Less Risk, More Effort: How Overconfidence Reshapes Tournament Strategies

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Abstract

In many competitive settings, players must decide not only how hard to work but also how much risk to take. This paper shows that overconfidence—the tendency to overestimate one's own ability—can lead to surprising strategic behavior in tournaments in which players make both risk and effort choices. We find two key results. First, overconfident players may adopt less risky strategies than rational ones, defying the common belief that overconfidence necessarily drives risk-taking. Second, when overconfident players adopt less risky strategies, they may exert greater effort, revealing a new mechanism by which overconfidence can enhance effort provision.

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JEL classification: J3, L1, M5

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1 Introduction

The evidence that humans are overconfident is widespread and well-established. For example, most drivers believe they are better than the median driver (Svenson, 1981). Overconfidence affects the behavior of decision-makers such as CEOs (Malmendier and Tate, 2005, 2008, 2015), fund managers (Menkhoff et al., 2006), poker and chess players (Park and Santos-Pinto, 2010), and CFOs (Ben-David et al., 2013). These decision-makers are often engaged in tournaments, that is, competitions where rewards are based on relative performance. For example, who gets promoted to CEO, who wins a prize in an athletic competition (Szymanski, 2003), or who wins a sales bonus (Murphy et al., 2004).[1]

Studies analyzing the role of overconfidence in tournaments have so far focused on how it influences either risk taking (Goel and Thakor, 2008) or effort provision (Santos-Pinto (2010); Santos-Pinto and Sekeris (2025)). However, in many tournaments, players decide not only how much risk they take but also how much effort they exert. For example, a CEO can choose whether her firm has an innovative or a conservative research and development strategy in addition to how hard she works. A fund manager can choose the risk exposure of her portfolio and how much time and resources to spend on collecting and analyzing stock information. A researcher chooses between pursuing a safe, mainstream project or engaging in a riskier, multidisciplinary one and then decides how many hours to work. Similarly, a poker player chooses risk taking and the effort she puts into computing conditional probabilities.

This paper investigates the implications of overconfidence for behavior in tournaments where players choose both risk and effort. We ask the following questions: Does overconfidence lead players to adopt more or less risky strategies? Does overconfidence raise or lower effort provision? Can tournament organizers benefit from the players' overconfidence?

To answer these questions we consider a two player tournament in which the

[1] Moore and Healy (2008) distinguish between three types of overconfidence: (i) overestimation of one's absolute performance, (ii) overestimation of one's relative performance (overplacement), and (iii) excessive confidence in the precision of one's private information, estimates, and forecasts (overprecision or miscalibration). In our study, we use the term overconfidence in the sense of overestimation of absolute and relative performance in a tournament.

players' production functions are additively separable in ability, effort, and a random shock. The tournament consists of two stages. In the first stage (risk stage), the players simultaneously choose the risk of their production functions which can be either low or high. In the second stage (effort stage), each player observes the chosen risks and then decides how much effort to exert. The player attaining the highest output or performance wins the tournament and receives the winner's prize whereas the other player receives the loser's prize. The players are homogeneous in ability, cost of effort, and confidence. An overconfident player overestimates the contribution of his ability to output. For analytical tractability, we focus on normally distributed random shocks and an exponential cost of effort. Since this is a two-stage game of complete information we look for subgame perfect equilibria (SPE).

We obtain two main results. First, overconfident players can adopt less risky strategies than rational ones. The intuition behind this rather counter-intuitive result is as follows. In a tournament between two rational players, both players choose the high-risk strategy in the first stage and low effort in the second stage (Hvide, 2002). Rational players share a common incentive to increase the level of risk to lessen the importance of differences in effort to the winning probability. This allows them to lower effort and save on effort costs. Now, consider a tournament between two overconfident players. Due to the (mis)perceived ability advantage, each player mistakenly believes he is the favorite, while viewing the opponent as the underdog. As a consequence, there are two effects from decreased risk taking for each player. The first, the favorable *likelihood effect*, is that decreased risk increases a player's perceived probability of winning as differences in perceived ability have a larger impact under lower risk. The second, the unfavorable *effort effect*, is that a player's equilibrium effort increases, resulting in higher effort cost without enhancing the probability of winning as the rival's effort rises by the same amount in response to the decreased risk. When overconfidence is large–a player perceives a significant ability advantage–the first effect dominates the second, resulting in the selection of a low-risk strategy in equilibrium. Conversely, when overconfidence is small–a player perceives a modest ability advantage over the opponent–the first effect is dominated by the second, leading to the selection of a high-risk strategy.

Our first main finding shows that in strategic settings such as tournaments, whether overconfidence increases or decreases risk taking hinges on how big the perceived ability gap is. It goes against the prevailing notion that overconfidence always leads to greater risk taking such as retaining stock options for too long, or investing in new technologies (See Malmendier and Tate (2008); Goel and Thakor (2008); Nosić and Weber (2010); Hirshleifer et al. (2012); Goldberg et al. (2020)). On the contrary, it reveals that overconfident players who perceive a significant ability advantage in a tournament actually take fewer risks than rational players.

Second, when overconfident players adopt a low-risk strategy, they may exert more effort than rational ones. To fully grasp this result, it's crucial to recognize that overconfidence has two distinct effects on effort. On the one hand, holding risk strategies constant, higher overconfidence reduces effort: once a player perceives an ability advantage, he feels comfortable scaling back on effort, thereby saving on effort costs. On the other hand, if heightened overconfidence prompts players to shift from a high-risk to a low-risk strategy, then it increases effort: by reducing randomness, the lower risk makes the tournament's outcome more sensitive to effort, thus encouraging players to work harder. As shown earlier, overconfident players with a large bias choose the low-risk strategy in the first stage. When the bias is not very large, the negative effect of overconfidence on effort provision is smaller than the positive effect due to lower risk taking. Consequently, overconfident players exert higher effort than rational ones. When the bias is very large, overconfident players exert less effort than rational ones as the negative effect of overconfidence on effort is larger than the positive effect due to lower risk taking.

Our second main finding provides a novel mechanism throught which overconfidence, by leading to lower risk taking, raises effort provision. This new mechanism stands in contrast to previous explanations that rely on the complementarity between self-confidence and effort (Bénabou and Tirole (2002, 2003); Gervais and Goldstein (2007); Santos-Pinto (2010); Santos-Pinto and Sekeris (2025)). Importantly, a reduction in risk coupled with increased player effort benefits the tournament organizer. Suppose the organizer is risk-neutral, unaware of any player biases, and aims to maximize profits—the difference between expected output and total prize costs. Overconfident players, by exerting more effort than fully rational ones, generate higher output without increasing compensation costs, thereby improving overall profitability for the tournament organizer.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 sets up the model. Section 4 derives the equilibrium and performs comparative statics. Section 5 describes what happens when an overconfident player faces a rational player. Section 6 concludes the paper. All proofs are in the Appendix.

2 Related Literature

Our paper contributes to two main strands of research.

First, it adds to the growing literature on how overconfidence affects labor and financial markets. Seminal work by Malmendier and Tate (2005, 2008, 2015) shows that overconfident CEOs overestimate their ability to raise their companies' stock prices, often holding stock options too long. Goel and Thakor (2008) examine overconfident and rational managers competing for promotion and find that overconfident managers, who underestimate project risk, are more likely to become CEOs. Santos-Pinto (2008) and De la Rosa (2011) demonstrate that overconfident workers can benefit firms by raising effort provision, even when effort is unobservable. Daniel and Hirshleifer (2015) focus on miscalibrated investors who overestimate the precision of their information, leading to aggressive trading. Hoffman and Burks (2020) show that truck drivers' persistent overestimation of productivity can benefit firms via reduced turnover. Our study contributes to this literature by showing that overconfidence can lower risk taking in tournaments which runs counter to the prevailing idea that overconfident individuals always take more risks than rational ones. In addition, we uncover a new mechanism through which worker overconfidence makes the firm better off: By lowering risk taking, overconfidence can raise workers' effort provision. This finding is in line with previous studies that identify positive effects of worker overconfidence on firms (Fang and Moscarini (2005); Gervais and Goldstein (2007); Santos-Pinto (2008); De la Rosa (2011)).

Second, we contribute to the tournament literature inaugurated by Lazear and Rosen (1981). Bronars (1986) was the first to analyze tournaments where risk taking is an explicit strategic choice. In his sequential tournament framework, a trailing player tends to adopt a high-risk strategy to catch up, while a leading player prefers a low-risk strategy to secure her advantage. Hvide (2002) shows that when players choose both risk and effort, a tournament can collapse into maximum risk and minimal effort. Kräkel and Sliwka (2004) analyze how risk affects effort and winning odds when players differ in ability, while Kräkel (2008) studies players with different risk aversion. Santos-Pinto (2010) studies how overconfidence affects effort and how the tournament organizer can exploit players' overconfidence. Santos-Pinto and Sekeris (2022) analyze the role of confidence heterogeneity on effort and performance in tournaments. We extend this literature by being the first to analyze the interaction between overconfidence, risk taking, and effort provision. Our results show that overconfidence may lead to lower risk taking and higher effort, thus avoiding the breakdown described by Hvide (2002).

3 Set-up

We consider a tournament with two players, where each player first chooses risk and then selects effort. The winner of the tournament, the player who attains the highest output or performance, receives the winner's prize y_w and derives utility u_w from it, while the other receives the loser's prize y_l, and derives utility u_l, with $0 \leq u_l < u_w$. The players are expected utility maximizers and have utility functions that are separable in the valuation of prizes and cost of effort. Effort a_i carries a cost $c(a_i)$ to player i, with $c'(a_i) > 0$, and $c''(a_i) > 0$. For simplicity, each player's outside option is normalized to zero.

Player i's output is additive in ability $t \geq 0$, effort a_i, and an individual noise term ϵ_i. Hence, when player i exerts effort a_i his output is

$$Q_i = t + a_i + \epsilon_i,$$

where the random variable ϵ_i is unimodal and symmetric about zero. Moreover, the random variables ϵ_i and ϵ_j are independent, have variances σ_i^2 and σ_j^2, and their probability distributions are known to both players. Since the difference between ϵ_i and ϵ_j will be critical, we define the random variable $x = \epsilon_j - \epsilon_i$, with cumulative distribution function $G(x)$ and density $g(x)$. Since ϵ_i and ϵ_j are independent and each is symmetric about zero, their difference x is also symmetric about zero. This implies $G(x) = 1-G(-x)$ for all x. Moreover, we assume $G(x)$ is continuous and twice differentiable. Note that continuity together with symmetry about 0 imply $g(x) = g(-x)$ for all x. Further, $g(x)$ satisfies $g'(x) > 0$ for $x < 0$ and $g'(x) < 0$ for $x > 0$. Finally, observe that unimodality, symmetry about zero, alongside continuity imply $g'(0) = 0$.[2]

Accordingly, player i's probability of winning the tournament is

$$\begin{aligned} P_i(a_i, a_j, \sigma_i^2, \sigma_j^2) &= \Pr(Q_i \geq Q_j) \\ &= \Pr(t + a_i + \epsilon_i \geq t + a_j + \epsilon_j) \\ &= \Pr(\epsilon_j - \epsilon_i \leq a_i - a_j) \\ &= G(a_i - a_j; \sigma_i^2, \sigma_j^2). \end{aligned}$$

An overconfident player i mistakenly perceives his output to be given by

$$\tilde{Q}_i = \lambda + t + a_i + \epsilon_i,$$

where $\lambda > 0$ is the parameter that captures player i's overconfidence. An overconfident player correctly perceives the rival's output function. Accordingly, player i's perceived probability of winning the tournament is

$$\begin{aligned} \widetilde{P}_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2) &= \Pr(\widetilde{Q}_i \geq Q_j) \\ &= \Pr(\lambda + t + a_i + \epsilon_i \geq t + a_j + \epsilon_j) \\ &= \Pr(\epsilon_j - \epsilon_i \leq \lambda + a_i - a_j) \\ &= G(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2). \end{aligned}$$

The timing of players' decisions is as follows. In the first stage (risk stage) players simultaneously choose their risk exposure $\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}$, where $i = 1, 2$ and $0 < \sigma_L^2 < \sigma_H^2 < \infty$. By choosing the high-risk strategy, a player induces a mean preserving spread of his output through an increase of the variance of ϵ_i.

[2]This specification is chosen for its analytical simplicity and is often used in the tournament literature (see Lazear and Rosen (1981), Green and Stokey (1983), and Akerlof and Holden (2012)).

In the second stage (effort stage), players observe the chosen risks and simultaneously decide their effort. This timing reflects real-world tournaments where risk choices often precede effort decisions. For instance, managers have the option to implement a new (and riskier) production technology or to stick to the old, more standard technology before they actually start producing. Also in many sports, players (or coaches) formulate a game plan before they decide how much effort to exert during the game. The game plan can be seen as a risk choice when the player (or coach) decides on an offensive (riskier) or defensive (less risky) strategy.

To be able to compute equilibria when players hold mistaken beliefs we assume that: (1) a player facing an overconfident opponent is aware that the latter's perception of his own ability is mistaken, (2) each player thinks that his own perception of his ability is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players' agree to disagree about their abilities. This approach follows Heifetz et al. (2007a,b) for games with complete information, and Squintani (2006) for games with incomplete information.[3] The solution concept we employ is SPE. We start by solving the second stage (effort stage) and then solve the first stage (risk stage).

4 Equilibrium

In the second stage, player i, $i \in \{1,2\}$, chooses the optimal level of effort that maximizes his perceived expected utility

$$E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)] = u_l + G(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u - c(a_i),$$

where $\Delta u = u_w - u_l$ represents the utility prize spread.

The first-order condition of player i is

$$\frac{\partial E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i} = g(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u - c'(a_i) = 0.$$

[3]These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin and Ross, 2002; Pronin and Kugler, 2007).

The second-order condition of player i is

$$\frac{\partial^2 E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i^2} = g'(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u - c''(a_i) < 0.$$

Hence, a sufficient condition for a pure-strategy Nash equilibrium to exist at the effort stage is that

$$g'(\lambda + a_i - a_j; \sigma_i^2, \sigma_j^2)\Delta u < c''(a_i), \ \forall a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2.$$

As established in the tournament literature, a pure-strategy Nash equilibrium only exists if there is sufficient noise and the cost function $c(a)$ is sufficiently convex (Lazear and Rosen, 1981). Therefore, existence of a pure-strategy Nash equilibrium is ensured when

$$\Delta u \sup_x g'(x) < \inf_{a>0} c''(a). \tag{1}$$

Condition (1) ensures that the second-order conditions are satisfied. Note that the lower is $\sup_x g'(x)$ the flatter is $g(x)$ and hence the higher is the noise in the tournament. Note also that $0 < c_0 = \inf_{a>0} c''(a)$ defines a class of cost functions with a second derivative bounded away from zero.

The pure-strategy Nash equilibrium of the effort stage (a_1^*, a_2^*) satisfies the two first-order conditions simultaneously and is given by

$$g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_1^*)$$

and

$$g(a_1^* - a_2^* - \lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a_2^*).$$

Lemma 1 shows that there exists a unique symmetric pure-strategy equilibrium of the effort stage.

Lemma 1. *The effort stage has a unique symmetric pure-strategy equilibrium.*

Lemma 1 tells us that in any SPE, for any risk strategy profile (σ_1^2, σ_2^2) chosen in the risk stage, the players exert the same effort in the effort stage. Since $a_1^* = a_2^* = a^*$ the first-order condition in the effort stage becomes

$$g(\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*). \tag{2}$$

Equation (2) shows that in equilibrium players should increase their effort level up to the point where the perceived marginal benefit of doing so - the perceived marginal probability of winning the tournament times the utility prize spread - equals its incremental cost - the marginal disutility of effort. Differentiating (2) gives us

$$\frac{\partial a^*}{\partial \lambda} = \frac{g'(\lambda;\sigma_1^2,\sigma_2^2)\Delta u}{c''(a^*)}. \tag{3}$$

Since Δu and $c''(a^*)$ are positive, the relation between overconfidence and effort is given by the sign of $g'(\lambda;\sigma_1^2,\sigma_2^2)$, that is, how overconfidence influences the perceived marginal probability of winning the tournament for any given risk strategy profile (σ_1^2,σ_2^2). Since $\lambda > 0$ it follows that $g'(\lambda;\sigma_1^2,\sigma_2^2) < 0$ and therefore $\partial a^*/\partial\lambda < 0$. Hence, holding first stage risk strategies constant, the second stage equilibrium effort decreases in the overconfidence bias λ. In other words, in the second stage, self-confidence and effort are substitutes. Intuitively, the higher a player's overconfident bias is, the greater his (misperceived) ability advantage to get himself a lead in the tournament. Consequently, he decreases his effort to save on effort costs.

In the first stage, players choose their risk strategies simultaneously. Hence, players 1 and 2 solve the following maximization problems

$$\max_{\sigma_1^2\in\{\sigma_L^2,\sigma_H^2\}} u_l + G(\lambda + a_1^* - a_2^*;\sigma_1^2,\sigma_2^2)\Delta u - c(a_1^*),$$

$$\max_{\sigma_2^2\in\{\sigma_L^2,\sigma_H^2\}} u_l + \left[1 - G(a_1^* - a_2^* - \lambda;\sigma_1^2,\sigma_2^2)\right]\Delta u - c(a_2^*),$$

respectively. Since in any SPE $a_1^* = a_2^* = a^*$ for any given (σ_1^2,σ_2^2), and the symmetry of $G(x)$ implies $G(\lambda) = 1 - G(-\lambda)$, the two problems are identical and the first stage maximization problem becomes

$$\max_{\sigma_i^2\in\{\sigma_L^2,\sigma_H^2\}} u_l + G(\lambda;\sigma_1^2,\sigma_2^2)\Delta u - c(a^*(\lambda,\sigma_1^2,\sigma_2^2)). \tag{4}$$

Problem (4) shows that a player's risk choice has two effects on his perceived expected utility. On the one hand, it changes the player's perceived winning probability (likelihood effect). On the other hand, it changes the player's effort in the second stage and therefore the cost of effort (effort effect).[4]

[4] This terminology for the two effects was introduced by Kräkel and Sliwka (2004),

Since the tournament is symmetric, we have two possible candidates for pure-strategy risk strategy profiles in a SPE: (σ_H^2, σ_H^2) and (σ_L^2, σ_L^2). It follows from (4) that both players choose the high-risk strategy as long as

$$c(a^*(\lambda, \sigma_L^2, \sigma_H^2)) - c(a^*(\lambda, \sigma_H^2, \sigma_H^2)) \geq \left[G(\lambda; \sigma_L^2, \sigma_H^2) - G(\lambda; \sigma_H^2, \sigma_H^2)\right] \Delta u. \quad (5)$$

Inequality (5) tells us that a player chooses the high-risk strategy when a unilateral deviation to a low-risk strategy raises the cost of effort more than it increases the perceived probability of winning times the utility prize spread. In other words, a player chooses the high-risk strategy when the unfavorable effort cost effect is greater than the favorable likelihood effect of switching to a low-risk strategy. Hence, when inequality (5) holds, there exists a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$. It also follows from (4) that both players choose the low-risk strategy as long as

$$\left[G(\lambda; \sigma_L^2, \sigma_L^2) - G(\lambda; \sigma_H^2, \sigma_L^2)\right] \Delta u \geq c(a^*(\lambda, \sigma_L^2, \sigma_L^2)) - c(a^*(\lambda, \sigma_H^2, \sigma_L^2)) \quad (6)$$

Inequality (6) tells us that a player chooses the low-risk strategy when a unilateral deviation to a high-risk strategy lowers the cost of effort less than it lowers the perceived probability of winning times the utility prize spread. In other words, a player chooses the low-risk strategy when the favorable effort cost effect is smaller than the unfavorable likelihood effect of switching to a high-risk strategy. Hence, when inequality (6) holds, there exists a SPE where $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$.

In following analysis, we establish the existence of threshold values for the bias λ such that inequalities (5) and (6) hold as equalities. Moreover, we demonstrate that when the players' bias is small, both choose the high-risk strategy but, when the players' bias is large, both choose the low-risk strategy. To formalize this, we specialize the model by assuming that ϵ_1 and ϵ_2 follow a normal distribution with zero mean and variance σ_1^2 and σ_2^2, respectively. Consequently, the difference $x = \epsilon_1 - \epsilon_2$ is also normally distributed with zero mean and variance $\sigma_1^2 + \sigma_2^2$. We denote the cumulative distribution function of x by $\Phi(x)$ and its density by $\phi(x)$. In addition, we assume an exponential cost of effort given by $c(a_i) = e^{a_i}$.[5]

[5]Note, that this specific cost function exhibits fixed costs as $c(0) > 0$. Fixed costs can be motivated by the fact that tournament players often face costs before participating in the actual tournament. Athletes may have to pay for a gaming license or travel to a specific sports contest. Also, the preparation in form of training prior to a tournament can be seen as fixed costs (Kräkel and Sliwka, 2004).

This specification allow us to derive a closed-form solution for the equilibrium effort and to obtain unique threshold values for λ under which (5) and (6) hold as equalities. To ensure the equilibrium effort remains positive for all values of λ we assume

$$\Delta u > 2\sqrt{\pi}\max\left\{\sigma_L e^{\frac{\lambda^2}{4\sigma_L^2}}, \sigma_H e^{\frac{\lambda^2}{4\sigma_H^2}}\right\}. \tag{7}$$

Proposition 1 provides the equilibrium effort for the specialized model.

Proposition 1. *In a tournament between two overconfident players where* ϵ_1 *and* ϵ_2 *are normally distributed with zero mean and variances* σ_1^2 *and* σ_2^2*, respectively, and the cost of effort is exponential, the equilibrium effort is given by*

$$a_1^*(\lambda, \sigma_1^2, \sigma_2^2) = a_2^*(\lambda, \sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}. \tag{8}$$

The equilibrium effort is decreasing in the overconfidence bias, λ*, and increasing in the utility prize spread,* Δu*. Furthermore, the equilibrium effort is decreasing in the sum of risks* $\sigma_1^2 + \sigma_2^2$ *when* $\lambda^2 < \sigma_1^2 + \sigma_2^2$*.*

Let us now consider stage 1, the risk stage. As we have seen, the level of risk affects a player's perceived winning probability (likelihood effect) as well as his effort (effort effect). Depending on the size and direction of these two effects we obtain different SPE outcomes. This finding is summarized below.

Proposition 2. *Consider a tournament between two overconfident players where* ϵ_1 *and* ϵ_2 *are normally distributed with zero mean and variances* σ_1^2 *and* σ_2^2*, respectively, and the cost of effort is exponential. Let* $\bar{\lambda}_1$ *denote the unique solution to (5) and* $\bar{\lambda}_2$ *the unique solution to (6).*
(i) If $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$*, then there is a unique SPE where both players choose the high-risk strategy.*
(ii) If $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$*, then there is a unique SPE where both players choose the low-risk strategy.*
In the above SPE, the players' equilibrium effort is given by (8).

To understand the intuition behind Proposition 2, let us start by looking at the SPE of a tournament between two rational players. In the second stage, the equilibrium effort is obtained by setting $\lambda = 0$ in equation (8):

$$a_1^*(\sigma_1^2, \sigma_2^2) = a_2^*(\sigma_1^2, \sigma_2^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right).$$

Since the equilibrium effort in the second stage is negatively related to the sum of risks, choosing a high-risk strategy in the first stage is a dominant strategy. In other words, a unilateral deviation to a low-risk strategy in the first stage does not alter a player's probability of winning but raises the cost of effort. Hence, in a tournament between two rational players, there is a unique SPE where both players choose the high-risk strategy in the first stage, i.e., $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$. This result is in line with Hvide (2002).

Now consider a tournament between two overconfident players. When the players' bias satisfies $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, a player's perceived advantage over the opponent is small. In this case, a player perceives the outcome of the tournament does not dependent much on the ability gap. Hence, it is beneficial for the players to limit the effort exerted, which they achieve by selecting the high-risk strategy. In contrast, when the players' bias satisfies $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$, a player's perceived advantage over the opponent is large, making him perceive the outcome of the tournament to be highly dependent on the ability gap. In this case, players seek to reduce the influence of risk to avoid jeopardizing their large perceived ability advantage. They can do so by selecting the low-risk strategy.[6]

Next, we compare the equilibrium efforts in a tournament with two overconfident players to those in a tournament with two rational players.

Proposition 3. *Consider a tournament between two overconfident players where ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is exponential. Let $\bar{\lambda}_1$ denote the unique solution*

[6]When $\lambda \in (\bar{\lambda}_1, \bar{\lambda}_2)$ there exists a unique SPE where players mix between the low and the high-risk strategy. In addition, when $\lambda \in (\bar{\lambda}_2, \bar{\lambda}_1)$ there exist one pure-strategy SPE where players choose the high-risk strategy, one pure-strategy SPE where players choose the low-risk strategy, and one SPE where players mix between the low and high-risk strategies. We focus throughout on pure-strategy SPE and hence skip the characterization of mixed-strategy SPE.

to (5) and $\bar{\lambda}_2$ *the unique solution to (6).*
(i) If $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$*, then effort provision is lower than if both players were rational.*
(ii) If $\lambda \in (\max\{\bar{\lambda}_1, \bar{\lambda}_2\}, 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)})$*, then effort provision is higher than if both players were rational.*
(iii) If $\lambda > 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)}$*, then effort provision is lower than if both players were rational.*

As we have seen, rational players choose the high-risk strategy in the first stage and low equilibrium effort in the second stage. We know from part (i) of Proposition 2 that overconfident players choose the high-risk strategy when $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$. In such instances, part (i) of Proposition 3 shows that overconfident players exert less effort than rational ones due to the negative effect of overconfidence on effort provision described by equation (3) and Proposition 1.

We also know from part (ii) of Proposition 2 that overconfident players choose the low-risk strategy when $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$. In such instances two cases can arise. First, when the bias is not too large, i.e., $\lambda \in (\max\{\bar{\lambda}_1, \bar{\lambda}_2\}, 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)})$, the positive effect of lower risk taking on effort provision dominates the negative effect of overconfidence on effort provision. In this case, part (ii) of Proposition 3 shows that overconfident players exert higher effort than rational ones. Second, when the bias is too large, i.e., $\lambda > 2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)}$, the positive effect of lower risk taking on effort provision is dominated by the negative effect of overconfidence on effort provision. In this case, part (iii) of Proposition 3 shows that overconfident players exert less effort than rational ones.

Figure 1 illustrates Proposition 3. The equilibrium effort of overconfident players is depicted by the plain curve and that of rational players by the dashed horizontal line. When the bias is low and players choose a high-risk strategy, overconfident players exert lower effort than rational players. When the bias leads the players to shift from the high-risk to the low-risk strategy, there is a jump in effort provision and they exert more effort than rational ones. As the bias increases further effort provision of overconfidence players decreases but is still higher than of effort provision of rational players. However, as the bias increases further, effort provision of overconfident players falls below that of rational players.

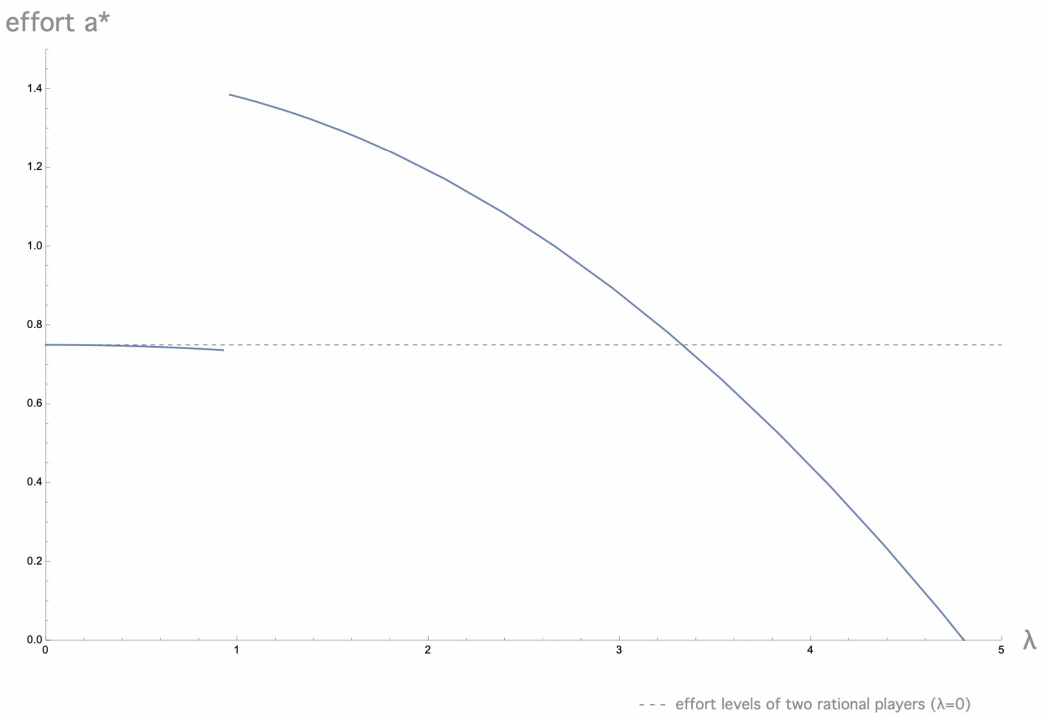

Figure 1: Relationship between equilibrium effort and overconfidence level, for $(\sigma_L^2, \sigma_H^2) = (4, 16)$ and $\Delta u = 30$. In this case, the thresholds $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are equal to 0.963 and 0.927, respectively, and $2\sigma_L\sqrt{\ln(\sigma_H/\sigma_L)} = 4\sqrt{ln2} = 3.3302$.

5 Heterogeneity in Confidence

In the Online Appendix, we investigate a tournament where an overconfident player faces a rational opponent, yielding four main findings.

First, in any SPE the overconfident player exerts less effort than the rational player. When players are equally talented but one is overconfident and the other is rational, the tournament is asymmetric and so is any SPE. Since the overconfident player thinks, mistakenly, that he has a talent advantage over his opponent, he prefers to lower his effort to save on effort costs.

Second, the overconfident player is less likely to win the tournament when risk is normally distributed. As we have seen, in any SPE the overconfident player exerts less effort than the rational player in the second stage. Hence, in any SPE where both workers choose the same risk strategy in the first stage, risk taking cancels out and the rational player has a higher objective probability of winning the tournament due to her higher effort. Matters are not so straightforward in any SPE where players choose different risk strategies. However, when the random shocks are normally distributed, only the sum of risks matters to determine how risk taking influences the players' objective probabilities of winning the tournament. Since both players face the same sum of risks, regardless of their chosen risk strategies, it follows that the overconfident player, by exerting lower effort, has a smallest objective winning probability than the rational player.

Third, the overconfident player may choose a less risky strategy than the rational player but the reverse cannot happen. The intuition behind this result is as follows. As the overconfident player becomes increasingly overconfident, both players exert lower efforts, and the effort gap increases. When the overconfident player's bias is large, he thinks, mistakenly, that he has a large talent advantage over the rational player, even considering that he ends up exerting less effort than the rational player. Thinking, mistakenly, that he is the favorite, the overconfident player chooses the low risk strategy. The rational player, aware of her opponent's overconfidence, and knowing that the effort gap in her favor is small, still prefers the high risk strategy. Therefore, when the overconfident player's bias is large, the overconfident player chooses the low risk strategy and the rational player the high risk strategy.

Fourth, when the overconfident player's bias is very large, the rational player takes less risk. In such instances, the overconfident player chooses a low risk strategy since he thinks, mistakenly, that his very large talent advantage more than compensates the large effort gap in favor of the rational player. The rational player, aware that her opponent's overconfidence, and knowing that the effort gap in her favor is large, thinks correctly she has a clear advantage and switches from the high to the low risk strategy. Thus, when the overconfident player's bias is very large, both players choose the low risk strategy. This finding shows that overconfident individuals can lead rational individuals to innovate less than they would if everyone were rational. This stands in contrast to the idea that overconfident individuals spur innovation.

6 Conclusion

Our study shows that overconfidence can fundamentally alter the equilibrium of a tournament in which players make both risk and effort choices. An overconfident player who mistakenly believes he has a high enough ability advantage over his rival will chose a low-risk strategy to avoid jeopardizing his advantage. This finding challenges the conventional notion that overconfident individuals take more risks than rational ones. In addition, if the overconfidence bias is not too high, the lower risk taking leads to increased effort. By uncovering a new mechanism through which overconfidence lowers risk and boosts effort, our study offers a fresh perspective on how biased self-beliefs can reshape competitive behavior.

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7 Appendix

Proof of Lemma 1

Assume, by contradiction, $a_1^* > a_2^*$. This implies $\lambda + a_1^* - a_2^* > a_1^* - a_2^* - \lambda > -\lambda - a_1^* + a_2^*$. Since $g(x) = g(-x)$ we have $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = g(-\lambda - a_1^* + a_2^*; \sigma_1^2, \sigma_2^2)$. Since $g'(x) < 0$ for $x > 0$ and $g'(x) > 0$ for $x < 0$ it follows from $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) = g(-\lambda - a_1^* + a_2^*; \sigma_1^2, \sigma_2^2)$ that $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) < g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) < c'(a_2^*)$ which contradicts $c'(a_1^*) > c'(a_2^*)$. Now, assume, by contradiction $a_1^* < a_2^*$. This implies $\lambda + a_2^* - a_1^* > \lambda + a_1^* - a_2^* > -\lambda + a_1^* - a_2^*$. Since $g(x) = g(-x)$ we have $g(\lambda + a_2^* - a_1^*; \sigma_1^2, \sigma_2^2) = g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$.Since $g'(x) < 0$ for $x > 0$ and $g'(x) > 0$ for $x < 0$ it follows from $g(\lambda + a_2^* - a_1^*; \sigma_1^2, \sigma_2^2) = g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$ that $g(\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2) > g(-\lambda + a_1^* - a_2^*; \sigma_1^2, \sigma_2^2)$. This inequality and the first-order conditions imply $c'(a_1^*) > c'(a_2^*)$ which contradicts $c'(a_1^*) < c'(a_2^*)$. Hence, the unique pure-strategy equilibrium of the effort stage is given by $a_1^* = a_2^*$. It is easy to see that this symmetric equilibrium satisfies the first-order conditions. Setting $a_1^* = a_2^* = a^*$ in the first-order conditions we obtain

$$g(\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*)$$

and

$$g(-\lambda; \sigma_1^2, \sigma_2^2)\Delta u = c'(a^*).$$

These two first-order conditions are equivalent since symmetry of $G(x)$ implies $g(x) = g(-x)$.

Proof of Proposition 1

When ϵ_1 and ϵ_2 are normally distributed with zero mean and variances σ_1^2 and σ_2^2, respectively, and the cost of effort is $c(a_i) = e^{a_i}$, the perceived expected utility of player $i = 1, 2$ is

$$E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)] = u_l + \Phi\left(\frac{\lambda + a_i - a_j}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)\Delta u - e^{a_i}.$$

The first-order condition of the effort stage for player $i = 1, 2$ is

$$\frac{\partial E[U_i(a_i, a_j, \lambda, \sigma_i^2, \sigma_j^2)]}{\partial a_i} = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{(\lambda + a_i - a_j)^2}{2(\sigma_1^2 + \sigma_2^2)}} \Delta u - e^{a_i} = 0.$$

We know from Lemma 1 that at the unique equilibrium we have $a_1 = a_2$. Hence, the equilibrium effort a^* satisfies

$$e^{a^*} = \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}}.$$

Taking logs we have

$$a^* = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) - \frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}.$$

This equilibrium effort is positive if the utility prize spread Δu is sufficiently big, i.e.,

$$\begin{aligned} \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}}\right) &> \frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)} \\ \frac{\Delta u}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} &> e^{\frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}} \\ \Delta u &> \sqrt{2\pi(\sigma_1^2 + \sigma_2^2)} e^{\frac{\lambda^2}{2(\sigma_1^2 + \sigma_2^2)}}. \end{aligned}$$

It is easy to check that this inequality holds given assumption (7), the right-hand side being convex in the sum of risks, and the fact that $\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}$, for $i = 1, 2$. The equilibrium effort a^* is decreasing with the overconfidence bias as $\partial a^* / \partial \lambda < 0$, and increasing with the utility prize spread as $\partial a^* / \partial \Delta u < 0$. To determine how a change in risk affects the equilibrium effort a^* let $\sigma^2 = \sigma_1^2 + \sigma_2^2$. We have

$$\frac{\partial a^*}{\partial \sigma} = \frac{\partial}{\partial \sigma}\left[\ln\left(\frac{\Delta u}{\sqrt{2\pi}}\right) - \ln \sigma - \frac{\lambda^2}{2\sigma^2}\right] = -\frac{1}{\sigma} + \frac{\lambda^2}{\sigma^3} = \frac{1}{\sigma}\left[-1 + \left(\frac{\lambda}{\sigma}\right)^2\right].$$

Hence, the equilibrium effort a^* decreases with risk as long as $\lambda^2 < \sigma^2 = \sigma_1^2 + \sigma_2^2$.

Proof of Proposition 2

For the specialized model, the perceived expected utility of player $i = 1, 2$ evaluated at the stage two equilibrium efforts is

$$E[U_i(a_i^*, a_j^*, \lambda, \sigma_i^2, \sigma_j^2)] = u_l + \Phi\left(\frac{\lambda + a_i^* - a_j^*}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)\Delta u - e^{a_i^*}$$

$$= u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)\Delta u - e^{\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_i^2+\sigma_j^2)}}\right) - \frac{\lambda^2}{2(\sigma_i^2+\sigma_j^2)}}$$

$$= u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_i^2+\sigma_j^2)}} e^{-\frac{\lambda^2}{2(\sigma_i^2+\sigma_j^2)}}$$

Thus, the maximization problem of player $i = 1, 2$ at the risk stage is

$$\max_{\sigma_i^2 \in \{\sigma_L^2, \sigma_H^2\}} u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_i^2 + \sigma_j^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_i^2+\sigma_j^2)}} e^{-\frac{\lambda^2}{2(\sigma_i^2+\sigma_j^2)}}$$

Proof of part (i): The high-risk equilibrium $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$ takes place when the perceived expected utility of the high-risk strategy is higher than a unilateral deviation to the low-risk strategy, that is, if

$$u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}$$
$$\geq u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} \geq \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2 + \sigma_H^2}}\right). \quad (9)$$

Setting $\lambda = 0$ in the LHS of (9) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (9) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Hence, the inequality is satisfied when $\lambda = 0$, i.e., when both players are rational they both choose the high-risk strategy. Note that RHS of (9) is always non-negative. Note also that the LHS of (9) is equal to zero when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} = \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}} = e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}+\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$\frac{1}{2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right) = -\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)} + \frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)},$$

or

$$\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right) = \lambda^2\left(\frac{1}{\sigma_L^2+\sigma_H^2} - \frac{1}{\sigma_H^2+\sigma_H^2}\right),$$

or

$$\lambda = \sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}.$$

Taking the derivative of the LHS of (9) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}LHS(\lambda) &= \frac{\partial}{\partial\lambda}\left(\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) \\
&= -\frac{\frac{\lambda}{\sigma_L^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} + \frac{\frac{\lambda}{\sigma_H^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} \\
&= -\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)^3}} + \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)^3}}
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\left.\frac{\partial}{\partial\lambda}LHS(\lambda)\right|_{\lambda=0} = 0$$

The derivative is negative when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{(\sigma_H^2+\sigma_H^2)^3}} < \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{(\sigma_L^2+\sigma_H^2)^3}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} < \sqrt{\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)^3},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} - \frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)} < \frac{3}{2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{(\sigma_H^2+\sigma_H^2)-(\sigma_L^2+\sigma_H^2)}{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{\sigma_H^2-\sigma_L^2}{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\lambda < \sqrt{3\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}.$$

Taking the derivative of the RHS of (9) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}RHS(\lambda) &= \frac{\partial}{\partial\lambda}\left[\Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)-\Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2+\sigma_H^2}}\right)\right] \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \frac{1}{\sqrt{\sigma_H^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_H^2+\sigma_H^2}}\right) \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} - \frac{1}{\sqrt{\sigma_H^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} \\
&= \frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} - \frac{1}{(\sigma_H^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}.
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\left.\frac{\partial}{\partial\lambda}RHS(\lambda)\right|_{\lambda=0} = \frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}} - \frac{1}{(\sigma_H^2+\sigma_H^2)\sqrt{2\pi}} > 0.$$

The derivative is positive when

$$\frac{1}{\sigma_L^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} > \frac{1}{\sigma_H^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}} < \frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2},$$

or

$$\frac{\lambda^2}{2\left(\sigma_L^2+\sigma_H^2\right)} - \frac{\lambda^2}{2\left(\sigma_H^2+\sigma_H^2\right)} < \ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{\lambda^2}{\sigma_L^2+\sigma_H^2} - \frac{\lambda^2}{\sigma_H^2+\sigma_H^2} < 2\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\frac{\sigma_H^2-\sigma_L^2}{(\sigma_H^2+\sigma_L^2)\,(\sigma_H^2+\sigma_H^2)}\lambda^2 < 2\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right),$$

or

$$\lambda < \sqrt{2\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}.$$

We have shown the LHS of (9) is strictly positive at $\lambda = 0$, decreases in λ, and is equal to zero at $\lambda = \sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}$. Furthermore, we have shown the RHS of (9) is non-negative, is equal to zero at $\lambda = 0$, first increases and then decreases in λ, and attains its maximum at $\lambda = \sqrt{2\frac{(\sigma_L^2+\sigma_H^2)(\sigma_H^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_H^2+\sigma_H^2}{\sigma_L^2+\sigma_H^2}\right)}$. Hence, it follows that there is a unique positive value for λ that satisfies (9) as an equality. Let $\bar{\lambda}_1$ denote the unique solution to:

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_H^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}} = \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_H^2+\sigma_H^2}}\right).$$

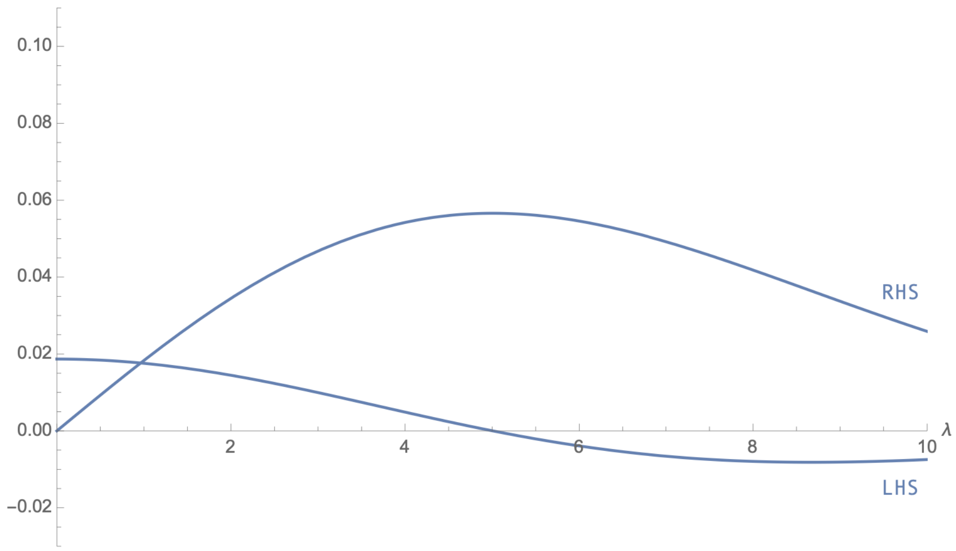

Figure 2: LHS and RHS of inequality (9)

Figure 2 shows the LHS and RHS of (9) for $(\sigma_L^2, \sigma_H^2) = (4, 16)$. As the plot shows, the LHS and RHS of (9) cross only once. We, thus, have a unique threshold for the overconfidence bias λ for which (9) holds as an equality. The point of intersection between the LHS and RHS represents the threshold $\bar{\lambda}_1$.

Hence, we have shown that if $\lambda < \bar{\lambda}_1$, then there exists a pure-strategy SPE where both players choose the high-risk strategy in the first stage and where the equilibrium effort in the second stage is given by (8) with $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$.

Proof of part (ii): The low-risk equilibrium $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$ takes place when the perceived expected utility of the low-risk strategy is higher than a unilateral deviation to the high-risk strategy, that is, if

$$u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}$$
$$\geq u_l + \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right)\Delta u - \frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} \leq \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_L^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2 + \sigma_H^2}}\right). \quad (10)$$

Setting $\lambda = 0$ in the LHS of (10) we obtain

$$LHS(\lambda = 0) = \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} - \frac{1}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}} > 0.$$

Setting $\lambda = 0$ in the RHS of (10) we obtain

$$RHS(\lambda = 0) = \Phi(0) - \Phi(0) = 0.5 - 0.5 = 0.$$

Hence, the inequality is violated when $\lambda = 0$, in other words, when both players are rational the low-risk equilibrium does not exist. Note that the RHS of (10) is always non-negative. Note also that the LHS of (10) is equal to zero when

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}} = \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2 + \sigma_H^2)}},$$

or

$$\sqrt{\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}} = e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}+\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}},$$

or

$$\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right) = \left(\frac{1}{\sigma_L^2+\sigma_L^2} - \frac{1}{\sigma_L^2+\sigma_H^2}\right)\lambda^2,$$

or

$$\lambda = \sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_L^2+\sigma_L^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}.$$

Taking the derivative of the LHS of (10) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}LHS(\lambda) &= \frac{\partial}{\partial\lambda}\left(\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}\right)\\
&= -\frac{\frac{\lambda}{\sigma_L^2+\sigma_L^2}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} + \frac{\frac{\lambda}{\sigma_L^2+\sigma_H^2}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}\\
&= -\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)^3}} + \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)^3}}
\end{aligned}$$

Evaluating this derivative at $\lambda = 0$ we have

$$\left.\frac{\partial}{\partial\lambda}LHS(\lambda)\right|_{\lambda=0} = 0$$

The derivative is negative when

$$\frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)^3}} > \frac{\lambda e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)^3}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} < \sqrt{\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)^3},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)} - \frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} < \frac{3}{2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{(\sigma_L^2+\sigma_H^2)-(\sigma_L^2+\sigma_L^2)}{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{\sigma_H^2-\sigma_L^2}{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}\lambda^2 < 3\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\lambda < \sqrt{3\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}.$$

Taking the derivative of the RHS of (10) with respect to λ we obtain

$$\begin{aligned}
\frac{\partial}{\partial\lambda}RHS(\lambda) &= \frac{\partial}{\partial\lambda}\left[\Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)-\Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right)\right] \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_L^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right)-\frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right) \\
&= \frac{1}{\sqrt{\sigma_L^2+\sigma_L^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}-\frac{1}{\sqrt{\sigma_L^2+\sigma_H^2}}\frac{1}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} \\
&= \frac{1}{(\sigma_L^2+\sigma_L^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}-\frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}.
\end{aligned}$$

Evaluating this derivative at $\lambda=0$ we have

$$\left.\frac{\partial}{\partial\lambda}RHS(\lambda)\right|_{\lambda=0} = \frac{1}{(\sigma_L^2+\sigma_L^2)\sqrt{2\pi}}-\frac{1}{(\sigma_L^2+\sigma_H^2)\sqrt{2\pi}} > 0.$$

The derivative is positive when

$$\frac{1}{(\sigma_L^2+\sigma_L^2)}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}} > \frac{1}{(\sigma_L^2+\sigma_H^2)}e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}},$$

or

$$e^{\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}} < \frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2},$$

or

$$\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)} < \ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\frac{\lambda^2}{(\sigma_L^2+\sigma_L^2)}-\frac{\lambda^2}{(\sigma_L^2+\sigma_H^2)} < 2\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right),$$

or

$$\lambda < \sqrt{2\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}.$$

We have shown the LHS of (10) is strictly positive at $\lambda=0$, decreases in λ, and is equal to zero at $\lambda=\sqrt{\frac{(\sigma_L^2+\sigma_H^2)(\sigma_L^2+\sigma_L^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}$. Furthermore, we have shown the RHS of (10) is non-negative, is equal to zero at $\lambda=0$, first increases and then decreases in λ, and attains its maximum at $\lambda = \sqrt{2\frac{(\sigma_L^2+\sigma_L^2)(\sigma_L^2+\sigma_H^2)}{\sigma_H^2-\sigma_L^2}\ln\left(\frac{\sigma_L^2+\sigma_H^2}{\sigma_L^2+\sigma_L^2}\right)}$. Hence, it follows that there is a unique positive value for λ that satisfies (10) as an equality. Let $\bar{\lambda}_2$ denote the unique solution to:

$$\frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}} - \frac{e^{-\frac{\lambda^2}{2(\sigma_L^2+\sigma_H^2)}}}{\sqrt{2\pi(\sigma_L^2+\sigma_H^2)}} = \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_L^2}}\right) - \Phi\left(\frac{\lambda}{\sqrt{\sigma_L^2+\sigma_H^2}}\right).$$

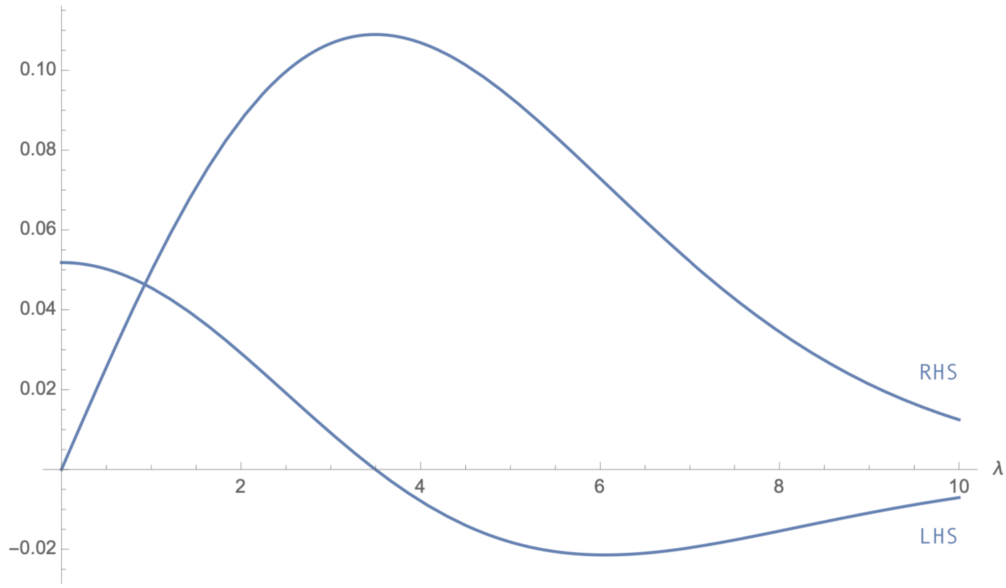

Figure 3: LHS and RHS of inequality (10)

Figure 3 shows the LHS and RHS of (10) for $(\sigma_L^2, \sigma_H^2) = (4, 16)$. As the plot shows, the LHS and RHS of (10) cross only once. We, thus, have a unique threshold for the overconfidence bias λ for which (10) holds as an equality. The point of intersection between the LHS and RHS represents the threshold $\bar{\lambda}_2$.

Hence, we have shown that if $\lambda > \bar{\lambda}_2$, then there exists a pure-strategy SPE where both players choose the low-risk strategy in the first stage and where the equilibrium effort in the second stage is given by (8) with $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$.

Since there no guarantee that $\bar{\lambda}_1$ is always bigger (or smaller) than $\bar{\lambda}_2$, to ensure that there is a unique SPE where both players choose the high-risk strategy in part (i) we impose $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$. Similarly, to ensure that there is a unique SPE where both players choose the low-risk strategy in part (ii) we impose $\lambda > \max\{\bar{\lambda}_1, \bar{\lambda}_2\}$.

We now discuss what is the SPE for the remaining values of λ. We distinguish between two cases: (a) $\bar{\lambda}_1 < \lambda < \bar{\lambda}_2$ and (b) $\bar{\lambda}_2 < \lambda < \bar{\lambda}_1$. In case (a) there is no symmetric pure-strategy SPE. However, existence is guaranteed by standard arguments. Hence, there exists a symmetric SPE where players mix between the low and the high risk strategies in the first stage and where the equilibrium effort in the second stage is given by (8). In case (b) there exist three SPE. There is one pure-strategy SPE where both players choose the high-risk strategy in the first stage and where the equilibrium effort in the second stage is given by (8) with $(\sigma_1^2, \sigma_2^2) = (\sigma_H^2, \sigma_H^2)$. There is another pure-strategy SPE where both players choose the low-risk strategy in the first stage and where the equilibrium effort in the second stage is given by (8) with $(\sigma_1^2, \sigma_2^2) = (\sigma_L^2, \sigma_L^2)$. Finally, there is a SPE where players mix between the low and the high-risk strategies in the first stage and where the equilibrium effort in the second stage is given by (8).

Proof of Proposition 3

The equilibrium effort in a tournament with rational players is

$$a^*(\sigma_H^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right). \tag{11}$$

The equilibrium effort in a tournament with overconfident players in a SPE where both players choose the high-risk strategy is

$$a^*(\lambda, \sigma_H^2, \sigma_H^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2 + \sigma_H^2)}}\right) - \frac{\lambda^2}{2(\sigma_H^2 + \sigma_H^2)} \tag{12}$$

The equilibrium effort in a tournament with overconfident players in a SPE where both players choose the low-risk strategy is

$$a^*(\lambda, \sigma_L^2, \sigma_L^2) = \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2 + \sigma_L^2)}}\right) - \frac{\lambda^2}{2(\sigma_L^2 + \sigma_L^2)} \tag{13}$$

It follows directly from (11) and (12) that if $\lambda < \min\{\bar{\lambda}_1, \bar{\lambda}_2\}$, then $a^*(\sigma_H^2, \sigma_H^2) > a^*(\lambda, \sigma_H^2, \sigma_H^2)$. This proves part (i). It follows from (11) and (13) that for $a^*(\lambda, \sigma_L^2, \sigma_L^2) > a^*(\sigma_H^2, \sigma_H^2)$ we must have

$$\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right) - \frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)} > \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right),$$

or

$$\ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_L^2+\sigma_L^2)}}\right) - \ln\left(\frac{\Delta u}{\sqrt{2\pi(\sigma_H^2+\sigma_H^2)}}\right) > \frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)},$$

or

$$\ln\frac{1}{\sqrt{\sigma_L^2}} - \ln\frac{1}{\sqrt{\sigma_H^2}} > \frac{\lambda^2}{2(\sigma_L^2+\sigma_L^2)},$$

or

$$\ln\frac{1}{\sigma_L} - \ln\frac{1}{\sigma_H} > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$\ln\sigma_H - \ln\sigma_L > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$\ln\frac{\sigma_H}{\sigma_L} > \frac{\lambda^2}{4\sigma_L^2},$$

or

$$4\sigma_L^2\ln\frac{\sigma_H}{\sigma_L} > \lambda^2,$$

or

$$\lambda < 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}.$$

Hence, if $\max\{\bar{\lambda}_1, \bar{\lambda}_2\} < \lambda < 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}$, then $a^*(\lambda, \sigma_L^2, \sigma_L^2) > a^*(\sigma_H^2, \sigma_H^2)$. This proves part (ii). Finally, this last result implies that if $\lambda > 2\sigma_L\sqrt{\ln\frac{\sigma_H}{\sigma_L}}$, then $a^*(\lambda, \sigma_L^2, \sigma_L^2) < a^*(\sigma_H^2, \sigma_H^2)$. This proves part (iii).