Overconfidence in Tullock Contests

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Abstract

We investigate the role of overconfidence in Tullock contests. An overconfident player overestimates the impact of his effort on the outcome of the contest. We find that when overconfidence is high relative to the number of players, an increase in overconfidence lowers equilibrium efforts. However, the opposite happens when overconfidence is low relative to the number of players. We demonstrate that overdissipation can occur due to overconfidence. Finally, we show that an increase in overconfidence unambiguously leads to an increase in the number of entrants.

JEL Codes: D60; D69; D91
Keywords: Overconfidence; Contests; Rent Dissipation; Entry.

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1 Introduction

This paper investigates the role of overconfidence in Tullock contests. This question is of relevance since evidence from psychology and economics shows that humans tend to be overconfident. A majority of people believe they are better than others in a wide variety of positive traits and skills (Myers 1996, Santos-Pinto and Sobel 2005). Examples include entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006), currency traders (Oberlechner and Osler 2008), or poker and chess players (Park and Santos-Pinto 2010).

Competitions often take the form of contests. For example, an R&D race to be the first to develop or get a patent in new product or technology, election campaigns, rent-seeking games, competitions for monopolies, litigation, and wars, are examples of contests. Overconfidence matters for entry and performance in competitions and for labor markets (Camerer and Lovallo 1999, Niederle and Vesterlund 2007, Moore and Healy 2008, Dohmen and Falk 2011, Malmendier and Taylor 2015, Huffman et al. 2019, Santos-Pinto and de la Rosa 2020). Overconfidence also seems to play a role in mate competition and acquisition (Waldman 1994, Murphy et al. 2015). Interestingly, Lyons et al. (2020) provide evidence that high-status lobbyists working for private interest groups in Washington, DC, USA tend to be overconfident: they overate their achievements and their success. This empirical finding is in line with the experimental findings of Niederle and Vesterlund (2007) and Dohmen and Falk (2011) according to which overconfident participants tend to self select more into more competitive environments.

What is the effect of players' overconfidence on their effort provision and on rent dissipation? Does overconfidence lead to more entry in a contest? These are important questions since although the extant literature has characterized in depth equilibria in contests, behavioral biases have so far received limited attention by scholars (e.g. Baharad and Nitzan 2008, Santos-Pinto and Sekeris 2025).

To address these questions, we employ a generalized n player Tullock contest

(1980) where v is the prize being contested, a_i the effort of player i, and $c(a_i)$ the cost of effort to player i. Player i's probability of winning the contest is $P(a_i, a_{-i}) = \frac{q(a_i)}{q(a_i)+\sum_{j\neq i} q(a_j)}$, where $q(a_i)$ is often referred to as the impact function (Ewerhart 2015). In an environment with fully rational players, the expected utility of player i is given by $E[U_i(a_i, a_{-i})] = P_i(a_i, a_{-i})v - c(a_i)$.

An earlier study conceptualizes overconfidence as an underestimation of the cost of effort: $E[U_i(a_i, a_{-i}; \gamma)] = P_i(a_i, a_{-i})v - \gamma c(a_i)$, where $0 < \gamma < 1$ (Ludwig et al. 2011). Likewise, overconfidence can also be modeled as an overestimation of the rival's cost of effort (Deng et al. 2024). These approaches to modeling overconfidence are isomorphic. We follow a novel approach by assuming an overconfident player i thinks, mistakenly, his impact function is $\lambda q(a_i)$, where $\lambda > 1$, and has correct beliefs about his rivals' impact functions. Accordingly, an overconfident player's perceived winning probability is $P_i(a_i, a_{-i}; \lambda) = \frac{\lambda q(a_i)}{\lambda q(a_i)+\sum_{j\neq i} q(a_j)}$, which is larger than his actual winning probability. Since the impact function embeds a player's ability, we conceptualize overconfidence as an overestimation of the effect of one's effort on contest outcomes—a common definition of overconfidence in the literature (e.g. Bénabou and Tirole 2002, 2003, Santos-Pinto 2008, 2010)—while assuming that players accurately assess their cost of effort. Importantly, our findings regarding the effect of overconfidence in contests run counter to earlier research.

We consider a symmetric $n \geq 2$ player Tullock contest where all players are overconfident. We demonstrate that the number of players as well as the degree of overconfidence matters in terms of understanding the effects of overconfidence on effort provision and rent dissipation in contests. On the one hand, overconfidence reduces individual and aggregate efforts when λ is larger than $n - 1$. In this case all players expect to be highly likely to win the contest. Hence, an increase in overconfidence lowers the perceived marginal probability of winning, which pushes players' efforts downwards. On the other hand, overconfidence raises individual and aggregate efforts if λ is smaller than $n - 1$. In such instances, all players expect to win the contest with a low probability. Therefore, an increase in overconfidence will raise the perceived marginal probability of winning, which pushes players' efforts upwards. This stands out as a novel contribution of our work compared to the existing literature which has exclusively focused on 2 player contests.

We next demonstrate that overconfidence can lead to overdissipation, i.e. situations where players' aggregate cost of effort is strictly larger than the value of the prize. In particular, we show that there is a threshold value of the number of players above which overdissipation can always occur, provided players are sufficiently overconfident. Moreover, we show that as the number of players goes up, overdissipation can be observed for values of λ close to 1.

Last we inquire how overconfidence affects entry in a contest. In order to answer this question, we assume $N \geq 2$ symmetric potential entrants that have an outside option. Overconfidence affects incentives to enter the contest through two channels. First, it raises the perceived winning probability, and thus the benefit of entry for given efforts of players. Second, it incentivizes players to modify their equilibrium efforts, thereby indirectly impacting the potential entrants' payoffs. We show that even when an increase in overconfidence raises players' individual efforts, and the two effects then go in opposite directions, higher overconfidence always results in more entry.

The paper is organized as follows. Section 2 discusses related literature. Section 3 sets-up the contest model. In Section 4 we derive the equilibrium and perform comparative statics. Section 5 studies entry, and Section 6 concludes the paper. All proofs are in the Appendix.

2 Related Literature

This study relates to two strands of literature. First, it contributes to the literature on behavioral biases in contests and tournaments.

Ludwig et al. (2011) analyze a Tullock contest where an overconfident player competes against a rational player. The overconfident player is assumed to underestimate his cost of effort. Ludwig et al. (2011) find that the overconfident player exerts more effort and the rational player exerts less effort than if both players were rational. They also find that the bias makes the contest organizer better off since the overconfident player's increase in effort more than compensates the rational player's decrease in effort. The intuition of these results is that an overconfident player has a lower perceived marginal cost of effort for any given marginal utility from winning the contest which leads him to put more effort. In turn, the rational player reduces his own effort because of strategic substitutability. Our results show that when overconfidence is an overestimation of the impact of one's effort, its effects on equilibrium efforts are quite different than those in found Ludwig et al. (2011). The differences in the results are driven by the fact that overconfidence in our setup raises the marginal perceived probability from winning for low values of effort whereas it lowers it for high value of effort. As a consequence, and in contrast to Ludwig et al. (2011), in our study, overconfidence shifts a player's best response function in a non-monotonic way as shown in Lemma 2. Our definition of overconfidence is adequate when both the monetary value of winning the contest and the cost of effort are known before entry.

Bansah et al. (2024) explore the role of overconfidence on a 2 player Tullock contest with linear impact and cost functions. Overconfidence is modeled as an overestimation of the winning probability, rather than an overestimation of the impact of one's effort. Observe, however, that their definition of overconfidence does not satisfy the property that the perceived winning probabilities are well defined for any value of the bias. In addition to our distinct approach to modeling overconfidence, our study employs more general impact and cost functions and extends the analysis to n players.

Santos-Pinto and Sekeris (2025) study how confidence heterogeneity affects effort and performance in tournaments and contests with two players. They demonstrate a non-monotonic effect of confidence on equilibrium relative efforts and winning probabilities. In the present paper, we use the same definition of overconfidence as in Santos-Pinto and Sekeris (2025), but rather than focusing on a 2 player contest, we consider contests with n players, and we explore the implications of overconfidence on rent dissipation and on entry.

The aforementioned studies analyze Tullock contests under the assumption that players have complete information about their abilities or costs, yet overconfident players overestimate their abilities or underestimate their costs. In contrast, Deng et al. (2024) consider a Tullock contest with incomplete information about the players' costs. In their model, a newly hired employee has private information about his cost of effort, while the incumbent employee has biased beliefs on the former's cost-type, i.e. he holds a biased prior belief on whether his rival is a low-cost or a high-cost type. They study how the asymmetry in beliefs affects aggregate expected effort provision, and whether a contest organizer should disclose or conceal information on the new hire's cost of effort to the incumbent. We instead model overconfidence as an overestimation of the impact of one's effort in a setup where there is no uncertainty about the players' true types.[1] Observe that, as explained above, overestimating one's own ability deeply differs from misestimating one's effort cost, and we equally extend the analysis to n player contests.

Santos Pinto (2010) studies how a tournament organizer optimally sets the prizes in a Lazear and Rosen (1981) rank-order tournament with overconfident players. We adopt the same definition of overconfidence and equilibrium concept. Observe, however, that although players' winning probabilities in both Lazear-Rosen tournaments and Tullock contests are logistic functions, the way in which noise affects

[1] Overconfidence has been studied using two approaches. One approach assumes incomplete information about players' types and models overconfidence as a shift in the belief distribution that places greater weight on types with higher ability or lower effort cost. (Bénabou and Tirole 2002, 2003, Santos-Pinto 2008, De la Rosa 2011, Deng et al. 2024). In contrast, the other approach assumes complete information and posits that overconfidence is an overestimation of ability or an underestimation of effort cost. (Santos-Pinto 2010, Ludwig et al. 2011, Bansah et al. 2024, Santos-Pinto and Sekeris 2025). The first approach imposes an upper limit on the bias, whereas the second approach does not.

the mapping of players' efforts to winning probabilities differs. As a consequence, overconfidence shifts players' best response functions differently in these two models. Santos Pinto (2010) finds in a symmetric two player tournament that an increase in overconfidence raises the equilibrium efforts of players. In contrast, we find the opposite in a two player contest, while we equally consider more than two players in our study.

Baharad and Nitzan (2008) and Keskin (2018) amend the standard model of contests by introducing probability weighting in line with Tversky and Kahneman's (1992) Cumulative Prospect Theory. This behavioral bias is modeled with an inverse S-shaped probability weighting function, i.e., a function where the marginal increase in the (perceived) subjective probability is higher for extreme (i.e. low and high) probabilities. Our own approach assumes a constant bias in players' beliefs that they are better than they really are at contesting their opponents. We thus see our approach as complementary to these earlier works since nothing precludes players from both assigning 'weights' to probabilities and be subject to an overconfidence bias. Notice that in terms of contribution to the literature on behavioral biases, our approach has the advantage to be flexible enough to accommodate a very large family of contest success functions while also allowing for any possible heterogeneities among players. Last, whereas Baharad and Nitzan (2008) and Keskin (2018)'s approach applies exclusively to probabilistic setups, our own model is equally suited to describe sharing contests that have gained in importance over the years (e.g. Dickson et al. 2018).[2]

Second, our study relates to the experimental literature on behavior in contests. Scholars have also long tried to explain the puzzle that contestants in lab experiments spend significantly higher amounts than the game's Nash equilibrium (Chowdhury et al. 2014, Price and Sheremeta 2015, Mago et al. 2016), and even overdissipation

[2]Other scholars have equally focused on the effect of behavioural biases on equilibrium outcomes in the presence of uncertainty. Kelsey and Melkonyan (2018) consider both optimistic and pessimistic attitudes to ambiguity, while Cornes and Hartley (2003) and Fu et al. (2022) introduce loss aversion in probabilistic contests.

can occur (Sheremeta 2011). The theoretical literature has attempted to explain overspending, but also extreme manifestations of such phenomena where contestants overdissipate the rent by expending on aggregate more resources than the value of the prize that is contested. Overspending has so far been attributed to players' risk attitudes (Jindapon and Whaley 2015) or to mixed strategy equilibria where overspending occurs with some probability but not in expectation (Baye et al. 1999). Our paper demonstrates that with overconfident contestants, overspending and even overdissipation can result when the number of players is sufficiently large and the overconfidence bias is relatively mild; overconfident players individually expend more effort than rational players when their odds of winning are low because of the high number of participants.

3 Set-up

To study the role of overconfidence in contests we consider a generalized n player Tullock contest. The effort cost is $c(a_i)$ with $c(0) = 0$, $c'(a_i) > 0$ and $c''(a_i) \geq 0$. Following Baik (1994) we assume the CSF is:

$$P_i(a_i, a_{-i}) = \begin{cases} q(a_i)/\sum_j q(a_j) & \text{if } \sum_j q(a_j) > 0 \\ 1/n & \text{if } \sum_j q_j(a_j) = 0 \end{cases},$$

where a_{-i} designates the vector of player i's competitors' efforts, $q(0) \geq 0$, $q'(a_i) > 0$ and $q''(a_i) \leq 0$. The overconfident player i mistakenly perceives his impact function to be $\lambda q(a_i)$, with $\lambda > 1$, and correctly perceives the rivals' impact functions. This way of modelling overconfidence in a contest implies that an overconfident player i's perceived winning probability is equal to

$$P_i(a_i, a_{-i}; \lambda) = \begin{cases} \lambda q(a_i)/[\lambda q(a_i) + \sum_{j \neq i} q(a_j)] & \text{if } \lambda q(a_i) + \sum_{j \neq i} q(a_j) > 0 \\ 1/n & \text{if } \lambda q(a_i) + \sum_{j \neq i} q(a_j) = 0 \end{cases}.$$

This specification of overconfidence in a contest satisfies four desirable properties. First, contests where players have heterogeneous productivity of effort are modelled similarly, that is, the players are assumed to have heterogeneous impact functions (Baik 1994, Singh and Wittman 2001, Stein 2002, Fonseca 2009). Second, the overconfident player's perceived winning probability is well defined for any value of $\lambda > 1$.[3] Third, the overconfident player's perceived winning probability is increasing in λ. Fourth, overestimating one's impact function is equivalent to underestimating the rivals' impact functions since $\lambda q(a_i)/[\lambda q(a_i) + \sum_{j\neq i} q(a_j)] = q(a_i)/[q(a_i) + \sum_{j\neq i} q(a_j)/\lambda]$. In other words, all our results are unaffected if overconfidence is instead modeled as underestimation of the impact of an opponent's effort.[4]

To be able to compute equilibria when players hold mistaken beliefs we assume that: (1) a player who faces a biased opponent is aware that the latter's perception of his own impact function (and probability of winning) is mistaken, (2) each player thinks that his own perception of his impact function (and probability of winning) is correct, and (3) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players agree to disagree about their impact functions (and winning probabilities). This approach follows Heifetz et al. (2007a,2007b) for games with complete information, and Squintani (2006) for games with incomplete information.

These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007). As stated by Pronin et al. (2002: 369) "people recognize the existence, and the impact, of most of the biases that social and cognitive psychologists have described over the past few decades. What they *lack* recognition of, we argue, is the role that those same biases play in governing their *own* judgments and inferences." For example, Libby

[3]This is not the case with alternative specifications. For example, if one assumes an overconfident player's perceived winning probability is $P_i(a_i, a_{-i}; \lambda) = \lambda q(a_i)/[q(a_i) + \sum_{j\neq i} q(a_j)]$, with $\lambda > 1$, then $P_i(a_i, a_{-i}; \lambda)$ is not a well defined probability for any value of $\lambda > 1$.

[4]This way of modeling overconfidence is often used in studies that analyze the impact of overconfidence on contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011).

and Rennekamp (2012) conduct a survey which shows that experienced financial managers believe that other managers are likely to be overconfident while failing to recognize their own overconfidence. Hoffman (2016) runs a field experiment which finds that internet businesspeople recognize others tend to be overconfident while being unaware of their own overconfidence.[5]

4 Equilibrium

Any player i chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_{-i}; \lambda)] = P_i(a_i, a_{-i}; \lambda)v - c(a_i) = \frac{\lambda q(a_i)}{\lambda q(a_i) + \sum_{j\neq i} q(a_j)} v - c(a_i).$$

The first-order condition is

$$\frac{\partial E[U_i(a_i, a_{-i}; \lambda)]}{\partial a_i} = \frac{\lambda q'(a_i) \sum_{j\neq i} q(a_j)}{\left[\lambda q(a_i) + \sum_{j\neq i} q(a_j)\right]^2} v - c'(a_i) = 0. \tag{1}$$

The second-order condition is

$$\frac{\partial^2 E[U_i(a_i, a_{-i}; \lambda)]}{\partial a_i^2} = \frac{q''(a_i)[\lambda q(a_i) + \sum_{j\neq i} q(a_j)] - 2\lambda [q'(a_i)]^2}{[\lambda q(a_i) + \sum_{j\neq i} q(a_j)]^3} \lambda \sum_{j\neq i} q(a_j) v - c''(a_i) < 0, \tag{2}$$

and the above inequality is satisfied since $q''(a_i) \leq 0$ and $c''(a_i) \geq 0$.

Let $R_i(Q_{-i})$ denote player i's best response to the aggregate effective effort of the rivals Q_{-i}, where $Q_{-i} = \sum_{j\neq i} q(a_j)$. Accordingly, $R_i(Q_{-i})$ is defined by the value of a_i satisfying (1), or

$$\lambda q'(a_i) Q_{-i} v = c'(a_i) \left[\lambda q(a_i) + Q_{-i}\right]^2. \tag{3}$$

Lemma 1 describes the shapes of the players' best responses.

[5] Ludwig and Nafziger (2011) conduct a lab experiment that elicits participants' beliefs about own and others' overconfidence and abilities. On the one hand they find that the largest group of participants thinks that they are themselves better at judging their ability correctly than others. On the other hand, they find that with a few exceptions, most people believe that others are unbiased.

Lemma 1. *$R_i(Q_{-i})$ is quasi-concave in Q_{-i} and reaches a maximum for $Q_{-i} = \lambda q(a_i)$.*

Lemma 1 tells us that the players' best responses are non-monotonic. Given high aggregate effective effort of the rivals, Q_{-i}, a player reacts to an increase in Q_{-i} by decreasing effort; given low aggregate effective effort of the rivals, a player reacts to an increase in Q_{-i} by increasing effort.

A second useful lemma describes how the players' best responses change with the overconfidence parameter λ.

Lemma 2. *An increase in λ leads to a contraction of player i's best response function, $\frac{\partial R_i(Q_{-i})}{\partial \lambda} < 0$, for $Q_{-i} < \lambda q(a_i)$ and to an expansion of his best response function, $\frac{\partial R_i(Q_{-i})}{\partial \lambda} > 0$, for $Q_{-i} > \lambda q(a_i)$. Moreover, the maximum value of a player's best response, a^{max} implicitly defined by $\frac{q'(a^{max})}{4q(a^{max})} v = c'(a^{max})$, is independent of λ.*

Lemma 2 characterizes the best response function of players who are subject to an overconfidence bias. For a high aggregate effective efforts of the rivals, an increase in overconfidence raises player i's effort level, while for low aggregate effective efforts of the rivals, an increase in overconfidence lowers player i's effort level. Moreover, the maximal value taken by player i's best response is independent of the overconfidence bias.

Making use of these results, and assuming for the time being that participation in the contest is guaranteed, we can establish equilibrium uniqueness in the following lemma:

Lemma 3. *A Tullock contest featuring n overconfident players admits a unique equilibrium.*

To guarantee participation by all n players for any overconfidence parameter λ, we impose the following assumption:

Assumption 1. *$\frac{v}{2} - c(a^{max}) \geq 0$.*

We next present our first proposition that uncovers the effect of overconfidence on equilibrium efforts.

Proposition 1. *In a Tullock contest with $n \geq 2$ symmetric players, individual and aggregate efforts decrease (increase) with overconfidence if $\lambda > (<) n - 1$.*

Proposition 1 uncovers that the effect of overconfidence on individual efforts is not the same in a Tullock contest with few versus many players. This result is driven by how overconfidence affects a player's perceived marginal winning probability.

In Figure 1 we depict with the plain curve a rational player i's winning probability, and with a dashed curve an overconfident player i's perceived winning probability, for fixed efforts of the rivals. In Figure 2, we accordingly depict the corresponding marginal winning probabilities of the two types of players. As we can see on Figure 2, the concavity of the perceived winning probability in own effort, a_i, implies that the perceived marginal winning probability of the overconfident player is higher than the one of the rational player for low efforts of player i, and therefore for low winning probabilities of player i, as seen on Figure 1. In contrast, the perceived marginal winning probability of the overconfident player is lower than the one of the rational player for high efforts.

Consider first a situation where overconfidence is high relative to the number of player, i.e. $\lambda > n - 1$. In such instances, all players expect to be highly likely to win the contest, which implies, as observed on Figure 2 that their perceived marginal winning probability is low. An increase in overconfidence will then reduce players' perceived marginal probability of winning and this incentivizes players to reduce their effort for a given expected (equilibrium) effort of their opponents: the high expected winning probability can now be achieved at lower cost. The exact opposite mechanism is at play when the degree of overconfidence is low compared to the number of players, i.e. $\lambda < n - 1$. In such instances, all players expect to have a small probability of winning the contest. In this case, an increase in overconfidence raises the players' perceived marginal probability of winning and this incentivizes them to increase effort.

Interestingly, if $n = 2$, the effect of an increase in overconfindence is unambiguous, as it always lowers players' equilibrium efforts. This finding is quite intuitive given that more confident players expect to have a high winning probability for given effort, and one can then expect that they save on cost of effort. However, for $n \geq 3$, we obtain the unexpected result that this is no longer necessarily the case.

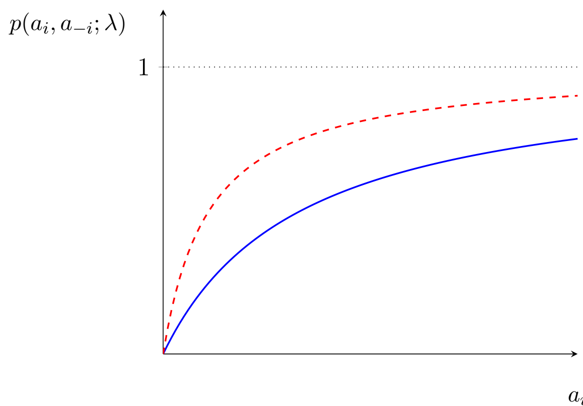

Figure 1: Perceived winning probabilities with rational (—) and overconfident (– –) players

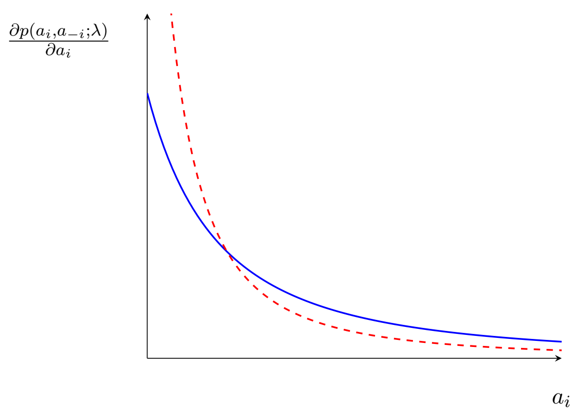

Figure 2: Perceived marginal winning probabilities with rational (—) and overconfident (– –) players

The above results allow us to state the following corollary which relates overconfidence to rent dissipation:

Corollary 1. *With $n \geq 2$ symmetric players, the maximal rent dissipation is attained when $\lambda = n-1$. For an overconfidence bias $\lambda > 1$, there always exists a finite $n^D(\lambda)$ such that overdissipation (i.e. the sum of players' effort costs is greater than the value of the prize) can be observed at equilibrium for $n > n^D(\lambda)$.*

It is widely known in the literature on contests that with rational players overdissipation can never be observed at equilibrium if the player's valuation of the prize is equal to the actual value of the prize.[6] Although the dissipation ratio, defined as the ratio of total expenditures (or sum of players' effort costs) to the value of the prize, $D = \frac{\sum_i c(a_i)}{v}$, does increase in the number of players, it is bounded by unity because individual equilibrium effort drops as the number of contestants increases. Indeed, a larger number of contestants implies that the competitors' aggregate effort is expected to be higher, thence reducing the marginal return to investing in the contest, which in turn pushes all contestants to individually contract their equilibrium effort. In Proposition 1 we demonstrated, however, that some overconfidence may push players to increase their equilibrium effort compared to a setup with fully rational players. Corollary 1 shows that there always exists a degree of overconfidence such that equilibrium individual efforts of overconfident players will equal the maximal equilibrium individual efforts that can be obtained in the game, i.e., the individual efforts produced in setups with fully rational players. Consequently, with sufficiently many overconfident players the aggregate effort can be higher than the value of the contested prize.

To visualize these two results, in Figure 3 we depict the individual equilibrium effort of (symmetric) contestants as a function of their overconfidence parameter in the most simple contest where players' payoffs are given by:

$$E[U_i(a_i, a_{-i}; \lambda)] = \frac{\lambda a_i}{\lambda a_i + \sum_{j \neq i} a_j} - a_i,$$

[6] See Dickson et al. (2022) for instances where players' valuation of the prize differs from the actual value of the prize.

With $n = 2$ and $\lambda = 1$, the equilibrium efforts are equal to $1/4$. If we consider contests with more players, the individual efforts can be kept equal to $1/4$ if $\lambda = n-1$. Consequently, under such circumstances, full dissipation can result with $n = 4$ and $\lambda = 3$, and overdissipation can therefore obtain for any $n > 4$.

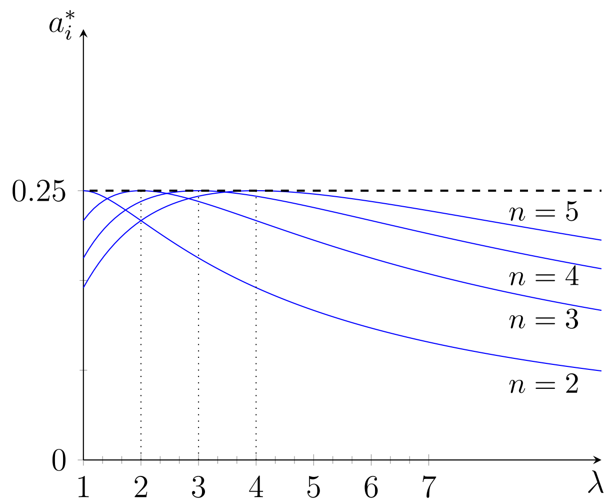

Figure 3: Individual equilibrium efforts as a function of λ with $q(a_i) = a_i$, $v = 1$ and $c(a_i) = a_i$.

It is important at this stage to underline that although for overdissipation to be observed it is necessary to have $n > n^D > 2$ players, the required degree of overconfidence may be quite low. Indeed, to visualize this we consider again the previous basic contest setup, and we impose for the sake of the argument the parameter restriction $\lambda < n - 1$, for $n \geq 3$. Since $a^* = \frac{\lambda(n-1)}{(\lambda+n-1)^2}$, this parameter restriction can easily be shown to imply that $\partial na^*/\partial n > 0$, $\partial^2 na^*/\partial n^2 < 0$, and $\partial a^*/\partial \lambda > 0$. We then plot the equilibrium aggregate effort, na^*, against the number of players, n, for various levels of overconfidence in Figure 4. It is well known that, with rational players, as n becomes arbitrarily large the dissipation ratio converges to unity, without ever reaching total rent dissipation. We know from Corollary 1 that for any number of players $n > n^D(\lambda) > 2$, there always exists a degree of overconfidence conducive to overdissipation. For example, Figure 4 shows that with $n = 6$ overdissipation is already observed when $\lambda = 1.5$, which corresponds to a perceived winning probability of 0.231 as opposed to the actual winning probability of 1/6. Increasing the number of players to, say, $n = 8$ implies that overdissipation can be achieved with an even lower degree of overconfidence (e.g. $\lambda = 1.25$). It is immediate to deduce that as the number of players becomes arbitrarily large in this setup, the required degree of overconfidence for observing overdissipation will become arbitrarily small (i.e. λ close to 1).

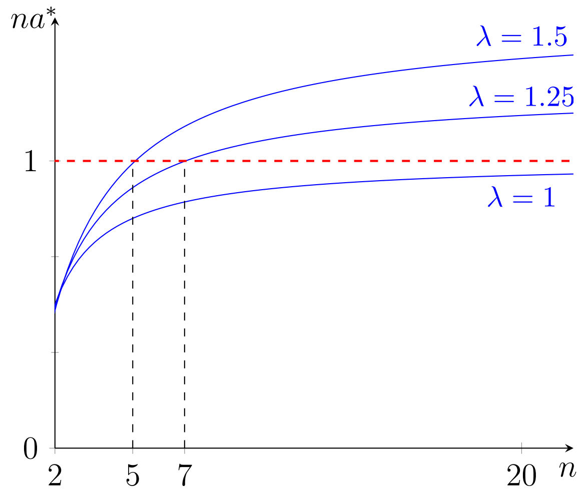

Figure 4: Equilibrium aggregate effort na^* as a function of n.

5 Entry

We now study the effect of overconfidence on entry in symmetric Tullock contests. The analysis so far assumes that players' outside option is zero. However, if the outside option is high enough, it is possible that the perceived expected utility of participating to the contest is too low to make entry attractive. To analyze how confidence affects entry, we assume there exist $N \geq 2$ symmetric potential entrants, and designate by n the number of players that enter the contest. Moreover, all potential entrants have an outside option equal to $\bar{v} < v$. This assumption guarantees that there is an incentive for at least one player to enter the contest. Further, we focus on pure strategy subgame perfect equilibria and on instances where at least two players have incentives to enter the contest. At the symmetric equilibrium, the equilibrium number of entrants, n^*, satisfies the equation:

$$\frac{\lambda}{\lambda + n^* - 1} v - c(a^*) = \bar{v} \tag{4}$$

Our last result describes how overconfidence affects n^*.

Proposition 2. *In a Tullock contest with a pool of $N \geq 2$ symmetric potential entrants, the equilibrium number of entrants n^* increases in overconfidence λ.*

An increase in overconfidence affects the incentives to enter the contest in two ways. First, it increases the players' perceived probability of winning for given efforts, which makes entry more attractive. Second, we know from Proposition 1 that for a fixed number of entrants, an increase in overconfidence raises (lowers) equilibrium individual efforts for $\lambda < (>) n - 1$, which makes entry less (more) attractive. Consequently, for high values of λ (higher than $n - 1$), an increase in overconfidence unambiguously makes entry more attractive. However, for low values of λ, the two effects go in opposite direction. Proposition 2 shows that the former effect always dominates the latter.

6 Conclusion

This paper studies the impact of overconfidence on Tullock contests. We assume an overconfident player overestimates the impact of his effort on the outcome of the contest while holding a correct assessment of the winning prize and his cost of effort. We demonstrate that in a symmetric $n > 2$ player contest, an increase in overconfidence increases the efforts of all players provided that the bias is small relative to the number of players. With sufficiently high levels of overconfidence, on the other hand, an increase in overconfidence will lead to lower equilibrium efforts. Our paper also provides conditions under which overspending and even overdissipation can result from overconfidence. Finally, we show that higher overconfidence always results in more entry at equilibrium.

Over the past years, Tullock contests have been extensively studied in laboratory experiments (e.g. Dechenaux et al. 2015). Our novel results highlight the importance of accounting for players' overconfidence when drawing predictions about behavior in Tullock contests. Our findings can be tested in a controlled laboratory experiment where self-confidence biases as well as the number of players can both be exogenously manipulated. We leave that for future research.

References

Baharad, E., and Nitzan, S. (2008). "Contest Efforts in Light of Behavioural Considerations," *The Economic Journal*, 118, 2047-2059.

Baik, K. (1994). "Effort Levels in Contests with Two Asymmetric Players," *Southern Economic Journal*, 61(2), 367-378.

Bansah, M., Neilson, W., and J.S. Lee (2024). "Overconfidence, Underconfidence, and Entry in Contests," *Managerial and Decision Economics*, 45, 19-33.

Baye, M.R., Kovenock, D., and C.G. De Vries (1999). "The Incidence of Overdissipation in Rent-Seeking Contests," *Public Choice*, 99, 439-454.

Bénabou, R., and J. Tirole (2002). "Self-Confidence and Personal Motivation," *The Quarterly Journal of Economics*, 117, 871-915.

Bénabou, R., and J. Tirole (2003). "Intrinsic and Extrinsic Motivation," *The Review of Economic Studies*, 70, 489-520.

Brozynski, T., Menkhoff, L., and U. Schmidt (2006). "The Impact of Experience on Risk Taking, Overconfidence, and Herding of Fund Managers: Complementary Survey Evidence," *European Economic Review*, 50, 1753-1766.

Camerer, C., and D. Lovallo (1999). "Overconfidence and Excess Entry: An Experimental Approach," *American Economic Review*, 89(1), 306-318.

Chowdhury, S.M., Sheremeta, R., and T.L. Turocyd (2014). "Overbidding and Overspreading in Rent-Seeking Experiments: Cost Structure and Prize Allocation Rules," *Games and Economic Behavior*, 87, 224–238.

Cooper, A., Woo, C., and Dunkelberg, W. (1988). "Entrepreneurs' Perceived Chances for Success," *Journal of Business Venturing*, 3, 97-108.

Cornes, R. C., Hartley, R. (2003). "Loss Aversion and the Tullock Paradox," Keele Economics Research Paper 2003/06.

Dechenaux , E., Kovenock, D., and R. Sheremeta (2015). "A Survey of Experimental Research on Contests, All-Pay Auctions and Tournaments," *Experimental Economics*, 18, 609-669.

de la Rosa, L.E. (2011). "Overconfidence and Moral Hazard," *Games and Economic Behavior*, 73(2), 429-451.

Deng, S., Fu, Q., Fang, H., and Z. Wu (2024). "Confidence Management in Contests," *Journal of Economics and Management Strategy*, forthcoming.

Dickson, A., MacKenzie, I.A., and P.G. Sekeris. (2018). "Rent-Seeking Incentives in Share Contests," *Journal of Public Economics* 166: 53-62.

Dickson, A., MacKenzie, I.A., and P.G. Sekeris (2022). "Rent Dissipation in Simple Tullock Contests," *Games*, 13(6), 83.

Dohmen, T., and A. Falk (2011). "Performance Pay and Multidimensional Sorting: Productivity, Preferences, and Gender," *American Economic Review*, 101(2), 556-590.

Ewerhart, C. (2015). "Contest Success Functions: The Common-Pool Perspective," Working Paper 195, Working Paper Series, University of Zurich.

Fonseca, M. (2009). "An Experimental Investigation of Asymmetric Contests," *International Journal of Industrial Organization*, 27(5), 582-591.

Fu, Q., Lyu, Y., Wu, Z., and Y. Zhang (2022). "Expectations-Based Loss Aversion in Contests," *Games and Economic Behavior* 133: 1-27.

Gervais, S. and I. Goldstein (2007). "The Positive Effects of Biased Self-perceptions in Firms," *Review of Finance*, 11(3), 453-496.

Guthrie, C., Rachlinski, J.J., and A.J. Wistrich (2001). "Inside the Judicial Mind," *Cornell Law Review*, 86, 777-830.

Heifetz, A., Shannon, C., and Spiegel, Y. (2007a). "The Dynamic Evolution of Preferences," *Economic Theory*, 32(2), 251-286.

Heifetz, A., Shannon, C., and Spiegel, Y. (2007b). "What to Maximize if You Must," *Journal of Economic Theory*, 133, 31-57.

Hoffman, M. (2016). "How is Information Valued? Evidence from Framed Field Experiments," *The Economic Journal*, 126, 1884-1911.

Jindapon, P., and Whaley, C.A. (2015). "Risk Lovers and the Rent Over-investment Puzzle," *Public Choice*, 164, 87-101.

Kelsey, D. and Melkonyan, T. (2018). "Contests with Ambiguity," *Oxford Economic Papers* 70(4): 1148-1169.

Keskin, K. (2018). "Cumulative Prospect Theory in Rent-Seeking Contests," *Mathematical Social Sciences* 96: 85-91.

Lazear, E.P. and S. Rosen (1981). "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy,*, 89(5), 841-864.

Libby, R., and K. Rennekamp (2012). "Self-Serving Attribution Bias, Overconfidence, and the Issuance of Management Forecasts, " *Journal of Accounting Research*, 50(1), 197-231.

Ludwig, S., and J. Nafziger (2011). "Beliefs About Overconfidence, " *Theory and Decision*, 70, 475-500.

Ludwig, S., Wichardt, P.C., and H. Wickhorst (2011). "Overconfidence Can Improve a player's Relative and Absolute Performance in Contests, " *Economics Letters*, 110(3), 193-196.

Lyons, B.A., McKay, A.M., and J. Reifler (2020). "High-Status Lobbyists are Most Likely to Overrate their Success, " *Nature Human Behaviour*, 4, 153-159.

Mago, S.D., Samak, A.C., and Sheremeta, R.M. (2016). "Facing your Opponents: Social Identification and Information Feedback in Contests, " *Journal of Conflict Resolution*, 60(3), 459-481.

Malmendier, U., and G. Tate (2005). "CEO Overconfidence and Corporate Investment," *The Journal of Finance*, 60(6), 2661-2700.

Malmendier, U., and G. Tate (2008). "Who Makes Acquisitions? CEO Overconfidence and the Market's Reaction," *Journal of Financial Economics*, 89(1), 20-43.

Malmendier, U., and T. Taylor (2015). "On the Verges of Overconfidence," *Journal of Economic Perspectives*, 29(4), 3-8.

Moore, Don A., and P.J. Healy (2008). "The Trouble with Overconfidence," *Psychological Review*, 115(2), 502.

Murphy, S.C., von Hippel, W., Dubbs, S.L., Angiletta, M.J., Wilson, R.S., Trivers, R., and F.K. Barlow (2015). "The Role of Overconfidence in Romantic Desirability and Competition," *Personality and Social Psychology Bulletin*, 41(8), 1036-1052.

Myers, D. (1996). *Social Psychology*, New York: McGraw Hill.

Niederle, M., and L. Vesterlund (2007). "Do Women Shy Away from Competition? Do Men Compete too Much?" *The Quarterly Journal of Economics*, 122(3), 1067-1101.

Oberlechner, T., and C. Osler (2008). "Overconfidence in Currency Markets," Available at SSRN 1108787.

Park, Y.J., and L. Santos-Pinto (2010). "Overconfidence in Tournaments: Evidence from the Field," *Theory and Decision*, 69(1), 144-166.

Price, C.R., and R.M. Sheremeta (2015). "Endowment Origin, Demographics Effects, and Individual Preferences in Contests, " *Journal of Economics & Management Strategy*, 24 (3), 597-619.

Pronin, E., and M.B. Kugler (2007). "Valuing Thoughts, Ignoring Behavior: The Introspection Illusion as a Source of the Bias Blind Spot, " *Journal of Experimental Social Psychology*, 43, 565-578.

Pronin, E., Lin, D.Y. and L. Ross (2002). "The Bias Blind Spot: Perceptions of Bias in Self Versus Others, " *Personality and Social Psychology Bulletin*, 28(3), 369-381.

Santos-Pinto, L. and J. Sobel (2005). "A Model of Positive Self-image in Subjective Assessments," *American Economic Review*, 95(5), 1386-1402.

Santos-Pinto, L. (2008). "Positive Self-image and Incentives in Organisations," *The Economic Journal*, 118(531), 1315-1332.

Santos-Pinto, L. (2010). "Positive Self-Image in Tournaments," *International Economic Review*, 51(2), 475-496.

Santos-Pinto, L., and E. L. de la Rosa (2020). "Overconfidence in Labor Markets," K. Zimmermann (Ed), *Handbook of Labor, Human Resources and Population Economics*. Springer-Verlag.

Santos-Pinto, L., and P.G. Sekeris (2025). "How Confidence Heterogeneity Shapes Effort and Performance in Tournaments and Contests," *Journal of Mathematical Economics*, 116, 103069.

Sheremeta, R. (2011). "Contest Design: An Experimental Investigation," *Economic Inquiry*, 49(2), 573-590.

Singh, N., and D. Wittman (2001). "Contests Where there is Variation in the Marginal Productivity of Effort," *Economic Theory*, 18(3), 711-744.

Squintani, F. (2006). "Equilibrium and Mistaken Self-Perception," *Economic Theory*, 27(3), 615-641.

Stein, W. (2002). "Asymmetric Rent-Seeking with More than Two Contestants," *Public Choice*, 113, 325-336.

Tullock, G. (1980). "Efficient Rent Seeking," In James Buchanan, R. T. and Tullock, G., editors, *Towards a Theory of the Rent-Seeking Society*, 97-112. Texas A&M University Press, College Station, TX.

Tversky A., and D. Kahneman (1992). "A Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, 5: 297-323.

Waldman, M. (1994). "Systematic Errors and the Theory of Natural Selection," *American Economic Review*, 84(3), 482-497.

7 Appendix

Proof of Lemma 1: The best response of player i, is defined implicitly by (3). Hence, the slope of the best response of player i, $R_i'(Q_{-i})$ is given by

$$-\frac{\partial R_i/\partial Q_{-i}}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial Q_{-i}}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda q(a_i)-Q_{-i}}{[\lambda q(a_i)+Q_{-i}]^3}\lambda q'(a_i)v}{\frac{q''(a_i)[\lambda q(a_i)+Q_{-i}]-2\lambda[q'(a_i)]^2}{[\lambda q(a_i)+Q_{-i}]^3}\lambda Q_{-i}v - c''(a_i)}. \tag{5}$$

The denominator is negative because player i's second-order condition is satisfied. Therefore, the sign of the slope of player i's best response is only determined by the sign of the numerator which only depends on $\lambda q(a_i) - Q_{-i}$. Hence, $R_i'(Q_{-i})$ is positive for $\lambda q(a_i) > Q_{-i}$, zero for $\lambda q(a_i) = Q_{-i}$, and negative for $\lambda q(a_i) < Q_{-i}$. This implies that $R_i(Q_{-i})$ increases in Q_{-i} for $\lambda q(a_i) > Q_{-i}$, reaches the maximum at $\lambda q(a_i) = Q_{-i}$, and decreases in Q_{-i} for $\lambda q(a_i) < Q_{-i}$.

Proof of Lemma 2: (This proof follows Baik 1994) Player i's best response is implicitly defined by:

$$\frac{\lambda q'(a_i)Q_{-i}}{[\lambda q(a_i) + Q_{-i}]^2}v - c'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(Q_{-i})}{\partial \lambda} = \frac{Q_{-i} - \lambda q(a_i)}{[\lambda q(a_i) + Q_{-i}]^3}q'(a_i)Q_{-i}v.$$

We see that $\partial R_i(Q_{-i})/\partial\lambda \gtreqless 0$ for $Q_{-i} \gtreqless \lambda q(a_i)$. We also know from Lemma 1 that $sign\{R_i'(Q_{-i})\} = -sign\left\{\frac{\partial R_i(Q_{-i})}{\partial\lambda}\right\}$.

Substituting next $q(Q_{-i}) = \lambda q(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a^{max} we obtain

$$\frac{\lambda q'(a^{max})\lambda q(a^{max})}{[\lambda q(a^{max}) + \lambda q(a^{max})]^2}v = c'(a^{max}),$$

or

$$\frac{\lambda^2 q'(a^{max})q(a^{max})}{4\lambda^2\,[q(a^{max})]^2}v = c'(a^{max}),$$

or

$$\frac{q'(a^{max})}{4q(a^{max})}v = c'(a^{max}). \tag{6}$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a^{max}, does not depend on λ.

Proof of Lemma 3:

To prove that the equilibrium is unique, and bearing in mind that $q(a_i)$ is monotonically increasing in a_i, we can rewrite the optimization problem as a function $q_i = q(a_i)$, so that $a_i = a^{-1}(q_i)$, derive the equilibrium value of q_i, and deduce the equilibrium value of a_i. If there is a unique equilibrium in the space $(q_1, q_2, \ldots, q_n)$, then there is a unique equilibrium in the space $(a_1, a_2, \ldots, a_n)$.

$$\max_{q_i} \frac{\lambda q_i}{\lambda q_i + Q_{-i}} v - \phi(q_i),$$

where $\phi(q_i) = c(a^{-1}(q_i))$. Accordingly, $\phi'(q_i) > 0$, and since $a^{-1''}(q_i) > 0$, it is immediate to deduce that $\phi''(q_i) > 0$.

Optimizing, we obtain:

$$\frac{\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^2} v - \phi'(q_i) = 0,$$

and this expression implicitly defines the best response of player i, $B(Q_{-i})$.

To prove that the equilibrium is unique it is then sufficient to show that the product of the slopes of the best response functions is less than 1: $\Gamma = B_1'(Q_{-1}) \circ B_2'(Q_{-2}) \ldots \circ B_n'(Q_{-n}) < 1$. We first derive the slope of the best response of player i:

$$B_i'(Q_{-i}) = -\frac{\frac{\lambda(\lambda q_i + Q_{-i}) - 2\lambda Q_i}{(\lambda q_i + Q_{-i})^3} v}{-\frac{2\lambda^2 Q_{-i}}{(\lambda q_i + Q_{-i})^3} v - \phi''(q_i)} = \frac{\frac{\lambda q_i - Q_i}{(\lambda q_i + Q_{-i})^3} v}{\frac{2\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^3} v + \frac{\phi''(q_i)}{\lambda}}.$$

Observe that if for an odd number of players $B_i'(Q_{-i}) < 0$, and that for the remaining players the best responses are positively slopped, then we necessarily deduce that $\Gamma < 1$. Second, if for an even number of players $B_i'(Q_{-i}) < 0$, and that for the remaining players the best responses are positively slopped, then we wish to prove that:

$$\Pi_{i=1}^{n} \left\{ \frac{\frac{\lambda q_i - Q_i}{(\lambda q_i + Q_{-i})^3} v}{\frac{2\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^3} v + \frac{\phi''(q_i)}{\lambda}} \right\} < 1.$$

Since $B_i^{'}(Q_{-i})$ is decreasing in $\phi''(q_i)$, it is thus sufficient to establish the result for $c''(q_i) = 0$. Rewriting the product of the contestants' best responses with this restriction, and simplifying, we thus want to show that:

$$\Pi_{i=1}^{n}\left\{\frac{\lambda q_i - Q_{-i}}{2\lambda Q_{-i}}\right\} < 1.$$

This expression is necessarily true if Q_{-i} is set to zero in the numerator, therefore implying that the expression is always verified since $\Pi_{i=1}^{n} q_i < 2\Pi_{i=1}^{n} Q_{-i}$.

The above reasoning guarantees that if all n players participate to the contest, then the game admits a unique equilibrium. It is immediate to observe that a symmetric equilibrium exists, therefore implying that the game's unique equilibrium is indeed symmetric.

Proof of Proposition 1: At the unique symmetric equilibrium the first-order condition (1) reads as:

$$\frac{\lambda q'(a^*)(n-1)q(a^*)}{\left[\lambda q(a^*) + (n-1)q(a^*)\right]^2} v - c'(a^*) = 0,$$

or

$$\frac{\lambda(n-1)q'(a^*)}{(\lambda+n-1)^2\, q(a^*)} v - c'(a^*) = 0. \tag{7}$$

To inspect the sign of $\partial a^*/\partial\lambda$ we apply the implicit function theorem to the above expression to obtain:

$$\begin{aligned}
\frac{\partial a^*}{\partial \lambda} &= -\frac{\frac{(n-1)(\lambda+n-1)^2 - 2(\lambda+n-1)\lambda(n-1)}{(\lambda+n-1)^4} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)} \\
&= -\frac{\frac{(n-1)(n-1-\lambda)}{(\lambda+n-1)^3} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)}.
\end{aligned}$$

Since the denominator of this expression is unambiguously negative, the sign of the expression is therefore given by the sign of $(n-1-\lambda)$.

Last, we need to guarantee that all n players are willing to participate to the contest. The perceived expected utility of any contestant i is given by:

$$E[U(a_i, a_{-i}; \lambda)] = \frac{\lambda q(a_i)}{\lambda q(a_i) + \sum_{j \neq i} q(a_j)} v - c(a_i).$$

Therefore, an increase in λ changes the perceived expected utility as follows:

$$\frac{dE[U(a_i, a_{-i}; \lambda)]}{d\lambda} = \frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_i} \frac{da_i}{d\lambda} + \sum_{j \neq i} \frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_j} \frac{da_j}{d\lambda}.$$

By the Enveloppe theorem, we know that the first term of the above expression is nil. Consequently, and since $\frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_j} < 0$, $\forall j \neq i$, at equilibrium, the sign of $\frac{dE[U(a_i, a_{-i}; \lambda)]}{d\lambda}$ is given by the sign of $\frac{da^*}{d\lambda}$, which has been shown to be given by the sign of $n - 1 - \lambda$. We then deduce that the lowest perceived expected utility for symmetric equilibrium efforts is attained when $\lambda = n - 1$. To ensure participation of all n players, we then require that the following holds:

$$E[U(a_i^*, a_{-i}^*; \lambda)] = \frac{\lambda}{\lambda + n - 1} v - c(a^*) \geq 0.$$

Replacing for $\lambda = n - 1$ this condition reads as:

$$E[U(a_i^*, a_{-i}^*; \lambda)] = \frac{v}{2} - c(a^*) \geq 0.$$

Last, since the maximal value of a^* has been proven to equal a^{max}, participation by all n contestants is always guaranteed by Assumption 1.

Proof of Corollary 1: At the unique symmetric equilibrium, players' equilibrium effort is given by equation (7). We know that the value of the maximal equilibrium effort is defined by a^{max} as implicitly defined by (6), and that when $\lambda = n - 1$, then $a^* = a^{max}$. Accordingly, for any n, there exist a $\lambda = n - 1$ such that the dissipation ratio is given by:

$$D^{max} = \frac{nc(a^{max})}{v},$$

and this characterizes the highest possible dissipation ratio. Since a^{max} is independent of n, that for $n = 2$, $D^{max} < 1$, and that D^{max} is monotonically increasing in n, there must always exist a $n^D(\lambda) > 2$ such that $D^{max} > 1$.

Proof of Proposition 2: We can re-write equation (4) as:

$$\psi = \frac{\lambda}{\lambda + n^* - 1} v - c(a^*) - \bar{v} = 0.$$

Consequently, the effect of overconfidence on the number of entrants is given by:

$$\frac{dn^*}{d\lambda} = -\frac{\frac{\partial \psi}{\partial \lambda}}{\frac{\partial \psi}{\partial n^*}} = -\frac{\frac{(n^*-1)}{(\lambda+n^*-1)^2} v - c'(a^*)\frac{\partial a^*}{\partial \lambda}}{-\frac{\lambda}{(\lambda+n^*-1)^2} v - c'(a^*)\frac{\partial a^*}{\partial n^*}}.$$

We can separately compute the following two expressions:

$$\partial a^*/\partial \lambda = \frac{\frac{[n-1][\lambda-n+1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}},$$

and,

$$\partial a^*/\partial n = -\frac{\frac{\lambda[\lambda-n+1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}},$$

where $\psi = \frac{\lambda(n-1)q'(a^*)}{(\lambda+n-1)^2 q(a^*)} v - c'(a^*) = 0$ as given in equation (7).

Substituting these two expressions in $dn^*/d\lambda$, we obtain:

$$\frac{dn^*}{d\lambda} = -\frac{(n-1)\left[\frac{1}{(\lambda+n^*-1)^2} v + c'(a^*)\frac{\frac{[-\lambda+n-1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}}\right]}{\lambda\left[-\frac{1}{(\lambda+n^*-1)^2} v - c'(a^*)\frac{\frac{[-\lambda+n-1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}}\right]} = \frac{n-1}{\lambda}.$$