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Transforming talent into performance: Efficiency heterogeneity, strategic behavior, and welfare in sports leagues

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A B S T R A C T

Understanding why some sports clubs consistently outperform others despite similar financial resources remains a central question in sports economics. This paper develops a contest-theoretic model of a sports league in which clubs differ in both their financial capacity and their efficiency in transforming player talent into on-field performance. Each club chooses its optimal level of talent investment under either profit-maximizing or win-maximizing objectives. The model explicitly distinguishes between two types of heterogeneity-market size and efficiency, allowing us to study how these asymmetries jointly shape equilibrium talent demand, competitive balance, and welfare. The results reveal that profit-maximizing clubs may reduce talent investment when efficiency improves, while win-maximizing clubs respond in the opposite direction. Efficiency differences also affect large and small clubs asymmetrically, with small clubs often expanding investment under conditions in which large clubs contract. Welfare implications depend critically on league orientation: in profit-oriented leagues, welfare improves when small clubs are less efficient and large clubs are more efficient, whereas the opposite holds in win-oriented leagues. By integrating contest theory with the literature on club efficiency, the paper demonstrates that efficiency heterogeneity is not inherently detrimental. Under certain conditions, it can yield strategic advantages and even enhance league welfare, offering new insights for both academic research and league policy.

1. Introduction

Players form the foundation of every sports club, and a team's success ultimately depends on how effectively playing talent is transformed into on-field performance. While talent is difficult to observe directly, realized performance reflects not only player quality but also the organizational context in which it is deployed. Coaching, tactical systems, complementarities among teammates, medical support, and club culture all shape the extent to which playing resources translate into success. Consequently, identical levels of playing talent may yield vastly different outcomes across clubs. Some highly regarded players fail to deliver after a transfer, while others thrive in more suitable environments. These differences highlight systematic heterogeneity in how clubs convert talent into performance.

Efficiency in this transformation matters because player-related expenditures dominate the cost structures of professional clubs. In Europe's "Big Five" leagues, the wage-to-revenue ratio ranged between 65 % and 98 % in 2021 (Deloitte, 2023). Clubs therefore face strong incentives to allocate resources efficiently, yet outcomes depend not only on how much talent they acquire but also on how productively they deploy it. A club with high efficiency derives superior performance from a given wage bill, while a less efficient club may underperform relative to its investment. These systematic differences in efficiency carry important implications for talent demand, financial sustainability, and competitive balance.

Existing contest-theoretic models of sports leagues typically abstract from such heterogeneity, assuming that one unit of talent translates into the same performance contribution regardless of where it is employed. Meanwhile, a rich empirical literature has documented persistent differences in how clubs convert financial and human resources into sporting outcomes, using methods such as data envelopment analysis (DEA) or stochastic frontier analysis (SFA). To date, however, these two literatures have developed largely in parallel.

This paper bridges these strands and extends the theoretical framework in two important ways. First, we incorporate differences in *talent-to-performance efficiency* across clubs, capturing heterogeneity in how effectively organizations transform player ability into sporting success. Second, we combine this with asymmetry in *market size*,

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allowing clubs to differ both in financial capacity and in the productivity of their talent investments. To the best of our knowledge, this is the first contest-theoretic analysis applied to sports that jointly models these two dimensions of heterogeneity within a unified framework. This dual-asymmetry approach extends classic models such as Szymanski and Késenne (2004) and Szymanski (2003) and enables us to examine how market size and efficiency interact to shape equilibrium talent demand, competitive balance, and league welfare.

To analyze these interactions, we develop a contest-theoretic model of a two-club league in which each organization invests in talent to maximize either profits or wins. By relaxing the standard homogeneous productivity assumption, the model allows clubs to differ systematically in how efficiently they transform talent into performance. This setup provides a tractable benchmark for studying how efficiency heterogeneity affects equilibrium talent allocation, club profits, competitive balance, and overall welfare.

Our analysis yields several novel insights. First, relative efficiency affects talent demand asymmetrically depending on whether clubs maximize profits or wins. Second, the effects differ between large and small clubs, generating asymmetric responses that shape league-level dynamics. Third, welfare implications are ambiguous: in profit-maximizing leagues, welfare improves when small clubs are less efficient and large clubs are more efficient, whereas the opposite holds in win-maximizing leagues.

By integrating contest theory with the efficiency literature, this paper offers a new perspective on why some clubs consistently outperform or underperform relative to their wage bills, and on the conditions under which such heterogeneity may prove beneficial rather than detrimental to the functioning of sports leagues.

2. Literature review

Our paper draws on two strands of the sports economics literature: contest theory applied to sports leagues and research on efficiency and productivity in professional football. By integrating these perspectives, we highlight the importance of club heterogeneity not only in market size but also in the efficiency with which talent is transformed into on-field performance.

The seminal work by Tullock (1980) introduced contest theory, in which agents exert costly effort to increase their probability of winning. This framework has been widely applied in sports economics to model how clubs allocate resources to maximize wins or profits (Szymanski, 2003). Building on earlier contributions by Rottenberg (1956) and Neale (1964), contest models have become central to analyzing the implications of revenue disparities and market size heterogeneity for competitive balance in sports leagues (Fort and Quirk, 1995; Szymanski and Késenne, 2004). A substantial body of work has examined how league structures, revenue sharing, and salary caps affect the allocation of talent and competitive balance (Dietl et al., 2009; Humphreys, 2002; Késenne, 2000; Scelles et al., 2013).

Recent theoretical and empirical work has extended this line of inquiry to modern policy instruments. Sela (2023) analyzes how balance constraints such as salary caps or effort limits can simultaneously promote competitive balance and aggregate effort within contests, providing microfoundations for league-level regulations. Empirically, Caglio et al. (2023) evaluate UEFA's Financial Fair Play (FFP) reforms and demonstrate that while these rules improved clubs' financial discipline, they also reinforced competitive asymmetries between large and small clubs. Bognar et al. (2024) further emphasize that competitive balance is a multi-dimensional construct encompassing both within-season dispersion and across-season mobility, implying that policy interventions may improve one dimension while worsening another.

Existing contest-theoretic models applied to sports, however, typically assume a uniform productivity assumption: a given unit of playing talent yields the same contribution to on-field performance regardless of the club that employs it. While recent contest models have begun to consider how institutional constraints shape effort and balance (e.g., Sela, 2023), they continue to treat clubs as homogeneous in their ability to transform talent into outcomes. This abstraction ignores the fact that clubs differ systematically in how effectively they deploy talent, a gap our model addresses.

A second relevant literature investigates the efficiency with which clubs convert financial and human resources into sporting outcomes. Several studies use frontier methods such as DEA or SFA to quantify how effectively clubs transform financial inputs (e.g., wage bills) into sporting outputs (points or wins) (e.g., Barros and Barrio, 2008; Escuer and á Cebrián, 2019; Garcia Sánchez, 2007). These studies consistently find substantial heterogeneity across clubs: some systematically outperform their financial inputs, while others underperform relative to expectations.

Other research emphasizes organizational and managerial sources of productivity differences. Dawson et al. (2000) estimate "coaching efficiency" in English football using stochastic frontiers, highlighting how managerial quality and tactical systems shape club-level performance. Franck and Nüesch (2012) show that superstar performance reflects not only individual talent but also organizational complementarities, while Peeters (2015) analyze how league governance and revenue-sharing arrangements affect clubs' strategic use of talent. Together, these studies point to persistent club-specific differences in how effectively talent is deployed.

At the aggregate level, wage bills are strongly correlated with performance. For example, Hall et al. (2002) document a causal link from wages to team success in English soccer. Frick (2007) reviews evidence across major European leagues and notes systematic deviations from this wage-performance relationship, reflecting persistent variation in club efficiency. At the policy level, this heterogeneity interacts with regulatory regimes. The evidence from Caglio et al. (2023) indicates that financial regulation can alter competitive balance by constraining resource allocation, while Bognar et al. (2024) highlight that such shifts should be evaluated across multiple dimensions of balance. Our theoretical framework enables an assessment of how efficiency heterogeneity shapes these regulatory outcomes.

This body of work motivates our modeling choice to introduce a club-specific efficiency parameter. Conceptually, this parameter captures how effectively a club converts acquired talent into realized on-field performance, reflecting a wide range of organizational, managerial, and contextual factors.

While contest theory has been widely used to study competitive balance and welfare in sports leagues, and empirical work has documented significant variation in club efficiency, these two literatures have developed largely in parallel. To our knowledge, no theoretical work has explicitly incorporated heterogeneity in talent-to-performance efficiency into a contest framework to examine its effects on equilibrium talent demand, competitive balance, and welfare. Our paper addresses this gap by introducing a club-specific efficiency parameter into a contest-theoretic model of a professional sports league. This extension allows us to analyze how asymmetries in both market size and productivity jointly shape strategic interactions between clubs and aggregate league-level outcomes.

3. Model

3.1. Talent and player performance

We consider a model of a two-club league where each club engages in a non-cooperative game to optimize outcomes–either in terms of profits or on-field success. In sports economics, clubs are commonly modeled as either profit maximizers or win maximizers. Profit-maximizing clubs seek to maximize financial returns, functioning analogously to firms in traditional industries (Zimbalist, 2003). By contrast, win-maximizing clubs prioritize sporting success, often investing heavily in talent even at the expense of short-term profits (Késenne, 2006).[1]

Following the standard approach in the contest-theory literature (Dietl et al., 2011a, 2009), we assume that each club independently invests in talent. Talent is modeled as a homogeneous, perfectly divisible input hired in a competitive labor market.[2] Let $t_i \geq 0$ denote the amount of talent acquired by club $i \in \{1,2\}$, at a constant unit cost $c \geq 0$.

The crucial assumption is that clubs differ systematically in how efficiently they convert talent into on-field performance. Some organizations are able to extract more performance from a given stock of talent due to superior coaching, tactical systems, medical and conditioning staff, or complementarities among players. Others may extract less, either because of weaker organizational structures, poor fit between players and tactical requirements, or limited support systems.

We capture this by modeling club i's performance as a deterministic function of talent:

$$P_i(t_i) = k_i t_i, \tag{1}$$

where $k_i > 0$ is a club-specific efficiency parameter. This parameter measures the performance contribution per unit of talent at club i. A higher k_i implies that a given talent investment translates more productively into on-field performance, while a lower k_i reflects weaker talent-to-performance conversion.

In this setup, k_i does not represent random shocks, valuation errors, or temporary biases. Rather, it summarizes persistent and systematic differences in organizational productivity-arising from factors such as tactical alignment, managerial quality, or the broader institutional environment. While individual players may over- or underperform in specific situations, our model abstracts from such idiosyncratic variation and focuses on the aggregate, club-level efficiency of transforming talent into performance.

3.2. Win percentages and competitive balance

We assume that a club's success on the field depends on its efficiency-adjusted performance levels. Specifically, the probability that club i wins against club j is given by a contest-success function (CSF) in logit form:

$$w_i(t_i, t_j) = \frac{P_i(t_i)}{P_i(t_i) + P_j(t_j)} = \frac{k_i t_i}{k_i t_i + k_j t_j}, \tag{2}$$

where $i, j \in \{1,2\}$ and $i \neq j$. This specification maintains the adding-up constraint $w_i + w_j = 1$. The logit CSF is widely used in the game-theoretic modeling of sports leagues, originally introduced by Tullock (1980) and further developed by Skaperdas (1996) and Clark and Riis (1998).

Following Szymanski and Valletti (2003), Dietl and Lang (2008), and Vrooman (2008), we define competitive balance (CB) as the product of the two clubs' win percentages:

$$CB(t_i, t_j) = w_i(t_i, t_j) \cdot w_j(t_i, t_j). \tag{3}$$

The maximum value of $CB = 1/4$ occurs when both clubs have an equal chance of winning, i.e., $w_1 = w_2 = 1/2$. A deviation from this benchmark reflects an imbalance in league competitiveness.

3.3. Fan demand and match quality

Each club generates revenue R_i, which depends on fan demand. Fan demand is driven by the perceived match quality, which in turn is influenced by the efficiency-adjusted performance of the clubs. We follow the framework established by Dietl et al. (2009), assuming that fans form their expectations based on the average quality of play.

Let a match between clubs i and j be characterized by match quality q_i, which reflects the performance of club i. Each fan k has a preference parameter $\theta_k \sim U[0,1]$, and derives utility from attending the match as $\max\{\theta_k q_i - p_i, 0\}$ (Peeters, 2015).

At a ticket price $p_i \geq 0$,[3] the marginal fan θ^* is indifferent between attending and abstaining, defined by $\theta^* = p_i/q_i$. Thus, the proportion of fans who attend the match is:

$$1 - \theta^* = \frac{q_i - p_i}{q_i}.$$

The resulting demand function for club $i \in \{1,2\}$ is:

$$d_i(p_i, q_i) = m_i \left(1 - \frac{p_i}{q_i}\right),$$

where $m_i > 0$ denotes the market size for club i.

3.4. Club revenues and profits

By normalizing all non-labor costs to zero, the revenue of club i is given by:[4]

$$R_i = \frac{m_i}{4} q_i. \tag{4}$$

Following Dietl et al. (2009), we assume that match quality q_i is influenced by two main factors: the probability of club i's success and the uncertainty of the match outcome. We model match quality as a linear combination of these components-win percentage and competitive balance-resulting in a specification that is standard in sports economics (e.g., Dietl et al., 2011b; Hoehn and Szymanski, 1999; Késenne, 2007; Szymanski, 2003; Szymanski and Késenne, 2004; Vrooman, 2007, 2008).

Specifically, incorporating the win percentage from Eq. (2) and the competitive balance from Eq. (3), we define match quality as:

$$q_i(t_i, t_j) = w_i(t_i, t_j) + w_i(t_i, t_j) \cdot w_j(t_i, t_j), \tag{5}$$

where $i, j \in \{1,2\}$, $i \neq j$. Substituting Eq. (5) into the revenue expression (4), and using the identity $w_j = 1 - w_i$, the revenue function becomes:

$$R_i(t_i, t_j) = \frac{m_i}{4}\left[2w_i(t_i, t_j) - w_i(t_i, t_j)^2\right].$$

Assuming a competitive labor market, the unit cost of talent c is common across clubs. The cost function for club $i \in \{1,2\}$ is linear:

$$C(t_i) = ct_i,$$

where $c > 0$ is the marginal cost of talent.[5]

Each club's profit is given by the difference between revenue and cost:

$$\pi_i(t_i, t_j) = R_i(t_i, t_j) - C(t_i).$$

[1] A club's objective function has important implications for league outcomes and social welfare. For example, leagues composed entirely of profit-maximizing clubs may achieve higher welfare levels than "mixed" leagues that include both profit- and win-oriented clubs (Dietl et al., 2009). Similarly, revenue-sharing schemes affect competitive balance differently depending on whether clubs pursue profits or wins (Dietl and Lang, 2008; Késenne, 2005).

[2] We consider an open league with flexible talent supply, such that investing in talent is equivalent to hiring additional or better players (Winfree and Fort, 2012).

[3] This price p_i could reflect either the cost of a ticket to attend the game or a subscription fee for media access.

[4] The optimal pricing strategy for club i is to set the price $p_i^* = q_i/2$, which maximizes revenue $R_i = p_i \cdot d_i(p_i, q_i)$.

[5] For simplicity, we abstract from non-labor costs and normalize fixed capital costs to zero. For more general cost structures, see Vrooman (1995) and Késenne (2007).

3.5. Social welfare

We define social welfare as the sum of aggregate consumer (fan) surplus, aggregate player salaries, and aggregate club profits (see, e.g., Dietl et al., 2017). To compute the aggregate consumer surplus, we sum the consumer surplus of club 1 and club 2 fans. The consumer surplus CS_i from the fans of club $i \in \{1,2\}$ is determined by integrating the demand function $d_i(p_i, q_i)$ from the equilibrium price $p_i^* = q_i/2$ to the maximum price $\overline{p}_i = q_i$ that fans are willing to pay for match quality q_i:

$$CS_i = \int_{p_i^*}^{\overline{p}_i} d_i(p_i, q_i)\, dp_i = \int_{\frac{q_i}{2}}^{q_i} m_i \frac{q_i - p_i}{q_i}\, dp_i = \frac{m_i}{8} q_i.$$

Assuming that player utility is equivalent to their salaries, the total player utility is represented by the aggregate salary payments $PS = ct_1 + ct_2$ in the league.

Therefore, social welfare, comprising aggregate consumer surplus, aggregate salary payments, and aggregate club profits, is given by:

$$W(t_1, t_2) = \frac{3}{8}\left[m_1 q_1(t_1, t_2) + m_2 q_2(t_1, t_2)\right].$$

Note that salary payments do not directly affect social welfare since they simply represent a transfer from clubs to players.

4. Analysis

We analyze a one-shot game in which each club chooses its talent investment t_i to maximize either profits or win percentage, taking the rival's decision as given. Performance is modeled deterministically as $P_i(t_i) = k_i t_i$, so heterogeneity arises only through the club-specific efficiency parameter k_i.

To simplify exposition, we normalize the model as follows:

$$\begin{aligned} k_1 &= k, \quad k_2 = 1, \\ m_1 &= m, \quad m_2 = 1. \end{aligned} \tag{6}$$

Thus, k can be interpreted as club 1's efficiency relative to club 2, and m can be interpreted as club 1's market size relative to club 2. These normalizations are without loss of generality and allow us to isolate the role of asymmetries with respect to efficiency and market size.

Based on the above normalization, we derive the following lemma:

Lemma 1.

(i) If $k < 1$, club 1 is less efficient than club 2;
(ii) If $k = 1$, both clubs have equal efficiency;
(iii) If $k > 1$, club 1 is more efficient than club 2.

Proof. Straightforward and therefore omitted. □

We now derive the Nash equilibrium in talent investment under two strategic objectives: profit maximization and win maximization.

4.1. Profit-maximizing clubs

We first consider the case where both clubs aim to maximize profits. For club $i \in \{1,2\}$, the optimization problem is:

$$\max_{t_i \geq 0} \pi_i(t_i, t_j) = R_i(t_i, t_j) - C(t_i),$$

where $i \neq j$. Revenue depends on win probability, which in turn is based on efficiency-adjusted performance.

To derive the Nash equilibrium in talent investment, we solve for each club's best-response function by setting marginal revenue equal to marginal cost. The first-order conditions (FOCs) for an interior maximum are:

$$\frac{\partial \pi_1}{\partial t_1} = c \quad \text{and} \quad \frac{\partial \pi_2}{\partial t_2} = c.$$

Substituting the profit expressions and applying the CSF and revenue formulations, we obtain:

$$\frac{\partial \pi_1}{\partial t_1} = \frac{mkt_2^2}{2(kt_1 + t_2)^3} - c = 0, \quad \frac{\partial \pi_2}{\partial t_2} = \frac{k^2 t_1^2}{2(kt_1 + t_2)^3} - c = 0.$$

These FOCs characterize each club's optimal investment given the rival's strategy.

The second-order conditions (SOCs) are given by

$$\frac{\partial^2 \pi_1}{\partial t_1^2} = -\frac{3}{2}\frac{mk^2 t_2^2}{(kt_1 + t_2)^4} < 0, \qquad \frac{\partial^2 \pi_2}{\partial t_2^2} = -\frac{3}{2}\frac{k^2 t_1^2}{(kt_1 + t_2)^4} < 0.$$

Thus, the SOCs for a maximum are satisfied.

Solving the system yields the unique Nash equilibrium talent levels (t_1^P, t_2^P) in closed form:[6]

$$t_1^P = \frac{km^3}{2c\left(km^2(3 + km) + \sqrt{km^3}\,(1 + 3km)\right)}, \tag{7}$$

$$t_2^P = \frac{k\left(-m(1 + 3km) + (3 + km)\sqrt{km^3}\right)}{2c\,(km - 1)^3}. \tag{8}$$

These equilibrium investments imply the following win probabilities:

$$(w_1^P, w_2^P) = \left(\frac{km^2}{km^2 + \sqrt{km^3}}, \frac{m}{m + \sqrt{km^3}}\right). \tag{9}$$

4.2. Win-maximizing clubs

Next, suppose both clubs are win-maximizers. Each club independently determines how much talent to acquire, with the goal of maximizing its talent stock subject to a budget constraint, where the budget is determined endogenously by revenue. Specifically, the maximization problem for club $i = 1, 2$ is given by:

$$\max_{t_i \geq 0} t_i \quad \text{subject to} \quad ct_i \leq R_i(t_i, t_j),$$

with $i, j \in \{1,2\}, i \neq j$. As before, we use $k_1 = k, k_2 = 1$ and $m_1 = m, m_2 = 1$.

The first-order conditions for this constrained optimization problem yield:

$$\begin{aligned} 1 - \lambda_i\left(c - \frac{\partial R_i}{\partial t_i}\right) &\leq 0, \\ t_i\left(1 - \lambda_i\left(c - \frac{\partial R_i}{\partial t_i}\right)\right) &= 0, \\ R_i - ct_i &\geq 0, \\ \lambda_i(R_i - ct_i) &= 0, \end{aligned}$$

where λ_i is the Lagrange multiplier associated with the budget constraint.

We establish the following lemma:

Lemma 2. *Each club will spend all its revenue on playing talent, leading to the following equilibrium talent levels in a league with win-maximizing clubs:*

$$(t_1^W, t_2^W) = \left(\frac{3m(2km - 1)}{4c(km + 1)^2}, \frac{3km(2 - km)}{4c(km + 1)^2}\right). \tag{10}$$

Proof. See Appendix. □

To ensure positive investments, the difference in market size between the two clubs should not be too large. Formally, the market size parameter must lie within a feasible range:

$$m \in (\underline{m}, \overline{m}) = \left(\frac{1}{2k}, \frac{2}{k}\right).$$

[6] It can easily be shown that $t_i^P > 0$.

Substituting the equilibrium talent levels into the win probability function, we derive the following equilibrium win percentages:

$$(w_1^W, w_2^W) = \left(\frac{2km-1}{km+1}, \frac{2-km}{km+1}\right).$$

5. Results

In this section, we examine how heterogeneity in talent-to-performance efficiency affects talent demand, competitive balance, club profits, and overall social welfare. Recall that k denotes club 1's relative efficiency, with $k = 1$ indicating efficiency parity. We refer to club 1, at which efficiency varies, as the focal club.

5.1. Talent demand

In a league with profit-maximizing clubs, it is not a priori obvious whether lower or higher efficiency at one club induces both teams to expand or contract their talent investments. We establish the following lemma:

Lemma 3. *In a league with profit-maximizing clubs, equilibrium talent demand* (t_1^P, t_2^P) *is maximized at the efficiency levels*

$$(\underline{k}^P, \overline{k}^P) = \left(\frac{1}{4m}, \frac{4}{m}\right).$$

Proof. See Appendix. □

We derive the following results concerning the effect of relative efficiency on talent demand:

Proposition 1.

(i) *Suppose clubs are highly heterogeneous in market size ($m < \frac{1}{4}$), and efficiency varies at the small club. When the small club becomes less efficient ($k < 1$), both clubs decrease their talent demand. Conversely, when the small club becomes more efficient ($k > 1$), both clubs increase their demand until the maximum is reached for $(\underline{k}^P, \overline{k}^P)$.*

(ii) *Suppose clubs are highly heterogeneous in market size ($m > 4$), and efficiency varies at the large club. When the large club becomes less efficient ($k < 1$), both clubs increase their talent demand until the maximum is reached for $(\underline{k}^P, \overline{k}^P)$. Conversely, when the large club becomes more efficient ($k > 1$), both clubs reduce their talent demand.*

(iii) *Suppose clubs are relatively homogeneous in market size with $m \in (1/4, 4)$. When the focal club's efficiency falls ($k < 1$), its own demand increases (up to the turning point at $\underline{k}^P$), while the rival's decreases. When the focal club's efficiency rises ($k > 1$), its own demand decreases while the rival's demand increases (up to the turning point at $\overline{k}^P$).*

Proof. See Appendix. □

Fig. 1 illustrates the results of the proposition by showing the effect of relative efficiency on equilibrium talent demand in three market-size scenarios. Panel (a) considers the case in which clubs are highly heterogeneous in market size and the focal club is the small-market club ($m < 1$), Panel (b) clubs are relatively homogeneous in market size, and Panel (c) the case in which clubs are again highly heterogeneous in market size and the focal club is the large-market club ($m > 1$). The horizontal axis is the relative efficiency k (with $k < 1$: lower efficiency at the focal club; $k > 1$: higher efficiency at the focal club). The vertical line at $k = 1$ marks the equal-efficiency benchmark; the vertical axis reports the corresponding equilibrium talent demands. 'VE' denotes 'variation in efficiency' and is used to mark the club whose efficiency differs from the benchmark.

For example, at $k = 1$ (equal efficiency), the large-market club demands more talent than the small-market club, as expected from $m \neq 1$. Moving left from $k = 1$ represents a decline in the focal club's efficiency; moving right represents an increase. This visualization makes transparent how departures from equal efficiency shape both clubs' optimal investments across different market-size configurations.

According to part (i), when clubs are very different in size, both clubs will demand more (less) talent when the small club becomes more (less) efficient in transforming talent into on-field performance (see Panel (a) of Fig. 1).

In practical terms, Panel (a) can be interpreted as a comparison between a financially dominant club such as Real Madrid or Manchester City and a smaller-market competitor such as Girona or Brentford. When a small club like Girona or Brentford improves its efficiency-by using data analytics, smart recruitment, or tactical innovation to extract more performance per euro spent-it raises its demand for talent despite limited financial capacity. In turn, large clubs respond strategically by increasing their own investment to maintain competitive advantage, leading to higher total talent spending across the league. This mirrors the model's prediction that greater efficiency at the small club stimulates investment on both sides. Conversely, when the small club operates inefficiently-for instance, through poor transfer decisions or managerial turnover-its reduced performance potential dampens not only its own spending but also the large club's incentive to invest heavily, since the expected competitive pressure diminishes. This aligns with the theoretical result that when efficiency at the small club falls, both clubs reduce their talent demand.

The intuition for this result is as follows: An increase in a club's efficiency k-its ability to convert talent into performance-has two opposing effects. First, efficiency raises the marginal productivity of talent, tending to increase optimal investment. However, it also softens strategic incentives: when a club can win more easily for any given level of talent, it faces diminishing marginal returns to additional investment because each extra unit of talent adds less to the probability of winning. Anticipating this, a profit-maximizing club scales back its talent spending once efficiency is sufficiently high. This "strategic softening" effect dominates at high k, generating the negative $\partial t_i / \partial k$ observed in the proposition. By contrast, when k is low, the club must invest aggressively to remain competitive, producing the positive portion of the comparative static.

Talent demand reaches its peak for high efficiency levels $(\underline{k}^P, \overline{k}^P)$. In this regime, the turning points lie to the right of parity: $\underline{k}^P = 1/(4m) > 1$ and $\overline{k}^P = 4/m \gg 1$. In addition, $\underline{k}^P < \overline{k}^P$, which means that club 1's talent demand is maximized with lower efficiency levels compared to club2′s talent demand. It is important to note that the maximum talent demand occurs at efficiency levels $k > 2$, which lie beyond the plotted range in Panel (a).

According to part (ii) of the proposition, in a league with highly heterogeneous clubs, both clubs will demand more talent when the large club is less efficient (see Panel (c) of Fig. 1). Interestingly, the small club might demand more talent than the large club if the small club is very inefficient ($m < 1/4$ and k small) or the large club is highly efficient ($m > 4$ and k large).

Part (iii) of the proposition deals with a league with homogeneous clubs (see Panel (b) of Fig. 1). When a club becomes less efficient, it will increase its demand, prompting a corresponding decrease in talent demand from its competitor. Conversely, when a club becomes more efficient, it will reduce its demand, prompting the competitor to increase its demand. This dynamic helps maintain competitive balance within the league and enables each club to optimize its profits.

In the next step, we examine a league with win-maximizing clubs and establish the following lemma:

Lemma 4. *In a league with win-maximizing clubs, equilibrium talent demand* (t_1^W, t_2^W) *is maximized at the efficiency levels*

$$(\underline{k}^W, \overline{k}^W) = \left(\frac{2}{m}, \frac{1}{2m}\right).$$

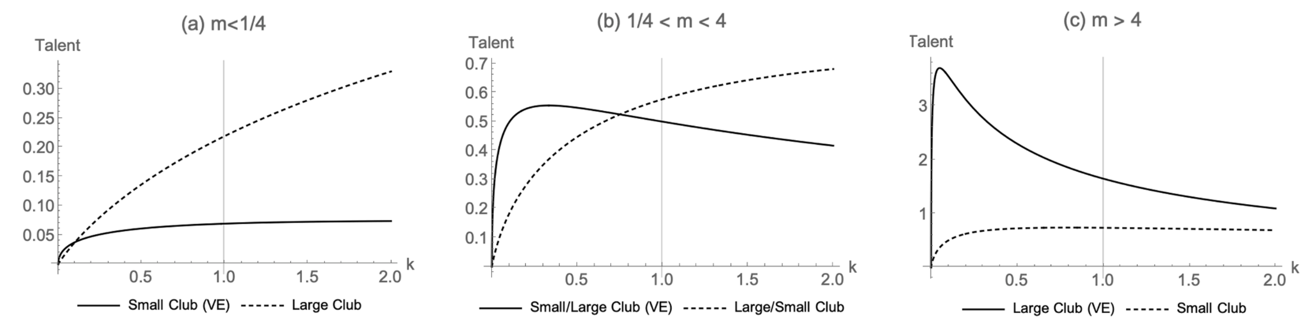

Fig. 1. Effect of relative efficiency on talent demand (profit-max).

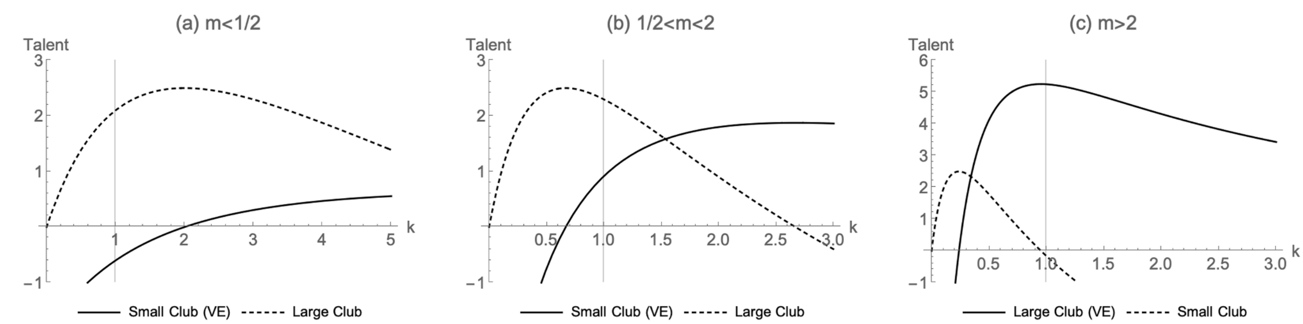

Fig. 2. Effect of relative efficiency on talent demand (win-max).

Proof. See Appendix. □

The following results concerning talent demand are obtained:

Proposition 2.

(i) *Suppose the clubs are highly heterogeneous in market size ($m < 1/2$) and efficiency varies at the small club. When the small club becomes less efficient ($k < 1$), both clubs reduce their talent demand. Conversely, when the small club becomes more efficient ($k > 1$), both clubs increase their talent demand until the maximum is reached for ($\underline{k}^W, \overline{k}^W$).*

(ii) *Suppose the clubs are highly heterogeneous in market size ($m > 2$) and efficiency varies at the large club. When the small club becomes less efficient ($k < 1$), both clubs increase their talent demand until the maximum is reached for ($\underline{k}^W, \overline{k}^W$). Conversely, when the small club becomes more efficient ($k > 1$), both clubs decrease their talent demand.*

(iii) *Suppose clubs are relatively homogeneous in market size with $m \in (1/4, 4)$. When the focal club's efficiency falls ($k < 1$), its own demand decreases, while the rival's increases (up to the turning point at $\overline{k}^W$). Conversely, when the focal club's efficiency rises ($k > 1$), its own demand increases (up to the turning point at $\underline{k}^W$) while the rival's demand decreases.*[7]

Proof. See Appendix. □

Fig. 2 illustrates the effect of relative efficiency on talent demand in a league with win-maximizing clubs.

According to parts (i) and (ii) of the proposition, when clubs are highly heterogeneous, the qualitative effects on talent demand mirror those in profit-maximizing leagues. However, for relatively homogeneous clubs (part (iii)), the pattern reverses: the focal club's demand moves in the same direction as its efficiency, while the rival adjusts in the opposite direction.

Finally, consistent with prior work, clubs invest more in talent when they maximize wins than when they maximize profits (Dietl et al., 2009), reflecting the priority that win-maximizing clubs place on on-field success over financial margins.

5.2. Competitive balance

In this subsection, we examine the effect of relative efficiency on competitive balance. There are no qualitative differences between profit-maximizing and win-maximizing scenarios.

Competitive balance is maximized for efficiency levels given by:

$$k^{CB} = \frac{1}{m}.$$

We differentiate two cases: (i) $m < 1$, which implies that efficiency varies at the small club, and (ii) $m > 1$, which implies that efficiency varies at the large club. The following proposition summarizes the results:

Proposition 3.

(i) *Suppose $m < 1$. When the small club becomes less efficient, competitive balance decreases. When the small club becomes more efficient, competitive balance increases until the league is perfectly balanced at an efficiency level of k^{CB}.*

(ii) *Suppose $m > 1$. When the large club becomes more efficient, competitive balance decreases. When the large club becomes less efficient, competitive balance increases until the league is perfectly balanced at an efficiency level of k^{CB}.*

Proof. See Appendix. □

Fig. 3 illustrates how competitive balance reacts when efficiency varies at the small club and at the large club (Panel (b)). The x-axis is the relative efficiency k (with $k < 1$: lower efficiency at the focal club; $k > 1$: higher efficiency at the focal club). The y-axis shows the win percentage of the clubs.

The figure illustrates that, in line with basic intuition, lower efficiency is beneficial to competitive balance when it occurs at the large club, and higher efficiency is beneficial when it occurs at the small club. Both scenarios contribute to a more balanced league, provided that the efficiency differences are not excessively large.

[7] Note that part (iii) holds regardless of whether efficiency varies at the small or large club. The only precondition is that the clubs are sufficiently homogeneous.

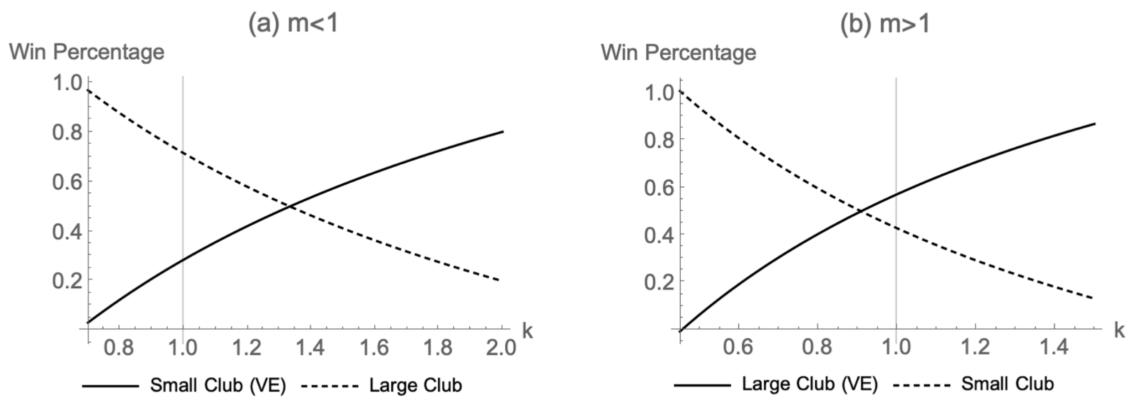

Fig. 3. Effect of relative efficiency on competitive balance.

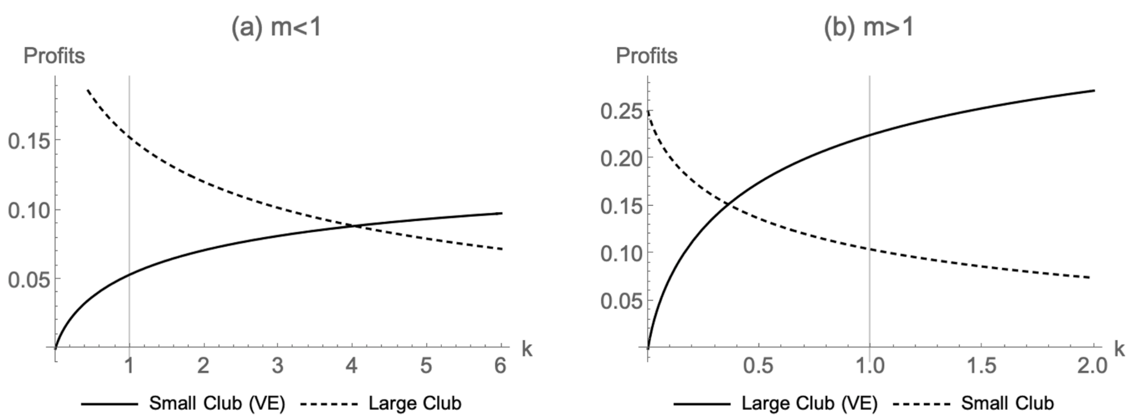

Fig. 4. Effect of relative efficiency on club profits.

This mechanism can be linked to real-world patterns in European football. When a dominant club such as Bayern Munich, Real Madrid, or Manchester City experiences a temporary decline in organizational efficiency-due to coaching changes or poor talent integration-the league tends to become more competitive as smaller clubs can narrow the gap. Conversely, when smaller-market teams like Brighton, Girona, or Union Berlin operate with exceptional efficiency in scouting or tactics, they enhance overall balance by challenging traditional powerhouses without matching their financial scale.

5.3. Club profits

In this subsection, we examine the effect of relative efficiency on club profits. We focus on a league with profit-maximizing clubs, as win-maximizing clubs, by definition, do not prioritize profits. We derive the following results:

Proposition 4. *Regardless of whether efficiency varies at small or large clubs, a club with higher efficiency typically realizes higher profits, while a club with lower efficiency experiences lower profits. The opposite effect is observed for the competing club.*

Proof. See Appendix. □

Fig. 4 illustrates the results from the proposition, with Panel (a) showing the effects when efficiency varies at the small club and Panel (b) showing the effects when it varies at the large club. The x-axis again represents relative efficiency, while the y-axis shows the resulting club profits.

The results from the proposition align with intuitive economic principles: transforming talent into on-field performance with higher efficiency results in greater output per unit of cost, thereby boosting profits. This occurs because the club acquires high-quality players at a lower cost, leading to better performance and higher revenues relative to their expenditures. On the other hand, low efficiency leads to excessive expenditures relative to player performance, diminishing a club's profits.

Moreover, this dynamic impacts the competing club as well. When one club has higher efficiency and realizes higher profits, the competing club may face increased competition, potentially reducing its profits. Conversely, when a club has lower efficiency and suffers lower profits, the competing club benefits from relatively lower competition, which can enhance its profitability.

In practical terms, these relationships can be observed when contrasting financially powerful yet efficiently run clubs-such as Manchester United and Tottenham Hotspur-with large clubs that temporarily mismanage resources, for instance through unsuccessful transfers or managerial instability. Efficient operations translate into sustained profitability despite high wage bills, whereas inefficiencies quickly erode margins. Similarly, smaller but well-managed clubs like Brighton, Atalanta, or Freiburg can remain financially viable by converting limited budgets into strong sporting outcomes, while less efficient peers struggle to balance performance ambitions with fiscal discipline.

5.4. Social welfare

In this final subsection, we examine the impact of relative efficiency on social welfare, focusing first on a league with profit-maximizing clubs. We derive the following results:

Proposition 5.

(i) Suppose $m < 1$. When the small club becomes less efficient, social welfare increases until it reaches a maximum at an efficiency level of $k = m$.

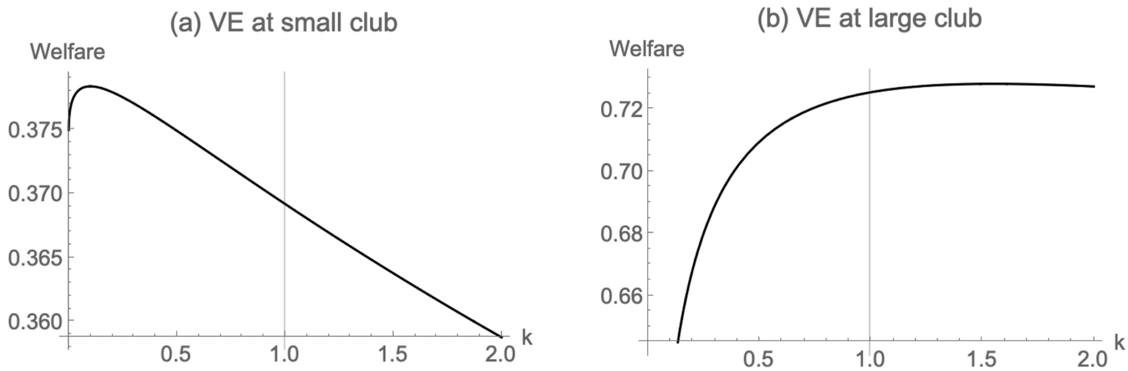

Fig. 5. Effect of relative efficiency on welfare (profit-max).

Conversely, when the small club becomes more efficient, social welfare decreases.

(ii) *Suppose $m > 1$. When the large club becomes less efficient, social welfare decreases. Conversely, when the large club becomes more efficient, social welfare increases until it reaches a maximum at an efficiency level of $k = m$.*

Proof. See Appendix. □

Fig. 5 illustrates the results from the proposition, with Panel (a) showing the effects when efficiency varies at the small club and Panel (b) showing the effects when it varies at the large club. The x-axis again represents relative efficiency, while the y-axis shows the resulting social welfare.

In a league with profit-maximizing clubs, efficiency asymmetries can raise social welfare even when they reduce competitive balance. Specifically, low efficiency at the small club (Panel (a) of Fig. 5) and high efficiency at the large club (Panel (b)) both increase welfare up to a maximum at $k = m$. These results highlight that, in profit-oriented leagues, social welfare does not always move in parallel with balance: situations in which large clubs are highly efficient and small clubs are less efficient can improve overall welfare despite widening the performance gap. This finding is consistent with earlier research showing that in leagues with profit-maximizing clubs, less balanced competition can sometimes enhance social welfare (Dietl and Lang, 2008; Dietl et al., 2009).

In practical terms, Fig. 5 captures a pattern typical of commercially oriented leagues such as the English Premier League, where clubs act as profit-maximizers and financial returns are central to decision-making. When a financially dominant, professionally managed club-such as Manchester United or Tottenham Hotspur-operates very efficiently by aligning coaching, recruitment, and brand strategy, overall welfare can rise even if the title race becomes less even. Highly efficient large clubs lift the league's international profile and media rights income. Symmetrically, if a smaller club operates below the efficiency frontier, its underperformance can, up to a point, still raise aggregate welfare in a profit-oriented setting: resources are concentrated where they generate the highest monetary return. Welfare peaks when the efficiency advantage of large clubs roughly matches their market-size advantage ($k = m$); beyond that point, excessive dominance depresses interest and welfare. Thus, in profit-maximizing leagues, moderate efficiency asymmetries-high efficiency at major clubs and lower efficiency at small clubs-can raise social welfare despite widening performance gaps.

Next, we derive the welfare implications of relative efficiency in a league with win-maximizing clubs.

Proposition 6.

(i) *Suppose $m < 1$. When the small club becomes less efficient, social welfare decreases. Conversely, when the small club becomes more efficient, social welfare increases until it reaches a maximum at an efficiency level of $k = \frac{1+2m}{m(2+m)}$.*

(ii) *Suppose $m > 1$. When the large club becomes less efficient, social welfare increases until it reaches a maximum at an efficiency level of $k = \frac{1+2m}{m(2+m)}$. Conversely, when the large club becomes more efficient, social welfare decreases.*

Proof. See Appendix. □

Fig. 6 illustrates the results of the proposition:

In a league with win-maximizing clubs, the welfare implications are more intuitive. Higher efficiency can benefit social welfare if it occurs at the small club, while lower efficiency can be beneficial if it happens at the large club. In this setting, welfare gains are aligned with more balanced competition: efficiency heterogeneity that favors small clubs and disadvantages large clubs strengthens both balance and social welfare. This complements existing findings that leagues with win-maximizing clubs achieve greater welfare when competitive outcomes are more evenly distributed (Dietl and Lang, 2008; Dietl et al., 2009).

In practical terms, Fig. 6 illustrates a different logic, one more relevant to leagues or clubs that prioritize sporting success over profits-such as FC Barcelona under member ownership, Paris Saint-Germain, or Bayern Munich, whose strategies emphasize winning titles and prestige. In such win-maximizing environments, social welfare rises when efficiency gains occur at smaller clubs, for example through Atalanta's data-driven recruitment, Girona's tactical innovations, or Freiburg's academy model. These efficiency improvements narrow the competitive gap and heighten fan engagement, broadcasting appeal, and league-wide interest. Conversely, temporary inefficiencies at dominant clubs-such as PSG or Bayern during transitional seasons-can also enhance welfare by making championships less predictable. In short, whereas profit-maximizing leagues benefit from efficient large clubs, win-maximizing leagues gain most when efficiency favours the underdogs.

6. Robustness

We conducted several robustness checks to ensure the validity and reliability of our results.

First, we examined the sensitivity of our findings to alternative specifications of the match quality function. While incorporating aggregate talent into the quality function increases model complexity, it does not qualitatively alter the outcomes. The fundamental relationships between efficiency, competitive balance, club profits, and social welfare remain consistent.

Second, our base model assumes that efficiency varies at only one club. To test the robustness of this assumption, we extended the model to allow for efficiency heterogeneity at both clubs. This extension likewise does not qualitatively change our results. The dynamics of higher and lower efficiency and their impacts on competitive balance and social

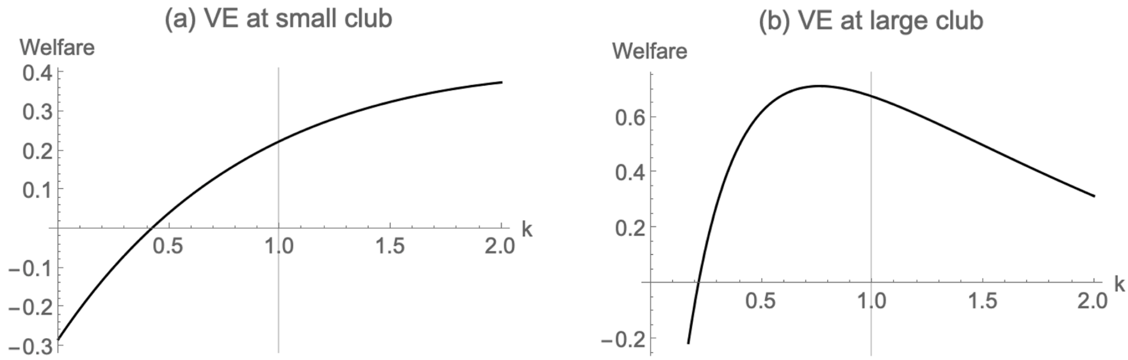

Fig. 6. Effect of relative efficiency on welfare (win-max).

welfare hold regardless of whether one or both clubs exhibit efficiency differences.

Third, we explored variations in market size heterogeneity. By adjusting the market size parameters to reflect different levels of disparity between clubs, we confirmed that our model's core findings remain robust. The effects of relative efficiency on talent demand, competitive balance, and social welfare persist across a wide range of market size distributions.

These robustness checks demonstrate that our findings are not sensitive to specific model assumptions or functional forms, ensuring the credibility of our conclusions across various scenarios and configurations.

7. Conclusion

Despite its importance, the literature in sports economics has largely overlooked the effects of efficiency heterogeneity across clubs. While prior research has focused on individual players or market inefficiencies, the aggregate consequences of systematic efficiency asymmetries at the squad level remain underexplored. This paper addresses this gap by modeling how clubs transform talent into performance with varying degrees of efficiency and how these differences affect strategic behavior and league outcomes. Rather than assuming a deterministic relationship between talent and outcomes, our framework models clubs making consistent but heterogeneous transformations of talent into performance within an uncertain environment.

Four key findings emerge. First, efficiency is not neutral background noise-it systematically influences strategic decisions. Profit-maximizing clubs may scale back investment when efficiency improves, while win-maximizing clubs tend to increase it. Second, efficiency differences affect large and small clubs asymmetrically: large clubs reduce investment when they are highly efficient, whereas small clubs often expand investment under the same conditions. Third, welfare implications depend critically on league orientation: in profit-oriented leagues, welfare improves when small clubs are less efficient and large clubs are more efficient, while the reverse holds in win-oriented leagues. Fourth, our analysis suggests that policies aimed at narrowing persistent efficiency gaps-through data sharing, managerial training, or subsidized analytics infrastructure-could help rebalance competition and improve overall league welfare.

More broadly, our model demonstrates that heterogeneity in organizational efficiency is not inherently detrimental. Under certain conditions, it generates strategic advantages and can even enhance league welfare. This finding challenges the conventional view that efficiency disparities are uniformly harmful and opens new avenues for both research and policy.

This paper contributes to the contest-based literature on sports leagues by jointly incorporating two distinct forms of heterogeneity: differences in *market size* and differences in *efficiency* in transforming talent into performance. While earlier theoretical studies, such as Szymanski (2003) and Dietl et al. (2009), focused primarily on market-size asymmetries, we introduce efficiency asymmetry as an additional and independent source of strategic variation. To the best of our knowledge, this is the first analysis that simultaneously models both dimensions within a unified contest-theoretic framework. This dual-asymmetry approach allows us to uncover novel comparative-static and welfare implications regarding how efficiency heterogeneity interacts with market structure and strategic behavior in professional sports leagues.

Our simplified model has several limitations that could be alleviated by future research. For example, one promising direction is to consider hybrid objectives, where clubs behave as profit-maximizers up to a financial threshold and then pursue win-maximization, reflecting more realistic behavior. Another extension would be to model repeated interactions across seasons, capturing how clubs update their efficiency-enhancing systems and strategic behavior over time. Future work could also extend the analysis to an N-club league. While closed-form solutions are not tractable beyond the two-club case, such an extension would allow for richer forms of heterogeneity and interaction among clubs of different sizes. In addition, incorporating institutional mechanisms such as revenue sharing, salary caps, or player-transfer systems could alter the comparative-static relationships identified here by changing clubs' effective budget constraints and strategic incentives. Finally, our welfare analysis assumes equal weights on club profits, player salaries, and consumer surplus. This simplifying assumption follows standard practice in the contest and sports economics literature and ensures analytical transparency when comparing welfare across league types. However, in reality, different stakeholders may contribute unequally to overall welfare-fan utility, for example, may carry greater social or economic importance than player compensation. Exploring such weighted welfare functions, possibly calibrated to empirical data on fan engagement or broadcasting revenues, represents an important avenue for future research.

In sum, recognizing and incorporating efficiency heterogeneity provides a richer theoretical framework for analyzing professional sports leagues. It not only explains why clubs with similar wage bills achieve different outcomes but also clarifies how these differences shape league competitiveness and welfare.

Declaration of competing interest

The authors declare that they have no competing interests.

Appendix A

A.1. Proof of Lemma 2

In a first step, we show that each club spends all its revenue on playing talent.

Suppose that at an optimum $R_i > ct_i$ (slack budget). Then by complementary slackness $\lambda_i = 0$, and the stationarity condition gives $1 \leq 0$, a contradiction. Thus, if $t_i > 0$, it must hold that:

$ct_i = R_i(t_i, t_j)$.

If $t_i = 0$, then $R_i = 0$, but a small increase in t_i would raise both the objective and R_i, contradicting optimality. Hence, in equilibrium $t_i > 0$ and the budget binds:

$ct_i = R_i(t_i, t_j)$.

In a second step, we are solving for equilibrium talent levels. Using $w_1 = \frac{kt_1}{kt_1 + t_2}$ and $w_2 = 1 - w_1$, the binding budget constraints become:

$$ct_1 = \frac{m}{4}\left(2w_1 - w_1^2\right), \tag{11}$$

$$ct_2 = \frac{1}{4}\left(2w_2 - w_2^2\right) = \frac{1}{4}(1 - w_1^2). \tag{12}$$

From the CSF we have:

$$w_1 = \frac{kt_1}{kt_1 + t_2} \quad \Longrightarrow \quad t_2 = kt_1 \frac{1 - w_1}{w_1}.$$

Substitute this into (12):

$$ckt_1 \frac{1 - w_1}{w_1} = \frac{1}{4}(1 - w_1^2) = \frac{1}{4}(1 - w_1)(1 + w_1).$$

If $w_1 \neq 1$, divide both sides by $(1 - w_1)$:

$$\frac{ckt_1}{w_1} = \frac{1}{4}(1 + w_1).$$

From (11) we have $t_1 = \frac{m}{4c}\left(2w_1 - w_1^2\right) = \frac{m}{4c} w_1(2 - w_1)$. Substitute into the previous equation:

$$\frac{ck}{w_1} \cdot \frac{m}{4c} w_1(2 - w_1) = \frac{1}{4}(1 + w_1) \quad \Rightarrow \quad km(2 - w_1) = 1 + w_1.$$

Solving for w_1 yields:

$$w_1 = \frac{2km - 1}{km + 1}.$$

From (11):

$$t_1 = \frac{m}{4c} w_1(2 - w_1).$$

Since $2 - w_1 = \frac{3}{km + 1}$ and $w_1 = \frac{2km - 1}{km + 1}$, we obtain:

$$t_1^W = \frac{3\,m(2km - 1)}{4c(km + 1)^2}.$$

From (12):

$$ct_2 = \frac{1}{4}(1 - w_1^2) = \frac{1}{4}\frac{(km + 1)^2 - (2km - 1)^2}{(km + 1)^2} = \frac{3km(2 - km)}{4(km + 1)^2}.$$

Hence,

$$t_2^W = \frac{3km(2 - km)}{4c(km + 1)^2}.$$

Together, equilibrium talent levels for win-maximizing clubs are:

$$(t_1^W, t_2^W) = \left(\frac{3\,m(2km - 1)}{4c(km + 1)^2}, \frac{3km(2 - km)}{4c(km + 1)^2}\right).$$

A.2. Proof of Lemma 3

Assume $m > 0$ and $c > 0$. We know

$$t_1^P(k) = \frac{km^3}{2c\left(km^2(3 + km) + \sqrt{km^3}\,(1 + 3km)\right)},$$

$$t_2^P(k) = \frac{k\left[-m(1 + 3km) + (3 + km)\sqrt{km^3}\right]}{2c\,(km - 1)^3}.$$

A convenient change of variables is

$$u := \sqrt{k\,m} \quad \Longleftrightarrow \quad k = \frac{u^2}{m}, \qquad \sqrt{k\,m^3} = mu,$$

with $u > 0$ since $k > 0$.

In a first step, substituting $k = \frac{u^2}{m}$ and $\sqrt{km^3} = mu$ gives

$$t_1^P(k) = \frac{u^2\,m^2}{2c\left[u^2\,m(3 + u^2) + mu(1 + 3u^2)\right]} = \frac{um}{2c\,(u^3 + 3u^2 + 3u + 1)} = \frac{um}{2c\,(u + 1)^3}.$$

Similarly, writing

$$J = -m(1 + 3km) + (3 + km)\sqrt{km^3} = m\left[-1 - 3u^2 + 3u + u^3\right] = m(u - 1)^3,$$

we have

$$t_2^P(k) = \frac{kJ}{2c\,(km - 1)^3} = \frac{\frac{u^2}{m} \cdot m(u - 1)^3}{2c\,(u^2 - 1)^3} = \frac{u^2}{2c\,(u + 1)^3}, \qquad u \neq 1.$$

(The apparent singularity at $u = 1$ is removable.)

Hence, in u-space:

$$t_1^P(u) = \frac{um}{2c\,(u + 1)^3}, \qquad t_2^P(u) = \frac{u^2}{2c\,(u + 1)^3}.$$

In a second step, differentiate with respect to u:

$$\frac{dt_1^P}{du} = \frac{m}{2c} \cdot \frac{(u + 1)^3 - 3u(u + 1)^2}{(u + 1)^6} = \frac{m}{2c} \cdot \frac{1 - 2u}{(u + 1)^4}.$$

Thus

$$\frac{dt_1^P}{du} = 0 \iff u = \frac{1}{2}, \qquad \operatorname{sign}\left(\frac{dt_1^P}{du}\right) = \begin{cases} +, & 0 < u < \frac{1}{2}, \\ 0, & u = \frac{1}{2}, \\ -, & u > \frac{1}{2}. \end{cases}$$

Because $\frac{du}{dk} = \frac{m}{2u} > 0$, the sign of $\partial t_1^P / \partial k$ matches that of $\frac{dt_1^P}{du}$. Hence

$$\frac{\partial t_1^P}{\partial k} < 0 \iff u > \frac{1}{2} \iff k > \frac{1}{4m}.$$

At $u = \frac{1}{2}$,

$$\underline{k}^P = \frac{u^2}{m} = \frac{1}{4m}, \qquad t_1^P(\underline{k}^P) = \frac{2m}{27c}.$$

The sign change from + to - confirms that $\underline{k}^P$ maximizes t_1^P.

For $t_2^P(u) = \frac{u^2}{2c(u+1)^3}$,

$$\frac{dt_2^P}{du} = \frac{(2u)(u + 1)^3 - u^2 \cdot 3(u + 1)^2}{2c\,(u + 1)^6} = \frac{u(2 - u)}{2c\,(u + 1)^4}.$$

Thus

$$\frac{dt_2^P}{du} = 0 \iff u \in \{0, 2\}, \quad \text{and} \quad \operatorname{sign}\left(\frac{dt_2^P}{du}\right) = \begin{cases} +, & 0 < u < 2, \\ 0, & u = 2, \\ -, & u > 2. \end{cases}$$

Hence

$$\frac{\partial t_2^P}{\partial k} < 0 \iff u > 2 \iff k > \frac{4}{m}.$$

At $u = 2$,

$$\overline{k}^P = \frac{u^2}{m} = \frac{4}{m}, \qquad t_2^P(\overline{k}^P) = \frac{2}{27c},$$

and the sign change $+ \to -$ implies $\overline{k}^P$ maximizes t_2^P.

In a third step, since $m > 0$,

$$\underline{k}^P = \frac{1}{4m} < \frac{4}{m} = \overline{k}^P.$$

A.3. Proof of Proposition 1

To prove Proposition 1, we first compute:

$$\underline{k}^P > 1 \Leftrightarrow m < \frac{1}{4}.$$

and derive the following results:

(i) If $m < \frac{1}{4}$, a lower efficiency at club 1 will always induce this club to reduce its talent demand. A higher efficiency at club 1 will induce this club to increase its talent demand until the maximum is reached for $k = \underline{k}^P > 1$.
(ii) If $m > \frac{1}{4}$, a lower efficiency at club 1 will induce this club to increase its talent demand until the maximum is reached for $k = \underline{k}^P < 1$. A higher efficiency at club 1 will always induce this club to reduce its talent demand.

The behavior of the other club (club 2) in response to efficiency variations at club 1 also depends on the magnitude of the efficiency change and the market size. If $k > \overline{k}^P = \frac{4}{m}$, then $\frac{\partial t_2^P}{\partial k} < 0$, i.e., increasing efficiency decreases the talent demand of club 2. We derive:

$$\overline{k}^P > 1 \Leftrightarrow m < 4.$$

and derive the following results:

(i) If $m < 4$, a lower efficiency at club 1 will induce the other club (club 2) to reduce its talent demand. A higher efficiency at club 1 will induce the other club (club 2) to increase its talent demand until the maximum is reached for $k = \overline{k}^P > 1$.
(ii) If $m > 4$, a lower efficiency at club 1 will induce the other club (club 2) to increase its talent demand until the maximum is reached for $k = \overline{k}^P < 1$. A higher efficiency at club 1 will induce the other club (club 2) to reduce its talent demand.

By combining the results above, we derive the conclusions presented in Proposition 1.

A.4. Proof of Lemma 4

Let $t_1^W(k,m)$ and $t_2^W(k,m)$ be given by *(11)* and define the positive constant $K \equiv \frac{3m}{4c} > 0$.

In a first step, for club 1, using the product rule,

$$\begin{aligned}\frac{\partial t_1^W}{\partial k} &= K\left[2m(km+1)^{-2} - 2m(2km-1)(km+1)^{-3}\right]\\ &= K \cdot 2m\,(km+1)^{-3}\left[(km+1) - (2km-1)\right]\\ &= K \cdot 2m\,(km+1)^{-3}\,(2-km).\end{aligned}$$

Hence

$$\operatorname{sign}\left(\frac{\partial t_1^W}{\partial k}\right) = \operatorname{sign}(2-km), \tag{13}$$

so $\frac{\partial t_1^W}{\partial k} > 0$ iff $km < 2$ and $\frac{\partial t_1^W}{\partial k} < 0$ iff $km > 2$.

For club 2, using the product rule,

$$\begin{aligned}\frac{\partial t_2^W}{\partial k} &= K\left[2(1-km)(km+1)^{-2} - 2m(2k-k^2 m)(km+1)^{-3}\right]\\ &= K \cdot 2\,(km+1)^{-3}\left[(1-km)(km+1) - m(2k-k^2 m)\right]\\ &= K \cdot 2\,(km+1)^{-3}\,(1-2km).\end{aligned}$$

Hence

$$\operatorname{sign}\left(\frac{\partial t_2^W}{\partial k}\right) = \operatorname{sign}(1-2km), \tag{14}$$

so $\frac{\partial t_2^W}{\partial k} > 0$ iff $km < \frac{1}{2}$ and $\frac{\partial t_2^W}{\partial k} < 0$ iff $km > \frac{1}{2}$.

In a second step, from (13), $k = \underline{k}^W \equiv 2/m$ is the only interior point where $\partial t_1^W/\partial k = 0$. Differentiate $\partial t_1^W/\partial k$ once more:

$$\begin{aligned}\frac{\partial^2 t_1^W}{\partial k^2} &= K \cdot 2m\,\frac{\mathrm{d}}{\mathrm{d}k}\left[(km+1)^{-3}(2-km)\right]\\ &= K \cdot 2m\left[-3m(2-km)(km+1)^{-4} - m(km+1)^{-3}\right].\end{aligned}$$

Evaluated at $k = \underline{k}^W$ (i.e. $km = 2$), the first term vanishes and the second term is strictly negative:

$$\left.\frac{\partial^2 t_1^W}{\partial k^2}\right|_{k=2/m} = K \cdot 2m\left[-m(2+1)^{-3}\right] < 0.$$

Thus $k = \underline{k}^W$ is a strict local maximizer of $t_1^W(k)$. Combined with the sign pattern in (13) (increasing for $km < 2$, decreasing for $km > 2$), this critical point is the *unique global* maximizer on $k > 0$.

Similarly, from (14), $k = \overline{k}^W \equiv 1/(2m)$ is the only interior point where $\partial t_2^W/\partial k = 0$. Differentiating $\partial t_2^W/\partial k$ and evaluating at $k = \overline{k}^W$ (i.e. $km = \frac{1}{2}$) yields

$$\left.\frac{\partial^2 t_2^W}{\partial k^2}\right|_{k=1/(2m)} = K \cdot 2\left[0 - 2m\left(\frac{3}{2}\right)^{-3}\right] < 0,$$

so $k = \overline{k}^W$ is a strict local maximizer. Using the monotonicity in (14) (increasing for $km < 1/2$, decreasing for $km > 1/2$), it is the *unique global* maximizer on $k > 0$.

In a third step, since $m > 0$, we have $\underline{k}^W = \frac{2}{m} > \frac{1}{2m} = \overline{k}^W$.

A.5. Proof of Proposition 2

To prove Proposition 2, we derive that if $k > \underline{k}^W = \frac{2}{m}$, then $\frac{\partial t_1^W}{\partial k} < 0$, meaning that increasing efficiency decreases the talent demand of club 1. We obtain

$$\underline{k}^W > 1 \Leftrightarrow m < 2.$$

Thus, if efficiency varies at the small club ($m < 1$), we know that $\underline{k}^W > 1$. We derive the following results:

(i) If $m < 2$, a lower efficiency at club 1 will always induce this club to reduce its talent demand. A higher efficiency at club 1 will induce this club to increase its talent demand until the maximum is reached for $k = \underline{k}^W > 1$.
(ii) If $m > 2$, a lower efficiency at club 1 will induce this club to increase its talent demand until the maximum is reached for $k = \underline{k}^W < 1$. A higher efficiency at club 1 will always induce this club to reduce its talent demand.

Similarly, the behavior of the other club (club 2) in response to efficiency variations at club 1 also depends on the magnitude of the efficiency change and the market size. If $k > \overline{k}^W = \frac{1}{2m}$, then $\frac{\partial t_2^W}{\partial k} < 0$, meaning that increasing efficiency decreases the talent demand of club 2. We compute

$$\overline{k}^W > 1 \Leftrightarrow m < \frac{1}{2}.$$

and derive the following results:

(i) If $m < \frac{1}{2}$, a lower efficiency at club 1 will induce the other club (club 2) to reduce its talent demand. A higher efficiency at club 1 will induce the other club (club 2) to increase its talent demand until the maximum is reached for $k = \overline{k}^W > 1$.
(ii) If $m > \frac{1}{2}$, a lower efficiency at club 1 will induce the other club (club 2) to increase its talent demand until the maximum is reached for $k = \overline{k}^W < 1$. A higher efficiency at club 1 will induce the other club (club 2) to reduce its talent demand.

By combining the results from above, we derive the conclusions presented in Proposition 2.

A.6. Proof of Proposition 3

To prove Proposition 3, we consider the case of win-maximizing clubs. The proof for profit-maximizing clubs is similar.

Recall that the equilibrium win percentages in a league with win-maximizing clubs are given by:

$$(w_1^W, w_2^W) = \left(\frac{2km-1}{km+1}, \frac{2-km}{km+1}\right).$$

We compute

$$CB(t_1^W, t_2^W) = \frac{(2-km)(2km-1)}{(km+1)^2}$$

and

$$\frac{\partial CB(t_1^W, t_2^W)}{\partial k} \Leftrightarrow k^{CB} = \frac{1}{m}$$

The results from Proposition 3 follow by noting that

$$k^{CB} = \frac{1}{m} > 1 \Leftrightarrow m < 1.$$

and the fact that the second-order conditions for a maximum are satisfied.

A.7. Proof of Proposition 4

In a league with profit-maximizing clubs, profits are given by

$$\pi_i(t_1, t_2) = R_i(t_1, t_2) - C(t_1),$$

with $i = 1, 2$.

By plugging equilibrium talent demand (t_1^P, t_2^P) into the profit function, we obtain

$$\pi_1(t_1^P, t_2^P) = \frac{km^2\left(km^2(1+3km)+3\theta\right)}{4(1+3km)\left(km^2(3+km)+\theta\right)} \tag{15}$$

$$\pi_2(t_1^P, t_2^P) = \frac{-1-8k\theta+3km(1+km(7+3km))}{4(-1+km)^3(1+3km)}, \tag{16}$$

with $\theta = \sqrt{km^3(1+3km)^2}$. We compute

$$\frac{\partial \pi_1(t_1^P, t_2^P)}{\partial k} = \frac{3km^4(m(1+km)(1+3km)+2\theta)}{4(1+3km)\left(km^2(3+km)+\theta\right)^2} > 0,$$

$$\frac{\partial \pi_2(t_1^P, t_2^P)}{\partial k} = \frac{12\theta - 3\,m(1-4k\theta+km(9+km(19+3km)))}{4(-1+km)^4(1+3km)} < 0,$$

with $\theta = \sqrt{km^3(1+3km)^2}$.

The claim from the proposition follows by noting that the second-order conditions for a maximum are satisfied.

A.8. Proof of Proposition 5

In a league with profit-maximizing clubs, social welfare is given by

$$W(t_1, t_2) = \frac{3}{8}\left[m_1 q_1(t_1, t_2) + m_2 q_2(t_1, t_2)\right].$$

By plugging equilibrium talent demand (t_1^P, t_2^P) into the welfare function, we obtain

$$W(t_1^P, t_2^P) = \frac{3+6\theta+6k\theta+3km^2(-3+k(-9+m(-8+3km)))}{8(-1+km)^2(1+3km)},$$

with $\theta = \sqrt{km^3(1+3km)^2}$.

We compute

$$\frac{\partial W(t_1^P, t_2^P)}{\partial k} = 0 \Leftrightarrow k = m.$$

The claim from the proposition follows by noting that the second-order conditions for a maximum are satisfied.

A.9. Proof of Proposition 6

In a league with win-maximizing clubs, social welfare in the equilibrium (t_1^W, t_2^W) is given by

$$W(t_1^W, t_2^W) = -\frac{9m(1-2k+(-2+k)km)}{8(1+km)^2}.$$

We compute

$$\frac{\partial W(t_1^W, t_2^W)}{\partial k} = 0 \Leftrightarrow k = \frac{1+2\,m}{m(2+m)}.$$

The claim from the proposition follows by noting that the second-order conditions for a maximum are satisfied.

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