Overconfidence in Tullock Contests

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Abstract

We investigate the role of overconfidence in Tullock contests. An overconfident player overestimates the impact of his effort on the outcome of the contest. We find that when overconfidence is high relative to the number of players, an increase in overconfidence lowers equilibrium efforts. However, the opposite happens when overconfidence is low relative to the number of players. We demonstrate that overdissipation can occur due to overconfidence. We show that an increase in overconfidence unambiguously leads to an increase in the number of entrants. We finally introduce heterogeneity in confidence and ability. We show that overconfidence can deter rational players from participating, and—when coupled with lower true ability—leads overconfident players to exert lower equilibrium effort.

JEL Codes: D60; D69; D91
Keywords: Overconfidence; Contests; Rent Dissipation; Entry.

1 Introduction

This paper examines how overconfidence shapes behavior in n player Tullock contests—a pertinent question, since psychology and economics repeatedly show that people systematically overrate their own abilities. Most individuals place themselves above average on a wide range of skills and traits (Myers 1996; Santos-Pinto and Sobel 2005). Such overconfidence has been documented among entrepreneurs (Cooper et al. 1988), judges (Guthrie et al. 2001), CEOs (Malmendier and Tate 2005, 2008), fund managers (Brozynski et al. 2006), poker and chess players (Park and Santos-Pinto 2010), currency traders (Oberlechner and Osler 2012), and store managers (Huffman et al. 2022).

Competitions often take the form of contests. For instance, an R&D race to be the first to develop or get a patent in new product or technology, election campaigns, rent-seeking games, competitions for monopolies, litigation, and wars, are examples of contests. Overconfidence matters for entry and performance in competitions and for labor markets (Camerer and Lovallo 1999, Niederle and Vesterlund 2007, Moore and Healy 2008, Dohmen and Falk 2011, Malmendier and Taylor 2015, Huffman et al. 2019, Santos-Pinto and de la Rosa 2020). Overconfidence also seems to play a role in mate competition and acquisition (Waldman 1994, Murphy et al. 2015). Interestingly, Lyons et al. (2020) provide evidence that high-status lobbyists working for private interest groups in Washington, DC, USA tend to be overconfident: they overate their achievements and their success. This empirical finding is in line with the experimental findings of Niederle and Vesterlund (2007) and Dohmen and Falk (2011) according to which overconfident participants tend to self select more into more competitive environments.

What is the effect of players' overconfidence on their effort provision and on rent dissipation? Does overconfidence lead to more entry in a contest? These are important questions since although the extant literature has characterized in depth equilibria in contests, behavioral biases have so far received limited attention by scholars (e.g. Baharad and Nitzan 2008, Santos-Pinto and Sekeris 2025).

To address these questions, we employ a generalized n player Tullock contest (1980) where v is the prize being contested, a_i the effort of player i, and $c(a_i)$ the cost of effort to player i. Player i's probability of winning the contest is $P(a_i, a_{-i}) = \frac{q(a_i)}{q(a_i)+\sum_{j\neq i} q(a_j)}$, where $q(a_i)$ is often referred to as the impact function (Ewerhart 2015). In an environment with fully rational players, the expected utility of player i is given by $E[U_i(a_i, a_{-i})] = P_i(a_i, a_{-i})v - c(a_i)$.

An earlier study conceptualizes overconfidence as an underestimation of the cost of effort: $E[U_i(a_i, a_{-i}; \gamma)] = P_i(a_i, a_{-i})v - \gamma c(a_i)$, where $0 < \gamma < 1$ (Ludwig et al. 2011). Likewise, overconfidence can also be modeled as an overestimation of the rival's cost of effort (Deng et al. 2024). These approaches to modeling overconfidence are isomorphic. We instead model overconfidence as a misperception of a player' probability of winning rather than as a misperception of effort costs or prize value. Empirically, overconfident individuals rarely believe that rewards are larger than they are, or that exerting effort is intrinsically cheaper; instead, they overestimate their chances of securing the existing prize. Political leaders, entrepreneurs, and managers typically understand the objective stakes and the effort required, but systematically judge their own prospects more favorably than warranted (Johnson 2004, Moore and Healy 2008, Charness et al. 2018, Huffman et al. 2022).

Accordingly, we assume an overconfident player i thinks, mistakenly, his impact function is $\lambda q(a_i)$, where $\lambda > 1$, and has correct beliefs about his rivals' impact functions. Hence, an overconfident player's perceived winning probability is $P_i(a_i, a_{-i}; \lambda) = \frac{\lambda q(a_i)}{\lambda q(a_i)+\sum_{j\neq i} q(a_j)}$, which is larger than his actual winning probability. Since the impact function embeds a player's ability, we conceptualize overconfidence as an overestimation of the effect of one's effort on contest outcomes—a common definition of overconfidence in the related literature (e.g. Bénabou and Tirole 2002, 2003, Santos-Pinto 2008, 2010). Importantly, our findings regarding the effect of overconfidence in contests run counter to earlier research.

We begin by considering a symmetric $n \geq 2$ player Tullock contest where all players are overconfident. We demonstrate that the number of players as well as the degree of overconfidence matters in terms of understanding the effects of overconfidence on effort provision and rent dissipation in contests. On the one hand, overconfidence reduces individual and aggregate efforts when λ is larger than $n-1$. In this case all players expect to be highly likely to win the contest. Hence, an increase in overconfidence lowers the perceived marginal probability of winning, which pushes players' efforts downwards. On the other hand, overconfidence raises individual and aggregate efforts if λ is smaller than $n-1$. In such instances, all players expect to win the contest with a low probability. Therefore, an increase in overconfidence will raise the perceived marginal probability of winning, which pushes players' efforts upwards. This stands out as a novel contribution of our work compared to the existing literature which has exclusively focused on 2 player contests.

We next demonstrate that overconfidence can lead to overdissipation, i.e. situations where players' aggregate cost of effort is strictly larger than the value of the prize. In particular, we show that there is a threshold value of the number of players above which overdissipation can always occur, provided players are sufficiently overconfident. Moreover, we show that as the number of players goes up, overdissipation can be observed for values of λ close to 1.

We then inquire how overconfidence affects entry in a contest. In order to answer this question, we assume $N \geq 2$ symmetric potential entrants that have an outside option. Overconfidence affects incentives to enter the contest through two channels. First, it raises the perceived winning probability, and thus the benefit of entry for given efforts of players. Second, it incentivizes players to modify their equilibrium efforts, thereby indirectly impacting the potential entrants' payoffs. We show that even when an increase in overconfidence raises players' individual efforts, and the two effects then go in opposite directions, higher overconfidence always results in more entry.

Last, we explore the effect of heterogeneity in confidence and abilities in our setup. When players are of equal abilities, some are overconfident, and the remaining are rational, we find that for low levels of overconfidence, the overconfident players invest higher effort at equilibrium. For high levels of overconfidence, the overconfident players slack and end up investing less effort at equilibrium. We also show that the presence of overconfident players can disincentivize rational players from participating in a contest. Finally, we assume that overconfident players are less able, and yet believe they have the same ability as rational players do. In such setups, we show that overconfident players will always exert less effort at equilibrium.

The paper is organized as follows. Section 2 discusses related literature. Section 3 sets-up the contest model. In Section 4 we derive the equilibrium and perform comparative statics. Section 5 studies entry. Section 6 considers heterogeneity in confidence and abilities, and Section 7 concludes the paper. All proofs are in the Appendix.

2 Related Literature

This study relates to two strands of literature. First, it contributes to the literature on behavioral biases in contests and tournaments.

Ludwig et al. (2011) examine a Tullock contest featuring an overconfident player who underestimates their effort cost against a rational opponent. They find that overconfidence increases the biased player's effort while reducing the rational player's effort, ultimately benefiting the contest organizer as the former's increased effort outweighs the latter's reduction. This occurs because the overconfident player perceives a lower marginal cost of effort, prompting higher exertion, which strategically discourages the rational player. In contrast, our analysis models overconfidence as an overestimation of one's impact on the probability of winning, resulting in qualitatively different equilibrium effects. Specifically, overconfidence raises the perceived marginal winning probability at low effort levels but lowers it at high effort, yielding a non-monotonic best response shift. This formulation is appropriate when both the contest's monetary value and effort costs are common knowledge pre-entry.

Bansah et al. (2024) explore the role of overconfidence on a 2 player Tullock contest with linear impact and cost functions. Overconfidence is modeled as an overestimation of the winning probability, rather than an overestimation of the impact of one's effort. Observe, however, that their definition of overconfidence does not satisfy the property that the perceived winning probabilities are well defined for any value of the bias. In addition to our distinct approach to modeling overconfidence, our study employs more general impact and cost functions and extends the analysis to n players.

Santos-Pinto and Sekeris (2025) study how confidence heterogeneity affects effort and performance in tournaments and contests with 2 players. They demonstrate a non-monotonic effect of confidence on equilibrium relative efforts and winning probabilities. In the present paper, we use the same definition of overconfidence as in Santos-Pinto and Sekeris (2025), but rather than focusing on a 2 player contest, we consider contests with $n \geq 2$ players, and we explore the implications of overconfidence on rent dissipation and on entry. The extension to $n > 2$ players is crucial since while with 2 players overconfidence always lowers players' equilibrium effort, with $n > 2$ players this may no longer be the case.

The aforementioned studies analyze Tullock contests under the assumption that players have complete information about their abilities or costs, yet overconfident players overestimate their abilities or underestimate their costs. In contrast, Deng et al. (2024) consider a Tullock contest with incomplete information about the players' costs. In their model, a newly hired employee has private information about his cost of effort, while the incumbent employee has biased beliefs on the former's cost-type, i.e. he holds a biased prior belief on whether his rival is a low-cost or a high-cost type. They study how the asymmetry in beliefs affects aggregate expected effort provision, and whether a contest organizer should disclose or conceal information on the new hire's cost of effort to the incumbent. We instead model overconfidence as an overestimation of the impact of one's effort in a setup where there is no uncertainty about the players' true types.[1] Observe that, as explained above, overestimating

[1]Overconfidence has been studied using two approaches. One approach assumes incomplete

one's own ability deeply differs from misestimating one's effort cost, and we equally extend the analysis to n player contests.

Santos Pinto (2010) studies how a tournament organizer optimally sets the prizes in a Lazear and Rosen (1981) rank-order tournament with overconfident players. We adopt the same definition of overconfidence and equilibrium concept. Observe, however, that although players' winning probabilities in both Lazear-Rosen tournaments and Tullock contests are logistic functions, the way in which noise affects the mapping of players' efforts to winning probabilities differs. As a consequence, overconfidence shifts players' best response functions differently in these two models. Santos Pinto (2010) finds in a symmetric two player tournament that an increase in overconfidence raises the equilibrium efforts of players. In contrast, we find the opposite in a two player contest, while we equally consider more than two players in our study.

Baharad and Nitzan (2008) and Keskin (2018) extend standard contest models by letting players distort objective win-probabilities with an inverse S-shaped weighting rule from Cumulative Prospect Theory: people overweight small and underweight large probabilities. Baharad and Nitzan (2008) and Keskin (2018) show that probability distortions can lead to rent over-dissipation in contests with a relatively large number of players, because small winning probabilities are overweighted. Our approach is complementary: we assume a bias in players' beliefs that they are better at contesting their opponents than they really are. We show that overconfidence can also lead to rent over-dissipation when the number of players is relatively large. Although this result looks similar to that in Baharad and Nitzan (2008) and

information about players' types and models overconfidence as a shift in the belief distribution that places greater weight on types with higher ability or lower effort cost. (Bénabou and Tirole 2002, 2003, Santos-Pinto 2008, De la Rosa 2011, Deng et al. 2024). In contrast, the other approach assumes complete information and posits that overconfidence is an overestimation of ability or an underestimation of effort cost. (Santos-Pinto 2010, Ludwig et al. 2011, Bansah et al. 2024, Santos-Pinto and Sekeris 2025). The first approach imposes an upper limit on the bias, whereas the second approach does not.

Keskin (2018), it is driven by a different mechanism. In our setting, overconfidence leads players to overestimate their marginal winning probabilities when the perceived winning probabilities are small, whereas in Baharad and Nitzan (2008) and Keskin (2018) probability distortion leads players to overestimate small winning probabilities. Observe that whereas their approach applies exclusively to probabilistic setups, our own model is equally suited to describe sharing contests that have gained in importance over the years (e.g. Dickson et al. 2018).[2]

Second, our study relates to the experimental literature on behavior in contests. Scholars have also long tried to explain the puzzle that contestants in lab experiments spend significantly higher amounts than the game's Nash equilibrium (Chowdhury et al. 2014, Price and Sheremeta 2015, Mago et al. 2016), and even overdissipation can occur (Sheremeta 2011). The theoretical literature has attempted to explain overspending, but also extreme manifestations of such phenomena where contestants overdissipate the rent by expending on aggregate more resources than the value of the prize that is contested. Overspending has so far been attributed to players' risk attitudes (Jindapon and Whaley 2015) or to mixed strategy equilibria where overspending occurs with some probability but not in expectation (Baye et al. 1999). Our paper demonstrates that overconfidence cannot be the reason why overspending is observed in two player contests, yet it uncovers that in contests with more than two players overspending and even overdissipation can result when the number of players is sufficiently large and the overconfidence bias is relatively mild. Indeed, overconfident players individually expend more effort than rational players when their odds of winning are low because of the high number of participants.

[2] Other scholars have equally focused on the effect of behavioural biases on equilibrium outcomes in the presence of uncertainty. Kelsey and Melkonyan (2018) consider both optimistic and pessimistic attitudes to ambiguity, while Cornes and Hartley (2003) and Fu et al. (2022) introduce loss aversion in probabilistic contests.

3 Set-up

To study the role of overconfidence in contests we consider a generalized Tullock contest with n symmetric players. The effort cost is $c(a_i)$ with $c(0) = 0$, $c'(a_i) > 0$ and $c''(a_i) \geq 0$. Following Baik (1994) we assume the CSF is:

$$P_i(a_i, a_{-i}) = \begin{cases} q(a_i)/\sum_j q(a_j) & \text{if } \sum_j q(a_j) > 0 \\ 1/n & \text{if } \sum_j q(a_j) = 0 \end{cases},$$

where a_{-i} designates the vector of player i's competitors' efforts, $q(0) \geq 0$, $q'(a_i) > 0$ and $q''(a_i) \leq 0$. Any player i mistakenly perceives his impact function to be $\lambda q(a_i)$, with $\lambda > 1$, and correctly perceives the rivals' impact functions. This way of modeling overconfidence in a contest implies that player i's perceived winning probability is equal to

$$P_i(a_i, a_{-i}; \lambda) = \begin{cases} \lambda q(a_i)/[\lambda q(a_i) + \sum_{j \neq i} q(a_j)] & \text{if } \lambda q(a_i) + \sum_{j \neq i} q(a_j) > 0 \\ 1/n & \text{if } \lambda q(a_i) + \sum_{j \neq i} q(a_j) = 0 \end{cases}.$$

This specification of overconfidence in a contest satisfies three desirable properties. First an overconfident player's perceived winning probability is well defined for any value of $\lambda > 1$.[3] Second, an overconfident player's perceived winning probability is increasing in λ. Third, overestimating one's impact function is equivalent to underestimating the rivals' impact functions since $\lambda q(a_i)/[\lambda q(a_i) + \sum_{j \neq i} q(a_j)] = q(a_i)/[q(a_i) + \sum_{j \neq i} q(a_j)/\lambda]$. In other words, all our results are unaffected if overconfidence is instead modeled as underestimation of the impact of an opponent's effort.[4]

[3] This is not the case with alternative specifications. For example, if one assumes an overconfident player's perceived winning probability is $P_i(a_i, a_{-i}; \lambda) = \lambda q(a_i)/[q(a_i) + \sum_{j \neq i} q(a_j)]$, with $\lambda > 1$, then $P_i(a_i, a_{-i}; \lambda)$ is not a well defined probability for any value of $\lambda > 1$.

[4] The way we model overconfidence is often used in studies that analyze the impact of overconfidence on contracts (Bénabou and Tirole 2002 and 2003, Gervais and Goldstein 2007, Santos-Pinto 2008 and 2010, and de la Rosa 2011).

To be able to compute equilibria when players hold mistaken beliefs we assume that: (1) when facing biased opponents, players are aware that the former's perceptions of their own impact function (and probability of winning) are mistaken, (2) each player thinks that his own perception of his impact function (and probability of winning) is correct, and (3) players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponents' beliefs. Hence, players agree to disagree about their impact functions (and winning probabilities). This approach follows Heifetz et al. (2007a,2007b) for games with complete information, and Squintani (2006) for games with incomplete information.

These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007). As stated by Pronin et al. (2002: 369) "people recognize the existence, and the impact, of most of the biases that social and cognitive psychologists have described over the past few decades. What they *lack* recognition of, we argue, is the role that those same biases play in governing their *own* judgments and inferences." For example, Libby and Rennekamp (2012) conduct a survey which shows that experienced financial managers believe that other managers are likely to be overconfident while failing to recognize their own overconfidence. Hoffman (2016) runs a field experiment which finds that internet businesspeople recognize others tend to be overconfident while being unaware of their own overconfidence.[5]

[5]Ludwig and Nafziger (2011) conduct a lab experiment that elicits participants' beliefs about own and others' overconfidence and abilities. On the one hand they find that the largest group of participants thinks that they are themselves better at judging their ability correctly than others. On the other hand, they find that with a few exceptions, most people believe that others are unbiased.

4 Equilibrium

Any player i chooses the optimal effort level that maximizes his perceived expected utility:

$$E[U_i(a_i, a_{-i}; \lambda)] = P_i(a_i, a_{-i}; \lambda)v - c(a_i) = \frac{\lambda q(a_i)}{\lambda q(a_i) + \sum_{j \neq i} q(a_j)} v - c(a_i).$$

The first-order condition is

$$\frac{\partial E[U_i(a_i, a_{-i}; \lambda)]}{\partial a_i} = \frac{\lambda q'(a_i) \sum_{j \neq i} q(a_j)}{\left[\lambda q(a_i) + \sum_{j \neq i} q(a_j)\right]^2} v - c'(a_i) = 0. \tag{1}$$

The second-order condition is

$$\frac{\partial^2 E[U_i(a_i, a_{-i}; \lambda)]}{\partial a_i^2} = \frac{q''(a_i)[\lambda q(a_i) + \sum_{j \neq i} q(a_j)] - 2\lambda [q'(a_i)]^2}{[\lambda q(a_i) + \sum_{j \neq i} q(a_j)]^3} \lambda \sum_{j \neq i} q(a_j) v - c''(a_i) < 0, \tag{2}$$

and the above inequality is satisfied since $q''(a_i) \leq 0$ and $c''(a_i) \geq 0$.

Let $R_i(Q_{-i})$ denote player i's best response to the aggregate effective effort of the rivals Q_{-i}, where $Q_{-i} = \sum_{j \neq i} q(a_j)$. Accordingly, $R_i(Q_{-i})$ is defined by the value of a_i satisfying (1), or

$$\lambda q'(a_i) Q_{-i} v = c'(a_i) \left[\lambda q(a_i) + Q_{-i}\right]^2. \tag{3}$$

In Lemmas 1-3 in the Appendix we generalize to symmetric n players the Lemmas 4-6 of Santos-Pinto and Sekeris (2025) who focus on contests featuring 2 asymmetric players. In Lemma 1 we show that the best response functions are quasi-concave, Lemma 2 describes how changes in overconfidence shift a player's best response function, and Lemma 3 demonstrates the existence of a unique equilibrium which is symmetric.

To guarantee participation by all n players for any overconfidence parameter λ, we impose the following assumption:

Assumption 1. $\frac{\lambda}{n-1+\lambda} v \geq c(a^{max})$, *where* $\frac{q'(a^{max})}{4q(a^{max})} v = c'(a^{max})$.

This assumption states that even when players expend the highest effort they would be willing to invest in a symmetric contest with overconfident players, their perceived expected utility is positive. Observe that the equivalent condition is always satisfied if players were rational in our setup (see e.g. Nti 1997). With rational players, although the expected share of the prize drops with the number of contestants, their individual equilibrium efforts also contract in such a way that the expected utility remains positive. In our setup, however, overconfident players may have incentives to invest high effort even when the number of players is large, thence implying that a positive perceived expected utility cannot be taken for granted.

We next present our first proposition that uncovers the effect of overconfidence on equilibrium efforts.

Proposition 1. *In a Tullock contest with symmetric overconfident players, individual and aggregate efforts decrease (increase) with overconfidence if* $\lambda > (<) n - 1$.

Proposition 1 establishes that the effect of overconfidence on individual efforts is not the same in a Tullock contest with few versus many players. This result is driven by how overconfidence affects a player's perceived marginal winning probability.

In Figure 1a we depict with the plain curve what a rational player i's winning probability would be, for fixed efforts of the rivals, while the dashed curve represents an overconfident player i's perceived winning probability. In Figure 1b, we accordingly depict the corresponding marginal winning probabilities of the two types of players. As we can see on Figure 1b, the concavity of the perceived winning probability in own effort, a_i, implies that the perceived marginal winning probability of an overconfident player is higher than the one of a rational player for low efforts of player i, and therefore for low winning probabilities of player i, as seen on Figure 1a. In contrast, the perceived marginal winning probability of an overconfident player is lower than the one of a rational player for high efforts.

Consider first a situation where overconfidence is high relative to the number of players, i.e. $\lambda > n - 1$. In such instances, all players expect to be highly likely to win the contest, which implies, as observed on Figure 1b that their perceived marginal winning probability is low. An increase in overconfidence will then reduce players' perceived marginal probability of winning and this incentivizes players to reduce their effort for a given expected (equilibrium) effort of their opponents: the high expected winning probability can now be achieved at lower cost. The exact opposite mechanism is at play when the degree of overconfidence is low compared to the number of players, i.e. $\lambda < n-1$. In such instances, all players expect to have a small probability of winning the contest. In this case, an increase in overconfidence raises the players' perceived marginal probability of winning and this incentivizes them to increase effort.

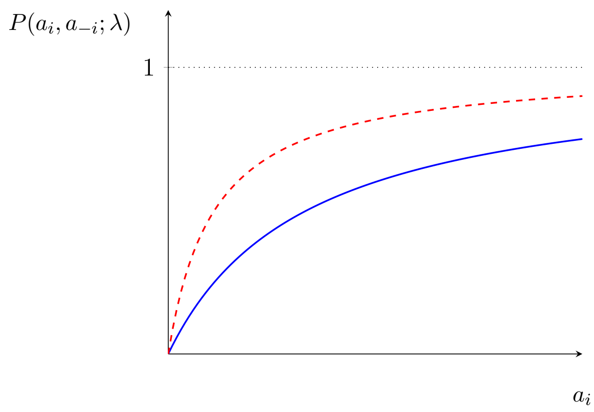
Interestingly, if $n = 2$, the effect of an increase in overconfindence is unambiguous, as it always lowers players' equilibrium efforts. This finding is quite intuitive given that more confident players expect to have a high winning probability for given effort, and one can then expect that they save on cost of effort. However, for $n \geq 3$, we obtain the unexpected result that this is no longer necessarily the case.

The above results allow us to state the following corollary which relates overconfidence to rent dissipation:

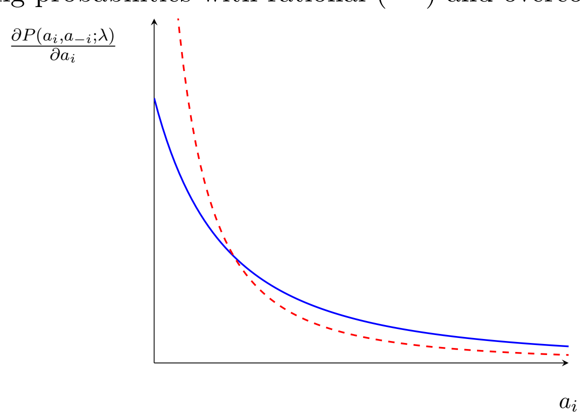
Corollary 1. *In a Tullock contest with symmetric overconfident players, there always exists a finite $n^D(\lambda)$ such that overdissipation (i.e. the sum of players' effort costs is greater than the value of the prize) can be observed at equilibrium for $n > n^D(\lambda) > 2$.*

It is widely known in the literature on contests that with rational players overdissipation can never be observed at equilibrium if the players' valuation of the prize is equal to the actual value of the prize.[6] Although the dissipation ratio, defined as the ratio of total expenditures (or sum of players' effort costs) to the value of the prize, $D = \frac{\sum_i c(a_i)}{v}$, does increase in the number of players, it is bounded by unity because individual equilibrium effort drops as the number of contestants increases. Indeed, a larger number of contestants implies that the competitors' aggregate effort is expected to be higher, thence reducing the marginal return to investing in the contest,

[6]See Dickson et al. (2022) for instances where players' valuation of the prize differs from the actual value of the prize.



(a) Perceived winning probabilities with rational (—) and overconfident (- -) players



(b) Perceived marginal winning probabilities with rational (—) and overconfident (- -) players

which in turn pushes all contestants to individually contract their equilibrium effort. In Proposition 1 we demonstrated, however, that some overconfidence may push players to increase their equilibrium effort compared to a setup with fully rational players. Corollary 1 shows that there always exists a degree of overconfidence such that equilibrium individual efforts of overconfident players will equal the maximal equilibrium individual efforts that can be obtained in the game, i.e., the individual efforts produced in setups with fully rational players. Consequently, with sufficiently many overconfident players the aggregate effort can be higher than the value of the contested prize. Observe that for a contest organizer aiming at maximizing players' efforts—a question of relevance elsewhere in the literature (e.g. Konrad 2009, Beviá and Corchón 2024)—it is optimal that players be subject to some overconfidence bias if $n > 2$.

To visualize these two results, in Figure 2 we depict the individual equilibrium effort of (symmetric) contestants as a function of their overconfidence parameter in the simplest contest where players' payoffs are given by:

$$E[U_i(a_i, a_{-i}; \lambda)] = \frac{\lambda a_i}{\lambda a_i + \sum_{j \neq i} a_j} - a_i, \tag{4}$$

With $n = 2$ and $\lambda = 1$, the equilibrium efforts are equal to $1/4$. If we consider contests with more players, the individual efforts can be kept equal to $1/4$ if $\lambda = n-1$. Consequently, under such circumstances, full dissipation can result with $n = 4$ and $\lambda = 3$, and overdissipation can therefore obtain for any $n > 4$.

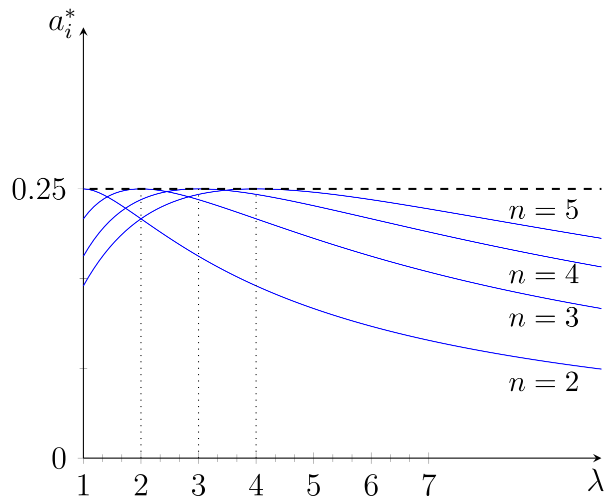


Figure 2: Individual equilibrium efforts as a function of λ with $q(a_i) = a_i$, $v = 1$ and $c(a_i) = a_i$.

It is important at this stage to underline that although for overdissipation to be observed it is necessary to have $n > n^D(\lambda) > 2$ players, the required degree of overconfidence may be quite low. Indeed, to visualize this we consider again the previous specific contest setup. One can easily derive $a^* = \frac{\lambda(n-1)}{(\lambda+n-1)^2}$. Observe then that the maximal individual effort players invest at equilibrium, a^{max}, is reached for $\lambda = n-1$. Accordingly, the threshold $n^D(\lambda)$ is obtained after solving for $min_n \left\{ n : n\frac{\lambda(n-1)}{(\lambda+n-1)^2} = 1 \right\}$ given that $\lambda = n-1$, the solution to which yields $n^D(\lambda) = 4$. To visualize the effect of overconfidence on rent dissipation, we plot on Figure 3 the aggregate effort na^* as a function of the number of players for different levels of overconfidence, λ. It is well known that, with rational players ($\lambda = 1$), as n becomes arbitrarily large the dissipation ratio converges to unity ($na^* \to 1$), without ever reaching total rent dissipation. We know from Corollary 1 that for any number of players $n > n^D(\lambda) = 4$, there always exists a degree of overconfidence conducive to overdissipation. Since overdissipation will be observed for $na^* > 1$, setting $na^* = 1$, substituting for a^* and deriving the roots for λ, we obtain:

$$\lambda_1 = \frac{(n-1)\left[(n-2) - \sqrt{n(n-4)}\right]}{2},$$

and,

$$\lambda_2 = \frac{(n-1)\left[(n-2) + \sqrt{n(n-4)}\right]}{2}.$$

Overdissipation will then occur for any $\lambda \in]\lambda_1, \lambda_2[$, while rent dissipation is maximized for $\lambda = n-1$.

With $n = 5$ full dissipation is already observed when $\lambda \approx 1.528$, which corresponds to a perceived winning probability of 0.27 as opposed to the actual winning probability of 0.2. Increasing overconfidence to $\lambda = 3$, the number of players resulting in full dissipation drops to 4. It is immediate to observe that as the number of players becomes arbitrarily large in this setup, the required degree of overconfidence for observing overdissipation will become arbitrarily small (i.e. λ close to 1).

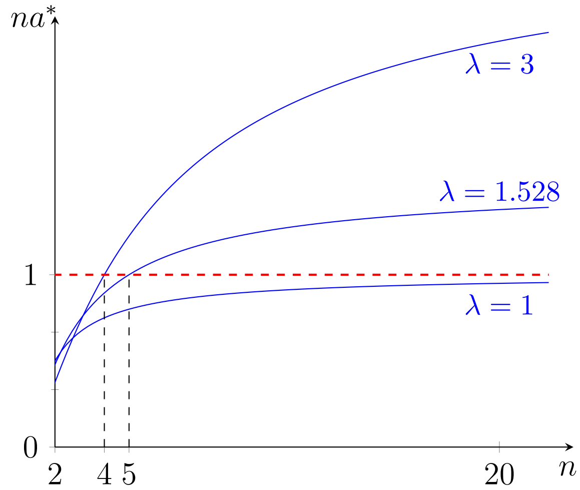


Figure 3: Equilibrium aggregate effort na^* as a function of n.

5 Entry

We now study the effect of overconfidence on entry in symmetric Tullock contests. The analysis so far assumes that players' outside option is zero. However, if the outside option is high enough, it is possible that the perceived expected utility of participating in the contest is too low to make entry attractive. To analyze how confidence affects entry, we assume there exist $N \geq 2$ symmetric potential entrants, and designate by n the number of players that enter the contest. Moreover, all potential entrants have an outside option equal to $\bar{v} < v$. This assumption guarantees that there is an incentive for at least one player to enter the contest. Further, we focus on pure strategy subgame perfect equilibria and on instances where at least two players have incentives to enter the contest. At the symmetric equilibrium, the equilibrium number of entrants, n^*, satisfies the equation:

$$\frac{\lambda}{\lambda + n^* - 1} v - c(a^*) = \bar{v} \tag{5}$$

Our last result describes how overconfidence affects n^*.

Proposition 2. *In a Tullock contest with a pool of $N \geq 2$ symmetric potential entrants, the equilibrium number of entrants n^* increases in overconfidence λ.*

An increase in overconfidence affects the incentives to enter the contest in two ways. First, it increases the players' perceived probability of winning for given efforts, which makes entry more attractive. Second, we know from Proposition 1 that for a fixed number of entrants, an increase in overconfidence raises (lowers) equilibrium individual efforts for $\lambda < (>) n - 1$, which makes entry less (more) attractive. Consequently, for high values of λ (higher than $n - 1$), an increase in overconfidence unambiguously makes entry more attractive. However, for low values of λ, the two effects go in opposite directions. Proposition 2 shows that the former effect always dominates the latter.

6 Extensions

In this section we discuss two extensions of the model.

6.1 Heterogeneity in Confidence

So far we have considered a symmetric setup with overconfident players. We now extend the model by assuming that $n_r \geq 1$ players are rational and $n_o \geq 1$ are overconfident, with $n = n_r + n_o$.

Any rational player i chooses the effort a_i that maximizes his expected utility as given by:

$$E\left[U(a_i, a_{-i})\right] = \frac{q(a_i)}{q(a_i) + \sum_{j \neq i} q(a_j)} v - c(a_i).$$

Likewise, an overconfident player i, maximizes his perceived expected utility:

$$E\left[U(a_i, a_{-i}; \lambda)\right] = \frac{\lambda q(a_i)}{\lambda q(a_i) + \sum_{j \neq i} q(a_j)} v - c(a_i).$$

Optimizing these expressions, we can derive the next result:

Proposition 3. *For any Tullock contest with* $n \geq 3$ *players,* $n_r \geq 1$ *rational players, and* $n_o \geq 1$ *overconfident players, there exists a unique overconfidence threshold* $\bar{\lambda} = (n-1)^2$ *such that* $\lambda \lesseqgtr \bar{\lambda} \Leftrightarrow a_o^* \gtreqless a_r^*$.

This proposition tells us that overconfident players will exert higher effort at equilibrium than rational ones provided that their overconfidence bias is not too large, but excessive bias ultimately reverses relative equilibrium efforts. The intuition underlying this result is similar to the one behind Proposition 1. Indeed, high levels of overconfidence incentivize players to slack, while with low levels of overconfidence, as players expect to have a small probability of winning the contest, their perceived marginal probability of winning is accordingly higher due to their bias.

In Proposition 3 we show that overconfident players exert higher effort than rational ones for $\lambda < (n-1)^2$. Since efforts are strategic substitutes in such situations for rational players, this implies that as overconfident players become more aggressive, rational players increasingly reduce their effort, and may eventually have incentive not to participate to the contest. The next proposition clarifies under which conditions this will happen:

Proposition 4. *Rational players decide not to participate to the contest if*

$$\frac{\lambda n_o(n_o-1)}{(\lambda+n_o-1)^2} > \frac{q'(0)}{c'(0)}\frac{c'(a_o^*)}{q'(a_o^*)},$$

where a_o^* *is an overconfident player's equilibrium effort when rational players do not participate to the contest. This inequality is easier to satisfy the larger is* n_o*, the lower is* v*, while the effect of* λ *is ambiguous.*

It follows from Proposition 4 that, all else equal, a larger number of overconfident players makes the contest less attractive for rational players. This effect is entirely driven by the fact that with more overconfident players in the game, the aggregate effort of overconfident contestants, $n_o a_o^*$, will be higher, which in turn reduces the incentives of the rational players to invest effort in the contest.

Next, although a lower prize, v, pushes downward the equilibrium efforts of overconfident contestants, a_o^*, the contest is also less attractive to rational players who have then less incentives to invest effort in the contest.

Last, the effect of an increase in overconfidence is ambiguous. To gain some intuition on the effect of overconfidence on rational players' participation to the contest, we next consider the simplest contest with unit marginal cost of effort as in Equation (4). In such instances, $q'(0) = q'(a_o^*) = c'(0) = c'(a_o*) = 1$, and thus the right-hand side of the inequality in Proposition 4 is equal to 1.

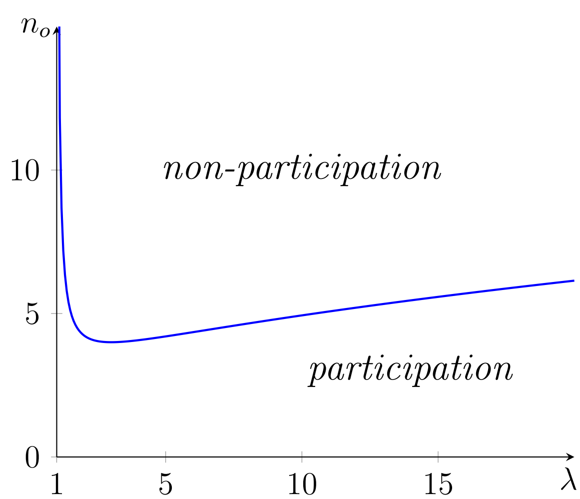


Figure 4: Equilibrium participation of rational players.

Figure 4 depicts the combination of values of overconfidence, on the x-axis, and number of overconfident players, on the y-axis, inducing rational players to participate or not to the contest. Since we consider the simplest contest, the inequality in Proposition 4 is straightforwardly shown to be true if $n_o > \frac{(3\lambda-2)+\lambda\sqrt{4\lambda-3}}{2(\lambda-1)}$.

In situations where there are few overconfident players, rational players always find it optimal participate to the contest. For larger number of overconfident players, however, their overconfidence level crucially impacts rational players' participation decision. Indeed, for very low levels of overconfidence, rational and overconfident players behave almost alike, and all players have incentives to participate to the contest. For intermediate levels of overconfidence, overconfident players invest overly high effort levels, and this makes participation unattractive to rational players. Finally, for high levels of overconfidence, overconfident players slack, and this makes participation attractive to rational players.

6.2 Heterogeneity in Ability and Confidence

So far we have assumed that all players have the same ability. Indeed for any effort a_i by player i, his effective effort is given by the impact function $q(a_i)$. To introduce heterogeneity in abilities, we assume that the effective effort of player i is given by $\theta_i q(a_i)$, where $\theta_i > 0$. We focus on the case where the population is composed of low and high ability players, denoted, respectively, by θ_L and θ_H, with $\theta_L < \theta_H$. Low ability players, L, are overconfident and believe that they have high ability θ_H, whereas high ability players are rational and correctly evaluate their ability θ_H. We denote the number of low-ability overconfident players by $n_L \geq 1$ and the number of high-ability rational players by $n_H \geq 2$, so that $n = n_L + n_H \geq 3$.[7]

Given this setup, the expected utility of a rational player i is:

$$E\left[U_i^H(a_i, a_{-i})\right] = \frac{\theta_H q(a_i)}{\theta_H q(a_i) + \sum_{j=1, j\neq i}^{n_H} \theta_H q(a_j) + \sum_{k=1}^{n_L} \theta_L q(a_k)} v - c(a_i).$$

The expected utility of an overconfident player i is:

$$E\left[U_i^L(a_i, a_{-i})\right] = \frac{\theta_H q(a_i)}{\theta_H q(a_i) + \sum_{j=1}^{n_H} \theta_H q(a_j) + \sum_{k=1, k\neq i}^{n_L} \theta_L q(a_k)} v - c(a_i).$$

Optimizing these expressions, we can derive our last result:

Proposition 5. *In any Tullock contest featuring $n_L \geq 1$ low-ability overconfident players who mistakenly believe to be of high ability, and $n_H \geq 2$ high-ability rational players who correctly evaluate their ability, overconfident players' equilibrium effort is lower than rational players' equilibrium effort, $a_L^* < a_H^*$. Consequently overconfident players always have a lower equilibrium probability of winning the contest.*

[7]The particular case where $n_L = n_H = 1$ is studied in Santos-Pinto and Sekeris (2025).

Unlike our earlier analysis where overconfident players mistakenly believed to have a higher ability than their rivals, we are here imposing that the overconfident players believe that they have the same ability to their rational rivals, while their true ability is lower. This result is rooted in the asymmetry of players' abilities. If all players were rational, low ability players would be investing lower effort at equilibrium. Overconfident players increase their investments compared to what they would have invested if they were rational, yet this increase does not totally close the gap separating their investments from those of rational high ability players.

7 Conclusion

This paper studies the impact of overconfidence on Tullock contests. We assume an overconfident player overestimates the impact of his effort on the outcome of the contest while holding a correct assessment of the winning prize and his cost of effort. We demonstrate that in a symmetric $n > 2$ player contest, an increase in overconfidence increases the efforts of all players provided that the bias is small relative to the number of players. With sufficiently high levels of overconfidence, on the other hand, an increase in overconfidence will lead to lower equilibrium efforts. Our paper also provides conditions under which overspending and even overdissipation can result from overconfidence. Finally, we show that higher overconfidence always results in more entry at equilibrium.

Over the past years, Tullock contests have been extensively studied in laboratory experiments (e.g. Dechenaux et al. 2015). Our novel results highlight the importance of accounting for players' overconfidence when drawing predictions about behavior in Tullock contests. Our findings can be tested in a controlled laboratory experiment where self-confidence biases as well as the number of players can both be exogenously manipulated. We leave that for future research.

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8 Appendix

Lemma 1. *$R_i(Q_{-i})$ is quasi-concave in Q_{-i} and reaches a maximum for $Q_{-i} = \lambda q(a_i)$.*

Proof of Lemma 1: The best response of player i, is defined implicitly by (3). Hence, the slope of the best response of player i, $R_i'(Q_{-i})$ is given by

$$-\frac{\partial R_i/\partial Q_{-i}}{\partial R_i/\partial a_i} = -\frac{\frac{\partial^2 E[U_i]}{\partial a_i \partial Q_{-i}}}{\frac{\partial^2 E[U_i]}{\partial a_i^2}} = -\frac{\frac{\lambda q(a_i)-Q_{-i}}{[\lambda q(a_i)+Q_{-i}]^3}\lambda q'(a_i)v}{\frac{q''(a_i)[\lambda q(a_i)+Q_{-i}]-2\lambda[q'(a_i)]^2}{[\lambda q(a_i)+Q_{-i}]^3}\lambda Q_{-i}v - c''(a_i)}. \qquad (6)$$

The denominator is negative because player i's second-order condition is satisfied. Therefore, the sign of the slope of player i's best response is only determined by the sign of the numerator which only depends on $\lambda q(a_i) - Q_{-i}$. Hence, $R_i'(Q_{-i})$ is positive for $\lambda q(a_i) > Q_{-i}$, zero for $\lambda q(a_i) = Q_{-i}$, and negative for $\lambda q(a_i) < Q_{-i}$. This implies that $R_i(Q_{-i})$ increases in Q_{-i} for $\lambda q(a_i) > Q_{-i}$, reaches the maximum at $\lambda q(a_i) = Q_{-i}$, and decreases in Q_{-i} for $\lambda q(a_i) < Q_{-i}$.

Lemma 2. *An increase in λ leads to a contraction of player i's best response function, $\frac{\partial R_i(Q_{-i})}{\partial \lambda} < 0$, for $Q_{-i} < \lambda q(a_i)$ and to an expansion of his best response function, $\frac{\partial R_i(Q_{-i})}{\partial \lambda} > 0$, for $Q_{-i} > \lambda q(a_i)$. Moreover, the maximum value of a player's best response, a^{max} implicitly defined by $\frac{q'(a^{max})}{4q(a^{max})}v = c'(a^{max})$, is independent of λ.*

Proof of Lemma 2: (This proof follows Baik 1994) Player i's best response is implicitly defined by:

$$\frac{\lambda q'(a_i)Q_{-i}}{[\lambda q(a_i) + Q_{-i}]^2}v - c'(a_i) = 0.$$

Hence, we have

$$\frac{\partial R_i(Q_{-i})}{\partial \lambda} = \frac{Q_{-i} - \lambda q(a_i)}{[\lambda q(a_i) + Q_{-i}]^3}q'(a_i)Q_{-i}v.$$

We see that $\partial R_i(Q_{-i})/\partial \lambda \gtreqless 0$ for $Q_{-i} \gtreqless \lambda q(a_i)$. We also know from Lemma 1 that $sign\{R_i'(Q_{-i})\} = -sign\left\{\frac{\partial R_i(Q_{-i})}{\partial \lambda}\right\}$.

Substituting next $Q_{-i} = \lambda q(a_i)$ into the first-order condition of player i and denoting the maximal effort he is willing to invest in the contest by a^{max} we obtain

$$\frac{\lambda q'(a^{max})\lambda q(a^{max})}{\left[\lambda q(a^{max}) + \lambda q(a^{max})\right]^2} v = c'(a^{max}),$$

or

$$\frac{\lambda^2 q'(a^{max}) q(a^{max})}{4\lambda^2 \left[q(a^{max})\right]^2} v = c'(a^{max}),$$

or

$$\frac{q'(a^{max})}{4q(a^{max})} v = c'(a^{max}). \tag{7}$$

This implies that the value of a_i corresponding to the maximum value of the player's best response, a^{max}, does not depend on λ.

Lemma 3. *A Tullock contest featuring n overconfident players willing to participate in the contest admits a unique equilibrium which is symmetric.*

Proof of Lemma 3: In what follows we first demonstrate existence of an equilibrium without imposing symmetry, and then prove that any equilibrium in this setup is symmetric and unique.

Assuming for the time being that participation in the contest is guaranteed, we can establish equilibrium uniqueness. To prove that the equilibrium is unique, and bearing in mind that $q(a_i)$ is monotonically increasing in a_i, we can rewrite the optimization problem as a function $q_i = q(a_i)$, so that $a_i = a^{-1}(q_i)$, derive the equilibrium value of q_i, and deduce the equilibrium value of a_i. If there is a unique equilibrium in the space $(q_1, q_2, \ldots, q_n)$, then there is a unique equilibrium in the space $(a_1, a_2, \ldots, a_n)$.

$$\max_{q_i} \frac{\lambda q_i}{\lambda q_i + Q_{-i}} v - \phi(q_i),$$

where $\phi(q_i) = c(a^{-1}(q_i))$. Accordingly, $\phi'(q_i) > 0$, and since $a^{-1''}(q_i) > 0$, it is immediate to deduce that $\phi''(q_i) > 0$.

Optimizing, we obtain:

$$\frac{\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^2} v - \phi'(q_i) = 0,$$

and this expression implicitly defines the best response of player i, $R_i(Q_{-i})$.

To prove that the equilibrium is unique it is then sufficient to show that the product of the slopes of the best response functions is less than 1: $\Gamma = R_1'(Q_{-1}) \circ R_2'(Q_{-2}) \ldots \circ R_n'(Q_{-n}) < 1$. We first derive the slope of the best response of player i:

$$R_i'(Q_{-i}) = -\frac{\frac{\lambda(\lambda q_i + Q_{-i}) - 2\lambda Q_i}{(\lambda q_i + Q_{-i})^3} v}{-\frac{2\lambda^2 Q_{-i}}{(\lambda q_i + Q_{-i})^3} v - \phi''(q_i)} = \frac{\frac{\lambda q_i - Q_i}{(\lambda q_i + Q_{-i})^3} v}{\frac{2\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^3} v + \frac{\phi''(q_i)}{\lambda}}.$$

Observe that if for an odd number of players $R_i'(Q_{-i}) < 0$, and that for the remaining players the best responses are positively slopped, then we necessarily deduce that $\Gamma < 1$. Second, if for an even number of players $R_i'(Q_{-i}) < 0$, and that for the remaining players the best responses are positively slopped, then we wish to prove that:

$$\Pi_{i=1}^n \left\{ \frac{\frac{\lambda q_i - Q_i}{(\lambda q_i + Q_{-i})^3} v}{\frac{2\lambda Q_{-i}}{(\lambda q_i + Q_{-i})^3} v + \frac{\phi''(q_i)}{\lambda}} \right\} < 1.$$

Since $R_i^{'}(Q_{-i})$ is decreasing in $\phi''(q_i)$, it is thus sufficient to establish the result for $\phi''(q_i) = 0$. Rewriting the product of the contestants' best responses with this restriction, and simplifying, we thus want to show that:

$$\Pi_{i=1}^n \left\{ \frac{\lambda q_i - Q_{-i}}{2\lambda Q_{-i}} \right\} < 1.$$

If the above expression is true when setting Q_{-i} equal to zero in the numerator, then the expression is necessarily true for any Q_{-i}. Setting therefore Q_{-i} in the numerator equal to zero, the expression is always verified since $\Pi_{i=1}^n q_i < 2\Pi_{i=1}^n Q_{-i}$. This completes the proof of existence.

To demonstrate that the equilibrium is symmetric, we proceed by contradiction. Assume the equilibrium is asymmetric, then there are at least two players, 1 and 2 such that $q_1 \neq q_2$. Denote by $\tilde{Q}$ the aggregate effective effort of the remaining players. Since the first-order condition for both players need to be satisfied, we deduce that at equilibrium we have:

$$\frac{\lambda(q_2 + \tilde{Q})}{(\lambda q_1 + q_2 + \tilde{Q})^2} \frac{1}{\phi'(q_1)} = \frac{\lambda(q_1 + \tilde{Q})}{(\lambda q_2 + q_1 + \tilde{Q})^2} \frac{1}{\phi'(q_2)},$$

or,

$$\frac{(\lambda q_2 + q_1 + \tilde{Q})^2}{(\lambda q_1 + q_2 + \tilde{Q})^2} = \frac{(q_1 + \tilde{Q})}{(q_2 + \tilde{Q})} \frac{\phi'(q_1)}{\phi'(q_2)}$$

Assume with loss of generality that $q_1 > q_2$. Then since $\lambda > 1$, the LHS is smaller than 1, thence implying that we require

$$(q_1 + \tilde{Q})\phi'(q_1) < (q_2 + \tilde{Q})\phi'(q_2)$$

But since $\phi'(q)$ is increasing in q, this is impossible, therefore implying that any equilibrium is symmetric.

Last, to establish uniqueness, denote any symmetric equilibrium efforts by q^*. The first order condition of any player is then equivalent to:

$$\frac{\lambda(n-1)}{(\lambda + n - 1)^2} v - q^* \phi'(q^*) = 0.$$

And since $\phi'(q^*)$ is increasing in q^* there can be only a single such value.

Proof of Proposition 1: At the unique symmetric equilibrium the first-order condition (1) reads as:

$$\frac{\lambda q'(a^*)(n-1)q(a^*)}{[\lambda q(a^*) + (n-1)q(a^*)]^2} v - c'(a^*) = 0,$$

or

$$\frac{\lambda(n-1)q'(a^*)}{(\lambda + n - 1)^2 q(a^*)} v - c'(a^*) = 0. \tag{8}$$

To inspect the sign of $\partial a^* / \partial \lambda$ we apply the implicit function theorem to the above expression to obtain:

$$\begin{aligned} \frac{\partial a^*}{\partial \lambda} &= -\frac{\frac{(n-1)(\lambda+n-1)^2 - 2(\lambda+n-1)\lambda(n-1)}{(\lambda+n-1)^4} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)} \\ &= -\frac{\frac{(n-1)(n-1-\lambda)}{(\lambda+n-1)^3} v \frac{q'(a^*)}{q(a^*)}}{\frac{\lambda(n-1)}{(\lambda+n-1)^2} v \frac{q''(a^*)q(a^*) - [q'(a^*)]^2}{q^2(a^*)} - c''(a^*)}. \end{aligned}$$

Since the denominator of this expression is unambiguously negative, the sign of the expression is therefore given by the sign of $(n-1-\lambda)$.

Last, we need to guarantee that all n players are willing to participate in the contest. The perceived expected utility of any contestant i is given by:

$$E[U(a_i, a_{-i}; \lambda)] = \frac{\lambda q(a_i)}{\lambda q(a_i) + \sum_{j \neq i} q(a_j)} v - c(a_i).$$

Therefore, an increase in λ changes the perceived expected utility as follows:

$$\frac{dE[U(a_i, a_{-i}; \lambda)]}{d\lambda} = \frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_i} \frac{da_i}{d\lambda} + \sum_{j \neq i} \frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_j} \frac{da_j}{d\lambda}.$$

By the Envelope theorem, we know that the first term of the above expression is nil. Consequently, and since $\frac{\partial E[U(a_i, a_{-i}; \lambda)]}{a_j} < 0$, $\forall j \neq i$, at equilibrium, the sign of $\frac{dE[U(a_i, a_{-i}; \lambda)]}{d\lambda}$ is given by the sign of $\frac{da^*}{d\lambda}$, which has been shown to be given by the sign of $n-1-\lambda$. To ensure participation of all n players, we then require that the following holds:

$$E[U(a_i^*, a_{-i}^*; \lambda)] = \frac{\lambda}{\lambda + n - 1} v - c(a^*) \geq 0.$$

Since the maximal value of a^* has been proven to equal a^{max}, participation by all n contestants is always guaranteed by Assumption 1.

Proof of Corollary 1: At the unique symmetric equilibrium, players' equilibrium effort is given by equation (8). We know that the value of the maximal equilibrium effort is defined by a^{max} as implicitly defined by (7), and that when $\lambda = n-1$, then $a^* = a^{max}$. Accordingly, for any n, there exist a $\lambda = n-1$ such that the dissipation ratio is given by:

$$D^{max} = \frac{nc(a^{max})}{v},$$

and this characterizes the highest possible dissipation ratio. Since a^{max} is independent of n, it follows that for $n = 2$, $D^{max} < 1$, and that D^{max} is monotonically increasing in n, there must always exist a $n^D(\lambda) > 2$ such that $D^{max} > 1$.

Proof of Proposition 2: We can re-write equation (5) as:

$$\psi = \frac{\lambda}{\lambda + n^* - 1} v - c(a^*) - \bar{v} = 0.$$

Consequently, the effect of overconfidence on the number of entrants is given by:

$$\frac{dn^*}{d\lambda} = -\frac{\frac{\partial \psi}{\partial \lambda}}{\frac{\partial \psi}{\partial n^*}} = -\frac{\frac{(n^*-1)}{(\lambda+n^*-1)^2}v - c'(a^*)\frac{\partial a^*}{\partial \lambda}}{-\frac{\lambda}{(\lambda+n^*-1)^2}v - c'(a^*)\frac{\partial a^*}{\partial n^*}}.$$

We can separately compute the following two expressions:

$$\partial a^*/\partial \lambda = \frac{\frac{[n-1][\lambda-n+1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}},$$

and,

$$\partial a^*/\partial n = -\frac{\frac{\lambda[\lambda-n+1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}},$$

where $\psi = \frac{\lambda(n-1)q'(a^*)}{(\lambda+n-1)^2 q(a^*)}v - c'(a^*) = 0$ as given in equation (8).

Substituting these two expressions in $dn^*/d\lambda$, we obtain:

$$\frac{dn^*}{d\lambda} = -\frac{(n-1)\left[\frac{1}{(\lambda+n^*-1)^2}v + c'(a^*)\frac{\frac{[-\lambda+n-1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}}\right]}{\lambda\left[-\frac{1}{(\lambda+n^*-1)^2}v - c'(a^*)\frac{\frac{[-\lambda+n-1]q'(a^*)v}{[\lambda+n-1]^3 q(a^*)}}{\frac{\partial \psi}{\partial a^*}}\right]} = \frac{n-1}{\lambda}.$$

Proof of Proposition 3: The first-order condition for the rational player i is:

$$\frac{q'(a_i)\sum_{j\neq i} q(a_j)}{\left(q(a_i) + \sum_{j\neq i} q(a_j)\right)^2}v - c'(a_i) = 0.$$

Proceeding likewise for an overconfident player i, we obtain:

$$\frac{q'(a_i)\lambda\sum_{j\neq i} q(a_j)}{\left(\lambda q(a_i) + \sum_{j\neq i} q(a_j)\right)^2}v - c'(a_i) = 0.$$

Considering the symmetric equilibrium, where all rational players exert the same effort a_r, and all overconfident players exert the same effort a_o, these two conditions now read as:

$$\frac{q'(a_r)[(n_r-1)q(a_r) + n_o q(a_o)]}{(n_r q(a_r) + n_o q(a_o))^2}v - c'(a_r) = 0, \quad (9)$$

and,

$$\frac{q'(a_o)\lambda\left[(n_r q(a_r) + (n_o - 1)q(a_o)\right]}{\left(\lambda q(a_o) + (n_o - 1)q(a_o) + n_r q(a_r)\right)^2} v - c'(a_o) = 0. \tag{10}$$

Assume that the equilibrium is such that $a_r^* = a_o^* = a^*$. For this to be the case, it is then necessary that, combining the above two conditions, we have:

$$\frac{q'(a^*)[(n_r - 1)q(a^*) + n_o q(a^*)]}{\left(n_r q(a^*) + n_o q(a^*)\right)^2} v = \frac{q'(a^*)\lambda(n_r q(a^*) + (n_o - 1)q(a^*)}{\left(\lambda q(a^*) + (n_o - 1)q(a^*) + n_r q(a^*)\right)^2} v,$$

or,

$$\frac{1}{n^2} = \frac{\lambda}{(\lambda + n - 1)^2}.$$

Solving for λ, we deduce that at equilibrium both types of players invest the same effort if either $\lambda = 1$ or $\lambda = (n-1)^2$. Having imposed that $\lambda > 1$, we define $\bar{\lambda} = (n-1)^2$.

Let next $R_r(a_o)$ denote the rational type-symmetric reaction function, i.e. the optimal choice of a rational player when all rational players are restricted to choose the same action a_r and all overconfident players choose a_o. Likewise, we also define $R_o(a_r)$. To complete the proof, we need to show that $R_r'(a_o) < 0$ when evaluated at $a_r = a_o = a$, and, observing that $R_r(a_o)$ is independent of λ, that $\partial R_o(a_r)/\partial\lambda < 0$ when $\lambda = \bar{\lambda}$. This in turn implies that for any $\lambda < \bar{\lambda}$, $a_o^* > a_r^*$, and that for any $\lambda > \bar{\lambda}$, $a_o^* < a_r^*$.

Inspecting first the sign of $R_r'(a_o)$, by implicit differentiation of (9), we have:

$$R_r'(a_o) = -\frac{\frac{q'(a_r)(n_o q'(a_o)(n_r q(a_r) + n_o q(a_o)) - 2n_o q'(a_o)((n_r - 1)q(a_r) + n_o q(a_o)))}{\left(n_r q(a_r) + n_o q(a_o)\right)^3}}{\frac{(q''(a_r)((n_r - 1)q(a_r) + n_o q(a_o)) + (q'(a_r))^2(n_r - 1))(n_r q(a_r) + n_o q(a_o)) - 2n_r(q'(a_r))^2((n_r - 1)q(a_r) + n_o q(a_o))}{\left(n_r q(a_r) + n_o q(a_o)\right)^3}}.$$

Observe that the denominator of this expression is negative since $q''(a_r) \leq 0$, and:

$$(n_r - 1)(n_r q(a_r) + n_o q(a_o)) - 2n_r((n_r - 1)q(a_r) + n_o q(a_o)) < 0,$$

since,

$$-(n_r - 1)n_r q(a_r) - n_o(n_r + 1)q(a_o) < 0.$$

We thus deduce that $R_r'(a_o) < 0$ if:

$$(n_o(n_r q(a_r) + n_o q(a_o)) - 2n_o((n_r - 1)q(a_r) + n_o q(a_o))) < 0.$$

Evaluating this term at $a_r = a_o = a$, we have:

$$q(a)\left(n_o(n_r + n_o) - 2n_o((n_r - 1) + n_o)\right) < 0.$$

and this expression is always true since it reads as $-n_0(n-2) < 0$.

Second, by implicit differentiation of (10), we have that $\frac{\partial R_o(a_r)}{\partial \lambda}$ equals:

$$\frac{\frac{q'(a_o)[(n_r q(a_r)+(n_o-1)q(a_o)][\lambda q(a_o)+(n_o-1)q(a_o)+n_r q(a_r)]-2q(a_o)q'(a_o)\lambda[n_r q(a_r)+(n_o-1)q(a_r)]}{(\lambda q(a_o)+(n_o-1)q(a_o)+n_r q(a_r))^3}}{\frac{[q''(a_o)\lambda[n_r q(a_r)+(n_o-1)q(a_o)]+(q'(a_o))^2\lambda(n_o-1)][\lambda q(a_o)+(n_o-1)q(a_o)+n_r q(a_r)]-2(\lambda+n_o-1)(q'(a_o)^2\lambda[n_r q(a_r)+(n_o-1)q(a_r)]}{(\lambda q(a_o)+(n_o-1)q(a_o)+n_r q(a_r))^3}}$$

As above, the sign of the denominator is negative since $q''(a_o) \leq 0$. Therefore the sign of $\partial R_o(a_r)/\partial\lambda$ at $\lambda = \bar{\lambda}$, which implies $a_r = a_o = a$, is negative if:

$$q'(a)q(a)^2(n-1)\left(n - 1 - \bar{\lambda}\right) < 0.$$

And this is true since $\bar{\lambda} = (n-1)^2$.

Proof of Proposition 4: For a rational player to invest zero effort at equilibrium when overconfident players exert effort a_o, it must be the case that the first order condition of any rational player is satisfied when $a_r^* = 0$, or,

$$\frac{q'(0)[(n_r - 1)q(0) + n_o q(a_o)]}{(n_r q(0) + n_o q(a_o))^2} v - c'(0) = 0.$$

Recalling that $q(0) = 0$, and solving this equation for $q(a_o)$ we obtain:

$$q(a_o) = \frac{q'(0)}{c'(0)} \frac{v}{n_o}. \tag{11}$$

Moreover, for any larger value of a_o, rational players do not participate to the contest.

If rational players do not participate to the contest, the first order condition of overconfident players after imposing symmetry is given by:

$$\frac{\lambda q'(a_o^*)(n_o - 1)}{(\lambda + n_o - 1)^2 q(a_o^*)} v = c'(a_o^*),$$

or,

$$q(a_o^*) = \frac{n_o - 1}{(\lambda + n_o - 1)^2} \frac{q'(a_o^*)}{c'(a_o^*)} v\lambda. \tag{12}$$

We thus conclude that when $a_o = a_o^*$, rational players refrain from participating to the contest if:

$$\frac{n_o - 1}{(\lambda + n_o - 1)^2} \frac{q'(a_o^*)}{c'(a_o^*)} v\lambda > \frac{q'(0)}{c'(0)} \frac{v}{n_o},$$

or,

$$\frac{\lambda n_o(n_o - 1)}{(\lambda + n_o - 1)^2} > \frac{q'(0)}{c'(0)} \frac{c'(a_o^*)}{q'(a_o^*)},$$

and this expression is true for any $\lambda > 1$.

Observe that from (12), we immediately deduce that $\partial a_o^* / \partial n_o < 0$, which implies that the right-hand side of the inequality is decreasing in n_0 since $c''() \geq 0$ and $q''() \leq 0$. Second, the left-hand side of the inequality is increasing in n_o since:

$$(2n_o - 1)(\lambda + n_o - 1) - 2n_o(n_o - 1) > 0,$$

or,

$$\lambda(2n_o - 1) - (n_o - 1) > 0,$$

Increasing next the value of v, it is immediate to observe that $\partial a_o^* / \partial v > 0$, thence implying that the right-hand side of the inequality is increasing.

Last, we know from Proposition 1 that the effect of λ on a_o^* is positive when $\lambda < n_o - 1$. Therefore, for $\lambda < n_o - 1$, the right-hand side of the above inequality is increasing with λ. Second, the left-hand side of the inequality is increasing in λ if:

$$(\lambda + n_o - 1) - 2\lambda > 0 \Leftrightarrow n_o - 1 > \lambda.$$

Hence an increase in λ has an ambiguous effect on the sign of the inequality.

Proof of Proposition 5: The first-order condition for a rational player i is:

$$\frac{\theta_H q'(a_i) \left[\sum_{j=1, j\neq i}^{n_H} \theta_H q(a_j) + \sum_{k=1}^{n_L} \theta_L q(a_k)\right]}{\left[\theta_H q(a_i) + \sum_{j=1, j\neq i}^{n_H} \theta_H q(a_j) + \sum_{k=1}^{n_L} \theta_L q(a_k)\right]^2} v - c'(a_i) = 0.$$

Likewise the first-order condition for an overconfident player i is:

$$\frac{\theta_H q'(a_i)\left[\sum_{j=1}^{n_H}\theta_H q(a_j)+\sum_{k=1,k\neq i}^{n_L}\theta_L q(a_k)\right]}{\left[\theta_H q(a_i)+\sum_{j=1}^{n_H}\theta_H q(a_j)+\sum_{k=1,k\neq i}^{n_L}\theta_L q(a_k)\right]^2}v-c'(a_i)=0.$$

Considering the symmetric case where all rational players invest the same effort a_H, and all overconfident players invest the same effort a_L, the above two conditions become:

$$\frac{\theta_H q'(a_H)\left[(n_H-1)\theta_H q(a_H)+n_L\theta_L q(a_L)\right]}{\left[n_H\theta_H q(a_H)+n_L\theta_L q(a_L)\right]^2}=c'(a_H)/v,$$

and,

$$\frac{\theta_H q'(a_L)\left[n_H\theta_H q(a_H)+(n_L-1)\theta_L q(a_L)\right]}{\left[(\theta_H-\theta_L)q(a_L)+n_H\theta_H q(a_H)+n_L\theta_L q(a_L)\right]^2}=c'(a_L)/v.$$

Let next $R_H(a_L)$ denote the rational type-symmetric reaction function, i.e. the optimal choice of a rational player when all rational players are restricted to choose the same action a_H and all overconfident players choose a_L. Likewise, we also define $R_L(a_H)$. These functions can straightforwardly be shown to be concave. Hence, to demonstrate that $a_L^* < a_H^*$, it is sufficient to show that $R_L(a_H)$ crosses the 45^o line at lower values of effort than does $R_H(a_L)$. For this to be true, it is necessary that when $a_H = a_L = a$, if the first-order condition for a high ability player H is satisfied, then $R_L(a) < a$. Denoting $A = n_H\theta_H q(a_H) + n_L\theta_L q(a_L)$, this is the case if:

$$\frac{q'(a)\theta_H(A-\theta_L q(a))}{((\theta_H-\theta_L)q(a)+A)^2}<\frac{q'(a)\theta_H(A-\theta_H q(a))}{A^2}.$$

Simplifying, this inequality is true if:

$$A^2<(\theta_H-\theta_L)q(a)(A-\theta_H q(a))+2A(A-\theta_H q(a))$$

Yet, for $a_H = a_L = a$, $A = (n_H\theta_H + n_L\theta_L)q$, and so the above condition becomes:

$$(n_H\theta_H+n_L\theta_L)^2q^2<(\theta_H-\theta_L)q^2((n_H\theta_H+n_L\theta_L)-\theta_H)+2(n_H\theta_H+n_L\theta_L)q^2((n_H\theta_H+n_L\theta_L)-\theta_H),$$

which simplifies to:

$$(n_H\theta_H+n_L\theta_L)^2 < (\theta_H-\theta_L)((n_H\theta_H+n_L\theta_L)-\theta_H)+2(n_H\theta_H+n_L\theta_L)((n_H\theta_H+n_L\theta_L)-\theta_H),$$

or,

$$\theta_H(\theta_H - \theta_L) < (n_H\theta_H + n_L\theta_L)((n_H - 1)\theta_H + (n_L - 1)\theta_L).$$

And the above expression is always true since for $n_H \geq 2$ and $n_L \geq 1$:

$$\theta_H < n_H\theta_H + n_L\theta_L,$$

and,

$$\theta_H - \theta_L < (n_H - 1)\theta_H + (n_L - 1)\theta_L.$$