Online Appendix
Less Risk, More Effort: How Overconfidence Reshapes Tournament Strategies

Noëmi Jacober Luís Santos-Pinto

This Online Appendix studies tournaments where an overconfident player 1 competes against a rational player 2. We start by solving the second stage (effort stage) and continue to solve the first stage (risk stage). In the second stage, player 1 chooses the optimal effort level that maximizes his perceived expected utility

$$E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] = u(y_l) + G(\lambda + a_1 - a_2; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1),$$

and player 2 chooses the optimal effort level that maximizes her expected utility

$$\begin{aligned} E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)] &= u(y_l) + \Pr(Q_2 \geq Q_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(a_2 + \epsilon_2 \geq a_1 + \epsilon_1)\Delta u - c(a_2) \\ &= u(y_l) + \Pr(\epsilon_2 - \epsilon_1 \geq a_1 - a_2)\Delta u - c(a_2) \\ &= u(y_l) + \left[1 - G(a_1 - a_2; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2). \end{aligned}$$

The pure-strategy Nash equilibrium (a_1^*, a_2^*) of the second stage satisfies the two first-order conditions simultaneously and is given by

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*),$$

and

$$g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*).$$

The second-order conditions of the effort stage are satisfied when the cost function is sufficiently convex. Our next result characterizes the pure-strategy equilibrium efforts (a_1^*, a_2^*).

Proposition 4. *In a tournament where player 1 is overconfident and player 2 is rational, the overconfident player 1 exerts less effort than the rational player 2, i.e., $a_1^* < a_2^*$. Moreover, the efforts of both players are decreasing in player 1's overconfidence bias λ, with $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$, such that the effort gap increases in λ, i.e., $\partial(a_2^* - a_1^*)/\partial\lambda > 0$.*

The intuition for this result is that while trusting his (perceived) advantage in ability to get himself a lead in the tournament, the overconfident player becomes slack relative to the rational player. This effect holds for any risk strategy profiles $\left(\sigma_1^2, \sigma_2^2\right)$.

Proposition 5. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the overconfident player 1 has a lower objective probability of winning the tournament than the rational player 2.*

To understand the intuition behind Proposition 5, let P_i denote the objective winning probability of player i, with $i = 1, 2$. Note that, in any pure-strategy SPE, the overconfident player 1 wins the tournament with probability $P_1 = G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$ and the rational player 2 with probability $P_2 = G(a_2^* - a_1^*; \sigma_1^2, \sigma_2^2)$.

When both players are rational, the tournament is symmetric and players exert the same efforts given by $a_1^* = a_2^* = a^*$ where a^* solves $g(0; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*)$. Symmetry of $g(x)$ implies $P_1 = P_2 = G(0; \sigma_1^2, \sigma_2^2) = 1/2$. Hence, when both players are rational, each player is equally likely to win the tournament (i.e., the winner is purely random). This is true for any cumulative distribution G that satisfies the assumptions we made.

When player 1 is overconfident and player 2 is rational, the tournament is asymmetric and the overconfident player 1 exerts less effort than the rational player 2, i.e., $a_1^* < a_2^*$. Hence, in any pure-strategy SPE where both players choose the same risk strategy, the overconfident player 1 is less likely to win the tournament due to his lower effort. However, in any pure-strategy SPE where the players choose different risk strategies that might no longer be the case due to different likelihood effects. Still, Proposition 5 shows that when G is the normal cumulative distribution only the sum of risks $\sigma^2 = \sigma_1^2 + \sigma_2^2$ matters to determine the likelihood effect. Since both players face the same sum of risks, the likelihood effect is identical and the overconfident player 1, who exerts less effort, has a lower objective probability of winning the tournament than the rational player 2.

In the first stage, players 1 and 2 solve the following maximization problems, respectively,

$$\max_{\sigma_1^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + G(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1^*) \tag{1}$$

and

$$\max_{\sigma_2^2 \in \{\sigma_L^2, \sigma_H^2\}} u(y_l) + \left[1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\right] \Delta u - c(a_2^*). \tag{2}$$

Problems (1) and (2) show that a player's risk choice influences his perceived expected utility through the effort and likelihood effects identified previously. However, these two effects are now interrelated: risk taking influences both the shape of the perceived cumulative distribution function G and the position of $a_1^* - a_2^*$ at which the perceived winning probability is computed, namely the gap between the equilibrium efforts. This interrelatedness is due to the heterogeneity in players' beliefs about talent and has important consequences for the SPE as we shall illustrate next.

It follows from (1) and (2) that both players choose the high risk strategy as long as

$$\begin{aligned} &G(\lambda + a_1^*(\sigma_H^2, \sigma_H^2) - a_2^*(\sigma_H^2, \sigma_H^2); \sigma_H^2, \sigma_H^2)\Delta u - c(a_1^*(\sigma_H^2, \sigma_H^2)) \\ &\geq G(\lambda + a_1^*(\sigma_L^2, \sigma_H^2) - a_2^*(\sigma_L^2, \sigma_H^2); \sigma_L^2, \sigma_H^2)\Delta u - c(a_1^*(\sigma_L^2, \sigma_H^2)), \end{aligned} \tag{3}$$

and

$$\begin{aligned} &\left[1 - G(a_1^*(\sigma_H^2, \sigma_H^2) - a_2^*(\sigma_H^2, \sigma_H^2); \sigma_H^2, \sigma_H^2)\right] \Delta u - c(a_2^*(\sigma_H^2, \sigma_H^2)) \\ &\geq \left[1 - G(a_1^*(\sigma_H^2, \sigma_L^2) - a_2^*(\sigma_H^2, \sigma_L^2); \sigma_H^2, \sigma_L^2)\right] \Delta u - c(a_2^*(\sigma_H^2, \sigma_L^2)). \end{aligned} \tag{4}$$

Both players choose the low risk strategy as long as

$$\begin{aligned} &G(\lambda + a_1^*(\sigma_L^2, \sigma_L^2) - a_2^*(\sigma_L^2, \sigma_L^2); \sigma_L^2, \sigma_L^2)\Delta u - c(a_1^*(\sigma_L^2, \sigma_L^2)) \\ &\geq G(\lambda + a_1^*(\sigma_H^2, \sigma_L^2) - a_2^*(\sigma_H^2, \sigma_L^2); \sigma_H^2, \sigma_L^2)\Delta u - c(a_1^*(\sigma_H^2, \sigma_L^2)), \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\left[1 - G(a_1^*(\sigma_L^2, \sigma_L^2) - a_2^*(\sigma_L^2, \sigma_L^2); \sigma_L^2, \sigma_L^2)\right] \Delta u - c(a_2^*(\sigma_L^2, \sigma_L^2)) \\ &\geq \left[1 - G(a_1^*(\sigma_L^2, \sigma_H^2) - a_2^*(\sigma_L^2, \sigma_H^2); \sigma_L^2, \sigma_H^2)\right] \Delta u - c(a_2^*(\sigma_L^2, \sigma_H^2)). \end{aligned} \tag{6}$$

Player 1 chooses the low risk strategy and player 2 the high risk strategy when inequalities (3) and (6) hold in opposite directions. Finally, player 2 chooses the low risk strategy, and player 1 the high risk strategy when inequalities (4) and (5) hold in opposite directions.

To be able to characterize the SPE of this asymmetric tournament, we specialize the model as in the previous section. Our next result characterizes the equilibrium efforts in the specialized model.

Proposition 6. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, the equilibrium efforts are:*

$$a_1^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_1^2 + \sigma_2^2))^2}{8(\sigma_1^2 + \sigma_2^2)(\lambda + (\sigma_1^2 + \sigma_2^2))^2} \tag{7}$$

$$a_2^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^4}{8(\sigma_1^2 + \sigma_2^2)(\lambda + (\sigma_1^2 + \sigma_2^2))^2} \tag{8}$$

The equilibrium efforts are strictly increasing in Δu and decreasing in player 1's overconfidence bias λ.

Indeed, in equilibrium, the overconfident player always exerts less effort, i.e.,

$a_2^* > a_1^*$. Note, in the case where $\lambda = 0$, we reach the symmetric case of [**?**], in which both players choose identical efforts in equilibrium. However, as the asymmetry grows stronger, i.e. λ increases, both efforts decrease. As the efforts decrease with different speeds, the tournament outcome becomes more asymmetric with higher overconfidence levels, and the effort gap increases. While the overconfidence of player 1 is not affecting the best response of player 2, an increase in λ shifts the best response function of player 1 and thereby the asymmetric equilibrium away from the symmetric equilibrium.

Before moving on to the risk stage, we take a closer look at the effect of risk taking on players' equilibrium efforts.

Lemma 2. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the cost of effort is exponential, and $\lambda^2 \leq \sigma^2 = \sigma_1^2 + \sigma_2^2$, the equilibrium efforts are decreasing in risk taking, that is,*

$$\frac{\partial a_1^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_1^2} = \frac{\partial a_1^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_2^2} < 0 \text{ and } \frac{\partial a_2^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_1^2} = \frac{\partial a_2^*(\sigma_1^2, \sigma_2^2)}{\partial \sigma_2^2} < 0.$$

Thus, for a given risk taking r_j of the other player, we have $a_i^*(\sigma_H^2, \sigma_{r_j}^2) \leq a_i^*(\sigma_L^2, \sigma_{r_j}^2)$.

Lemma 2 describes the effect of risk taking on the equilibrium effort levels.[1] If the level of overconfidence of player 1 is sufficiently small, i.e. $\lambda^2 \leq \sigma^2$, the high risk strategy will reduce the equilibrium effort of both players. Lemma 3 describes the effect of risk taking on the equilibrium effort gap.

Lemma 3. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, the equilibrium effort gap is decreasing in the sum of risks, i.e., $\partial(a_2^* - a_1^*)/\partial\sigma^2 < 0$.*

Lemma 3 shows that an increase in the sum of risks lowers the equilibrium effort gap. Finally, Lemma 4 describes how a change in the sum of risks affects the players' perceived winning probabilities.

Lemma 4. *In a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential, a player's probability of winning decreases in the sum of risks $\sigma_{r_1}^2 + \sigma_{r_2}^2$, independently of player 1's overconfidence.*

Ultimately, the SPE outcome depends on the relative importance of effort and likelihood effects and their interrelatedness. Proposition 7 characterizes the SPE

[1] Note, that the derivatives with respect to σ_i^2 always coincide with the derivatives with respect to σ^2. For this reason, what matters ultimately is the total variance.

of the specialized model.

Proposition 7. *Consider a tournament where player 1 is overconfident and player 2 is rational, ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, the cost of effort is exponential, and $\lambda < \sigma^2 = \sigma_1^2 + \sigma_2^2$. Let $\bar{\lambda}_{hh1}$ denote the unique solution to (3), $\bar{\lambda}_{hh2}$ the unique solution to (4), $\bar{\lambda}_{ll1}$ the unique solution to (5), and $\bar{\lambda}_{ll2}$ the unique solution to (6).*
(i) If $\lambda < \min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}$, then there is a unique SPE where both players choose the high risk strategy.
(ii) If $\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$, then there is a unique SPE where the overconfident player chooses the low risk strategy and the rational player the high risk strategy.
(iii) If $\lambda < \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\}$, then there is a unique SPE where both players choose the low risk strategy.
In all of the above SPE the equilibrium efforts of players 1 and 2 are given by (7) and (8), respectively.

In Proposition 7 we show that there is a unique asymmetric SPE for any overconfidence bias λ. Depending on the size of λ, this SPE consists of different strategy profiles. To give some intuition, consider a small bias, i.e., $\lambda < \min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}$. In this case, both players choose the high risk strategy. When player 1 is just slightly overconfident, both players see themselves as being almost equally talented and therefore choose very similar efforts and have very similar winning probabilities. In such a situation, the outcome of the tournament is less dependent on the perceived talent gap but more so on effort. Hence, it is beneficial for the players to limit the effort exerted. They do so by selecting the high risk strategy.

Now, consider a large bias, i.e., $\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$. In this case, the overconfident player chooses the low risk strategy whereas the rational player chooses a high risk strategy. Now player 1 thinks, mistakenly, that he has a large talent advantage over player 2. This mistaken perception holds even after player 1 takes into account that player 2 will exert more effort than him. Since player 1 thinks, mistakenly, that he has a large advantage, he goes for the low risk strategy. Player 2, being aware that 1 is overconfident, knows that she is going to exert only a slightly higher effort than player 1. Since, from player 2's perspective, talent is the same and efforts are very close, she chooses the high risk strategy.

Finally, consider a very large bias, i.e., $\lambda > \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\}$. In this case, both players opt for the low risk strategy. For player 1 the above reasoning applies again. Player 2 knows she exerts much more effort than player 1. This large effort advantage makes her want to play it safe and therefore she opts for the low risk strategy. Also, player 2, by choosing the low risk strategy, increases the effort gap even further which pushes up player 2's probability of winning.

We show in the proof of Proposition 7 that for any overconfidence bias $\lambda < \sigma^2 = \sigma_1^2 + \sigma_2^2$, there does not exist a SPE where the overconfident player chooses a high risk strategy and the rational player a low risk strategy. Hence, Proposition 7 shows that the idea that overconfident individuals choose riskier strategies than rational ones does not hold in our model. Instead, either both players choose the same risk strategy or, when overconfidence is large, the overconfident player chooses the low risk strategy while the rational player chooses the high risk strategy.

Proof of Proposition 4

The first-order conditions of players 1 and 2 are

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*),$$

and

$$g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*),$$

respectively.

Assume, by contradiction, $a_1^* = a_2^*$. This, $\lambda > 0$, and $g'(x; \sigma_1^2, \sigma_2^2) < 0$ for $x > 0$ imply $g(\lambda; \sigma_1^2, \sigma_2^2) < g(0; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) = c'(a_2^*)$. Next, asssume, by contradiction, $a_1^* > a_2^*$. Since, $\lambda > 0$ this implies $\lambda + a_1^* - a_2^* > a_1^* - a_2^* > 0$. However, this and $g'(x; \sigma_1^2, \sigma_2^2) < 0$ for $x > 0$, in turn, imply $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) > c'(a_2^*)$. Hence, it must be that $a_1^* < a_2^*$. Finally, note that $a_1^* < a_2^*$ and the first-order conditions imply $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality, $\lambda + a_1^* - a_2^* > a_1^* - a_2^*$ and $g'(x; \sigma_1^2, \sigma_2^2) > 0$ for $x < 0$ imply $\lambda + a_1^* - a_2^* > 0$. Hence, in equilibrium we have $\lambda + a_1^* > a_2^* > a_1^*$.

The impact of overconfidence on the pure-strategy Nash equilibrium efforts is obtained from total differentiation of the first-order conditions of players 1 and 2:

$$g'(\lambda + a_1^* - a_2^*)(\partial\lambda + \partial a_1^* - \partial a_2^*)\Delta u = c''(a_1^*)\partial a_1^*$$

and

$$g'(a_1^* - a_2^*)(\partial a_1^* - \partial a_2^*)\Delta u = c''(a_2^*)\partial a_2^*.$$

Diving both equations by $\partial\lambda$ we obtain

$$g'(\lambda + a_1^* - a_2^*)\left(1 + \frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial\lambda}, \tag{9}$$

and

$$g'(a_1^* - a_2^*)\left(\frac{\partial a_1^*}{\partial\lambda} - \frac{\partial a_2^*}{\partial\lambda}\right)\Delta u = c''(a_2^*)\frac{\partial a_2^*}{\partial\lambda}. \tag{10}$$

Solving (10) for $\partial a_2^*/\partial\lambda$ we have

$$\frac{\partial a_2^*}{\partial\lambda} = \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial\lambda}. \tag{11}$$

Substituting (11) into (9) we obtain

$$g'(\lambda + a_1^* - a_2^*)\left[1 + \frac{\partial a_1^*}{\partial \lambda} - \frac{g'(a_1^* - a_2^*)\Delta u}{g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)}\frac{\partial a_1^*}{\partial \lambda}\right]\Delta u = c''(a_1^*)\frac{\partial a_1^*}{\partial \lambda}$$

Solving this equation for $\partial a_1^*/\partial\lambda$ we obtain

$$\frac{\partial a_1^*}{\partial \lambda} = \frac{1}{D^*}\left[g'(a_1^* - a_2^*)\Delta u + c''(a_2^*)\right] g'(\lambda + a_1^* - a_2^*)\Delta u, \tag{12}$$

where

$$D^* = \left[g'(\lambda + a_1^* - a_2^*)\Delta u - c''(a_1^*)\right]\left[-g'(a_1^* - a_2^*)\Delta u - c''(a_2^*)\right] + g'(\lambda + a_1^* - a_2^*)g'(a_1^* - a_2^*)(\Delta u)^2.$$

Substituting (12) into (11) we obtain

$$\frac{\partial a_2^*}{\partial \lambda} = \frac{1}{D^*}g'(a_1^* - a_2^*)g'(\lambda + a_1^* - a_2^*)(\Delta u)^2. \tag{13}$$

Note that the two terms inside square brackets in D^* are the second-order conditions of workers 1 and 2, respectively, and their signs are negative. Hence, the sign of the product of the terms inside square brackets is positive. Now, $\lambda + a_1^* > a_2^* > a_1^*$ implies $g'(\lambda + a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$. Hence, the last term in D^* is negative. However, simplifying D^* we obtain

$$D^* = -g'(\lambda + a_1^* - a_2^*)c''(a_2^*)\Delta u + g'(a_1^* - a_2^*)c''(a_1^*)\Delta u + c''(a_1^*)c''(a_2^*). \tag{14}$$

When $g'(\lambda + a_1^* - a_2^*) < 0$ and $g'(a_1^* - a_2^*) > 0$, the first and second terms in (14) are positive. The third term in (14) also is positive since $c'' > 0$. Hence, if $\lambda + a_1^* > a_2^* > a_1^*$, then $D^* > 0$. Thus, we have shown that $D^* > 0$. It follows from (12), (13), $\lambda + a_1^* > a_2^* > a_1^*$, and $D^* > 0$, that $\partial a_1^*/\partial\lambda < \partial a_2^*/\partial\lambda < 0$.

Proof of Proposition 5

When the random terms are normally distributed, players 1 and 2 objective probabilities of winning the tournament are

$$P_1(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right),$$

and

$$P_2(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = 1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = 1 - \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right),$$

respectively. We know from Proposition 4 that in the pure-strategy Nash equilibrium of the effort stage the rational player 2 exerts higher effort than the overconfident player 1, i.e., $a_2^* > a_1^*$. This implies

$$P_1(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2) = \Phi\left(\frac{a_1^* - a_2^*}{\sqrt{(\sigma_1^2 + \sigma_2^2)}}\right) < \Phi(0) = \frac{1}{2}.$$

Proof of Proposition 6

Player 1 chooses the optimal effort level that maximizes

$$E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)] = u(y_l) + \Phi\left(\frac{\lambda + a_1 - a_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - e^{a_1},$$

and player 2 chooses the optimal effort level that maximizes

$$E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)] = u(y_l) + \left[1 - \Phi\left(\frac{a_1 - a_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - e^{a_2}.$$

The first-order conditions for players 1 and 2, respectively, are

$$\frac{\partial E[U_1(a_1, a_2, \lambda, \sigma_1^2, \sigma_2^2)]}{\partial a_1} = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{(\lambda + a_1 - a_2)^2}{2(\sigma_1^2 + \sigma_2^2)}} \Delta u - e^{a_1} = 0 \tag{15}$$

$$\frac{\partial E[U_2(a_1, a_2, \sigma_1^2, \sigma_2^2)]}{\partial a_2} = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{(a_1 - a_2)^2}{2(\sigma_1^2 + \sigma_2^2)}} \Delta u - e^{a_2} = 0. \tag{16}$$

Taking logs and rearranging the first-order conditions yields the following expressions.

$$a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 - 2\ln(r)\sigma^2 = 0 \tag{17}$$

$$a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2 - 2\ln(r)\sigma^2 = 0, \tag{18}$$

where $r = \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2$. Rearranging further and we obtain

$$a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 = 2\ln(r)\sigma^2$$

$$a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2 = 2\ln(r)\sigma^2$$

Thus, it must be that

$$a_1^2 + \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 + a_2^2 = a_2^2 + \left(2\sigma^2 - 2a_1\right) a_2 + a_1^2$$

Simplifying this expression yields

$$\begin{aligned} \left(2\sigma^2 + 2\lambda - 2a_2\right) a_1 + \lambda^2 - 2\lambda a_2 &= \left(2\sigma^2 - 2a_1\right) a_2 \\ 2a_1\sigma^2 + 2a_1\lambda - 2a_1 a_2 + \lambda^2 - 2\lambda a_2 &= 2a_2\sigma^2 - 2a_1 a_2 \\ 2a_1\sigma^2 + 2a_1\lambda + \lambda^2 - 2\lambda a_2 &= 2a_2\sigma^2 \\ a_1(2\sigma^2 + 2\lambda) + \lambda^2 &= a_2(2\sigma^2 + 2\lambda) \end{aligned}$$

Thus,

$$a_2 = a_1 + \frac{\lambda^2}{2\sigma^2 + 2\lambda}, \tag{19}$$

where we define $z = \frac{\lambda^2}{2\sigma^2+2\lambda}$. Inserting into (18) we obtain

$$
\begin{aligned}
(a_1 + z)^2 + \left(2\sigma^2 - 2a_1\right)(a_1 + z) + a_1^2 - 2\ln(r)\sigma^2 &= 0 \\
a_1^2 + 2a_1 z + z^2 + 2a_1\sigma^2 + 2z\sigma^2 - 2a_1^2 - 2a_1 z + a_1^2 - 2\ln(r)\sigma^2 &= 0 \\
z^2 + 2a_1\sigma^2 + 2z\sigma^2 - 2\ln(r)\sigma^2 &= 0
\end{aligned}
$$

Solving for a_1 yields

$$
\begin{aligned}
a_1 &= \ln r - \frac{(2\sigma^2 + z)}{2\sigma^2} z \\
&= \ln(r) - \frac{(2\sigma^2 + \frac{\lambda^2}{2\sigma^2+2\lambda})}{2\sigma^2} \left(\frac{\lambda^2}{2\sigma^2 + 2\lambda}\right) \\
&= \ln(r) - \frac{2\sigma^2(2\sigma^2 + 2\lambda) + \lambda^2}{2\sigma^2(2\sigma^2 + 2\lambda)} \left(\frac{\lambda^2}{2\sigma^2 + 2\lambda}\right) \\
&= \ln(r) - \frac{2\sigma^2(2\sigma^2 + 2\lambda) + \lambda^2}{2\sigma^2(2\sigma^2 + 2\lambda)^2} \lambda^2 \\
&= \ln(r) - \frac{4\sigma^4 + 4\sigma^2\lambda + \lambda^2}{8\sigma^2(\sigma^2 + \lambda)^2} \lambda^2 \\
&= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2(2\sigma^2 + \lambda)^2}{8\sigma^2(\sigma^2 + \lambda)^2}.
\end{aligned}
$$

Which coincides with equation (7). The equilibrium effort level for player 2 we obtain be inserting the previous result into (19).

$$
\begin{aligned}
a_2 &= \ln(r) - \frac{\lambda^2(2\sigma^2 + \lambda)^2}{8\sigma^2(\sigma^2 + \lambda)^2} + \frac{\lambda^2}{2\sigma^2 + 2\lambda} \\
&= \ln(r) - \frac{\lambda^2(2\sigma^2 + \lambda)^2}{8\sigma^2(\sigma^2 + \lambda)^2} + \frac{4\sigma^2\lambda^2(\sigma^2 + \lambda)}{8\sigma^2(\sigma^2 + \lambda)^2} \\
&= \ln(r) - \frac{\lambda^2(4\sigma^4 + 4\sigma^2\lambda + \lambda^2) - 4\sigma^4\lambda^2 - 4\sigma^2\lambda^3}{8\sigma^2(\sigma^2 + \lambda)^2} \\
&= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^4}{8\sigma^2(\sigma^2 + \lambda)^2}.
\end{aligned}
$$

Which coincides with equation (8). When analyzing the effect of the overconfidence level λ on the equilibrium values, we find a negative relationship, independent of the size of λ:

$$
\begin{aligned}
\frac{\partial a_1^*(\sigma_1^2,\sigma_2^2)}{\partial\lambda} &= -\frac{\left(2\lambda(2\sigma^2+\lambda)^2+2\lambda^2(2\sigma^2+\lambda)\right)8\sigma^2(\sigma^2+\lambda)^2-\lambda^2(2\sigma^2+\lambda)^2 16\sigma^2(\sigma^2+\lambda)}{(8\sigma^2(\sigma^2+\lambda)^2)^2} \\
&= -\frac{\left(\lambda(2\sigma^2+\lambda)^2+\lambda^2(2\sigma^2+\lambda)\right)(\sigma^2+\lambda)-\lambda^2(2\sigma^2+\lambda)^2}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\frac{\left(\lambda(2\sigma^2+\lambda)\right)\left(\left(2\sigma^2+2\lambda\right)(\sigma^2+\lambda)-\lambda(2\sigma^2+\lambda)\right)}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\frac{\left(2\lambda\sigma^2+\lambda^2\right)\left(2\sigma^4+4\lambda\sigma^2+2\lambda^2-2\lambda\sigma^2-\lambda^2\right)}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\underbrace{\frac{\left(2\lambda\sigma^2+\lambda^2\right)\left(2\sigma^4+2\lambda\sigma^2+\lambda^2\right)}{4\sigma^2(\sigma^2+\lambda)^3}}_{>0} < 0 \\
\frac{\partial a_2^*(\sigma_1^2,\sigma_2^2)}{\partial\lambda} &= -\frac{32\lambda^3\sigma^2(\sigma^2+\lambda)^2-16\lambda^4\sigma^2(\sigma^2+\lambda)}{(8\sigma^2(\sigma^2+\lambda)^2)^2} \\
&= -\frac{2\lambda^3(\sigma^2+\lambda)-\lambda^4}{4\sigma^2(\sigma^2+\lambda)^3} \\
&= -\underbrace{\frac{2\lambda^3\sigma^2+\lambda^4}{4\sigma^2(\sigma^2+\lambda)^3}}_{>0} < 0
\end{aligned}
$$

Proof of Lemma 2

Consider the derivatives of the equilibrium efforts of player 1 and 2 with respect to their respective risk choice.

$$
\begin{aligned}
\frac{\partial a_1^*(\sigma_1^2,\sigma_2^2)}{\partial\sigma_1^2} &= \frac{\partial}{\partial\sigma_1^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right)-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}\right) \\
&= \frac{\partial}{\partial\sigma^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right)-\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\frac{\Delta u}{\sqrt{2\pi}}\left(-\frac{1}{2}\right)}{\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\left(\sigma^2\right)^{\frac{3}{2}}}-\frac{32\lambda^2(\lambda+2\sigma^2)\sigma^2(\lambda+\sigma^2)^2-8\lambda^2(\lambda+2\sigma^2)^2\left((\lambda+\sigma^2)^2+2\sigma^2(\lambda+\sigma^2)\right)}{\left(8\sigma^2(\lambda+\sigma^2)^2\right)^2} \\
&= \frac{1}{2\sigma^2}\left[-1+\frac{\lambda^2}{\sigma^2}\frac{4\sigma^8+8\sigma^6\lambda+7\sigma^4\lambda^2+4\sigma^2\lambda^3+\lambda^4}{4\sigma^8+16\sigma^6\lambda+24\sigma^4\lambda^2+16\sigma^2\lambda^3+4\lambda^4}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial a_2^*(\sigma_1^2,\sigma_2^2)}{\partial \sigma_2^2} &= \frac{\partial}{\partial \sigma_2^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\right) - \frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}\right) \\
&= \frac{\partial}{\partial \sigma^2}\left(\ln\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right) - \frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\frac{\Delta u}{\sqrt{2\pi}}\left(-\frac{1}{2}\right)}{\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\left(\sigma^2\right)^{\frac{3}{2}}} - \frac{-8\lambda^4\left((\lambda+\sigma^2)^2+2\sigma^2(\lambda+\sigma^2)\right)}{\left(8\sigma^2(\lambda+\sigma^2)^2\right)^2} \\
&= \frac{1}{2\sigma^2}\left[-1+\frac{\lambda^2}{\sigma^2}\frac{3\sigma^4\lambda^2+4\sigma^2\lambda^3+\lambda^4}{4\sigma^8+16\sigma^6\lambda+24\sigma^4\lambda^2+16\sigma^2\lambda^3+4\lambda^4}\right]
\end{aligned}
$$

A sufficient condition for the derivatives $\partial a_1^2(.)/\partial \sigma_1^2$ and $\partial a_2^*(.)/\partial \sigma_2^2$ to be negative is that

$$
-1+\frac{\lambda^2}{\sigma^2} \leq 0
$$

and thus

$$
\lambda^2 \leq \sigma^2 = \sigma_1^2 + \sigma_2^2. \tag{20}
$$

Hence, if we assume (20), it follows that

$$
a_1^*(\sigma_H^2,\sigma_H^2) < a_1^*(\sigma_L^2,\sigma_H^2) = a_1^*(\sigma_H^2,\sigma_L^2) < a_1^*(\sigma_L^2,\sigma_L^2),
$$

and

$$
a_2^*(\sigma_H^2,\sigma_H^2) < a_2^*(\sigma_L^2,\sigma_H^2) = a_2^*(\sigma_H^2,\sigma_L^2) < a_2^*(\sigma_L^2,\sigma_L^2) < 0.
$$

Proof of Lemma 3

$$
\begin{aligned}
\frac{\partial a_2^* - a_1^*}{\partial \sigma^2} &= \frac{\partial}{\partial \sigma^2}\left(\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2} - \frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\partial}{\partial \sigma^2}\left(\frac{4\lambda^2\sigma^2(\lambda+\sigma^2)}{8\sigma^2(\lambda+\sigma^2)^2}\right) \\
&= \frac{\partial}{\partial \sigma^2}\left(\frac{\lambda^2}{2(\lambda+\sigma^2)}\right) \\
&= -\frac{\lambda^2}{2(\lambda+\sigma^2)^2}
\end{aligned}
$$

Proof of Lemma 4

We determine how player 1's probability of winning is influenced by the total variance. If only considering the probability of winning the tournament, the overconfident player chooses the low risk instead of the high risk if

$$G(\lambda + a_1^* - a_2; \sigma_L^2, \sigma_{r_2}^2) > G(\lambda + a_1^* - a_2; \sigma_H^2, \sigma_{r_2}^2)$$

$$\frac{1}{2}\left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_L^2+\sigma_{r_2}^2))\sqrt{2(\sigma_L^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau\right) > \frac{1}{2}\left(1 + \frac{2}{\sqrt{\pi}} \int_0^{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_H^2+\sigma_{r_2}^2))\sqrt{2(\sigma_H^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau\right)$$

$$\int_0^{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_L^2+\sigma_{r_2}^2))\sqrt{2(\sigma_L^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau > \int_0^{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_{r_2}^2))}{2(\lambda+(\sigma_H^2+\sigma_{r_2}^2))\sqrt{2(\sigma_H^2+\sigma_{r_2}^2)}}} e^{-\tau^2} d\tau$$

The left hand side is greater if its integral's upper limit is larger than the one of the right hand side. Thus, if we have that

$$\frac{\lambda(\lambda + 2(\sigma_L^2 + \sigma_{r_2}^2))}{2(\lambda + (\sigma_L^2 + \sigma_{r_2}^2))\sqrt{2(\sigma_L^2 + \sigma_{r_2}^2)}} > \frac{\lambda(\lambda + 2(\sigma_H^2 + \sigma_{r_2}^2))}{2(\lambda + (\sigma_H^2 + \sigma_{r_2}^2))\sqrt{2(\sigma_H^2 + \sigma_{r_2}^2)}}$$

As it is the total variance that matter, the probability of winning is the same in either of the two asymmetric risk choice profiles, i.e. $G(\lambda + a_1^* - a_2; \sigma_L^2, \sigma_H^2) = G(\lambda + a_1^* - a_2; \sigma_H^2, \sigma_L^2)$. Further, as $\frac{\partial G(\lambda + a_1^* - a_2^*, \sigma_1^2; \sigma_2^2)}{\partial \sigma_1^2} < 0$, player 1's probability of winning decreases with higher values of the total variance. The same applies for the probability of winning of player 2. If only taking into account the probability of winning the tournament, player 2 chooses the low risk instead of the high risk if

$$G(a_1^* - a_2; \sigma_{r_1}^2, \sigma_L^2) > G(a_1^* - a_2; \sigma_{r_1}^2, \sigma_H^2)$$

$$\frac{1}{2}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}}} e^{-\tau^2} d\tau\right) > \frac{1}{2}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}} e^{-\tau^2} d\tau\right)$$

$$\int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_L^2))\sqrt{2(\sigma_{r_1}^2+\sigma_L^2)}}} e^{-\tau^2} d\tau < \int_0^{\frac{-\lambda^2}{2(\lambda+(\sigma_{r_1}^2+\sigma_H^2))\sqrt{2(\sigma_{r_1}^2+\sigma_H^2)}}} e^{-\tau^2} d\tau$$

The right hand side is greater if its integral's upper limit is larger than the one of the left hand side. Thus, we have that

$$\frac{-\lambda^2}{2(\lambda + (\sigma_{r_1}^2 + \sigma_L^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_L^2)}} < \frac{-\lambda^2}{2(\lambda + (\sigma_{r_1}^2 + \sigma_H^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_H^2)}}$$

$$\frac{1}{(\lambda + (\sigma_{r_1}^2 + \sigma_L^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_L^2)}} > \frac{1}{(\lambda + (\sigma_{r_1}^2 + \sigma_H^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_H^2)}}$$

$$(\lambda + (\sigma_{r_1}^2 + \sigma_H^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_H^2)} > (\lambda + (\sigma_{r_1}^2 + \sigma_L^2))\sqrt{2(\sigma_{r_1}^2 + \sigma_L^2)}$$

Proof of Proposition 7

Given the Nash equilibrium efforts of the effort stage (7) and (8), we can write the perceived expected utilities of both players at the risk stage as

$$\begin{aligned}
E[U_1(a_1^*, a_2^*, \lambda, \sigma_1^2, \sigma_2^2)] &= u(y_l) + G(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u - c(a_1^*) \\
&= u(y_l) + \Phi\left(\frac{\lambda + a_1^*(\sigma_1^2, \sigma_2^2) - a_2^*(\sigma_1^2, \sigma_2^2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - e^{a_1^*(\sigma_1^2, \sigma_2^2)} \\
&= u(y_l) + \Phi\left(\frac{\lambda + \frac{\lambda^4 - \lambda^2(\lambda + 2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u \\
&= u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_1^2+\sigma_2^2))}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u,
\end{aligned}$$

and

$$\begin{aligned}
E[U_2(a_1^*, a_2^*, \sigma_1^2, \sigma_2^2)] &= u(y_l) + [1 - G(a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)]\Delta u - c(a_2^*) \\
&= u(y_l) + \left[1 - \Phi\left(\frac{a_1^*(\sigma_1^2, \sigma_2^2) - a_2^*(\sigma_1^2, \sigma_2^2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_1^2, \sigma_2^2)} \\
&= u(y_l) + \left[1 - \Phi\left(\frac{\frac{\lambda^4 - \lambda^2(\lambda + 2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u \\
&= u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u.
\end{aligned}$$

Thus, the maximization problems of the risk stage for players 1 and 2, respectively, are

$$\max_{\sigma_1^2 \in \{\sigma_L^2, \sigma_H^2\}} \quad u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_1^2+\sigma_2^2))}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_1^2+\sigma_2^2))^2}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u$$

$$\max_{\sigma_2^2 \in \{\sigma_L^2, \sigma_H^2\}} \quad u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_1^2+\sigma_2^2))}}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_1^2+\sigma_2^2)(\lambda+(\sigma_1^2+\sigma_2^2))^2}}}{\sqrt{2\pi(\sigma_1^2+\sigma_2^2)}}\Delta u$$

(i) Let us consider a SPE where

$$(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2),$$

and

$$
\begin{aligned}
a_1^*(\sigma_H^2, \sigma_H^2) &= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) - \frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2} \\
a_2^*(\sigma_H^2, \sigma_H^2) &= \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) - \frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}.
\end{aligned}
$$

In a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, that is,

$$
E[U_1(a_1^*(\sigma_H^2,\sigma_H^2), a_2^*(\sigma_H^2,\sigma_H^2), \lambda, \sigma_H^2, \sigma_H^2)] \geq E[U_1(a_1^*(\sigma_L^2,\sigma_H^2), a_2^*(\sigma_L^2,\sigma_H^2), \lambda, \sigma_L^2, \sigma_H^2)],
$$

or

$$
\begin{aligned}
& u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_H^2))}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\Delta u \\
& \geq u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_H^2))}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}\Delta u,
\end{aligned}
$$

or

$$
\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} \geq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_H^2))}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_H^2))}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right). \tag{21}
$$

Setting $\lambda = 0$ in the LHS of (21) we obtain

$$
LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} > 0.
$$

Setting $\lambda = 0$ in the RHS of (21) we obtain

$$
RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.
$$

Note that the RHS of (21) is non-negative. Note also that the LHS of (21) is equal to zero when

$$
\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} = \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}},
$$

or

$$
\sqrt{\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}} = e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2} + \frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}},
$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)=\frac{(\lambda+2(\sigma_L^2+\sigma_H^2))^2}{(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}-\frac{(\lambda+2(\sigma_H^2+\sigma_H^2))^2}{(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}. \tag{22}$$

The LHS and the RHS of (22) are both strictly positive. Moreover, the LHS and the RHS of (22) are both monotonically decreasing in λ. Since the LHS starts at a higher value and decreases at a faster rate with λ than the RHS, there is a unique value for λ that satisfies (22). Denote this value by $\bar{\lambda}_{0hh1}$. Since the LHS of (21) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0hh1})$ such that (21) holds as an equality. Denote this value by $\bar{\lambda}_{hh1}$. Hence, inequality (21) is satisfied when $\lambda < \bar{\lambda}_{hh1}$ and is violated when $\lambda > \bar{\lambda}_{hh1}$.

In a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$, that is,

$$E[U_2(a_1^*(\sigma_H^2,\sigma_H^2),a_2^*(\sigma_H^2,\sigma_H^2),\sigma_H^2,\sigma_H^2)]\geq E[U_2(a_1^*(\sigma_H^2,\sigma_L^2),a_2^*(\sigma_H^2,\sigma_L^2),\sigma_H^2,\sigma_L^2)],$$

or

$$\begin{aligned}
&u(y_l)+\left[1-\Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\right]\Delta u-\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\Delta u\\
&\geq u(y_l)+\left[1-\Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right)\right]\Delta u-\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}\Delta u,
\end{aligned}$$

or

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}-\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\geq\Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_H^2))}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)-\Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right). \tag{23}$$

Setting $\lambda = 0$ in the LHS of (23) we obtain

$$LHS(\lambda=0)=\frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}-\frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}>0.$$

Setting $\lambda = 0$ in the RHS of (23) we obtain

$$RHS(\lambda=0)=\Phi(0)-\Phi(0)=0.5-0.5=0.$$

Note that the RHS of (23) is non-negative. Note also that the LHS of (23) is equal to zero when

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}=\frac{e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2+\sigma_H^2}{\sigma_H^2+\sigma_L^2}} = e^{-\frac{\lambda^4}{8(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}+\frac{\lambda^4}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_H^2+\sigma_L^2}\right) = \frac{\lambda^2}{(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2} - \frac{\lambda^2}{(\sigma_H^2+\sigma_H^2)(\lambda+(\sigma_H^2+\sigma_H^2))^2}. \tag{24}$$

The LHS and the RHS of (24) are both strictly positive. Moreover, the LHS of (24) is monotonically decreasing in λ whereas the RHS of (24) is monotonically increasing in λ. Since the LHS starts at a higher value than the RHS, there is a unique value for λ that satisfies (24). Denote this value by $\bar{\lambda}_{0hh2}$. Since the LHS of (23) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0hh2})$ such that (23) holds as an equality. Denote this value by $\bar{\lambda}_{hh2}$. Hence, inequality (23) is satisfied when $\lambda < \bar{\lambda}_{hh2}$ and is violated when $\lambda > \bar{\lambda}_{hh2}$.
(iii) Let us consider a SPE where

$$(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2),$$

and

$$a_1^*(\sigma_L^2,\sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right) - \frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}$$
$$a_2^*(\sigma_L^2,\sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right) - \frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}.$$

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$, that is,

$$E[U_1(a_1^*(\sigma_L^2,\sigma_L^2), a_2^*(\sigma_L^2,\sigma_L^2), \lambda, \sigma_L^2, \sigma_L^2)] \geq E[U_1(a_1^*(\sigma_H^2,\sigma_L^2), a_2^*(\sigma_H^2,\sigma_L^2), \lambda, \sigma_H^2, \sigma_L^2)],$$

or

$$u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\Delta u$$
$$\geq u(y_l) + \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right)\Delta u - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}}\Delta u,$$

or

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2+\sigma_L^2)}} \leq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right) - \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+(\sigma_H^2+\sigma_L^2))}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right). \tag{25}$$

Setting $\lambda = 0$ in the LHS of (25) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (25) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Note that the RHS of (21) is non-negative. Note also that the LHS of (25) is equal to zero when

$$\frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} = \frac{e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2 + \sigma_L^2}{\sigma_L^2 + \sigma_L^2}} = e^{-\frac{\lambda^2(\lambda+2(\sigma_H^2+\sigma_L^2))^2}{8(\sigma_H^2+\sigma_L^2)(\lambda+(\sigma_H^2+\sigma_L^2))^2}+\frac{\lambda^2(\lambda+2(\sigma_L^2+\sigma_L^2))^2}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_H^2 + \sigma_L^2}{\sigma_L^2 + \sigma_L^2}\right) = \frac{(\lambda + 2(\sigma_L^2 + \sigma_L^2))^2}{(\sigma_L^2 + \sigma_L^2)(\lambda + (\sigma_L^2 + \sigma_L^2))^2} - \frac{(\lambda + 2(\sigma_H^2 + \sigma_L^2))^2}{(\sigma_H^2 + \sigma_L^2)(\lambda + (\sigma_H^2 + \sigma_L^2))^2}. \tag{26}$$

The LHS and the RHS of (26) are both strictly positive. Moreover, the LHS and the RHS of (26) are both monotonically decreasing in λ. Since the LHS starts at a higher value and decreases at a faster rate with λ than the RHS, there is a unique value for λ that satisfies (26). Denote this value by $\bar{\lambda}_{0ll1}$. Since the LHS of (25) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0ll1})$ such that (25) holds as an equality. Denote this value by $\bar{\lambda}_{ll1}$. Hence, inequality (25) is satisfied when $\lambda > \bar{\lambda}_{ll1}$ and is violated when $\lambda < \bar{\lambda}_{ll1}$.

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, that is,

$$E[U_2(a_1^*(\sigma_L^2, \sigma_L^2), a_2^*(\sigma_L^2, \sigma_L^2), \sigma_L^2, \sigma_L^2)] \geq E[U_2(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \sigma_L^2, \sigma_H^2)],$$

or

$$\begin{aligned} u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}}\Delta u \\ \geq u(y_l) + \left[1 - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right)\right]\Delta u - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\Delta u, \end{aligned}$$

or

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} \leq \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_H^2))}}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right) - \Phi\left(\frac{\frac{-\lambda^2}{2(\lambda+(\sigma_L^2+\sigma_L^2))}}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right). \tag{27}$$

Setting $\lambda = 0$ in the LHS of (27) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (27) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Note that the RHS of (27) is non-negative. Note also that the LHS of (27) is equal to zero when

$$\frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} = \frac{e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_L^2 + \sigma_H^2}{\sigma_L^2 + \sigma_L^2}} = e^{-\frac{\lambda^4}{8(\sigma_L^2+\sigma_H^2)(\lambda+(\sigma_L^2+\sigma_H^2))^2} + \frac{\lambda^4}{8(\sigma_L^2+\sigma_L^2)(\lambda+(\sigma_L^2+\sigma_L^2))^2}},$$

or

$$\frac{4}{\lambda^2}\ln\left(\frac{\sigma_L^2 + \sigma_H^2}{\sigma_L^2 + \sigma_L^2}\right) = \frac{\lambda^2}{8(\sigma_L^2 + \sigma_L^2)(\lambda + (\sigma_L^2 + \sigma_L^2))^2} - \frac{\lambda^2}{8(\sigma_L^2 + \sigma_H^2)(\lambda + (\sigma_L^2 + \sigma_H^2))^2}. \tag{28}$$

The LHS and the RHS of (28) are both strictly positive. Moreover, the LHS of (28) is monotonically decreasing in λ whereas the RHS of (28) is monotonically increasing in λ. Since the LHS starts at a higher value than the RHS, there is a unique value for λ that satisfies (28). Denote this value by $\bar{\lambda}_{0ll2}$. Since the LHS of (27) is positive and the RHS is equal to zero when $\lambda = 0$, it follows that there is a unique $\lambda \in (0, \bar{\lambda}_{0ll2})$ such that (27) holds as an equality. Denote this value by $\bar{\lambda}_{ll2}$. Hence, inequality (27) is satisfied when $\lambda > \bar{\lambda}_{ll2}$ and is violated when $\lambda < \bar{\lambda}_{ll2}$.
(ii) Let us consider a SPE where

$$(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2),$$

and

$$a_1^*(\sigma_L^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_L^2 + \sigma_H^2))^2}{8(\sigma_L^2 + \sigma_H^2)(\lambda + (\sigma_L^2 + \sigma_H^2))^2}$$
$$a_2^*(\sigma_L^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}}\right) - \frac{\lambda^4}{8(\sigma_L^2 + \sigma_H^2)(\lambda + (\sigma_L^2 + \sigma_H^2))^2}.$$

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, player 1 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$, that is,

$$E[U_1(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \lambda, \sigma_L^2, \sigma_L^2)] \geq E[U_1(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \lambda, \sigma_H^2, \sigma_L^2)]. \tag{29}$$

In SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_H^2)$, player 2 cannot gain with a deviation to $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$, that is,

$$E[U_2(a_1^*(\sigma_L^2, \sigma_H^2), a_2^*(\sigma_L^2, \sigma_H^2), \sigma_L^2, \sigma_H^2)] \geq E[U_2(a_1^*(\sigma_L^2, \sigma_L^2), a_2^*(\sigma_L^2, \sigma_L^2), \sigma_L^2, \sigma_L^2)]. \tag{30}$$

Note that inequality (29) states the contrary of inequality (25) and that inequality (30) states the contrary of inequality (27). Hence, when $\lambda \in (\min\{\bar{\lambda}_{hh1}, \bar{\lambda}_{hh2}\}, \max\{\bar{\lambda}_{ll1}, \bar{\lambda}_{ll2}\})$ this SPE holds.

Finally, we show that the strategy profile

$$(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2),$$

and

$$a_1^*(\sigma_H^2, \sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}}\right) - \frac{\lambda^2(\lambda + 2(\sigma_H^2 + \sigma_L^2))^2}{8(\sigma_H^2 + \sigma_L^2)(\lambda + (\sigma_H^2 + \sigma_L^2))^2}$$
$$a_2^*(\sigma_H^2, \sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_L^2)}}\right) - \frac{\lambda^4}{8(\sigma_H^2 + \sigma_L^2)(\lambda + (\sigma_H^2 + \sigma_L^2))^2}.$$

cannot be a SPE. If $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$ were a SPE, then

$$E[U_1(a_1^*(\sigma_H^2, \sigma_L^2), a_2^*(\sigma_H^2, \sigma_L^2), \lambda, \sigma_H^2, \sigma_L^2)] \geq E[U_1(a_1^*(\sigma_L^2, \sigma_L^2), a_2^*(\sigma_L^2, \sigma_L^2), \lambda, \sigma_L^2, \sigma_L^2)],$$

and

$$E[U_2(a_1^*(\sigma_H^2, \sigma_L^2), a_2^*(\sigma_H^2, \sigma_L^2), \sigma_H^2, \sigma_L^2)] \geq E[U_2(a_1^*(\sigma_H^2, \sigma_H^2), a_2^*(\sigma_H^2, \sigma_H^2), \sigma_H^2, \sigma_H^2)].$$

These inequalities are given by

$$\Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_H^2+\sigma_L^2))}{2(\lambda+\sigma_H^2+\sigma_L^2)}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right)\Delta u - e^{a_1^*(\sigma_H^2, \sigma_L^2)} \geq \Phi\left(\frac{\frac{\lambda(\lambda+2(\sigma_L^2+\sigma_L^2))}{2(\lambda+\sigma_L^2+\sigma_L^2)}}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)\Delta u - e^{a_1^*(\sigma_L^2, \sigma_L^2)}, \tag{31}$$

and

$$\left[1 - \Phi\left(\frac{-\frac{\lambda^2}{2(\lambda+\sigma_H^2+\sigma_L^2)}}{\sqrt{\sigma_H^2+\sigma_L^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_H^2, \sigma_L^2)} \geq \left[1 - \Phi\left(\frac{-\frac{\lambda^2}{2(\lambda+\sigma_H^2+\sigma_H^2)}}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\right]\Delta u - e^{a_2^*(\sigma_H^2, \sigma_H^2)}, \tag{32}$$

respectively. For (31) to be satisfied, we need that $\Phi(.)\Delta u - e^{a_1^*(.)}$ is increasing in the sum of risks σ^2, i.e.,

$$\frac{\partial}{\partial\sigma^2}\left[\Phi\left(\frac{\frac{\lambda(\lambda+2\sigma^2)}{2(\lambda+\sigma^2)}}{\sqrt{\sigma^2}}\right)\Delta u - e^{\log\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right)-\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}}\right] > 0$$

$$-e^{-\frac{\lambda^2(\lambda+2\sigma^2)^2}{8\sigma^2(\lambda+\sigma^2)^2}}\left[\frac{\lambda(\lambda^2+\lambda\sigma^2+2\sigma^4)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}} - \frac{1}{2\sigma^2\sqrt{2\pi\sigma^2}} + \frac{\lambda^2(\lambda+2\sigma^4)(\lambda^2+\lambda\sigma^2+2\sigma^2)}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}}\right]\Delta u > 0$$

$$-\frac{\lambda(\lambda^2+\lambda\sigma^2+2\sigma^4)}{2(\lambda+\sigma^2)^2} + 1 - \frac{\lambda^2(\lambda+2\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4)}{4\sigma^2(\lambda+\sigma^2)^3} > 0$$

$$4\sigma^2(\lambda+\sigma^2)^3 > \lambda\sigma^2(\lambda+\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4) + \lambda^2(\lambda+2\sigma^2)(\lambda^2+\lambda\sigma^2+2\sigma^4)$$

$$4\sigma^2(\lambda+\sigma^2)^3 > \lambda(\lambda^2+\lambda\sigma^2+2\sigma^4)\left(\sigma^2(\lambda+\sigma^2)+\lambda(\lambda+2\sigma^2)\right) \tag{33}$$

For (32) to hold, we need that $[1-\Phi(.)]\Delta u - e^{a_2^*(.)}$ is decreasing in the sum of risks σ^2, i.e.,

$$\frac{\partial}{\partial\sigma^2}\left[\left[1-\Phi\left(\frac{\frac{-\lambda^2}{2\lambda+\sigma^2}}{\sqrt{\sigma^2}}\right)\right]\Delta u - e^{\log\left(\frac{\Delta u}{\sqrt{2\pi\sigma^2}}\right)-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\right] < 0$$

$$-e^{-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\frac{\lambda^2(\lambda+3\sigma^2)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}}\Delta u - e^{-\frac{\lambda^4}{8\sigma^2(\lambda+\sigma^2)^2}}\frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}}\Delta u < 0$$

$$-\frac{\lambda^2(\lambda+3\sigma^2)}{4\sigma^2(\lambda+\sigma^2)^2\sqrt{2\pi\sigma^2}} - \frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{8\sigma^4(\lambda+\sigma^2)^3\sqrt{2\pi\sigma^2}} < 0$$

$$-\lambda^2(\lambda+3\sigma^2) - \frac{\lambda^4(\lambda+3\sigma^2)-4\sigma^2(\lambda+\sigma^2)^3}{2\sigma^2(\lambda+\sigma^2)} < 0$$

$$4\sigma^2(\lambda+\sigma^2)^3 < 2\sigma^2\lambda^2(\lambda+\sigma^2)(\lambda+3\sigma^2) + \lambda^4(\lambda+3\sigma^2)$$

$$4\sigma^2(\lambda+\sigma^2)^3 < \lambda^2(\lambda+3\sigma^2)\left(2\sigma^2(\lambda+\sigma^2)+\lambda^2\right) \tag{34}$$

For (33) and (34) to be satisfied and we would need that,

$$\begin{aligned}
\lambda\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)\left(\sigma^2(\lambda+\sigma^2)+\lambda(\lambda+2\sigma^2)\right) &< \lambda^2(\lambda+3\sigma^2)\left(2\sigma^2(\lambda+\sigma^2)+\lambda^2\right)\\
\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)\left(\lambda^2+3\lambda\sigma^2+\sigma^4\right) &< \lambda\left(\lambda^2+2\lambda\sigma^2+2\sigma^4\right)(\lambda+3\sigma^2)\\
\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)\left(\lambda+3\sigma^2\right)\lambda+(\lambda^2+\lambda\sigma^2+2\sigma^4)\sigma^4 &< \lambda\left(\lambda^2+\lambda\sigma^2+2\sigma^4\right)(\lambda+3\sigma^2)+\lambda^2\sigma^2(\lambda+3\sigma^2)\\
(\lambda^2+\lambda\sigma^2+2\sigma^4)\sigma^4 &< \lambda^2\sigma^2(\lambda+3\sigma^2)\\
(\lambda^2+\lambda\sigma^2+2\sigma^4)\sigma^2 &< \lambda^2(\lambda+3\sigma^2)\\
\lambda^2\sigma^2+\lambda\sigma^4+2\sigma^6 &< \lambda^3+3\lambda^2\sigma^2\\
\lambda\sigma^4+2\sigma^6 &< \lambda^3+2\lambda^2\sigma^2\\
\frac{\sigma^2}{\lambda}+\frac{2\sigma^4}{\lambda^2} &< \frac{\lambda}{\sigma^2}+2\\
0 &< \frac{\lambda}{\sigma^2}-\frac{\sigma^2}{\lambda}+2\left(1-\frac{\sigma^4}{\lambda^2}\right)\\
0 &< \frac{\lambda^2-\sigma^4}{\lambda\sigma^2}+2\frac{\lambda^2-\sigma^4}{\lambda^2}\\
0 &< (\lambda^2-\sigma^4)\frac{1}{\lambda}\left(\frac{1}{\sigma^2}+\frac{2}{\lambda}\right)
\end{aligned}$$

Since we consider values of λ such that $\lambda < \sigma^2$ this is a contradiction. Therefore, there are no values of $\lambda < \sigma^2$ for which (31) and (32) hold simultaneously and $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_L^2)$ can be a part of a SPE.