

Success, Learning, and Overconfidence in Elimination Contests*

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Abstract

We study a two-stage elimination contest in which a newcomer, whose ability is unknown to all, himself included, faces incumbents of known ability. After a semifinal win, the newcomer and his rival in the final both revise their beliefs about his ability, so his overconfidence evolves endogenously: it grows when his ex-ante prospects are poor and shrinks when they are good. The more overconfident the newcomer, the more he values reaching the final and the harder he fights in the semifinal relative to his rival. Because a victory obtained through greater effort reveals less about ability, this raises his chance of reaching the final but lowers his chance of winning it; the first effect dominates, so overconfidence raises his overall probability of winning. The model clarifies when success feeds further overconfidence biases and helps explain why overconfident individuals often rise to the top in organizational or competitive environments.

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1 Introduction

Why do overconfident individuals—who misjudge their abilities—so often reach the top in organizations? For instance, empirical studies show that approximately 40 percent of CEOs of companies listed in the Standard & Poor’s 1500 index are overconfident (Malmendier and Tate, 2015), and that managers in a workplace tournament with performance bonuses are persistently overconfident about how well they will perform (Huffman et al., 2022). In this paper we offer a new explanation: individuals who are both uncertain about their ability and overconfident may work harder than their rivals in a multistage elimination contest.

To illustrate, consider a lawyer who joins a firm mid-career and competes for partnership in an up-or-out promotion contest: he must first be preferred to a long-tenured associate for promotion to senior associate, and then prevail over another senior associate for the single partnership slot. The partners can rely on years of information about the incumbent associates (Rebitzer and Taylor, 2007), whereas the newcomer’s ability to perform at this level is uncertain to all, the newcomer included, and he overestimates his chances of being a high-ability lawyer.

Two consequences follow. First, since the newcomer expects to fare better in the partnership contest than he objectively should, reaching it appears more valuable to him, and he works harder in the first stage relative to his rival. This raises his probability of making it to the next stage of the contest. A promotion obtained through greater effort is less informative about talent, so he enters the second stage of the contest with slightly weaker true prospects, but the gain in the first stage dominates: overconfidence raises his overall probability of becoming a partner.

Second, his overconfidence does not remain constant. After the first promotion, everyone revises upward their assessment of the newcomer, but not by the same amount. When he is a long shot, his rival attributes the promotion largely to luck and barely revises his view, whereas the newcomer, who already gave himself a real chance, reads it as confirmation of his talent, and his overconfidence grows. When he is a promising lawyer, his own belief has little room left to rise while his rival’s still does, and the same promotion tempers his overconfidence.

The law firm is not an isolated case. Promotions to CEO positions display a similar dynamic: boards weigh candidates from inside the firm, whose past performance is well documented, against external recruits whose ability to lead the organization remains largely untested (Parrino, 1997; Zhang and Rajagopalan, 2004). In politics, primaries followed by a general election place a newcomer, uncertain about how voters perceive her, first against rivals from her own party and then against a seasoned opponent from another party, whose reputation is already well established (Crutzen et al., 2010; Andreottola, 2021). Likewise, in competitive grant schemes with successive rounds of evaluation, junior

researchers compete alongside senior colleagues whose publication records are broadly known (Azoulay et al., 2011). In sports, a tennis player who comes through qualifying faces seeded players whose level is documented by years of results, while his own ability to compete at the top level is untested. In each case, the elimination format creates settings where early successes shape expectations about an untested entrant, including the entrant’s own.¹

Are overconfident contenders more likely to win a contest than their unbiased rivals? Does earning a promotion increase a contender’s overconfidence? To address these questions, we develop a formal model that embeds overconfidence—defined as an overestimation of the probability of being high ability—within a two-stage elimination contest with incomplete information. A newcomer, whose ability is unknown to all, himself included, competes against incumbents of known ability, and a semifinal victory leads everyone to revise their beliefs about the newcomer’s ability.

Our analysis answers both questions. First, overconfidence increases a contender’s expected probability of winning an elimination contest. Indeed, a more overconfident newcomer values reaching the final more, and therefore raises his semifinal effort relative to his rival’s. A stronger bias thus increases his probability of winning the semifinal. However, it decreases his probability of winning the final, because a semifinal victory obtained through greater relative effort is less informative about ability. The first effect dominates, so the newcomer’s overall probability of winning the contest increases with his bias.

Second, a promotion can either increase or decrease a contender’s overconfidence. Victories obtained when everyone expects the newcomer to be of low ability boost his overconfidence, whereas victories obtained when everyone expects him to be of high ability dampen it. Early success therefore feeds overconfidence precisely among those least expected to succeed. Unlike existing explanations, this mechanism requires neither biased updating nor selective memory: it arises with perfect recall and Bayesian updating.

The rest of the paper proceeds as follows. Section 2 discusses related literature. Section 3 sets up the model. Section 4 studies the elimination contest with an unbiased newcomer. Section 5 considers the case of an overconfident newcomer. Section 6 concludes the paper. All proofs can be found in the Appendix.

2 Related Literature

Our paper contributes to three strands of the literature: elimination contests, the dynamics of overconfidence, and the effect of learning in contests and tournaments.

First, our contribution to elimination contests is most closely related to Rosen (1986)

¹In organizations and in politics, seniority, internal networks, and party machinery also play a role, but they strengthen the incumbents’ position rather than affect the mechanism we study.

and Chen and Santos-Pinto (2025).² The main focus of Rosen (1986) is to explain why contest organizers set increasingly larger prizes as players advance in an elimination contest. Although most of the analysis is performed under complete information, Rosen (1986) also discusses the extension of his model to an incomplete information elimination contest. Rosen (1986) finds that uncertainty about ability is a force that dampens incentives to perform in the early stages since it creates incentives to experiment to discover own strength. Unlike Rosen, we focus on the effect of overconfidence on effort provision and winning probabilities in elimination contests. We find that uncertainty about ability coupled with overconfidence can instead induce an overconfident player to invest more in the early stage of the elimination contest, thereby revealing that overconfidence can reverse Rosen’s prediction for early stages.

Chen and Santos-Pinto (2025) provide the first formal analysis of overconfidence in elimination contests. They show that in the second stage the overconfident player always exerts less effort than an unbiased rival, and if the prize spread is large and confidence moderate, may exert more effort in the first stage. In their model there is no uncertainty and so the overconfidence bias remains fixed across stages. In our framework instead, there is uncertainty about the overconfident player’s ability, and as a consequence the overconfidence bias changes endogenously from the first to the second stage.³ We show that winning the first stage amplifies the bias when the player’s true prior probability of being high-ability is low and dampens it otherwise. This novel mechanism yields new predictions for efforts and outcomes. In the second stage, when the newcomer is on average weaker, as measured by the geometric mean of his possible abilities, the equilibrium effort of the overconfident newcomer is larger than his rival. The opposite result obtains if the newcomer is on average stronger. Interestingly, therefore, we derive conditions for obtaining the counter-intuitive result of Chen and Santos-Pinto (2025), but also conditions under which the more intuitive prediction that overconfidence raises second stage effort prevails. In addition, we show that the overconfident newcomer’s true probability of winning the contest, measured as the product of the first and second stages true winning probabilities, can increase with his first stage bias.

Second, we contribute to the literature on the emergence and evolution of overconfidence. Gervais and Odean (2001) show that early success always raises confidence about ability when individuals overweight successes relative to failures. Compte and Postlewaite (2004) likewise link confidence to past outcomes through biased recollection, and show that overconfidence can decline with experience. By contrast, our model assumes biased

²Related analyses of elimination contests examine aspects of optimal design and organization, such as rent-seeking structures (Gradstein and Konrad, 1999), information revelation (Zhang and Wang, 2009), optimal seeding (Groh et al., 2012), multi-stage design (Fu and Lu, 2012), and sabotage (Klunover, 2021).

³Another fundamental difference is that in Chen and Santos-Pinto (2025) the players’ winning probabilities are determined by Alcalde and Dahm’s (2007) CSF while here we consider a Tullock CSF. This is important given the prevalence and wide use of the Tullock CSF in multistage contests.

priors but perfect recall and Bayesian updating, and shows that early success can either amplify or reduce overconfidence depending on the newcomer’s ex-ante probability of being high ability; in particular, for a highly overconfident newcomer, winning the semifinal can reduce the bias. Like Bénabou and Tirole (2002), we highlight how effort choices affect beliefs about ability, but our mechanism does not rely on time-inconsistency: players may strategically restrain semifinal effort to boost others’ beliefs and expected final-stage payoffs. Finally, whereas Zábojník (2004) and Benoît and Dubra (2011) study how rational overconfidence can arise from exogenous signals, we instead examine how preexisting confidence biases evolve endogenously through effort choices in a dynamic elimination contest.

Last, our paper is related to the literature on learning in dynamic contests and tournaments. Denter et al. (2022) study how one-sided asymmetric information about effort costs affects behavior in contests, allowing an informed newcomer to signal his type to an uninformed incumbent through costly signal. They show that only newcomers with sufficiently low marginal effort costs benefit from revealing their type in equilibrium, and they also allow for overconfidence. Deng et al. (2024) analyze a contest in which beliefs are distorted about the newcomer’s cost of effort: overconfidence arises when the rival perceives the newcomer’s marginal cost of effort to be lower than it actually is. They show that a firm, by strategically disclosing its private information about the newcomer’s type, can shape these beliefs and thereby influence effort incentives and contest outcomes, with confidence distortions sometimes raising total effort. Our paper differs in three main respects: we model overconfidence as an overestimation of the probability of being high ability, we assume symmetric information, and we analyze a two-stage elimination contest rather than a one-shot contest. We show that overconfidence can improve the newcomer’s chances of winning the elimination contest.

In Altmann et al. (2012), Kubitz (2023), Barbieri and Serena (2025), and Catepillán et al. (2025), each player has private information about either his ability, cost of effort, valuation of the prize, or more generally objective function. We instead assume that nobody (including the newcomer himself) knows the newcomer’s ability. This distinction is essential, since unlike private information setups where players attempt signaling or concealing their known identity, in our setup the newcomer’s first stage effort influences everyone’s beliefs about his ability at the start of the second stage. Indeed, an early victory achieved with little effort leads to a sharper updating of beliefs, since such success is more likely attributed to the winner’s unknown — and potentially high — ability than it would be if the effort had been greater. Such behavior therefore potentially trumps everyone, the newcomer included, and may confer a strategic advantage to the newcomer if he reaches the subsequent round. This mechanism is highlighted in Krähmer (2007) in a repeated contest with symmetric information about players’ abilities. In contrast to Krähmer (2007), we focus on the role of overconfidence in an elimination contest.

3 Set-up

3.1 Players, abilities, and payoffs

Four players compete in a two-stage elimination contest. In the first stage, player 1 faces player 3 in one semifinal and player 2 faces player 4 in the other. The two semifinal winners meet in the final, whose winner obtains a prize worth $v > 0$; all other players obtain 0.⁴

Player 1, the *newcomer*, has either low ability θ_L or high ability θ_H , with $0 < \theta_L < 1 < \theta_H$. Players 2, 3, and 4, the *incumbents*, have ability normalized to 1, and this is common knowledge. The product $\Theta \equiv \theta_L \theta_H$ plays a central role in the analysis: $\sqrt{\Theta}$ is the geometric mean of the newcomer's two possible abilities. We say that the newcomer is *on average weaker* than the incumbents when $\Theta < 1$, *on average as strong* when $\Theta = 1$, and *on average stronger* when $\Theta > 1$. When $\Theta = 1$, a high ability newcomer is as much stronger than the incumbents as a low ability newcomer is weaker than them, in the sense that $\theta_H/1 = 1/\theta_L$.

In each match, player i chooses effort $a_i \geq 0$ at cost ca_i , with $c > 0$. We denote semifinal efforts by a_i^s and final efforts by a_i^f . When player i with ability θ_i faces player j with ability θ_j , player i wins with probability given by the Tullock contest success function

$$P_i(a_i, a_j; \theta_i, \theta_j) = \frac{\theta_i a_i}{\theta_i a_i + \theta_j a_j}. \quad (1)$$

3.2 Beliefs and overconfidence

Nobody, the newcomer included, observes the newcomer's ability. Its true probability of being high is $\pi \in (0, 1)$, and the incumbents hold this correct prior. We compare two cases. An *unbiased* newcomer holds the same prior π . An *overconfident* newcomer instead holds the prior $\tilde{\pi} = \pi + b^s$, where $b^s \in (0, 1 - \pi]$ is his bias at the start of the contest. We therefore model overconfidence as an upwardly biased belief about the probability of being high ability, rather than as an overestimate of a known deterministic ability, effort productivity, or winning probability. This formulation is particularly suited to our setting because the newcomer is uncertain about his own type and learns about it from contest outcomes. It allows the newcomer and his rivals to update differently after the same observed success, so that the newcomer's overconfidence can evolve endogenously across stages. By contrast, an overestimate of a known ability or productivity parameter would naturally remain fixed absent an additional learning mechanism, while directly biasing

⁴All results are unchanged in a three-player setting where the winner of the newcomer's contest faces an incumbent who advances directly to the final. We nevertheless assume that there are four players because this matches the applications discussed in the introduction and the semifinal-final structure they share.

winning probabilities would abstract from the inference about ability that is central to our analysis.

Following Squintani (2006), we assume that players agree to disagree: (i) the incumbents know b^s and consider the newcomer's belief mistaken; (ii) the newcomer considers his own belief correct; and (iii) each player's prior is common knowledge.⁵

3.3 Information and timing

The game unfolds as follows:

1. Nature draws the newcomer's ability; no player observes it.
2. In each semifinal, the two players choose their efforts simultaneously.
3. Semifinal efforts and outcomes are publicly observed.
4. The finalists update their beliefs about the newcomer's ability by Bayes' rule.
5. The two finalists choose their final efforts simultaneously.
6. The winner of the final obtains the prize.

3.4 Belief updating

Let $P_1^s(\theta) \equiv \theta a_1^s / (\theta a_1^s + a_3^s)$ denote the probability that the newcomer wins his semifinal if his ability is θ . If the newcomer wins, the incumbent he faces in the final holds the posterior

$$\mu = \frac{\pi P_1^s(\theta_H)}{\pi P_1^s(\theta_H) + (1 - \pi) P_1^s(\theta_L)}. \quad (2)$$

An unbiased newcomer shares this posterior. An overconfident newcomer instead updates from his own prior $\tilde{\pi}$ and holds the posterior

$$\tilde{\mu} = \frac{\tilde{\pi} P_1^s(\theta_H)}{\tilde{\pi} P_1^s(\theta_H) + (1 - \tilde{\pi}) P_1^s(\theta_L)}. \quad (3)$$

His bias in the final, $b^f \equiv \tilde{\mu} - \mu$, is therefore endogenous: it depends on his initial bias and on semifinal efforts. Comparing b^f with b^s tells us whether a semifinal victory amplifies or dampens overconfidence. If the newcomer loses his semifinal, beliefs about his ability play no further role.

3.5 Equilibrium

We use the perfect Bayesian equilibrium concept, adapted to heterogeneous priors. Games in which players hold different priors are well defined in Harsanyi's framework, which

⁵This approach has been used in related work, e.g., Yildiz (2007) and Santos-Pinto and Sekeris (2025). These assumptions are consistent with the psychology literature on the "blind spot bias," according to which individuals believe that others are more susceptible to biases than themselves (Pronin and Ross, 2002; Pronin and Kugler, 2007).

does not require a common prior (Morris, 1995). Given the belief structure described in Section 3.2, players disagree only about Nature’s move, and correctly anticipate each other’s strategies. A strategy profile specifies each player’s semifinal effort and, for every observed semifinal history, the finalists’ efforts. Together with a belief system, it forms an equilibrium if: (a) after every semifinal history, each finalist’s belief about the newcomer’s ability is obtained from his or her own prior by Bayes’ rule; (b) final efforts form a Bayesian Nash equilibrium (BNE) given these beliefs, each player maximizing expected utility under his or her own beliefs; and (c) semifinal efforts maximize each player’s expected utility under his or her own prior, anticipating the continuation play in (b). Because each semifinal outcome occurs with positive probability for any positive efforts, Bayes’ rule pins down beliefs after every history, including after deviations.⁶ We solve the game by backward induction.

4 Unbiased Newcomer

Before turning to overconfidence, it is useful to understand how uncertainty about ability and learning shape behavior in the elimination contest with an unbiased newcomer. This benchmark isolates a dynamic force that will remain central once overconfidence is introduced: semifinal effort affects not only the probability of advancing, but also what a victory reveals about the newcomer’s ability and hence the value of reaching the final. Characterizing this learning channel first allows us to distinguish the effects generated by uncertainty itself from those generated by overconfident beliefs. The results in this section therefore provide both the benchmark and several of the building blocks for the analysis of the overconfident newcomer in Section 5.

We proceed by backward induction. First, we derive the equilibrium efforts in a final between a newcomer and an incumbent. Second, using the final stage solution, we characterize the equilibrium efforts in the semifinal between a newcomer and an incumbent.

4.1 Final

We study the final between the newcomer, player 1, and the incumbent player 2, without loss of generality. The expected utility of player $i = 1, 2$ in the final is

$$E[U_i^f(a_i^f, a_j^f)] = P_i^f(a_i^f, a_j^f)v - ca_i^f.$$

⁶Because semifinal efforts are observed, a deviation by the newcomer in the semifinal affects the incumbent’s posterior and hence her effort in the final. If efforts were unobservable, the incumbent’s posterior would be computed from equilibrium efforts and would not respond to a deviation. This distinction is immaterial when $\Theta = 1$, since final efforts are then independent of beliefs; all results for that case hold under either assumption. When $\Theta \neq 1$, equilibrium semifinal efforts would differ, as discussed in Section 5.5.

Player 1's expected utility in the final is therefore

$$E[U_1^f(a_1^f, a_2^f)] = \left[\mu \frac{\theta_H a_1^f}{\theta_H a_1^f + a_2^f} + (1 - \mu) \frac{\theta_L a_1^f}{\theta_L a_1^f + a_2^f} \right] v - c a_1^f, \quad (4)$$

and player 2's expected utility in the final is

$$E[U_2^f(a_1^f, a_2^f)] = \left[\mu \frac{a_2^f}{\theta_H a_1^f + a_2^f} + (1 - \mu) \frac{a_2^f}{\theta_L a_1^f + a_2^f} \right] v - c a_2^f, \quad (5)$$

where μ is the players' common posterior belief that player 1 has high ability, given by

$$\mu = \frac{\pi P_1^s(a_1^s, a_3^s; \theta_H)}{\pi P_1^s(a_1^s, a_3^s; \theta_H) + (1 - \pi) P_1^s(a_1^s, a_3^s; \theta_L)} = \frac{\pi \theta_H (\theta_L a_1^s + a_3^s)}{\theta_L \theta_H a_1^s + [\pi \theta_H + (1 - \pi) \theta_L] a_3^s}, \quad (6)$$

where (a_1^s, a_3^s) are the semifinal efforts for players 1 and 3. Note that μ is decreasing with a_1^s since

$$\frac{\partial \mu}{\partial a_1^s} = - \frac{\pi(1 - \pi) \theta_L \theta_H (\theta_H - \theta_L) a_3^s}{[\theta_L \theta_H a_1^s + [\pi \theta_H + (1 - \pi) \theta_L] a_3^s]^2} < 0. \quad (7)$$

Although winning the semifinal leads to a higher posterior belief that the newcomer has high ability, if such a victory is achieved with a higher effort, the posterior belief will be smaller than if it is achieved with lower effort.

Our first result characterizes the Bayesian Nash equilibrium of the final.

Proposition 1. *In a final between an unbiased newcomer and an incumbent, the equilibrium efforts are symmetric and given by:*

$$a_1^{f*} = a_2^{f*} = a^{f*} = \left[\mu \frac{\theta_H}{(\theta_H + 1)^2} + (1 - \mu) \frac{\theta_L}{(\theta_L + 1)^2} \right] \frac{v}{c}. \quad (8)$$

The proof of Proposition 1 follows directly from the combination of players' first-order conditions. Recalling that $\Theta = \theta_L \theta_H$, the equilibrium effort (i) is unaffected by μ when $\Theta = 1$, (ii) increases in μ for $\Theta < 1$, and (iii) decreases in μ when $\Theta > 1$. The intuition of this result is the following. In any complete information Tullock contest with linear costs and asymmetric abilities, equilibrium efforts are symmetric across players. Moreover, the higher the asymmetry in abilities, the lower the equilibrium efforts (see e.g. Corchón 2000). In our setup where there is incomplete information about the newcomer's ability, players can be seen as optimizing a weighted average of two complete information Tullock contests with linear costs, where the weights are given by the posterior beliefs μ and $(1 - \mu)$. Accordingly, the players will invest equal efforts at equilibrium.

Second, observe that with probability $(1 - \mu)$, the newcomer has an ability of θ_L and the incumbent an ability of 1. Alternatively, by dividing the numerator and the denominator of the Tullock contest success function by θ_L , one can re-interpret this as the newcomer

having an ability of 1 and the incumbent having an ability of $1/\theta_L$. Therefore if $\Theta = 1$, so that $\theta_H = 1/\theta_L$, this implies that the players invest the same equilibrium efforts in the two “degenerate” cases where $\mu = 0$ and $\mu = 1$. Consequently, for any probability μ the players invest these same equilibrium efforts as explained above in point (i).

Extending this logic, we deduce that $\theta_H < 1/\theta_L$ implies that when $\mu = 1$ equilibrium efforts are higher than when $\mu = 0$. Hence, equilibrium efforts are monotonically increasing in μ when $\Theta < 1$, as stated above in point (ii). Likewise we deduce observation (iii) according to which equilibrium efforts are monotonically decreasing in μ when $\Theta > 1$.

It follows from (8) that player 1’s equilibrium probability of winning the final is:⁷

$$P_1^f(a_1^{f*}, a_2^{f*}) = \mu \frac{\theta_H}{\theta_H + 1} + (1 - \mu) \frac{\theta_L}{\theta_L + 1}.$$

It also follows from (8) that player 1’s equilibrium expected utility of the final is

$$E[U_1^f(a_1^{f*}, a_2^{f*})] = \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu\chi \right] v, \quad (9)$$

where $\chi = \left(\frac{\theta_H}{\theta_H + 1} \right)^2 - \left(\frac{\theta_L}{\theta_L + 1} \right)^2$. From equation (9) we can determine how a change in μ affects player 1’s equilibrium expected utility of the final. We have $\frac{\partial E[U_1^f(a_1^{f*}, a_2^{f*})]}{\partial \mu} = \chi v > 0$. Hence, $E[U_1^f(a_1^{f*}, a_2^{f*})]$ is increasing with μ .

4.2 Semifinal

We now consider the semifinal between the unbiased newcomer and the incumbent player 3. Observe that in the semifinal opposing incumbents 2 and 4, the players’ expected utility of reaching the final is a weighted average of their expected utility when facing either player 1 or player 3. Yet, given the symmetry of incumbents 2 and 4, their continuation value is identical, and their equilibrium semifinal effort as well. Although their equilibrium efforts eventually depend on their expectation of whom they will meet in the final, the identity of the winner is irrelevant, and we can then assume, without loss of generality, that the winner of the semifinal opposing players 1 and 3 will face player 2 in the final.

Player 1’s expected utility of the semifinal is

$$\begin{aligned} E[U_1^s(a_1^s, a_3^s)] &= P_1^s(a_1^s, a_3^s) E[U_1^f(a_1^{f*}, a_2^{f*})] - ca_1^s \\ &= \left[\pi \frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1 - \pi) \frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s} \right] \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu\chi \right] v - ca_1^s, \end{aligned}$$

⁷Throughout the analysis we slightly abuse notation for presentation reasons. We designate the optimal efforts in the final for player i by a_i^{f*} , omitting its implicit dependence on the semifinal efforts (a_1^s, a_3^s) .

where μ is the posterior belief that player 1 has high ability and is given by equation (6).

Player 3's expected utility of the semifinal is

$$\begin{aligned} E[U_3^s(a_1^s, a_3^s)] &= P_3^s(a_1^s, a_3^s)E[U_3^f(a_3^{f*}, a_2^{f*})] - ca_3^s \\ &= \left[\pi \frac{a_3^s}{\theta_H a_1^s + a_3^s} + (1 - \pi) \frac{a_3^s}{\theta_L a_1^s + a_3^s} \right] \frac{v}{4} - ca_3^s. \end{aligned}$$

If player 3 reaches the final and the opponent is player 2, then both players exert effort $a^{f*} = \frac{v}{4c}$ and player 3's equilibrium expected utility of the final is $E[U_3^f(a_3^{f*}, a_2^{f*})] = v/4$.

The first-order conditions of players 1 and 3 in the semifinal are

$$\frac{\partial P_1^s}{\partial a_1^s} \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu\chi \right] v + P_1^s(a_1^s, a_3^s) \chi v \frac{\partial \mu}{\partial a_1^s} = c, \quad (10)$$

and

$$\frac{\partial P_3^s}{\partial a_3^s} \frac{v}{4} = c, \quad (11)$$

respectively. The first term in the left-hand side of equation (10) is the standard contest incentive: increasing semifinal effort raises the probability of reaching the final. The second term captures a strategic learning effect. Conditional on reaching the final (as reflected by $P_1^s(a_1^s, a_3^s)$), higher semifinal effort affects the posterior belief about the newcomer's ability. Since $\frac{\partial \mu}{\partial a_1^s} < 0$, greater semifinal effort lowers the posterior probability that the newcomer is of high ability. Because the continuation value of the final is increasing in this belief at rate $v\chi > 0$, this reduces the expected utility from the final. In other words, everything else constant, a semifinal victory under higher effort by the unbiased newcomer is interpreted as a negative signal of ability ("a truly strong player would not need to try so hard"), thereby lowering the posterior belief μ which, in turn, reduces the unbiased newcomer's expected utility of the final. Hence, higher semifinal effort has not only a positive effect on the probability of advancing, but also a negative effect through beliefs that lowers the continuation value of the final.

To show existence of a pure-strategy equilibrium we demonstrate in the Appendix that the second-order conditions are verified whenever the first-order conditions are satisfied.

Proposition 2. *In the semifinal between an unbiased newcomer and an incumbent of a two-stage elimination contest, there exists a unique value $\bar{\pi} \in (0, 1)$ of the ex-ante probability π that the newcomer has high ability such that the players' equilibrium efforts satisfy $a_1^{s*} < a_3^{s*}$ if $\pi < \bar{\pi}$, $a_1^{s*} = a_3^{s*}$ if $\pi = \bar{\pi}$, and $a_1^{s*} > a_3^{s*}$ if $\pi > \bar{\pi}$.*

To better grasp Proposition 2, we consider the two extreme cases where π is either close to 0 or to 1. If π is close to 0, the incentives for the newcomer to invest in the semifinal are very low, since both the likelihood of winning the semifinal (for any effort)

and the expected utility of the final are low. The incumbent is equally incentivized to invest little effort in the semifinal, yet, she will invest more than the newcomer since both her winning probability and expected utility of the final are much higher. Combined, this implies that for low values of π the incumbent invests higher effort in the semifinal.

Consider next a situation where π is close to 1. In such instances the incentives of the newcomer and the incumbent are reversed since the newcomer holds both a high probability of winning the semifinal (for a given effort) and has a higher expected utility of the final compared to the incumbent. Accordingly, for high values of π the newcomer exerts higher effort in the semifinal.

To complete the analysis of the model with an unbiased newcomer, we discuss how the newcomer's probability of winning the contest depends on the newcomer's potential abilities when the prior beliefs induce equal efforts in the semifinal.

Proposition 3. *Let $\bar{\pi} \in (0, 1)$ be the unique value such that the semifinal equilibrium efforts satisfy $a_1^{s*} = a_3^{s*}$, and let $\mu(\bar{\pi})$ denote the posterior belief at $\pi = \bar{\pi}$. Then the newcomer's equilibrium probability of winning the contest, $P_1^{f*}(\mu(\bar{\pi}))$, is:*

- (i) *equal to $1/4$ if $\Theta = 1$,*
- (ii) *less than $1/4$ if $\Theta < 1$,*
- (iii) *greater than $1/4$ if $\Theta > 1$.*

Moreover, $\bar{\pi} < 1/2$ for $\Theta = 1$, which implies that $P_1^{s}(\bar{\pi}) < 1/2$ and $P_1^{f*}(\mu(\bar{\pi})) > 1/2$.*

When the newcomer is on average as strong as the incumbents ($\Theta = 1$), his equilibrium probability of winning the elimination contest is $1/4$. Interestingly, in such cases the prior belief that induces equal efforts in the semifinal is less than $1/2$. This in turn implies that in such instances the probability that the newcomer wins the semifinal is less than $1/2$, while the probability he wins the final is greater than $1/2$. In other words, although the newcomer has a relatively low probability of having a high ability, which drives down his probability of winning the semifinal, the learning effect increases sufficiently the players' posterior belief so as to make the newcomer the most likely winner of the final. Notice that in the absence of learning by all players, the newcomer would then have a lower probability of winning both the semifinal and the final.

5 Overconfident Newcomer

This section analyzes the model with an overconfident newcomer. First, we characterize the equilibrium efforts in a final between the newcomer and an incumbent. Second, we analyze the first-order conditions that determine the equilibrium efforts in the semifinal between the newcomer and an incumbent. Third, we specialize the model to the case $\Theta = 1$ which allows us to fully characterize how overconfidence affects winning probabilities.

5.1 Final

Assume, as before, that player 2 reaches the final. The perceived expected utilities of players 1 and 2 in the final are given by

$$\tilde{E}[U_1^f(a_1^f, a_2^f)] = \tilde{P}_1^f(a_1^f, a_2^f)v - ca_1^f = \left[\tilde{\mu} \frac{\theta_H a_1^f}{\theta_H a_1^f + a_2^f} + (1 - \tilde{\mu}) \frac{\theta_L a_1^f}{\theta_L a_1^f + a_2^f} \right] v - ca_1^f, \quad (12)$$

and

$$E[U_2^f(a_1^f, a_2^f)] = P_2^f(a_1^f, a_2^f)v - ca_2^f = \left[\mu \frac{a_2^f}{\theta_H a_1^f + a_2^f} + (1 - \mu) \frac{a_2^f}{\theta_L a_1^f + a_2^f} \right] v - ca_2^f. \quad (13)$$

In equation (12), $\tilde{\mu}$ is player 1's perceived posterior belief of having high ability given by

$$\tilde{\mu} = \frac{\tilde{\pi} \theta_H (\theta_L a_1^s + a_3^s)}{\theta_L \theta_H a_1^s + [\tilde{\pi} \theta_H + (1 - \tilde{\pi}) \theta_L] a_3^s}. \quad (14)$$

Likewise, in equation (13), μ designates player 2's posterior belief that player 1 has high ability and is given by equation (6).

Recall that player 1's final stage overconfidence bias is $b^f = \tilde{\mu} - \mu$. Note that whereas the semifinal stage bias b^s is exogenous, the final stage bias b^f is endogenous because it depends on the posterior beliefs $\tilde{\mu}$ and μ which are determined by the equilibrium efforts exerted in the semifinal. Furthermore, since $\tilde{\mu}$ is a function of $\tilde{\pi}$, which itself is influenced by the semifinal stage bias b^s , and the semifinal equilibrium efforts depend on b^s , the final stage bias b^f depends on b^s as well. Comparing player 1's overconfidence biases in the two stages, b^s and b^f , we can make the following observation.

Result 1. *For any semifinal efforts, there exists a unique value $\hat{\pi} \in (0, 1)$ of the ex-ante probability π that the newcomer has high ability such that $b^f > b^s$ if $\pi < \hat{\pi}$, $b^f = b^s$ if $\pi = \hat{\pi}$, and $b^f < b^s$ if $\pi > \hat{\pi}$.*

This result says that in an elimination contest, winning the semifinal amplifies the overconfidence bias of a player whose ex-ante probability of being high ability is low, and dampens it otherwise. First observe that when the overconfident player 1 wins the semifinal, both player 1 and his rival in the final, player 2, revise upwards their beliefs about player 1 having high ability, i.e., $\tilde{\mu} > \tilde{\pi}$ and $\mu > \pi$. Given that both players revise their beliefs upwards, we wish to understand what makes either player revise his beliefs the most. When π is close to 0—meaning the unbiased incumbent is almost certain that his opponent has low ability—a win by player 1 in the semifinal does little to change player 2's belief. The strong prior prevents the unbiased incumbent from significantly revising his expectations. For intermediate values of π ,—meaning the unbiased incumbent is very uncertain about his opponent's ability—a win by player 1 in the semifinal leads to a

large upwards revision in the beliefs of player 2. Last, when π is close to 1, there is little room for further increasing players' beliefs. The overconfident player mistakenly assigns a higher probability to having high ability. This, in turn, leads the overconfident player to update upwards his beliefs by a large amount when π is low. Consequently, for low values of π the update from the overconfident player will be larger than the one from the unbiased incumbent. For intermediate values of π , the overconfident player believes his probability of being a high type is very high, and the update of beliefs will therefore be small. Accordingly, it will be the unbiased incumbent who will update his beliefs the most for such priors.

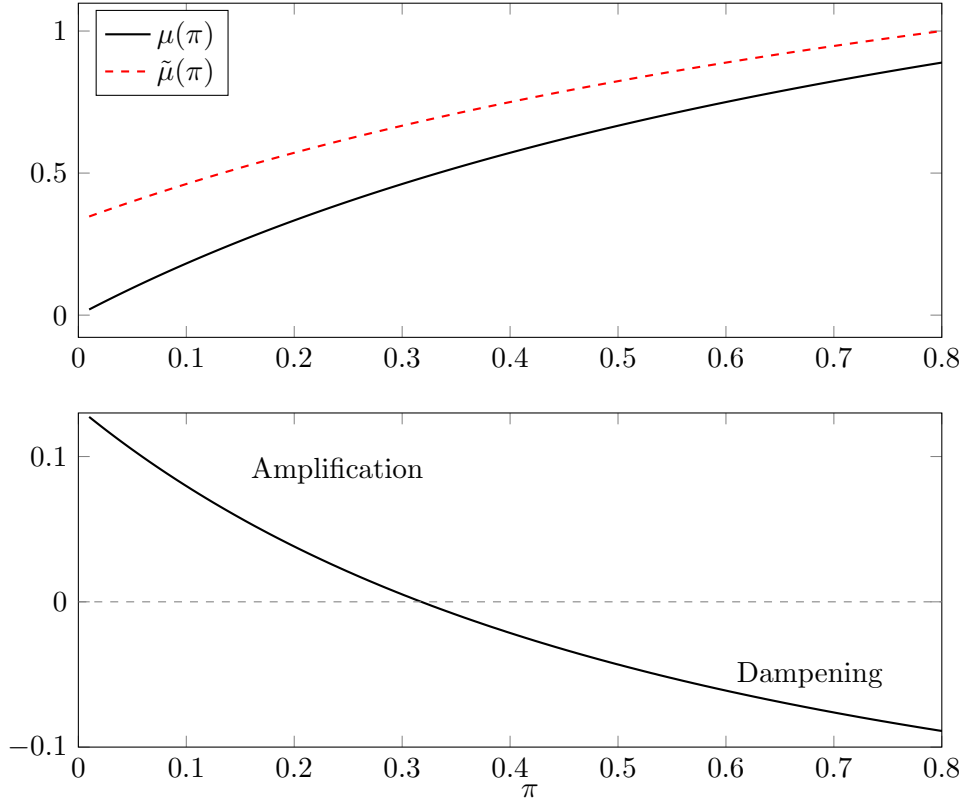


Figure 1: Top panel: unbiased ($\mu(\pi)$) and biased ($\tilde{\mu}(\pi)$) posterior beliefs. Bottom panel: change in the overconfidence bias: $b^f - b^s$. The two panels are produced by imposing $a_1^s = a_3^s$, $\theta_H = 2$, $\theta_L = 0.5$, and $b^s = 0.2$.

Figure 1 illustrates Result 1. On the top panel the plain curve displays the posterior belief of the unbiased incumbent on the newcomer having high ability following a semifinal victory by the newcomer, $\mu(\pi)$. The dashed curve depicts the overconfident newcomer's posterior belief of being high ability, $\tilde{\mu}(\pi)$. Both posterior beliefs increase with π , but as the prior beliefs that the newcomer is high ability gets higher, the scope for improvement of both beliefs shrinks. On the bottom panel we depict the change in the overconfidence bias of the newcomer, i.e. $b^f - b^s$.

We show in the Appendix that the final admits a unique pure strategy BNE. We can then establish the next proposition.

Proposition 4. *In a final between an overconfident newcomer and an incumbent, the equilibrium efforts depend on the product of the newcomer's possible abilities as follows:*

- (i) *If $\Theta = 1$, then $a_1^{f*} = a_2^{f*}$,*
- (ii) *If $\Theta < 1$, then $a_1^{f*} > a_2^{f*}$,*
- (iii) *If $\Theta > 1$, then $a_1^{f*} < a_2^{f*}$.*

Proposition 4 highlights what changes in the final once the newcomer is overconfident. Under unbiased beliefs, Proposition 1 showed that the newcomer and the incumbent always choose the same equilibrium effort. The reason is that they share the same posterior belief about the newcomer's ability and therefore evaluate the two possible ability states using the same weights. Overconfidence breaks this symmetry: the newcomer assigns probability $\tilde{\mu}$ to being high ability, whereas the incumbent assigns the lower probability μ . This disagreement over the likelihood of the two ability states can generate asymmetric equilibrium efforts. Which player exerts more effort, however, depends only on the relative degree of ability asymmetry in the two states, summarized by $\Theta = \theta_L \theta_H$, and not on the particular values of μ and $\tilde{\mu}$.

To see why, recall the intuition underlying Proposition 1. The final can be viewed as a weighted average of two complete-information Tullock contests. If the newcomer is high ability, his relative ability is θ_H . If he is low ability, dividing both players' effective abilities by θ_L allows us to reinterpret the contest as one in which the newcomer has ability 1 and the incumbent has relative ability $1/\theta_L$. In a complete-information Tullock contest with linear costs, equilibrium effort falls as the asymmetry in abilities becomes larger. The disagreement between the newcomer and the incumbent therefore matters because they attach different weights to contests with different degrees of ability asymmetry.

When the newcomer is on average as strong as the incumbent, we have $\Theta = 1$, so that $\theta_H = 1/\theta_L$. The degree of ability asymmetry is then the same in the two possible states: in one state the newcomer is the stronger player by a factor θ_H , while in the other the incumbent is the stronger player by exactly the same factor $1/\theta_L = \theta_H$. Consequently, the complete-information equilibrium effort associated with the two states is the same. Different beliefs about which state is more likely therefore have no effect on relative incentives. Even though the newcomer is overconfident and $\tilde{\mu} > \mu$, both players choose the same equilibrium effort, just as in Proposition 1.

When the newcomer is on average weaker than the incumbent, we have $\Theta < 1$, so that $\theta_H < 1/\theta_L$. The high-ability state is therefore the less asymmetric of the two contests and is associated with higher complete-information equilibrium effort. Because $\tilde{\mu} > \mu$, the overconfident newcomer places relatively more weight than the incumbent on precisely this high-effort state. The incumbent, conversely, places relatively more weight on the low-ability state, in which the asymmetry $1/\theta_L$ is larger and equilibrium effort is lower. This difference in perceived state weights leads the newcomer to exert more effort than the incumbent.

When the newcomer is on average stronger than the incumbent, we have $\Theta > 1$, and the argument is reversed. Now $\theta_H > 1/\theta_L$, so the high-ability state is the more asymmetric state and is associated with lower complete-information equilibrium effort. Since the overconfident newcomer assigns relatively more weight to this state, while the incumbent assigns relatively more weight to the less asymmetric low-ability state, the newcomer exerts less effort than the incumbent.

The comparison with Proposition 1 is therefore particularly revealing. With an unbiased newcomer, uncertainty about ability by itself does not generate an effort asymmetry in the final: common beliefs make the players choose equal efforts for every posterior μ . Overconfidence changes this conclusion because the players no longer agree on the probabilities of the underlying ability states. Whether this disagreement raises or lowers the newcomer's effort relative to the incumbent depends on which state the newcomer overweights and on whether that state is associated with greater or smaller ability asymmetry. Thus, overconfidence does not mechanically increase effort. It increases the newcomer's effort when $\Theta < 1$, leaves relative effort unchanged when $\Theta = 1$, and decreases it when $\Theta > 1$.

Proposition 4 therefore also uncovers an interesting implication for one-shot contests: an overconfident player may exert more effort than his rival. This differs from the prediction of Chen and Santos-Pinto (2025) for the final stage of an elimination contest, where the overconfident player exerts less effort. The difference stems from how overconfidence is modeled. In Chen and Santos-Pinto (2025), the player overestimates a deterministic ability. Here, the newcomer is uncertain about his own ability and overestimates the probability that it is high. Overconfidence therefore changes the weights he attaches to different possible competitive environments, and this can either increase or decrease his equilibrium effort depending on the relative ability asymmetry across those environments.

5.2 Semifinal

Player 1's perceived expected utility of the semifinal is

$$\begin{aligned}\tilde{E}[U_1^s(a_1^s, a_3^s)] &= \tilde{P}_1^s(a_1^s, a_3^s) \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] - ca_1^s \\ &= \left[\tilde{\pi} \frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1 - \tilde{\pi}) \frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s} \right] \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] - ca_1^s,\end{aligned}$$

where $a_1^{f*} = h_1(\tilde{\mu}, \mu)$, $a_2^{f*} = h_2(\tilde{\mu}, \mu)$, $\tilde{\mu}$ is given by equation (14) and μ by equation (6).

The first-order condition of player 1 in the semifinal is

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} \tilde{E}[U_1^{f*}] + \tilde{P}_1^s(a_1^s, a_3^s) \left[\frac{d\tilde{E}[U_1^{f*}]}{d\mu} \frac{\partial \mu}{\partial a_1^s} + \frac{d\tilde{E}[U_1^{f*}]}{d\tilde{\mu}} \frac{\partial \tilde{\mu}}{\partial a_1^s} \right] = c. \quad (15)$$

The first-order condition for player 3 in the semifinal is the same as in the previous section and thus given by:

$$\frac{\partial P_3^s}{\partial a_3^s} \frac{v}{4} = c. \quad (16)$$

To show existence of a pure-strategy equilibrium we demonstrate in the Appendix that the second-order conditions are verified whenever the first-order conditions are satisfied for the specific case $\Theta = 1$. By continuity of the players' perceived expected utilities in θ_L and θ_H it follows that there is an interval $[\underline{\Theta}, \bar{\Theta}]$, with $\underline{\Theta} < 1$ and $\bar{\Theta} > 1$, such that for any Θ in that interval the second-order conditions are verified.

Making use of the Envelope theorem, we can rewrite equation (15) as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} \tilde{E}[U_1^{f*}] + \tilde{P}_1^s(a_1^s, a_3^s) \left[\frac{\partial \tilde{E}[U_1^{f*}]}{\partial a_2^f} \frac{\partial a_2^{f*}}{\partial \mu} \frac{\partial \mu}{\partial a_1^s} + \frac{\partial \tilde{E}[U_1^{f*}]}{\partial a_2^f} \frac{\partial a_2^{f*}}{\partial \tilde{\mu}} \frac{\partial \tilde{\mu}}{\partial a_1^s} + \frac{\partial \tilde{E}[U_1^{f*}]}{\partial \tilde{\mu}} \frac{\partial \tilde{\mu}}{\partial a_1^s} \right] = c. \quad (17)$$

The newcomer's first-order condition equates the perceived marginal benefit of effort in the semifinal to the constant marginal cost c . The perceived marginal benefit of effort is itself composed of two terms. The first term describes the effect of semifinal effort on the perceived probability of winning the semifinal weighted by the perceived expected utility of reaching the final. The sign of this term is positive. The second term measures how a change in semifinal effort affects the perceived expected utility of reaching the final, weighted by the perceived probability of winning the semifinal. Observe that the perceived expected utility of reaching the final is affected by the newcomer's first period effort through three channels. Indeed, true posterior beliefs μ are (negatively) impacted by an increase in a_1^s , in turn affecting the equilibrium efforts of the rival in the final, a_2^{f*} . Perceived posterior beliefs $\tilde{\mu}$ are also (negatively) impacted by an increase in a_1^s , in turn affecting the expected utility of the newcomer in the final both directly, and indirectly through its effect on the equilibrium efforts of the rival, a_2^{f*} .

Given the generality of our setup, it is not possible to derive closed form solutions and to further characterize the equilibrium, except when $\Theta = 1$ as we show next. The cases where $\Theta \neq 1$ are discussed in Section 5.4.

5.3 The Model with $\Theta = 1$

We now compare equilibrium semifinal effort under unbiased and overconfident beliefs when the newcomer is on average as strong as the incumbents. In this case, the ranking of the newcomer's effort relative to the incumbent's can be characterized by a single threshold value of the prior probability of high ability. Proposition 5 shows how the position of the unbiased prior π and the biased prior $\tilde{\pi}$ relative to this threshold determines whether overconfidence changes the ranking of equilibrium efforts.

Proposition 5. *In the semifinal between an overconfident newcomer and an incumbent of a two-stage elimination contest where $\Theta = 1$, there exists a unique value*

$$\tilde{\pi} = \frac{\theta_H + 3}{4(\theta_H + 1)}$$

for the ex-ante probability π the newcomer has high ability such that:

- (i) For $\pi < \tilde{\pi} < \tilde{\pi}$, both an unbiased and an overconfident newcomer exert less effort than the incumbent at equilibrium in the semifinal.*
- (ii) For $\pi < \tilde{\pi} < \tilde{\pi}$, an overconfident newcomer exerts more effort than the incumbent at equilibrium in the semifinal, while an unbiased newcomer exerts less effort.*
- (iii) For $\tilde{\pi} < \pi < \tilde{\pi}$, both an unbiased and an overconfident newcomer exert more effort than the incumbent at equilibrium in the semifinal.*

When $\Theta = 1$ we have the following three scenarios. First, when the newcomer's ex-ante probability of having high ability is low and his biased prior belief $\tilde{\pi}$ is smaller than the threshold value, $\pi < \tilde{\pi} < \tilde{\pi}$, both an unbiased and an overconfident newcomer exert less effort at equilibrium in the semifinal than the incumbent. This is intuitive, the small bias leads to a small change in effort provision and both an unbiased and an overconfident newcomer exert less effort than the incumbent for the same reasons highlighted after Proposition 2 in Section 4.2.

Second, when the newcomer's ex-ante probability of having high ability is low and his biased prior belief $\tilde{\pi}$ is greater than the threshold value, $\pi < \tilde{\pi} < \tilde{\pi}$, an overconfident newcomer exerts more effort at equilibrium in the semifinal than the incumbent whereas an unbiased newcomer exerts less effort. In this case the higher bias leads to a large effort provision since the newcomer overestimates his probability of winning the semifinal as well as his expected utility of the final. Consequently the overconfident newcomer exerts more effort than the incumbent when an unbiased newcomer would have exerted less effort than the incumbent.

Third, when the newcomer's ex-ante probability of having high ability is greater than the threshold value, $\tilde{\pi} < \pi < \tilde{\pi}$, both an unbiased and an overconfident newcomer exert more effort at equilibrium in the semifinal than the incumbent.

Last we analyze the impact of the newcomer's semifinal bias b^s on his true equilibrium probabilities of winning the final and semifinal, P_1^{f*} and P_1^{s*} , respectively. This allows us to determine how the semifinal bias b^s changes the newcomer's true equilibrium probability of winning the elimination contest $P_1^* = P_1^{s*}P_1^{f*}$.

Proposition 6. *In a two-stage elimination contest where $\Theta = 1$, the overconfident newcomer's true equilibrium probability of winning the final, P_1^{f*} , decreases in his overconfidence bias b^s , and his true equilibrium probability of winning the semifinal, P_1^{s*} , increases in b^s . His true equilibrium probability of winning the contest, $P_1^*P_1^{f*}$, increases in his*

overconfidence bias b^s .

When $\Theta = 1$, the newcomer's true equilibrium probability of winning the final is given by

$$P_1^{f*} = \frac{1 + \mu^*(\theta_H - 1)}{\theta_H + 1}, \quad (18)$$

where

$$\mu^* = \pi \frac{\frac{a_1^{s*}}{a_3^{s*}} + \theta_H}{\frac{a_1^{s*}}{a_3^{s*}} + \pi\theta_H + (1 - \pi)/\theta_H}.$$

Since

$$\partial\mu^*/\partial(a_1^{s*}/a_3^{s*}) = -\frac{(1 - \pi)(\theta_H - 1/\theta_H)}{(a_1^{s*}/a_3^{s*} + \pi\theta_H + (1 - \pi)/\theta_H)^2} < 0,$$

and since P_1^{f*} is increasing in μ^* , it follows that the newcomer's true equilibrium probability of winning the final is decreasing in a_1^{s*}/a_3^{s*} .

The newcomer's true equilibrium probability of winning the semifinal is given by

$$P_1^{s*} = \pi \frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*} + a_3^{s*}} + (1 - \pi) \frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H + a_3^{s*}}, \quad (19)$$

and we immediately observe that P_1^{s*} is increasing with a_1^{s*}/a_3^{s*} . Consequently, a change in the equilibrium semifinal relative effort induces P_1^{f*} and P_1^{s*} to move in opposite directions. We show, in the proof of Proposition 6, that an increase in the newcomer's semifinal bias b^s raises the semifinal equilibrium relative effort a_1^{s*}/a_3^{s*} . Hence, an increase in the newcomer's semifinal bias b^s lowers his true equilibrium probability of winning the final while it raises his true equilibrium probability of winning the semifinal. We demonstrate in Proposition 6 that the net effect of the increase in P_1^{s*} dominates.

Figure 2 illustrates Proposition 6. The curves for $\Theta = 1$ depict P_1^{s*} and P_1^{f*} (left panel) and their product P_1^* (right panel) as functions of b^s when $\pi = 1/4$, $\theta_L = 1/2$, $\theta_H = 2$, $v = 10$, and $c = 1$.

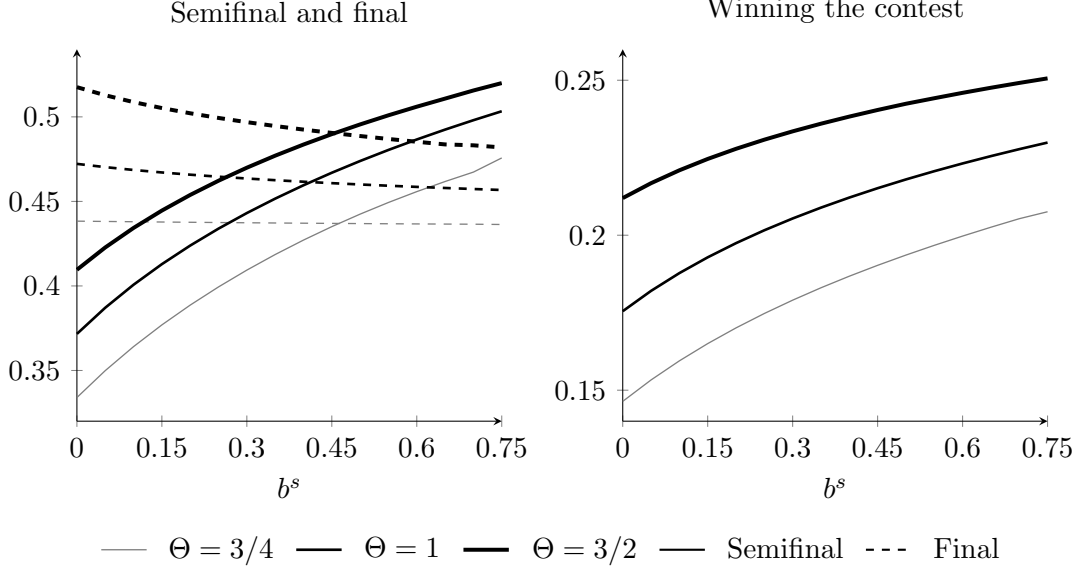


Figure 2: Newcomer’s true probabilities of winning the semifinal and the final (left panel) and the entire elimination contest (right panel), as functions of his initial bias b^s , for $\pi = 1/4$, $\theta_L = 1/2$, $v = 10$, $c = 1$, and $\theta_H \in \{3/2, 2, 3\}$.

5.4 Relaxing the Restriction $\Theta = 1$

Proposition 6 is obtained when imposing the restriction $\Theta = 1$. This simplification eliminates asymmetries in equilibrium efforts in the final stage, and simplifies significantly the analysis of the semifinal stage. Observe, however, that it does not constitute a knife-edge condition for the qualitative result of Proposition 6. Indeed, all primitives of the model are smooth, and equilibrium efforts in both stages are interior and characterized by first-order conditions satisfying standard regularity conditions. Hence, by standard continuity arguments, equilibrium effort levels and the induced winning probabilities vary continuously with the parameters θ_L and θ_H . Since in the specific case where $\Theta = 1$, P_1^{f*} strictly decreases in b^s , P_1^{s*} strictly increases in b^s , and P_1^* strictly increases in b^s , by continuity this implies that there exists an open neighborhood of this value—both for $\Theta < 1$ and for $\Theta > 1$ —in which the overall effect of overconfidence on the true probability of winning the contest remains unchanged in sign.

To complement this continuity argument, we turn to numerical simulations that illustrate the robustness of the result when the restriction $\Theta = 1$ is relaxed. Figure 2 also depicts the true equilibrium winning probabilities for the same parameter values, except that $\theta_H = 3/2$, such that $\Theta = 3/4$ (thin curves), and $\theta_H = 3$, such that $\Theta = 3/2$ (bold curves). In both cases, the probability of winning the semifinal increases in b^s , the probability of winning the final decreases in b^s , and the probability of winning the whole contest increases in b^s . The results thus remain qualitatively unchanged for values of Θ that do not lie in the vicinity of 1.

5.5 Unobservable efforts

Throughout this paper we have assumed that semifinal efforts are publicly observable. We now discuss how our results change when they are not. With unobservable efforts, beliefs along the equilibrium path are unaffected: since equilibrium efforts are correctly anticipated, the incumbent who reaches the final forms the same posterior μ as before. What changes is the incentive to deviate. If the newcomer departs from his equilibrium semifinal effort, the incumbent does not observe it, and her posterior, and hence her effort in the final, remain those computed from equilibrium efforts. The newcomer, by contrast, knows his actual effort, so his own posterior still responds to it. Information about ability remains symmetric along the equilibrium path, but a deviation creates private information.

Formally, the terms of the newcomer's first-order condition (17) that run through the incumbent's final effort, $\partial a_2^{f*}/\partial \mu$ and $\partial a_2^{f*}/\partial \tilde{\mu}$, disappear, while the term running through the newcomer's own posterior is unchanged. The implications depend on Θ .

When $\Theta = 1$, final efforts are independent of beliefs, so these terms are zero in any case. The equilibrium is identical under both assumptions, and Propositions 5 and 6 hold without change. When $\Theta \neq 1$, the equilibrium differs, and the reason is best understood through what a semifinal victory tells the incumbent. A newcomer who wins his semifinal with little effort appears strong, since his victory is then more likely due to ability than to effort. With observable efforts, the newcomer can therefore shape his rival's view of him, and through it the intensity of the final.

Whether appearing strong is desirable depends on how the newcomer compares with the incumbents on average. When he is on average stronger ($\Theta > 1$), a final against a newcomer believed to be strong is an unbalanced contest, in which both finalists invest little effort. Appearing strong thus makes the final cheaper for the newcomer, and with observable efforts he has an incentive to hold back in the semifinal in order to win it convincingly. When efforts are unobservable, holding back no longer impresses anyone, since the incumbent cannot see it; this motive disappears, and the newcomer exerts more effort in the semifinal. When the newcomer is on average weaker ($\Theta < 1$), the logic is reversed. A final against a newcomer believed to be strong is a more balanced and therefore fiercer contest. With observable efforts, the newcomer thus prefers not to appear too strong, which leads him to work harder in the semifinal; when efforts are unobservable, this concern disappears, and he exerts less effort.⁸ For the overconfident newcomer, the same forces are at work, but the incumbent's reaction now depends on both players' beliefs, and the direction of the change is ambiguous.

⁸Formally, a higher posterior raises the unbiased newcomer's continuation value at rate χv with observable efforts, and at rate Δv , where $\Delta \equiv \frac{\theta_H}{\theta_H+1} - \frac{\theta_L}{\theta_L+1}$, with unobservable efforts. Since $\chi = \Delta(\frac{\theta_H}{\theta_H+1} + \frac{\theta_L}{\theta_L+1})$ and the term in parentheses exceeds 1 if and only if $\Theta > 1$, the two rates coincide only when $\Theta = 1$.

In sum, relaxing observability affects the semifinal, not the final. The final-stage results are stated for given posterior beliefs, and hold unchanged, as does Result 1, which holds for any semifinal efforts. What changes when $\Theta \neq 1$ is the newcomer’s semifinal effort: he has an incentive to exert more effort when he is on average stronger than the incumbents, and less when he is on average weaker. Through the semifinal efforts, this shift in turn changes the posterior beliefs with which the finalists enter the final, and hence the newcomer’s probability of winning the contest. When $\Theta = 1$, none of this applies: semifinal efforts, final efforts, and winning probabilities are the same under both assumptions, and our main results, Propositions 5 and 6, hold whether or not efforts are observed.

6 Conclusion

This paper studies how overconfidence shapes behavior and outcomes in a two-stage elimination contest in which a newcomer, whose ability is unknown to all, himself included, competes against incumbents of known ability. Two results stand out. First, overconfidence raises the newcomer’s probability of winning the contest. The more overconfident the newcomer, the more he values reaching the final, and the harder he fights in the semifinal relative to his rival. This raises his chance of reaching the final by more than it lowers his chance of winning it once there. Second, overconfidence is not a fixed trait but evolves with success. Because everyone learns from the newcomer’s results, a semifinal victory amplifies his overconfidence when his ex-ante prospects are poor, and dampens it when they are good.

Our analysis has implications for organizations. Promotion contests that pit external hires against long-tenured employees are natural settings for our mechanism, and our results suggest that such contests may systematically favor overconfident newcomers. Early promotions, moreover, inflate overconfidence most among those who were least expected to succeed. Since overconfident executives make different investment and financing decisions from their unbiased counterparts (Malmendier and Tate, 2015), this matters not only for who reaches the top but also for how organizations are run once they get there. More broadly, the widespread overconfidence observed at the top of organizations may be both a cause and a consequence of success, and our model generates both channels at once.

Our results carry lessons for the study of contests with behavioral biases. First, biases should not be treated as fixed parameters in dynamic contests. When contestants learn from their results, the bias itself is shaped by the contest, and the same early success can inflate or deflate it depending on how promising a contestant initially appeared. Second, the consequences of overconfidence depend on what contestants are uncertain about. Overestimating the probability of being able is not equivalent to overestimating a known

ability, and the two can lead to opposite effort choices in the same contest. Third, in multi-stage contests, the effects of a bias can differ across stages and even pull in opposite directions. Assessing whether a bias helps or hurts a contestant therefore requires looking at the contest as a whole, rather than at any single stage.

Finally, the model delivers several empirically testable predictions. Among newcomers with the same ex-ante probability of being high ability, overconfident ones should reach the final stages of elimination contests more often than unbiased ones, without being more successful once there. Moreover, a first success should raise overconfidence among newcomers who looked weak at the outset, but lower it among those who looked strong. Because these predictions are stage-specific, they lend themselves naturally to laboratory experiments, in which beliefs can be elicited before and after each round.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT and Claude in order to improve the text and assist the authors in the simulations. After using these tools/services, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

References

- Alcalde, J. and Dahm, M. (2007). Tullock and Hirshleifer: A meeting of the minds. *Review of Economic Design*, 11:101–124.
- Altmann, S., Falk, A., and Wibral, M. (2012). Promotions and incentives: The case of multistage elimination tournaments. *Journal of Labor Economics*, 30(1):149–174.
- Andreottola, G. (2021). Signaling valence in primary elections. *Games and Economic Behavior*, 126:1–32.
- Azoulay, P., Graff Zivin, J. S., and Manso, G. (2011). Incentives and creativity: Evidence from the academic life sciences. *RAND Journal of Economics*, 42(3):527–554.
- Barbieri, S. and Serena, M. (2025). Repeated contests with commitment types. *European Economic Review*, 177:105055.
- Benoît, J. and Dubra, J. (2011). Apparent overconfidence. *Econometrica*, 79(5):1591–1625.
- Catepillán, J., Figueroa, N., and Lemus, J. (2025). Signaling in dynamic contests with heterogeneous rivals. *Journal of Mathematical Economics*, 116:103080.
- Chen, Y. and Santos-Pinto, L. (2025). Overconfidence and strategic behavior in elimination contests: Implications for CEO selection. *Review of Economic Design*.

- Compte, O. C. and Postlewaite, A. (2004). Confidence-enhanced performance. *American Economic Review*, 94:1536–1557.
- Corchón, L. C. (2000). On the allocative effects of rent seeking. *Journal of Public Economic Theory*, 2(4):483–491.
- Crutzen, B. S. Y., Castanheira, M., and Sahuguet, N. (2010). Party organization and electoral competition. *Journal of Law, Economics, & Organization*, 26(2):212–242.
- Deng, S., Fang, H., Fu, Q., and Wu, Z. (2024). Confidence management in contests. *Journal of Economics & Management Strategy*, 33(4):1007–1028.
- Denter, P., Morgan, J., and Sisak, D. (2022). Showing off or laying low? The economics of psych-outs. *American Economic Journal: Microeconomics*, 14(1):529–580.
- Fu, Q. and Lu, J. (2012). The optimal multi-stage contests. *Economic Theory*, 51(2):351–371.
- Goel, A. M. and Thakor, A. V. (2008). Overconfidence, CEO selection, and corporate governance. *Journal of Finance*, 63:2737–2784.
- Gradstein, M. and Konrad, K. A. (1999). Orchestrating rent seeking contests. *Economic Journal*, 109(458):536–545.
- Groh, C., Moldovanu, B., Sela, A., and Sunde, U. (2012). Optimal seedings in elimination tournaments. *Economic Theory*, 51(2):401–420.
- Huffman, D., Raymond, C., and Shvets, J. (2022). Persistent overconfidence and biased memory: Evidence from managers. *American Economic Review*, 112(10):3141–3175.
- Klunover, D. (2021). When sabotage fails. *Mathematical Social Sciences*, 113:164–168.
- Krähmer, D. (2007). Equilibrium learning in simple contests. *Games and Economic Behavior*, 59(1):105–131.
- Kubitz, G. (2023). Two-stage contests with private information. *American Economic Journal: Microeconomics*, 15(1):239–287.
- Malmendier, U. and Tate, G. (2015). Behavioral CEOs: The role of managerial overconfidence. *Journal of Economic Perspectives*, 29(4):37–60.
- Morris, S. (1995). The common prior assumption in economic theory. *Economics and Philosophy*, 11(2):227–253.
- Parrino, R. (1997). CEO turnover and outside succession: A cross-sectional analysis. *Journal of Financial Economics*, 46(2):165–197.
- Pronin, E. and Kugler, M. (2007). Valuing thoughts, ignoring behavior: The introspection illusion as a source of the bias blind spot. *Journal of Experimental Social Psychology*, 43:565–578.
- Pronin, E., L. D. and Ross, L. (2002). The bias blind spot: Perceptions of bias in self versus others. *Personality and Social Psychology Bulletin*, 28(3):369–381.

- Rebitzer, J. B. and Taylor, L. J. (2007). When knowledge is an asset: Explaining the organizational structure of large law firms. *Journal of Labor Economics*, 25(2):201–229.
- Rosen, S. (1986). Prizes and incentives in elimination tournaments. *The American Economic Review*, 76(4):701–715.
- Santos-Pinto, L. and Sekeris, P. (2025). How confidence heterogeneity shapes effort and performance in tournaments and contests. *Journal of Mathematical Economics*, 116(103069).
- Squintani, F. (2006). Equilibrium and mistaken self-perception. *Economic Theory*, 27(3):615–641.
- Yildiz, M. (2007). Wishful thinking in strategic environments. *The Review of Economic Studies*, 74(1):319–344.
- Zábojník, J. (2004). A model of rational bias in self-assessments. *Economic Theory*, 23(2):259–282.
- Zhang, J. and Wang, R. (2009). The role of information revelation in elimination contests. *The Economic Journal*, 119(536):613–641.
- Zhang, Y. and Rajagopalan, N. (2004). When the known devil is better than an unknown god: An empirical study of the antecedents and consequences of relay CEO successions. *Academy of Management Journal*, 47(4):483–500.

7 Appendix

Proof of Proposition 1: The first-order conditions of players 1 and 2 in the final are given by

$$\left[\mu \frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1 - \mu) \frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2} \right] a_2^f v = c, \quad (20)$$

and

$$\left[\mu \frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1 - \mu) \frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2} \right] a_1^f v = c. \quad (21)$$

It is immediately observed that $a_1^{f*} = a_2^{f*}$. It follows from the first-order conditions that the final has a unique pure-strategy Bayesian-Nash equilibrium.

Second-order conditions of the semifinal with an unbiased newcomer: The first derivative of the newcomer's expected utility in the semifinal is:

$$\frac{\partial E[U_1^s]}{\partial a_1^s} = \frac{\partial P_1^s}{\partial a_1^s} \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu \chi \right] v + P_1^s \chi v \frac{\partial \mu}{\partial a_1^s} - c.$$

The second derivative is therefore given by:

$$\frac{\partial^2 E[U_1^s]}{(\partial a_1^s)^2} = \frac{\partial^2 P_1^s}{(\partial a_1^s)^2} \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \mu \chi \right] v + 2 \frac{\partial P_1^s}{\partial a_1^s} \chi v \frac{\partial \mu}{\partial a_1^s} + P_1^s \chi v \frac{\partial^2 \mu}{(\partial a_1^s)^2}.$$

Since $\frac{\partial^2 P_1^s}{(\partial a_1^s)^2} < 0$, this expression is a fortiori negative if:

$$\left[\frac{\partial^2 P_1^s}{(\partial a_1^s)^2} \mu + 2 \frac{\partial P_1^s}{\partial a_1^s} \frac{\partial \mu}{\partial a_1^s} + P_1^s \frac{\partial^2 \mu}{(\partial a_1^s)^2} \right] \underbrace{\chi v}_{>0} < 0.$$

We thus need to show that the term inside square brackets is negative. Substituting for the appropriate terms, and simplifying, the term inside square brackets is given by:

$$-\frac{2\pi\theta_H^2 a_3^s}{(\theta_H a_1^s + a_3^s)^3} < 0.$$

Hence, the second-order condition for the newcomer is satisfied. It is immediate to verify that the second-order condition for the incumbent is also satisfied.

Proof of Proposition 2: To prove this result, we consider an effort level of a player that maximizes his semifinal payoff. Accordingly, that effort level must satisfy his first-order condition. We next fix the other player's effort at the same level, i.e. $a_1^s = a_3^s$, and deduce that $a_1^s \leq a_3^s \Leftrightarrow \partial E[U_1^s(a_1^s, a_3^s)]/\partial a_1 \leq \partial E[U_3^s(a_1^s, a_3^s)]/\partial a_3$, for $a_1^s = a_3^s = a > 0$. More

specifically, we define the difference of the players' first order derivatives when evaluated at $a_1^s = a_3^s$ as $\Psi(\pi)$, and we show that $\Psi(0) < 0$, $\Psi(1) > 0$, and $\Psi'(\pi) > 0$ on $\pi \in [0, 1]$, thence implying that there is a unique ex-ante probability the newcomer has high ability $\bar{\pi}$ for which players 1 and 3 exert the same equilibrium effort in the semifinal.

We start by simplifying the first-order condition of player 1 given by equation (10):

$$\frac{\partial P_1^s}{\partial a_1^s} E[U_1^f(a_1^{f*}, a_2^{f*})] + P_1^s(a_1^s, a_3^s) \chi v \frac{\partial \mu}{\partial a_1^s} = c.$$

We know that

$$\frac{\partial P_1^s}{\partial a_1^s} = \frac{\partial P_3^s}{\partial a_3^s} \frac{a_3^s}{a_1^s}.$$

Using this last equation, the first-order condition of player 3 given by equation (11) becomes

$$\frac{\partial P_1^s}{\partial a_1^s} \frac{a_1^s}{a_3^s} \frac{v}{4} = c.$$

Using these first-order conditions, we can next express $\Psi(\pi)$ as:

$$\begin{aligned} \Psi(\pi) &= \left. \frac{\partial E[U_1^s(a_1^s, a_3^s)]}{\partial a_1^s} \right|_{a_1^s=a_3^s} - \left. \frac{\partial E[U_3^s(a_1^s, a_3^s)]}{\partial a_3^s} \right|_{a_1^s=a_3^s} \\ &= \left\{ \frac{\partial P_1^s}{\partial a_1^s} E[U_1^f(a_1^{f*}, a_2^{f*})] + P_1^s(a_1^s, a_3^s) \chi v \frac{\partial \mu}{\partial a_1^s} - \frac{\partial P_1^s}{\partial a_1^s} \frac{a_1^s}{a_3^s} \frac{v}{4} \right\} \bigg|_{a_1^s=a_3^s} \\ &= \left\{ \frac{\partial P_1^s}{\partial a_1^s} \left[E[U_1^f(a_1^{f*}, a_2^{f*})] - \frac{a_1^s}{a_3^s} \frac{v}{4} \right] + P_1^s(a_1^s, a_3^s) \chi v \frac{\partial \mu}{\partial a_1^s} \right\} \bigg|_{a_1^s=a_3^s}. \end{aligned}$$

We have

$$P_1^s(a_1^s, a_3^s) = \pi \frac{\theta_H a_1^s}{\theta_H a_1^s + a_3^s} + (1 - \pi) \frac{\theta_L a_1^s}{\theta_L a_1^s + a_3^s}.$$

So, we have

$$\frac{\partial P_1^s}{\partial a_1^s} = \pi \frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} + (1 - \pi) \frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2}$$

Observe that when imposing $a_1^s = a_3^s = a$, we have:

$$\begin{aligned} P_1^s(a_1^s, a_3^s) \bigg|_{a_1^s=a_3^s=a} &= \frac{\theta_L}{1 + \theta_L} + \pi \left(\frac{\theta_H}{1 + \theta_H} - \frac{\theta_L}{1 + \theta_L} \right), \\ \frac{\partial P_1^s}{\partial a_1^s} \bigg|_{a_1^s=a_3^s=a} &= \left[\frac{\theta_L}{(\theta_L + 1)^2} + \pi \left(\frac{\theta_H}{(\theta_H + 1)^2} - \frac{\theta_L}{(\theta_L + 1)^2} \right) \right] \frac{1}{a}, \\ E[U_1^f(a_1^{f*}, a_2^{f*})] \bigg|_{a_1^s=a_3^s=a} &= \left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \frac{\pi \theta_H (\theta_L + 1)}{\theta_L \theta_H + \pi \theta_H + (1 - \pi) \theta_L} \chi \right] v, \end{aligned}$$

and

$$\left. \frac{\partial \mu}{\partial a_1^s} \right|_{a_1^s = a_3^s = a} = - \frac{\pi(1-\pi)\theta_L\theta_H(\theta_H - \theta_L)}{[\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L]^2 a}.$$

Substituting in Ψ for the appropriate terms we then obtain:

$$\begin{aligned} \Psi(\pi) &= \left[\frac{\theta_L}{(\theta_L + 1)^2} + \pi \left(\frac{\theta_H}{(\theta_H + 1)^2} - \frac{\theta_L}{(\theta_L + 1)^2} \right) \right] \frac{1}{a} \left[\left[\left(\frac{\theta_L}{\theta_L + 1} \right)^2 + \frac{\pi\theta_H(\theta_L + 1)}{\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L} \chi \right] - \frac{1}{4} \right] v \\ &\quad - \left[\frac{\theta_L}{1 + \theta_L} + \pi \left(\frac{\theta_H}{1 + \theta_H} - \frac{\theta_L}{1 + \theta_L} \right) \right] \chi v \frac{\pi(1-\pi)\theta_L\theta_H(\theta_H - \theta_L)}{[\theta_L\theta_H + \pi\theta_H + (1-\pi)\theta_L]^2 a} \end{aligned}$$

After some manipulations we obtain

$$\begin{aligned} \Psi(\pi) &= \frac{1}{(\theta_H + 1)^4(\theta_L + 1)^4} \left[-\theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2) \right. \\ &\quad \left. + \left(\theta_H(\theta_L + 1)^4(3\theta_H^2 - 2\theta_H - 1) + \theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2) \right) \pi \right] \frac{v}{4a} \end{aligned}$$

Hence,

$$\Psi'(\pi) = \frac{\theta_H(\theta_L + 1)^4(3\theta_H^2 - 2\theta_H - 1) + \theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2)}{(\theta_H + 1)^4(\theta_L + 1)^4} \frac{v}{4a},$$

where for $\theta_H > 1$,

$$3\theta_H^2 - 2\theta_H - 1 = (3\theta_H + 1)(\theta_H - 1) > 0.$$

and for $0 < \theta_L < 1$,

$$1 + 2\theta_L - 3\theta_L^2 = 3(1 - \theta_L)\left(\theta_L + \frac{1}{3}\right) > 0.$$

Therefore

$$\Psi'(\pi) > 0.$$

Setting $\Psi(\pi) = 0$, and solving for π , we obtain:

$$\bar{\pi} = \frac{\theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2)}{\theta_H(\theta_L + 1)^4(3\theta_H^2 - 2\theta_H - 1) + \theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2)}.$$

Proof of Proposition 3: At $\pi = \bar{\pi}$ we have $a_1^{s*} = a_3^{s*}$, so $r^s \equiv a_1^{s*}/a_3^{s*} = 1$. Hence the newcomer's equilibrium probability of winning the semifinal is

$$P_1^{s*}(\bar{\pi}) = \bar{\pi} \frac{\theta_H}{\theta_H + 1} + (1 - \bar{\pi}) \frac{\theta_L}{\theta_L + 1}.$$

Moreover, when $r^s = 1$ the posterior belief μ simplifies to

$$\mu(\pi) = \frac{\pi\theta_H(\theta_L + 1)}{\theta_L\theta_H + \pi\theta_H + (1 - \pi)\theta_L},$$

so the newcomer's equilibrium probability of winning the final, evaluated at $\pi = \bar{\pi}$, is

$$P_1^{f*}(\mu(\bar{\pi})) = \mu(\bar{\pi}) \frac{\theta_H}{\theta_H + 1} + (1 - \mu(\bar{\pi})) \frac{\theta_L}{\theta_L + 1}.$$

Define $\Delta \equiv \frac{\theta_H}{\theta_H + 1} - \frac{\theta_L}{\theta_L + 1}$. Then

$$P_1^s(\bar{\pi}) = \frac{\theta_L}{\theta_L + 1} + \bar{\pi} \Delta, \quad P_1^f(\mu(\bar{\pi})) = \frac{\theta_L}{\theta_L + 1} + \mu(\bar{\pi}) \Delta.$$

Substituting the closed-form expression of $\bar{\pi}$ and simplifying yields the factorization

$$P_1^{f*}(\mu(\bar{\pi})) P_1^{s*}(\bar{\pi}) - \frac{1}{4} = \frac{(\theta_H - 1)(3\theta_H + 1)(\theta_L - 1)(3\theta_L + 1)(\theta_H \theta_L - 1)}{4(3\theta_H \theta_L + \theta_H + \theta_L - 1)(\theta_H^2 \theta_L^2 - \theta_H^2 \theta_L - \theta_H \theta_L^2 - 8\theta_H \theta_L - 3\theta_H - 3\theta_L - 1)}.$$

Under $\theta_H > 1 > \theta_L > 0$, the factors $(\theta_H - 1)$, $(3\theta_H + 1)$, and $(3\theta_L + 1)$ are positive, while $(\theta_L - 1)$ is negative. Also, one checks that

$$3\theta_H \theta_L + \theta_H + \theta_L - 1 > 0 \quad \text{and} \quad \theta_H^2 \theta_L^2 - \theta_H^2 \theta_L - \theta_H \theta_L^2 - 8\theta_H \theta_L - 3\theta_H - 3\theta_L - 1 < 0.$$

Therefore the denominator in the last display is negative, and the product $(\theta_H - 1)(3\theta_H + 1)(\theta_L - 1)(3\theta_L + 1)$ is negative, so the sign of $P_1^{f*}(\mu(\bar{\pi})) P_1^{s*}(\bar{\pi}) - \frac{1}{4}$ coincides with the sign of $(\theta_H \theta_L - 1)$. This proves

$$P_1^{f*}(\mu(\bar{\pi})) P_1^{s*}(\bar{\pi}) \gtrless \frac{1}{4} \iff \theta_H \theta_L \gtrless 1.$$

To complete the proof, we show that $\bar{\pi} < 1/2$ if $\Theta = 1$. We fix $\Theta = 1$ and show that $\bar{\pi} < 1/2$, which is true if the following expression is true:

$$\theta_L(\theta_H + 1)^4(1 + 2\theta_L - 3\theta_L^2) < \theta_H(\theta_L + 1)^4(3\theta_H^2 - 2\theta_H - 1),$$

or, replacing $\theta_L = 1/\theta_H$,

$$\frac{(\theta_H + 1)^4}{\theta_H(3\theta_H^2 - 2\theta_H - 1)} < \frac{\left(\frac{1}{\theta_H} + 1\right)^4}{\frac{1}{\theta_H} \left(1 + 2\frac{1}{\theta_H} - 3\frac{1}{\theta_H^2}\right)},$$

or,

$$\frac{(\theta_H + 1)^4}{\theta_H(3\theta_H^2 - 2\theta_H - 1)} < \frac{(\theta_H + 1)^4}{\theta_H(\theta_H^2 + 2\theta_H - 3)},$$

or

$$\theta_H^2 + 2\theta_H - 3 < 3\theta_H^2 - 2\theta_H - 1,$$

which is equivalent to

$$0 < 2(\theta_H - 1)^2,$$

which is always true.

Next, we wish to show that $P_1^{s*} < 1/2 < P_1^{f*}$. Bearing in mind that $a_1^{s*} = a_3^{s*}$, to establish that $P_1^{s*} < 1/2$ it is sufficient to show that:

$$\bar{\pi} \frac{\theta_H}{\theta_H + 1} + (1 - \bar{\pi}) \frac{\theta_L}{\theta_L + 1} < 1/2,$$

when $\theta_L = 1/\theta_H$. It is immediate to show that this is true if and only if $\bar{\pi} < 1/2$. Since $P_1^{s*} P_1^{f*} = 1/4$ for $\theta_L = 1/\theta_H$, and bearing in mind that $a_1^{f*} = a_2^{f*}$, we therefore conclude that $\bar{\pi} < 1/2 \Rightarrow P_1^{s*} < 1/2 < P_1^{f*}$.

Proof of Result 1: The gap between the final and the semifinal stage biases is given by:

$$\begin{aligned} b^f - b^s &= (\tilde{\mu} - \mu) - (\tilde{\pi} - \pi) \\ &= (\tilde{\mu} - \tilde{\pi}) - (\mu - \pi) \\ &= \tilde{\pi} \left[\frac{P_1^s(a_1^s, a_3^s; \theta_H)}{\tilde{\pi} P_1^s(a_1^s, a_3^s; \theta_H) + (1 - \tilde{\pi}) P_1^s(a_1^s, a_3^s; \theta_L)} - 1 \right] \\ &\quad - \pi \left[\frac{P_1^s(a_1^s, a_3^s; \theta_H)}{\pi P_1^s(a_1^s, a_3^s; \theta_H) + (1 - \pi) P_1^s(a_1^s, a_3^s; \theta_L)} - 1 \right] \\ &= \tilde{\pi}(1 - \tilde{\pi}) \frac{P_1^s(a_1^s, a_3^s; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L)}{\tilde{\pi} P_1^s(a_1^s, a_3^s; \theta_H) + (1 - \tilde{\pi}) P_1^s(a_1^s, a_3^s; \theta_L)} \\ &\quad - \pi(1 - \pi) \frac{P_1^s(a_1^s, a_3^s; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L)}{\pi P_1^s(a_1^s, a_3^s; \theta_H) + (1 - \pi) P_1^s(a_1^s, a_3^s; \theta_L)}. \end{aligned}$$

Thence, the sign of $b^f - b^s$ is given by:

$$\begin{aligned} \text{sgn}\{b^f - b^s\} &= \text{sgn}\{(1 - \tilde{\pi})(1 - \pi) P_1^s(a_1^s, a_3^s; \theta_L) - \pi \tilde{\pi} P_1^s(a_1^s, a_3^s; \theta_H)\} \\ &= \text{sgn}\{(1 - \pi - b^s)(1 - \pi) P_1^s(a_1^s, a_3^s; \theta_L) - \pi(\pi + b^s) P_1^s(a_1^s, a_3^s; \theta_H)\} \\ &= \text{sgn}\left\{ -[P_1^s(a_1^s, a_3^s; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L)] \pi^2 \right. \\ &\quad \left. - [2P_1^s(a_1^s, a_3^s; \theta_L) + b^s(P_1^s(a_1^s, a_3^s; \theta_H) - P_1^s(a_1^s, a_3^s; \theta_L))] \pi \right. \\ &\quad \left. + (1 - b^s) P_1^s(a_1^s, a_3^s; \theta_L) \right\}. \end{aligned}$$

Since for $\pi = 0$ the above expression is always positive, for $\pi = 1 - b^s$ it is always negative, and given the quadratic nature of the expression there exists a unique threshold $\hat{\pi} \in (0, 1 - b^s)$ such that $\pi \lesseqgtr \hat{\pi} \Leftrightarrow b^f \gtrless b^s$.

Proof of Uniqueness of a Pure Strategy Bayesian-Nash Equilibrium: Optimizing (12) and (13), we obtain the two following first-order conditions:

$$\left[\tilde{\mu} \frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1 - \tilde{\mu}) \frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2} \right] a_2^f v = c, \quad (22)$$

and

$$\left[\mu \frac{\theta_H}{(\theta_H a_1^f + a_2^f)^2} + (1 - \mu) \frac{\theta_L}{(\theta_L a_1^f + a_2^f)^2} \right] a_1^f v = c. \quad (23)$$

To show that the best response functions of players in the final, $R_i^f(a_{-i}^f)$, $i \in \{1, 2\}$, are quasi-concave, we focus on the best response of the overconfident player, player 1, and the reasoning extends to the rival player 2. We show that (i) the slope of player 1's best response function is strictly positive for $a_2^f = 0$, i.e. $(\partial R_1^f / \partial a_2^f)|_{a_2^f=0} > 0$, (ii) it is strictly negative for $a_2^f \rightarrow \infty$, i.e. $(\partial R_1^f / \partial a_2^f)|_{a_2^f \rightarrow \infty} < 0$, and (iii) whenever $\partial R_1^f / \partial a_2^f = 0$, then $\partial^2 R_1^f / (\partial a_2^f)^2 < 0$.

By implicit differentiation of the first-order condition of player 1 as given by equation (22), we deduce that the sign of the slope of player 1's best response is given by:

$$\text{sgn} \left\{ \frac{\partial R_1^f}{\partial a_2^f} \right\} = \text{sgn} \left\{ \frac{\partial^2 E[U_1^f(a_1^f, a_2^f)]}{\partial a_1^f \partial a_2^f} \right\} = \text{sgn} \left\{ \frac{\tilde{\mu} \theta_H (\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1 - \tilde{\mu}) \theta_L (\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3} \right\}. \quad (24)$$

Points (i) and (ii) are immediately deduced upon observing the above expression. Turning next to (iii), define first $\phi(a_2^f) = \frac{\partial^2 E[U_1^f(a_1^f, a_2^f)]}{\partial a_1^f \partial a_2^f}$. To establish (iii) it is then sufficient to show that when $\phi(a_2^f) = 0$, then $\phi'(a_2^f) < 0$. We thus compute $\phi'(a_2^f)$ which is given by:

$$\phi'(a_2^f) = -\frac{\tilde{\mu} \theta_H}{(\theta_H a_1^f + a_2^f)^3} - \frac{3\tilde{\mu} \theta_H (\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^4} - \frac{(1 - \tilde{\mu}) \theta_L}{(\theta_L a_1^f + a_2^f)^3} - \frac{3(1 - \tilde{\mu}) \theta_L (\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^4}.$$

Substituting for $\phi(a_2^f) = 0$, we can show that the above expression can be re-expressed as:

$$\phi'(a_2^f) = \frac{\tilde{\mu} \theta_H}{(\theta_H a_1^f + a_2^f)^3} \left(\frac{(\theta_H - \theta_L) a_1^f}{\theta_L a_1^f - a_2^f} - 3(\theta_H a_1^f - a_2^f) \left(\frac{1}{\theta_H a_1^f + a_2^f} - \frac{1}{\theta_L a_1^f + a_2^f} \right) \right),$$

or,

$$\phi'(a_2^f) = \frac{\tilde{\mu} \theta_H 4(\theta_H - \theta_L) a_1^f}{(\theta_H a_1^f + a_2^f)^3 (\theta_L a_1^f - a_2^f)} < 0,$$

with the sign following from the observation that to have $\phi(a_2^f) = 0$, it is necessary that $\theta_L a_1^f - a_2^f < 0$.

To prove uniqueness, we show that the contraction mapping $\partial R_1^f(a_2^f) / \partial a_2^f \cdot \partial R_2^f(a_1^f) / \partial a_1^f$ is smaller than 1. The slopes of $R_1^f(a_2^f)$ and $R_2^f(a_1^f)$ are respectively given by:

$$\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} = \frac{\frac{\tilde{\mu} \theta_H (\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1 - \tilde{\mu}) \theta_L (\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3}}{2a_2^f \left[\frac{\tilde{\mu} \theta_H^2}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1 - \tilde{\mu}) \theta_L^2}{(\theta_L a_1^f + a_2^f)^3} \right]},$$

and,

$$\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} = -\frac{\frac{\mu\theta_H(\theta_H a_1^f - a_2^f)}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\mu)\theta_L(\theta_L a_1^f - a_2^f)}{(\theta_L a_1^f + a_2^f)^3}}{2a_1^f \left[\frac{\mu\theta_H}{(\theta_H a_1^f + a_2^f)^3} + \frac{(1-\mu)\theta_L}{(\theta_L a_1^f + a_2^f)^3} \right]}.$$

Observe first that if $\theta_L a_1^f \geq a_2^f$, then $\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} > 0$ and $\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} < 0$. Next, observe that if $\theta_H a_1^f \leq a_2^f$, then $\frac{\partial R_1^f(a_2^f)}{\partial a_2^f} < 0$ and $\frac{\partial R_2^f(a_1^f)}{\partial a_1^f} > 0$. In both cases the product of the slopes of the best responses is negative and the contraction mapping is smaller than 1. A necessary condition for the slopes of the best responses at equilibrium to be of equal sign is that

$$\theta_H a_1^f - a_2^f > 0 > \theta_L a_1^f - a_2^f. \quad (25)$$

We now demonstrate that at equilibrium it is impossible for both best responses to be negatively sloped. Denote $\Psi_K = \frac{\theta_K(\theta_K a_1^f - a_2^f)}{(\theta_K a_1^f + a_2^f)^3}$, $K = \{H, L\}$. Since at equilibrium a_1^{f*} and a_2^{f*} cannot be negative, when condition (25) holds we have $\Psi_H > 0$ and $\Psi_L < 0$. Accordingly, the sign of the slope of $R_1^f(a_2^{f*})$ is given by the sign of $\tilde{\mu}\Psi_K + (1-\tilde{\mu})\Psi_L$, and the sign of the slope of $R_2^f(a_1^{f*})$ is given by the sign of $\mu\Psi_K + (1-\mu)\Psi_L$. Assume then that the sign of the slope of $R_2^f(a_1^{f*})$ is negative. For this to be the case when $\Psi_H > 0$ and $\Psi_L < 0$ we must have $-\frac{\Psi_H}{\Psi_L} > \frac{1-\mu}{\mu}$. Thence, for $R_1^f(a_2^{f*})$ to also be negative when $\Psi_H > 0$ and $\Psi_L < 0$, we need that $-\frac{\Psi_H}{\Psi_L} < \frac{1-\tilde{\mu}}{\tilde{\mu}}$. For both these inequalities to hold, we need that $\frac{1-\tilde{\mu}}{\tilde{\mu}} > \frac{1-\mu}{\mu}$, which contradicts $\tilde{\mu} > \mu$. Hence, at equilibrium it is impossible for both best responses to be negatively sloped.

We are left with the case where at equilibrium both best responses are positively sloped. To next show that the contraction mapping is smaller than 1. This is equivalent to inquiring whether the following inequality is true:

$$\left(\frac{\tilde{\mu}\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) \left(\frac{\mu\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\mu)\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) + 4a_1^{f*}a_2^{f*} \left(\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) \left(\frac{\mu\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\mu)\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) > 0.$$

We then drop from the above expression the following two positive terms:

$$\left(\frac{\tilde{\mu}\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) \frac{\mu\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + 4a_1^{f*}a_2^{f*} \left(\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1-\tilde{\mu})\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) \frac{\mu\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3}.$$

The original expression is then necessarily true if:

$$\begin{aligned} & \left(\frac{\tilde{\mu}\theta_H(\theta_H a_1^{f*} - a_2^{f*})}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1 - \tilde{\mu})\theta_L(\theta_L a_1^{f*} - a_2^{f*})}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) (\theta_L a_1^{f*} - a_2^{f*}) \\ & + 4a_1^{f*} a_2^{f*} \left(\frac{\tilde{\mu}\theta_H^2}{(\theta_H a_1^{f*} + a_2^{f*})^3} + \frac{(1 - \tilde{\mu})\theta_L^2}{(\theta_L a_1^{f*} + a_2^{f*})^3} \right) > 0. \end{aligned}$$

or,

$$\begin{aligned} & \frac{\tilde{\mu}\theta_H}{(\theta_H a_1^{f*} + a_2^{f*})^3} \left((\theta_H a_1^{f*} - a_2^{f*})(\theta_L a_1^{f*} - a_2^{f*}) + 4a_1^{f*} a_2^{f*} \theta_H \right) \\ & + \frac{(1 - \tilde{\mu})\theta_L}{(\theta_L a_1^{f*} + a_2^{f*})^3} \left((\theta_L a_1^{f*} - a_2^{f*})^2 + 4a_1^{f*} a_2^{f*} \theta_L \right) > 0. \end{aligned}$$

It is then sufficient to prove that the first expression is true, or:

$$\theta_L \theta_H (a_1^{f*})^2 + (a_2^{f*})^2 - \theta_L a_1^{f*} a_2^{f*} + 3a_1^{f*} a_2^{f*} \theta_H > 0,$$

and this expression is always true since $\theta_H > \theta_L$.

Proof of Proposition 4: Let $\phi_K = \frac{\theta_K}{(\theta_K a_1^f + a_2^f)^2}$, $K = \{L, H\}$. The first-order conditions become

$$[\tilde{\mu}\phi_H + (1 - \tilde{\mu})\phi_L] a_2^f v = c,$$

and

$$[\mu\phi_H + (1 - \mu)\phi_L] a_1^f v = c.$$

Let us study the sign of

$$\begin{aligned} \Xi &= [\tilde{\mu}\phi_H + (1 - \tilde{\mu})\phi_L] - [\mu\phi_H + (1 - \mu)\phi_L] \\ &= \tilde{\mu}(\phi_H - \phi_L) - \mu(\phi_H - \phi_L) \\ &= (\tilde{\mu} - \mu)(\phi_H - \phi_L) \\ &= (\tilde{\mu} - \mu)(\theta_H - \theta_L) \frac{(a_2^f)^2 - \theta_L \theta_H (a_1^f)^2}{(\theta_H a_1^f + a_2^f)^2 (\theta_L a_1^f + a_2^f)^2} \end{aligned}$$

If we know the sign of Ξ , then we know which player exerts higher effort as $\Xi = 0$ implies $a_1^{f*} = a_2^{f*}$, $\Xi > 0$ implies $a_1^{f*} > a_2^{f*}$, and $\Xi < 0$ implies $a_1^{f*} < a_2^{f*}$. Note that $\tilde{\pi} > \pi$ implies $\tilde{\mu} - \mu > 0$. Hence, to determine the sign of Ξ we only need to consider the sign of the numerator of the third term. To do that we consider three possible situations: (i) $\theta_L \theta_H = 1$; (ii) $\theta_L \theta_H < 1$; and (iii) $\theta_L \theta_H > 1$.

Consider case (i): $\theta_L \theta_H = 1$. The sign of Ξ is given by $(a_2^f)^2 - (a_1^f)^2$. If $a_2^f = a_1^f$, then $\Xi = 0$ and both first-order conditions are satisfied. If $a_2^f > a_1^f$, then $\Xi > 0$, and the

first-order conditions are violated. If $a_2^f < a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. Hence, when $\theta_L \theta_H = 1$ the equilibrium satisfies $a_1^{f*} = a_2^{f*} = v\theta_H/c(\theta_H + 1)^2$.

Consider case (ii): $\theta_L \theta_H < 1$. The sign of Ξ is given by $(a_2^f)^2 - \theta_L \theta_H (a_1^f)^2$. If $a_2^f = a_1^f$, then $\Xi > 0$, and the first-order conditions are violated. If $a_2^f > a_1^f$, then $\Xi > 0$, and the first-order conditions are violated. For $a_2^f < a_1^f$ there are three subcases: (a) if $\theta_L \theta_H (a_1^f)^2 < (a_2^f)^2 < (a_1^f)^2$, then $\Xi > 0$, and there will exist values of a_1^f and a_2^f that satisfy the first-order conditions; (b) if $(a_2^f)^2 = \theta_L \theta_H (a_1^f)^2$, then $\Xi = 0$, and the first-order conditions are violated; (c) if $(a_2^f)^2 < \theta_L \theta_H (a_1^f)^2$, then $\Xi < 0$, and the first-order conditions are violated. Hence, when $\theta_L \theta_H \in (0, 1)$ the equilibrium satisfies $a_1^{f*} > a_2^{f*}$.

Consider case (iii): $\theta_L \theta_H > 1$. The sign of Ξ is given by $(a_2^f)^2 - \theta_L \theta_H (a_1^f)^2$. If $a_2^f = a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. If $a_2^f < a_1^f$, then $\Xi < 0$, and the first-order conditions are violated. For $a_2^f > a_1^f$ there are three subcases: (a) if $(a_2^f)^2 < \theta_L \theta_H (a_1^f)^2$, then $\Xi < 0$, and there will exist values of a_1^f and a_2^f that satisfy the first-order conditions; (b) if $(a_2^f)^2 = \theta_L \theta_H (a_1^f)^2$, then $\Xi = 0$, and the first-order conditions are violated; (c) if $(a_2^f)^2 > \theta_L \theta_H (a_1^f)^2$, then $\Xi > 0$, and the first-order conditions are violated. Hence, when $\theta_L \theta_H > 1$ the equilibrium satisfies $a_2^{f*} > a_1^{f*}$.

Second-order conditions of the semifinal with an overconfident newcomer when $\theta_L \theta_H = 1$: The first derivative of the newcomer's perceived expected utility in the semifinal is:

$$\frac{\partial \tilde{E}[U_1^s]}{\partial a_1^s} = \frac{\partial \tilde{P}_1^s}{\partial a_1^s} \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \tilde{\mu} \right] v + \tilde{P}_1^s(a_1^s, a_3^s) \frac{\theta_H - 1}{\theta_H + 1} v \frac{\partial \tilde{\mu}}{\partial a_1^s} - c.$$

The second derivative is therefore given by:

$$\frac{\partial^2 \tilde{E}[U_1^s]}{(\partial a_1^s)^2} = \frac{\partial^2 \tilde{P}_1^s}{(\partial a_1^s)^2} \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \tilde{\mu} \right] v + 2 \frac{\partial \tilde{P}_1^s}{\partial a_1^s} \frac{\theta_H - 1}{\theta_H + 1} v \frac{\partial \tilde{\mu}}{\partial a_1^s} + \tilde{P}_1^s \frac{\theta_H - 1}{\theta_H + 1} v \frac{\partial^2 \tilde{\mu}}{(\partial a_1^s)^2}.$$

Since $\frac{\partial^2 \tilde{P}_1^s}{(\partial a_1^s)^2} < 0$, this expression is a fortiori negative if:

$$\left[\frac{\partial^2 \tilde{P}_1^s}{(\partial a_1^s)^2} \tilde{\mu} + 2 \frac{\partial \tilde{P}_1^s}{\partial a_1^s} \frac{\partial \tilde{\mu}}{\partial a_1^s} + \tilde{P}_1^s \frac{\partial^2 \tilde{\mu}}{(\partial a_1^s)^2} \right] \underbrace{\frac{\theta_H - 1}{\theta_H + 1} v}_{>0} < 0.$$

We thus need to show that the term inside square brackets is negative. Substituting for the appropriate terms, and simplifying, we can show that the term inside square brackets is given by:

$$-\frac{2\tilde{\pi}\theta_H^2 a_3^s}{(\theta_H a_1^s + a_3^s)^3} < 0.$$

Hence, the second-order condition for the newcomer is satisfied. It is immediate to verify

that the second-order condition for the incumbent is also satisfied.

Proof of Proposition 5: When $\Theta = 1$, we know that $a_1^{f*} = a_2^{f*}$, which using player 1's first-order condition is shown to equal:

$$a^{f*} = \left[\tilde{\mu} \frac{\theta_H}{(\theta_H + 1)^2} + (1 - \tilde{\mu}) \frac{\theta_L}{(\theta_L + 1)^2} \right] \frac{v}{c}$$

Substituting for $\theta_L = 1/\theta_H$, this expression reads as:

$$a^{f*} = \frac{\theta_H}{(\theta_H + 1)^2} \frac{v}{c}.$$

Player 1's equilibrium perceived expected utility of the final is then equal to:

$$\begin{aligned} \tilde{E}[U_1^f(a_1^{f*}, a_2^{f*}; \tilde{\mu})] &= \left[\tilde{\mu} \frac{\theta_H}{\theta_H + 1} + (1 - \tilde{\mu}) \frac{1}{\theta_H + 1} \right] v - c a^{f*} \\ &= \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \tilde{\mu} \right] v, \end{aligned}$$

where, imposing $\theta_L = 1/\theta_H$, we have

$$\tilde{\mu} = \frac{\tilde{\pi} \theta_H (a_1^s + \theta_H a_3^s)}{\theta_H a_1^s + [\tilde{\pi} \theta_H^2 + (1 - \tilde{\pi})] a_3^s}.$$

We know that when $\Theta = 1$, $\partial a_2^{f*} / \partial \tilde{\mu} = \partial a_2^{f*} / \partial \mu = 0$, and hence the first-order condition (17) can be written as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \tilde{\mu} \right] v + \tilde{P}_1^s(a_1^s, a_3^s) \frac{\theta_H - 1}{\theta_H + 1} v \frac{\partial \tilde{\mu}}{\partial a_1^s} = c. \quad (26)$$

Computing each term of (26) separately, we have:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \frac{\partial \tilde{P}_1^s(a_1^s, a_3^s; \tilde{\pi})}{\partial a_1^s} = \left[\frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2} + \tilde{\pi} \left(\frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} - \frac{\theta_L a_3^s}{(\theta_L a_1^s + a_3^s)^2} \right) \right]$$

Imposing $\theta_L = 1/\theta_H$, this reads as:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \left[\frac{\theta_H a_3^s}{(a_1^s + \theta_H a_3^s)^2} + \tilde{\pi} \left(\frac{\theta_H a_3^s}{(\theta_H a_1^s + a_3^s)^2} - \frac{\theta_H a_3^s}{(a_1^s + \theta_H a_3^s)^2} \right) \right]$$

Evaluating this expression at $a_1^s = a_3^s = a$, this expression becomes:

$$\frac{\partial \tilde{P}_1^s}{\partial a_1^s} = \frac{\theta_H}{a(\theta_H + 1)^2}.$$

Turning next to the square-bracketed term of expression (26), since a_2^{f*} is independent of posterior beliefs, the first multiplicative term is then nil. Focusing next on the last term of expression (26), we have that when evaluated at $a_1^s = a_3^s = a$, then:

$$\begin{aligned} \frac{\partial \tilde{E}[U_1^f]}{\partial \tilde{\mu}} \frac{\partial \tilde{\mu}}{\partial a_1^s} &= - \left(\frac{\theta_H}{\theta_H + 1} - \frac{\theta_L}{\theta_L + 1} \right) \frac{\tilde{\pi}(1 - \tilde{\pi})\theta_L\theta_H(\theta_H - \theta_L)}{[\theta_L\theta_H + \tilde{\pi}\theta_H + (1 - \tilde{\pi})\theta_L]^2 a} v \\ &= - \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi}(1 - \tilde{\pi})\theta_H(\theta_H^2 - 1)}{[\theta_H + \tilde{\pi}\theta_H^2 + (1 - \tilde{\pi})]^2 a} v \\ &= - \frac{\tilde{\pi}(1 - \tilde{\pi})\theta_H(\theta_H - 1)^2}{[\theta_H + \tilde{\pi}\theta_H^2 + (1 - \tilde{\pi})]^2 a} v \\ &= - \frac{\tilde{\pi}(1 - \tilde{\pi})\theta_H(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)^2 a} v \end{aligned}$$

The first-order condition of player 1 when evaluated at $a_1^s = a_3^s = a$ is then equal to:

$$\frac{\theta_H}{a(\theta_H + 1)^2} \frac{1 - \tilde{\pi} + \tilde{\pi}\theta_H^3}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)} v - \frac{1 - \tilde{\pi} + \tilde{\pi}\theta_H}{\theta_H + 1} \frac{\tilde{\pi}(1 - \tilde{\pi})\theta_H(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)^2 a} v = c,$$

or

$$\frac{\theta_H}{a(\theta_H + 1)^2} \frac{1 - \tilde{\pi} + \tilde{\pi}\theta_H^3}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)} v - \frac{\theta_H}{\theta_H + 1} \frac{\tilde{\pi}(1 - \tilde{\pi})(\theta_H - 1)^2}{(\theta_H + 1)^2(1 - \tilde{\pi} + \tilde{\pi}\theta_H)a} v = c,$$

or

$$\frac{\theta_H(1 - \tilde{\pi} + \tilde{\pi}\theta_H^2)}{(\theta_H + 1)^4} \frac{v}{a} = c$$

We next explore the first-order condition of player 3 which, when evaluated at $a_1^s = a_3^s = a$, is given by:

$$\frac{\theta_H}{4(\theta_H + 1)^2} \frac{v}{a} = c.$$

Using the first-order derivatives, we can next express $\Psi(\tilde{\pi})$ evaluated at $a_1^s = a_3^s = a$ as:

$$\begin{aligned} \Psi(\tilde{\pi}) &= \left. \frac{\partial \tilde{E}[U_1^s(a_1^s, a_3^s; \tilde{\pi})]}{\partial a_1^s} \right|_{a_1^s = a_3^s = a} - \left. \frac{\partial E[U_3^s(a_1^s, a_3^s)]}{\partial a_3^s} \right|_{a_1^s = a_3^s = a} \\ &= \left[\frac{\theta_H(1 - \tilde{\pi} + \tilde{\pi}\theta_H^2)}{(\theta_H + 1)^4} - \frac{\theta_H}{4(\theta_H + 1)^2} \right] \frac{v}{a} \\ &= - \frac{\theta_H(\theta_H - 1)[\theta_H + 3 - 4\tilde{\pi}(\theta_H + 1)]}{4(\theta_H + 1)^4} \frac{v}{a}, \end{aligned}$$

with $\Psi(\tilde{\pi}) \gtrless 0 \Leftrightarrow a_1^{s*} \gtrless a_3^{s*}$. It follows that $\Psi(\tilde{\pi}) = 0$ when

$$\tilde{\pi} = \frac{\theta_H + 3}{4(\theta_H + 1)}$$

So, when the prior belief of the overconfident player 1, $\tilde{\pi}$, is higher than $\tilde{\pi}$, he exerts more effort than the rival in the semifinal. Note that $\theta_H > 1$ implies $\tilde{\pi} \in (1/4, 1/2)$ and that the higher is θ_H the closer is $\tilde{\pi}$ to $1/4$.

Proof of Proposition 6: To prove this result, we first show that $P_1^{s*}P_1^{f*}$ is monotonically increasing in the ratio a_1^{s*}/a_3^{s*} , and we then show that this ratio is monotonically increasing in b^s .

Making use of equations (18) and (19), the newcomer's ex-ante true probability of winning the elimination contest is given by:

$$P_1^{s*}P_1^{f*} = \left[\pi \frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*} + a_3^{s*}} + (1 - \pi) \frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H + a_3^{s*}} \right] \frac{1 + \mu^*(\theta_H - 1)}{\theta_H + 1},$$

which after substituting for μ^* as given by equation (6) becomes:

$$P_1^{s*}P_1^{f*} = \left[\pi \frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*} + a_3^{s*}} + (1 - \pi) \frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H + a_3^{s*}} \right] \frac{1 + \frac{\pi \theta_H (a_1^{s*}/\theta_H + a_3^{s*})}{a_1^{s*} + (\pi \theta_H + (1 - \pi)/\theta_H) a_3^{s*}} (\theta_H - 1)}{\theta_H + 1}.$$

Rewriting the above expression as a function of $x = a_1^{s*}/a_3^{s*}$, we obtain:

$$\begin{aligned} P_1^{s*}P_1^{f*} &= \left[\frac{\pi \theta_H}{\theta_H x + 1} + \frac{(1 - \pi)}{x + \theta_H} \right] \left[1 + \frac{\pi \theta_H (x/\theta_H + 1)(\theta_H - 1)}{x + (\pi \theta_H + (1 - \pi)/\theta_H)} \right] \frac{x}{\theta_H + 1} \\ &= \frac{\theta_H x + \pi \theta_H^2 + (1 - \pi)}{(\theta_H x + 1)(x + \theta_H)} \left[\frac{\theta_H x + \pi \theta_H^2 + (1 - \pi) + \pi \theta_H (x + \theta_H)(\theta_H - 1)}{\theta_H x + \pi \theta_H^2 + (1 - \pi)} \right] \frac{x}{\theta_H + 1} \\ &= \frac{(1 - \pi)(\theta_H x + 1) + \pi \theta_H^2 (x + \theta_H)}{(\theta_H x + 1)(x + \theta_H)} \frac{x}{\theta_H + 1} \\ &= \left[(1 - \pi) \frac{x}{x + \theta_H} + \pi \theta_H \frac{\theta_H x}{\theta_H x + 1} \right] \frac{1}{\theta_H + 1}. \end{aligned}$$

Since the two terms inside the squared brackets are increasing in x , we deduce that $P_1^{s*}P_1^{f*}$ is equally increasing in x .

We next prove that $dx/db^s > 0$. At optimality, the first order conditions of both players 1 and 3 ought to be simultaneously satisfied. The first order condition of player 1 is given by:

$$\begin{aligned} &\left[\tilde{\pi} \frac{\theta_H a_3^{s*}}{(\theta_H a_1^{s*} + a_3^{s*})^2} + (1 - \tilde{\pi}) \frac{a_3^{s*}/\theta_H}{(a_1^{s*}/\theta_H + a_3^{s*})^2} \right] \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi} \theta_H (a_1^{s*}/\theta_H + a_3^{s*})}{a_1^{s*} + (\tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H) a_3^{s*}} \right] v \\ &- \left[\tilde{\pi} \frac{\theta_H a_1^{s*}}{\theta_H a_1^{s*} + a_3^{s*}} + (1 - \tilde{\pi}) \frac{a_1^{s*}/\theta_H}{a_1^{s*}/\theta_H + a_3^{s*}} \right] \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi} (1 - \tilde{\pi}) (\theta_H - 1/\theta_H) a_3^{s*}}{[a_1^{s*} + (\tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H) a_3^{s*}]^2} v = c \end{aligned}$$

Rewriting this condition as a function of x , we obtain:

$$\begin{aligned} & \left[\tilde{\pi} \frac{\theta_H}{(\theta_H x + 1)^2} + (1 - \tilde{\pi}) \frac{1/\theta_H}{(x/\theta_H + 1)^2} \right] \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi} \theta_H (x/\theta_H + 1)}{x + \tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H} \right] \frac{v}{a_3^{s*}} \\ & - \left[\tilde{\pi} \frac{\theta_H x}{\theta_H x + 1} + (1 - \tilde{\pi}) \frac{x/\theta_H}{x/\theta_H + 1} \right] \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi}(1 - \tilde{\pi})(\theta_H - 1/\theta_H)}{[x + \tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H]^2} \frac{v}{a_3^{s*}} = c. \end{aligned}$$

Proceeding likewise for player 3, we obtain:

$$\begin{aligned} & \left[\pi \frac{\theta_H a_1^{s*}}{(\theta_H a_1^{s*} + a_3^{s*})^2} + (1 - \pi) \frac{a_1^{s*}/\theta_H}{(a_1^{s*}/\theta_H + a_3^{s*})^2} \right] \frac{v}{4} = c \\ & \left[\pi \frac{\theta_H x}{(\theta_H x + 1)^2} + (1 - \pi) \frac{x/\theta_H}{(x/\theta_H + 1)^2} \right] \frac{v}{4a_3^s} = c. \end{aligned}$$

Combining these two first-order conditions, we obtain:

$$\begin{aligned} \mathcal{A}(x) &= \left[\tilde{\pi} \frac{\theta_H}{(\theta_H x + 1)^2} + (1 - \tilde{\pi}) \frac{1/\theta_H}{(x/\theta_H + 1)^2} \right] \left[\left(\frac{1}{\theta_H + 1} \right)^2 + \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi} \theta_H (x/\theta_H + 1)}{x + \tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H} \right] \\ &- \left[\tilde{\pi} \frac{\theta_H x}{\theta_H x + 1} + (1 - \tilde{\pi}) \frac{x/\theta_H}{x/\theta_H + 1} \right] \frac{\theta_H - 1}{\theta_H + 1} \frac{\tilde{\pi}(1 - \tilde{\pi})(\theta_H - 1/\theta_H)}{[x + \tilde{\pi} \theta_H + (1 - \tilde{\pi})/\theta_H]^2} \\ &- \left[\pi \frac{\theta_H x}{(\theta_H x + 1)^2} + (1 - \pi) \frac{x/\theta_H}{(x/\theta_H + 1)^2} \right] \frac{1}{4} = 0. \end{aligned}$$

This expression can be re-written as:

$$\mathcal{A}(x) = -\frac{\theta_H}{4(\theta_H + 1)^2(\theta_H x^2 + \theta_H^2 x + x + \theta_H)^2} [c_1 x^3 + c_2 x^2 + c_3 x + c_4] = 0,$$

where:

$$\begin{aligned} c_1 &= \pi + 2\pi\theta_H + \theta_H^2 + 2(1 - \pi)\theta_H^3 + (1 - \pi)\theta_H^4 \\ c_2 &= 2\theta_H(\theta_H^2 + 1) \\ c_3 &= 8\tilde{\pi}\theta_H - 6\theta_H - 2\pi\theta_H - \pi + \theta_H^2 + 2\pi\theta_H^3 + \pi\theta_H^2 - 8\tilde{\pi}\theta_H^3 + 1 \\ c_4 &= -4(1 - \tilde{\pi} + \tilde{\pi}\theta_H^4) \end{aligned}$$

Observe that the fraction in $\mathcal{A}(x)$ is always positive, thence implying that the first-order conditions are satisfied if $\mathcal{B}(x) = c_1 x^3 + c_2 x^2 + c_3 x + c_4 = 0$, thence implying that this condition defines the equilibrium value of x . We are interested in the sign of dx/db^s which is the same as the sign of $dx/d\tilde{\pi}$. Applying implicit differentiation to $\mathcal{B}(x)$, we have:

$$\frac{dx}{d\tilde{\pi}} = -\frac{\frac{\partial \mathcal{B}(x)}{\partial \tilde{\pi}}}{\frac{\partial \mathcal{B}(x)}{\partial x}} = -\frac{4(2\theta_H x(1 - \theta_H^2) + 1 - \theta_H^4)}{3c_1 x^2 + 2c_2 x + c_3}.$$

The sign of the numerator is negative, thence implying that the sign of the entire expression is given by the sign of the denominator. Exploiting the fact that $\mathcal{B}(x) = 0$ implies:

$$c_3 = -\frac{c_4 + c_1x^3 + c_2x^2}{x}.$$

Substituting for c_3 in the denominator of the above expression, we deduce that $dx/d\tilde{\pi} > 0$ if:

$$3c_1x^2 + 2c_2x - \frac{c_4}{x} - c_1x^2 - c_2x > 0.$$

Since $x > 0$, simplifying the above expression and multiplying by x , we deduce that $dx/d\tilde{\pi} > 0$ if:

$$2c_1x^3 + c_2x^2 - c_4 > 0,$$

which is necessarily true since $c_1 > 0$, $c_2 > 0$, and $c_4 < 0$.