Success, Learning, and Overconfidence in Elimination Contests*

Luís Santos-Pinto† and Petros G. Sekeris‡

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Abstract

The paper analyzes a two-stage elimination contest in which an overconfident newcomer, uncertain about his ability, sees his overconfidence bias evolving endogenously following an early success. A first stage win amplifies the newcomer's overconfidence bias when his ex-ante probability of being high ability is low, and dampens it otherwise. Overconfidence can raise the newcomer's equilibrium effort in both stages and thus increase his chance of winning the contest. The model clarifies when success feeds further overconfidence biases and helps explain why overconfident individuals often rise to the top in organizational or competitive environments.

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†Faculty of Business and Economics, University of Lausanne, Internef 535, CH-1015, Lausanne, Switzerland. Email: luispedro.santospinto@unil.ch

‡TBS Business School, 1 Place Jourdain, 31000 Toulouse, France. Email: p.sekeris@tbs-education.fr

1 Introduction

Why do overconfident individuals—who misjudge their abilities—so often reach the top in organizations? For instance, empirical studies show that approximately 40 percent of CEOs of companies listed in the Standard & Poor's 1500 index are overconfident (Malmendier and Tate, 2015), and that managers in a workplace tournament with performance bonuses are persistently overconfident about how well they will perform (Huffman et al., 2022). Existing theories suggest that overconfident individuals may rise to the top because their bias can serve as a commitment device that attracts like-minded employees (Van den Steen, 2005), align incentives through higher leverage (Hackbarth, 2008), promote risk taking that increases promotion chances (Goel and Thakor, 2008), or signal greater effort and learning intensity valued by firms (Gervais et al., 2011). In this paper we offer a new explanation: individuals who are both uncertain about their ability and overconfident may work harder than their rivals in a multistage elimination contest.

Uncertainty about ability is common in organizations. Indeed, organizations typically staff open positions by promoting from within or by drawing on the external labor market (Bidwell and Keller, 2014). In such cases, the promotion contest sets a newcomer whose ability is largely untested against incumbents whose ability is already known. Early victories in these contests not only advance the newcomer but also reshape beliefs: both incumbents and the newcomer update their assessments of the latter's true talent.

Examples abound. Consider an external hire in a law firm who, after joining mid-career, must compete with long-tenured associates for a partnership slot in an up-or-out promotion contest. The partners can rely on years of information about their incumbent associates, whereas the newcomer enters with uncertain prospects (Rebitzer and Taylor, 2007). Promotions to CEO positions often display a similar dynamic: boards weigh candidates from inside the firm, whose past performance is well documented, against external recruits whose ability to lead the organization remains largely untested (Parrino, 1997; Zhang and Rajagopalan, 2004). In politics, two-round elections—where candidates first compete in their party for party leadership before facing off a leader from a rival party in the general election—often place a political newcomer, uncertain about how voters perceive her abilities, against seasoned opponents whose reputations are already well established (Crutzen et al., 2010; Andreottola, 2021). Similarly, in academic grant applications, junior faculty face funding tournaments alongside senior faculty whose publication records and expertise are broadly known (Azoulay et al., 2011). In each case, the eliminatory nature of the process creates settings where early successes affect expectations about an unknown entrant.

Crucially, this uncertainty about a newcomer's true ability opens the door to overconfidence. When individuals face incomplete information about how their skills compare to others', initial victories can inflate their beliefs about their own competence beyond what is objectively warranted. Indeed, overconfidence is most likely to emerge precisely when the contestant is uncertain about his type (Benoît and Dubra, 2011).

Does earning a promotion increase a newcomer's overconfidence? How does overconfidence shape contestants' efforts across the successive stages of a promotion contest? Are overconfident newcomers more likely to win the contest than their rational rivals?

To address these questions, we develop a formal model that embeds overconfidence—defined as an overestimation of the probability of being high ability—within a two-stage elimination contest with incomplete information. In the first stage (semifinal stage), four players are matched pairwise, and each pair competes in one semifinal. The semifinal winners go on to the second stage and compete in the final. In each pairwise interaction the players choose their efforts simultaneously and their winning probabilities are determined by their efforts and abilities via a Tullock contest success function (CSF). We consider a winner-take-all contest where players' utility of the prize is v and their constant marginal cost of effort is $c > 0$.

Player 1, the *newcomer*, can have either low, θ_L, or high, θ_H, ability, with $0 \leq \theta_L < 1 < \theta_H$. The ex-ante probability player 1 has high ability is $\pi \in (0, 1)$, and this is common knowledge. We sequentially analyze two scenarios. In the first scenario, the newcomer is rational, while in the second he is overconfident. A rational newcomer holds the correct prior belief that his ability is high with probability π. An overconfident newcomer holds the mistaken prior belief that his ability is high with probability $\tilde{\pi} = \pi + b^s$, where b^s is the newcomer's semifinal stage bias which satisfies $b^s \in (0, 1 - \pi]$. Players 2, 3, and 4, the *incumbents*, possess identical ability normalized to 1, and this value is common knowledge. The incumbents know the newcomer's ability is θ_H with ex-ante probability π and θ_L with ex-ante probability $1 - \pi$, and that an overconfident newcomer perceives his ability is θ_H with probability $\tilde{\pi}$ and θ_L with probability $1 - \tilde{\pi}$. Hence, we are considering an incomplete information setup where players hold no private information.

A semifinal win prompts the newcomer to update his self-belief via Bayes' rule, while the incumbent who reaches the final also updates her belief about the newcomer's type via Bayes' rule. A rational newcomer's posterior belief that his ability is high is denoted by μ and that of an overconfident one by $\tilde{\mu}$. Posterior beliefs about the newcomer's ability are determined by the equilibrium semifinal efforts and prior beliefs.[1] Accordingly, an overconfident newcomer's final-stage bias b^f is determined endogenously and equals $b^f = \tilde{\mu} - \mu$. Given the resulting posterior beliefs, the semifinal winners choose their efforts in the final. Comparing b^s to b^f reveals whether a semifinal victory amplifies or attenuates the newcomer's overconfidence bias.

Our main findings are as follows. First, a semifinal victory *amplifies* the newcomer's

[1] At equilibrium with incomplete but symmetric information about ability, semifinal efforts do not need to be observable for finalists to form posterior beliefs about ability as semifinal efforts are perfectly anticipated through the equilibrium strategy.

overconfidence bias when his ex-ante probability of being high ability is low: $b^f > b^s$ whenever π is small. Conversely, the same win *dampens* the newcomer's overconfidence bias when his ex-ante probability of being high ability is high: $b^f < b^s$ for large π. Thus an early victory in an elimination contest can either heighten or temper a player's overconfidence bias depending on the ex-ante likelihood of being high ability. The mechanism behind this unexpected result is straightforward once beliefs are traced. When π is close to zero, the incumbent's posterior hardly moves if the newcomer wins his semifinal. On the other hand, following a semifinal victory, an overconfident newcomer significantly upgrades his self-belief because $\tilde{\pi} = \pi + b^s$ shifts appreciably away from zero: this unexpected win is mistakenly attributed in the newcomer's mind to a significantly higher probability he is of high type. By contrast, when π is near one, both players already regard the newcomer as almost certainly high-ability, leaving little scope for further upward revision, so the bias naturally contracts. Overall, a semifinal victory amplifies overconfidence in players with lower expected ability, while attenuating it in those with higher expected ability. This result carries implications for the players' behavior and outcomes in the final stage as described below.

Second, in our first scenario where the final involves a rational newcomer and an incumbent, both players select the same effort at equilibrium regardless of how the newcomer's possible abilities compare to that of the incumbent.[2] In contrast, in our second scenario where the final involves an overconfident newcomer and an incumbent, the players' efforts at equilibrium are sensitive to the ability comparison: the newcomer's effort exceeds the incumbent's only when the product of the newcomer's possible abilities $\theta_L\theta_H$ lies below the incumbent's ability (normalized to 1). The intuition behind this result is as follows. When $\theta_L\theta_H \in (0, 1)$, the overconfident newcomer attributes a higher weight than the incumbent to a scenario where the gap between the players' abilities is not too large. This in turn incentivizes the overconfident newcomer to invest more effort than the incumbent. Moreover, the overconfident newcomer's equilibrium effort as well as his true probability of winning the final rise monotonically with his bias in the final, b^f. The opposite happens when $\theta_L\theta_H > 1$ and the overconfident newcomer attributes a higher weight than the incumbent to a scenario where the gap between the players' abilities is large. This in turn incentivizes him to invest less effort than the incumbent.

Third, in our first scenario where the semifinal involves a rational newcomer and an incumbent, the identity of the higher-effort player hinges on the ex-ante probability π that the newcomer is high ability: when π is small the rational newcomer expends less effort than the incumbent at equilibrium, but when π is large he expends more. The intuition for this result can be grasped by considering the two extreme cases where π is

[2]This is a well known result in Tullock contests with complete information. With identical linear costs, heterogeneous abilities cancel out in the first-order conditions, forcing identical effort. In our model with a rational newcomer, where both players hold the same posterior beliefs, they can be seen as optimizing a weighted average of two complete information contests.

either close to 0 or to 1. If π is close to 0, the incentives for the newcomer to invest in the semifinal are very low, since both the likelihood of winning the semifinal (for any effort) and the expected utility of the final are low. The incumbent is equally incentivized to invest little effort in the semifinal, yet, she will invest more than the newcomer since both her winning probability and expected utility of the final are much higher. Combined, this implies that for low values of π the incumbent invests higher effort in the semifinal. When π is close to 1, the incentives of the newcomer and the incumbent are reversed since the newcomer holds both a high probability of winning the semifinal (for a given effort) and has a higher expected utility of the final compared to the incumbent. Accordingly, for high values of π the rational newcomer exerts higher effort in the semifinal.

Fourth, in our second scenario where the semifinal is played between an overconfident newcomer and an incumbent, an increase in the newcomer's semifinal stage bias b^s raises his prior belief $\tilde{\pi}$ which has similar effects as an increase in π in the rational newcomer case. In addition, an increase in b^s changes the wedge between the newcomer's posterior belief $\tilde{\mu}$ and the incumbent's posterior belief μ as previously described. This introduces a new linkage between the semifinal and the final whereby, the overconfident newcomer's choice of effort in the semifinal affects b^f which, in turn, affects the newcomer's rival choice of effort in the final. The complexity of the problem prevents us from deriving general results for any value of θ_L and θ_H. However, imposing $\theta_L\theta_H = 1$ we are able to characterize the equilibrium of the semifinal since the final stage equilibrium efforts are identical and unaffected by the newcomer's final stage bias b^f. We find that when π is low and the bias b^s is large enough, an overconfident newcomer exerts higher semifinal effort than the incumbent whereas, in our first scenario, for such low values of π a rational newcomer exerts lower semifinal effort than the incumbent. Indeed, when π is low, a rational newcomer anticipates a low expected utility from winning the semifinal. By contrast, an overconfident newcomer misattributes a semifinal victory to his own ability, thereby reinforcing his overconfidence and making advancement to the final appear more attractive. This in turn incentivizes the overconfident newcomer to exert higher effort than his rival in the semifinal.

Fifth, we demonstrate that overconfidence raises the newcomer's true equilibrium probability of winning the elimination contest when $\theta_L\theta_H = 1$. An increase in the overconfident newcomer's semifinal stage bias b^s raises the newcomer's semifinal equilibrium relative effort. This raises the newcomer's true equilibrium probability of winning the semifinal. However, the increase in the newcomer's semifinal relative effort also lowers μ which, in turn, reduces the newcomer's true equilibrium probability of wining the final. Still, we are able to show that the increase in the probability of winning the semifinal dominates the drop in the probability of winning the final. By continuity, this result also holds in the vicinity of $\theta_L\theta_H = 1$, and simulations show that it remains true even when $\theta_L\theta_H$ is substantially different from 1.

The rest of the paper proceeds as follows. Section 2 discusses related literature. Section 3 sets-up the model. Section 4 studies the elimination contest with a rational newcomer. Section 5 considers the case of an overconfident newcomer. Section 6 concludes the paper. All proofs can be found in the Appendix.

2 Related Literature

Our paper contributes to three strands of the literature: elimination contests, the dynamics of overconfidence, and the effect of learning in contests and tournaments.

First, our contribution to elimination contests is most closely related to Rosen (1986) and Chen and Santos-Pinto (2025).[3] The main focus of Rosen (1986) is to explain why contest organizers set increasingly larger prizes as players advance in an elimination contest. Although most of the analysis is performed under complete information, Rosen (1986) also discusses the extension of his model to an incomplete information elimination contest. Rosen (1986) finds that uncertainty about ability is a force that dampens incentives to perform in the early stages since it creates incentives to experiment to discover own strength. Unlike Rosen, we focus on the effect of overconfidence on effort provision and winning probabilities in elimination contests. We find that uncertainty about ability coupled with overconfidence can instead induce an overconfident player to invest more in the early stage of the elimination contest, thereby revealing that overconfidence can totally reverse Rosen's results for incomplete information.

Chen and Santos-Pinto (2025) provide the first formal analysis of overconfidence in elimination contests. They show that in the second stage the overconfident player always exerts less effort than a rational rival, and if the prize spread is large and confidence moderate, may exert more effort in the first stage. In their model there is no uncertainty and so the overconfidence bias remains fixed across stages. In our framework instead, there is uncertainty about the overconfident player's ability, and as a consequence the overconfidence bias changes endogenously from the first to the second stage.[4] We show that winning the first stage amplifies the bias when the player's true prior probability of being high-ability is low and dampens it otherwise. This novel mechanism yields new predictions for efforts and outcomes. In the second stage, when $\theta_L \theta_H \in (0, 1)$, the equilibrium effort of the overconfident newcomer is larger than his rational rival, and both his effort and true winning probability increase with his final stage bias b^f. The

[3]Related analyses of elimination contests examine aspects of optimal design and organization, such as rent-seeking structures (Gradstein and Konrad, 1999), information revelation (Zhang and Wang, 2009), optimal seeding (Groh et al., 2012), multi-stage design (Fu and Lu, 2012), and sabotage (Klunover, 2021).

[4]Another fundamental difference is that in Chen and Santos-Pinto (2025) the players' winning probabilities are determined by Alcalde and Dahm's (2007) CSF while here we consider a Tullock CSF. This is important given the prevalence and wide use of the Tullock CSF in multistage contests.

opposite result obtains if $\theta_L \theta_H > 1$. In addition, we show that the overconfident player's true probability of winning the contest, measured as the product of the first and second stages true winning probabilities, can increase with his first stage bias.

Second, we contribute to the literature on the emergence and evolution of overconfidence. Gervais and Odean (2001) show that early success always raises confidence about ability when individuals overweight successes relative to failures. Compte and Postlewaite (2004) likewise link confidence to past outcomes through biased recollection, and show that overconfidence can decline with experience. By contrast, our model assumes biased priors but perfect recall and Bayesian updating, and shows that early success can either amplify or reduce overconfidence depending on the newcomer's ex-ante probability of being high ability; in particular, for a highly overconfident newcomer, winning the semifinal can reduce the bias. Like Bénabou and Tirole (2002), we highlight how effort choices affect beliefs about ability, but our mechanism does not rely on time-inconsistency: players may strategically restrain semifinal effort to boost others' beliefs and expected final-stage payoffs. Finally, whereas Zábojník (2004) and Benoît and Dubra (2011) study how rational overconfidence can arise from exogenous signals, we instead examine how preexisting confidence biases evolve endogenously through effort choices in a dynamic elimination contest.

Last, our paper is related to the literature on learning in dynamic contests and tournaments. Denter et al. (2022) study how one-sided asymmetric information about effort costs affects behavior in contests, allowing an informed newcomer to signal his type to an uninformed incumbent through costly signal. They show that only newcomers with sufficiently low marginal effort costs benefit from revealing their type in equilibrium, and they also allow for overconfidence. Deng et al. (2024) analyze a contest in which beliefs are distorted about the newcomer's cost of effort: overconfidence arises when the rival perceives the newcomer's marginal cost of effort to be lower than it actually is. They show that a firm, by strategically disclosing its private information about the newcomer's type, can shape these beliefs and thereby influence effort incentives and contest outcomes, with confidence distortions sometimes raising total effort. Our paper differs in two main respects: we assume symmetric information and analyze a two-stage elimination contest rather than a one-shot contest. We show that overconfidence can improve the newcomer's chances of winning the elimination contest.

In Altmann et al. (2012), Kubitz (2023), Barbieri and Serena (2025), and Catepillán et al. (2025), each player has private information about either his ability, cost of effort, valuation of the prize, or more generally objective function. We instead assume that nobody (including the newcomer himself) knows the newcomer's ability. This distinction is essential, since unlike private information setups where players attempt signaling or concealing their known identity, in our setup the newcomer's first stage effort influences everyone's beliefs about his ability at the start of the second stage. Indeed, an early victory achieved with little effort lead to a sharper updating of beliefs, since such success is more likely attributed to the winner's unknown — and potentially high — ability than it would be if the effort had been greater. Such behavior therefore potentially trumps everyone, the newcomer included, and may confer a strategic advantage to the newcomer if he reaches the subsequent round. This mechanism is highlighted in Krähmer (2007) in a repeated contest with symmetric incomplete information about players' abilities. In contrast to Krähmer (2007), we focus on the role of overconfidence in an elimination contest.

3 Set-up

Consider a two-stage elimination contest where players 1 and 3 compete in one semifinal and 2 and 4 in the other semifinal. The semifinal winners move on to the final. Player 1, the newcomer, can have either low, θ_L, or high, θ_H, ability, with $0 \leq \theta_L < 1 < \theta_H$. The ex-ante probability player 1 has high ability is $\pi \in (0, 1)$. The abilities of players 2, 3, and 4 are common knowledge, identical, and normalized to 1. Players 2, 3 and 4 know that the ex-ante probability player 1 has high ability is π.

The utility of the winning prize is v and the utility of the losers' prize is normalized to 0. Player i's cost of exerting effort a_i is equal to $C(a_i) = ca_i$, with $c > 0$.

The players' winning probabilities in any pairwise interaction are determined according to a Tullock contest success function. Moreover, the probability player i's wins against player j when player i has ability θ_i and player j has ability θ_j is as follows:

$$P_i(a_i, a_j; \theta_i, \theta_j) = \frac{\theta_i a_i}{\theta_i a_i + \theta_j a_j}, \tag{1}$$

where $\theta_1 \in \{\theta_L, \theta_H\}$, $\theta_2 = \theta_3 = \theta_4 = 1$, and $j \neq i$.

Player 1 is rational when his prior belief of having high ability is equal to π. To model overconfidence, we assume player 1 has a (subjective) prior belief of having high ability equal to $\tilde{\pi} = \pi + b^s$, where $b^s \in (0, 1 - \pi]$ is the overconfidence bias in the semifinal. Player 1's rivals know that player 1 is overconfident, that is they know b^s, but think, correctly, player 1 is mistaken.

We work with the perfect Bayesian equilibrium concept (PBE) and solve the game by backwards induction. A semifinal victory prompts the newcomer to update his belief about his own type via Bayes' rule, while an incumbent who also reaches the final also updates her belief about the newcomer's type via Bayes' rule. The players' posterior beliefs are a function of their prior beliefs and the semifinal efforts. Given the posterior beliefs, we derive the Bayesian-Nash equilibrium (BNE) of the final and compute the corresponding equilibrium payoffs (continuation values). Finally, given the continuation values, we then solve for the BNE of each semifinal. The resulting strategy profile and belief system jointly satisfy the requirements of a PBE for the two-stage elimination contest.

To be able to compute the equilibrium taking into account that players can hold mistaken beliefs we assume: (i) a player who faces a biased opponent is aware that the latter's perception is mistaken, (ii) each player thinks that his own perception is correct, and (iii) both players have a common understanding of each other's beliefs, despite their disagreement on the accuracy of their opponent's beliefs. Hence, players agree to disagree about their perceptions. This approach was introduced by Squintani (2006) and has been implemented in related literature (e.g., Yildiz 2007, Santos-Pinto and Sekeris, 2025).[5]

4 Rational Newcomer

In this section we analyze the model under the assumption that the newcomer is rational. We proceed by backward induction. First, we derive the equilibrium efforts in a final between a newcomer and an incumbent. Second, using the final stage solution, we characterize the equilibrium efforts in the semifinal between a newcomer and an incumbent.

4.1 Final

We study the final between the newcomer, player 1, and the incumbent player 2, without loss of generality. The expected utility of player $i = 1, 2$ in the final is

$$E[U_i^f(a_i^f, a_j^f)] = P_i^f(a_i^f, a_j^f)v - ca_i^f.$$

Player 1's expected utility in the final is therefore

$$E[U_1^f(a_1^f, a_2^f)] = \left[\mu \frac{\theta_H a_1^f}{\theta_H a_1^f + a_2^f} + (1-\mu)\frac{\theta_L a_1^f}{\theta_L a_1^f + a_2^f}\right] v - ca_1^f, \tag{2}$$

and player 2's expected utility in the final is

$$E[U_2^f(a_1^f, a_2^f)] = \left[\mu \frac{a_2^f}{\theta_H a_1^f + a_2^f} + (1-\mu)\frac{a_2^f}{\theta_L a_1^f + a_2^f}\right] v - ca_2^f, \tag{3}$$

where μ is the players' common posterior belief that player 1 has high ability, given by

$$\mu = \frac{\pi P_1^s(a_1^s, a_3^s; \theta_H)}{\pi P_1^s(a_1^s, a_3^s; \theta_H) + (1-\pi) P_1^s(a_1^s, a_3^s; \theta_L)} = \frac{\pi\theta_H\left(\theta_L a_1^s + a_3^s\right)}{\theta_L\theta_H a_1^s + \left[\pi\theta_H + (1-\pi)\theta_L\right] a_3^s}, \tag{4}$$

[5]These assumptions are consistent with the psychology literature on the "Blind Spot Bias" according to which individuals believe that others are more susceptible to behavioral biases than themselves (Pronin et al. 2002, Pronin and Kugler 2007).

where (a_1^s, a_3^s) are the semifinal efforts for players 1 and 3. Note that μ is decreasing with a_1^s since

$$\frac{\partial \mu}{\partial a_1^s} = -\frac{\pi(1-\pi)\theta_L\theta_H(\theta_H-\theta_L)a_3^s}{\left[\theta_L\theta_H a_1^s + \left[\pi\theta_H + (1-\pi)\theta_L\right]a_3^s\right]^2} < 0. \tag{5}$$

Although winning the semifinal leads to a higher posterior belief that the newcomer has high ability, if such a victory is achieved with a higher effort, the posterior belief will be smaller than if it is achieved with lower effort.

The first-order conditions of players 1 and 2 in the final are given by

$$\left[\mu\frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1-\mu)\frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2}\right] a_2^f v = c, \tag{6}$$

and

$$\left[\mu\frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1-\mu)\frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2}\right] a_1^f v = c. \tag{7}$$

It follows from the first-order conditions that the final has a unique pure-strategy Bayesian-Nash equilibrium. Our first result characterizes the BNE equilibrium of the final.

Proposition 1. *In a final between a rational newcomer and an incumbent, the equilibrium efforts are symmetric and given by:*

$$a_1^{f*} = a_2^{f*} = a^{f*} = \left[\mu\frac{\theta_H}{(\theta_H+1)^2} + (1-\mu)\frac{\theta_L}{(\theta_L+1)^2}\right]\frac{v}{c}. \tag{8}$$

The proof of Proposition 1 follows directly from the combination of first-order conditions (6) and (7). Denote the product of the newcomer's possible abilities by $\Theta = \theta_L\theta_H$. Observe that the equilibrium effort (i) is unaffected by μ when $\Theta \in \{0, 1\}$, (ii) increases in μ for $\Theta \in (0, 1)$, and (iii) decreases in μ when $\Theta > 1$. The intuition of this result is the following. In any complete information Tullock contest with linear costs and asymmetric abilities, equilibrium efforts are symmetric across players. Moreover, the higher the asymmetry in abilities, the lower the equilibrium efforts (see e.g. Corchón 2000). In our setup where there is incomplete information about the newcomer's ability, players can be seen as optimizing a weighted average of two complete information Tullock contests with linear costs, where the weights are given by the posterior beliefs μ and $(1-\mu)$. Accordingly, the players will invest equal efforts at equilibrium.

Second, observe that with probability $(1-\mu)$, the newcomer has an ability of θ_L and the incumbent an ability of 1. Alternatively, by dividing the numerator and the denominator of the Tullock contest success function by θ_L, one can re-interpret this as the newcomer having an ability of 1 and the incumbent having an ability of $1/\theta_L$. Therefore if $\Theta = 1$, so that $\theta_H = 1/\theta_L$, this implies that the players invest the same equilibrium efforts in the two "degenerate" cases where $\mu = 0$ and $\mu = 1$. Consequently, for any probability μ the players invest these same equilibrium efforts as explained above in point (i).

Extending this logic, we deduce that $\theta_H < 1/\theta_L$ implies that when $\mu = 1$ equilibrium efforts are higher than when $\mu = 0$. Hence, equilibrium efforts are monotonically increasing in μ when $\Theta \in (0, 1)$, as stated above in point (ii). Likewise we deduce observation (iii) according to which equilibrium efforts are monotonically decreasing in μ when $\Theta > 1$.

It follows from (8) that player 1's equilibrium probability of winning the final is:[6]

$$P_1^f(a_1^{f*}, a_2^{f*}) = \mu \frac{\theta_H}{\theta_H + 1} + (1 - \mu) \frac{\theta_L}{\theta_L + 1}.$$

It also follows from (8) that player 1's equilibrium expected utility of the final is

$$E[U_1^f(a_1^{f*}, a_2^{f*})] = \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu\chi \right] v, \tag{9}$$

where $\chi = \left(\frac{\theta_H}{\theta_H+1} \right)^2 - \left(\frac{\theta_L}{\theta_L+1} \right)^2$. From equation (9) we can determine how a change in μ affects payer 1's equilibrium expected utility of the final. We have $\frac{\partial E[U_1^f(a_1^{f*}, a_2^{f*})]}{\partial \mu} = \chi v > 0$. Hence, $E[U_1^f(a_1^{f*}, a_2^{f*})]$ is increasing with μ.

4.2 Semifinal

We now consider the semifinal between the rational newcomer and the incumbent player 3. Observe that in the semifinal opposing incumbents 2 and 4, the players' expected utility of reaching the final is a weighted average of their expected utility when facing either player 1 or player 3. Yet, given the symmetry of incumbents 2 and 4, their continuation value is identical, and their equilibrium semifinal effort as well. Although their equilibrium efforts eventually depend on their expectation of whom they will meet in the final, the identity of the winner is irrelevant, and we can then assume, without loss of generality, that the winner of the semifinal opposing players 1 and 3 will face player 2 in the final.

Player 1's expected utility of the semifinal is

$$\begin{aligned} E[U_1^s(a_1^s, a_3^s)] &= P_1^s(a_1^s, a_3^s) E[U_1^f(a_1^{f*}, a_2^{f*})] - c a_1^s \\ &= \left[\pi \frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1 - \pi) \frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s} \right] \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu\chi \right] v - c a_1^s, \end{aligned}$$

where μ is the posterior belief that player 1 has high ability and is given by equation (4).

[6] Throughout the analysis we slightly abuse notation for presentation reasons. We designate the optimal efforts in the final for player i by a_i^{f*}, omitting its implicit dependence on the semifinal efforts (a_1^s, a_3^s).

Player 3's expected utility of the semifinal is

$$E[U_3^s(a_1^s, a_3^s)] = P_3^s(a_1^s, a_3^s)E[U_3^f(a_3^{f*}, a_2^{f*})] - ca_3^s$$
$$= \left[\pi \frac{a_3^s}{\theta_H a_1^s + a_3^s} + (1-\pi)\frac{a_3^s}{\theta_L a_1^s + a_3^s}\right]\frac{v}{4} - ca_3^s.$$

If player 3 reaches the final and the opponent is player 2, then both players exert effort $a^{f*} = \frac{v}{4c}$ and player 3's equilibrium expected utility of the final is $E[U_3^f(a_3^{f*}, a_2^{f*})] = v/4$.

The first-order conditions of players 1 and 3 in the semifinal are

$$\frac{\partial P_1^s}{\partial a_1^s}\left[\left(\frac{\theta_L}{\theta_L+1}\right)^2 + \mu\chi\right]v + P_1^s(a_1^s, a_3^s)\chi v \frac{\partial \mu}{\partial a_1^s} = c, \tag{10}$$

and

$$\frac{\partial P_3^s}{\partial a_3^s}\frac{v}{4} = c, \tag{11}$$

respectively. The first term in the left-hand side of equation (10) is the standard contest incentive: increasing semifinal effort raises the probability of reaching the final. The second term captures a strategic learning effect. Conditional on reaching the final (as reflected by $P_1^s(a_1^s, a_3^s)$), higher semifinal effort affects the posterior belief about the newcomer's ability. Since $\frac{\partial \mu}{\partial a_1^s} < 0$, greater semifinal effort lowers the posterior probability that the newcomer is of high ability. Because the continuation value of the final is increasing in this belief at rate $v\chi > 0$, this reduces the expected utility from the final. In other words, everything else constant, a semifinal victory under higher effort by the rational newcomer is interpreted as a negative signal of ability ("a truly strong player would not need to try so hard"), thereby lowering the posterior belief μ which, in turn, reduces the rational newcomer's expected utility of the final. Hence, higher semifinal effort has not only a positive effect on the probability of advancing, but also a negative effect through beliefs that lowers the continuation value of the final.

To show existence of a pure-strategy equilibrium we demonstrate in the Appendix that the second-order conditions are verified whenever the first-order conditions are satisfied.

Proposition 2. *In the semifinal between a rational newcomer and an incumbent of a two-stage elimination contest, there exists a unique value $\bar{\pi} \in (0,1)$ for the ex-ante probability π that the newcomer has high ability such that when $\pi \lesseqgtr \bar{\pi}$, the players' equilibrium efforts satisfy $a_1^{s*} \lesseqgtr a_3^{s*}$.*

To better grasp Proposition 2, we consider the two extreme cases where π is either close to 0 or to 1. If π is close to 0, the incentives for the newcomer to invest in the semifinal are very low, since both the likelihood of winning the semifinal (for any effort) and the expected utility of the final are low. The incumbent is equally incentivized to invest little effort in the semifinal, yet, she will invest more than the newcomer since both her winning probability and expected utility of the final are much higher. Combined, this implies that for low values of π the incumbent invests higher effort in the semifinal.

Consider next a situation where π is close to 1. In such instances the incentives of the newcomer and the incumbent are reversed since the newcomer holds both a high probability of winning the semifinal (for a given effort) and has a higher expected utility of the final compared to the incumbent. Accordingly, for high values of π the newcomer exerts higher effort in the semifinal.

To complete the analysis of the model with a rational newcomer, we discuss how the newcomer's probability of winning the contest depends on the newcomer's potential abilities when the prior beliefs induce equal efforts in the semifinal.

Proposition 3. *Let $\bar{\pi} \in (0,1)$ be the unique value such that in the semifinal equilibrium the efforts satisfy $a_1^{s*} = a_3^{s*}$, and let $\mu(\bar{\pi})$ denote the posterior belief at $\pi = \bar{\pi}$. Then*

$$P_1^{f*}\big(\mu(\bar{\pi})\big)\, P_1^{s*}(\bar{\pi}) \gtreqless \frac{1}{4} \quad \Leftrightarrow \quad \Theta \gtreqless 1.$$

Moreover, $\bar{\pi} < 1/2$ for $\Theta = 1$, which implies that $P_1^{s*}(\bar{\pi}) < 1/2$ and $P_1^{f*}(\mu(\bar{\pi})) > 1/2$.

When the product of the newcomer's abilities is 1, his equilibrium probability of winning the elimination contest is 1/4. Interestingly, in such cases the prior belief that induces equal efforts in the semifinal is less than 1/2. This in turn implies that in such instances the probability that the newcomer wins the semifinal is less than 1/2, while the probability he wins the final is greater than 1/2. In other words, although the newcomer has a relatively low probability of having a high ability, which drives down his probability of winning the semifinal, the learning effect increases sufficiently the players' posterior belief so as to make the newcomer the most likely winner of the final. Notice that in the absence of learning by all players, the newcomer would then have a lower probability of winning both the semifinal and the final.

5 Overconfident Newcomer

This section analyzes the model with an overconfident newcomer. First, we characterize the equilibrium efforts in a final between the newcomer and an incumbent. Second, we analyze the first-order conditions that determine the equilibrium efforts in the semifinal between the newcomer and an incumbent. Third, we specialize the model to the case $\Theta = 1$ which allows us characterize how overconfidence affects winning probabilities.

5.1 Final

Assume, as before, that player 2 reaches the final. The perceived expected utilities of players 1 and 2 in the final are given by

$$\tilde{E}[U_1^f(a_1^f, a_2^f)] = \tilde{P}_1^f(a_1^f, a_2^f)v - ca_1^f = \left[\tilde{\mu}\frac{\theta_H a_1^f}{\theta_H a_1^f + a_2^f} + (1-\tilde{\mu})\frac{\theta_L a_1^f}{\theta_L a_1^f + a_2^f}\right] v - ca_1^f, \quad (12)$$

and

$$E[U_2^f(a_1^f, a_2^f)] = P_2^f(a_1^f, a_2^f)v - ca_2^f = \left[\mu\frac{a_2^f}{\theta_H a_1^f + a_2^f} + (1-\mu)\frac{a_2^f}{\theta_L a_1^f + a_2^f}\right] v - ca_2^f. \quad (13)$$

In equation (12), $\tilde{\mu}$ is player 1's perceived posterior belief of having high ability given by

$$\tilde{\mu} = \frac{\tilde{\pi}\theta_H\big(\theta_L a_1^s + a_3^s\big)}{\theta_L\theta_H a_1^s + \big[\tilde{\pi}\theta_H + (1-\tilde{\pi})\theta_L\big]a_3^s}. \quad (14)$$

Likewise, in equation (13), μ designates player 2's posterior belief that player 1 has high ability and is given by equation (4).

Define player 1's final stage overconfidence bias as $b^f = \tilde{\mu} - \mu$. Note that whereas the semifinal stage bias b^s is exogenous, the final stage bias b^f is endogenous because it depends on the posterior beliefs $\tilde{\mu}$ and μ which are determined by the equilibrium efforts exerted in the semifinal. Furthermore, since $\tilde{\mu}$ is a function of $\tilde{\pi}$, which itself is influenced by the semifinal stage bias b^s, and the semifinal equilibrium efforts depend on b^s, the final stage bias b^f depends on b^s as well. Comparing player 1's overconfidence biases in the two stages, b^s and b^f, we can make the following observation.

Result 1. *There exists a unique value $\hat{\pi} \in (0,1)$ for the ex-ante probability π the newcomer has high ability such that when $\pi \lesseqgtr \hat{\pi}$ then $b^f \gtreqless b^s$.*

This results says that in an elimination contest, winning the semifinal amplifies the overconfidence bias of a player whose ex-ante probability of being high ability is low, and dampens it otherwise. First observe that when the overconfident player 1 wins the semifinal, both player 1 and his rival in the final, player 2, revise upwards their beliefs about player 1 having high ability, i.e., $\tilde{\mu} > \tilde{\pi}$ and $\mu > \pi$. Given that both players revise their beliefs upwards, we wish to understand what makes either player revise his beliefs the most. When π is close to 0—meaning the rational player is almost certain that his opponent has low ability—a win by player 1 in the semifinal does little to change player 2's belief. The strong prior prevents the rational player to significantly revise his expectations. For intermediate values of π,—meaning the rational player is very uncertain about his opponent's ability—a win by player 1 in the semifinal leads to a large upwards revision in the beliefs of player 2. Last, when π is close to 1, there is little room for further increasing players' beliefs. The overconfident player mistakenly assigns a higher probability to having high ability. This, in turn, leads the overconfident player to update upwards his beliefs by a large amount when π is low. Consequently, for low values of π the update from the overconfident player will be larger that the one from the rational player. For intermediate values of π, the overconfident player believes his probability of being a high type is very high, and the update of beliefs will therefore be small. Accordingly, it will be the rational player who will update his beliefs the most for such priors.

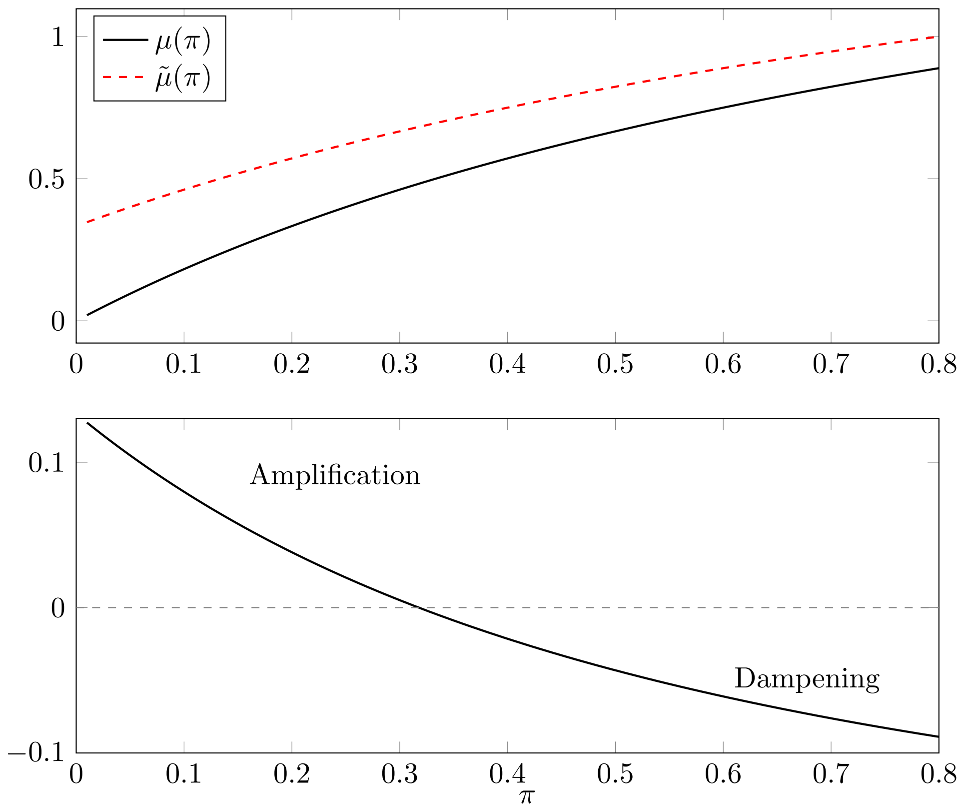

Figure 1: Top panel: rational ($\mu(\pi)$) and biased ($\tilde{\mu}(\pi)$) posterior beliefs. Bottom panel: change in the overconfidence bias: $b^f - b^s$. The two panels are produced by imposing $a_1^s = a_3^s$, $\theta_H = 2$, $\theta_L = 0.5$, and $b^s = 0.2$.

Figure 1 illustrates Result 1. On the top panel the plain curve displays the posterior belief of the rational incumbent on the newcomer having high ability following a semifinal victory by the newcomer, $\mu(\pi)$. The dashed curve depicts the overconfident newcomer's posterior belief of being high ability, $\tilde{\mu}(\pi)$. Both posterior beliefs increase with π, but as the prior beliefs that the newcomer is high ability gets higher, the scope for improvement of both beliefs shrinks. On the bottom panel we depict the change in the overconfidence bias of the newcomer, i.e. $b_f - b_s$.

Optimizing (12) and (13), we obtain the two following first-order conditions:

$$\left[\tilde{\mu}\frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1-\tilde{\mu})\frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2}\right] a_2^f v = c, \tag{15}$$

and

$$\left[\mu\frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1-\mu)\frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2}\right] a_1^f v = c. \tag{16}$$

From these expressions, we can derive the next two Lemmas:

Lemma 1. *The players' best response functions in the final are quasi-concave.*

Lemma 2. *The final admits a unique pure strategy Bayesian-Nash equilibrium.*

Having shown that the final admits a unique pure strategy BNE, we can establish the next proposition.

Proposition 4. *In a final between an overconfident newcomer and an incumbent, the equilibrium efforts depend on the product of the newcomer's possible abilities as follows:*
(i) If $\Theta \in \{0, 1\}$*, then* $a_1^{f*} = a_2^{f*}$*;*
(ii) If $\Theta \in (0, 1)$*, then* $a_1^{f*} > a_2^{f*}$*;*
(iii) If $\Theta > 1$*, then* $a_1^{f*} < a_2^{f*}$*.*

Proposition 4 shows that whether the overconfident newcomer invests more than the incumbent in the final depends on how the incumbent's fixed ability, which equals 1, compares with the product of the newcomer's possible abilities, Θ. Interestingly, the posterior beliefs that the newcomer is of high ability, μ and $\tilde{\mu}$, are irrelevant in determining which player exerts higher effort at equilibrium.

Following the reasoning underlying Proposition 1, we know that when $\Theta = 1$, i.e. $\theta_H = 1/\theta_L$, the players can be seen as optimizing a weighted average of two Tullock contests where the most able player is either the newcomer with an ability θ_H, or the incumbent with an ability $1/\theta_L = \theta_H$. Hence, the highest ability player will always have the same ability for any weights μ or $\tilde{\mu}$. Accordingly, for any weighing (μ and $\tilde{\mu}$) the equilibrium efforts will be the same for both players, and equal to the efforts they would invest if information was complete. When $\Theta = 0$, which requires $\theta_L = 0$, the newcomer can only reach the final if he has high ability, consequently the posterior beliefs are $\tilde{\mu} = \mu = 1$ (there is certainty in the final), and both players choose the same effort.

When $\Theta \in (0, 1)$, i.e. $\theta_H < 1/\theta_L$, since $\tilde{\mu} > \mu$, the newcomer puts more weight than the incumbent in the scenario where his relative ability is θ_H, while the incumbent puts more weight than the newcomer in the scenario where his own relative ability is $1/\theta_L$. Yet, we know that the more unequal the abilities of the player, the lower their equilibrium efforts in a complete information setup. Consequently, when $\theta_H < 1/\theta_L$, the newcomer puts more weight on the scenario where players invest higher equilibrium efforts, whereas the incumbent puts more weight on the scenario where the players invest lower equilibrium efforts. Accordingly, at equilibrium the newcomer will invest more effort than the incumbent.

When $\Theta > 1$, i.e. $\theta_H > 1/\theta_L$, the opposite holds true. In such instances, the newcomer puts more weight on the scenario where the players invest lower equilibrium efforts, since θ_H is indeed higher to $1/\theta_L$. Therefore, at equilibrium the newcomer will invest less effort than the incumbent.

Proposition 4 uncovers a interesting result: in a one-shot contest, an overconfident player may expend more effort than his rational rival. This finding differs from earlier work on overconfidence in elimination contests by Chen and Santos-Pinto (2025) that predicts lower effort by the overconfident player in the final stage. The divergence arises from how the bias is specified: whereas Chen and Santos-Pinto (2025) assume an overconfident player overestimates his deterministic ability, we here treat the overconfident player as uncertain about his ability and prone to exaggerating the chance that it is high.

We now introduce a lemma that will be helpful to state our next proposition.

Lemma 3. *An increase in the newcomer's posterior belief $\tilde{\mu}$ leads to a contraction of his best response function in the final, $\partial R_1^f(a_2^f)/\partial\tilde{\mu} < 0$, for $a_2^f < a_1^f\sqrt{\theta_L\theta_H}$, and to an expansion of his best response function in the final otherwise.*

The next result studies how posterior beliefs shape equilibrium outcomes in the final. First, it analyzes the effect of the newcomer's posterior belief $\tilde{\mu}$, holding the incumbent's belief μ fixed, on both players' equilibrium efforts, on the newcomer's winning probabilities, and on his perceived expected utility. It then examines how changes in the incumbent's posterior belief μ, holding $\tilde{\mu}$ fixed, affect equilibrium efforts, the newcomer's winning probabilities, and his perceived expected utility.[7] Note that the newcomer's true equilibrium probability of winning the final is

$$P_1^{f*} = \mu\frac{\theta_H a_1^{f*}}{\theta_H a_1^{f*} + a_2^{f*}} + (1-\mu)\frac{\theta_L a_1^{f*}}{\theta_L a_1^{f*} + a_2^{f*}},$$

and his perceived equilibrium probability of winning the final is

$$\tilde{P}_1^{f*} = \tilde{\mu}\frac{\theta_H a_1^{f*}}{\theta_H a_1^{f*} + a_2^{f*}} + (1-\tilde{\mu})\frac{\theta_L a_1^{f*}}{\theta_L a_1^{f*} + a_2^{f*}}.$$

Proposition 5. *Consider a final between an overconfident newcomer and an incumbent.*
(i) If $\Theta = 1$, the players' efforts are unaffected by posterior beliefs ($\partial a_1^{f*}/\partial\tilde{\mu} = \partial a_2^{f*}/\partial\tilde{\mu} = 0$, and $\partial a_1^{f*}/\partial\mu = \partial a_2^{f*}/\partial\mu = 0$). The newcomer's true winning probability increases with μ and is unaffected by $\tilde{\mu}$. The newcomer's perceived winning probability and perceived expected utility $\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]$ increase with $\tilde{\mu}$ and are unaffected by μ.
(ii) If $\Theta \in (0,1)$, the newcomer's effort increases with his posterior belief $\tilde{\mu}$ for a fixed μ ($\partial a_1^{f*}/\partial\tilde{\mu} > 0$). Fixing μ, the incumbent's effort satisfies $\partial a_2^{f*}/\partial\tilde{\mu} < 0$ if $\mu > \bar{\mu}$ and

[7] Notice that although μ and $\tilde{\mu}$ are endogenous to the semifinal efforts, in the final they are treated as given.

$\partial a_2^{f*}/\partial\tilde{\mu} > 0$ if $\mu < \bar{\mu}$; moreover, $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$ if $\mu > \bar{\mu}$, while the effect is undetermined otherwise. Fixing $\tilde{\mu}$, we have $\partial a_2^{f*}/\partial\mu > 0$ and $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\mu < 0$. The newcomer's true and perceived winning probabilities increase with $\tilde{\mu}$.

(iii) If $\Theta > 1$, the newcomer's effort decreases with his posterior belief $\tilde{\mu}$ for a fixed μ ($\partial a_1^{f*}/\partial\tilde{\mu} < 0$). Fixing μ, the incumbent's effort satisfies $\partial a_2^{f*}/\partial\tilde{\mu} < 0$ if $\mu < \bar{\mu}$ and $\partial a_2^{f*}/\partial\tilde{\mu} > 0$ if $\mu > \bar{\mu}$; moreover, $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$ if $\mu < \bar{\mu}$, while the effect is undetermined otherwise. Fixing $\tilde{\mu}$, we have $\partial a_2^{f*}/\partial\mu < 0$ and $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\mu > 0$. The newcomer's true winning probability decreases with $\tilde{\mu}$, while the effect of $\tilde{\mu}$ on his perceived winning probability is undetermined.

Proposition 5 shows how a change in the posterior beliefs of one player ($\tilde{\mu}$ or μ), for fixed posterior beliefs of the other player (μ or $\tilde{\mu}$), shape equilibrium outcomes in the final.[8] When $\Theta \in (0, 1)$ an increase in the newcomer's posterior belief $\tilde{\mu}$ raises his equilibrium effort and this, in turn, increases his true and perceived equilibrium probabilities of winning the final. In contrast, when $\Theta > 1$ an increase in $\tilde{\mu}$, lowers the newcomer's equilibrium effort which, in turn, reduces his true equilibrium probability of winning the final. Note that in this case the impact of an increase in $\tilde{\mu}$ on the newcomer's perceived probability of winning the final is undetermined since $\tilde{\mu}$ goes up but the newcomer's effort goes down.

Bearing in mind that the newcomer's incentives in the semifinal depend on his perceived expected utility of the final, it is important to explore how this is affected by changes in the players' posterior beliefs. When $\Theta = 1$, the equilibrium efforts are equal and unaffected by the newcomer's posterior belief. In such cases, an increase in $\tilde{\mu}$ raises the equilibrium perceived expected winning probability, which makes the final more attractive to the newcomer. When $\Theta \in (0, 1)$, and the incumbent's posterior belief is high, $\mu > \bar{\mu}$, an increase in $\tilde{\mu}$ makes the final more attractive to the newcomer. In such instances, higher values of $\tilde{\mu}$ incentivize player 1 to increase his equilibrium effort, while also pushing player 2 to strategically reduce his equilibrium effort. In contrast, if the incumbent's posterior belief is low, $\mu < \bar{\mu}$, efforts are strategic complements from player 2's perspective, thus implying that an increase in $\tilde{\mu}$ raises player 2's equilibrium effort. In such instances, the effect of $\tilde{\mu}$ on the newcomer's equilibrium perceived expected utility is undetermined. Finally, when $\Theta > 1$, and the incumbent's posterior belief is low, $\mu < \bar{\mu}$, an increase $\tilde{\mu}$ makes the final more attractive to the newcomer.

Finally, Proposition 5 isolates the effect of a change in the incumbent's posterior belief μ, on her equilibrium effort in the final a_2^{f*}, as well as on the newcomer's perceived expected utility of the final. When $\Theta \in (0, 1)$, an increase in the posterior belief μ incentivizes the incumbent to invest higher effort at equilibrium, thence resulting in a lower equilibrium perceived expected utility for the newcomer. In contrast, when $\Theta > 1$,

[8] Bear in mind that an increase in $\tilde{\mu}$ for a given μ is equivalent to an increase in the final stage bias b^f.

the incumbent is incentivized to reduce her equilibrium effort, thereby leading to an increase of the newcomer's perceived payoff.

5.2 Semifinal

Player 1's perceived expected utility of the semifinal is

$$\begin{aligned}\tilde{E}[U_1^s(a_1^s, a_3^s)] &= \tilde{P}_1^s(a_1^s, a_3^s)\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] - ca_1^s \\ &= \left[\tilde{\pi}\frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1-\tilde{\pi})\frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s}\right]\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] - ca_1^s,\end{aligned}$$

where $a_1^{f*} = h_1(\tilde{\mu}, \mu)$, $a_2^{f*} = h_2(\tilde{\mu}, \mu)$, $\tilde{\mu}$ is given by equation (14) and μ by equation (4).

The first-order condition of player 1 in the semifinal is

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s}\tilde{E}\big[U_1^{f*}\big] + \tilde{P}_1^s(a_1^s, a_3^s)\left[\frac{d\tilde{E}[U_1^{f*}]}{d\mu}\frac{\partial \mu}{\partial a_1^s} + \frac{d\tilde{E}[U_1^{f*}]}{d\tilde{\mu}}\frac{\partial \tilde{\mu}}{\partial a_1^s}\right] = c. \tag{17}$$

The first-order condition for player 3 in the semifinal is the same as in the previous section and thus given by:

$$\frac{\partial P_3^s}{\partial a_3^s}\frac{v}{4} = c. \tag{18}$$

To show existence of a pure-strategy equilibrium we demonstrate in the Appendix that the second-order conditions are verified whenever the first-order conditions are satisfied for the specific case $\Theta = 1$. By continuity of the players' perceived expected utilities in θ_L and θ_H it follows that there is an interval $[\underline{\Theta}, \bar{\Theta}]$, with $\underline{\Theta} < 1$ and $\bar{\Theta} > 1$, such that for any Θ in that interval the second-order conditions are verified.

Making use of the Envelope theorem, we can rewrite equation (17) as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s}\tilde{E}\big[U_1^{f*}\big] + \tilde{P}_1^s(a_1^s, a_3^s)\left[\frac{\partial \tilde{E}\big[U_1^{f*}\big]}{\partial a_2^f}\frac{\partial a_2^{f*}}{\partial \mu}\frac{\partial \mu}{\partial a_1^s} + \frac{\partial \tilde{E}\big[U_1^{f*}\big]}{\partial a_2^f}\frac{\partial a_2^{f*}}{\partial \tilde{\mu}}\frac{\partial \tilde{\mu}}{\partial a_1^s} + \frac{\partial \tilde{E}\big[U_1^{f*}\big]}{\partial \tilde{\mu}}\frac{\partial \tilde{\mu}}{\partial a_1^s}\right] = c. \tag{19}$$

The newcomer's first-order condition equates the perceived marginal benefit of effort in the semifinal to the constant marginal cost c. The perceived marginal benefit of effort is itself composed of two terms. The first term describes the effect of semifinal effort on the perceived probability of winning the semifinal weighted by the perceived expected utility of reaching the final. The sign of this term is positive. The second term measures how a change in semifinal effort affects the perceived expected utility of reaching the final, weighted by the perceived probability of winning the semifinal. Observe that the perceived expected utility of reaching the final is affected by the newcomer's first period effort through three channels. Indeed, true posterior beliefs μ are (negatively) impacted by an increase in a_1^s, in turn affecting the equilibrium efforts of the rival in the final, a_2^{f*}. Perceived posterior beliefs $\tilde{\mu}$ are also (negatively) impacted by an increase in a_1^s, in turn affecting the expected utility of the newcomer in the final both directly, and indirectly through its effect on the equilibrium efforts of the rival, a_2^{f*}.

Proposition 5 uncovers the sign of several effects contained in equation (19). For $\Theta = 1$, we know that $\partial a_2^{f*}/\partial\tilde{\mu} = \partial a_2^{f*}/\partial\mu = 0$, and that $\partial\tilde{E}[U_1^{f*}]/\partial\tilde{\mu} > 0$. Consequently, since $\partial\tilde{\mu}/\partial a_1^s < 0$, we deduce that the squared-bracketed term of equation (19) is negative. In words, increasing semifinal effort, lowers the newcomer's perceived expected utility of reaching the final, thence containing his incentives to invest in a_1^s. For $\Theta \neq 1$, on the other hand, we know from Proposition 5 that the sign of the squared-bracketed term is ambiguous.

Given the generality of our setup, it is not possible to derive closed form solutions and to further characterize the equilibrium, except when $\Theta = 1$ as we show next. The cases where $\Theta \neq 1$ are discussed in Section 5.4.

5.3 The Model with $\Theta = 1$

When $\Theta = 1$, we know that $a_1^{f*} = a_2^{f*}$, which using player 1's first-order condition is shown to equal:

$$a^{f*} = \left[\tilde{\mu}\frac{\theta_H}{(\theta_H+1)^2} + (1-\tilde{\mu})\frac{\theta_L}{(\theta_L+1)^2}\right]\frac{v}{c}$$

Substituting for $\theta_L = 1/\theta_H$, this expression reads as:

$$a^{f*} = \frac{\theta_H}{(\theta_H+1)^2}\frac{v}{c}.$$

Player 1's equilibrium perceived expected utility of the final is then equal to:

$$\begin{aligned}\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] &= \left[\tilde{\mu}\frac{\theta_H}{\theta_H+1} + (1-\tilde{\mu})\frac{1}{\theta_H+1}\right]v - ca^{f*} \\ &= \left[\left(\frac{1}{\theta_H+1}\right)^2 + \frac{\theta_H-1}{\theta_H+1}\tilde{\mu}\right]v,\end{aligned}$$

where, imposing $\theta_L = 1/\theta_H$, we have

$$\tilde{\mu} = \frac{\tilde{\pi}\theta_H\big(a_1^s + \theta_H a_3^s\big)}{\theta_H a_1^s + \big[\tilde{\pi}\theta_H^2 + (1-\tilde{\pi})\big]a_3^s}.$$

We know from Proposition 5 that when $\Theta = 1$, $\partial a_2^{f*}/\partial\tilde{\mu} = \partial a_2^{f*}/\partial\mu = 0$, and hence the first-order condition (19) can be written as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s}\left[\left(\frac{1}{\theta_H+1}\right)^2+\frac{\theta_H-1}{\theta_H+1}\tilde{\mu}\right]v+\tilde{P}_1^s(a_1^s,a_3^s)\frac{\theta_H-1}{\theta_H+1}v\frac{\partial \tilde{\mu}}{\partial a_1^s}=c. \tag{20}$$

Using the incumbent's first-order condition in equation (11), alongside the overconfident newcomer's first-order condition in equation (20), we can state the next result.

Proposition 6. *In the semifinal between an overconfident newcomer and an incumbent of a two-stage elimination contest where $\Theta = 1$, there exists a unique value*

$$\breve{\pi}=\frac{\theta_H+3}{4(\theta_H+1)}$$

for the ex-ante probability π the newcomer has high ability such that:
(i) For $\pi < \tilde{\pi} < \breve{\pi}$, both a rational and an overconfident newcomer exert less effort than the incumbent at equilibrium in the semifinal.
(ii) For $\pi < \breve{\pi} < \tilde{\pi}$, an overconfident newcomer exerts more effort than the incumbent at equilibrium in the semifinal, while a rational newcomer exerts less effort.
(iii) For $\breve{\pi} < \pi < \tilde{\pi}$, both a rational and an overconfident newcomer exert more effort than the incumbent at equilibrium in the semifinal.

When $\Theta = 1$ we have the following three scenarios. First, when the newcomer's ex-ante probability of having high ability is low and his biased prior belief $\tilde{\pi}$ is smaller than the threshold value, $\pi < \tilde{\pi} < \breve{\pi}$, both a rational and an overconfident newcomer exert less effort at equilibrium in the semifinal than the incumbent. This is intuitive, the small bias leads to a small change in effort provision and both a rational and an overconfident newcomer exert less effort than the incumbent for the same reasons highlighted after Proposition 2 in Section 4.2.

Second, when the newcomer's ex-ante probability of having high ability is low and his biased prior belief $\tilde{\pi}$ is greater than the threshold value, $\pi < \breve{\pi} < \tilde{\pi}$, an overconfident newcomer exerts more effort at equilibrium in the semifinal than the incumbent whereas a rational newcomer exerts less effort. In this case the higher bias leads to a large effort provision since the newcomer overestimates his probability of winning the semifinal as well at his expected utility of the final. Consequently the overconfident newcomer exerts more effort than the incumbent when a rational newcomer would have exerted less effort than the incumbent.

Third, when the newcomer's ex-ante probability of having high ability is greater than the threshold value, $\breve{\pi} < \pi < \tilde{\pi}$, both a rational and an overconfident newcomer exert more effort at equilibrium in the semifinal than the incumbent.

Last we analyze the impact of the newcomer's semifinal bias b^s on his true equilibrium probabilities of winning the final and semifinal, P_1^{f*} and P_1^{s*}, respectively. This allows us to determine how the semifinal bias b^s changes the newcomer's true equilibrium probability of winning the elimination contest $P_1^* = P_1^{s*} P_1^{f*}$.

Proposition 7. *In a two-stage elimination contest where $\Theta = 1$, the overconfident newcomer's true equilibrium probability of winning the final, P_1^{f*}, decreases in his overconfidence bias b^s, and his true equilibrium probability of winning the semifinal, P_1^{s*}, increases in b^s. His true equilibrium probability of winning the contest, $P_1^{s*} P_1^{f*}$, increases in his overconfidence bias b^s.*

When $\Theta = 1$, the newcomer's true equilibrium probability of winning the final is given by

$$P_1^{f*} = \frac{1 + \mu(\theta_H - 1)}{\theta_H + 1}, \tag{21}$$

where

$$\mu = \pi \frac{\frac{a_1^s}{a_3^s} + \theta_H}{\frac{a_1^s}{a_3^s} + \pi\theta_H + (1-\pi)/\theta_H}.$$

Since

$$\partial\mu / \partial(a_1^s / a_3^s) = -\frac{(1-\pi)(\theta_H - 1/\theta_H)}{(a_1^s/a_3^s + \pi\theta_H + (1-\pi)/\theta_H)^2} < 0,$$

and since P_1^{f*} is increasing in μ, it follows that the newcomer's true equilibrium probability of winning the final is decreasing in a_1^s/a_3^s.

The newcomer's true equilibrium probability of winning the semifinal is given by

$$P_1^{s*} = \pi \frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*} + a_3^{s*}} + (1-\pi) \frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H + a_3^{s*}}, \tag{22}$$

and we immediately observe that P_1^{s*} is increasing with a_1^{s*}/a_3^{s*}. Consequently, a change in the equilibrium semifinal relative effort induces P_1^{f*} and P_1^{s*} to move in opposite directions. We show, in the proof of Proposition 7, that an increase in the newcomer's semifinal bias b^s raises the semifinal equilibrium relative effort a_1^{s*}/a_3^{s*}. Hence, an increase in the newcomer's semifinal bias b^s lowers his true equilibrium probability of winning the final while it raises his true equilibrium probability of winning the semifinal. We demonstrate in Proposition 7 that the net effect of the increase in P_1^{s*} dominates.

Figure 2 illustrates Proposition 7. It depicts P_1^{s*}, P_1^{f*}, and their product P_1^*, as a function of b^s when $\pi = 1/4$, $\theta_L = 1/2$, $\theta_H = 2$, $v = 10$, and $c = 1$.

5.4 Relaxing the Restriction $\Theta = 1$

Proposition 7 is obtained when imposing the restriction $\Theta = 1$. This simplification eliminates asymmetries in equilibrium efforts in the final stage, and simplifies significantly the analysis of the semifinal stage. Observe, however, that it does not constitute a knife-edge condition for the qualitative result of Proposition 7. Indeed, all primitives of the

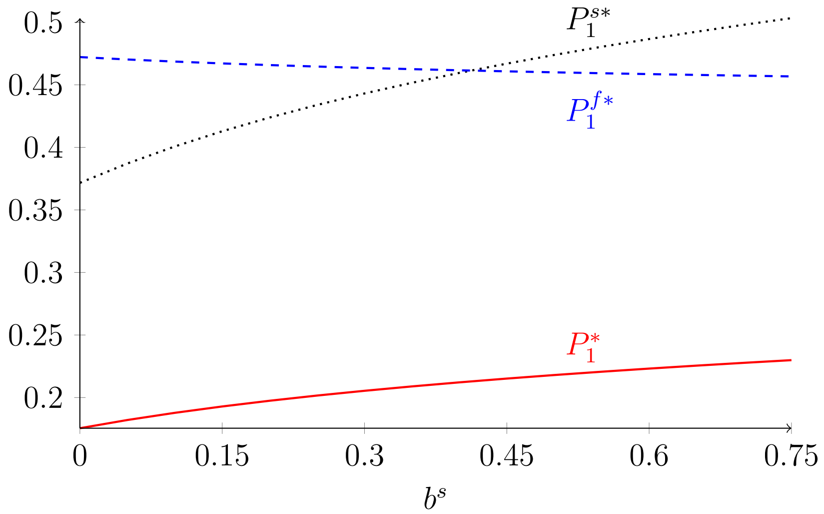

Figure 2: Newcomer's true winning probabilities in the elimination contest when $\Theta = 1$.

model are smooth, and equilibrium efforts in both stages are interior and characterized by first-order conditions satisfying standard regularity conditions. Hence, by standard continuity arguments, equilibrium effort levels and the induced winning probabilities vary continuously with the parameters θ_L and θ_H. Since in the specific case where $\Theta = 1$, P_1^{f*} strictly decreases in b^s, P_1^{s*} strictly increases in b^s, and P_1^* strictly increases in b^s, by continuity this implies that there exists an open neighborhood of this value—both for $\Theta < 1$ and for $\Theta > 1$—in which the overall effect of overconfidence on the true probability of winning the contest remains unchanged in sign.

To complement this continuity argument, we now turn to numerical simulations that illustrate the robustness of the result when the restriction $\Theta = 1$ is relaxed. On Figure 3 we depict numerical values for true equilibrium winning probabilities using the same parameter values as in Figure 2, except that $\theta_H = 3/2$ such that $\Theta = 3/4$. On Figure 4 we reproduce the exercise for the case where $\theta_H = 3$ such that $\Theta = 3/2$. As one can see, the results remain qualitatively unchanged for values of Θ that do not lie in the vicinity of 1.

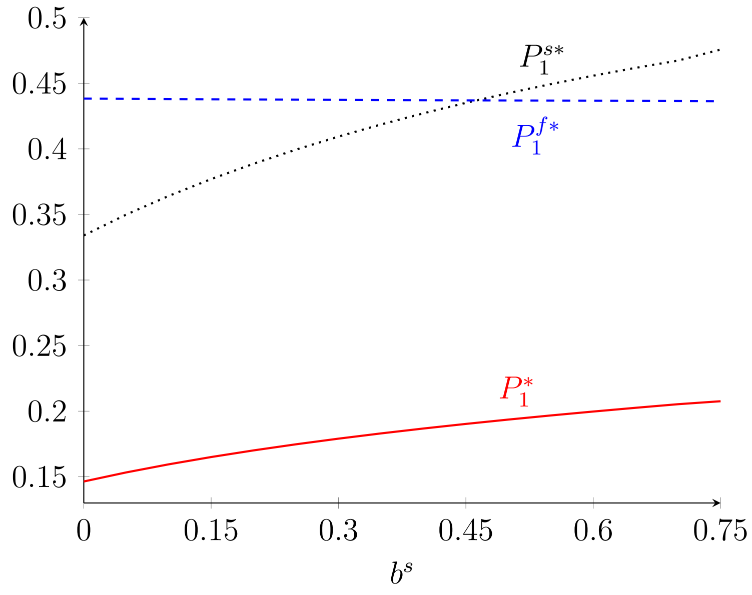

Figure 3: Newcomer's true winning probabilities in the elimination contest when $\Theta < 1$.

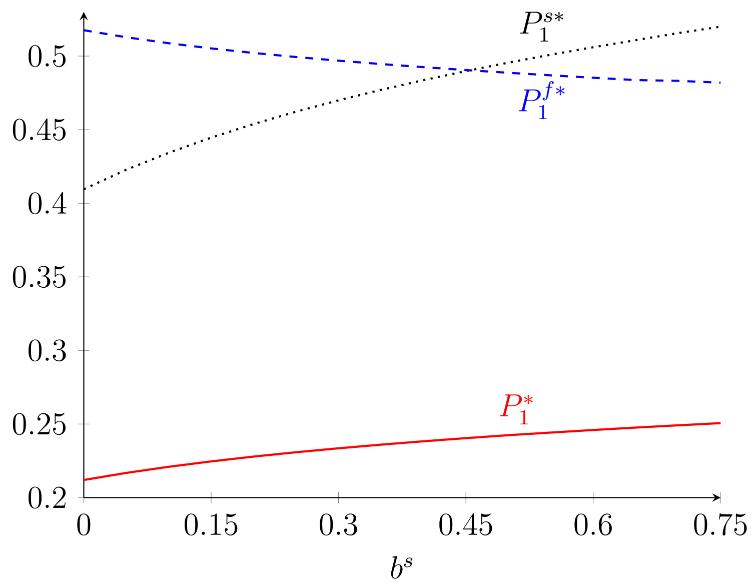

Figure 4: Newcomer's true winning probabilities in the elimination contest when $\Theta > 1$.

6 Conclusion

This paper analyzes how overconfidence shapes behavior and winning probabilities in a two-stage elimination contest with incomplete information. We start by showing that following a first stage victory the newcomer's overconfidence bias is boosted when his ex-ante probability of having high ability is low, otherwise the bias is dampened. This new mechanism has implications for behavior in the second stage. When the product of the newcomer's possible abilities is low, the equilibrium effort of the overconfident newcomer is larger than his rational rival, and both his effort and true winning probability increase with his bias. The opposite result obtains if the product of the newcomer's possible abilities is high. In addition, we show that the overconfident player's true probability of winning the contest, measured as the product of the first and second stages true winning probabilities, can increase with his first stage bias. Our results clarify under which conditions success breeds further overconfidence or tames it. They also provide an explanation for why overconfident individuals so frequently attain the upper levels of promotion contests.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT in order to improve the text and assist the authors in the simulations. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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7 Appendix

Second-order conditions of the semifinal with a rational newcomer: The first derivative of the newcomer's expected utility in the semifinal is:

$$\frac{\partial E[U_1^s]}{\partial a_1^s} = \frac{\partial P_1^s}{\partial a_1^s}\left[\left(\frac{\theta_L}{\theta_L+1}\right)^2 + \mu\chi\right] v + P_1^s \chi v \frac{\partial \mu}{\partial a_1^s} - c.$$

The second derivative is therefore given by:

$$\frac{\partial^2 E[U_1^s]}{(\partial a_1^s)^2} = \frac{\partial^2 P_1^s}{(\partial a_1^s)^2}\left[\left(\frac{\theta_L}{\theta_L+1}\right)^2 + \mu\chi\right] v + 2\frac{\partial P_1^s}{\partial a_1^s}\chi v \frac{\partial \mu}{\partial a_1^s} + P_1^s \chi v \frac{\partial^2 \mu}{(\partial a_1^s)^2}.$$

Since $\frac{\partial^2 P_1^s}{(\partial a_1^s)^2} < 0$, this expression is a fortiori negative if:

$$\left[\frac{\partial^2 P_1^s}{(\partial a_1^s)^2}\mu + 2\frac{\partial P_1^s}{\partial a_1^s}\frac{\partial \mu}{\partial a_1^s} + P_1^s \frac{\partial^2 \mu}{(\partial a_1^s)^2}\right] \underbrace{\chi v}_{>0} < 0.$$

We thus need to show that the term inside squared brackets is negative. Substituting for the appropriate terms, and simplifying, the term inside squared brackets is given by:

$$-\frac{2\pi\theta_H^2 a_3^s}{(\theta_H a_1^s + a_3^s)^3} < 0.$$

Hence, the second-order condition for the newcomer is satisfied. It is immediate to verify that the second-order condition for the incumbent is also satisfied.

Proof of Proposition 2: To prove this result, we consider an effort level of a player that maximizes his semifinal payoff. Accordingly, that effort level must satisfy his first-order condition. We next fix the other player's effort at the same level, i.e. $a_1^s = a_3^s$, and deduce that $a_1^s \lesseqgtr a_3^s \Leftrightarrow \partial E[U_1^s(a_1^s, a_3^s)]/\partial a_1 \lesseqgtr \partial E[U_3^s(a_1^s, a_3^s)]/\partial a_3$, for $a_1^s = a_3^s = a > 0$. More specifically, we define the difference of the players' first order derivatives when evaluated at $a_1^s = a_3^s$ as $\Psi(\pi)$, and we show that $\Psi(0) < 0$, $\Psi(1) > 0$, and $\Psi'(\pi) > 0$ on $\pi \in [0, 1]$, thence implying that there is a unique ex-ante probability the newcomer has high ability $\bar{\pi}$ for which players 1 and 3 exert the same equilibrium effort in the semifinal.

We start by simplifying the first-order condition of player 1 given by equation (10):

$$\frac{\partial P_1^s}{\partial a_1^s} E\big[U_1^f(a_1^{f*}, a_2^{f*})\big] + P_1^s(a_1^s, a_3^s)\chi v \frac{\partial \mu}{\partial a_1^s} = c.$$

We know that

$$\frac{\partial P_1^s}{\partial a_1^s} = \frac{\partial P_3^s}{\partial a_3^s}\frac{a_3^s}{a_1^s}.$$

Using this last equation, the first-order condition of player 3 given by equation (11) becomes

$$\frac{\partial P_1^s}{\partial a_1^s}\frac{a_1^s}{a_3^s}\frac{v}{4} = c.$$

Using these first-order conditions, we can next express $\Psi(\pi)$ as:

$$\begin{aligned}
\Psi(\pi) &= \left.\frac{\partial E[U_1^s(a_1^s, a_3^s)]}{\partial a_1^s}\right|_{a_1^s=a_3^s} - \left.\frac{\partial E[U_3^s(a_1^s, a_3^s)]}{\partial a_3^s}\right|_{a_1^s=a_3^s} \\
&= \left.\left\{\frac{\partial P_1^s}{\partial a_1^s} E\big[U_1^f(a_1^{f*}, a_2^{f*})\big] + P_1^s(a_1^s, a_3^s)\chi v \frac{\partial \mu}{\partial a_1^s} - \frac{\partial P_1^s}{\partial a_1^s}\frac{a_1^s}{a_3^s}\frac{v}{4}\right\}\right|_{a_1^s=a_3^s} \\
&= \left.\left\{\frac{\partial P_1^s}{\partial a_1^s}\left[E\big[U_1^f(a_1^{f*}, a_2^{f*})\big] - \frac{a_1^s}{a_3^s}\frac{v}{4}\right] + P_1^s(a_1^s, a_3^s)\chi v \frac{\partial \mu}{\partial a_1^s}\right\}\right|_{a_1^s=a_3^s}.
\end{aligned}$$

We have

$$P_1^s(a_1^s, a_3^s) = \pi\frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1-\pi)\frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s}.$$

So, we have

$$\frac{\partial P_1^s}{\partial a_1^s} = \pi\frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} + (1-\pi)\frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2}$$

Observe that when imposing $a_1^s = a_3^s = a$, we have:

$$\left.P_1^s(a_1^s, a_3^s)\right|_{a_1^s=a_3^s=a} = \frac{\theta_L}{1+\theta_L} + \pi\left(\frac{\theta_H}{1+\theta_H} - \frac{\theta_L}{1+\theta_L}\right),$$

$$\left.\frac{\partial P_1^s}{\partial a_1^s}\right|_{a_1^s=a_3^s=a} = \left[\frac{\theta_L}{(\theta_L+1)^2} + \pi\left(\frac{\theta_H}{(\theta_H+1)^2} - \frac{\theta_L}{(\theta_L+1)^2}\right)\right]\frac{1}{a},$$

$$\left.E[U_1^f(a_1^{f*}, a_2^{f*})]\right|_{a_1^s=a_3^s=a} = \left[\left(\frac{\theta_L}{\theta_L+1}\right)^2 + \frac{\pi\theta_H\big(\theta_L+1\big)}{\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L}\chi\right]v,$$

and

$$\left.\frac{\partial \mu}{\partial a_1^s}\right|_{a_1^s=a_3^s=a} = -\frac{\pi(1-\pi)\theta_L\theta_H(\theta_H-\theta_L)}{\big[\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L\big]^2 a}.$$

Substituting in Ψ for the appropriate terms we then obtain:

$$\begin{aligned}
\Psi(\pi) &= \left[\frac{\theta_L}{(\theta_L+1)^2} + \pi\left(\frac{\theta_H}{(\theta_H+1)^2} - \frac{\theta_L}{(\theta_L+1)^2}\right)\right]\frac{1}{a}\left[\left[\left(\frac{\theta_L}{\theta_L+1}\right)^2 + \frac{\pi\theta_H\big(\theta_L+1\big)}{\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L}\chi\right] - \frac{1}{4}\right]v \\
&- \left[\frac{\theta_L}{1+\theta_L} + \pi\left(\frac{\theta_H}{1+\theta_H} - \frac{\theta_L}{1+\theta_L}\right)\right]\chi v\frac{\pi(1-\pi)\theta_L\theta_H(\theta_H-\theta_L)}{\big[\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L\big]^2 a}
\end{aligned}$$

After some manipulations we obtain

$$\Psi(\pi) = \frac{1}{(\theta_H+1)^4(\theta_L+1)^4}\Big[-\theta_L(\theta_H+1)^4(1+2\theta_L-3\theta_L^2) + \Big(\theta_H(\theta_L+1)^4(3\theta_H^2-2\theta_H-1)+\theta_L(\theta_H+1)^4(1+2\theta_L-3\theta_L^2)\Big)\pi\Big]\frac{v}{4a}$$

Hence,

$$\Psi'(\pi) = \frac{\theta_H(\theta_L+1)^4(3\theta_H^2-2\theta_H-1) \ + \ \theta_L(\theta_H+1)^4(1+2\theta_L-3\theta_L^2)}{(\theta_H+1)^4(\theta_L+1)^4}\frac{v}{4a},$$

where for $\theta_H > 1$,

$$3\theta_H^2 - 2\theta_H - 1 = (3\theta_H+1)(\theta_H-1) > 0.$$

and for $0 \leq \theta_L < 1$,

$$1+2\theta_L-3\theta_L^2 = 3(1-\theta_L)\Big(\theta_L+\tfrac{1}{3}\Big) > 0.$$

Therefore

$$\Psi'(\pi) > 0.$$

Setting $\Psi(\pi) = 0$, and solving for π, we obtain:

$$\bar{\pi} = \frac{\theta_L(\theta_H+1)^4(1+2\theta_L-3\theta_L^2)}{\theta_H(\theta_L+1)^4(3\theta_H^2-2\theta_H-1)+\theta_L(\theta_H+1)^4(1+2\theta_L-3\theta_L^2)}.$$

Proof of Result 1: There exists a unique prior belief $\hat{\pi} \in [0,1]$ which is such that when $\pi \lesseqgtr \hat{\pi}$ then $b^f \gtreqless b^s$.

$$\begin{aligned}
b^f - b^s &= (\tilde{\mu}-\mu) - (\tilde{\pi}-\pi) \\
&= (\tilde{\mu}-\tilde{\pi}) - (\mu-\pi) \\
&= \tilde{\pi}\left[\frac{P_1^s(a_1^s,a_3^s;\theta_H)}{\tilde{\pi}P_1^s(a_1^s,a_3^s;\theta_H)+(1-\tilde{\pi})P_1^s(a_1^s,a_3^s;\theta_L)} - 1\right] \\
&\quad - \pi\left[\frac{P_1^s(a_1^s,a_3^s;\theta_H)}{\pi P_1^s(a_1^s,a_3^s;\theta_H)+(1-\pi)P_1^s(a_1^s,a_3^s;\theta_L)} - 1\right] \\
&= \tilde{\pi}(1-\tilde{\pi})\frac{P_1^s(a_1^s,a_3^s;\theta_H)-P_1^s(a_1^s,a_3^s;\theta_L)}{\tilde{\pi}P_1^s(a_1^s,a_3^s;\theta_H)+(1-\tilde{\pi})P_1^s(a_1^s,a_3^s;\theta_L)} \\
&\quad - \pi(1-\pi)\frac{P_1^s(a_1^s,a_3^s;\theta_H)-P_1^s(a_1^s,a_3^s;\theta_L)}{\pi P_1^s(a_1^s,a_3^s;\theta_H)+(1-\pi)P_1^s(a_1^s,a_3^s;\theta_L)}.
\end{aligned}$$

Thence, the sign of $b^f - b^s$ is given by:

$$\begin{aligned}
\text{sgn}\big\{b^f - b^s\big\} &= \text{sgn}\{(1-\tilde{\pi})(1-\pi)\, P_1^s(a_1^s, a_3^s; \theta_L) - \pi\tilde{\pi}\, P_1^s(a_1^s, a_3^s; \theta_H)\} \\
&= \text{sgn}\{(1-\pi-b^s)(1-\pi)\, P_1^s(a_1^s, a_3^s; \theta_L) - \pi(\pi+b^s)\, P_1^s(a_1^s, a_3^s; \theta_H)\} \\
&= \text{sgn}\Big\{-\big[P_1^s(a_1^s, a_3^s; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L)\big]\pi^2 \\
&\qquad - \big[2 + (P_1(a_1, a_3; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L))\big]\, b^s\, \pi \\
&\qquad + (1-b^s)\, P_1^s(a_1^s, a_3^s; \theta_L)\Big\}.
\end{aligned}$$

Since for $\pi = 0$ the above expression is always positive, for $\pi = 1$ it is always negative, and given the quadratic nature of the expression there exists a unique threshold $\hat{\pi} \in (0,1)$ such that $\pi \lesseqgtr \hat{\pi} \Leftrightarrow b^f \gtreqless b^s$.

Proof of Proposition 3: At $\pi = \bar{\pi}$ we have $a_1^{s*} = a_3^{s*}$, so $r^s \equiv a_1^{s*}/a_3^{s*} = 1$. Hence the newcomer's equilibrium probability of winning the semifinal is

$$P_1^{s*}(\bar{\pi}) = \bar{\pi}\frac{\theta_H}{\theta_H+1} + (1-\bar{\pi})\frac{\theta_L}{\theta_L+1}.$$

Moreover, when $r^s = 1$ the posterior belief μ simplifies to

$$\mu(\pi) = \frac{\pi\theta_H(\theta_L+1)}{\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L},$$

so the newcomer's equilibrium probability of winning the final, evaluated at $\pi = \bar{\pi}$, is

$$P_1^{f*}\big(\mu(\bar{\pi})\big) = \mu(\bar{\pi})\frac{\theta_H}{\theta_H+1} + \big(1-\mu(\bar{\pi})\big)\frac{\theta_L}{\theta_L+1}.$$

Define $\Delta \equiv \frac{\theta_H}{\theta_H+1} - \frac{\theta_L}{\theta_L+1}$. Then

$$P_1^s(\bar{\pi}) = \frac{\theta_L}{\theta_L+1} + \bar{\pi}\,\Delta, \qquad P_1^f\big(\mu(\bar{\pi})\big) = \frac{\theta_L}{\theta_L+1} + \mu(\bar{\pi})\,\Delta.$$

Substituting the closed-form expression of $\bar{\pi}$ and simplifying yields the factorization

$$P_1^{f*}\big(\mu(\bar{\pi})\big)P_1^{s*}(\bar{\pi}) - \frac{1}{4} = \frac{(\theta_H-1)(3\theta_H+1)(\theta_L-1)(3\theta_L+1)\,(\theta_H\theta_L-1)}{4\,(3\theta_H\theta_L+\theta_H+\theta_L-1)\,(\theta_H^2\theta_L^2-\theta_H^2\theta_L-\theta_H\theta_L^2-8\theta_H\theta_L-3\theta_H-3\theta_L-1)}.$$

Under $\theta_H > 1 > \theta_L > 0$, the factors (θ_H-1), $(3\theta_H+1)$, and $(3\theta_L+1)$ are positive, while (θ_L-1) is negative. Also, one checks that

$$3\theta_H\theta_L + \theta_H + \theta_L - 1 > 0 \quad \text{and} \quad \theta_H^2\theta_L^2 - \theta_H^2\theta_L - \theta_H\theta_L^2 - 8\theta_H\theta_L - 3\theta_H - 3\theta_L - 1 < 0.$$

Therefore the denominator in the last display is negative, and the product $(\theta_H-1)(3\theta_H+$

$1)(\theta_L - 1)(3\theta_L + 1)$ is negative, so the sign of $P_1^{f*}(\mu(\bar{\pi}))P_1^{s*}(\bar{\pi}) - \frac{1}{4}$ coincides with the sign of $(\theta_H\theta_L - 1)$. This proves

$$P_1^{f*}\big(\mu(\bar{\pi})\big)\, P_1^{s*}(\bar{\pi}) \gtreqless \frac{1}{4} \quad \Longleftrightarrow \quad \theta_H\theta_L \gtreqless 1.$$

To complete the proof, we show that $\bar{\pi} < 1/2$ if $\Theta = 1$. We fix $\Theta = 1$ and show that $\bar{\pi} < 1/2$, which is true if the following expression is true:

$$\theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2) < \theta_H(\theta_L + 1)^4(3\theta_H - 2\theta_H - 1),$$

or, replacing $\theta_L = 1/\theta_H$,

$$\frac{(\theta_H + 1)^4}{\theta_H(3\theta_H^2 - 2\theta_H - 1)} < \frac{\left(\frac{1}{\theta_H} + 1\right)^4}{\frac{1}{\theta_H}\left(1 + 2\frac{1}{\theta_H} - 3\frac{1}{\theta_H^2}\right)},$$

or,

$$\frac{(\theta_H + 1)^4}{\theta_H(3\theta_H^2 - 2\theta_H - 1)} < \frac{(\theta_H + 1)^4}{\theta_H(\theta_H^2 + 2\theta_H - 3)},$$

or

$$\theta_H^2 + 2\theta_H - 3 < 3\theta_H^2 - 2\theta_H - 1,$$

which is equivalent to

$$0 < 2(\theta_H - 1)^2,$$

which is always true.

Next, we wish to show that $P_1^{s*} < 1/2 < P_1^{f*}$. Bearing in mind that $a_1^{s*} = a_3^{s*}$, to establish that $P_1^{s*} < 1/2$ it is sufficient to show that:

$$\bar{\pi}\frac{\theta_H}{\theta_H + 1} + (1 - \bar{\pi})\frac{\theta_L}{\theta_L + 1} < 1/2,$$

when $\theta_L = 1/\theta_H$. It is immediate to show that this is true if and only if $\bar{\pi} < 1/2$. Since $P_1^{s*}P_1^{f*} = 1/4$ for $\theta_L = 1/\theta_H$, and bearing in mind that $a_1^{f*} = a_2^{f*}$, we therefore conclude that $\bar{\pi} < 1/2 \Rightarrow P_1^{s*} < 1/2 < P_1^{f*}$.

Proof of Lemma 1: To show that the best response functions of players in the final, $R_i^f(a_{-i}^f)$, $i \in \{1, 2\}$, are quasi-concave, we focus on the best response of the overconfident player, player 1, and the reasoning extends to the rival player 2. We show that (i) the slope of player 1's best response function is strictly positive for $a_2^f = 0$, i.e. $(\partial R_1^f/\partial a_2^f)\big|_{a_2^f=0} > 0$, (ii) that it it strictly negative for $a_2^f \to \infty$, i.e. $(\partial R_1^f/\partial a_2^f)\big|_{a_2^f\to\infty} < 0$, and (iii) that whenever $\partial R_1^f/\partial a_2^f = 0$, then $\partial^2 R_1^f/(\partial a_2^f)^2 < 0$.

By implicit differentiation of the first-order condition of player 1 as given by equation

(15), we deduce that the sign of the slope of player 1's best response is given by:

$$\text{sgn}\left\{\frac{\partial R_1^f}{\partial a_2^f}\right\} = \text{sgn}\left\{\frac{\partial^2 E[U_1^f(a_1^f, a_2^f)]}{\partial a_1^f \partial a_2^f}\right\} = \text{sgn}\left\{\frac{\tilde{\mu}\theta_H(\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3}\right\}. \tag{23}$$

Points (i) and (ii) are immediately deduced upon observing the above expression. Turning next to (iii), define first $\phi(a_2^f) = \frac{\partial^2 E[U(a_1^f, a_2^f)]}{\partial a_1^f \partial a_2^f}$. To establish (iii) it is then sufficient to show that when $\phi(a_2^f) = 0$, then $\phi'(a_2^f) < 0$. We thus compute $\phi'(a_2^f)$ which is given by:

$$\phi'(a_2^f) = -\frac{\tilde{\mu}\theta_H}{(\theta_H a_1^f + a_2^f)^3} - \frac{3\tilde{\mu}\theta_H(\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^4} - \frac{(1-\tilde{\mu})\theta_L}{(\theta_L a_1^f + a_2^f)^3} - \frac{3(1-\tilde{\mu})\theta_L(\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^4}.$$

Substituting for $\phi(a_2^f) = 0$, we can show that the above expression can be re-expressed as:

$$\phi'(a_2^f) = \frac{\tilde{\mu}\theta_H}{(\theta_H a_1^f + a_2^f)^3}\left(\frac{(\theta_H - \theta_L)a_1^f}{\theta_L a_1^f - a_2^f} - 3(\theta_H a_1^f - a_2^f)\left(\frac{1}{\theta_H a_1^f + a_2^f} - \frac{1}{\theta_L a_1^f + a_2^f}\right)\right),$$

or,

$$\phi'(a_2^f) = \frac{\tilde{\mu}\theta_H 4(\theta_H - \theta_L)a_1^f}{(\theta_H a_1^f + a_2^f)^3(\theta_L a_1^f - a_2^f)} < 0,$$

with the sign following from the observation that to have $\phi(a_2^f) = 0$, it is necessary that $\theta_L a_1^f - a_2^f < 0$.

Proof of Lemma 2: To prove uniqueness, we show that the contraction mapping $\partial R_1^f(a_2^f)/\partial a_2^f . \partial R_2^f(a_1^f)/\partial a_1^f$ is smaller to 1. The slopes of $R_1^f(a_2^f)$ and $R_2^f(a_1^f)$ are respectively given by:

$$\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} = \frac{\frac{\tilde{\mu}\theta_H(\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3}}{2a_2^f\left[\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\tilde{\mu})\theta_L^2}{(\theta_L a_1^f + a_2^f)^3}\right]},$$

and,

$$\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} = -\frac{\frac{\mu\theta_H(\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\mu)\theta_L(\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3}}{2a_1^f\left[\frac{\mu\theta_H}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\mu)\theta_L}{(\theta_L a_1^f + a_2^f)^3}\right]}.$$

Observe first that if $\theta_L a_1^f \geq a_2^f$, then $\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} > 0$ and $\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} < 0$. Next, observe that if $\theta_H a_1^f \leq a_2^f$, then $\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} < 0$ and $\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} > 0$. In both cases the product of the slopes of the best responses is negative and the contraction mapping is smaller to 1. A necessary condition for the slopes of the best responses at equilibrium to be of equal sign is that

$$\theta_H a_1^f - a_2^f > 0 > \theta_L a_1^f - a_2^f. \tag{24}$$

We now demonstrate that at equilibrium it is impossible for both best responses to be negatively sloped. Denote $\Psi_K = \frac{\theta_K(\theta_K a_1^f - a_2^f)}{(\theta_K a_1^f + a_2^f)^3}$, $K = \{H, L\}$. Since at equilibrium a_1^{f*} and a_2^{f*} cannot be negative, when condition (24) holds we have $\Psi_H > 0$ and $\Psi_L < 0$. Accordingly, the sign of the slope of $R_1^f(a_2^{f*})$ is given by the sign of $\tilde{\mu}\Psi_K + (1-\tilde{\mu})\Psi_L$, and the sign of the slope of $R_2^f(a_1^{f*})$ is given by the sign of $\mu\Psi_K + (1-\mu)\Psi_L$. Assume then that the sign of the slope of $R_2^f(a_1^{f*})$ is negative. For this to be the case when $\Psi_H > 0$ and $\Psi_L < 0$ we must have $-\frac{\Psi_H}{\Psi_L} > \frac{1-\mu}{\mu}$. Thence, for $R_1^f(a_2^{f*})$ to also be negative when $\Psi_H > 0$ and $\Psi_L < 0$, we need that $-\frac{\Psi_H}{\Psi_L} < \frac{1-\tilde{\mu}}{\tilde{\mu}}$. For both these inequalities to hold, we need that $\frac{1-\tilde{\mu}}{\tilde{\mu}} > \frac{1-\mu}{\mu}$, which contradicts $\tilde{\mu} > \mu$. Hence, at equilibrium it is impossible for both best responses to be negatively sloped.

We are left with the case where at equilibrium both best responses are positively sloped. To next show that the contraction mapping is smaller to 1. This is equivalent to inquiring whether the following inequality is true:

$$\left(\frac{\tilde{\mu}\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)\left(\frac{\mu\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\mu)\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)$$
$$+4a_1^{f*}a_2^{f*}\left(\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)\left(\frac{\mu\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\mu)\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right) > 0.$$

We then drop from the above expression the following two positive terms:

$$\left(\frac{\tilde{\mu}\,\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\,\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)\frac{\mu\,\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3}$$
$$+\ 4a_1^{f*}a_2^{f*}\left(\frac{\tilde{\mu}\,\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\,\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)\frac{\mu\,\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3}.$$

The original expression is then necessarily true if:

$$\left(\frac{\tilde{\mu}\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right)(\theta_L a_1^{f*} - a_2^{f*})$$
$$+4a_1^{f*}a_2^{f*}\left(\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right) > 0.$$

or,

$$\frac{\tilde{\mu}\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3}\Big((\theta_H a_1^{f*} - a_2^{f*})(\theta_L a_1^{f*} - a_2^{f*}) + 4a_1^{f*}a_2^{f*}\theta_H\Big) + \frac{(1-\tilde{\mu})\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^3}\Big((\theta_L a_1^{f*} - a_2^{f*})^2 + 4a_1^{f*}a_2^{f*}\theta_L\Big) > 0.$$

It is then sufficient to prove that the first expression is true, or:

$$\theta_L\theta_H(a_1^{f*})^2 + (a_2^{f*})^2 - \theta_L a_1^{f*}a_2^{f*} + 3a_1^{f*}a_2^{f*}\theta_H > 0,$$

and this expression is always true since $\theta_H > \theta_L$.

Proof of Proposition 4: Let $\phi_K = \frac{\theta_K}{(\theta_K a_1^f + a_2^f)^2}$, $K = \{L, H\}$. The first-order conditions become

$$[\tilde{\mu}\phi_H + (1-\tilde{\mu})\phi_L]\, a_2^f v = c,$$

and

$$[\mu\phi_H + (1-\mu)\phi_L]\, a_1^f v = c.$$

Let us study the sign of

$$\begin{aligned}\Xi &= [\tilde{\mu}\phi_H + (1-\tilde{\mu})\phi_L] - [\mu\phi_H + (1-\mu)\phi_L]\\ &= \tilde{\mu}(\phi_H - \phi_L) - \mu(\phi_H - \phi_L)\\ &= (\tilde{\mu} - \mu)(\phi_H - \phi_L)\\ &= (\tilde{\mu} - \mu)(\theta_H - \theta_L)\frac{(a^f)_2^2 - \theta_L\theta_H(a_1^f)^2}{(\theta_H a_1^f + a_2^f)^2(\theta_L a_1^f + a_2^f)^2}\end{aligned}$$

If we know the sign of Ξ, then we know which player exerts higher effort as $\Xi = 0$ implies $a_1^{f*} = a_2^{f*}$, $\Xi > 0$ implies $a_1^{f*} > a_2^{f*}$, and $\Xi < 0$ implies $a_1^{f*} < a_2^{f*}$. Note that $\tilde{\pi} > \pi$ implies $\tilde{\mu} - \mu > 0$. Hence, to determine the sign of Ξ we only need to consider the sign of the numerator of the third term. To do that we consider three possible situations: (i) $\theta_L\theta_H \in \{0, 1\}$; (ii) $\theta_L\theta_H \in (0, 1)$; and (iii) $\theta_L\theta_H > 1$.

Consider case (i): $\theta_L\theta_H \in \{0, 1\}$. When $\theta_L\theta_H = 0$, then we must have $\theta_L = 0$. Substituting $\theta_L = 0$ in the expressions for the posterior beliefs we have $\tilde{\mu} = \mu = 1$, and hence $\Xi = 0$ and $a_1^{f*} = a_2^{f*} = v\theta_H/c(\theta_H + 1)^2$. When $\theta_L\theta_H = 1$, the sign of Ξ is given by $(a_2^f)^2 - (a_1^f)^2$. If $a_2^f = a_1^f$, then $\Xi = 0$ and both first-order conditions are satisfied. If $a_2^f > a_1^f$, then $\Xi > 0$, and the first-order conditions are violated. If $a_2^f < a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. Hence, when $\theta_L\theta_H = 1$ the equilibrium satisfies $a_1^{f*} = a_2^{f*} = v\theta_H/c(\theta_H + 1)^2$.

Consider case (ii): $\theta_L\theta_H \in (0, 1)$. The sign of Ξ is given by $(a_2^f)^2 - \theta_L\theta_H(a_1^f)^2$. If

$a_2^f = a_1^f$, then $\Xi > 0$, and the first-order conditions are violated. If $a_2^f > a_1^f$, then $\Xi > 0$, and the first-order conditions are violated. For $a_2^f < a_1^f$ there are three subcases: (a) if $\theta_L\theta_H(a_1^f)^2 < (a_2^f)^2 < (a_1^f)^2$, then $\Xi > 0$, and there will exist values of a_1^f and a_2^f that satisfy the first-order conditions; (b) if $(a_2^f)^2 = \theta_L\theta_H(a_1^f)^2$, then $\Xi = 0$, and the first-order conditions are violated; (c) if $(a_2^f)^2 < \theta_L\theta_H(a_1^f)^2$, then $\Xi < 0$, and the first-order conditions are violated. Hence, when $\theta_L\theta_H \in (0,1)$ the equilibrium satisfies $a_1^{f*} > a_2^{f*}$.

Consider case (iii): $\theta_L\theta_H > 1$. The sign of Ξ is given by $(a_2^f)^2 - \theta_L\theta_H(a_1^f)^2$. If $a_2^f = a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. If $a_2^f < a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. For $a_2^f > a_1^f$ there are three subcases: (a) if $(a_2^f)^2 < \theta_L\theta_H(a_1^f)^2$, then $\Xi < 0$, and there will exist values of a_1^f and a_2^f that satisfy the first-order conditions; (b) if $(a_2^f)^2 = \theta_L\theta_H(a_1^f)^2$, then $\Xi = 0$, and the first-order conditions are violated; (c) if $(a_2^f)^2 > \theta_L\theta_H(a_1^f)^2$, then $\Xi > 0$, and the first-order conditions are violated. Hence, when $\theta_L\theta_H > 1$ the equilibrium satisfies $a_2^{f*} > a_1^{f*}$.

Proof of Lemma 3: The sign of $\partial R_1^f(a_2^f)/\partial\tilde{\mu}$ is given by the sign of $\partial^2\tilde{E}[U_1^f(a_1^f, a_2^f)]/\partial a_1^f\partial\tilde{\mu}$, which is given by:

$$\begin{aligned} \operatorname{sgn}\{\theta_H(\theta_L a_1^f + a_2^f)^2 - \theta_L(\theta_H a_1^f + a_2^f)^2\} &= \operatorname{sgn}\{(\theta_H\theta_L^2 - \theta_L\theta_H^2)(a_1^f)^2 + (\theta_H - \theta_L)(a_2^f)^2\} \\ &= \operatorname{sgn}\{(\theta_H - \theta_L)((a_2^f)^2 - \theta_L\theta_H(a_1^f)^2)\}. \end{aligned}$$

Hence, if $a_2^f < a_1^f\sqrt{\theta_L\theta_H}$, then $\partial R_1^f(a_2^f)/\partial\tilde{\mu} < 0$. However, if $a_2^f > a_1^f\sqrt{\theta_L\theta_H}$, then $\partial R_1^f(a_2^f)/\partial\tilde{\mu} > 0$.

Proof of Proposition 5: Making use of the Envelope theorem, we deduce that the effect of the newcomer's posterior belief $\tilde{\mu}$ on his equilibrium perceived expected utility is given by:

$$\frac{d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{d\tilde{\mu}} = \frac{\partial\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{\partial\tilde{\mu}} + \frac{\partial\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{\partial a_2^{f*}}\frac{\partial a_2^{f*}}{\partial\tilde{\mu}}$$

$$= \underbrace{\left[\frac{\theta_H a_1^{f*}}{\theta_H a_1^{f*} + a_2^{f*}} - \frac{\theta_L a_1^{f*}}{\theta_L a_1^{f*} + a_2^{f*}}\right]}_{>0} v - \underbrace{\left[\tilde{\mu}\frac{\theta_H a_1^{f*}}{(\theta_H a_1^{f*} + a_2^{f*})^2} + (1-\tilde{\mu})\frac{\theta_L a_1^{f*}}{(\theta_L a_1^{f*} + a_2^{f*})^2}\right]}_{>0} v \underbrace{\frac{\partial a_2^{f*}}{\partial\tilde{\mu}}}_{?} \quad (25)$$

Making use of the Envelope theorem, we deduce that the effect of the incumbent's posterior belief μ on the newcomer's equilibrium perceived expected utility is given by:

$$\begin{aligned}\frac{d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})]}{d\mu} &= \frac{\partial \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})]}{\partial \mu} + \frac{\partial \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})]}{\partial a_2^{f*}} \frac{\partial a_2^{f*}}{\partial \mu} \\ &= -\underbrace{\left[\tilde{\mu}\frac{\theta_H a_1^{f*}}{(\theta_H a_1^{f*} + a_2^{f*})^2} + (1-\tilde{\mu})\frac{\theta_L a_1^{f*}}{(\theta_L a_1^{f*} + a_2^{f*})^2}\right]}_{>0} \frac{\partial a_2^{f*}}{\partial \mu}.\end{aligned}$$

Observe that the perceived expected utility of the newcomer in the final is a function of $\tilde{\mu}$, and that it depends on μ only through its effect on the players' equilibrium efforts. We therefore deduce that $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})]/d\mu \gtreqless 0$ if $\partial a_2^{f*}/\partial \mu \lesseqgtr 0$.

From the first-order condition (15) we know that $R_1^f(a_2^f)$ is independent of μ, for a fixed $\tilde{\mu}$. Therefore $\partial a_2^{f*}/\partial \mu \lesseqgtr 0 \Leftrightarrow \partial R_2^f(a_1^{f*})/\partial \mu \lesseqgtr 0$. Using the first-order condition of player 2 as given by (16), we deduce that the sign of $\partial R_2^f(a_1^{f*})/\partial \mu$ is given by:

$$\text{sgn}\left\{\frac{\partial R_2^f(a_1^{f*})}{\partial \mu}\right\} = \frac{\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^2} - \frac{\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^2},$$

which is given by:

$$\begin{aligned}\text{sgn}\{\theta_H(\theta_L a_1^{f*} + a_2^{f*})^2 - \theta_L(\theta_H a_1^{f*} + a_2^{f*})^2\} &= \text{sgn}\{(\theta_H\theta_L^2 - \theta_L\theta_H^2)(a_1^{f*})^2 + (\theta_H - \theta_L)(a_2^{f*})^2\} \\ &= \text{sgn}\{(\theta_H - \theta_L)((a_2^{f*})^2 - \theta_L\theta_H(a_1^{f*})^2)\}.\end{aligned}$$

Hence, if $a_2^{f*} < a_1^{f*}\sqrt{\theta_L\theta_H}$, then $\partial R_2^f(a_1^{f*})/\partial \mu < 0$. However, if $a_2^{f*} > a_1^{f*}\sqrt{\theta_L\theta_H}$, then $\partial R_2^f(a_1^{f*})/\partial \mu > 0$.

Case (i): $\theta_L\theta_H = 1$. From Proposition 4 part (i), we know that if $\theta_L\theta_H = 1$, then $a_1^{f*} = a_2^{f*}$. Substituting for $\theta_L = 1/\theta_H$ and $a^{f*} = a_1^{f*} = a_2^{f*}$ in the first-order condition (15), we obtain:

$$a^{f*} = \frac{\theta_H}{(1+\theta_H)^2}\frac{v}{c}.$$

We thus obtain that the players' equilibrium efforts are independent of their posterior beliefs, which necessarily implies that

$$\frac{d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{d\tilde{\mu}} = \frac{\partial \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{\partial \tilde{\mu}} = \frac{\theta_H - 1}{\theta_H + 1}v > 0,$$

and

$$\frac{d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]}{d\mu} = 0.$$

Case (ii): $\theta_L\theta_H \in (0, 1)$. Observe first that, for a fixed value of μ, $\tilde{\mu}$ does not affect the best response of player 2 as implicitly defined in (16). Second, we know from Lemma 3 that the best response function of player 1, as implicitly defined in (15), is shifting outwards with $\tilde{\mu}$, for a fixed value of μ, if and only if the sign of the following expression is positive:

$$\frac{\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^2} - \frac{\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^2}.$$

In the proof of Proposition 3 we demonstrate that the sign of this expression is positive if and only if $\theta_L\theta_H \in (0,1)$. Consequently, if $\theta_L\theta_H \in (0,1)$, then an increase in $\tilde{\mu}$ shifts outwards the best response of player 1 leaving the best response of player 2 unchanged, thence resulting in a higher equilibrium effort a_1^{f*}, and, given the quasi-concavity of player 2's best response function, the true as well as the perceived equilibrium winning probability of player 1 increases.

Assume that $\tilde{\mu} = \mu$, which implies that $a_1^{f*} = a_2^{f*}$. Using Lemma 2, the sign of the slope of the best response of player 2 at equilibrium, $\partial R_2^f(a_1^{f*})/\partial a_1^{f*}$, is given by:

$$\begin{aligned} \operatorname{sgn}\left\{\frac{R_2^f(a_1^{f*})}{\partial a_1^{f*}}\right\} &= -\operatorname{sgn}\left\{\mu\frac{\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + (1-\mu)\frac{\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3}\right\} \\ &= -\operatorname{sgn}\left\{\mu\frac{\theta_H(\theta_H - 1)}{(\theta_H+1)^3} + (1-\mu)\frac{\theta_L(\theta_L - 1)}{(\theta_L+1)^3}\right\}. \end{aligned}$$

Observe that the above expression is decreasing in μ since $\theta_H > 1 > \theta_L$. Moreover for $\mu = 0$, the expression is positive, and for $\mu = 1$ it is negative. We can then deduce that there exists a unique $\mu = \bar{\mu}$, such that for $\mu \lesseqgtr \bar{\mu}$, $\partial R_2^f(a_1^{f*})/\partial a_1^{f*} \gtreqless 0$. This $\bar{\mu}$ is defined as:

$$\bar{\mu} = \frac{\frac{\theta_L(1-\theta_L)}{(\theta_L+1)^3}}{\frac{\theta_L(1-\theta_L)}{(\theta_L+1)^3} + \frac{\theta_H(\theta_H-1)}{(\theta_H+1)^3}} = \frac{\theta_L(1-\theta_L)(\theta_H+1)^3}{\theta_L(1-\theta_L)(\theta_H+1)^3 + \theta_H(\theta_H-1)(\theta_L+1)^3}.$$

It follows that for $\mu = \bar{\mu}$, $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$.

Consider next any $\tilde{\mu} > \mu$. We know from above, that the slope of the best response of player 2 at the 45^o line is positive for $\mu < \bar{\mu}$, nil for $\mu = \bar{\mu}$, and negative for $\mu > \bar{\mu}$. Moreover, we know from Lemma 1 that the best response functions are quasi-concave. In addition, the best response of player 2 is independent of $\tilde{\mu}$. We can therefore deduce that for any $\tilde{\mu} > \mu$, and provided $\mu > \bar{\mu}$, the best response of player 1 intersects the best response of player 2 below the 45^o line where $a_1^{f*} > a_2^{f*}$ and where the best response of player 2 has a negative slope. Hence, for $\mu > \bar{\mu}$, $da_2^{f*}/d\tilde{\mu} < 0$, and $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$.

For $\mu < \bar{\mu}$, the slope of the best response of player 2 is positive on the 45^o line, in which case an increase in $\tilde{\mu}$ leads to an increase in a_2^{f*} when the slope of the best response of player 2 is positive at equilibrium. Consequently, we are unable to determine the effect of $\tilde{\mu}$ on the equilibrium perceived expected utility of player 1 in such instances.

Finally, observe that from Proposition 4 part (ii) we know that $\theta_L\theta_H \in (0,1)$ implies $a_2^{f*} > a_1^{f*}\sqrt{\theta_L\theta_H}$, which in turn, implies $\partial R_2^f(a_1^{f*})/\partial\mu > 0$. Hence, $\partial a_2^{f*}/\partial\mu > 0$ which, in turn, implies $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})] < 0$.

Case (iii): If $\theta_L\theta_H > 1$. From Proposition 4 part (iii) we know that $\theta_L\theta_H > 1$ implies $a_2^{f*} < a_1^{f*}\sqrt{\theta_L\theta_H}$, which in turn, thanks to Lemma 3 implies $\partial R_1^f(a_2^{f*})/\partial\tilde{\mu} < 0$. Since an increase in $\tilde{\mu}$, for a fixed value of μ, shifts the best response of player 1 inwards while leaving the best response of player 2 unchanged, this results in a lower equilibrium effort of player 1 as well as a lower true equilibrium winning probability. Yet, the effect on player 1's perceived equilibrium winning probability is undetermined in this case since, even though the increase in $\tilde{\mu}$ raises the perceived equilibrium winning probability, the change in players' efforts pushes the perceived equilibrium winning probability in the other direction.

If we then consider $\mu = \bar{\mu}$ as above, it follows that $E[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$. For $\mu < \bar{\mu}$, the slope of the best response of player 2 is positive above the 45^o line, and any increase in $\tilde{\mu}$ will then result in reductions of a_2^{f*}. Hence, for $\mu < \bar{\mu}$, $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})]/d\tilde{\mu} > 0$. Last for $\mu > \bar{\mu}$, for similar reasons to the ones in Case (ii), the effect of $\tilde{\mu}$ on the equilibrium expected utility of player 1 us undetermined.

Finally, observe that from Proposition 4 part (iii) we know that $\theta_L\theta_H > 1$ implies $a_2^{f*} < a_1^{f*}\sqrt{\theta_L\theta_H}$ which in turn implies $\partial R_2^f(a_1^{f*})/\partial\mu < 0$. Hence, $\partial a_2^{f*}/\partial\mu < 0$ which, in turn, implies $d\tilde{E}[U_1^f(a_1^{f*}, a_2^{f*})] > 0$.

Second-order conditions of the semifinal with an overconfident newcomer when $\theta_L\theta_H = 1$: The first derivative of the newcomer's perceived expected utility in the semifinal is:

$$\frac{\partial\tilde{E}[U_1^s]}{\partial a_1^s} = \frac{\partial\tilde{P}_1^s}{\partial a_1^s}\left[\left(\frac{1}{\theta_H+1}\right)^2 + \frac{\theta_H-1}{\theta_H+1}\tilde{\mu}\right]v + \tilde{P}_1^s(a_1^s, a_3^s)\frac{\theta_H-1}{\theta_H+1}v\frac{\partial\tilde{\mu}}{\partial a_1^s} - c.$$

The second derivative is therefore given by:

$$\frac{\partial^2\tilde{E}[U_1^s]}{(\partial a_1^s)^2} = \frac{\partial^2\tilde{P}_1^s}{(\partial a_1^s)^2}\left[\left(\frac{1}{\theta_H+1}\right)^2 + \frac{\theta_H-1}{\theta_H+1}\tilde{\mu}\right]v + 2\frac{\partial\tilde{P}_1^s}{\partial a_1^s}\frac{\theta_H-1}{\theta_H+1}v\frac{\partial\tilde{\mu}}{\partial a_1^s} + \tilde{P}_1^s\frac{\theta_H-1}{\theta_H+1}v\frac{\partial^2\tilde{\mu}}{(\partial a_1^s)^2}.$$

Since $\frac{\partial^2\tilde{P}_1^s}{(\partial a_1^s)^2} < 0$, this expression is a fortiori negative if:

$$\left[\frac{\partial^2\tilde{P}_1^s}{(\partial a_1^s)^2}\tilde{\mu} + 2\frac{\partial\tilde{P}_1^s}{\partial a_1^s}\frac{\partial\tilde{\mu}}{\partial a_1^s} + \tilde{P}_1^s\frac{\partial^2\tilde{\mu}}{(\partial a_1^s)^2}\right]\underbrace{\frac{\theta_H-1}{\theta_H+1}v}_{>0} < 0.$$

We thus need to show that the term inside squared brackets is negative. Substituting for the appropriate terms, and simplifying, we can show that the term inside squared brackets is given by:

$$-\frac{2\tilde{\pi}\theta_H^2 a_3^s}{(\theta_H a_1^s + a_3^s)^3} < 0.$$

Hence, the second-order condition for the newcomer is satisfied. It is immediate to verify that the second-order condition for the incumbent is also satisfied.

Proof of Proposition 6: Computing each term of (20) separately, we have:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \frac{\partial \tilde{P}_1^s(a_1^s, a_3^s; \tilde{\pi})}{\partial a_1^s} = \left[\frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2} + \tilde{\pi}\left(\frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} - \frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2}\right)\right]$$

Imposing $\theta_L = 1/\theta_H$, this reads as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \left[\frac{\theta_H a_3^s}{(a_1^s + \theta_H a_3^s)^2} + \tilde{\pi}\left(\frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} - \frac{\theta_H a_3^s}{(a_1^s + \theta_H a_3^s)^2}\right)\right]$$

Evaluating this expression at $a_1^s = a_3^s = a$, this expression becomes:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \frac{\theta_H}{a(\theta_H + 1)^2}.$$

Turning next to the squared-bracketed term of expression (20), since a_2^{f*} is independent of posterior beliefs, the first multiplicative term is then nil. Focusing next on the last term of expression (20), we have that when evaluated at $a_1^s = a_3^s = a$, then:

$$\begin{aligned}
\frac{\partial \tilde{E}\left[U_1^f\right]}{\partial \tilde{\mu}} g \frac{\partial \tilde{\mu}}{\partial a_1^s} &= -\left(\frac{\theta_H}{\theta_H + 1} - \frac{\theta_L}{\theta_L + 1}\right) \frac{\tilde{\pi}(1-\tilde{\pi})\theta_L\theta_H(\theta_H - \theta_L)}{\left[\theta_L\theta_H + \tilde{\pi}\theta_H + (1-\tilde{\pi})\theta_L\right]^2 a} v \\
&= -\frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi}(1-\tilde{\pi})\theta_H(\theta_H^2 - 1)}{\left[\theta_H + \tilde{\pi}\theta_H^2 + (1-\tilde{\pi})\right]^2 a} v \\
&= -\frac{\tilde{\pi}(1-\tilde{\pi})\theta_H(\theta_H - 1)^2}{\left[\theta_H + \tilde{\pi}\theta_H^2 + (1-\tilde{\pi})\right]^2 a} v \\
&= -\frac{\tilde{\pi}(1-\tilde{\pi})\theta_H(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)^2 a} v
\end{aligned}$$

The first-order condition of player 1 when evaluated at $a_1^s = a_3^s = a$ is then equal to:

$$\frac{\theta_H}{a(\theta_H + 1)^2} \frac{1 - \tilde{\pi} + \tilde{\pi}\,\theta_H^3}{(\theta_H + 1)^2\,(1 - \tilde{\pi} + \tilde{\pi}\,\theta_H)} v - \frac{1 - \tilde{\pi} + \tilde{\pi}\theta_H}{\theta_H + 1} \frac{\tilde{\pi}(1-\tilde{\pi})\theta_H(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)^2 a} v = c,$$

or

$$\frac{\theta_H}{a(\theta_H + 1)^2} \frac{1 - \tilde{\pi} + \tilde{\pi}\,\theta_H^3}{(\theta_H + 1)^2\,(1 - \tilde{\pi} + \tilde{\pi}\,\theta_H)} v - \frac{\theta_H}{\theta_H + 1} \frac{\tilde{\pi}(1-\tilde{\pi})(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H) a} v = c,$$

or

$$\frac{\theta_H \left(1-\tilde{\pi}+\tilde{\pi}\,\theta_H^2\right)}{(\theta_H+1)^4}\frac{v}{a}=c$$

We next explore the first-order condition of player 3 which, when evaluated at $a_1^s = a_3^s = a$, is given by:

$$\frac{\theta_H}{4(\theta_H+1)^2}\frac{v}{a}=c.$$

Using the first-order derivatives, we can next express $\Psi(\tilde{\pi})$ evaluated at $a_1^s = a_3^s = a$ as:

$$\begin{aligned}\Psi(\tilde{\pi}) &= \left.\frac{\partial \tilde{E}[U_1^s(a_1^s,a_3^s;\tilde{\pi})]}{\partial a_1^s}\right|_{a_1^s=a_3^s=a} - \left.\frac{\partial E[U_3^s(a_1^s,a_3^s)]}{\partial a_3^s}\right|_{a_1^s=a_3^s=a} \\ &= \left[\frac{\theta_H \left(1-\tilde{\pi}+\tilde{\pi}\,\theta_H^2\right)}{(\theta_H+1)^4}-\frac{\theta_H}{4(\theta_H+1)^2}\right]\frac{v}{a} \\ &= -\frac{\theta_H(\theta_H-1)\left[\theta_H+3-4\tilde{\pi}(\theta_H+1)\right]}{4(\theta_H+1)^4}\frac{v}{a},\end{aligned}$$

with $\Psi(\tilde{\pi}) \gtreqless 0 \Leftrightarrow a_1^{s*} \gtreqless a_3^{s*}$. It follows that $\Psi(\tilde{\pi}) = 0$ when

$$\breve{\pi}=\frac{\theta_H+3}{4(\theta_H+1)}$$

So, when the prior belief of the overconfident player 1, $\tilde{\pi}$, is higher than $\breve{\pi}$, he exerts more effort than the rational rival in the semifinal. Note that $\theta_H > 1$ implies $\breve{\pi} \in (1/4, 1/2)$ and that the higher is θ_H the closer is $\breve{\pi}$ to $1/4$.

Proof of Proposition 7: To prove this result, we first show that $P_1^{s*}P_1^{f*}$ is monotonically increasing in the ratio a_1^{s*}/a_3^{s*}, and we then show that this ratio is monotonically increasing in b^s.

Making use of equations (21) and (22), the newcomer's ex-ante true probability of winning the elimination contest is given by:

$$P_1^{s*}P_1^{f*}=\left[\pi\frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*}+a_3^{s*}}+(1-\pi)\frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H+a_3^{s*}}\right]\frac{1+\mu(\theta_H-1)}{\theta_H+1},$$

which after substituting for μ as given by Equation (4) becomes:

$$P_1^{s*}P_1^{f*}=\left[\pi\frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*}+a_3^{s*}}+(1-\pi)\frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H+a_3^{s*}}\right]\frac{1+\frac{\pi\theta_H(a_1^{s*}/\theta_H+a_3^{s*})}{a_1^{s*}+(\pi\theta_H+(1-\pi)/\theta_H)a_3^{s*}}(\theta_H-1)}{\theta_H+1}.$$

Rewriting the above expression as a function of $x = a_1^s/a_3^s$, we obtain:

$$\begin{aligned} P_1^{s*}P_1^{f*} &= \left[\frac{\pi\theta_H}{\theta_H x+1}+\frac{(1-\pi)}{x+\theta_H}\right]\left[1+\frac{\pi\theta_H(x/\theta_H+1)(\theta_H-1)}{x+(\pi\theta_H+(1-\pi)/\theta_H)}\right]\frac{x}{\theta_H+1} \\ &= \frac{\theta_H x+\pi\theta_H^2+(1-\pi)}{(\theta_H x+1)(x+\theta_H)}\left[\frac{\theta_H x+\pi\theta_H^2+(1-\pi)+\pi\theta_H(x+\theta_H)(\theta_H-1)}{\theta_H x+\pi\theta_H^2+(1-\pi)}\right]\frac{x}{\theta_H+1} \\ &= \frac{(1-\pi)(\theta_H x+1)+\pi\theta_H^2(x+\theta_H)}{(\theta_H x+1)(x+\theta_H)}\frac{x}{\theta_H+1} \\ &= \left[(1-\pi)\frac{x}{x+\theta_H}+\pi\theta_H\frac{\theta_H x}{\theta_H x+1}\right]\frac{1}{\theta_H+1}. \end{aligned}$$

Since the two terms inside the squared brackets are increasing in x, we deduce that $P_1^{s*}P_1^{f*}$ is equally increasing in x.

We next prove that $dx/db^s > 0$. At optimality, the first order conditions of both players 1 and 3 ought to be simultaneously satisfied. The first order condition of player 1 is given by:

$$\begin{aligned} &\left[\tilde{\pi}\frac{\theta_H a_3^{s*}}{(\theta_H a_1^{s*}+a_3^{s*})^2}+(1-\tilde{\pi})\frac{a_3^{s*}/\theta_H}{(a_1^{s*}/\theta_H+a_3^{s*})^2}\right]\left[\left(\frac{1}{\theta_H+1}\right)^2+\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}\theta_H(a_1^{s*}/\theta_H+a_3^{s*})}{a_1^{s*}+(\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H)a_3^{s*}}\right]v \\ &\qquad -\left[\tilde{\pi}\frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*}+a_3^{s*}}+(1-\tilde{\pi})\frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H+a_3^{s*}}\right]\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}(1-\tilde{\pi})(\theta_H-1/\theta_H)a_3^{s*}}{\left[a_1^{s*}+(\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H)a_3^{s*}\right]^2}v = c \end{aligned}$$

Rewritting this condition as a function of x, we obtain:

$$\begin{aligned} &\left[\tilde{\pi}\frac{\theta_H}{(\theta_H x+1)^2}+(1-\tilde{\pi})\frac{1/\theta_H}{(x/\theta_H+1)^2}\right]\left[\left(\frac{1}{\theta_H+1}\right)^2+\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}\theta_H(x/\theta_H+1)}{x+\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H}\right]\frac{v}{a_3^{s*}} \\ &\qquad -\left[\tilde{\pi}\frac{\theta_H x}{\theta_H x+1}+(1-\tilde{\pi})\frac{x/\theta_H}{x/\theta_H+1}\right]\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}(1-\tilde{\pi})(\theta_H-1/\theta_H)}{\left[x+\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H\right]^2}\frac{v}{a_3^{s*}} = c. \end{aligned}$$

Proceeding likewise for player 3, we obtain:

$$\begin{aligned} \left[\pi\frac{\theta_H a_1^{s*}}{(\theta_H a_1^{s*}+a_3^{s*})^2}+(1-\pi)\frac{a_1^{s*}/\theta_H}{(a_1^{s*}/\theta_H+a_3^{s*})^2}\right]\frac{v}{4} &= c \\ \left[\pi\frac{\theta_H x}{(\theta_H x+1)^2}+(1-\pi)\frac{x/\theta_H}{(x/\theta_H+1)^2}\right]\frac{v}{4a_3^s} &= c. \end{aligned}$$

Combining these two first-order conditions, we obtain:

$$\begin{aligned} \mathcal{A}(x) &= \left[\tilde{\pi}\frac{\theta_H}{(\theta_H x+1)^2}+(1-\tilde{\pi})\frac{1/\theta_H}{(x/\theta_H+1)^2}\right]\left[\left(\frac{1}{\theta_H+1}\right)^2+\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}\theta_H(x/\theta_H+1)}{x+\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H}\right] \\ &- \left[\tilde{\pi}\frac{\theta_H x}{\theta_H x+1}+(1-\tilde{\pi})\frac{x/\theta_H}{x/\theta_H+1}\right]\frac{\theta_H-1}{\theta_H+1}\frac{\tilde{\pi}(1-\tilde{\pi})(\theta_H-1/\theta_H)}{\left[x+\tilde{\pi}\theta_H+(1-\tilde{\pi})/\theta_H\right]^2} \\ &- \left[\pi\frac{\theta_H x}{(\theta_H x+1)^2}+(1-\pi)\frac{x/\theta_H}{(x/\theta_H+1)^2}\right]\frac{1}{4} = 0. \end{aligned}$$

This expression can be re-written as:

$$\mathcal{A}(x) = -\frac{\theta_H}{4(\theta_H+1)^2(\theta_H x^2 + \theta_H^2 x + x + \theta_H)^2}\left[c_1x^3 + c_2x^2 + c_3x + c_4\right] = 0,$$

where:

$$\begin{aligned}
c_1 &= \pi + 2\pi\theta_H + \theta_H^2 + 2(1-\pi)\theta_H^3 + (1-\pi)\theta_H^4 \\
c_2 &= 2\theta_H(\theta_H^2+1) \\
c_3 &= 8\tilde{\pi}\theta_H - 6\theta_H - 2\pi\theta_H - \pi + \theta_H^2 + 2\pi\theta_H^3 + \pi\theta_H^2 - 8\tilde{\pi}\theta_H^3 + 1 \\
c_4 &= -4\big(1 - \tilde{\pi} + \tilde{\pi}\theta_H^4\big)
\end{aligned}$$

Observe that the fraction in $\mathcal{A}(x)$ is always positive, thence implying that the first-order conditions are satisfied if $\mathcal{B}(x) = c_1x^3 + c_2x^2 + c_3x + c_4 = 0$, thence implying that this condition defines the equilibrium value of x. We are interested in the sign of dx/db^s which is the same to the sign of $dx/d\tilde{\pi}$. Applying implicit differentiation to $\mathcal{B}(x)$, we have:

$$\frac{dx}{d\tilde{\pi}} = -\frac{\frac{\partial\mathcal{B}(x)}{\partial\tilde{\pi}}}{\frac{\partial\mathcal{B}(x)}{\partial x}} = -\frac{4(2\theta_H x(1-\theta_H^2) + 1 - \theta_H^4)}{3c_1x^2 + 2c_2x + c_3}.$$

The sign of the numerator is negative, thence implying that the sign of the entire expression is given by the sign of the denominator. Exploiting the fact that $\mathcal{B}(x) = 0$ implies:

$$c_3 = -\frac{c_4 + c_1x^3 + c_2x^2}{x}.$$

Substituting for c_3 in the denominator of the above expression, we deduce that $dx/d\tilde{\pi} > 0$ if:

$$3c_1x^2 + 2c_2x - \frac{c_4}{x} - c_1x^2 - c_2x > 0.$$

Since $x > 0$, simplifying the above expression and multiplying by x, we deduce that $dx/d\tilde{\pi} > 0$ if:

$$2c_1x^3 + c_2x^2 - c_4 > 0,$$

which is necessarily true since $c_1 > 0$, $c_2 > 0$, and $c_4 < 0$.