$ is positively sloped in the interval $[0, \hat{e}_2^{smax}]$. Last, since $R_2^s(e_1)$ will never reach larger values than $\overline{e}_2^{smax}$, we deduce that at the equilibrium of the semifinal the best response of the overconfident player 1 is necessarily positively sloped.

To prove that the equilibrium is unique it is then sufficient to show that the composite function $\Gamma(e_1, e_2) = \frac{\partial R_1^s(e_2)}{\partial e_2} \circ \frac{\partial R_2^s(e_1)}{\partial e_1}$, has a slope smaller than 1 for any equilibrium pair (e_1^s, e_2^s), since the function is continuous on $\mathbb{R}$. If $\frac{\partial R_2^s(e_1)}{\partial e_1} < 0$, then since $\frac{\partial R_1^s(e_2)}{\partial e_2} > 0$, the condition is necessarily satisfied. If, on the other hand, $\frac{\partial R_2^s(e_1)}{\partial e_1} > 0$, then we simply need to prove that if $\frac{\partial R_i^s(e_j)}{\partial e_j} > 0$ for both players, then the product of the slopes of the best response functions is smaller than 1. Rewriting the product of the slopes of the best responses, and simplifying expressions, we thus want to show that:

$$\frac{\lambda_1 q'(e_1) q'(e_2)[\lambda_1 q(e_1) - q(e_2)]}{\lambda_1 q''(e_1)[\lambda_1 q(e_1) + q(e_2)] - 2\lambda_1^2 [q'(e_1)]^2 q(e_2)} \frac{q'(e_1) q'(e_2)[q(e_2) - q(e_1)]}{q''(e_2)[q(e_1) + q(e_2)] - 2[q'(e_2)]^2 q(e_1)} < 1$$

This is equivalent to

$$\frac{[q'(e_1)]^2[q'(e_2)]^2[\lambda_1 q(e_1)-q(e_2)][q(e_2)-q(e_1)]}{[q''(e_1)[\lambda_1 q(e_1)+q(e_2)]-2\lambda_1[q'(e_1)]^2q(e_2)][q''(e_2)[q(e_1)+q(e_2)]-2[q'(e_2)]^2q(e_1)]}<1$$

Since the left-hand side is decreasing in $q''(e_i)$, $i=1,2$, the above inequality is satisfied if

$$\frac{[q'(e_1)]^2[q'(e_2)]^2[\lambda_1 q(e_1)-q(e_2)][q(e_2)-q(e_1)]}{4\lambda_1[q'(e_1)]^2q(e_2)[q'(e_2)]^2q(e_1)}<1$$

which simplifies to

$$[\lambda_1 q(e_1)-q(e_2)][q(e_2)-q(e_1)]<4\lambda_1 q(e_1)q(e_2),$$

which is satisfied.

Proof of Proposition 1

At the 45 degree line, player 1's best response in the semifinal takes the value $\hat{e}_1$ given by

$$\frac{\lambda_1 q'(\hat{e}_1)}{(1+\lambda_1)^2 q(\hat{e}_1)}\widetilde{v}_1=c. \tag{27}$$

At 45 degree line, player 2's best response in the semifinal takes the value $\hat{e}_2$ given by

$$\frac{q'(\hat{e}_2)}{4q(\hat{e}_2)}v_2=c. \tag{28}$$

Inequality (16) and equations (27) and (28) imply

$$\frac{q'(\hat{e}_1)}{q(\hat{e}_1)}<\frac{q'(\hat{e}_2)}{q(\hat{e}_2)}.$$

Given that $q(.)$ is (weakly) concave, this inequality can only be satisfied provided $\hat{e}_1>\hat{e}_2$. This, the concavity of the players' best responses, and Lemma 1, imply the equilibrium in the semifinal lies below the 45 degree line, i.e., $e_1^s>e_2^s$.