Research Article

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The Effects of Introducing Advertising in Pay TV: A Model of Asymmetric Competition between Pay TV and Free TV

Abstract: The television broadcasting industry is of crucial economic and social importance. Traditionally, this industry has been dominated by free-to-air TV (FTV) but due to technological progress, subscription-based pay TV (PTV) has emerged as a competing business model. A key question for the PTV broadcasters is whether to air commercials in addition to charging subscription fees. Based on a theoretical model of asymmetric competition between a PTV and an FTV broadcaster, we examine the effects of placing PTV advertising on broadcaster market strategies, viewer demands, broadcaster profits and consumer surplus. We find that introducing advertising on PTV can induce a higher viewer demand on this channel but a lower viewer demand on the FTV channel. Surprisingly, consumers can benefit through the introduction of advertising in PTV and broadcaster profits can increase if the viewer disutility of advertising is sufficiently large. Our study provides an analytical framework for choosing and implementing an optimal PTV strategy when an FTV competitor preexists in the market. Furthermore, our study derives implications for policymakers and regulatory authorities by showing that additional PTV advertising is not necessarily socially undesirable due to the strategic market reactions.

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1 Introduction

The television broadcasting industry is of crucial economic and social importance. For example, in the US nearly every household owns a television set and the average American adult spends almost 6 h a day viewing television (TVB 2021). Television is also the most influential medium in a consumer's purchase decision, making television a very important vehicle for placing advertisements. Television advertising has become a multi-billion-dollar industry and North America is the world's largest market for TV advertising. In 2019, advertisers spent about $70 billion to promote their products and services on television (Guttmann 2020).

In general, there are two generic business models in the television broadcasting industry: free-to-air TV (FTV) and subscription-based pay TV (PTV). Traditionally, the television broadcasting industry has been dominated by FTV. In this business model, viewers are fully subsidized because broadcasters want to attract as many viewers as possible in order to maximize advertising revenues. Basically, a TV channel is more attractive for advertisers, the larger its viewership. On the other hand, viewers typically dislike advertising as it is perceived as a nuisance.

Due to technological progress (i.e. encryption techniques and digital decoding) PTV has emerged as a competing business model (Peitz and Valletti 2008). One can observe that various access systems and business models are chosen by PTV broadcaster such as pay-per-channel and pay-per view. In a pay-per-channel model, consumers subscribe to one or multiple channels for a given period of time and thus can freely switch between the subscribed channels, whereas in a pay-per-view model, subscribers pay for each single program. An additional business model called "flexible pay-per-channel" has been proposed by Sun, Chen, and Shieh (2008). The flexible pay-per-channel model combines the advantages of pay-per-channel and pay-per-view by allowing a consumer to subscribe in a flexible manner to her favorite channels with the possibility to unsubscribe or change the subscription at any time.

In addition to these different business models, PTV channels vary with respect to their subscription and advertising strategy. Some PTV channels choose to charge a higher subscription fee, but air little or no advertising, while other PTV channels combine both viewer and advertiser incomes. In general, sports-based PTV channels often feature advertising in addition to charging subscription fees, while movie-based PTV channels are commonly devoid of traditional advertising (Dietl, Fort, and Lang 2013). For example, live games on ESPN and Fox Sports come with commercial advertising just like regular FTV. On the other hand, movie-based PTV channels such as FX Movie Channel or Sundance Now are commercial free or have very limited commercials.

A common concern of PTV broadcasters when carrying commercial advertising is that it may shake one of the pillars of their own existence because PTV viewers are willing to pay a subscription fee to avoid annoying advertising on FTV. Thus, a key question for the PTV broadcasters is whether or not to air advertising on its channels and what the consequences are if they decide to do so. Or in more general terms: What are the economic effects of placing advertising on PTV and how will FTV broadcasters react? Will market shares of PTV increase or decrease? Whose profits will increase and whose will decrease? And, most importantly, will consumers benefit or not?

To answer these and related research questions, we develop a simple duopoly model of asymmetric competition between a PTV and an FTV broadcaster. We analyze the effects of placing PTV advertising on broadcaster market strategies (i.e. choice of subscription fee and advertising level), viewer demands, broadcaster profits and consumer surplus. So far, the literature has concentrated on the symmetric competition between either PTV or FTV broadcasters. Because the reality is often characterized by the coexistence of both PTV and FTV, more research that analyzes the competition between both business models is needed. Research on symmetric competition neglects the important aspect of consumer migration from one business model to the other. This paper contributes to partly fill this gap in the literature.

Our model identifies the conditions under which the PTV broadcaster has incentives to place advertising on its channel. We find that PTV advertising level can increase with a higher viewer disutility from advertising but the PTV channel will never attract a larger viewership than the FTV channel. Furthermore, we show that introducing advertising in PTV induces a decrease of the subscription fees on this channel and a decrease in the advertising level of FTV. We further find that introducing advertising on PTV can induce a higher viewer demand on this channel but a lower viewer demand on FTV. Surprisingly, consumers can benefit through the introduction of advertising in PTV and broadcaster profits can increase if the viewer disutility of advertising is sufficiently large.

The main contribution of our paper is to provide an analytical framework for how to choose and implement an optimal PTV strategy when an FTV competitor preexists in the market. In addition, our paper can also serve as a reference point for PTV broadcasters considering a possible PTV strategy change. Finally, our paper derives implications for policy makers and regulatory authorities. For example, we find additional PTV advertising is not necessarily socially undesirable due to the strategic market reactions.

The remainder of this paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 presents our duopoly model of asymmetric competition between a PTV and an FTV broadcaster. We distinguish two regimes:

In Regime A, we examine the case where the subscription-based PTV channel places ads on its channel and Regime B where the PTV channel only relies on subscription fees. Section 4 compares both regimes. Finally, Section 5 concludes the paper and provides implications for future research.

2 Related Literature

The literature on the economics of media markets is flourishing and has experienced a significant growth in recent years.[1] The focus of the extant literature is on the choice of optimal market strategies by media platforms (i.e. content differentiation, advertising level and viewer charge) and welfare implications in models with symmetric competition between either FTV or PTV. Some papers also consider two separate scenarios, in which competition takes place between PTV and FTV platforms, respectively, and they compare the two independent scenarios. The key distinction of our paper from previous research is that we explicitly model the direct competition between PTV and FTV, rather than treating them independently. Thus, our paper offers insights about a scenario, in which PTV and FTV exist at the same time and compete for the same viewership. Although in reality PTV and FTV often coexist, the literature has neglected to model this aspect of asymmetric competition.

The literature review is structured as follows: First, we review the literature on symmetric competition in media markets. Second, we describe the literature on asymmetric competition and show how our paper extends this stream of literature.

2.1 Symmetric Competition

In a model of symmetric competition, Anderson and Coate (2005) analyze the nature of market failure in the broadcasting industry. Commercial broadcasters provide advertising levels and programming amount that can be above or below socially optimal levels, depending on how strongly viewers dislike advertising. With the ability to price programming, broadcasters can internalize the nuisance of advertisements by substituting prices for advertising at the margin. However, this is not necessarily socially desirable because pricing may also result in some viewers being inefficiently excluded.

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1 See Anderson and Gabszewicz (2006) and Budzinski and Kuchinke (2020), who provide a comprehensive overview to the literature. Anderson, Waldfogel and Stromberg (2016) features a comprehensive collection of articles on media economics.

In a similar context, Peitz and Valletti (2008) also introduce endogenous content provision by competing TV broadcasters into their model.[2] They assume that PTV broadcasters generate revenues from both advertisers and viewers, whereas FTV broadcasters are solely funded by advertising. Their model shows that under FTV, the advertising level is higher than under PTV when viewers strongly dislike advertising. Under FTV, broadcasters tend to provide less differentiated content while they always maximally differentiate their content under PTV. However, they do not model asymmetric competition because they consider two separate regimes with two competing PTV channels and two competing FTV channels, respectively.

Rather than horizontal program diversity, Armstrong (2005) studies the endogenous choice of vertical program quality by competing broadcasters in a model of symmetric competition. He also considers two separate funding regimes, in which either two PTV channels or two FTV channels compete against each other and finds that the program quality level in the PTV regime is higher than under the FTV regime. Our model builds on Armstrong (2005), but we model asymmetric competition and focus on the effects of introducing advertising on PTV.

Moreover, Armstrong (2006) analyzes the competition between media firms in the broader concept of a two-sided market.[3] Inter alia, he analyzes a competitive bottleneck in a two-sided media market, in which media platforms compete for viewers but not for advertisers. If the consumers join only one specific media platform, advertisers have to place advertising on all competing platforms to reach all consumers. The model shows that the equilibrium price for the access to media platforms (i.e. magazines) depends on how advertising charges are levied. The equilibrium reader price and platforms' profit is lower (higher) if platforms charge advertisers on a lump-sum basis than under a per-reader basis in the case that readers like (dislike) advertising.

All papers mentioned above have assumed Hotelling duopoly competition. Crampes, Haritchabalet, and Jullien (2009) extend these studies by considering the number of active media platforms as endogenous. They develop a Salop-Vickrey style model of media competition with free entry by assuming that media platforms are financed from advertising receipts and consumer subscriptions

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2 For further work on the endogenous choice of content diversity between media platforms, see also Gabszewicz (2001, 2002, 2004) and Gal-Or and Dukes (2003).

3 Two-sided markets are markets in which the agents of two distinct market sides interact via a platform and exert indirect network externalities upon each other. General research on two-sided markets include Caillaud and Jullien (2003), Rochet and Tirole (2003), Wright (2004), Weyl (2010), and Grossmann, Lang, and Dietl (2021). For a recent overview of media platforms and network effects, see Belleflamme and Peitz (2018).

or only from advertising receipts, in the case of free media platforms.[4] Their model suggests that there is an excessive level of entry and an insufficient level of advertising under constant or increasing returns to scale in the audience size. Similarly, Kind, Nilssen, and Sorgard (2009) allows the number of media platforms to vary and investigates how media platforms raise revenue in a model with quadratic consumer preferences. They find that media firms' scope for raising advertising revenues is constrained by the number of media firms. Moreover, a low level of horizontal differentiation between two media firms, or rather their content, restricts the scope for raising revenues from direct consumer payment.

Finally, Ambrus, Calvano, and Reisinger (2016) develop a model of symmetric competition between two ad-financed media outlets (advertising networks and traditional broadcasting stations) that receive demand from consumers and advertisers to study the effects on advertising levels and the impact of entry and mergers. In their model, they identify novel forces which reflect the incentives of media outlets to control the composition of their customer base.[5]

2.2 Asymmetric Competition

Neither of the above mentioned papers on media competition has analyzed the asymmetric competition between PTV and FTV. To the best of our knowledge, only a few related papers have analyzed asymmetric competition between for-profit media firms. For example, Casadesus-Masanell and Zhu (2010) study the choice of optimal business model for an incumbent when it faces competition from an ad-sponsored entrant. The authors investigate four possible business models for the incumbent: a subscription-based model; an ad-sponsored model; a mixed model with both subscription and advertising revenues; and a dual model with two products (one based on the ad-sponsored model and the other based on the mixed model). They show that the incumbent's optimal business model depends on the cost difference between employing a pure business model and employing a hybrid business model, the marginal return on advertising, and the exogenous quality levels of incumbent product and entrant product. The analysis of the subscription-based model and the mixed model is related to our analysis of asymmetric competition between PTV and FTV. However, in contrast to Casadesus-Masanell and Zhu (2010) where consumers/viewers are differentiated in their willingness to pay for products (à la Shaked-Sutton), in our model, viewers are differentiated in their tastes for the different products (à la Hotelling). Moreover, our study focuses on the effects of placing advertising in

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4 Choi (2006) has presented a related Salop–Vickrey model with free entry.
5 For a related contribution, see Anderson, Foros, and Kind (2018).

PTV on different equilibrium outcomes, while Casadesus-Masanell and Zhu (2010) focus on the incumbent's optimal business model.

Gabszewicz, Laussel, and Sonnac (2012) present a two-sided market model in which two competing newspapers operate in two interrelated markets: the readers' and the advertisers' markets. They provide an explanation for the rise of free newspapers as well as the simultaneous survival of the traditional paid outlets. In their model, the media platforms (newspapers) are vertically, but not horizontally, differentiated, whereas the opposite is true in our model. Moreover, the consumers (readers) in their model are assumed to be advertisement-neutral while they dislike ads in our model.

Lin (2011) develops a duopoly model of asymmetric competition between PTV and FTV. His model focuses on the endogenous choice of program quality made by television broadcasters. The paper shows that the broadcasters vertically differentiate their channel programs if viewers strongly or weakly dislike advertising. Depending on the degree of horizontal differentiation, PTV offers higher or lower quality programming than FTV. Dietl, Lang, and Lin (2013) consider a model of asymmetric competition between a pay and a free media platform in which the free media platform can charge its advertisers either on a lump-sum basis or on a per-consumer basis. They demonstrated that in small media markets, social welfare (total surplus consisting of broadcaster profits, consumer and advertiser surplus) is always higher if advertisers are charged on a per-consumer basis, while in large media markets this claim is only true if the nuisance cost of advertising for consumers is sufficiently high. Calvano and Polo (2020) study competition among TV broadcasters and develop a model in which two originally identical broadcasters chose opposite business models. In their model, strategic substitutability is a strong driver that leads to such an asymmetric equilibrium.

However, contrary to our model, none of the above mentioned paper allow for the possibility that PTV can introduce advertising on its channel.

3 Model

We consider a TV viewing market with three types of agents: consumers (viewers), broadcasters, and advertisers. The market is served by one PTV broadcaster and one FTV broadcaster. TV viewers, who are of mass one, are uniformly distributed along the unit interval. The two competing channels are situated at the extremes of the interval with the pay channel located at $x = 0$ and the free channel located at $x = 1$. We consider the Hotelling model with linear transport costs of $t > 0$ per unit of length. Hence, the two channel programs are horizontally differentiated from the perspective of viewers and the parameter t can be interpreted as the

differentiation parameter. A lower value of t means that the channels or their program are perceived as closer substitutes by the viewers.

The PTV broadcaster charges its consumers a subscription fee for access to its channels whereas the FTV broadcaster gives free access to its channels with no further monetary charges. We differentiate two market strategies of the PTV broadcaster: In Regime A, the PTV broadcaster that charges its viewers via a subscription fee, in addition, places advertisements on its channel. Thus, in Regime A, the PTV broadcaster has two strategic variables at its disposal: the subscription fee and the level of advertising. For the FTV broadcaster, the level of advertising is the only choice variable. We compare this regime to a benchmark case (Regime B) where the PTV broadcaster only charges its viewers via a subscription fee and does not place advertisements on its channel.

Please note that more than the two regimes are imaginable. For example, another scenario would be to depart from a market with two symmetric FTV channels and to analyze the consequences when one of the channels starts charging a subscription fee. However, it might be technically very difficult and costly for an FTV channel to adopt a subscription system due to the broadcasting technology in place. Image an FTV channel with over-the-air broadcast so that any person with an antenna can view the content. For such an FTV channel to charge a subscription fee would require encrypting its broadcast or moving to the cable network. It might be much easier for a PTV channel to add commercial advertisements to its channel. In sum, it seems quite natural to assume that PTV broadcasters can introduce ads (which reflects our current setting), but it is less clear that FTV broadcasters can create a subscription fee.[6]

3.1 Regime A

In this section, we examine Regime A where the PTV broadcaster places advertisements on its channel and charges its viewers a subscription fee. The FTV broadcaster gives free access to its channel and places advertisements on its channel. The indirect utility of a viewer, located at point $x \in [0,1]$ when choosing the PTV channel denoted by a subscript p and the FTV channel denoted by a subscript f, respectively, is defined as

$$u_p = v - s_p - \gamma a_p - tx, \tag{1}$$

$$u_f = v - \gamma a_f - t(1-x), \tag{2}$$

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6 Nevertheless, for the sake of completeness, we examine the symmetric competition between two FTV channels in Appendix B.

where $v > 0$ represents the viewers' intrinsic value from watching TV (Tag 2009; Economides and Tag 2012).[7] The PTV broadcaster charges viewers a subscription fee $s_p > 0$ for the access to the channel programming (Bel and Calzada 2007; Calzada and Valletti 2008; Calzada 2009).[8] We assume that both broadcasters have the possibility to place advertising on their respective channels, where the amount of advertising placed on the PTV and FTV channel is given by a_p and a_f, respectively. The parameter $\gamma > 0$ describes the extent to which viewers dislike advertising because each advertisement produces a perceived nuisance cost of γ by the viewers.[9] Moreover, we make the assumption of full viewer market coverage, i.e. the viewers' intrinsic value from watching TV is sufficiently large such that all viewers will watch one program.

We further assume that viewers are single-homing, i.e. they choose only one channel to watch. The marginal viewer, who is indifferent between watching PTV and FTV, can be identified at the location $\bar{x} = \frac{1}{2}[1 + \frac{1}{t}(\gamma(a_f - a_p) - s_p)]$. All viewers to the left of $\bar{x}$, i.e. $x \in [0, \bar{x})$, decide to watch PTV programming and all viewers to the right of $\bar{x}$, i.e. $x \in (\bar{x}, 1]$, consume FTV programming. As a result, the viewer demand functions for the PTV and FTV channels, respectively, can be derived as[10]

$$n_p = \frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_f - a_p) - s_p)\right], \tag{3}$$

$$n_f = 1 - n_p = \frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_p - a_f) + s_p)\right]. \tag{4}$$

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7 An interesting extension of our paper would be to assume that PTV offers content of a superior quality so that the viewers' intrinsic value from watching TV is higher on PTV than on FTV. Higher intrinsic values for PTV could change the result of Proposition 3 so that the PTV channel could possibly attract a larger viewership than the FTV channel. In addition, heterogeneous intrinsic values between the channels could be another justification for our single-homing assumption.

8 Note that we assume a positive subscription fee. Otherwise, the model would collapse to one with symmetric competition between two FTV broadcasters.

9 Kaiser and Song (2009) use data from the German magazine market to empirically assess the extent to which readers (dis-)like advertising. They show that the readers' attitude toward advertising depends on the nature of advertisements. For example, readers in many magazine segments, such as women's magazines, business and politics magazines, and car magazines, in which advertisements are relatively more informative, appreciate advertising. However, advertising is perceived as a nuisance to readers of adult magazines, a segment, in which advertisements are particularly uninformative. Moreover, Rysman (2004) found in the market for Yellow Pages that consumers value advertising.

10 See Wilbur (2008) empirically estimates viewer demand for programs on one side and advertiser demand for audiences on the other side based on a two-sided model of the television industry.

Please note that each broadcaster captures a positive market share when the second-order conditions for a maximum are satisfied.

We assume that placing advertising on its channel produces costs for each broadcaster given by $c(a)$ where $c(a) \in C^2$ is a strictly convex cost function with $c'(a) > 0$ and $c''(a) > 0$ for $a > 0$ and $c'(0) = c''(0) = 0$.[11] These costs can be interpreted either as transaction costs when broadcasters negotiate, serve, and deal with the advertisers, or as direct costs for the operative/technical realization of advertising implementation. Alternatively, these costs can be interpreted as broadcasters' opportunity costs for given-up programming slots/time. For simplicity, we assume that there are no other costs.

The profit functions of the broadcaster are then given by

$$\pi_p = \left(s_p + ka_p\right) n_p - c\left(a_p\right), \tag{5}$$

$$\pi_f = ka_f n_f - c\left(a_f\right), \tag{6}$$

where $k > 0$ is a parameter that captures how much revenue a broadcaster can generate per ad a. We assume that the advertising market is competitive, and that the advertising price is exogenously given, which also means that advertisers could place their ads in other firms. The parameter k can be interpreted as a measure for the effectiveness of advertising. For tractability, we assume a linear specification of advertising revenue, i.e. a broadcaster generates revenues of $k \cdot a \cdot n$ if its channel programming attracts n viewers and a ads are placed on the channel.

Moreover, the consumers (viewers) of the PTV and FTV channel, respectively, obtain the following surpluses:

$$\text{CS}_p = \int_0^{n_p} \left(v - s_p - \gamma a_p - tz\right) \mathrm{d}z = \left(v - s_p - \gamma a_p\right) n_p - \frac{t}{2} n_p^2, \tag{7}$$

$$\text{CS}_f = \int_{n_p}^{1} \left(v - \gamma a_f - t(1-z)\right) \mathrm{d}z = (1 - n_p)\left(v - \gamma a_f - \frac{t}{2}(1 - n_p)\right). \tag{8}$$

Aggregate consumer (viewer) surplus is then given by the sum of PTV and FTV consumer surpluses, i.e. $\text{CS} = \text{CS}_p + \text{CS}_f$.

In a next step, we setup the maximization problems of both broadcasters. The PTV broadcaster has two strategic variables at its disposal: the subscription fee

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11 For the assumption that media firms incur costs for placing advertisements, see also Blair and Romano (1993) and Armstrong (2006).

s_p and the level of advertising a_p. For the FTV broadcaster, the level of advertising a_f is the only choice variable. Therefore, the broadcasters solve the following maximization problems:

$$\max_{(s_p,a_p)\geq 0} \left\{ \pi_p = \left[s_p + ka_p \right] n_p(s_p, a_p) - c\left(a_p\right) \right\}, \tag{9}$$

$$\max_{a_f>0} \left\{ \pi_f = ka_f n_f(a_f) - c\left(a_f\right) \right\}. \tag{10}$$

The first-order conditions for the PTV broadcaster are given by

$$\frac{\partial \pi_p}{\partial s_p} = \underbrace{n_p^A\left(s_p^A, a_p^A\right)}_{>0} + \underbrace{\left(s_p^A + ka_p^A\right)\frac{\partial n_p^A}{\partial s_p}}_{<0} = 0, \tag{11}$$

$$\frac{\partial \pi_p}{\partial a_p} = \underbrace{kn_p^A\left(s_p^A, a_p^A\right)}_{>0} + \underbrace{\left(s_p^A + ka_p^A\right)\frac{\partial n_p^A}{\partial a_p}}_{<0} - c'\left(a_p^A\right) = 0, \tag{12}$$

where $c'(a) \equiv \frac{\partial c(a)}{\partial a}$. To ensure that the second-order conditions for a maximum are satisfied, the differentiation parameter t has to be sufficiently large, i.e.[12]

$$t > t_{\text{SOC}} \equiv \frac{(k-\gamma)^2}{4c''\left(a_p^A\right)},$$

where $c''(a) \equiv \frac{\partial^2 c(a)}{\partial a^2}$. From the first-order conditions (11) and (12), we observe that increasing either the subscription fee or the advertising level triggers a positive revenue effect and a negative demand effect because PTV viewer demand decreases with s_p and a_p, respectively. Additionally, increasing the advertising level induces higher costs (cost effect).

For the FTV broadcaster, the first-order condition is given by[13]

$$\frac{\partial \pi_f}{\partial a_f} = kn_f^A\left(a_f^A\right) + ka_f^A\frac{\partial n_f^A}{\partial a_f} - c'\left(a_f^A\right) = 0. \tag{13}$$

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12 For a derivation of the second-order conditions, see the proof of Lemma 1 in Appendix A.1.

13 The second-order condition is satisfied because $\frac{\partial^2 \pi_f}{\partial a_f^2} = -\frac{k\gamma}{t} - \frac{\partial^2 c\left(a_f^*\right)}{\partial a_f^2} < 0$.

and has a similar interpretation as above. By solving this system of equations, we can establish the following lemma:

Lemma 1. (i) *For the PTV channel, the subscription fee s_p^A and advertising level a_p^A in equilibrium are implicitly given by*

$$s_p^A = \frac{1}{2}(\gamma a_f^A - (k + \gamma)a_p^A + t) \text{ and } c'\left(a_p^A\right) = \frac{k - \gamma}{4t}(\gamma a_f^A + (k - \gamma)a_p^A + t), \quad (14)$$

and for the FTV channel the equilibrium advertising level a_f^A is implicitly given by

$$c'\left(a_f^A\right) = \frac{k}{4t}(-3\gamma a_f^A - (k - \gamma)a_p^A + 3t). \quad (15)$$

(ii) *The equilibrium $\left(s_p^A, a_p^A, a_f^A\right)$ exists and is unique if the differentiation parameter t is sufficiently large with $t > t^A \equiv \max\{t_{\text{SOC}}, t_{eq}, t_{s_p}, t_{a_f}\}$ and the viewers' disutility from advertising does not exceed the marginal return on advertising $\gamma \leq k$.*

Proof. See Appendix A.1.   □

Regarding Part (i), we assume that $t > t_{s_p} \equiv ka_p^A + \gamma\left(a_p^A - a_f^A\right)$ to ensure that the PTV broadcaster sets a positive subscription fee $s_p^A > 0$. Hence, if the channel programs are not sufficiently differentiated, the viewers are not willing to pay an additional subscription fee for watching PTV containing advertising because they can enjoy similar programs on FTV. As a result our model would collapse to one with symmetric competition between two FTV broadcasters (for an analysis of this scenario, see Appendix B).[14] Moreover, to ensure that the FTV channel sets a positive advertising level, the differentiation parameter t has to be sufficiently large, with $t > t_{a_f} \equiv \gamma a_f^A + \frac{k-\gamma}{3}a_p^A$.

According to Part (ii), a necessary assumption to ensure the existence and uniqueness of the equilibrium is also a sufficiently large differentiation parameter with $t > t_{eq}$. This assumption guarantees that the advertising level, which PTV would have to set such that the FTV broadcaster has no incentives to place advertising on its channel (i.e. FTV is driven out of the market) is larger than the PTV advertising level if FTV refrains from advertising. As shown in the proof of Lemma 1, we need this technical property to show that the reaction functions have at least one intersection point, which ensures the existence of an equilibrium. The

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14 Please note that $t = t_{\text{SOC}}$ also results in a subscription fee s_p^A of zero, yielding the symmetric model with two FTV broadcasters that we analyze in Appendix B. Hence, $t > \max\{t_{\text{SOC}}, t_{s_p}\}$ ensures that the PTV broadcaster sets a positive subscription fee.

uniqueness then follows from the strictly monotonous reaction functions. There-fore, in the subsequent analysis, we assume that the differentiation parameter is sufficiently large with $t > t^A \equiv \max\{t_{\text{SOC}}, t_{\text{eq}}, t_{s_p}, t_{a_f}\}$.

Proposition 1. (i) *For $\gamma = k$, the PTV broadcaster will not place advertising on its channel, while for $\gamma \in [0, k)$, the PTV broadcaster places advertising on its channel and its level of advertising is always below the corresponding level on the FTV channel.*

(ii) The level of advertising a_f^A on the FTV channel always decreases with a higher viewer disutility γ from advertising. For the PTV channel, the level of advertising a_p^A increases with a higher viewer disutility if this disutility is sufficiently small. Formally, $\frac{\partial a_f^A}{\partial \gamma} < 0$ and $\frac{\partial a_p^A}{\partial \gamma} > 0 \Leftrightarrow \gamma < \widehat{\gamma} \equiv \frac{k\left(a_f^A - 2a_p^A\right) - t}{2\left(a_f^A - a_p^A\right)}$.

Proof. See Appendix A.2.   □

Part (i) of the proposition indicates that a PTV broadcaster has incentives to place advertising on its channel in addition to a positive subscription fee, if the marginal return on advertising exceeds the viewers' disutility from advertising, i.e. $k > \gamma$. In other words, for a given level of k, placing advertising is beneficial for the PTV broadcaster's profits whenever the viewers' disutility from advertising is not too strong. If $\gamma = k$, the PTV broadcaster is better off in terms of profits when relying only on subscription fees. It implies that there is no scope for PTV to place advertising when the viewers' disutility from advertising is too strong ($\gamma = k$). Otherwise, viewers will simply migrate away from PTV where they originally intend to avoid annoying advertising by paying a subscription fee. If the viewers' disutility from advertising is below the marginal return on advertising, then the PTV broadcaster is able to choose an optimal income mix from both subscription and advertising revenues. In this case, the PTV channel will always set a lower advertising level than the FTV channel. The reason is that subscription and advertising revenues are substitutes. Therefore, a higher subscription fee leads to less advertising and the PTV channel will always offer a lower advertising level than the FTV channel.

Our result for the case $\gamma < k$ also rejects the claim of so-called "profit neutrality." For example, Peitz and Valletti (2008), demonstrate that there is a full pass-through of advertising revenues to lower subscription fees, which implies that advertising revenues do not affect the profits of two media platforms in equi-librium. However, this finding seems to be valid only for the case of symmetric competition between two PTV channels both using advertising. We have shown,

under certain circumstances, profit neutrality does not apply to the PTV broadcaster under asymmetric competition with an FTV channel because advertising revenues are not neutral, but rather increase overall profit.

Regarding the comparative statics results in Part (ii), we derive that the advertising level a_f^A of the FTV channel always decreases with a higher viewer disutility from advertising γ. To observe the intuition behind this result, we rearrange the first-order condition (13) of the FTV broadcaster and write

$$c'\left(a_f^A\right) = kn_f^A + ka_f^A \frac{\partial n_f^A}{\partial a_f}. \tag{16}$$

Recall that a higher FTV advertising level triggers a positive effect kn_f^A through higher advertising revenues and a negative effect $ka_f^A \frac{\partial n_f^A}{\partial a_f}$ through a lower viewer demand on profits. We derive that a higher γ diminishes the positive effect and strengthens the negative effect, which lowers the right-hand side (rhs) of (16).[15] As a result, the FTV broadcaster lowers its advertising level a_f^A to ensure a decrease of the left-hand side (lhs).

Regarding the behavior of the PTV broadcaster with respect to a higher disutility parameter γ, we rearrange its first-order condition (12) and write

$$c'\left(a_p^A\right) = kn_p^A + \left(s_p^A + ka_p^A\right) \frac{\partial n_p^A}{\partial a_p}. \tag{17}$$

Similarly to above, a higher FTV advertising level triggers a positive effect kn_p^A through higher advertising revenues and a negative effect $\left(s_p^A + ka_p^A\right) \frac{\partial n_p^A}{\partial a_p}$ through a lower viewer demand on profits. Contrary to above, a higher viewer disutility from advertising strengthens both the positive and the negative effect. Hence, the effect of a higher γ on the rhs of (17) is ambiguous and depends on the level of γ. First, suppose that the threshold $\widehat{\gamma}$ is positive, which implies that the differentiation parameter t is sufficiently small, i.e. $t < \widehat{t} \equiv k\left(a_f^A - 2a_p^A\right)$. If γ is sufficiently small with $\gamma < \widehat{\gamma}$, then the rhs increases with a higher viewer disutility from advertising such that the PTV broadcaster increases a_p to ensure an increase of the lhs. If $\gamma > \widehat{\gamma}$, the reverse is true and the advertising level in PTV decreases with a higher γ. However, if the differentiation parameter t is sufficiently large, i.e. $t > \widehat{t}$, then the threshold $\widehat{\gamma}$ is negative, and hence, a higher viewer disutility from advertising induces a decrease in the advertising level in PTV independent of γ.

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15 Formally, with $\tau_1 \equiv kn_f$ and $\tau_2 \equiv ka_f \frac{\partial n_f}{\partial a_f}$, we derive $\frac{\partial \tau_1}{\partial \gamma} = \frac{k}{4t}(a_p - a_f) < 0$ and $\frac{\partial \tau_2}{\partial \gamma} = -ka_f \frac{1}{4t} <$ 0. Hence, the positive effect is diminished and the strength of the negative effect increases.

With the subscription fee s_p^A, the equilibrium demands of the viewers for the PTV and FTV channels, respectively, are given by[16]

$$n_p^A = \frac{1}{2}\left[1 + \frac{1}{2t}(\gamma\left(a_f^A - a_p^A\right) + ka_p^A - t)\right], \tag{18}$$

$$n_f^A = \frac{1}{2}\left[1 + \frac{1}{2t}(\gamma\left(a_p^A - a_f^A\right) - ka_p^A + t)\right]. \tag{19}$$

Proposition 2. *The PTV channel never attracts a larger viewership than the FTV channel independent of the viewer disutility from advertising, i.e. $n_f^A \geq n_p^A$.*

Proof. See Appendix A.3.   □

The proposition indicates that the FTV channel never attracts a lower viewership than the PTV channel even though viewers might derive a comparatively large disutility from advertising and the advertising level in FTV is always higher than in PTV. To observe the intuition behind this result, consider the case where viewers do not perceive advertisement as a nuisance, i.e. $\gamma = 0$. In this case, it is not surprising that the FTV channel attracts more viewers than the PTV channel because viewers are charged via the subscription fee for PTV. Increasing the disutility parameter γ has the following effects on PTV viewer demand. Total differentiation of $n_p^A = \frac{1}{2}\left[1 + \frac{1}{t}(\gamma\left(a_f^A - a_p^A\right) - s_p^A)\right]$ yields

$$\frac{\mathrm{d}n_p^A}{\mathrm{d}\gamma} = \frac{1}{2t}\left[\underbrace{\left(a_f^A - a_p^A\right)}_{>0} + \gamma\underbrace{\frac{\partial\left(a_f^A - a_p^A\right)}{\partial\gamma}}_{\gtreqless 0} - \underbrace{\frac{\partial s_p^A}{\partial\gamma}}_{>0}\right].$$

Given that the FTV channel has a higher level of advertising than the PTV channel, a higher γ pronounces this difference and therefore has a direct positive effect $a_f^A - a_p^A$ on viewer demand. However, it is ambiguous whether the difference in advertising levels $a_f^A - a_p^A$ shrinks or expends such that the sign of the second effect is undetermined. Finally, a higher γ induces an increase in the subscription fee s_p^A, which has a negative effect on PTV viewer demand. Overall, it is ambiguous whether PTV viewer demand follows a U-shaped or an inverted U-shaped pattern in the disutility parameter γ. However, we know if the differentiation parameter t is sufficiently small, i.e. $t < \hat{t}$,[17] then PTV and FTV viewer

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16 Note that $t > t^*$ also ensures that n_p^* and n_f^* are both positive.
17 Note that $t < \hat{t}$ implies $\hat{\gamma} > 0$.

demand follows an inverted U-shaped and U-shaped pattern in γ, respectively, because a higher viewer disutility from advertising induces a decrease in the PTV advertising level. Formally, $\partial n_p^A / \partial \gamma \big|_{\gamma=0} = \left[\left(a_f^A - a_p^A \right) + k \left(\partial a_p^A / \partial \gamma \right) \right] / (4t) > 0$ if $\partial a_p^A / \partial \gamma > 0$. Finally, we derive that the PTV channel attracts the same viewership as the FTV channel if the disutility parameter γ is given by $\gamma = \frac{t - k a_p^A}{a_f^A - a_p^A}$.

By substituting $\left(n_p^A, n_f^A, s_p^A \right)$ into the profit functions (5) and (6), we derive the equilibrium profits of the PTV and FTV broadcaster, respectively, as

$$\pi_p^A = \frac{1}{8t}(\gamma a_f^A + (k - \gamma)a_p^A + t)^2 - c\left(a_p^A \right),$$

$$\pi_f^A = \frac{k a_f^A}{4t}(-\gamma a_f^A - (k - \gamma)a_p^A + 3t) - c\left(a_f^A \right).$$

Similarly, we obtain the consumer surpluses in equilibrium as

$$CS_p^A = \frac{1}{32t}(\gamma a_f^A + (k - \gamma)a_p^A + t)(8\upsilon - 5\gamma a_f^A + 3(k - \gamma)a_p^A - 5t),$$

$$CS_f^A = \frac{1}{32t}(-\gamma a_f^A - (k - \gamma)a_p^A + 3t)(8\upsilon - 7\gamma a_f^A + (k - \gamma)a_p^A - 3t).$$

3.2 Regime B

In Regime B, we present the benchmark scenario in which the PTV broadcaster only charges a subscription fee and does not place advertisements on its channel.[18] Here, the PTV broadcaster has only the subscription fee s_p as a choice variables so that the maximization problems of both broadcasters are then given by

$$\max_{s_p \geq 0} \left\{ \pi_p = s_p n_p(s_p) \right\} \text{ and } \max_{a_f > 0} \left\{ \pi_f = k a_f n_f(a_f) - c\left(a_f \right) \right\} \tag{20}$$

with the corresponding first-order conditions

$$\frac{\partial \pi_p}{\partial s_p} = n_p^B \left(s_p^B \right) + s_p^B \frac{\partial n_p^B}{\partial s_p} = 0 \text{ and } \frac{\partial \pi_f}{\partial a_f} = k n_f^B \left(a_f^B \right) + k a_f^B \frac{\partial n_f^B}{\partial a_f} - c'\left(a_f^B \right) = 0.$$
$$\tag{21}$$

By solving this system of equations, we can establish the optimality conditions as follows:

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18 We provide only a short analysis of this regime because it can be easily calculated from Regime A.

$$s_p^B = \frac{1}{2}\left(\gamma a_f^B + t\right) \text{ and } c'\left(a_f^B\right) = \frac{3k}{4t}\left(t - \gamma a_f^B\right). \tag{22}$$

The result is summarized in the following proposition:

Proposition 3. *In Regime B, the equilibrium demands of the viewers for the PTV and FTV channels, respectively, are given by*

$$n_p^B = \frac{1}{2}\left[1 + \frac{1}{2t}\left(\gamma a_f^B - t\right)\right] \text{ and } n_f^B = \frac{1}{2}\left[1 + \frac{1}{2t}\left(t - \gamma a_f^B\right)\right], \tag{23}$$

so that the equilibrium profits of the PTV and FTV broadcaster as well as the consumer surpluses yield

$$\pi_p^B = \frac{1}{8t}\left(\gamma a_f^B + t\right)^2 \text{ and } \pi_f^B = \frac{k a_f^B}{4t}\left(-\gamma a_f^B + 3t\right) - c\left(a_f^B\right),$$

$$CS_p^B = \frac{1}{32t}\left(\gamma a_f^B + t\right)(8v - 5\gamma a_f^B - 5t) \text{ and } CS_f^B$$

$$= \frac{1}{32t}\left(-\gamma a_f^B + 3t\right)(8v - 7\gamma a_f^B - 3t).$$

Proof. Straightforward and therefore omitted. □

4 Comparison of the Regimes

In this section, we compare Regime A in which the PTV broadcaster places advertisements on its channel with the benchmark Regime B in which the PTV broadcaster does not place ads. For tractability, we conduct the analysis in this section for a quadratic cost function $c(a) = \frac{\theta}{2}a^2$ where $\theta > 0$ is a constant.

First, we analyze the effects of introducing PTV advertising on the FTV advertising level, the PTV subscription fee and viewer demands. We establish the following proposition.

Proposition 4. (i) *The subscription fee on the PTV channel is lower in Regime A than in Regime B, i.e.* $s_p^A < s_p^B$.

(ii) The advertising level on the FTV channel is lower in Regime A than in Regime B, i.e. $a_f^A < a_f^B$.

(ii) Viewer demand for FTV (PTV) is lower (higher) in Regime A than in Regime B, i.e. $n_f^A < n_f^B$ *and* $n_p^A > n_p^B$.

Proof. See Appendix A.4. □

Part (i) of the proposition states that the subscription fee on the PTV channel decreases if the PTV broadcaster decides to place advertising on its channel. Introducing ads on the PTV channel has two effects on the subscription fee: a direct effect on the fee and an indirect effect via a change in the ads level of the FTV channel. Both effects are negative, yielding a lower subscription fee. That is, to compensate the viewers for watching advertising, the PTV broadcaster lowers its subscription fee so that the fee is lower in Regime A than in Regime B. In more common terms of the two-sided market literature, additional advertising revenues will be passed through to the viewers in the form of a lower subscription fee.[19] However, the total advertising level is higher in Regime A than in Regime B, i.e. $a_f^A + a_p^A > a_f^B$. Thus, the lower advertising level on the FTV channel in Regime A is compensated for by the additional advertising level on the PTV channel. When interpreting the subsequent results in this section, it is important to keep in mind that advertisers are spending more when the PTV channel introduces advertising (Regime A) compared to the benchmark case (Regime B). In addition, due to the single-homing assumption, the ads of the two firms do not affect the same consumers. Indeed, the costs for the consumers is the sum of the disutility from the ads and the subscription fee.

Part (ii) of the proposition posits that the FTV channel always reduces its own advertising level when the PTV channel places advertising. Basic intuition might suggest that when the pay channel introduces advertising, the free channel faces less fierce competition because the viewers dislike advertising. As a result, the free channel enjoys a competitive advantage and hence, tends to increase its own advertising level. However, as we have shown in Part (i), there is also an accompanying effect from PTV due to its lower subscription fee. The latter effect dominates and the FTV channel reduces its own advertising level so that the corresponding level is lower in Regime A than in Regime B.

Part (iii) of the proposition indicates that introducing advertising in PTV will actually help the PTV broadcaster to gain additional market share. As a direct consequence, the competing FTV broadcaster loses market share. The intuition for this result is as follows. On the one hand, introducing advertising in PTV has a direct positive effect on the viewer demand for PTV because the lower subscription fee overcompensates for the additional advertising. On the other hand, placing PTV advertising induces a lower FTV advertising level and thus exerts an indirect negative effect on the viewer demand for PTV. The positive effect dominates the

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19 Using data from the German magazine market, Kaiser and Wright (2006) find results which are consistent with this result. That is, advertisers value readers more than readers value advertisements, and thus, magazines subsidize cover prices (i.e. subscription fees) and make their profits from advertisers.

negative effect so that overall effect of introducing PTV advertising on the PTV viewer demand is positive. Since there is a market stealing effect in the Hotelling model, the viewer demand for FTV decreases at the same time. In sum, TV viewers value a reduction in the level of advertising on FTV less than a decrease in the subscription fee for PTV. As a result, PTV becomes relatively more attractive for the viewers and more viewers will choose pay over FTV.

In the next proposition, we compare broadcaster profits and consumer surplus.

Proposition 5. (i) *Aggregate consumer surplus is higher in Regime A than in Regime B, i.e. $CS_p^A + CS_f^A > CS_p^B + CS_f^B$.*

(ii) The profit of the FTV (PTV) broadcaster is lower (higher) in Regime A than in Regime B, i.e. $\pi_f^A < \pi_f^B$ and $\pi_p^A > \pi_p^B$.

Proof. See Appendix A.5.   □

Part (i) shows that introducing PTV advertising has an unambiguously positive effect on the aggregate consumer surplus. The intuition for this result is as follows. Although the PTV broadcaster places advertising on its channel in Regime A, PTV consumers benefit due to the lower subscription fee. In addition, PTV advertising exerts a positive effect on the FTV consumers through a lower FTV advertising level. In sum, the additional benefit derived by the viewers from a lower subscription fee and a lower FTV advertising level exceeds the disutility from additional PTV advertising. It is important to mention that the demand effects are neutral due to the Hotelling specification.

Regarding the effects on PTV broadcaster profits stated in Part (ii) of the proposition, we know that the profit-maximizing advertising level for the PTV channel is positive, i.e. $a_p^A > 0$, if the viewers' disutility from advertising is smaller than the marginal return of advertising, i.e. $\gamma < k$. Hence, it is clear that profits of the PTV channel must increase when switching from a strategy that solely relies on subscription fees to a strategy consisting of both subscription and advertising income. Otherwise, the PTV broadcaster would opt not to place any advertising and choose $a_p^A = 0$. Hence, for $\gamma < k$, additional advertising revenues will overcompensate the loss in subscription revenues and additional costs.

However, how does the PTV's strategy change affect the profit of the FTV broadcaster? We know that the introduction of PTV advertising leads to a reduction of the advertising level on the FTV channel. This reduction in the advertising level triggers a negative effect through lower advertising revenues and a positive effect through lower advertising costs. Moreover, the introduction of advertising on PTV also reduces the viewer demand for FTV and thus exerts an additional negative

effect on the revenues of the FTV broadcaster. Yet, the negative revenue effect caused by a lower advertising level and a shrinking viewership dominates the positive cost effect induced through lower advertising costs such that the overall FTV broadcaster's profit is lower in Regime A than in Regime B.

Regarding aggregate broadcaster profits, we obtain the following result:

Proposition 6. *Aggregate broadcaster profits are higher in Regime A than in Regime B if the viewers' disutility from advertising is sufficiently large, i.e. $\pi_p^A + \pi_f^A > \pi_p^B + \pi_f^B \Leftrightarrow \gamma > \gamma^*$*

Proof. We have to rely on numerical simulations because it is analytical not tractable to derive the results regarding aggregate profits in closed form.   □

Figure 1 displays the difference $\Pi^A - \Pi^B = \left(\pi_p^A + \pi_f^A \right) - \left(\pi_p^B + \pi_f^B \right)$ in aggregate broadcaster profits between Regimes A and B (y-axis) as a function of the viewers' disutility γ from advertising (x-axis). We set the parameters as follows: $k = 3$, $t = 12$ and $\theta = 1$ and plot the whole range of $\gamma \in [0, k]$. The figure shows that for $\gamma < \gamma^*$ the difference in aggregate profits is negative, whereas for $\gamma > \gamma^*$ the difference is positive. Thus, only if the viewers sufficiently dislike advertising then aggregate profits are higher in the case that the PTV broadcaster places ads on its channel. The intuition for this result is as follows. In the benchmark
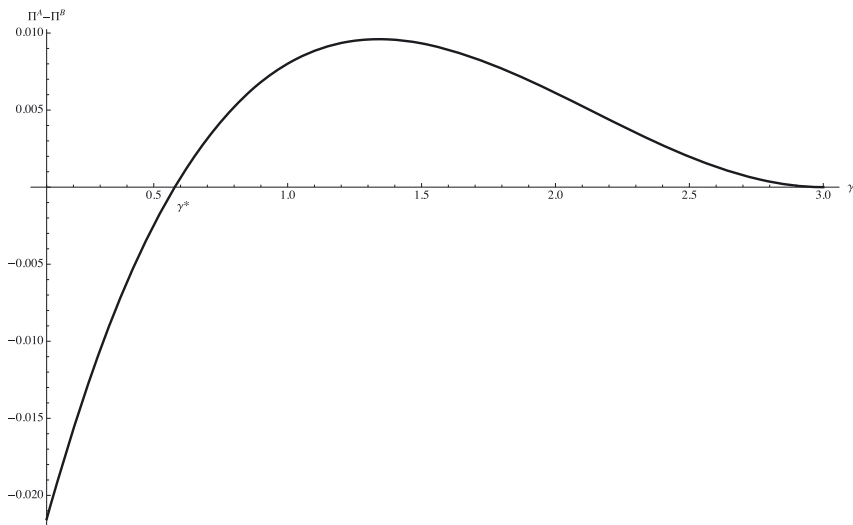
Figure 1: Difference in profits.

regime B, the PTV broadcaster profits always increase with the viewers' disutility from advertising γ, whereas the FTV broadcaster profits always decrease with a higher γ. With increasing γ more viewers tend to switch to the PTV channel where they can avoid ads, yielding an increase of the PTV profits. However, this picture changes in regime A because now there are also ads on the PTV channel. With the numerical simulations, we can show that when γ exceeds a critical level, the additional gain in the PTV broadcaster profit overcompensates for a potential decrease in the FTV broadcaster profit so that aggregate profits are higher in regime A than in regime B.

5 Conclusion

This paper develops a model of asymmetric competition between a PTV broadcaster and an FTV broadcaster to analyze the effects of introducing advertising in PTV. Our model shows that the PTV broadcaster will place advertising on its channel if the viewers' disutility of watching advertising is not too strong. Introducing advertising on the PTV channel leads to a lower subscription fee because additional advertising revenues will be passed through to the viewers. As a strategic reaction, the FTV channel reduces its own advertising level. In addition, placing advertising in PTV induces a higher viewer demand on this channel but a lower viewer demand on the FTV channel because the viewers value a decrease of the subscription fee in PTV more than a reduction of the advertising level in FTV.

Given that the demand effects are neutral in our Hotelling model, the overall consumer surplus on both channels increases through the introduction of PTV advertising. That is, the consumers benefit more from a lower subscription fee and a lower FTV advertising level than they suffer from additional PTV advertising. Moreover, if the marginal return from advertising exceeds the viewers' disutility from advertising then introducing advertising in PTV increases profits of this channel. Regarding the FTV profits, we find that placing advertising in PTV has a negative effect on the FTV broadcaster's profit because the positive cost effect induced through lower advertising costs cannot compensate for the negative revenue effect caused by a lower advertising level and a shrinking viewership.

Because the model in this paper is a stylized one, we have made some simplifying assumptions. For example, viewers are supposed to join either PTV or FTV. It is reasonable that FTV viewers without paying the subscription fee are excluded from the PTV programming such that they are "forced" to single-home. By contrast, PTV viewers still have the opportunity to watch FTV programming

and thus they have the possibility to multi-home.[20] However, the single-homing assumption of viewers is widely adopted in the existing literature when it comes to capture the competition for viewer market share between media firms. One can justify the assumption by pointing out that viewers cannot physically watch two broadcasts at the same time (at least without decreasing the entertainment value) and thus have to make a choice for watching only one channel.

Another simplifying assumption is that the broadcaster's marginal return on advertising and the viewers' disutility parameter are symmetric between channels. An interesting avenue for further research would be to allow for heterogeneity in the effectiveness of advertising and the disutility parameter, respectively.

Finally, it would be interesting to extend our framework to explicitly model the advertiser's market.[21] In our current setup, the return on advertising (or advertising price) is exogenously given and advertisers are not restricted in how much advertising space they can buy at this price. However, in reality, advertisers are constrained by the amount of advertising they can contract and thus interesting interactions with the broadcasters might arise. In addition, explicitly modelling the advertiser's market and including the advertiser's surplus would allow conducting a full-fledged welfare analysis which was not possible with the current setup.

Appendix A: Proofs

A.1 Proof of Lemma 1

First, we derive the second-order conditions. By noting that $\frac{\partial n_p}{\partial s_p} = -\frac{1}{2t}$, $\frac{\partial^2 n_p}{\partial s_p^2} = 0$, $\frac{\partial n_p}{\partial a_p} = -\frac{\gamma}{2t}$, $\frac{\partial^2 n_p}{\partial a_p^2} = 0$, $\frac{\partial c}{\partial a_p} > 0$ and $\frac{\partial^2 c}{\partial a_p^2} > 0$, the second-order conditions for a maximum require

(a) $\dfrac{\partial^2 \pi_p}{\partial (s_p)^2} = -\dfrac{1}{t} < 0$ and $\dfrac{\partial^2 \pi_p}{\partial a_p^2} = -\dfrac{k\gamma}{t} - \dfrac{\partial^2 c}{\partial a_p^2} < 0,$

(b) $\dfrac{\partial^2 \pi_p}{\partial s_p^2} \dfrac{\partial^2 \pi_p}{\partial a_p^2} - \dfrac{\partial^2 \pi_p}{\partial s_p \partial a_p} \dfrac{\partial^2 \pi_p}{\partial a_p \partial s_p} = \dfrac{1}{t}\left(\dfrac{k\gamma}{t} + \dfrac{\partial^2 c}{\partial a_p^2}\right) - \left(-\dfrac{k+\gamma}{2t}\right)^2 > 0.$

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20 Note that PTV packages can also include FTV channels.
21 See, e.g. Reisinger (2012) who develops a two-sided market model in which platforms compete for advertisers and users.

It is easy to see that (a) is satisfied by definition. To ensure that (b) is satisfied, we must assume that $t > \frac{(k-\gamma)^2}{4c''(a_p^*)}$, where $c''(a_p) = \frac{\partial^2 c}{\partial a_p^2}$.

Second, we derive the optimality conditions in Part (i). By solving $\frac{\partial \pi_p}{\partial s_p} = 0$ for s_p with $\frac{\partial n_p}{\partial s_p} = -\frac{1}{2t}$, we obtain $s_p^* = \frac{1}{2}(\gamma a_f^* - (k+\gamma)a_p^* + t)$. Plugging s_p^* into $\frac{\partial \pi_p}{\partial a_p} = kn_p^*\left(s_p^*, a_p^*\right) + \left(s_p^* + ka_p^*\right)\frac{\partial n_p^*}{\partial a_p} - c'\left(a_p^*\right) = 0$ and $\frac{\partial \pi_f}{\partial a_f} = kn_f^*\left(a_f^*\right) + ka_f^*\frac{\partial n_f^*}{\partial a_f} - c'\left(a_f^*\right) = 0$ and rearranging these equations produces the optimality conditions for PTV and FTV.

Part (ii). To prove the existence and uniqueness of the equilibrium $\left(s_p^A, a_p^A, a_f^A\right)$, we proceed as follows. First, we show that the reaction function $R_f(a_p)$ of FTV is a monotonous decreasing function in a_p and the reaction function $R_p(a_f)$ of PTV is a monotonous increasing function in a_f. We rearrange the optimality conditions of PTV and FTV and we define

$$F_p\left(a_p^A, a_f^A\right) = \frac{k-\gamma}{4t}(\gamma a_f^A + (k-\gamma)a_p^A + t) - c'\left(a_p^A\right),$$

$$F_f\left(a_p^A, a_f^A\right) = \frac{k}{4t}(-3\gamma a_f^A - (k-\gamma)a_p^A + 3t) - c'\left(a_f^A\right).$$
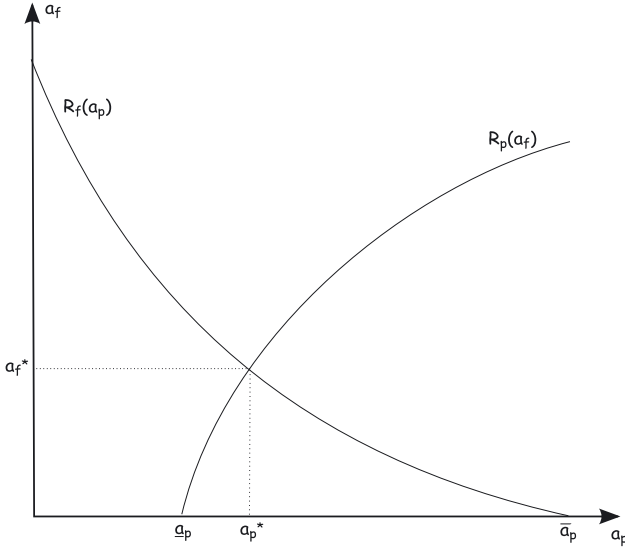
With the implicit function theorem, we derive

$$\frac{da_p^A}{da_f} = -\frac{\partial F_p/\partial a_f}{\partial F_p/\partial a_p} = -\frac{\gamma(k-\gamma)}{(k-\gamma)^2 - 4tc''\left(a_p^A\right)} \overset{t>t_{SOC}}{>} 0,$$

$$\frac{da_f^A}{da_p} = -\frac{\partial F_f/\partial a_p}{\partial F_f/\partial a_f} = -\frac{k(k-\gamma)}{3k\gamma + 4tc''\left(a_f^A\right)} < 0.$$

Hence, PTV reacts with a higher advertising level to an increase in the FTV advertising level, while the opposite is true for the reaction of FTV. We illustrate the reaction functions in Figure 2.

We define $\overline{a}_p$ as the advertising level, which PTV would have to set such that the FTV broadcaster has no incentives to places advertising on its channel, i.e. $R_f(\overline{a}_p) = 0$. Moreover, $\underline{a}_p$ is the PTV advertising level if FTV refrains from advertising, i.e. $\underline{a}_p = R_p(0)$. Now, we have to show that $\overline{a}_p > \underline{a}_p$. This guarantees that the monotonous reaction function have exactly one intersection point, which characterizes the unique equilibrium. Throughout the paper, we assume that the viewers' disutility from advertising does not exceed the marginal return on advertising $\gamma \leq k$.

Because $c'\left(a_f^A\right) = \frac{k}{4t}(-3\gamma a_f^A - (k-\gamma)a_p^A + 3t)$ is the optimality condition for FTV, we derive that if $a_p = \frac{3t}{k-\gamma}$ then $a_f = 0$. Hence, $\overline{a}_p(t) = \frac{3t}{k-\gamma}$ and thus $\frac{\partial \overline{a}_p}{\partial t} > 0$.

Figure 2: Reaction functions

Moreover, $\underline{a}_p$ is implicitly characterized by the PTV's optimality condition $c'(\underline{a}_p) = \frac{k-\gamma}{4t}((k-\gamma)\underline{a}_p + t)$. With the implicit function theorem, we derive

$$\frac{\partial \underline{a}_p}{\partial t} = \frac{\underline{a}_p (k-\gamma)^2}{t\left[(k-\gamma)^2 - 4tc''(\underline{a}_p)\right]} < 0,$$

for t sufficiently large. Because $\overline{a}_p(t)$ is continuous and a monotonously increasing function in t and $\underline{a}_p(t)$ is continuous and a monotonously decreasing function in t, there exists a value t_{eq} such that $\overline{a}_p(t) > \underline{a}_p(t)$ for all $t > t_{\text{eq}}$. Hence, we have shown that for a sufficiently large differentiation parameter, i.e. $t > t_{\text{eq}}$, an unique equilibrium $\left(a_p^A, a_f^A\right)$ exists, which is implicitly defined by $c'\left(a_p^A\right) = \frac{k-\gamma}{4t}(\gamma a_f^A + (k-\gamma)a_p^A + t)$ and $c'\left(a_f^A\right) = \frac{k}{4t}(-3\gamma a_f^A - (k-\gamma)a_p^A + 3t)$. Note that a_p^A is always larger than zero, while t has to be sufficiently large to ensure that a_f^A is larger than zero, i.e. $a_f^A > 0 \Leftrightarrow t > t_{a_f} \equiv \gamma a_f^A + \frac{k-\gamma}{3}a_p^A$.

In a next step, we will derive the conditions under which a positive equilibrium subscription fee s_p^A exists and is unique. According to the first-order conditions, the subscription fee s_p^A is implicitly defined by $s_p^A = \frac{1}{2}(\gamma a_f^A - (k+\gamma)a_p^A + t)$. Hence, $s_p^A > 0 \Leftrightarrow t > t_{s_p} \equiv ka_p^A + \gamma\left(a_p^A - a_f^A\right)$. Because for $t > \max\{t_{\text{eq}}, t_{a_f}, t_{SOC}\}$, a unique equilibrium $\left(a_p^A, a_f^A\right)$ exists and is positive, we conclude that $t > t_{s_p} \equiv ka_p^A + \gamma\left(a_p^A - a_f^A\right)$ ensures that a unique subscription fee exists and is positive.

For the PTV channel, the subscription fee s_p^A and advertising level a_p^A in equilibrium are implicitly given by

$$s_p^A = \frac{1}{2}(\gamma a_f^A - (k + \gamma)a_p^A + t) \text{ and } c'\left(a_p^A\right) = \frac{k - \gamma}{4t}(\gamma a_f^A + (k - \gamma)a_p^A + t). \quad (24)$$

For example, for a quadratic cost function $c(a) = 1/2a^2$, the advertising level $\underline{a}_p(t)$ is given by

$$\underline{a}_p(t) = \frac{t(k - \gamma)}{4t - (k - \gamma)^2},$$

and thus

$$\overline{a}_p(t) > \underline{a}_p(t) \Leftrightarrow t > t_{eq} \equiv \frac{(k - \gamma)^2}{3}.$$

A.2 Proof of Proposition 1

To prove Part (i), we differentiate three cases:
1. Suppose that $\gamma = k$. In this case, we derive $c'\left(a_p^A\right) = 0$ and hence $a_p^A = 0$. The subscription fee is then given by $s_p^A = \frac{1}{2}\left(ka_f^A + t\right)$ and a_f^A is implicitly defined by $c'\left(a_f^A\right) = \frac{3k}{4t}\left(-ka_f^A + t\right)$. To ensure that the FTV broadcaster sets a positive advertising level, it must hold $t > ka_f^A$ and hence $a_f^A > a_p^A$ for $\gamma = k$.
2. Suppose that $\gamma = 0$. In this case, the subscription fee is given by $s_p^A = \frac{1}{2}\left(-ka_p^A + t\right)$. To ensure a positive fee, we assume that $t > ka_p^A$. The advertising level $\left(a_p^A, a_f^A\right)$ are then implicitly defined by $c'\left(a_p^A\right) = \frac{k}{4t}\left(ka_p^A + t\right)$ and $c'\left(a_f^A\right) = \frac{k}{4t}\left(-ka_p^A + 3t\right)$. Since $t > ka_p^A$, the FTV broadcaster sets a positive advertising level because $-ka_p^A + 3t > 0$. Next, we derive

$$c'\left(a_p^A\right) - c'\left(a_f^A\right) = \frac{k\left(ka_p^A - t\right)}{2t} < 0,$$

   because $t > ka_p^A$. It follows that $a_f^A > a_p^A$ for $\gamma = 0$.
3. Suppose that $\gamma \in (0, k)$. We can show that a_p^A and a_f^A will never coincide in the interval $\gamma \in (0, k)$. Suppose that $a_p^A = a_f^A = a^A$ and hence $c\left(a_p^A\right) = c\left(a_f^A\right)$ such that $\frac{k-\gamma}{4t}(ka^A + t) = \frac{k}{4t}\left[-(2\gamma + k)a^A + 3t\right]$. This equality is satisfied if and only if $\gamma = -2k$, which is not in the interval of feasible γ.

   From 1.–3., it follows that $a_f^A > a_p^A$ for $\gamma \in (0, k]$.

Moreover, we can rule out the case $\gamma > k$. To show this claim, we provide a proof by contradiction. We know that $c'\left(a_p^A\right) = \frac{k-\gamma}{4t}(\gamma a_f^A + (k - \gamma)a_p^A + t) \geq 0$. Now, suppose that $\gamma > k$. Hence, it must be the case that $t + \gamma a_f^A \leq (\gamma - k)a_p^A$.

However, a positive subscription fee $s_p^A > 0$ implies $t + \gamma a_f^A > (\gamma + k)a_p^A$. That is, $t + \gamma a_f^A \in ((\gamma + k)a_p^A, (\gamma - k)a_p^A]$, which cannot be satisfied in equilibrium. Therefore, our assumption was wrong and it must hold $\gamma \leq k$.

To prove the comparative statics result in Part (ii), we rearrange (15) and (14) and define $F_p\left(\gamma, a_p^A\right) = \frac{k-\gamma}{4t}(\gamma a_f^A + (k - \gamma)a_p^A + t) - c'\left(a_p^A\right) = 0$ and $F_f\left(\gamma, a_f^A\right) = \frac{k}{4t}(-3\gamma a_f^A - (k - \gamma)a_p^A + 3t) - c'\left(a_f^A\right) = 0$. With the implicit function theorem, we derive

$$\frac{\partial a_f^A}{\partial \gamma} = -\frac{\partial F_f/\partial \gamma}{\partial F_f/\partial a_f} = -\frac{\left(3a_f^A - a_p^A\right)k}{3k\gamma + 4tc''\left(a_f^A\right)} < 0.$$

Hence, the advertising level on the FTV channel is always decreasing in γ. Regarding the advertising level on PTV, we derive

$$\frac{\partial a_p^A}{\partial \gamma} = -\frac{\partial F_p/\partial \gamma}{\partial F_p/\partial a_p} = \frac{\left(a_f^A - a_p^A\right)(2\gamma - k) + t + ka_p^A}{(k - \gamma)^2 - 4tc''\left(a_p^A\right)}.$$

The second-order conditions require $4tc''\left(a_p^A\right) > (k - \gamma)^2$, which implies that the denominator is negative. Moreover, it is always the case that $a_f^A > a_p^A$. As a result, we derive that for $\gamma < \hat{\gamma}$, the numerator will be negative, and hence,

$$\frac{\partial a_p^A}{\partial \gamma} > 0 \Leftrightarrow \gamma < \hat{\gamma} \equiv \frac{k\left(a_f^A - 2a_p^A\right) - t}{2\left(a_f^A - a_p^A\right)}.$$

A.3 Proof of Proposition 2

To show that $n_f^A \geq n_p^A \; \forall \gamma \in [0, k]$, we differentiate three cases:

1. Suppose that $\gamma = k$. In this case, we derive $n_f^A - n_p^A = \frac{1}{2t}\left(t - ka_f^A\right) > 0$ because $t > ka_f^A$.

2. Suppose that $\gamma = 0$. In this case, we derive $n_f^A - n_p^A = \frac{1}{2t}\left(t - ka_p^A\right) > 0$ because $t > ka_p^A$.

3. Suppose that $\gamma \in (0, k)$. To show that $n_f^A \geq n_p^A \; \forall \gamma \in (0, k)$, we provide a proof by contradiction. Suppose that n_f^A can fall below n_p^A in the interval $\gamma \in (0, k)$. Note that $n_f^A(\gamma)$ and $n_p^A(\gamma)$ are both continuous functions in γ. Hence, it must be the case that n_f^A and n_p^A intersect twice in the interval $\gamma \in (0, k)$ because $n_f^A(0) > n_p^A(0)$ and $n_f^A(k) > n_p^A(k)$. However, only one point of intersection exists because

$$n_f^A = n_p^A = \frac{1}{2} \Leftrightarrow \gamma = \frac{t - ka_p^A}{a_f^A - a_p^A}.$$

It follows that the assumption was wrong and $n_f^A \geq n_p^A \, \forall \gamma \in (0, k)$.

A.4 Proof of Proposition 3

For Proposition 3, we calculate the equilibrium solutions in Regimes A and B for the quadratic cost function $c(a) = \frac{\theta}{2}a^2$.

In Regime A, the maximization problems for the PTV and FTV broadcasters, respectively, are given by

$$\max_{(s_p, a_p) > 0} \left\{ \pi_p = (s_p + ka_p)\frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_f - a_p) - s_p)\right] - \frac{\theta}{2}a_p^2 \right\},$$

$$\max_{a_f > 0} \left\{ \pi_f = ka_f\frac{1}{2}\left[1 + \frac{1}{t}(\gamma(a_p - a_f) + s_p)\right] - \frac{\theta}{2}a_f^2 \right\}.$$

By deriving the corresponding first-order conditions and solving the system of equations, it is straightforward to calculate the subscription fee s_p^A and the level of advertising a_p^A for the PTV broadcaster in Regime A as

$$\left(s_p^A, a_p^A\right) = \left(\frac{1}{\lambda}t\,(2\theta t + 3k\gamma)\,(2\theta t - k^2 + k\gamma)\,,\, \frac{1}{\lambda}t\,(2\theta t + 3k\gamma)\,(k - \gamma)\right),$$

with $\lambda = 8\theta^2 t^2 - k\,(k - \gamma)^2\,\gamma - 2\theta t\,(k^2 - 5k\gamma + \gamma^2)$.

For the FTV broadcaster, the level of advertising a_f^A in Regime A is

$$a_f^A = \frac{1}{\lambda}2kt\left(3\theta t - (k - \gamma)^2\right).$$

The viewer demands on the PTV channel n_p^A and FTV channel n_f^A are

$$n_p^A = \frac{1}{\lambda}\theta t\,(2\theta t + 3k\gamma) \text{ and } n_f^A = \frac{1}{\lambda}\left(3\theta t - (k - \gamma)^2\right)(2\theta t + k\gamma).$$

As in the general model, we assume that the differentiation parameter is sufficiently large with $t > t^A$. This implies

$$\lambda > 0,\, 3\theta t > (k - \gamma)^2 \text{ and } 2\theta t + k\gamma > k^2. \tag{25}$$

In Regime B, the maximizations problems for the PTV and FTV broadcasters, respectively, are given by

$$\max_{s_p > 0} \left\{ \pi_p = [s_p] \frac{1}{2} \left[1 + \frac{1}{t}(\gamma a_f - s_p) \right] \right\},$$

$$\max_{a_f > 0} \left\{ \pi_f = k a_f \frac{1}{2} \left[1 + \frac{1}{t}(s_p - \gamma a_f) \right] - \frac{1}{2} \theta \left(a_f \right)^2 \right\}.$$

By deriving the corresponding first-order conditions and solving the system of equations, we calculate the subscription fee s_p^B for the PTV broadcaster in Regime B as

$$s_p^B = \frac{20 t^2 + 3 k t \gamma}{40 t + 3 k \gamma}.$$

For the FTV broadcaster the level of advertising a_f^B in Regime B is

$$a_f^B = \frac{3 k t}{40 t + 3 k \gamma},$$

so that the viewer demands in Regime B are given by

$$n_p^B = \frac{20 t + 3 k \gamma}{2(40 t + 3 k \gamma)} \text{ and } n_f^B = \frac{3 (20 t + k \gamma)}{2(40 t + 3 k \gamma)}.$$

To prove Part (i) of Proposition 3, we must show that

$$s_p^A = \frac{1}{\lambda} t \left(20 t + 3 k \gamma \right) \left(20 t - k^2 + k \gamma \right) < \frac{20 t^2 + 3 k t \gamma}{40 t + 3 k \gamma} = s_p^B.$$

Rearranging of the inequality yields $\left[t \left(20 t + 3 k \gamma \right) \left(20 t - k^2 + k \gamma \right) \right] (40 t + 3 k \gamma) - \lambda \left(20 t^2 + 3 k t \gamma \right) < 0$. We further simplify the inequality and obtain

$$-t \left(k - \gamma \right) \left(20 t + 3 k \gamma \right) \left[20 t \left(k + \gamma \right) + k \gamma \left(2 k + \gamma \right) \right] < 0,$$

which proves the claim in Part (i).

To prove Part (ii) of Proposition 3, we must show that

$$a_f^A = \frac{1}{\lambda} 2 k t \left(30 t - (k - \gamma)^2 \right) < \frac{3 k t}{40 t + 3 k \gamma} = a_f^B.$$

Rearranging of the inequality yields $[2 k t (30 t - (k - \gamma)^2)] (40 t + 3 k \gamma) - \lambda (3 k t) < 0$. We further simplify the inequality and obtain

$$-k t \left(k - \gamma \right)^2 \left(20 t + 3 k \gamma \right) < 0,$$

which proves the claim in Part (ii).

To prove Part (iii) of Proposition 3, we must show that

$$n_p^A = \frac{1}{\lambda} \theta t \left(20 t + 3 k \gamma \right) > \frac{20 t + 3 k \gamma}{2(40 t + 3 k \gamma)} = n_p^B.$$

Rearranging of the inequality yields $[\theta t\,(2\theta t + 3k\gamma)]\,[2(4\theta t + 3k\gamma)] - \lambda(2\theta t + 3k\gamma) > 0$. We further simplify the inequality and obtain

$$(k - \gamma)^2\,(2\theta t + k\gamma)\,(2\theta t + 3k\gamma) > 0.$$

Because $n_f = 1 - n_p$ it follows that $n_f^A < n_f^B$, which proves Part (iii) and thus completes the proof of the proposition.

A.5 Proof of Proposition 4

First, we will prove Part (i) of the proposition which states that aggregate consumer surplus is higher in Regime A than in Regime B, i.e. $CS_p^A + CS_f^A > CS_p^B + CS_f^B$.

In Regime A, we compute the aggregate consumer surplus $CS^A = CS_p^A + CS_f^A$ as

$$CS^A = v - \frac{5t}{2} + \frac{\theta t^2 \phi}{\lambda^2}$$

with

$$\lambda = 8\theta^2 t^2 - k\,(k - \gamma)^2\,\gamma - 2\theta t\,(k^2 - 5k\gamma + \gamma^2).$$
$$\phi = \theta t\,(2\theta t + 3k\gamma)^2 - 2\,(2k^2 - 8\theta t - 7k\gamma + 2\gamma^2)\,\lambda.$$

In Regime B, the aggregate consumer surplus $CS^B = CS_p^B + CS_f^B$ is given by

$$CS^B = v - \frac{5t}{4} + \frac{\theta t^2\,(13\theta t + 9k\gamma)}{(4\theta t + 3k\gamma)^2}.$$

To prove Part (i) of the proposition, we must show that $CS^A > CS^B$ or equivalently

$$4\theta t\,(4\theta t + 3k\gamma)^2\,\phi - 3\,(44\theta^2 t^2 + 52\theta kt\gamma + 15k^2\gamma^2)\,\lambda^2 > 0.$$

We further rearrange this inequality as

$$-(k - \gamma)^2\,(\tau_1 + \tau_2) > 0$$

with

$$\tau_1 = 45k^4\gamma^4\,(k - \gamma)^2 - 128\theta^5 t^5 + 16\theta^4 t^4\,((k^2 + \gamma^2) - 52k\gamma)$$
$$+ 16\theta^3 t^3 k\gamma\,(8(k^2 + \gamma^2) - 115k\gamma),$$
$$\tau_2 = 24\theta^2 k^2 t^2 \gamma^2\,(11(k^2 + \gamma^2) - 73k\gamma) + 12\theta t k^3 \gamma^3\,(16(k^2 + \gamma^2) - 59k\gamma).$$

Because of $k > \gamma$ the above inequality simplifies to $\tau_1 + \tau_2 < 0$. After rearrangements and simplifications we obtain $\beta_1 + \beta_2 + \beta_3 > 0$ with

$$\beta_1 = 16\theta^4 t^4 (8\theta t - \eta_2) + 128\theta^3 t^3 k\gamma (3\theta t - \eta_2),$$

$$\beta_2 = 264\theta^2 t^2 k^2 \gamma^2 (3\theta t - \eta_2) + 45 k^4 \gamma^4 (3\theta t - \eta_2) + 192\theta t k^3 \gamma^3 (3\theta t - \eta_2),$$

$$\beta_3 = 416\theta^4 t^4 k\gamma + 792\theta^3 t^3 k^2 \gamma^2 + 189\theta t k^4 \gamma^4 + 648\theta^2 t^2 k^3 \gamma^3.$$

and $\eta_1 = (k^2 + \gamma^2)$ and $\eta_2 = (k - \gamma)^2$. From our assumption $t > t^A$ it follows $3\theta t > \eta_2$ (see Eq. (25)) and thus it holds $\beta_1 + \beta_2 + \beta_3 > 0$, which proves Part (i) of the proposition.

Second, we will prove Part (ii) of the proposition which states that fhe profit of the FTV (PTV) broadcaster is lower (higher) in Regime A than in Regime B, i.e. $\pi_f^A < \pi_f^B$ and $\pi_p^A > \pi_p^B$.

In Regime A, we derive the profits of the PTV and FTV broadcasters, respectively, as

$$\pi_p^A = \frac{1}{2\lambda^2} \theta t^2 \left(4\theta t - (k - \gamma)^2 \right) (2\theta t + 3k\gamma)^2,$$

$$\pi_f^A = \frac{1}{\lambda^2} 2k^2 t \left((k - \gamma)^2 - 3\theta t \right)^2 (\theta t + k\gamma),$$

with

$$\lambda = 8\theta^2 t^2 - k(k - \gamma)^2 \gamma - 2\theta t (k^2 - 5k\gamma + \gamma^2).$$

In Regime B, the profits of the PTV and FTV broadcasters, respectively, are

$$\pi_p^B = \frac{t(2\theta t + 3k\gamma)^2}{2(4\theta t + 3k\gamma)^2} \text{ and } \pi_f^B = \frac{9k^2 t(\theta t + k\gamma)}{2(4\theta t + 3k\gamma)^2}.$$

To prove the claim that the profits of the PTV broadcaster is higher in Regime A than in Regime B, we show

$$\pi_p^A = \frac{1}{2\lambda^2} \theta t^2 \left(4\theta t - (k - \gamma)^2 \right) (2\theta t + 3k\gamma)^2 > \frac{t(2\theta t + 3k\gamma)^2}{2(4\theta t + 3k\gamma)^2} = \pi_p^B.$$

After rearranging the inequality we get

$$\left[\theta t^2 \left(4\theta t - (k - \gamma)^2 \right) (2\theta t + 3k\gamma)^2 \right] \left[2(4\theta t + 3k\gamma)^2 \right]$$

$$- (2\lambda^2) \left[t(2\theta t + 3k\gamma)^2 \right] > 0 \Leftrightarrow$$

$$\left[\theta t \left(4\theta t - (k - \gamma)^2 (4\theta t + 3k\gamma)^2 \right) - \lambda^2 \right] > 0$$

After rearrangements

$$-(k - \gamma)^2 \xi > 0.$$

with

$$\xi = \Big[-16\theta^3 t^3 + k^2\gamma^2 (k-\gamma)^2 + 4\theta^2 t^2 \left(k^2 - 6k\gamma + \gamma^2\right)$$
$$+ \theta k t \gamma \left(4k^2 - 11k\gamma + 4\gamma^2\right)\Big]$$

Because $k > \gamma$ the proof further simplifies to $\xi < 0$. After further rearrangements and by denoting $\eta_2 = (k-\gamma)^2$ we obtain

$$4\theta^2 t^2 \left(4\theta t - \eta_2 - 2k\gamma\right) + 4\theta t k\gamma \left(6\theta t - \eta_2 - 2k\gamma\right) + k^2\gamma^2 \left(11\theta t - \eta_2 - 2k\gamma\right)$$
$$+ 2k^3\gamma^3 > 0$$

Again, we rearrange and simplify the inequality to

$$4\theta^2 t^2 \left(4\theta t - \eta_2\right) + 4\theta t k\gamma \left(4\theta t - \eta_2\right) + k^2\gamma^2 \left(3\theta t - \eta_2\right) > 0.$$

From our assumption $t > t^A$ it follows $3\theta t > \eta_2$ (see (25)) and thus the above inequality is larger than zero, which proves the claim.

Next, we show that the profits of the FTV broadcaster is lower in Regime A than in Regime B, i.e.

$$\pi_f^A = \frac{1}{\lambda^2} 2k^2 t \left(\left(k-\gamma\right)^2 - 3\theta t\right)^2 \left(\theta t + k\gamma\right) < \frac{9k^2 t \left(\theta t + k\gamma\right)}{2\left(4\theta t + 3k\gamma\right)^2} = \pi_f^B.$$

After rearranging the inequality we get

$$\left[2k^2 t \left(\left(k-\gamma\right)^2 - 3\theta t\right)^2 \left(\theta t + k\gamma\right)\right]\left[2\left(4\theta t + 3k\gamma\right)^2\right] - \lambda^2 \left[9k^2 t \left(\theta t + k\gamma\right)\right] < 0.$$

We further simplify the inequality and obtain

$$-k^2 t \eta_2 \left(\theta t + k\gamma\right)\left(2\theta t + 3k\gamma\right)\left[48\theta^2 t^2 - 9k\eta_2\gamma - 2\theta t \left(7k^2 - 32k\gamma + 7\gamma^2\right)\right] < 0.$$

Thus, we only need to show that

$$48\theta^2 t^2 - 9k\eta_2\gamma - 2\theta t \left(7k^2 - 32k\gamma + 7\gamma^2\right) > 0 \Leftrightarrow$$
$$\theta t \left(48\theta t - 14\eta_2\right) + k\gamma \left(36\theta t - 9\eta_2\right) > 0.$$

From our assumption $t > t^A$ it follows $3\theta t > \eta_2$ (see (25)) and thus the above inequality is larger than zero, which proves the claim. This completes the proof of Proposition 4.

Appendix B: Symmetric Competition Between Two FTV Broadcasters

In this appendix, we analyze an alternative benchmark: the symmetric competition between two FTV broadcasters, indexed $i, j = \{1, 2\}$. We call this benchmark Regime C.

In Regime C, the maximization problems for the FTV broadcaster $i = \{1, 2\}$ is given by

$$\max_{a_{f,i} > 0} \left\{ \pi_{f,i} = k a_{f,i} \frac{1}{2} \left[1 + \frac{1}{t}(\gamma(a_{f,j} - a_{f,i})) \right] - \frac{\theta}{2} a_{f,i}^2 \right\}.$$

for the quadratic cost function $c(a) = \frac{\theta}{2} a^2$.

By deriving the corresponding first-order conditions and solving the system of equations, it is straightforward to calculate the symmetric level of advertising $a_{f,i}^C$ for the FTV broadcaster i in Regime C as

$$a_{f,i}^C = \frac{kt}{4\theta t + k\gamma}.$$

Given the symmetric competition, it is straightforward to derive viewer demands on the two FTV channels as

$$n_{f,i}^C = \frac{1}{2}.$$

The aggregate consumer surplus $\text{CS}^C = \text{CS}_{f,1}^C + \text{CS}_{f,2}^C$ is given by

$$\text{CS}^C = v - \frac{5t}{4} + \frac{4\theta t^2}{4\theta t + k\gamma}$$

and the profits of the FTV broadcaster i is

$$\pi_{f,i}^C = \frac{k^2 t (2\theta t + k\gamma)}{2 (4\theta t + k\gamma)^2}.$$

To compare between Regime A and Regime C, please note that the broadcaster indexed with 1 in Regime C is the FTV channel that decides to charge a subscription fee in addition to carrying ads and thus becomes a PTV channel in Regime A and is indexed with p.

The next proposition derives the impact on the advertising level and viewer demand through a switch from Regime C to Regime A:

Proposition 7. *When the FTV channel 1 decides to charge a subscription fee in addition to carrying ads, the following results are true:*

(i) The advertising level on this channel decreases and the advertising level on the other channel increases, i.e. $a_p^A < a_{f,1}^C$ *and* $a_f^A > a_{f,2}^C$.

(ii) Viewer demand on this channel decreases and viewer demand on the other channel increases, i.e. $n_p^A < n_{f,1}^C$ *and* $n_f^A > n_{f,2}^C$.

Proof. Remember that $\lambda = 8\theta^2 t^2 - k(k-\gamma)^2 \gamma - 2\theta t (k^2 - 5k\gamma + \gamma^2)$

To prove Part (i) of the proposition, we first show that

$$a_p^A = \frac{1}{\lambda} t (2\theta t + 3k\gamma)(k-\gamma) < \frac{kt}{4\theta t + k\gamma} = a_{f,1}^C$$

Rearranging this inequality yields

$$\frac{2kt(-6\theta t + (k-\gamma)^2)}{-32\theta^2 t^2 + k(k-\gamma)^2\gamma + 4\theta t(k^2 - 5k\gamma + \gamma^2)} > \frac{kt}{4\theta t + k\gamma}$$

Since $t > \frac{k(k-\gamma)}{4\theta}$ we can show that the inequality holds for all $k > \gamma$.

With a similar calculation, we can show that

$$a_f^A = \frac{1}{\lambda} 2kt \left(3\theta t - (k-\gamma)^2\right) > \frac{kt}{4\theta t + k\gamma} = a_{f,2}^C.$$

To prove Part (ii) of the proposition, we first show that

$$n_p^A = \frac{1}{\lambda} \theta t (2\theta t + 3k\gamma) < \frac{1}{2} = n_{f,1}^C$$

Rearranging this inequality yields

$$\frac{2\theta t(4\theta t + 3k\gamma)}{32\theta^2 t^2 - k(k-\gamma)^2\gamma - 4\theta t(k^2 - 5k\gamma + \gamma^2)} < \frac{1}{2}$$

Since $t > \frac{k(k-\gamma)}{4\theta}$ we can show that the inequality holds for all $k > \gamma$.

With a similar calculation, we can show that

$$n_f^A = \frac{1}{\lambda} \left(3\theta t - (k-\gamma)^2\right)(2\theta t + k\gamma) > \frac{1}{2} = n_{f,2}^C. \qquad \square$$

In the next proposition, we compare broadcaster profits and consumer surplus in Regime A and C:

Proposition 8. (i) *Each broadcaster realizes more profits in Regime A than in Regime C, i.e.* $\pi_p^A > \pi_{f,1}^C$ *and* $\pi_f^A > \pi_{f,2}^C$.

(ii) *Aggregate consumer surplus is higher in Regime A than in Regime C, i.e.* $CS^A = CS_p^A + CS_f^A > CS_{f,1}^C + CS_{f,2}^C = CS^C$.

Proof. Remember that $\lambda = 8\theta^2 t^2 - k(k-\gamma)^2 \gamma - 2\theta t (k^2 - 5k\gamma + \gamma^2)$.

To prove Part (i) of the proposition, we first show that

$$\pi_p^A = \frac{1}{2\lambda^2}\theta t^2 \left(4\theta t - (k-\gamma)^2\right)(2\theta t + 3k\gamma)^2 > \frac{k^2 t (2\theta t + k\gamma)}{2(4\theta t + k\gamma)^2} = \pi_{f,1}^C$$

Rearranging this inequality yields

$$\frac{\theta t^2 (8\theta t - (k-\gamma)^2)(4\theta t + 3k\gamma)^2}{(-32\theta^2 t^2 + k(k-\gamma)^2\gamma + 4\theta t(k^2 - 5k\gamma + \gamma^2))^2} > \frac{k^2 t(2\theta t + k\gamma)}{2(4\theta t + k\gamma)^2}$$

Since $t > \frac{k(k-\gamma)}{4\theta}$ we can show that the inequality holds for all $k > \gamma$.

With a similar calculation, we can show that

$$\pi_f^A = \frac{1}{\lambda^2}2k^2 t \left((k-\gamma)^2 - 3\theta t\right)^2 (\theta t + k\gamma) > \frac{k^2 t(2\theta t + k\gamma)}{2(4\theta t + k\gamma)^2} = \pi_{f,2}^C$$

To prove Part (ii), we show that

$$CS^A = v - \frac{5t}{2} + \frac{\theta t^2 \phi}{\lambda^2} > v - \frac{5t}{4} + \frac{4\theta t^2}{4\theta t + k\gamma}$$

with

$$\lambda = 8\theta^2 t^2 - k(k-\gamma)^2 \gamma - 2\theta t (k^2 - 5k\gamma + \gamma^2)$$

$$\phi = \theta t (2\theta t + 3k\gamma)^2 - 2 (2k^2 - 8\theta t - 7k\gamma + 2\gamma^2) \lambda$$

Rearranging this inequality yields

$$\frac{\phi}{\lambda^2} > \frac{5t}{2} + \frac{4}{4\theta t + k\gamma}$$

Since $t > \frac{k(k-\gamma)}{4\theta}$ we can show that the inequality holds for all $k > \gamma$. ☐

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