Overconfidence and Strategic Behavior in Elimination Contests: Implications for CEO Selection*

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Abstract

We analyze how overconfidence affects behavior in multistage elimination contests. Our findings reveal a nuanced interplay between overconfidence and effort exertion. An overconfident player exerts less effort in the final stage than a rational rival. However, this pattern can be inverted in the semifinals stage, where an overconfident player can exert more effort than a rational rival. We also uncover that an overconfident player can have the highest probability of winning an elimination contest. Our results offer a novel perspective on CEO overconfidence and highlight that high executive compensation renders the pursuit of CEO positions exceptionally appealing to overconfident managers.

Keywords: Overconfidence, Elimination Contest, Encouragement Effect, Complacency Effect.

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1 Introduction

Elimination contests are a common feature in organizations, politics, academia, and sports. In companies, managers compete for promotion to senior executive positions, and those who advance vie for a chief executive officer (CEO) role. In politics, politicians battle for top party positions, then compete to become high-level government officials. In academia, PhDs contend for assistant professorships and later vie for tenure. Tennis and many other sports also rely heavily on elimination contents.

Given how widespread these contests are, it is crucial to consider the impact of overconfidence—a well-documented bias detected in both laboratory and real-world settings.[1] Overconfidence influences economic behavior in labor markets (Spinnewijn 2013, 2015; Kőszegi 2014; Santos-Pinto and de la Rosa 2020), and a significant number of CEOs exhibit this bias, affecting corporate decisions (Malmendier and Tate 2005, 2008, 2015). This naturally raises the question of why overconfident CEOs attain these positions in the first place.

In this paper we analyze how overconfidence, conceptualized as overestimation of one's probability of winning, affects behavior in multistage elimination contests. We are interested in finding answers to the following questions. How does overconfidence affect effort provision in the different stages of an elimination contest? Is an overconfident player more or less likely to win an elimination contest than a rational player?

To address these questions we consider a two-stage elimination contest with four players. In the first stage, the players are matched pairwise, and each pair competes in one semifinal. The first-stage winners go on to the second stage of the contest and compete against each other in the final. The winner receives prize w_1, the runner-up prize w_2, and the first-stage losers receive nothing, with $w_1 > w_2 \geqslant 0$.

In each pairwise interaction the players choose their efforts simultaneously and their winning probabilities are determined by a contest success function. Players are homogeneous, except for their confidence levels. This allows us to zero in on the impact of overconfidence on players' incentives to exert effort and winning probabilities. An overconfident player overestimates his chances of winning at each stage but has a correct perception of the prizes and cost of effort. Furthermore, an overconfident player's bias is observable by his rivals.

[1] Moore and Healy (2008) distinguish between three types of overconfidence: overestimation of one's skill (absolute overconfidence), overplacement (relative overconfidence), and excessive precision (overprecision). This paper focuses on the first type of overconfidence.

We solve the elimination contest using backward induction, beginning with the final stage. In the final, each player selects their effort level based on the condition that the product of the perceived marginal probability of winning and the utility prize spread equals the marginal cost of effort. In the semifinal, a player similarly chooses an effort level where the perceived marginal benefit matches the marginal cost. Here, the perceived marginal benefit is calculated as the product of the perceived marginal probability of winning the semifinal and the perceived expected utility of reaching the final.

We start by solving the model under a general contest success function that satisfies standard properties of contests, namely, a player's winning probability is differentiable, increasing in own effort, decreasing in the rival's effort, and concave. To incorporate overconfidence into the model, we assume that a player's perceived probability of winning increases in his level of overconfidence. We show that, as Rosen (1986) initially suggested, overconfidence affects semifinal effort through two distinct channels. First, when a player's overconfidence causes his rival to reduce her final-stage effort, it increases his perceived expected utility of reaching the final, because he believes he has a higher chance of winning despite planning to exert less effort than his rival. Everything else equal, this incentivizes an overconfident player to exert higher effort in the semifinal. We call this the encouragement effect of overconfidence. Second, when an overconfident player believes his semifinal winning probability is high, the concavity and upper bound of his perceived winning probability imply that further increases in overconfidence lower his perceived marginal probability of winning, reducing the perceived payoff of exerting more effort. Everything else equal, this incentivizes an overconfident player to exert lower effort in the semifinal. We call this the complacency effect of overconfidence. Hence, the equilibrium effort exerted by an overconfident player in the semifinal depends on the dominance of one effect over the other: if the encouragement effect prevails, the player exerts more effort; if the complacency effect dominates, the player exerts less.

Next, we specialize the model by assuming the players' winning probabilities are determined by the contest success function proposed by Alcalde and Dahm (2007).[2] We assume one player is overconfident while the other three are rational. This setup reflects situations where a minority of players exhibit overconfidence, enabling us to analyze the effects of overconfidence in the most straightforward manner. We identify four key find-

[2]Alcalde and Dahm's CSF yields a tractable model for multi-stage games as the equilibrium efforts and payoffs of the subgames can be easily computed and plugged into earlier stages of the game. Section 5 explains Alcalde and Dahm's CSF in detail.

ings. First, in the final, the overconfident player exerts less effort than his rational rival at equilibrium. Intuitively, the (mis)perceived advantage of the overconfident player leads him to lower his effort. The rational player, anticipating the overconfident player will lower his effort, also lowers her effort but not as much. In addition, the overconfident player's equilibrium perceived expected utility of the final increases in his bias as this raises the player's perceived probability of winning the final and lowers his cost of effort.

Second, overconfidence generates an encouragement and a complacency effect with Alcalde and Dahm's CSF. As we have just seen, the overconfidence bias induces the rational rival to reduce her final-stage effort, making the final more appealing to the overconfident player and thus creating an encouragement effect. Furthermore, under Alcalde and Dahm's CSF, the encouragement effect grows with the utility prize spread and declines with overconfidence. We also demonstrate that the overconfident player believes his probability of winning the semifinal exceeds 1/2, and hence an increase in overconfidence reduces his perceived marginal probability of winning, leading to a complacency effect. Under Alcalde and Dahm's CSF, the complacency effect is exactly -1.

Third, when the utility prize spread is large enough and overconfidence is not too extreme, the encouragement effect dominates the complacency effect and the overconfident player exerts more effort in the semifinal than his rational rival. When either of the above conditions fails to hold, the encouragement effect is dominated by the complacency effect and the overconfident player exerts less effort in the semifinal than his rational rival.

Fourth, an overconfident player can emerge with the highest equilibrium probability of winning an elimination contest. This occurs when effort significantly influences winning probabilities, the utility prize spread is large, and the overconfidence bias is relatively small.

The rest of the paper is organized as follows. Section 2 reviews the related literature, while Section 3 introduces the general model. Section 4 outlines the encouragement and complacency effects. Section 5 specializes the model with Alcalde and Dahm's (2007) CSF. Section 6 concludes the paper. All proofs are in the Appendix.

2 Related Literature

Our study relates to four strands of literature. First, it contributes to the literature on CEO overconfidence. Empirical evidence documents that a substantial share of CEOs are overconfident (for a review see Malmendier and Tate, 2015). The seminal contribution to this literature is Malmendier and Tate (2005, 2008) who measure CEO overconfidence as the tendency to hold stock options longer before exercise. Malmendier and Tate (2015) use this measure together with additional controls and find that approximately 40 percent of CEOs of companies listed in the Standard & Poor's 1500 index are overconfident.

Several theories on the selection of managers into CEO positions have been proposed to explain CEO overconfidence. According to Van den Steen (2005), CEO overconfidence serves as a commitment device that helps attract and retain employees that share the same values as the CEO. For Hackbarth (2008), CEO overconfidence leads to high debt levels which prevent CEOs from diverting funds, which, in turn, increases firm value and reduces conflicts between CEOs and shareholders. Goel and Thakor (2008) study elimination tournaments where risk-averse managers compete to become CEO by choosing the level of risk of their projects. Some managers are rational while others are overconfident. An overconfident manager underestimates project risk which increases the propensity to take risky projects (e.g. R&D activities). Some of the more risky projects will be successful and hence, the higher risk taking of overconfident managers will improve their chances of promotion to CEO. Finally, Gervais et al. (2011), show that firms can find overconfident managers more attractive because they exert higher effort to learn about their projects.

Our findings offer a new perspective on why overconfident managers are promoted to CEO positions. In a seminal contribution, Rosen (1986), models the competition for promotion to a top executive role as a multistage elimination contest where in each stage fewer managers are selected for the next step of the career ladder. Our model suggests that when the prize differential across the corporate ladder is substantial, moderately overconfident managers are more likely to be promoted to CEO than their rational counterparts. This occurs because moderately overconfident managers tend to exert greater effort, such as working longer hours, early in their careers due to the encouragement effect provided by their overconfidence. The larger the disparity between the compensation of a lower-level manager and that of a CEO, the stronger this encouragement effect becomes. Consequently, our results emphasize the significant influence that large increases in executive compensation (Murphy 2013) can have in making the pursuit of a CEO position particularly appealing to overconfident individuals.

Second, our study contributes to the large literature on gender gaps in the labor market. Empirical evidence documents gender gaps in wages and in top business positions. For instance, in 2022 women in the US earn 82 percent of their male counterparts

(Kochhar 2023) and women represent only 6 percent of top business executives in the US (Keller et al. 2022). The wage gender gap is larger in high skilled work, and much of it seems to be caused by gaps in promotions (Blau and DeVaro 2007, Blau and Kahn 2017, Bronson and Thoursie 2019). Laboratory experiments show that gender differences in confidence and risk attitudes can account for gender gaps in behavior in tournaments and contests (Niederle and Vesterlund 2007, Kamas and Preston 2012, Gillen et al. 2019, Price 2020, Buser et al. 2021, van Veldhuizen 2022).

Our findings show that the large executive compensation spreads coupled with higher male confidence can make competing for a top business position much more attractive to male candidates. We also predict that much of the gender gap in promotions will take place early in workers' careers. This could place women at a further disadvantage besides the negative effects of childbirth and child-rearing (Bertrand et al. 2010, Goldin and Katz 2011, Goldin 2014).[3]

Third, our study also contributes to the literature on overconfidence, tournaments, and contests. Santos-Pinto (2010) shows how firms can optimally set tournament prizes to exploit workers' overconfidence, defined as overestimation of productivity of effort. Ludwig et al. (2011) show that an overconfident player, defined as someone who underestimates the cost of effort, exerts more effort than a rational player in a Tullock contest. Deng et al. (2024) study the effect of overconfidence on the firm's information disclosure policy in a Tullock contest between an incumbent and a new hire. Santos-Pinto and Sekeris (2025) study how heterogeneity in confidence, defined as perception of the winning probability, affects effort provision in tournaments and contests. They uncover a non-monotonic effect of confidence on equilibrium effort and winning probabilities. All of these studies focus on one-stage tournaments and contests. To the best of our knowledge, ours is the first study on overconfidence in a multistage setting.

Finally, our study contributes to the literature on multistage elimination contests. The seminal work is Rosen (1986) who studies how prizes affect players' effort across stages. There are many studies on multistage elimination contests regarding different aspects, such as effort in one stage being carried over to a subsequent stage (Baik and Lee, 2000), budget constraints on individual expenditures (Stein and Rapoport, 2005), the

[3] Many studies suggest that gender gap varies with culture (Gneezy et al, 2003 and 2009; Booth and Nolen, 2009 and 2014). In societies where gender equality is more promoted, gender gaps become less significant in many areas, including entry and performance in a competitive environments. Differences in work environment, characteristics of professions, and education also affect the magnitude of gender gaps.

discouragement effect (Konrad, 2012), optimal prize setting (Mago et al, 2013; Cheng et al, 2019; Coehn et al, 2018; Moldovanu and Sela, 2006), optimal contest structure (Gradstein and Konrad, 1999; Moldovanu and Sela, 2006; Fu and Lu, 2018; Hou and Zhang, 2021), heterogeneity in either abilities or valuations (Rosen, 1986; Baik and Lee, 2000; Stracke, 2013; Brown and Minor, 2014), seeding (Groh et al, 2012), and sabotage (Klunover, 2021).[4] Our paper expands this strand of literature by considering a new dimension: heterogeneity in confidence levels.

3 Set-up

Consider a two-stage elimination contest with four players. In the first stage, the players are matched pairwise, and each pair competes in one semifinal. The first-stage winners go on to the second stage of the contest and compete against each other in the final. The winner receives prize w_1, the runner-up receives prize w_2, and the first-stage losers receive nothing, with $w_1 > w_2 \geqslant 0$.

The players choose their efforts simultaneously to maximize their expected utilities in each pairwise interaction. The effort of player i in a pairwise interaction is denoted by e_i. Player i derives utility $u(w)$ from prize $w \geqslant 0$, where $u'(w) > 0$, $u''(w) \leqslant 0$, and $u(0) = 0$, and has cost of effort $c(e_i) = ce_i$, with $c \geqslant 1$, and $e_i \geqslant 0$. Player i's actual probability of winning when paired with j at stage $t \in \{s, f\}$ is modeled via a contest success function denoted by $p_{ij}^t(e_i, e_j)$. We assume $p_{ij}^t(e_i, e_j) \in [0, 1]$ for all i, $\sum_i p_{ij}^t(e_i, e_j) \leqslant 1$ for all (e_i, e_j), $p_{ij}^t(e_i, e_j)$ is differentiable, increasing in own effort, $\partial p_{ij}^t/\partial e_i = mgp_{ij}^t(e_i, e_j) \geqslant 0$, decreasing in the rival's effort, $\partial p_{ij}^t/\partial e_j \leqslant 0$, and concave, $\partial^2 p_{ij}^t/\partial e_i^2 < 0$.

We assume an overconfident player overestimates his probability of winning each pairwise interaction and has a correct perception of the prizes and cost of effort. This definition of overconfidence is in line with Santos-Pinto (2008, 2010) and Santos-Pinto and Sekeris (2025). Accordingly, player i's perceived probability of winning when paired with j at stage $t \in \{s, f\}$ is denoted by $\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i)$, where λ_i represents player i's confidence. We assume $\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i) \in [0, 1]$ for all i, $\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i)$ is differentiable, increasing in confidence, $\partial \widetilde{p}_{ij}^t/\partial \lambda_i \geqslant 0$, increasing in own effort, $\partial \widetilde{p}_{ij}^t/\partial e_i = mg\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i) \geqslant 0$, decreasing in the rival's effort, $\partial \widetilde{p}_{ij}^t/\partial e_j \leqslant 0$, and concave, $\partial^2 \widetilde{p}_{ij}^t/\partial e_i^2 < 0$.

[4] Few studies have focused on multistage elimination tournaments. One exception is Altmann et al. (2012), who study theoretically and experimentally a two-stage tournament in which four players compete simultaneously in the first stage, and the two players with the highest output advance to the second stage.

The solution concept is Subgame Perfect Equilibrium. We solve the elimination contest via backwards induction and determine the Nash equilibrium of the final before we determine the Nash equilibrium of each semifinal. To be able to compute the equilibrium taking into account that players can hold mistaken beliefs we assume: (i) a player who faces a biased opponent is aware that the latter's perception (and probability of winning) is mistaken, (ii) each player thinks that his own perception (and probability of winning) is correct, and (iii) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players agree to disagree about their perceptions (and probabilities of winning). This approach follows Heifetz et al. (2007a, 2007b) for games with complete information, and Squintani (2006) for games with incomplete information.[5] Finally, we assume that each player not only knows the confidence level of his direct rival in the semifinal but also the confidence levels of the other two potential rivals in the other semifinal.

4 Encouragement and Complacency Effects

This section reveals the two key effects that overconfidence has on a player's effort in the semifinal of an elimination contest.

In a final between i and j, player i chooses the level of effort e_i that maximizes his perceived expected utility:

$$\widetilde{E}^f[U_{ij}(e_i, e_j, \lambda_i)] = \widetilde{p}_{ij}^f u(w_1) + (1 - \widetilde{p}_{ij}^f)u(w_2) - ce_i = \widetilde{p}_{ij}^f \Delta u - ce_i + u(w_2),$$

where $\Delta u = u(w_1) - u(w_2)$ represents the utility prize spread. The first-order condition of player i in a final against j is $mg\widetilde{p}_{ij}^f(e_i, e_j, \lambda_i)\Delta u = c$. In the final, an overconfident player chooses the level of effort at which the perceived marginal benefit of effort equals the marginal cost.[6] The Nash equilibrium efforts in a final between i and j, (e_i^f, e_j^f), are the solution to

$$mg\widetilde{p}_{ij}^f(e_i^f, e_j^f, \lambda_i)\Delta u = c,$$

and

$$mg\widetilde{p}_{ji}^f(e_i^f, e_j^f, \lambda_j)\Delta u = c.$$

[5] These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007).

[6] The second-order condition is satisfied since

$$\frac{\partial^2 \widetilde{E}^f[U_{ij}(e_i, e_j, \lambda_i)]}{\partial e_i^2} = \frac{\partial^2 \widetilde{p}_{ij}^f}{\partial e_i^2}\Delta u < 0.$$

Now consider the semifinals stage. Let players i and j be seeded in one semifinal and players h and k be seeded in the other semifinal. If i wins his semifinal, then i faces h in the final with probability p_{hk}^s and k with probability $p_{kh}^s = 1 - p_{hk}^s$. Hence, player i's perceived expected utility of reaching the final (or perceived continuation value), $\widetilde{v}_i$, is:

$$\widetilde{v}_i = p_{hk}^s \widetilde{E}^f[U_{ih}(e_i^f, e_h^f, \lambda_i)] + p_{kh}^s \widetilde{E}^f[U_{ik}(e_i^f, e_k^f, \lambda_i)].$$

In the semifinal between i and j, player i chooses the level of effort e_i that maximizes his perceived expected utility:

$$\widetilde{E}^s[U_{ij}(e_i, e_j, \lambda_i)] = \widetilde{p}_{ij}^s \widetilde{v}_i - ce_i.$$

The first-order condition of player i in a semifinal against j is $mg\widetilde{p}_{ij}^s(e_i, e_j, \lambda_i)\widetilde{v}_i = c$.[7] The Nash equilibrium efforts in the semifinal between i and j, (e_i^s, e_j^s), are the solution to

$$mg\widetilde{p}_{ij}^s(e_i^s, e_j^s, \lambda_i)\widetilde{v}_i = c, \tag{1}$$

and

$$mg\widetilde{p}_{ji}^s(e_i^s, e_j^s, \lambda_j)\widetilde{v}_j = c. \tag{2}$$

To ensure the elimination contest has a unique Subgame Perfect Equilibrium we make the additional assumption

$$\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i \partial e_j} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_i \partial e_j} < \frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i^2} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_j^2}, \tag{3}$$

for $t \in \{s, f\}$.

Lemma 1 *The elimination contest has a unique Subgame Perfect Equilibrium.*

Differentiating (1) and (2) and solving for $\partial e_i^s / \partial \lambda_i$ we find how the player i's equilibrium effort changes with his bias

$$\frac{de_i^s}{d\lambda_i} = \frac{mg\widetilde{p}_{ij}^s}{\lambda_i} \frac{\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i} + \frac{\partial mg\widetilde{p}_{ij}^s}{\partial \lambda_i}\frac{\lambda_i}{mg\widetilde{p}_{ij}^s}}{-\frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_i} + \frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_j}\frac{\frac{\partial mg\widetilde{p}_{ji}^s}{\partial e_i}}{\frac{\partial mg\widetilde{p}_{ji}^s}{\partial e_j}}}, \tag{4}$$

[7]The second-order condition is satisfied since

$$\frac{\partial^2 \widetilde{E}^s[U_{ij}(e_i, e_j, \lambda_i)]}{\partial e_i^2} = \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2} \Delta u < 0.$$

where the sign of the denominator in equation (4) is positive.[8] Thus, whether overconfidence raises or lowers the equilibrium effort of the overconfident player in the semifinal depends on the signs and magnitudes of the two terms in the numerator in equation (4). The first term

$$\frac{\partial \widetilde{v}_i}{\partial \lambda_i} \frac{\lambda_i}{\widetilde{v}_i},$$

represents the elasticity of the perceived expected utility of reaching the final with respect to the bias, $\varepsilon_{\widetilde{v}_i,\lambda_i}$. If an increase in the bias raises the overconfident player's perceived expected utility of reaching the final, the first term is positive.

Meanwhile, the second term

$$\frac{\partial mg\widetilde{p}_{ij}^s}{\partial \lambda_i} \frac{\lambda_i}{mg\widetilde{p}_{ij}^s},$$

represents the elasticity of the perceived marginal probability of winning the semifinal with respect to the bias, $\varepsilon_{mg\widetilde{p}_{ij}^s,\lambda_i}$. If an increase in the bias lowers the overconfident player's perceived marginal probability of winning the semifinal, the second term is negative.

Hence, we see from equation (4) that the bias can have two effects on the overconfident player's incentives to exert effort in a semifinal. On the one hand, an increase in the bias can increase the overconfident player's perceived expected utility of reaching the final which motivates him to raise effort. We label this the encouragement effect of overconfidence. On the other hand, an increase in the bias can reduce the overconfident player's perceived marginal probability of winning the semifinal which motivates him to lower effort. We label this the complacency effect of overconfidence.

Note that sign of the encouragement effect is positive when

$$\frac{\partial \widetilde{v}_i}{\partial \lambda_i} = p_{hk}^s \frac{\partial \widetilde{E}^f[U_{ih}(e_i^f, e_h^f, \lambda_i)]}{\partial \lambda_i} + (1 - p_{hk}^s) \frac{\partial \widetilde{E}^f[U_{ik}(e_i^f, e_k^f, \lambda_i)]}{\partial \lambda_i} > 0,$$

where

$$\frac{\partial \widetilde{E}^f[U_{ih}(e_i^f, e_h^f, \lambda_i)]}{\partial \lambda_i} = \left(\frac{\partial \widetilde{p}_{ih}^f}{\partial \lambda_i} + \frac{\partial \widetilde{p}_{ih}^f}{\partial e_h} \frac{\partial e_h^f}{\partial \lambda_i} \right) \Delta u. \tag{5}$$

Since $\frac{\partial \widetilde{p}_{ih}^f}{\partial \lambda_i} > 0$ and $\frac{\partial \widetilde{p}_{ih}^f}{\partial e_h} < 0$, it follows from (5) that overconfidence generates an encouragement effect when an increase in the bias lowers a rival's equilibrium effort in the final.

[8] In the Appendix we derive equation (4) and show that its denominator is positive.

5 Elimination Contest with Alcalde and Dahm's (2007) CSF

This section solves the elimination contest using Alcalde and Dahm's (2007) CSF which, unlike the Tullock (1980) CSF, offers a very tractable model for a multistage elimination contest with heterogeneous players. By deriving closed-form solutions for the equilibrium efforts and winning probabilities, we can directly evaluate how overconfidence influences players' effort levels and their chances of winning the elimination contest.

Player i's actual winning probability when paired with j at stage $t \in \{s, f\}$ is

$$p_{ij}^t(e_i, e_j) = \begin{cases} 1 - \frac{1}{2}(\frac{e_j}{e_i})^\alpha & \text{if} \quad e_i \geqslant e_j \\ \frac{1}{2}(\frac{e_i}{e_j})^\alpha & \text{if} \quad e_i \leqslant e_j \end{cases}$$

where $\alpha > 0$ determines how sensitive Alcalde and Dahm's CSF is to effort.[9] We assume $0 < \alpha \leq 1$ which, as shown in Alcalde and Dahm (2007), implies that each subgame of the elimination contest has a unique pure strategy Nash equilibrium. Like the Tullock CSF, the Alcalde and Dahm CSF is homogeneous of degree zero in effort. This ensures that the probabilities of winning do not depend on the unit of measurement of efforts. The key distinction between the Alcalde and Dahm CSF and the Tullock CSF lies in how the winning probabilities are determined: under Alcalde and Dahm, they depend on percentage-based differences in effort, whereas under Tullock, they depend on ratio-based differences in effort.[10]

Following Santos-Pinto and Sekeris (2025), player i's perceived winning probability against j at stage $t \in \{s, f\}$ is

$$\widetilde{p}_{ij}^t(e_i, e_j, \lambda_i) = \begin{cases} 1 - \frac{1}{2}\frac{e_j^\alpha}{\lambda_i e_i^\alpha} & \text{if} \quad \lambda_i e_i^\alpha \geqslant e_j^\alpha \\ \frac{1}{2}\frac{\lambda_i e_i^\alpha}{e_j^\alpha} & \text{if} \quad \lambda_i e_i^\alpha \leqslant e_j^\alpha \end{cases} \tag{6}$$

where $\lambda_i \geqslant 1$. Player i is rational when $\lambda_i = 1$ and overconfident when $\lambda_i > 1$.

In the top panel of Figure 1, we plot the actual (blue) and perceived (red) winning probabilities of an overconfident player with $\lambda_i = 1.5$ under $\alpha = 0.9$ and $e_j = 1$. When both players exert the same effort, $e_i = e_j = 1$, his actual winning probability is $1/2$, but he perceives it as higher. He also erroneously believes that if he exerts $e_i = 0.64$ while his rival maintains $e_j = 1$, his winning probability stays at $1/2$. The bottom panel

[9] When $\alpha = 0$ the CSF is completely insensitive to effort and we obtain the extreme case of a (fair) lottery. As α increases, the CSF becomes more sensitive to effort, and the contest becomes more deterministic until the extreme case of an all-pay auction is reached when $\alpha \to \infty$.

[10] For a discussion of the similarities and differences between these two contest success functions see Corchón (2007).

of Figure 1 shows the overconfident player's actual (blue) and perceived (red) marginal probabilities of winning. At low effort levels, his perceived marginal probability exceeds the actual value; at high effort levels, it falls below it.

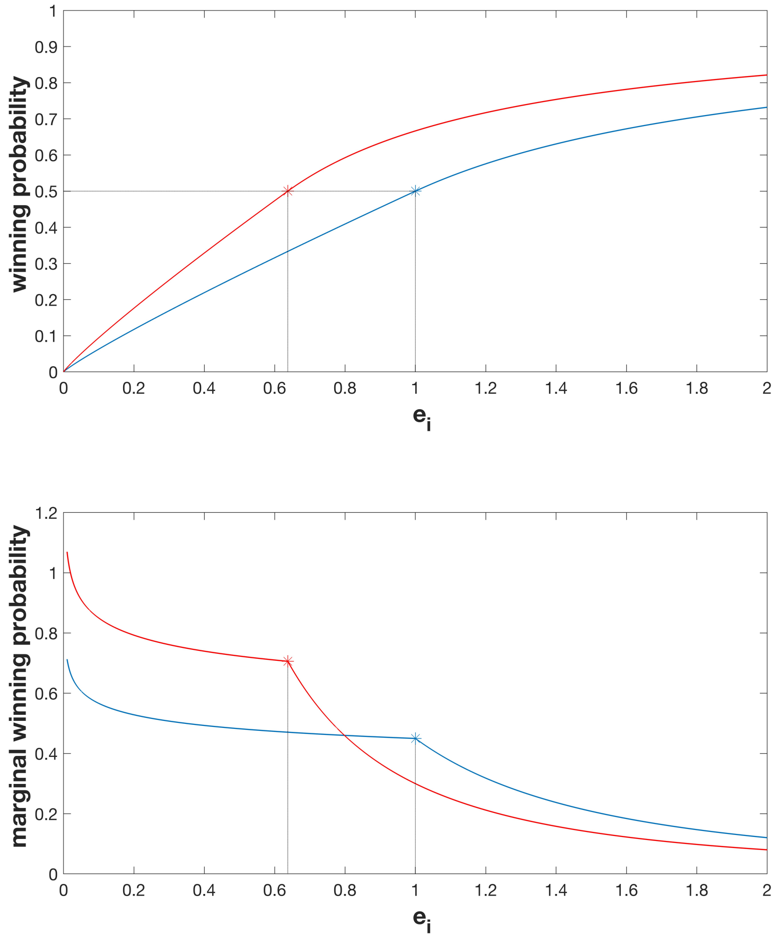

Figure 1: Winning probabilities with Alcalde and Dahm's CSF. The actual winning probabilities are depicted in blue, the perceived winning probabilities are depicted in red. We set $\alpha = 0.9$, $\lambda_i = 1.5$, and $e_j = 1$.

5.1 Rational Benchmark

We start by characterizing the Subgame Perfect Equilibrium of the elimination contest when all players are rational. This serves as a benchmark to which we compare our subsequent results.

From now we label the players as 1, 2, 3, and 4. Furthermore, we assume players 1 and 2 compete in one semifinal, while players 3 and 4 face off in the other semifinal. Since all players are identical, we consider a final between players 1 and 3 without loss of generality.

Proposition 1 *Consider an elimination contest with Alcalde and Dahm's CSF and four rational players. The equilibrium effort in the final is* $\overline{e}^f = \alpha\Delta u/2c$, *the equilibrium winning probability is* $\overline{p}^f = 0.5$, *and the equilibrium expected utility is*

$$\overline{E}^f[U(\overline{e}^f)] = \frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2).$$

The equilibrium effort in the semifinals is

$$\overline{e}^s = \frac{\alpha}{2c}\left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right],$$

the equilibrium winning probability is $\overline{p}^s = 0.5$, *and the equilibrium expected utility is*

$$\overline{E}^s[U(\overline{e}^s, \overline{e}^f)] = \frac{1-\alpha}{2}\left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right].$$

In a final featuring two rational players, equilibrium is reached when both exert the same effort, resulting in an equal winning probabilities. The equilibrium effort in the final increases in the utility prize spread, Δu, in the role that effort plays in determining the winner of the final, α, and decreases in the marginal cost of effort, c. Similarly, in a semifinal featuring two rational players, equilibrium is reached when both exert the same effort, resulting in an equal winning probabilities. The equilibrium effort in the semifinal is smaller than the equilibrium effort in the final for all $\alpha \in (0, 1]$ when $u(w_1) > 3u(w_2)$. Finally, we have $\overline{P} = \overline{p}^s\overline{p}^f = 1/4$. When all players are rational the elimination contest is symmetric and hence each has 1/4 probability of being the winner.

5.2 Equilibrium Efforts

We now characterize the Subgame Perfect Equilibrium of the elimination contest when player 1 is overconfident, $\lambda_1 > 1$, and players 2, 3, and 4 are rational, $\lambda_2 = \lambda_3 = \lambda_4 = 1$. Since players 3 and 4 are identical and face off in the same semifinal, we consider a final between an overconfident player 1 and a rational player 3 without loss of generality.

We start by introducing two lemmas that will be useful to understand the impact of overconfidence on the final stage. Lemma 2 describes the shape of the players' best responses in the final, and Lemma 3 describes how the overconfident player's best response in the final changes with his bias.

Lemma 2 $R_1^f(e_3)$ *is quasi-concave in* e_3 *and reaches a maximum at* $\lambda_1 e_1^\alpha = e_3^\alpha$. $R_3^f(e_1)$ *is quasi-concave in* e_1 *and reaches a maximum at* $e_1 = e_3$.

Lemma 2 tells us that the players' best responses in the final are non-monotonic. Given high effort of the rival, a player reacts to an increase in effort of the rival by decreasing effort; given low effort of the rival, a player reacts to an increase in effort of the rival by increasing effort.

Lemma 3 *An increase in player 1's overconfidence λ_1 leads to a contraction of his best response in the final, $\partial R_1^f(e_3)/\partial\lambda_1 \leqslant 0$, for $e_3^\alpha \leqslant \lambda_1 e_1^\alpha$, and to an expansion of his best response in the final, $\partial R_1^f(e_3)/\partial\lambda_1 \geqslant 0$, for $e_3^\alpha \geqslant \lambda_1 e_1^\alpha$. Moreover, the maximum value of player 1's best response in the final is independent of player 1's overconfidence.*

Lemma 3 describes how overconfidence shifts player 1's best response in the final. For a high effort of the rival, an increase in confidence raises player 1's effort level, while for low effort of the rival, an increase in confidence lowers player 1's effort level. Moreover, the maximal value taken by player 1's best response in the final is independent of his overconfidence bias.

Proposition 2 *In a final between an overconfident player 1 and a rational player 3 of an elimination contest with Alcalde and Dahm's CSF, the equilibrium efforts are $e_1^f = \alpha\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u/2c$, and $e_3^f = \alpha\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u/2c$, with $e_1^f < e_3^f < \overline{e}^f$. The perceived equilibrium winning probability of player 1 is $\widetilde{p}_{13}^f = 1-0.5\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}$, and the actual equilibrium winning probabilities are $p_{13}^f = 0.5\lambda_1^{-\frac{\alpha}{2\alpha+1}}$,* and *$p_{31}^f = 1-0.5\lambda_1^{-\frac{\alpha}{2\alpha+1}}$ with $\widetilde{p}_{13}^f > p_{31}^f > 1/2 > p_{13}^f$. The perceived equilibrium expected utility of player 1 is*

$$\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)] = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u$$

and the equilibrium expected utility of player 3 is

$$E^f[U_{31}(e_1^f, e_3^f)] = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u,$$

with $\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)] > E^f[U_{31}(e_1^f, e_3^f)] > \overline{E}^f[U(\overline{e}^f)]$.

Proposition 2 establishes that the overconfident player 1 exerts less effort in the final than his rational competitor, player 3. Intuitively, the overconfident player, given his perceived advantage, mistakenly thinks he can reduce his effort without endangering his prospects of success. The rational player, anticipating the overconfident player will lower his effort, also lowers her effort but not as much. Hence, both players exert lower effort than if both were rational.[11] At equilibrium, the overconfident player's perceived probability of winning the final is greater than $1/2$ whereas his actual winning probability is less than $1/2$ given the lower equilibrium effort.

We also see from Proposition 2 that the overconfident player's equilibrium perceived expected utility of the final, $\widetilde{E}^f[U_{13}(e_1^f, e_3^f, \lambda_1)]$, increases in his bias. An increase in the bias raises the overconfident player's perceived probability of winning the final and lowers his cost of effort. In addition, Proposition 2 shows that the rational player's equilibrium expected utility of the final, $E^f[U_{31}(e_1^f, e_3^f)]$, increases in the overconfident player's bias. An increase in the bias raises the rational player's probability of winning the final and lowers her cost of effort. Hence, the bias makes reaching the final more attractive not only to the overconfident player but also to a rational player. Finally, at equilibrium, the overconfident player's perceived expected utility of reaching the final is greater than that of the rational player.[12]

Figure 2 provides a visual depiction of Lemma 2, Lemma 3, and Proposition 2. It depicts the best responses and equilibrium efforts in a final where $\alpha = 0.9$, $\Delta u = 9$, $c = 1$, and $\lambda_1 = 1.1$. The equilibrium when players 1 and 3 are rational is depicted by point E at the 45 degree line. The equilibrium when player 1 is overconfident and player 3 is rational is depicted by point E' above the 45 degree line. We can see that player 1's overconfidence shifts his best response inwards in the area where the players' efforts are strategic complements. Hence, as overconfidence decreases player 1's effort in the final, the effort of player 3 also decreases.

[11] Note that as player 1's overconfidence increases, his best response, $R_1^f(e_3)$, shifts inwards for low values of e_3. Since player 3's best response, $R_3^f(e_1)$, is positively sloped for low values of e_1 and is unaffected by changes in player 1's bias, both players' equilibrium efforts in the final decrease in the bias of player 1.

[12] As the overconfident player's bias converges to infinity, the efforts of both players converge to zero, the overconfident player's perceived probability of winning the final converges to 1, his actual probability of winning the final converges to zero, his perceived expected utility of reaching the final converges to the utility of the winner's prize $u(w_1)$ and so does the rational player's expected utility of the final.

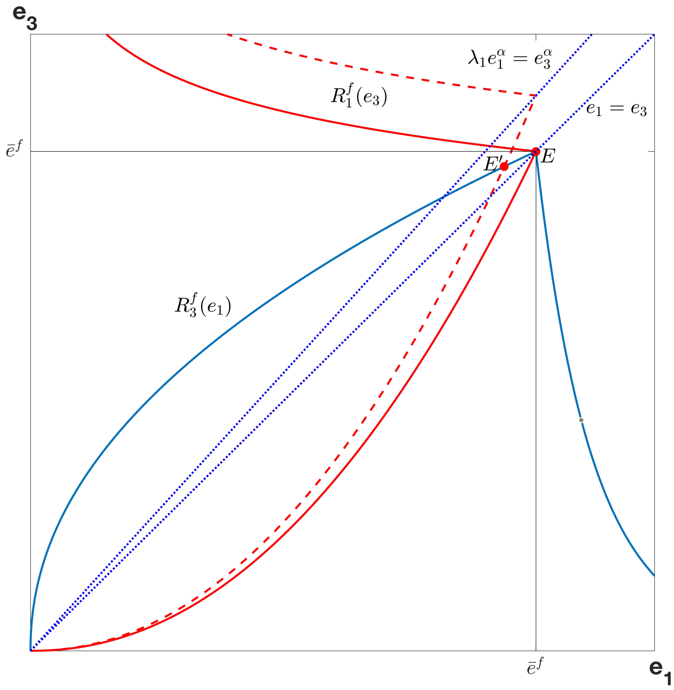

Figure 2: Equilibrium efforts in the final of an elimination contest with Alcalde and Dahm's CSF. The best response of a rational player 1 is depicted in solid red, and that of an overconfident player 1 in dashed red. Player 3 is rational and her best response is depicted in solid blue. Point E depicts the equilibrium when player 1 is rational and point E' when player 1 is overconfident. We set $\alpha = 0.9$, $\Delta u = 9$, $c = 1$, and $\lambda_1 = 1.1$.

We now move on to the semifinals. Note that, since players 3 and 4 are identical, player 1's perceived expected utility of reaching the final is:

$$\widetilde{v}_1 = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u. \tag{7}$$

It follows from (7) that

$$\frac{\partial \widetilde{v}_1}{\partial \lambda_1} = \frac{(\alpha+1)^2}{2(2\alpha+1)}\lambda_1^{-\frac{3\alpha+2}{2\alpha+1}}\Delta u > 0 \tag{8}$$

Hence, overconfidence generates an encouragement effect with Alcalde and Dahm's (2007) CSF. Moreover, the encouragement effect is given by:

$$\frac{\partial \widetilde{v}_1}{\partial \lambda_1}\frac{\lambda_1}{\widetilde{v}_1} = \frac{(\alpha+1)^2}{2(2\alpha+1)}\frac{1}{\frac{u(w_1)}{u(w_1)-u(w_2)}\lambda_1^{\frac{\alpha+1}{2\alpha+1}} - \frac{1+\alpha}{2}}. \tag{9}$$

Equation (9) shows that the size of the encouragement effect increases in the utility prize spread, decreases in the bias, and converges to zero when the bias converges to infinity (as $\lambda_1 \to \infty$ we have $\widetilde{v}_1 \to u(w_1)$).

We now introduce a lemma that characterizes how overconfidence shifts a player's best response in the semifinal.

Lemma 4 *An increase in player 1's overconfidence leads to a contraction of his best response in the semifinal,* $\partial R_1^s(e_2)/\partial\lambda_1 < 0$*, for* $e_2^\alpha < \lambda_1 e_1^\alpha$ *and* $\frac{\partial \widetilde{v}_1}{\partial \lambda_1}\frac{\lambda_1}{\widetilde{v}_1} < 1$*, otherwise, it leads to an expansion of his best response in the semifinal,* $\partial R_1^s(e_2)/\partial\lambda_1 > 0$*. Moreover, the maximum value of player 1's best response in the semifinal increases in player 1's overconfidence.*

Lemma 4 shows that an increase in confidence contracts player 1's best response in the semifinal when the rival exerts low effort and the encouragement effect is less than 1, otherwise, an increase in confidence expands player 1's best response in the semifinal. Moreover, the maximal value taken by player 1's best response in the semifinal is increasing in his overconfidence bias.

Next, we characterize the equilibrium of the semifinal between players 1 and 2.

Proposition 3 *Consider the semifinal between an overconfident player 1 and a rational player 2 of an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players.*
(i) If $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ *and* $\lambda_1 < \hat{\lambda}$ *where* $\hat{\lambda}$ *is given by* $\frac{1+\alpha}{2}\frac{u(w_1)-u(w_2)}{u(w_1)} = \frac{\hat{\lambda}-1}{\hat{\lambda}-\hat{\lambda}^{-\frac{\alpha+1}{2\alpha+1}}}$*, then the equilibrium efforts and winning probabilities satisfy* $e_1^s > \bar{e}^s > e_2^s$ *and* $\widetilde{p}_{12}^s > p_{12}^s > 1/2 > p_{21}^s$*.*
(ii) If either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ *or* $\lambda_1 \geqslant \hat{\lambda}$*, then the equilibrium efforts and winning probabilities satisfy* $e_1^s \leqslant e_2^s \leqslant \bar{e}^s$ *and* $\widetilde{p}_{12}^s > p_{21}^s \geqslant \frac{1}{2} \geqslant p_{12}^s$*.*

Proposition 3 shows that when the utility prize spread is large enough, $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$, and overconfidence is not too extreme, $\lambda_1 < \hat{\lambda}$, the encouragement effect dominates the complacency effect and therefore the overconfident player exerts more effort in the semifinal than his rational rival. When either of these two conditions is not met, the encouragement effect is dominated by the complacency effect, and the overconfident player exerts less effort in the semifinal than his rational rival.

The proof of Proposition 3 shows that, at the Nash equilibrium of the semifinal, the overconfident player 1 perceived winning probability is greater than 1/2, that is, $\widetilde{p}_{12}^s(e_1^s, e_2^s, \lambda_1) > 1/2$. From (6) this is equivalent to $\lambda_1(e_1^s)^\alpha \geqslant (e_2^s)^\alpha$ which, in turn, implies that the complacency effect is

$$\frac{\partial mg\widetilde{p}_{12}^{s}}{\partial\lambda_1}\frac{\lambda_1}{mg\widetilde{p}_{12}^{s}} = -\frac{\alpha}{2\lambda_1^2}\frac{(e_2^s)^{\alpha}}{(e_1^s)^{\alpha+1}}\frac{\lambda_1}{\frac{\alpha}{2\lambda_1}\frac{(e_2^s)^{\alpha}}{(e_1^s)^{\alpha+1}}} = -1. \tag{10}$$

It follows from equations (9) and (10) that when the bias is relatively small, i.e., λ_1 is close to 1, a necessary condition for the encouragement effect to dominate is that it is greater than 1 when $\lambda_1 = 1$. This is equivalent to $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$. Thus, the encouragement effect dominates when the utility prize spread is large enough and the bias is close to 1. As the bias increases, the size of the encouragement effect decreases and converges to zero while the size of the complacency effect is fixed at -1. Hence, there exists an upper bound for the bias above which the complacency effect dominates.

Figure 3 illustrates the first part of Proposition 3. It depicts the best responses and equilibrium efforts in a semifinal where $\alpha = 0.9$, $\Delta u = 9$, $u(w_2) = 9/4$, $c = 1$, and $\lambda_1 = 1.1$. These parameters satisfy $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda} = 2.436$. Point E at the 45 degree line depicts the equilibrium when players 1 and 2 are rational. Point E' below the 45 degree line depicts the equilibrium when player 1 is overconfident and player 2 is rational. We can see that player 1's overconfidence shifts his best response outwards for every possible effort level of player 2. The outward shift implies that the equilibrium E' is attained in an area where the players' efforts are strategic substitutes. Hence, as overconfidence increases player 1's effort in the semifinal, the effort of player 2 decreases. Because the encouragement effect outweighs the complacency effect, the overconfident player 1 exerts higher effort than his rational rival, player 2, in the semifinal.

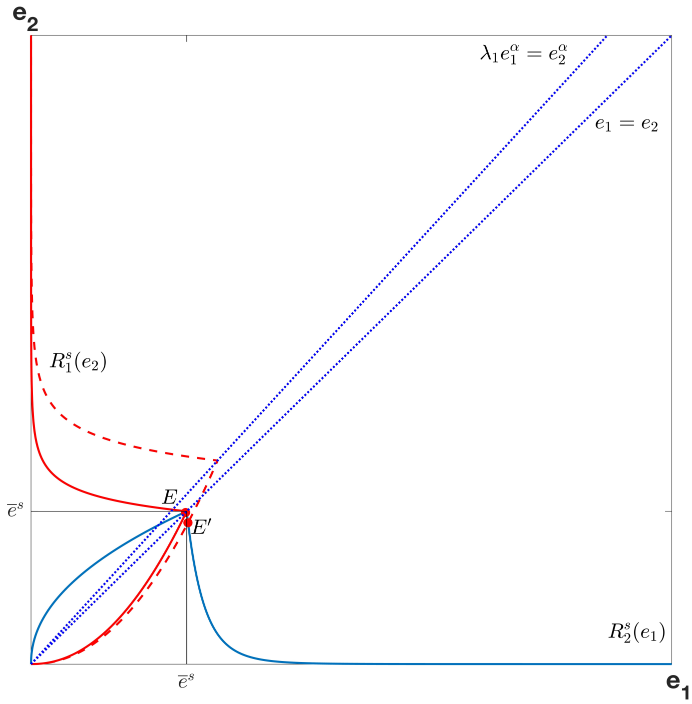

Figure 3: Equilibrium efforts in a semifinal of an elimination contest with Alcalde and Dahm's CSF. The best response of a rational player 1 is depicted in solid red, and that of an overconfident player 1 in dashed red. Player 2 is rational and her best response is depicted in solid blue. Point E depicts the equilibrium when player 1 is rational, and point E' when player 1 is overconfident. We set $\alpha = 0.9$, $\Delta u = 9$, $u(w_2) = 9/4$, $c = 1$, and $\lambda_1 = 1.1$. These values satisfy $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$.

Next, we characterize the equilibrium of the semifinal with rational players 3 and 4.

Proposition 4 *In the semifinal between rational players 3 and 4 of an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players, the equilibrium efforts and winning probabilities satisfy* $e_3^s = e_4^s > \bar{e}^s$ *and* $p_{34}^s = p_{43}^s = 1/2$.

Proposition 4 demonstrates that the presence of an overconfident player in an elimination contest has a spillover effect on the equilibrium efforts in the semifinal with two rational players. In that semifinal, both players have an incentive to exert higher effort when player 1 is overconfident than if player 1 were rational. This occurs because the winner of the semifinal with two rational players will face the overconfident player 1 in the final with probability $p_{12}^s > 0$ and this leads to a higher expected utility of reaching the final than if player 1 were rational.

5.3 Equilibrium Winning Probabilities

We now analyze how overconfidence affects the players' chances of winning the elimination contest. We are particularly interested in knowing if an overconfident player can be the one with the highest chances of winning an elimination contest.

We denote by P_i player i's equilibrium probability of winning the elimination contest. P_i is the product of player i's equilibrium probability of winning his semifinal and his equilibrium probability of winning the final:

$$P_i = p^s_{ij}(p^s_{hk}p^f_{ih} + p^s_{kh}p^f_{ik}).$$

Hence, player 1's equilibrium probability of winning the elimination contest is

$$P_1 = p^s_{12}(p^s_{34}p^f_{13} + p^s_{43}p^f_{14}) = p^s_{12}p^f_{13},$$

where the second equality follows from $p^f_{13} = p^f_{14}$. Player 2's equilibrium probability of winning the elimination contest is

$$P_2 = p^s_{21}(p^s_{34}p^f_{23} + p^s_{43}p^f_{24}) = p^s_{21}\frac{1}{2},$$

where the second equality follow from $p^s_{34} + p^s_{43} = 1$ and $p^f_{23} = p^f_{24} = 1/2$. Players 3 and 4 equilibrium probability of winning the elimination contest is

$$P_3 = P_4 = p^s_{34}(p^s_{12}p^f_{31} + p^s_{21}p^f_{32}) = \frac{1}{2}\left[p^s_{12}(1 - p^f_{13}) + (1 - p^s_{12})\frac{1}{2}\right].$$

From Proposition 2, we have $p^f_{13} < 1/2 < p^f_{31}$, indicating that the overconfident player is always the underdog in the final. By contrast, Proposition 3 shows that if the encouragement effect dominates, then $p^s_{12} > 1/2 > p^s_{21}$, meaning the overconfident player is the favorite in the semifinal. If the complacency effect instead prevails, the overconfident player becomes the underdog in the semifinal. Consequently, the overconfident player may be favored to win in the semifinal but remain the underdog in the final, or else be the underdog in both stages. Thus, the overconfident player is never the favorite throughout.[13]

It follows from the previous paragraph that a necessary condition for the overconfident player to have the highest equilibrium probability of winning the elimination contest is that his equilibrium probability of winning the semifinal p^s_{12} is greater than $1/2$. In other words, the overconfident player must exert higher effort in his semifinal than the rational

[13] The concepts of favorite and underdog were introduced by Dixit (1987) for one-shot Cournot-Nash games. Yildirim (2005) extends these concepts to contests where players can exert effort in multiple periods before the contest concludes.

player 2. When this is the case, rational player 2 has an equilibrium probability of winning which is less than 1/4 since $p^s_{21} = 1 - p^s_{12} < 1/2$. Rational players 3 and 4 have an equilibrium winning probability greater than 1/4 since they have a positive probability of facing the overconfident player 1 in the final. Hence, in equilibrium, the overconfident player 1 has the highest winning probability when $P_1 > P_3 = P_4$ which is equivalent to $6p^s_{12}p^f_{13} - p^s_{12} - 1 > 0$.

Proposition 5 *In an elimination contest with Alcalde and Dahm's CSF, one overconfident player, and three rational players, if* $\alpha > \frac{\sqrt{97}-5}{12}$ *and* $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$*, then there exist* $\lambda_1 \in (1, \hat{\lambda})$ *for which the overconfident player has the highest equilibrium probability of winning the elimination contest, i.e.,* $P_1 > P_3 = P_4 > 1/4 > P_2$.

Proposition 5 shows that an overconfident player can have the highest equilibrium probability of winning an elimination contest. Moreover, for this to be the case three conditions need to be met. First, the CSF's effort sensitivity parameter α must be greater than $\frac{\sqrt{97}-5}{12} \approx 0.404$. Hence, the role that effort plays in determining the winning probabilities must be sufficiently high. Second, the utility prize spread needs to be large enough, $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$. Third, the overconfidence bias must be relatively low.

6 Conclusion

We analyze how overconfidence affects behavior in multistage elimination contests. Our findings reveal a nuanced interplay between overconfidence and effort exertion in a two-stage elimination contest.

In the final stage, an overconfident player always exerts lower effort at equilibrium than a rational player. The (mis)perceived advantage of the overconfident player leads him to think, mistakenly, he can reduce his effort without endangering his prospects of success. The rational player, aware of the rival's bias, also lowers her effort but not as much. Hence, the bias lowers the overconfident player's probability of winning the final.

In the semifinals stage, the bias has two opposite effects on an overconfident player's incentives to exert effort. On the one hand, it raises the overconfident player's perceived expected utility of reaching the final. On the other hand, it leads to a decrease in the overconfident player's perceived marginal probability of winning the semifinal. The first effect encourages an overconfident player to raise effort whereas the second effect makes him complacent and leads him to lower effort. If the encouragement effect dominates, an overconfident player exert higher effort at equilibrium in a semifinal than a rational rival.

We show that for the encouragement effect to dominate two conditions have to be met. First, the prize spread needs to be large enough. Second, the overconfident player's bias cannot be too extreme. The intuition behind these two conditions is as follows. The higher is the prize spread, the higher is the perceived expected utility of reaching the final, and the higher is the encouragement effect. As the overconfident player's bias increases, the encouragement effect gets smaller whereas the complacency effect is unchanged. Hence, there exists an upper bound for the bias above which the complacency effect dominates.

We also find that an overconfident player can be the one with the highest probability of winning an elimination contest. For this to be the case, three conditions have to be met. First, the role that effort plays in determining the winning probabilities must be sufficiently high. Second, the prize spread needs to be large enough. Third, the overconfident player's bias needs to be small.

Our study contributes to the literature on CEO overconfidence by providing a novel explanation for why overconfident managers are selected to CEO positions. In addition, our results highlight the role that increases in executive compensation (interpreted as increases in the prize spread) can have in making elimination contests more attractive to overconfident managers. Our study also contributes to the literature on gender gaps in the labor market by showing that large executive compensation coupled with higher male confidence can make competing for top business positions more attractive to males.

Future research could explore elimination contests where players differ not only in confidence levels but also in abilities. If an overconfident player has lower ability than a rational player, the bias may increase both his perceived expected utility of reaching the final as well as his marginal probability of winning. Additionally, future studies could examine how contest designers should optimally structure prizes in the presence of overconfident players.

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Appendix

Proof of Lemma 1: The best response of player i in a final against player j, $R_i^f(e_j)$, is implicitly defined by

$$mg\widetilde{p}_{ij}^f(e_i, e_j, \lambda_i)\Delta u = c.$$

The best response of player i in a semifinal against player j, $R_i^s(e_j)$, is implicitly defined by

$$mg\widetilde{p}_{ij}^s(e_i, e_j, \lambda_i)\widetilde{v}_i = c.$$

Hence, the slope of the best response of player i in stage $t \in \{s, f\}$ is given by

$$-\frac{\partial R_i^t/\partial e_j}{\partial R_i^t/\partial e_i} = -\frac{\frac{\partial^2 \widetilde{E}^t[U_{ij}(e_i,e_j,\lambda_i)]}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{E}^t[U_{ij}(e_i,e_j,\lambda_i)]}{\partial e_i^2}} = -\frac{\frac{\partial^2 \widetilde{p}_{ij}^t(e_i,e_j,\lambda_i)}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{p}_{ij}^t(e_i,e_j,\lambda_i)}{\partial e_i^2}} = -\frac{\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i^2}}. \tag{11}$$

Using the contraction mapping theorem, we know that an equilibrium exists and is unique when the product of the slopes of the best responses is less than 1, that is,

$$\left(-\frac{\partial R_i^t/\partial e_j}{\partial R_i^t/\partial e_i}\right) \times \left(-\frac{\partial R_j^t/\partial e_i}{\partial R_j^t/\partial e_j}\right) < 1, \tag{12}$$

or, substituting (11) into (12),

$$\frac{\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i^2}} \times \frac{\frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_j^2}} < 1,$$

which is equivalent to assumption (3)

$$\frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i \partial e_j} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_i \partial e_j} < \frac{\partial^2 \widetilde{p}_{ij}^t}{\partial e_i^2} \frac{\partial^2 \widetilde{p}_{ji}^t}{\partial e_j^2}.$$

Since each subgame of the elimination contest has a unique Nash equilibrium, it follows that the elimination contest has a unique Subgame Perfect Equilibrium.

Derivation of equation (4): The first-order conditions (FOCS, from now on) in a semifinal between i and j are

$$\frac{\partial \widetilde{p}_{ij}^s(e_i, e_j, \lambda_i)}{\partial e_i}\widetilde{v}_i = c$$

$$\frac{\partial \widetilde{p}_{ji}^s(e_i, e_j, \lambda_j)}{\partial e_j}\widetilde{v}_j = c$$

Total differentiation with respect to e_i, e_j, and λ_i gives us

$$\left(\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2}\frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial e_j}\frac{de_j}{d\lambda_i} + \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial \lambda_i}\right)\widetilde{v}_i + \frac{\partial \widetilde{p}_{ij}^s}{\partial e_i}\frac{\partial \widetilde{v}_i}{\partial \lambda_i} = 0$$

$$\left(\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j \partial e_i}\frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j^2}\frac{de_j}{d\lambda_i}\right)\widetilde{v}_j = 0$$

Solving the second equation for $\frac{de_j}{d\lambda_i}$ we obtain

$$\frac{de_j}{d\lambda_i} = -\frac{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j \partial e_i}}{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j^2}}\frac{de_i}{d\lambda_i}$$

Replacing in the first equation we obtain

$$\left(\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2}\frac{de_i}{d\lambda_i} - \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial e_j}\frac{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j \partial e_i}}{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j^2}}\frac{de_i}{d\lambda_i} + \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial \lambda_i}\right)\widetilde{v}_i + \frac{\partial \widetilde{p}_{ij}^s}{\partial e_i}\frac{\partial \widetilde{v}_i}{\partial \lambda_i} = 0$$

Solving this equation for $\frac{de_i}{d\lambda_i}$ we have

$$\frac{de_i}{d\lambda_i} = -\frac{\frac{\partial \widetilde{p}_{ij}^s}{\partial e_i}\frac{\partial \widetilde{v}_i}{\partial \lambda_i} + \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial \lambda_i}\widetilde{v}_i}{\left(\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2} - \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial e_j}\frac{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j \partial e_i}}{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j^2}}\right)\widetilde{v}_i}$$

or

$$\frac{de_i}{d\lambda_i} = -\frac{1}{\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2}}\frac{\frac{\partial \widetilde{p}_{ij}^s}{\partial e_i}\frac{\partial \widetilde{v}_i}{\partial \lambda_i} + \frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial \lambda_i}\widetilde{v}_i}{\left(1 - \frac{\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i \partial e_j}}{\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2}}\frac{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j \partial e_i}}{\frac{\partial^2 \widetilde{p}_{ji}^s}{\partial e_j^2}}\right)\widetilde{v}_i} = -\frac{\frac{mg\widetilde{p}_{ij}^s}{\lambda_i}}{\frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_i}}\frac{\frac{\partial \widetilde{v}_i}{\partial \lambda_i}\frac{\lambda_i}{\widetilde{v}_i} + \frac{\partial mg\widetilde{p}_{ij}^s}{\partial \lambda_i}\frac{\lambda_i}{mg\widetilde{p}_{ij}^s}}{\left(1 - \frac{\frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_j}}{\frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_i}}\frac{\frac{\partial mg\widetilde{p}_{ji}^s}{\partial e_i}}{\frac{\partial mg\widetilde{p}_{ji}^s}{\partial e_j}}\right)} \tag{13}$$

Note that in equation (13) the sign of the denominator of the first term is positive due to the assumption $\frac{\partial^2 \widetilde{p}_{ij}^s}{\partial e_i^2} = \frac{\partial mg\widetilde{p}_{ij}^s}{\partial e_i} < 0$ and the sign of the denominator of the second term is positive due to assumption (3).

Proof of Proposition 1:

The final stage

$$p_{13}^f = \begin{cases} 1 - \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha} & \text{if } e_1^\alpha \geqslant e_3^\alpha \\ \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha} & \text{if } e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

$$p_{31}^f = \begin{cases} 1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha} & \text{if } e_3^\alpha \geqslant e_1^\alpha \\ \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha} & \text{if } e_3^\alpha \leqslant e_1^\alpha \end{cases}$$

Rational player 1

$$E^f(U_{13}) = p_{13}^f \Delta u + u(w_2) - ce_1 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\right)\Delta u + u(w_2) - ce_1 & \text{if } \quad e_1^\alpha \geqslant e_3^\alpha \\ \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha}\Delta u + u(w_2) - ce_1 & \text{if } \quad e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

Rational player 3

$$E^f(U_{31}) = p^f_{31}\Delta u + u(w_2) - ce_3 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha}\right)\Delta u + u(w_2) - ce_3 & \text{if } e_3^\alpha \geqslant e_1^\alpha \\ \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\Delta u + u(w_2) - ce_3 & \text{if } e_3^\alpha \leqslant e_1^\alpha \end{cases}$$

There are 2 cases:

$$\begin{cases} e_1^\alpha \geqslant e_3^\alpha \\ e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

1. Equilibrium efforts in the final

   (1) case 1: $e_1^\alpha \geqslant e_3^\alpha$

   Player 1 $\quad max \quad E^f(U_{13}) = \left(1 - \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\right)\Delta u + u(w_2) - ce_1$

   Player 3 $\quad max \quad E^f(U_{31}) = \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\Delta u + u(w_2) - ce_3$

   FOCS:

   $[e_1] \quad \frac{\alpha}{2}\frac{e_3^\alpha}{e_1^{\alpha+1}}\Delta u - c = 0$

   $[e_3] \quad \frac{\alpha}{2}\frac{e_3^{\alpha-1}}{e_1^\alpha}\Delta u - c = 0$

   SOCS:

   $[e_1] \quad \frac{\alpha}{2}(-\alpha - 1)\frac{e_3^\alpha}{e_1^{\alpha+2}}\Delta u < 0$

   $[e_3] \quad \frac{\alpha}{2}(\alpha - 1)\frac{e_3^{\alpha-2}}{e_1^\alpha}\Delta u < 0$

   Solving the FOCS, we get $e_1^f = e_3^f = \frac{\alpha}{2c}\Delta u$

   (2) case 2: $e_1^\alpha \leqslant e_3^\alpha$

   The same as the previous case.

   Thus the unique equilibrium is $\overline{e}^f = e_1^f = e_3^f = \frac{\alpha}{2c}\Delta u$,

2. Equilibrium winning probabilities in the final

$$\overline{p}^f = p^f_{13} = p^f_{31} = \frac{1}{2}$$

3. Expected utilities of final

$$\overline{E}^f(U) = E^f(U_{13}) = E^f(U_{31}) = \frac{1}{2}[u(w_1) + u(w_2)] - c\frac{\alpha}{2c}\Delta u = \frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)$$

   Since $0 < \alpha \leqslant 1$, we have $\overline{E}^f(U) \geqslant 0$. The participation constraints are satisfied.

The semifinals stage

1. Expected utilities of reaching the final

   Player 1's expected utility of reaching the final is given by

$$v_1 = p^s_{34} E^f(U_{13}) + p^s_{43} E^f(U_{14}) = \overline{E}^f(U) = \frac{1-\alpha}{2} u(w_1) + \frac{1+\alpha}{2} u(w_2)$$

   Since all 4 players are identical,

$$\overline{v} = v_1 = v_2 = v_3 = v_4 = \frac{1-\alpha}{2} u(w_1) + \frac{1+\alpha}{2} u(w_2).$$

2. Equilibrium efforts in the semifinal

   Using the extension of the equilibrium result in the final, we can get that

$$\overline{e}^s = e^s_1 = e^s_2 = e^s_3 = e^s_4 = \frac{\alpha}{2c}\overline{v} = \frac{\alpha}{2c}\left[\frac{1-\alpha}{2} u(w_1) + \frac{1+\alpha}{2} u(w_2)\right]$$

3. Equilibrium winning probabilities in the semifinal

$$\overline{p}^s = p^s_{12} = p^s_{21} = p^s_{34} = p^s_{43} = \frac{1}{2}$$

4. Expected utilities of semifinal

$$\overline{E}^s(U) = \frac{1}{2}\overline{v} - c\frac{\alpha}{2c}\overline{v} = \frac{1-\alpha}{2}\overline{v} = \frac{1-\alpha}{2}\left[\frac{1-\alpha}{2} u(w_1) + \frac{1+\alpha}{2} u(w_2)\right]$$

   Since $0 < \alpha \leqslant 1$, we have $\overline{E}^s(U) \geqslant 0$. The participation constraints are satisfied.

5. Note that if $\frac{u(w_1)}{u(w_2)} > \frac{3+\alpha}{1+\alpha}$, then $\overline{e}^s < \overline{e}^f$. Since $\alpha \in (0, 1]$, this inequality is satisfied for all α when $u(w_1) > 3u(w_2)$.

Proof of Lemma 2: Player 1's best response in a final with 3, $R^f_1(e_3)$, is defined by

$$\begin{cases} \frac{\alpha}{2\lambda_1}\frac{e_3^\alpha}{e_1^{\alpha+1}}\Delta u = c & \text{if } \lambda_1 e_1^\alpha \geqslant e_3^\alpha \\ \frac{\alpha}{2}\lambda_1\frac{e_1^{\alpha-1}}{e_3^\alpha}\Delta u = c & \text{if } \lambda_1 e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

Hence, the slope of player 1's best response in a final with 3 is

$$-\frac{\frac{\partial R^f_1(e_3)}{\partial e_3}}{\frac{\partial R^f_1(e_3)}{\partial e_1}} = -\frac{\frac{\partial^2 \widetilde{E}^f(U_{13})}{\partial e_1 \partial e_3}}{\frac{\partial^2 \widetilde{E}^f(U_{13})}{\partial e_1^2}} = \begin{cases} -\dfrac{\frac{\alpha^2}{2\lambda_1}\frac{e_3^{\alpha-1}}{e_1^{\alpha+1}}\Delta u}{-(1+\alpha)\frac{\alpha}{2\lambda_1}\frac{e_3^\alpha}{e_1^{\alpha+2}}\Delta u} = \frac{\alpha}{1+\alpha}\frac{e_1}{e_3} & \text{if } \lambda_1 e_1^\alpha > e_3^\alpha \\ -\dfrac{-\frac{\alpha^2}{2}\lambda_1\frac{e_1^{\alpha-1}}{e_3^{\alpha+1}}\Delta u}{-(1-\alpha)\frac{\alpha}{2}\lambda_1\frac{e_1^{\alpha-2}}{e_3^\alpha}\Delta u} = -\frac{\alpha}{1-\alpha}\frac{e_1}{e_3} & \text{if } \lambda_1 e_1^\alpha < e_3^\alpha \end{cases}$$

Therefore, the sign of the slope of player 1's best response in a final with 3 is positive for $\lambda_1 e_1^\alpha > e_3^\alpha$ and negative for $\lambda_1 e_1^\alpha < e_3^\alpha$. This implies that $R_1^f(e_3)$ increases in e_3 for $\lambda_1 e_1^\alpha > e_3^\alpha$, reaches the maximum at $\lambda_1 e_1^\alpha = e_3^\alpha$, and decreases in e_3 for $\lambda_1 e_1^\alpha < e_3^\alpha$. The analysis of player 3's best response in a final with 1, $R_3^f(e_1)$, is similar.

Proof of Lemma 3: From player 1's best response in the final (see Lemma 2) we have

$$\frac{\partial R_1^f(e_3)}{\partial \lambda_1} = \begin{cases} -\frac{\alpha}{2\lambda_1^2}\frac{e_3^\alpha}{e_1^{\alpha+1}}\Delta u & \text{if } \lambda_1 e_1^\alpha \geqslant e_3^\alpha \\ \frac{\alpha}{2}\frac{e_1^{\alpha-1}}{e_3^\alpha}\Delta u & \text{if } \lambda_1 e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

We see that $\partial R_1^f(e_3)/\partial\lambda_1 \leqslant 0$ for $\lambda_1 e_1^\alpha \geqslant e_3^\alpha$ and $\partial R_1^f(e_3)/\partial\lambda_1 \geqslant 0$ for $\lambda_1 e_1^\alpha \leqslant e_3^\alpha$. Substituting $e_3^\alpha = \lambda_1 e_1^\alpha$ into player 1's best response and denoting the maximal effort that 1 is willing to invest in the final by e_1^{fmax} we obtain

$$\frac{\alpha}{2\lambda_1}\frac{\lambda_1 (e_1^{fmax})^\alpha}{(e_1^{fmax})^{\alpha+1}}\Delta u = c$$

or

$$e_1^{fmax} = \frac{\alpha}{2c}\Delta u.$$

This implies that the value of e_1 corresponding to the maximum value of player 1's best response in the final, e_1^{fmax}, does not depend on λ_1.

Proof of Proposition 2: The perceived winning probabilities of the players are:

$$\widetilde{p}_{13}^f = \begin{cases} 1 - \frac{1}{2}\frac{e_3^\alpha}{\lambda_1 e_1^\alpha} & \text{if} \quad \lambda_1 e_1^\alpha \geqslant e_3^\alpha \\ \frac{1}{2}\frac{\lambda_1 e_1^\alpha}{e_3^\alpha} & \text{if} \quad \lambda_1 e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

$$p_{31}^f = \begin{cases} 1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha} & \text{if} \quad e_3^\alpha \geqslant e_1^\alpha \\ \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha} & \text{if} \quad e_3^\alpha \leqslant e_1^\alpha \end{cases}$$

Overconfident player 1

$$E^f(U_{13}) = \widetilde{p}_{13}^f\Delta u + u(w_2) - ce_1 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_3^\alpha}{\lambda_1 e_1^\alpha}\right)\Delta u + u(w_2) - ce_1 & \text{if} \quad \lambda_1 e_1^\alpha \geqslant e_3^\alpha \\ \frac{1}{2}\frac{\lambda_1 e_1^\alpha}{e_3^\alpha}\Delta u + u(w_2) - ce_1 & \text{if} \quad \lambda_1 e_1^\alpha \leqslant e_3^\alpha \end{cases}$$

Rational player 3

$$E^f(U_{31}) = p_{31}^f\Delta u + u(w_2) - ce_3 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha}\right)\Delta u + u(w_2) - ce_3 & \text{if} \quad e_3^\alpha \geqslant e_1^\alpha \\ \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\Delta u + u(w_2) - ce_3 & \text{if} \quad e_3^\alpha \leqslant e_1^\alpha \end{cases}$$

There are 4 cases:

$$\begin{cases} \lambda_1 e_1^\alpha \geqslant e_3^\alpha \quad and \quad e_3 \geqslant e_1 \\ \lambda_1 e_1^\alpha \geqslant e_3^\alpha \quad and \quad e_3 \leqslant e_1 \\ \lambda_1 e_1^\alpha \leqslant e_3^\alpha \quad and \quad e_3 \geqslant e_1 \\ \lambda_1 e_1^\alpha \leqslant e_3^\alpha \quad and \quad e_3 \leqslant e_1 \end{cases}$$

Since $\lambda_1 > 1$, the fourth case is impossible.

1. Equilibrium efforts

   (1) case 1: $\lambda_1 e_1^\alpha \geqslant e_3^\alpha \quad and \quad e_3 \geqslant e_1$

   Player 1 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_3^\alpha}{\lambda_1 e_1^\alpha}\right)\Delta u + u(w_2) - ce_1$

   Player 3 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha}\right)\Delta u + u(w_2) - ce_3$

   FOCS:

   $[e_1] \quad \frac{\alpha}{2\lambda_1}\frac{e_3^\alpha}{e_1^{\alpha+1}}\Delta u - c = 0$

   $[e_3] \quad \frac{\alpha}{2}\frac{e_1^\alpha}{e_3^{\alpha+1}}\Delta u - c = 0$

   SOCS:

   $[e_1] \quad \frac{\alpha}{2\lambda_1}(-\alpha - 1)\frac{e_3^\alpha}{e_1^{\alpha+2}}\Delta u < 0$

   $[e_3] \quad \frac{\alpha}{2}(-\alpha - 1)\frac{e_1^\alpha}{e_3^{\alpha+2}}\Delta u < 0$

   Solving the FOCS, we obtain

   $$e_1^f = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u$$

   $$e_3^f = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u$$

   Check the conditions $\lambda_1(e_1^f)^\alpha \geqslant (e_3^f)^\alpha$ and $e_3^f \geqslant e_1^f$: $\lambda_1(e_1^f)^\alpha \geqslant (e_3^f)^\alpha$ is equivalent to $\frac{\alpha+1}{2\alpha+1} \geqslant 0$ and $e_3^f \geqslant e_1^f$ is equivalent to $\frac{1}{2\alpha+1} \geqslant 0$, hence, the conditions are satisfied.

   (2) case 2: $\lambda_1 e_1^\alpha \geqslant e_3^\alpha \quad and \quad e_3 \leqslant e_1$

   Player 1 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_3^\alpha}{\lambda_1 e_1^\alpha}\right)\Delta u + u(w_2) - ce_1$

   Player 3 $\quad max \quad \frac{1}{2}\frac{e_3^\alpha}{e_1^\alpha}\Delta u + u(w_2) - ce_3$

   FOCS:

   $[e_1] \quad \frac{\alpha}{2\lambda_1}\frac{e_3^\alpha}{e_1^{\alpha+1}}\Delta u - c = 0$

$[e_3] \quad \frac{\alpha}{2}\frac{e_3^{\alpha-1}}{e_1^{\alpha}}\Delta u - c = 0$

divide the two FOCS, we get

$$\frac{e_3}{e_1} = \lambda_1 > 1$$

which contradicts the condition that $e_3 \leqslant e_1$

(3) case 3: $\lambda_1 e_1^\alpha \leqslant e_3^\alpha \quad and \quad e_3 \geqslant e_1$

Player 1 $\quad max \quad \frac{1}{2}\frac{\lambda_1 e_1^\alpha}{e_3^\alpha}\Delta u + u(w_2) - ce_1$

Player 3 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_3^\alpha}\right)\Delta u + u(w_2) - ce_3$

FOCS:

$[e_1] \quad \frac{\alpha\lambda_1}{2}\frac{e_1^{\alpha-1}}{e_3^{\alpha}}\Delta u - c = 0$

$[e_3] \quad \frac{\alpha}{2}\frac{e_1^{\alpha}}{e_3^{\alpha+1}}\Delta u - c = 0$

divide the two FOCS, we get

$$\frac{e_3}{e_1} = \frac{1}{\lambda_1} < 1$$

which contradicts the condition that $e_3 \geqslant e_1$

Thus the unique equilibrium is

$$e_1^f = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u$$
$$e_3^f = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u$$

where $e_1^f < \overline{e}^f$ and $e_3^f < \overline{e}^f$:

$$e_1^f < \overline{e}^f \iff \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u < \frac{\alpha}{2c}\Delta u \iff \lambda_1^{-\frac{\alpha+1}{2\alpha+1}} < 1$$

$$e_3^f < \overline{e}^f \iff \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u < \frac{\alpha}{2c}\Delta u \iff \lambda_1^{-\frac{\alpha}{2\alpha+1}} < 1$$

2. Equilibrium winning probabilities

The true winning probabilities are

$$p_{13}^f = \frac{1}{2}\left(\frac{e_1^f}{e_3^f}\right)^\alpha = \frac{1}{2}\left(\lambda_1^{-\frac{1}{2\alpha+1}}\right)^\alpha = \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}$$

$$p_{31}^f = 1 - p_{13}^f = 1 - \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}$$

The overconfident player 1's perceived winning probability is

$$\widetilde{p}_{13}^f = 1 - \frac{1}{2}\frac{(e_3^f)^\alpha}{\lambda_1(e_1^f)^\alpha} = 1 - \frac{1}{2}\frac{\left(\frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u\right)^\alpha}{\lambda_1\left(\frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u\right)^\alpha} = 1 - \frac{1}{2}\frac{\left(\lambda_1^{-\frac{\alpha}{2\alpha+1}}\right)^\alpha}{\lambda_1\left(\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^\alpha} = 1 - \frac{1}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}$$

We can easily get $\widetilde{p}_{13}^f > p_{31}^f > \frac{1}{2} > p_{13}^f$.

3. Expected utilities of final

$$\widetilde{E}^f(U_{13}) = \widetilde{p}_{13}^f u(w_1) + (1-\widetilde{p}_{13}^f)u(w_2) - ce_1^f = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u$$

$$E^f(U_{31}) = p_{31}^f u(w_1) + (1-p_{31}^f)u(w_2) - ce_3^f = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u$$

We can easily get that $\widetilde{E}^f(U_{13}) > E^f(U_{31}) > \overline{E}^f(U)$. Since $\overline{E}^f(U) \geqslant 0$, the participation constraints of both players are satisfied.

Proof of Lemma 4: The best response of player 1 in the semifinal with 2, $R_1^s(e_2)$, is defined by

$$\begin{cases} \frac{\alpha}{2\lambda_1}\frac{e_2^\alpha}{e_1^{\alpha+1}}\widetilde{v}_1 = c & \text{if } \lambda_1 e_1^\alpha \geqslant e_2^\alpha \\ \frac{\alpha}{2}\lambda_1\frac{e_1^{\alpha-1}}{e_2^\alpha}\widetilde{v}_1 = c & \text{if } \lambda_1 e_1^\alpha \leqslant e_2^\alpha \end{cases}$$

Hence,

$$\frac{\partial R_1^s(e_2)}{\partial \lambda_1} = \begin{cases} -\frac{\alpha}{2\lambda_1^2}\frac{e_2^\alpha}{e_1^{\alpha+1}}\widetilde{v}_1 + \frac{\alpha}{2\lambda_1}\frac{e_2^\alpha}{e_1^{\alpha+1}}\frac{\partial\widetilde{v}_1}{\partial\lambda_1} = \frac{\alpha}{2\lambda_1^2}\frac{e_2^\alpha}{e_1^{\alpha+1}}\left(-\widetilde{v}_1 + \lambda_1\frac{\partial\widetilde{v}_1}{\partial\lambda_1}\right) & \text{if } \lambda_1 e_1^\alpha \geqslant e_2^\alpha \\ \\ \frac{\alpha}{2}\frac{e_1^{\alpha-1}}{e_2^\alpha}\widetilde{v}_1 + \frac{\alpha}{2}\lambda_1\frac{e_1^{\alpha-1}}{e_2^\alpha}\frac{\partial\widetilde{v}_1}{\partial\lambda_1} = \frac{\alpha}{2}\frac{e_1^{\alpha-1}}{e_2^\alpha}\left(\widetilde{v}_1 + \lambda_1\frac{\partial\widetilde{v}_1}{\partial\lambda_1}\right) & \text{if } \lambda_1 e_1^\alpha \leqslant e_2^\alpha \end{cases} \quad (14)$$

Since $\frac{\partial\widetilde{v}_1}{\partial\lambda_1} > 0$ it follows from (14) that $\frac{\partial R_1^s}{\partial\lambda_1} > 0$ for $\lambda_1 e_1^\alpha \leqslant e_2^\alpha$. Since $\frac{\partial\widetilde{v}_1}{\partial\lambda_1} > 0$ it also follows from (14) that $\frac{\partial\widetilde{v}_1}{\partial\lambda_1}\frac{\lambda_1}{\widetilde{v}_1} < 1(> 1)$, then $\frac{\partial R_1^s}{\partial\lambda_1} < 0(> 0)$ for $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$. Substituting $e_2^\alpha = \lambda_1 e_1^\alpha$ into player 1's best response in the semifinal and denoting the maximal effort that 1 is willing to invest in the semifinal by e_1^{smax} we obtain

$$\frac{\alpha}{2\lambda_1}\frac{\lambda_1(e_1^{smax})^\alpha}{(e_1^{smax})^{\alpha+1}}\widetilde{v}_1 = c$$

or

$$e_1^{smax} = \frac{\alpha}{2c}\widetilde{v}_1.$$

Since $\widetilde{v}_1$ increases with λ_1, it follows from the last equality that e_1^{smax} increases with λ_1.

Proof of Proposition 3:

1. Perceived expected utilities of reaching the final

Using Proposition 2, we can get the perceived expected utilities of reaching the final of each player.

Overconfident player 1:

$$\widetilde{v}_1 = p^s_{34}\widetilde{E}^f(U_{13}) + p^s_{43}\widetilde{E}^f(U_{14})$$

Since player 3 and 4 are identical, $\widetilde{E}^f(U_{13}) = \widetilde{E}^f(U_{14})$

$$\widetilde{v}_1 = u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\Delta u$$

Since $\widetilde{E}^f(U_{13}) > \overline{E}^f(U)$, we can get $\widetilde{v}_1 > \overline{v}$.

Rational player 2:

$$v_2 = p^s_{34}E^f(U_{23}) + p^s_{43}E^f(U_{24})$$

Since players 3 and 4 are identical, $E^f(U_{23}) = E^f(U_{24})$

$$v_2 = \frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2) = \overline{v}$$

2. Equilibrium efforts and winning probabilities in the semifinal

Player 1

$$\widetilde{E}^s(U_{12}) = \widetilde{p}^s_{12}\widetilde{v}_1 - ce_1 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_2^\alpha}{\lambda_1 e_1^\alpha}\right)\widetilde{v}_1 - ce_1 & \text{if} \quad \lambda_1 e_1^\alpha \geqslant e_2^\alpha \\ \frac{1}{2}\frac{\lambda_1 e_1^\alpha}{e_2^\alpha}\widetilde{v}_1 - ce_1 & \text{if} \quad \lambda_1 e_1^\alpha \leqslant e_2^\alpha \end{cases}$$

Player 2

$$E^s(U_{21}) = p^s_{21}v_2 - ce_2 = \begin{cases} \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_2^\alpha}\right)v_2 - ce_2 & \text{if}\, e_2 \geqslant e_1 \\ \frac{1}{2}\frac{e_2^\alpha}{e_1^\alpha}v_2 - ce_2 & \text{if}\, e_2 \leqslant e_1 \end{cases}$$

There are 4 cases:

$$\begin{cases} \lambda_1 e_1^\alpha \geqslant e_2^\alpha \quad and \quad e_2 \leqslant e_1 \\ \lambda_1 e_1^\alpha \geqslant e_2^\alpha \quad and \quad e_2 \geqslant e_1 \\ \lambda_1 e_1^\alpha \leqslant e_2^\alpha \quad and \quad e_2 \geqslant e_1 \\ \lambda_1 e_1^\alpha \leqslant e_2^\alpha \quad and \quad e_2 \leqslant e_1 \end{cases}$$

Since $\lambda_1 > 1$, the fourth case is impossible.

(1) case 1: $\lambda_1 e_1^\alpha \geqslant e_2^\alpha \quad and \quad e_2 \leqslant e_1$, which corresponds to (i).

Player 1 $\quad max \quad \left(1-\frac{1}{2}\frac{e_2^\alpha}{\lambda_1 e_1^\alpha}\right)\widetilde{v}_1 - ce_1$

Player 2 $\quad max \quad \frac{1}{2}\frac{e_2^\alpha}{e_1^\alpha}v_2 - ce_2$

FOCS:

$[e_1] \quad \frac{\alpha}{2\lambda_1}\frac{e_2^\alpha}{e_1^{\alpha+1}}\widetilde{v}_1 - c = 0$

$[e_2] \quad \frac{\alpha}{2}\frac{e_2^{\alpha-1}}{e_1^\alpha}v_2 - c = 0$

SOCS:

$[e_1] \quad \frac{\alpha}{2\lambda_1}(-\alpha-1)\frac{e_2^\alpha}{e_1^{\alpha+2}}\widetilde{v}_1 < 0$

$[e_2] \quad \frac{\alpha}{2}(\alpha-1)\frac{e_2^{\alpha-2}}{e_1^\alpha}v_2 < 0$

Solving the two FOCS, we obtain

$$e_1 = \frac{\alpha}{2c}\lambda_1^{\alpha-1}(\widetilde{v}_1)^{1-\alpha}(v_2)^\alpha$$

$$e_2 = \frac{\alpha}{2c}\lambda_1^{\alpha}(\widetilde{v}_1)^{-\alpha}(v_2)^{\alpha+1}$$

$$\frac{e_2}{e_1} = \lambda_1\frac{v_2}{\widetilde{v}_1}$$

Check the conditions $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$ and $e_2 \leqslant e_1$:

① $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$

As long as $e_1 \geqslant e_2$ is satisfied, $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$ is satisfied.

② $e_1 \geqslant e_2$

$$e_1 \geqslant e_2 \Longleftrightarrow \frac{\widetilde{v}_1}{\lambda_1 v_2} \geqslant 1$$

$$\Longleftrightarrow \frac{\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)\Delta u}{\lambda_1\left[\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)\Delta u\right]} \geqslant 1$$

$$\Longleftrightarrow \frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \geqslant \lambda_1\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)$$

Let

$$f(\lambda_1) = \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right) - \lambda_1\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right),$$

we can easily get that $f(\lambda_1 = 1) = 0$ and $f(\lambda_1 \to \infty) < 0$.

$$f'(\lambda_1) = \frac{(1+\alpha)^2}{2(2\alpha+1)}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}-1} - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)$$

$$f'(\lambda_1) \lesseqqgtr 0 \iff \frac{(1+\alpha)^2}{2(2\alpha+1)}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}-1} \lesseqqgtr \frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}$$
$$\iff \frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)} \lesseqqgtr \lambda_1^{\frac{\alpha+1}{2\alpha+1}+1}$$
$$\iff \left[\frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)}\right]^{\frac{1}{\frac{\alpha+1}{2\alpha+1}+1}} \lesseqqgtr \lambda_1$$

Let

$$g(\alpha) = \left[\frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)}\right]^{\frac{1}{\frac{\alpha+1}{2\alpha+1}+1}}$$

a) If $g(\alpha) \leqslant 1$, then $f'(\lambda_1) < 0$ always holds. This means $f(\lambda_1) < 0$ always holds, and thus $\frac{e_1}{e_2} < 1$ always holds.

$$g(\alpha) \leqslant 1 \iff \frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)} \leqslant 1$$
$$\iff \frac{(1+\alpha)^2}{2\alpha+1} \leqslant \frac{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)}{u(w_1)-u(w_2)}$$
$$\iff (1+\alpha)^2\left(u(w_1)-u(w_2)\right) \leqslant (2\alpha+1)[(1-\alpha)u(w_1)+(1+\alpha)u(w_2)]$$
$$\iff (\alpha+3\alpha^2)u(w_1) \leqslant (2+5\alpha+3\alpha^2)u(w_2)$$
$$\iff \frac{u(w_1)}{u(w_2)} \leqslant \frac{2+5\alpha+3\alpha^2}{\alpha(1+3\alpha)}$$
$$\iff \frac{u(w_1)}{u(w_2)} - 1 \leqslant \frac{2+5\alpha+3\alpha^2}{\alpha(1+3\alpha)} - 1$$
$$\iff \frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$$

When $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$, the condition $e_1 \geqslant e_2$ is never satisfied given that $\lambda_1 > 1$.

b) If $g(\alpha) > 1$, then

$$f'(\lambda_1)\begin{cases} > 0 & \text{when } \lambda_1 < \left[\frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)}\right]^{\frac{1}{\frac{\alpha+1}{2\alpha+1}+1}} \\ < 0 & \text{when } \lambda_1 > \left[\frac{(1+\alpha)^2}{2\alpha+1}\frac{u(w_1)-u(w_2)}{(1-\alpha)u(w_1)+(1+\alpha)u(w_2)}\right]^{\frac{1}{\frac{\alpha+1}{2\alpha+1}+1}} \end{cases}$$

We now show that if $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$, then there exists a unique threshold $\hat{\lambda} > 1$ where $f(\lambda_1) = 0$, that is,

$$\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} = \lambda_1\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)$$

which is equivalent to

$$u(w_1) - \frac{1+\alpha}{2}\hat{\lambda}^{-\frac{\alpha+1}{2\alpha+1}}\Delta u = \hat{\lambda}\left[\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)\Delta u\right],$$

or

$$u(w_1) - \frac{1+\alpha}{2}\hat{\lambda}^{-\frac{\alpha+1}{2\alpha+1}}\Delta u = \hat{\lambda}\left(u(w_1) - \frac{1+\alpha}{2}\Delta u\right),$$

or

$$\frac{1+\alpha}{2}\frac{u(w_1)-u(w_2)}{u(w_1)} = \frac{\hat{\lambda}-1}{\hat{\lambda}-\hat{\lambda}^{-\frac{\alpha+1}{2\alpha+1}}}. \tag{15}$$

Since $\alpha \in (0,1]$ and $u(w_1) > u(w_2)$, the left-hand side of (15) takes a value in the interval $(0,1)$. The right-hand side of (15) is increasing in $\hat{\lambda}$ for $\lambda > 1$, its limit when $\hat{\lambda} \to 1$ is $\frac{2\alpha+1}{3\alpha+2}$, and its limit when $\hat{\lambda} \to \infty$ is 1. Hence, the threshold $\hat{\lambda}$ exists and is unique provided that

$$\frac{1+\alpha}{2}\frac{u(w_1)-u(w_2)}{u(w_1)} > \frac{2\alpha+1}{3\alpha+2}.$$

It is easy to show that this inequality is equivalent to

$$\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}.$$

Therefore, if $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$, then there exists a unique value for $\hat{\lambda}$, greater than 1, that satisfies (15). This, in turn, implies:

$$f(\lambda_1)\begin{cases} > 0 & \text{when } \lambda_1 < \hat{\lambda} \\ = 0 & \text{when } \lambda_1 = \hat{\lambda} \\ < 0 & \text{when } \lambda_1 > \hat{\lambda} \end{cases}$$

$$e_1 - e_2\begin{cases} > 0 & \text{when } \lambda_1 < \hat{\lambda} \\ = 0 & \text{when } \lambda_1 = \hat{\lambda} \\ < 0 & \text{when } \lambda_1 > \hat{\lambda} \end{cases}$$

The condition $e_1 \geqslant e_2$ is only satisfied when $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 \leqslant \hat{\lambda}$. And $e_1 > e_2$ is only satisfied when $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$.

Therefore the solution

$$e_1 = \frac{\alpha}{2c}\lambda_1^{\alpha-1}(\widetilde{v}_1)^{1-\alpha}(v_2)^{\alpha}$$

$$e_2 = \frac{\alpha}{2c}\lambda_1^{\alpha}(\widetilde{v}_1)^{-\alpha}(v_2)^{\alpha+1}$$

only applies when $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$.

(2) case 2: $\lambda_1 e_1^\alpha \geqslant e_2^\alpha \quad and \quad e_2 \geqslant e_1$, which corresponds to (ii).

Player 1 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_2^\alpha}{\lambda_1 e_1^\alpha}\right)\widetilde{v}_1 - ce_1$

Player 2 $\quad max \quad \left(1 - \frac{1}{2}\frac{e_1^\alpha}{e_2^\alpha}\right)v_2 - ce_2$

FOCS:

$[e_1] \quad \frac{\alpha}{2\lambda_1}\frac{e_2^\alpha}{e_1^{\alpha+1}}\widetilde{v}_1 - c = 0$

$[e_2] \quad \frac{\alpha}{2}\frac{e_1^\alpha}{e_2^{\alpha+1}}v_2 - c = 0$

SOCS:

$[e_1] \quad \frac{\alpha}{2\lambda_1}(-\alpha-1)\frac{e_2^\alpha}{e_1^{\alpha+2}}\widetilde{v}_1 < 0$

$[e_2] \quad \frac{\alpha}{2}(-\alpha-1)\frac{e_1^\alpha}{e_2^{\alpha+2}}v_2 < 0$

Solving the FOCS, we obtain

$$e_1 = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}(\widetilde{v}_1)^{\frac{\alpha+1}{2\alpha+1}}(v_2)^{\frac{\alpha}{2\alpha+1}}$$

$$e_2 = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}(\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}}(v_2)^{\frac{\alpha+1}{2\alpha+1}}$$

$$\frac{e_2}{e_1} = \lambda_1^{\frac{1}{2\alpha+1}}(\widetilde{v}_1)^{-\frac{1}{2\alpha+1}}(v_2)^{\frac{1}{2\alpha+1}}$$

Check the conditions $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$ and $e_2 \geqslant e_1$:

① $\lambda_1 e_1^\alpha \geqslant e_2^\alpha$

$$\lambda_1 e_1^\alpha \geqslant e_2^\alpha \Longleftrightarrow \frac{\lambda_1 e_1^\alpha}{e_2^\alpha} \geqslant 1 \Longleftrightarrow \lambda_1^{\frac{\alpha+1}{2\alpha+1}}(\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}}(v_2)^{-\frac{\alpha}{2\alpha+1}} \geqslant 1$$

Since $\lambda_1 > 1$ and $\widetilde{v}_1 > v_2$, the inequality is always satisfied. Therefore $\lambda_1 e_1^\alpha > e_2^\alpha$ always holds when $\lambda_1 > 1$.

② $e_2 \geqslant e_1$

$$\begin{aligned} e_2 \geqslant e_1 &\Longleftrightarrow \frac{e_2}{e_1} \geqslant 1 \\ &\Longleftrightarrow \lambda_1^{\frac{1}{2\alpha+1}}(\widetilde{v}_1)^{-\frac{1}{2\alpha+1}}(v_2)^{\frac{1}{2\alpha+1}} \geqslant 1 \\ &\Longleftrightarrow \left(\frac{\lambda_1 v_2}{\widetilde{v}_1}\right)^{\frac{1}{2\alpha+1}} \geqslant 1 \Longleftrightarrow \frac{\lambda_1 v_2}{\widetilde{v}_1} \geqslant 1 \end{aligned}$$

We have already seen in case (1) that $e_2 \geqslant e_1$ is satisfied when either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$.

Therefore the solution

$$e_1 = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}(\widetilde{v}_1)^{\frac{\alpha+1}{2\alpha+1}}(v_2)^{\frac{\alpha}{2\alpha+1}}$$

$$e_2 = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}(\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}}(v_2)^{\frac{\alpha+1}{2\alpha+1}}$$

only applies when either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$.

(3) case 3: $\lambda_1 e_1^\alpha \leqslant e_2^\alpha$ *and* $e_2 \geqslant e_1$

Player 1 *max* $\frac{1}{2}\frac{\lambda_1 e_1^\alpha}{e_2^\alpha}\widetilde{v}_1 - ce_1$

Player 2 *max* $[1-\frac{1}{2}(\frac{e_1}{e_2})^\alpha]v_2 - ce_2$

FOCS:

$[e_1]$ $\frac{\alpha\lambda_1}{2}\frac{e_1^{\alpha-1}}{e_2^\alpha}\widetilde{v}_1 - c = 0$

$[e_2]$ $\frac{\alpha}{2}\frac{e_1^\alpha}{e_2^{\alpha+1}}v_2 - c = 0$

dividing the two FOCS, we obtain

$$\frac{e_2}{e_1} = \frac{v_2}{\lambda_1\widetilde{v}_1} < 1$$

which contradicts the condition that $e_2 \geqslant e_1$

Therefore, the equilibrium in this semifinal:

(1) When $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$, which corresponds to (i)

$$e_1^s = \frac{\alpha}{2c}\lambda_1^{\alpha-1}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{1-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}\Delta u$$

$$e_2^s = \frac{\alpha}{2c}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{1+\alpha}\Delta u$$

where $e_1^s > e_2^s$.

$$p_{21}^s = \frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}$$

$$p_{12}^s = 1 - \frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}$$

$$\widetilde{p}_{12}^s = 1 - \frac{1}{2}\lambda_1^{\alpha-1}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}$$

(2) When either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$, which corresponds to (ii).

$$e_1^s = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{\frac{\alpha+1}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\frac{\alpha}{2\alpha+1}}\Delta u$$

$$e_2^s = \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{\frac{\alpha}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\frac{\alpha+1}{2\alpha+1}}\Delta u$$

where $\lambda_1 (e_1^s)^\alpha > (e_2^s)^\alpha$ and $e_1^s \leqslant e_2^s$.

$$p_{12}^s = \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \right)^{\frac{\alpha}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2} \right)^{-\frac{\alpha}{2\alpha+1}}$$

$$p_{21}^s = 1 - \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \right)^{\frac{\alpha}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2} \right)^{-\frac{\alpha}{2\alpha+1}}$$

$$\widetilde{p}_{12}^s = 1 - \frac{1}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}} \right)^{-\frac{\alpha}{2\alpha+1}} \left(\frac{u(w_1)}{u(w_1) - u(w_2)} - \frac{1+\alpha}{2} \right)^{\frac{\alpha}{2\alpha+1}}$$

3. Equilibrium efforts compared to benchmark

   (i) when $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

   We show that $e_1^s > \overline{e}^s > e_2^s$:

$$e_1^s > \overline{e}^s \Longleftrightarrow \frac{e_1^s}{\overline{e}^s} > 1 \Longleftrightarrow \frac{\frac{\alpha}{2c}\lambda_1^{\alpha-1}\widetilde{v}_1^{\,1-\alpha} v_2^\alpha}{\frac{\alpha}{2c}\overline{v}} > 1 \Longleftrightarrow \left(\frac{\widetilde{v}_1}{\lambda_1 v_2}\right)^{1-\alpha} > 1$$

$$e_2^s < \overline{e}^s \Longleftrightarrow \frac{e_2^s}{\overline{e}^s} < 1 \Longleftrightarrow \frac{\frac{\alpha}{2c}\lambda_1^{\alpha}\left(\widetilde{v}_1\right)^{-\alpha}\left(v_2\right)^{\alpha+1}}{\frac{\alpha}{2c}\overline{v}} < 1 \Longleftrightarrow \left(\frac{\lambda_1 v_2}{\widetilde{v}_1}\right)^{\alpha} < 1$$

   Since $\frac{\widetilde{v}_1}{\lambda_1 v_2} > 1$, we can get $e_1^s > \overline{e}^s$ and $e_2^s < \overline{e}^s$.

   (ii) when either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$

   We show that $e_1^s \leqslant e_2^s \leqslant \overline{e}^s$. Since we already showed that $e_1^s \leqslant e_2^s$ is satisfied under this condition, we only have to show $e_2^s \leqslant \overline{e}^s$.

$$\begin{aligned} e_2^s \leqslant \overline{e}^s &\Longleftrightarrow \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\widetilde{v}_1^{\frac{\alpha}{2\alpha+1}} v_2^{\frac{\alpha+1}{2\alpha+1}} \leqslant \frac{\alpha}{2c}\overline{v} \\ &\Longleftrightarrow \frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\widetilde{v}_1^{\frac{\alpha}{2\alpha+1}} v_2^{\frac{\alpha+1}{2\alpha+1}} \leqslant \frac{\alpha}{2c} v_2 \\ &\Longleftrightarrow \widetilde{v}_1^{\frac{\alpha}{2\alpha+1}} \leqslant \lambda_1^{\frac{\alpha}{2\alpha+1}} v_2^{\frac{\alpha}{2\alpha+1}} \\ &\Longleftrightarrow \widetilde{v}_1 \leqslant \lambda_1 v_2 \end{aligned}$$

   which always holds, thus $e_1^s \leqslant e_2^s \leqslant \overline{e}^s$ always holds.

4. Winning probabilities compared to benchmark

   (i) when $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

We show that $p_{21}^s < \frac{1}{2}$ and $\widetilde{p}_{12}^s > p_{12}^s > \frac{1}{2}$:

$$p_{21}^s < \frac{1}{2} \Longleftrightarrow \frac{1}{2}\left(\frac{e_2^s}{e_1^s}\right)^\alpha < \frac{1}{2} \Longleftrightarrow e_2^s < e_1^s$$

$$p_{12}^s = 1 - p_{21}^s > \frac{1}{2}$$

$$\widetilde{p}_{12}^s > p_{12}^s \Longleftrightarrow 1 - \frac{1}{2}\left(\frac{e_2^s}{\lambda_1 e_1^s}\right)^\alpha > 1 - \frac{1}{2}\left(\frac{e_2^s}{e_1^s}\right)^\alpha \Longleftrightarrow \lambda_1 > 1$$

(ii) when either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$

We show that $p_{12}^s \leqslant \frac{1}{2}$, $p_{21}^s \geqslant \frac{1}{2}$ and $\widetilde{p}_{12}^s > \frac{1}{2}$:

$$p_{12}^s \leqslant \frac{1}{2} \Longleftrightarrow \frac{1}{2}\left(\frac{e_1^s}{e_2^s}\right)^\alpha \leqslant \frac{1}{2} \Longleftrightarrow e_1^s \leqslant e_2^s$$

$$p_{21}^s = 1 - p_{12}^s \geqslant \frac{1}{2}$$

$$\begin{aligned}
\widetilde{p}_{12}^s > \frac{1}{2} &\Longleftrightarrow 1 - \frac{1}{2}\left(\frac{e_2^s}{\lambda_1 e_1^s}\right)^\alpha > \frac{1}{2} \\
&\Longleftrightarrow \frac{e_2^s}{\lambda_1 e_1^s} < 1 \\
&\Longleftrightarrow \frac{\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{\frac{\alpha+1}{2\alpha+1}}}{\lambda_1^{\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha+1}{2\alpha+1}} (v_2)^{\frac{\alpha}{2\alpha+1}}} < 1 \\
&\Longleftrightarrow \lambda_1^{-\frac{2\alpha}{2\alpha+1}} (\widetilde{v}_1)^{-\frac{1}{2\alpha+1}} (v_2)^{\frac{1}{2\alpha+1}} < 1 \\
&\Longleftrightarrow \lambda_1^{-\frac{2\alpha}{2\alpha+1}} \left(\frac{v_2}{\widetilde{v}_1}\right)^{\frac{1}{2\alpha+1}} < 1
\end{aligned}$$

5. Participation constraints

(1) When $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

$$\begin{aligned}
\widetilde{E}^s(U_{12}) &= \widetilde{p}_{12}^s \widetilde{v}_1 - c e_1^s \\
&> p_{12}^s \widetilde{v}_1 - c e_1^s \\
&= \left(1 - \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^\alpha\right) \widetilde{v}_1 - c\frac{\alpha}{2c}\lambda_1^{\alpha-1} (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha \\
&= \widetilde{v}_1 - \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha - \frac{\alpha}{2}\lambda_1^{\alpha-1} (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha \\
&> \widetilde{v}_1 - \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha - \frac{1}{2}\lambda_1^{\alpha-1} (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha \\
&> \widetilde{v}_1 - \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha - \frac{1}{2}\lambda_1^{\alpha} (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha \\
&= \widetilde{v}_1 - \lambda_1^\alpha (\widetilde{v}_1)^{1-\alpha} (v_2)^\alpha \\
&= (\widetilde{v}_1)^{1-\alpha}\left[(\widetilde{v}_1)^\alpha - \lambda_1^\alpha (v_2)^\alpha\right] > 0
\end{aligned}$$

$$\begin{aligned}E^s(U_{21}) &= p_{21}^s v_2 - ce_2^s \\ &= \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^\alpha v_2 - c\frac{\alpha}{2c}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^{\alpha+1} \\ &= \frac{1}{2}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^{1+\alpha} - \frac{\alpha}{2}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^{\alpha+1} \\ &= \frac{1-\alpha}{2}\lambda_1^\alpha (\widetilde{v}_1)^{-\alpha} (v_2)^{1+\alpha} \geqslant 0\end{aligned}$$

(2) When either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$

$$\widetilde{E}^s(U_{12}) = \widetilde{p}_{12}^s \widetilde{v}_1 - ce_1^s$$

Since $\widetilde{p}_{12}^s > \frac{1}{2}$, $\widetilde{v}_1 > \overline{v}$ and $e_1^s < \overline{e}^s$, we can get that $\widetilde{E}^s(U_{12}) > \overline{E}^s(U) \geqslant 0$.

$$\begin{aligned}E^s(U_{21}) &= p_{21}^s v_2 - ce_2^s \\ &= \left(1 - \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{-\frac{\alpha}{2\alpha+1}}\right) v_2 - c\frac{\alpha}{2c}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{\frac{\alpha+1}{2\alpha+1}} \\ &= v_2 - \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{\frac{\alpha+1}{2\alpha+1}} - \frac{\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{\frac{\alpha+1}{2\alpha+1}} \\ &= v_2 - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}} (v_2)^{\frac{\alpha+1}{2\alpha+1}} \\ &= (v_2)^{\frac{\alpha+1}{2\alpha+1}} \left((v_2)^{\frac{\alpha}{2\alpha+1}} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} (\widetilde{v}_1)^{\frac{\alpha}{2\alpha+1}}\right) \geqslant 0\end{aligned}$$

Proof of Proposition 4:

1. Expected utilities of reaching the final

   Rational player 3:

$$\begin{aligned}v_3 &= p_{12}^s E^f(U_{31}) + p_{21}^s E^f(U_{32}) \\ &= p_{12}^s \left[u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} \Delta u\right] + p_{21}^s \left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right]\end{aligned}$$

   Rational player 4:

$$\begin{aligned}v_4 &= p_{12}^s E^f(U_{41}) + p_{21}^s E^f(U_{42}) \\ &= p_{12}^s \left[u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} \Delta u\right] + p_{21}^s \left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right]\end{aligned}$$

   Since $E^f(U_{31}) = E^f(U_{41}) > \overline{E}^f(U) = E^f(U_{32}) = E^f(U_{42})$, we have $v_3 = v_4 > \overline{v}$.

2. Equilibrium efforts

   (1) When $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

$$e_3^s = e_4^s = \frac{\alpha}{2c}v_3 = \frac{\alpha}{2c}\left[p_{12}^s \left[u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}} \Delta u\right] + p_{21}^s \left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right]\right]$$

where
$p_{12}^s = 1 - \frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}.$

$$p_{34}^s = p_{43}^s = \frac{1}{2}$$

(2) When either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$

$$e_3^s = e_4^s = \frac{\alpha}{2c}v_3 = \frac{\alpha}{2c}\left[p_{12}^s\left[u(w_1) - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\Delta u\right] + p_{21}^s\left[\frac{1-\alpha}{2}u(w_1) + \frac{1+\alpha}{2}u(w_2)\right]\right]$$

where
$p_{12}^s = \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{\frac{\alpha}{2\alpha+1}}\left[\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right]^{-\frac{\alpha}{2\alpha+1}}.$

$$p_{34}^s = p_{43}^s = \frac{1}{2}$$

We show that $e_3^s = e_4^s > \overline{e}^s$ holds in both (1) and (2):

$$e_3^s = e_4^s > \overline{e}^s \Longleftrightarrow \frac{\alpha}{2c}v_3 > \frac{\alpha}{2c}\overline{v} \Longleftrightarrow v_3 > \overline{v}$$

3. Participation constraints

$$E^s(U_{34}) = p_{34}^s v_3 - c e_3^s = \frac{1}{2}v_3 - c\frac{\alpha}{2c}v_3 = \frac{1-\alpha}{2}v_3 \geqslant 0$$

$$E^s(U_{43}) = E^s(U_{34}) \geqslant 0$$

Proof of Proposition 5:

1. When $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ and $\lambda_1 < \hat{\lambda}$

   (1) True winning probability of player 1

$$\begin{aligned} P_1 &= p_{13}^f p_{12}^s \\ &= \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}\left[1 - \frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}\right] \end{aligned}$$

Let

$$f(\lambda_1) = P_1 - \frac{1}{4}.$$

We can get

$$f(\lambda_1 = 1) = \frac{1}{2} \times \frac{1}{2} - \frac{1}{4} = 0$$

$$f(\lambda_1 = \hat{\lambda}) = \frac{1}{2}\hat{\lambda}^{-\frac{\alpha}{2\alpha+1}} \times \frac{1}{2} - \frac{1}{4} < 0$$

$f(\lambda_1)$ can also be written as the following:

$$f(\lambda_1) = \frac{1}{2}\lambda_1^{-\frac{\alpha}{2\alpha+1}}$$
$$-\frac{1}{4}\lambda_1^{\alpha-\frac{\alpha}{2\alpha+1}}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha} - \frac{1}{4}$$

Taking derivative of $f(\lambda_1)$ we obtain

$$f'(\lambda_1) = -\frac{1}{2}\frac{\alpha}{2\alpha+1}\lambda_1^{-\frac{\alpha}{2\alpha+1}-1} - \frac{1}{4}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{\alpha}$$
$$\Bigg[\left(\alpha - \frac{\alpha}{2\alpha+1}\right)\lambda_1^{\alpha-\frac{\alpha}{2\alpha+1}-1}\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}$$
$$+\lambda_1^{\alpha-\frac{\alpha}{2\alpha+1}}(-\alpha)\left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha-1}$$
$$\left(-\frac{\alpha+1}{2}\right)\left(-\frac{\alpha+1}{2\alpha+1}\right)\lambda_1^{-\frac{\alpha+1}{2\alpha+1}-1}\Bigg]$$

$$f'(\lambda_1=1) = -\frac{1}{2}\frac{\alpha}{2\alpha+1}\left[1+\frac{1}{2}\left(2\alpha - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{-1}\frac{\alpha+1}{2}(1+\alpha)\right)\right]$$

$f'(\lambda_1=1)$ and $1+\frac{1}{2}\left(2\alpha - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{-1}\frac{\alpha+1}{2}(1+\alpha)\right)$ has the opposite sign.

When $1+\frac{1}{2}\left(2\alpha - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{-1}\frac{\alpha+1}{2}(1+\alpha)\right) < 0$, $f'(\lambda_1=1) > 0$.

Since

$$1+\frac{1}{2}\left(2\alpha - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{-1}\frac{\alpha+1}{2}(1+\alpha)\right) < 0$$
$$\iff 2\alpha - \left(\frac{u(w_1)}{u(w_1)-u(w_2)} - \frac{1+\alpha}{2}\right)^{-1}\frac{\alpha+1}{2}(1+\alpha) < -2$$
$$\iff \frac{4u(w_2)}{u(w_1)-u(w_2)} < 3\alpha-1$$

$f'(\lambda_1=1) > 0$ is only satisfied when $\alpha > \frac{1}{3}$ and $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{4}{3\alpha-1}$.

We now show $\frac{4}{3\alpha-1} > \frac{2(2\alpha+1)}{\alpha(3\alpha+1)}$:

$$\frac{4}{3\alpha-1} > \frac{2(2\alpha+1)}{\alpha(3\alpha+1)} \iff 4\alpha(3\alpha+1) > 2(2\alpha+1)(3\alpha-1)$$
$$\iff 12\alpha^2+4\alpha > 12\alpha^2+2\alpha-2$$
$$\iff 2\alpha+2 > 0$$

Thus we know that under the conditions $\alpha > \frac{1}{3}$ and $\frac{u(w_1)-u(w_2)}{u(w_2)} > \frac{4}{3\alpha-1}$, $f(\lambda_1)$ is positive when λ_1 is small and close to 1. Therefore there exist parameter configurations where the overconfident player's equilibrium winning probability P_1 is higher than the benchmark.

(2) True winning probability of player 2

We show that $P_2 < \frac{1}{4}$:

$$P_2 = p_{23}^f p_{21}^s = \frac{1}{2} p_{21}^s < \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(3) True winning probabilities of players 3 and 4

We show that $P_3 = P_4 > \frac{1}{4}$:

$$\begin{aligned}
P_3 = P_4 &= p_{12}^s p_{31}^f p_{34}^s + p_{21}^s p_{32}^f p_{34}^s \\
&= p_{12}^s p_{31}^f \frac{1}{2} + p_{21}^s \frac{1}{2}\frac{1}{2} \\
&= p_{12}^s p_{31}^f \frac{1}{2} + (1 - p_{12}^s)\frac{1}{2}\frac{1}{2} \\
&= p_{12}^s \left(p_{31}^f \frac{1}{2} - \frac{1}{4} \right) + \frac{1}{4} \\
&> p_{12}^s \left(\frac{1}{2}\frac{1}{2} - \frac{1}{4} \right) + \frac{1}{4} = \frac{1}{4}
\end{aligned}$$

(4) Comparing the true winning probabilities of players 1 and 3

$$\begin{aligned}
P_1 - P_3 &= p_{13}^f p_{12}^s - p_{12}^s p_{31}^f p_{34}^s - p_{21}^s p_{32}^f p_{34}^s \\
&= p_{13}^f p_{12}^s - p_{12}^s p_{31}^f p_{34}^s - (1 - p_{12}^s)\, p_{32}^f p_{34}^s \\
&= p_{13}^f p_{12}^s - \frac{1}{2} p_{12}^s p_{31}^f - \frac{1}{2}\frac{1}{2}(1 - p_{12}^s) \\
&= p_{13}^f p_{12}^s - \frac{1}{2} p_{12}^s \left(1 - p_{13}^f\right) - \frac{1}{4}(1 - p_{12}^s) \\
&= \frac{3}{2} p_{13}^f p_{12}^s - \frac{1}{4} p_{12}^s - \frac{1}{4}
\end{aligned}$$

The sign of $P_1 - P_3$ is the same as the sign of $6 p_{13}^f p_{12}^s - p_{12}^s - 1$

Let $f(\lambda_1) = 6 p_{13}^f p_{12}^s - p_{12}^s - 1$

$$f(\lambda_1 = 1) = 6 \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{2} - 1 = 0$$

$$f(\lambda_1 = \hat{\lambda}) = 6 \times \frac{1}{2} \hat{\lambda}^{-\frac{\alpha}{2\alpha+1}} \times \frac{1}{2} - \frac{1}{2} - 1 < 0$$

$$
\begin{aligned}
f(\lambda_1) &= 3\lambda_1^{-\frac{\alpha}{2\alpha+1}}\left[1-\frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{\alpha}\right]\\
&\quad-\left[1-\frac{1}{2}\lambda_1^{\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{\alpha}\right]-1\\
&=3\lambda_1^{-\frac{\alpha}{2\alpha+1}}-\frac{1}{2}\lambda_1^{\alpha}\left(3\lambda_1^{-\frac{\alpha}{2\alpha+1}}-1\right)\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\\
&\quad\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{\alpha}-2
\end{aligned}
$$

$$
\begin{aligned}
f'(\lambda_1) &= -3\frac{\alpha}{2\alpha+1}\lambda_1^{-\frac{\alpha}{2\alpha+1}-1}-\frac{1}{2}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{\alpha}\\
&\quad\left[\left(3\left(-\frac{\alpha}{2\alpha+1}+\alpha\right)\lambda_1^{-\frac{\alpha}{2\alpha+1}+\alpha-1}-\alpha\lambda_1^{\alpha-1}\right)\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha}\right.\\
&\quad+\lambda_1^{\alpha}\left(3\lambda_1^{-\frac{\alpha}{2\alpha+1}}-1\right)(-\alpha)\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}}\right)^{-\alpha-1}\\
&\quad\left.\frac{1+\alpha}{2}\frac{\alpha+1}{2\alpha+1}\lambda_1^{-\frac{\alpha+1}{2\alpha+1}-1}\right],
\end{aligned}
$$

or

$$
f'(\lambda_1=1)=\alpha\left(-\frac{3}{2}\frac{1}{2\alpha+1}-1+\frac{1+\alpha}{2}\frac{\alpha+1}{2\alpha+1}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{-1}\right)
$$

$$
\begin{aligned}
f'(\lambda_1=1)>0 &\iff -\frac{3}{2}\frac{1}{2\alpha+1}-1+\frac{1+\alpha}{2}\frac{\alpha+1}{2\alpha+1}\left(\frac{u(w_1)}{u(w_1)-u(w_2)}-\frac{1+\alpha}{2}\right)^{-1}>0\\
&\iff \frac{6\alpha^2+5\alpha-3}{2\,(4\alpha+5)}>\frac{u(w_2)}{u(w_1)-u(w_2)}
\end{aligned}
$$

This is satisfied when $\alpha>\frac{-5+\sqrt{97}}{12}$ and $\frac{u(w_1)-u(w_2)}{u(w_2)}>\frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$.

We now show $\frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}>\frac{2(2\alpha+1)}{\alpha(3\alpha+1)}$:

$$
\begin{aligned}
\frac{2\,(4\alpha+5)}{6\alpha^2+5\alpha-3}>\frac{2(2\alpha+1)}{\alpha(3\alpha+1)} &\iff \frac{4\alpha+5}{6\alpha^2+5\alpha-3}>\frac{2\alpha+1}{\alpha(3\alpha+1)}\\
&\iff (4\alpha+5)\,\alpha(3\alpha+1)>(2\alpha+1)\left(6\alpha^2+5\alpha-3\right)\\
&\iff 12\alpha^3+19\alpha^2+5\alpha>12\alpha^3+16\alpha^2-\alpha-3\\
&\iff 3\alpha^2+6\alpha+3>0
\end{aligned}
$$

Thus we know that under the conditions $\alpha>\frac{-5+\sqrt{97}}{12}$ and $\frac{u(w_1)-u(w_2)}{u(w_2)}>\frac{2(4\alpha+5)}{6\alpha^2+5\alpha-3}$, $f(\lambda_1)$ is positive when λ_1 is small and close to 1. Therefore there exist parameter configurations where the overconfident player's equilibrium winning probability P_1 is higher than that of the rational player in the other semifinal P_3.

2. When either $\frac{u(w_1)-u(w_2)}{u(w_2)} \leqslant \frac{2(1+2\alpha)}{\alpha(1+3\alpha)}$ or $\lambda_1 \geqslant \hat{\lambda}$

   (1) True winning probability of player 1

   Since player 3 and player 4 are identical, the equilibrium winning probability of player 1 is

$$P_1 = p_{13}^f p_{12}^s < \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

   (2) True winning probability of player 2

$$P_2 = p_{23}^f p_{21}^s = \frac{1}{2} p_{21}^s > \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

   (3) True winning probabilities of players 3 and 4

   $P_3 = P_4 > \frac{1}{4}$ still holds.