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Contrasting Trends in Firm Volatility†

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Over the past decades, the real and financial volatility of listed firms has increased, while the volatility of private firms has decreased. We first provide panel data evidence that, at the firm level, sales and employment volatility are impacted by changes in the degree of ownership concentration. We then construct a model with private and listed firms where risk-taking is a choice variable at the firm-level. Due to general equilibrium feedback, we find that both an increase in stock market participation and integration in international capital markets generate opposite trends in volatility for private and listed firms. (*JEL* G15, G32, L25)

Over the past decades, the idiosyncratic volatility of publicly listed US firms has risen considerably, be it computed from real or financial variables.[1] Existing explanations focus on product market competition.[2] Indeed the United States, like many other countries over the past 30 years, has experienced profound deregulation of many industries, a dramatic increase in international competition, and an acceleration in the pace of innovation on product markets. Yet privately held firms have experienced the opposite movement. In a recent paper, Steven J. Davis et al. (2007) show that employment growth volatility of nonlisted firms has decreased by about 50 percent between the early 1980s and the late 1990s. In our French census data, we find similar evolutions (see Figure 1). The volatilities of listed and private firms have evolved in opposite directions, a fact that competition-based theories cannot explain.

This paper proposes an explanation for these contrasting trends. Our starting point is that, at the firm level, risk-taking is a choice variable affected by risk sharing among shareholders (Elhanan Helpman and Assaf Razin 1978; Gilles Saint-Paul 1992; Maurice Obstfeld 1994). Recently the degree of risk sharing among owners of

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[1] John Y. Campbell et al. (2001) find that the volatility of stock returns, after filtering out aggregate shocks, has been multiplied by four. The ten year rolling standard deviation of sales or employment growth has doubled between 1955 and 2000 (Diego Comin and Thomas Philippon 2006, Paul J. Irvine and Jeffrey Pontiff 2009). Compared to the 1950s, industry leaders are now three times more likely to lose their preeminence.

[2] See Thesmar and Thoenig (2000), Comin and Philippon (2006), José-Miguel Gaspar and Massimo Massa (2006), and Irvine and Pontiff (2009).

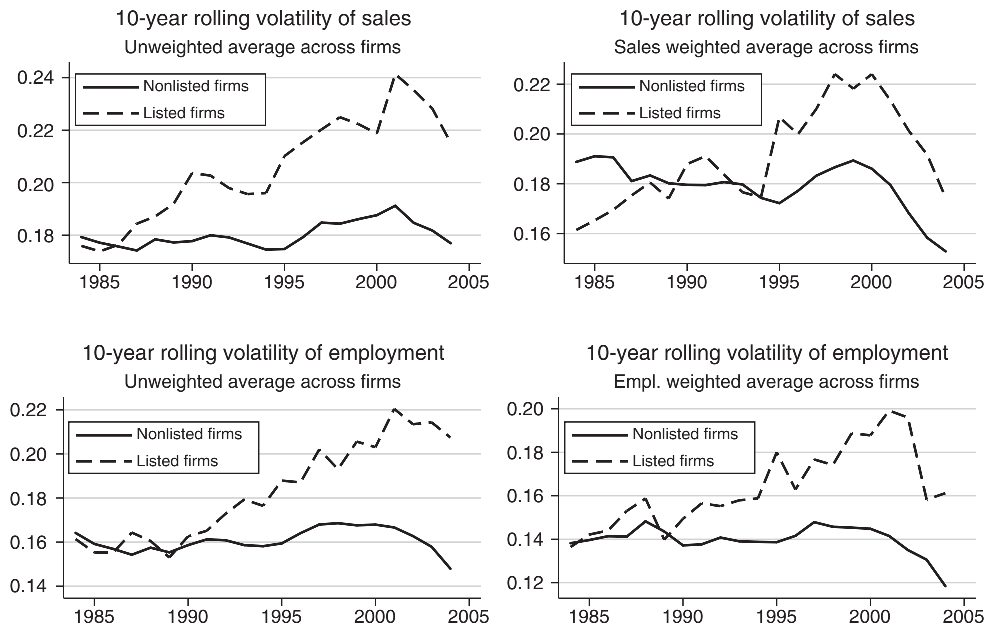

Figure 1. Trends in French Firm Volatility

listed firms has risen dramatically because of increased stock market participation (Luigi Guiso, Michael Haliassos, and Tullio Jappelli 2003), a rise in institutional ownership (Kenneth R. French 2008), and international capital market integration (Anusha Chari and Peter Blair Henry 2004). Against this background, publicly listed firms have taken on more operating risk by adopting ambitious, but risky, projects. This could explain the pattern of rising volatility for listed firms, but could this also lead to a *decrease* in volatility of privately held firms? Our theoretical analysis addresses this question.

To this purpose, we provide a model of endogenous risk-taking by listed and private firms in the presence of aggregate and idiosyncratic uncertainty.[3] Listed firms are owned by diffused shareholders (the "investors"), while private firms have only one owner (the "entrepreneur") who therefore bears all of idiosyncratic risk. Both listed and nonlisted firms are able to interact in our model despite them having distinct shareholders. They compete on the labor market and on product markets (which are segmented à la Dixit Stiglitz). In this setup, our main comparative static exercise relates to a increase in stock market participation. Following an exogenous increase in the number of investors, we show that investors' effective tolerance to risk increases and that public firms respond by becoming more volatile and, on average, more productive. The demand for production factors increases and this props

[3] In our model, risk-taking affects the firm-level demand curve only. It is clear that risk-taking also affects the firm-level supply curve. From a theoretical perspective, both channels deliver very similar results, and we thus choose to consider only the first channel. As a consequence, our analysis remains silent on productivity evolution. However, it is clear that our theoretical mechanism and our empirical results are perfectly compatible with the technology view elaborated in Comin and Sunil Mulani (2005) and Comin and Philippon (2006) (who show that sales per worker growth has also experienced an increase in volatility for listed firms).

up factors real prices. This negatively impacts the profits of privately held firms, and entrepreneurs become poorer. Since entrepreneurs have decreasing absolute risk aversion,[4] their risk tolerance decreases and private firms reduce their risk-taking. In summary, an increase in stock market participation generates two contrasting trends in firm volatility.

To gain tractability, we then study a log-linearized version of our model, assuming small aggregate and idiosyncratic shocks. This approach allows us to show that in spite of constrasting changes in their volatilities, listed and private firms both experience a drop in their real profits. Even though listed firms become more efficient, on average, competition prevents them from passing these gains to their shareholders. We also find that the utilities of investors and private entrepreneurs decline while workers' utilities go up.

Finally, we use our approximated model to generate an additional comparative static related to the so-called "Great Moderation." In our model, such a decrease in aggregate uncertainty is unable to generate the dual trend in firm volatility that we seek to explain. In our last extension, we also study the impact of international capital market integration. We do this because financial integration is viewed by some economists as a more significant source of risk sharing among investors than stock market participation (see discussion in Guiso, Haliassos, and Jappelli 2003). For instance, Chari and Henry (2004) show that stock market liberalizations increase stock prices by about 15 percent, of which 6.8 percent can be attributed to improved risk sharing. To this purpose, we extend our baseline model to the case with two countries and imperfectly correlated aggregate shocks. We find that capital market integration is able to generate contrasting trends in firm volatility by enhancing risk sharing among investors.

Beyond providing a unified rationale behind the opposite trends in firm volatility, our model also contributes to the literature on firm risk-taking and capital market development (Saint-Paul 1992, Obstfeld 1994, Daron Acemoglu and Fabrizio Zilibotti 1997). First, we augment this class of models by introducing firms that do not access capital markets and by explicitly modelling imperfect competition. Our model, in contrast to Saint-Paul (1992) or Obstfeld (1994), does not have the unsettling property that countries with more developed or integrated financial markets should display more aggregate output uncertainty (but less income fluctuations) as they specialize. It is indeed a well-established fact that more developed economies have lower gross domestic product (GDP) growth volatility (see Miklos Koren and Silvana Tenreyro 2007 for a discussion).

Before developing the model, we present in Section I some suggestive evidence based on a large panel of French firms active over the 1984–2004 period. Like
AQ 1 Davis, Haltiwanger, and Jarmin (2007), we find evidence of opposite trends in idiosyncratic volatility. The volatility of listed firms increases strongly, while the volatility of nonlisted firms declines. We also provide micro-level evidence that firms with more diversified shareholders tend to become more volatile. Given the influence of the theories referred to above, such evidence is surprisingly scarce in the empirical

[4] We assume that agents have CRRA utility in final consumption.

literature. Using a sample of publicly listed firms, David Sraer and Thesmar (2007) find that family firms are less volatile than nonfamily firms. In their sample of privately held Italian companies, Claudio Michelacci and Fabiano Schivardi (2008) find that the dispersion of productivity is smaller among family firms, a fact that they interpret as indicative of lower volatility. Such existing evidence is cross-sectional only and is therefore subject to strong omitted variable biases. Here, we exploit the panel dimension of our data in order to identify the within firm correlation between *changes* in shareholder concentration and *changes* in firm-level volatility. This allows us to test successfully our core theoretical assumption, namely that risk-taking is a decision variable at the firm level, and that risk-taking depends on the degree of ownership concentration. Our finding is confirmed in a very recent paper by Sebnem Kalemli-Ozcan, Bent Sorensen, and Vadym Volosovych (2010) who find a significant and positive effect of foreign ownership on firm-level volatility in a panel of European firms over the 1990–2006 period.

Existing literature based on US evidence shows that the selection process governing entry into the set of listed firms shifted toward riskier firms after 1979. This shift continued and intensified through the late 1990s (Eugene F. Fama and French 2004; Davis, Haltiwanger, and Jarmin 2007). With a relative influx of increasingly risky listed firms, the composition of the population of listed firms relative to the population of nonlisted firms changed dramatically. This composition effect contributed in an important way to the observed upward trend in the average volatility of listed firms in the United States. In this paper, we study an additional and complementary mechanism by focusing our firm-level analysis on endogenous risk-taking within a given listing status. For the purpose of the analysis, we abstract from any composition effect. By assumption the listing margin is exogenously fixed and firms can only adjust their risk-taking margin. Our approach is also motivated by evidence from French data, where the selection process into the set of listed firms has experienced a historical change that differs from the change observed in the United States.[5] However, we believe that in reality both margins matter, and we left for future work the building of a unified theory, where firms may adjust simultaneoulsy their level of risk-taking and their listing status.

The rest of the paper is devoted to the theoretical analysis. In Section II, we present the closed economy model and derive some comparative static properties. In Section III, we make the assumption that aggregate and idiosyncratic shocks are small, and linearize the model. This allows us to present additional properties of the equilibrium and comparative statics. In Section IV, we present the linearized model with capital market integration. We conclude in Section V.

[5] In many developed countries, the fraction of the economy that is listed on the stock market has significantly increased over the last three decades. For instance, Fama and French (2004) report that the number of new listings in the United States rose from 156 per year in the 1970s to 549 per year in the 1980–2001 period. This rise in the annual number of IPOs can be related to the creation of NASDAQ in the mid-1970s. Similar reforms were undertaken in European countries, as with the creation of the Second Marché and the Nouveau Marché in France, the Frankfurter Neue Markt in Germany, and the London AIM. These reforms had the effect of increasing the number of publicly held firms in the 1990s, albeit to a smaller extent than in the United States. Taking a longer view, the picture reverses itself in continental Europe. For instance, the number of listed firms in France decreases from about 30 per million inhabitants in the 1950s to only 15 in 2000.

I. Firm Level Volatility: Trends and Determinants

This section presents evidence based on French data. We describe our data in Section IA and provide evidence on the opposite trends of volatility for listed and private firms in Section IB. Finally, in Section IC, we successfully test the mechanism at the core of our theory by showing that in the panel dimension a *decrease* in ownership concentration is accompanied by an *increase* in firm-level idiosyncratic volatility.

AQ 2

A. *Data*

Our sample is composed of all French firms, active at some point between 1984 and 2004, that were never state owned,[6] and whose total sales exceed 30 million euros or whose labor force exceeds 500 employees for *at least three years* during the period. Each of these firms is tracked throughout the period. This leads to an unbalanced panel, that contains all large and many medium sized businesses in the French economy, be they privately held or publicly listed. This sample selection procedure ensures that our ownership data (see below) provide exhaustive coverage, which is necessary to measure both listed status and ownership concentration. Hence, our sample is not representative of the typical French firm, which has about one employee.[7]

For all these firms, information is gathered from two sources: accounting data and ownership data. Accounting data come from tax files used by the Ministry of Finance to collect corporate tax (BRN data) available from 1978 to 2004. The BRN data represents the universe of all French firms with more than 1 million euros of annual turnover. In terms of variables, BRN provides us with the balance sheet, profit statement, and employment record of these firms. This data source is used to construct three variables: the 4-digit industry, total employment, and total turnover. We end up with 148,789 observations (some 5,722 firms per year), corresponding to 9,294 different firms, since there is both entry to and exit from the sample.

Ownership information is obtained from the Financial Relation Survey (LIFI in French), conducted each year from 1984 to 2004 by the French Statistical office. Only 64,275 observations (some 3,200 firms per year) can be found in LIFI. From this survey, we build two variables. The first one, *LIST*, is a dummy equal to one when the firm belongs to a publicly listed business group or is itself listed on the French stock market. For business group membership, we define as the group leader the firm that owns, directly or indirectly, at least 50 percent of a given firm's equity. This information is retrieved using LIFI survey data and, for indirect ownership, an algorithm developed at the statistical office. We then checked by hand whether the firm itself or its group leader was listed on the French stock market for each year between 1984 and 2004. In our 1984–2004 sample, these directly or indirectly listed firms make up 13 percent of all observations, which corresponds, on average,

[6] This screen automatically removes privatized firms.
[7] However, it covers a large fraction of total private employment in France (about 20 percent).

to 815 firms out of 5,972 firms each year. Most of these firms are indirectly listed through their group leader.

Our second variable based on the LIFI survey is the *INDIV* dummy, which measures ownership concentration. From LIFI, we get, for each firm, the fraction of equity that is held by French individuals, foreign individuals, French firms, foreign firms, employees, the state, and "unknown" companies or persons. *INDIV* is equal to 1 when the self-reported fraction of equity held by French individuals is above 50 percent. This variable is not always reported, so the number of observations for which we have such information is 64,275 (3,213 observations per year). Given that the LIFI survey asks for ownership by "known" individuals, who cannot be too numerous, we believe that *INDIV* properly captures the set of firms that are controlled by a restricted set of shareholders.

B. *Trends in Firm-Level Volatility*

Our first measure of firm-level volatility, $rolvol_{it}$, is the rolling window standard deviation of sales growth, which is commonly used in the literature.[8] First, growth rates are computed following Davis et al. (2007):

$$g_{it} = 2\left(\frac{sales_{it} - sales_{it-1}}{sales_{it} + sales_{it-1}}\right). \tag{1}$$

We use the same formula for employment growth. From this definition, growth is bounded above by 2 and below by -2. We do not need to windsorize these growth rates. To compute g_{it}, we do require, however, that firm i is present in the data at t and $t + 1$, and therefore exclude entries $(g_{it} = 2)$ and exits $(g_{it} = -2)$ from our sample. [9]

We now define firm-level volatility. For firm i at date t, with a growth rate of g_{it}, we define rolling volatility as

$$rolvol_{it} = \left(\frac{N_{it}}{N_{it} - 1}\right)^{1/2} \left(\frac{1}{N_{it}} \sum_{t+6>t'>t-5} (g_{it'})^2 - \left(\frac{1}{N_{it}} \sum_{t+6>t'>t-5} g_{it'}\right)^2\right)^{1/2},$$

where N_{it} represents the number of observations for which g_{it} is defined between $t - 4$ and $t + 5$. This measure does not require the firm to have growth rates for ten years in a row around date t. In particular, it allows us to compute volatilities through to the end of the sample (2004), even though estimates of volatility there will be noisier.

[8] For recent empirical studies using this kind of rolling window measure, see Comin and Philippon (2006), Comin and Mulani (2006), and Davis et al. (2007). More generally, the literature has used two different measures of firm volatility: with the standard deviation of the time series performance of a firm over a (rolling) window and with the cross-sectional dispersion of firm performance across firms. The evolution of these two measures may in principle be different. The time-series volatility measure is better in that it removes the average growth rate of the firm and, hence, eliminates the bias in the evolution of firm volatility caused by a change in the distribution of the firm's growth potential.

[9] We do this because our data does not allow a good treatment of entry and exit. For the fairly large firms that we consider here, most of these movements correspond to divestitures, acquisitions, or even, most of the time, plain organizational restructuring.

Last, we compute, for each year, the mean of $rolvol_{it}$ separately for the groups of listed and nonlisted firms. We first compute equal-weighted average volatilities. We also compute sales-weighted (using lagged sales, $sales_{it-1}$) average sales volatility, and past-employment-weighted average employment volatility. In computing weighted averages, we take out the top 1 percent of the sales (or employment) distribution, to remove the disproportionate role of a few very large firms.[10] The evolution of volatility for all four measures is reported in Figure 1. Over the sample period, sales growth volatility of listed firms increases from 18 to 22 percent. For privately held firms, the pattern depends on the weighting scheme. The equal-weighted average volatility is essentially flat over the period, while the size-weighted average trends downward, from 19 to 16 percent. In both cases, the two trends diverge in an economically sizable and statistically significant way. The very same pattern is present when we look at employment volatility, with the comforting difference that employment volatility is somewhat smaller than sales volatility. All in all, these trends are smaller than in the US-based study by Davis et al. (2007, table 2), who find a decline of 19 percentage points in private-firm volatility and an increase by 11 percentage points in listed firm volatility. One possible reason is that they work on a representative sample of US firms, while, for data availability constraints, we have to focus on rather large French firms.

C. *Ownership Concentration and Firm-Level Volatility*

In this section, we look at the relation between ownership concentration and volatility. We implement the methodology used by Donald P. Morgan, Bertrand Rime, and Philip E. Strahan (2004) and Rui Castro, Gian Luca Clementi, and Glenn MacDonald (2009), who proxy volatility with the absolute difference between actual and expected sales growth. The advantage of this methodology is that it provides a volatility estimate every year. This allows us to fully exploit the panel dimension of our dataset and control for fixed unobserved variables.

In a first-stage, we run the following regression for firm i at date t:

$$g_{it} = \alpha_s + \delta_t + LISTED_{it} + INDIV_{it} + \log(sales_{it-1}) + \varepsilon_{it}, \tag{2}$$

where sales growth g_{it} is the left-hand-side variable computed as in equation (1). α_s is an industry fixed effect and δ_t is a year dummy designed to capture aggregate volatility. We estimate (2) through OLS and retrieve $|\hat{\varepsilon}_{it}|$, the absolute deviation of sales growth from its conditional mean. This will be our estimate of the volatility of firm i in year t.

In a second-stage, we estimate the following equation:

$$|\hat{\varepsilon}_{it}| = \alpha_i + \delta_t + OC_{it} + \log(sales_{it-1}) + \nu_{it}, \tag{3}$$

[10] The top 1 percent of the size distribution contributes to as much as 20–30 percent of the weighted averages, which is why we chose to remove these firms. Doing so delivers somewhat smoother estimates but does not qualitatively affect the results.

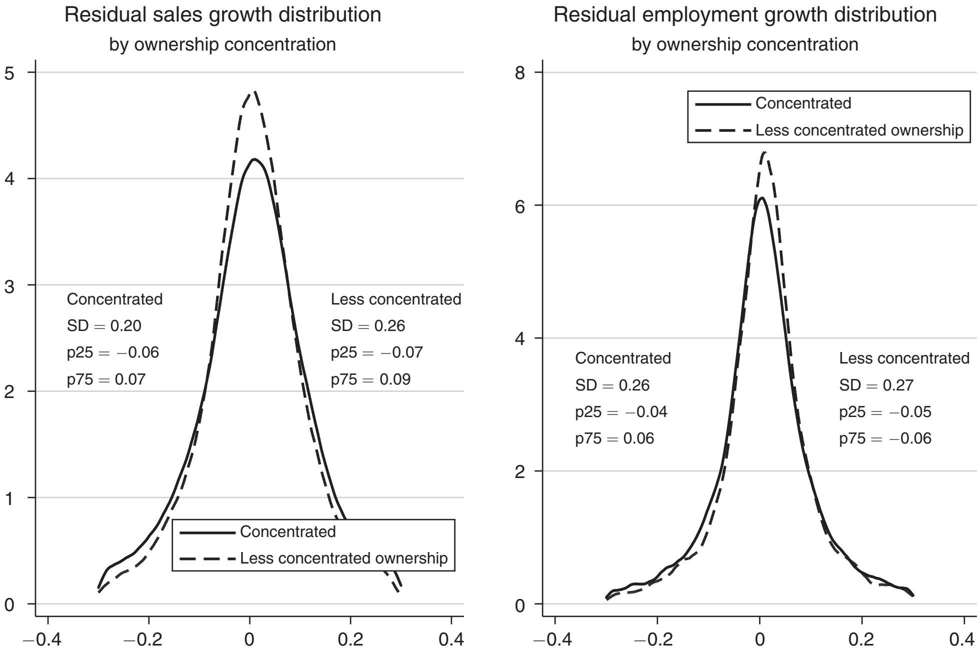

Figure 2. Sales Growth Dispersion by Level of Ownership Concentration

where γ_i is a firm fixed effect, δ_t is a year dummy, and ν_{it} is an error term that we allow to be correlated across observations within each firm (we cluster at the firm level). OC_{it} is the measure of ownership concentration. We also use two alternative measures of ownership concentration: the *INDIV* variable of individual ownership, and the *LISTED* variable.

Before running the regressions, we show that volatility is higher, in the cross section, for firms with less concentrated ownership. In Figure 2, we plot the distribution of residual growth $\hat{\varepsilon}_{it}$ according to the *INDIV* variable. It is quite apparent that the distributions of residual employment and sales growth are flatter for firms that report less than 50 percent of known individual shareholders. In Figure 3, we repeat the same exercise for listed and nonlisted firms. Here too, nonlisted firms seem to have a tighter distribution of growth residuals around their mean, but the difference is less eye-catching.

To quantify and statistically assess the effect of ownership concentration, we now turn to estimates of various specifications of equation (3), reported in Table 1. Let us first look at estimates using the *INDIV* variable (panels A and B). Panel A shows the estimate of sales growth volatility, while panel B reproduces the exercise for employment growth. The specification in column 1 (panels A and B) is estimated without firm fixed effect, and is therefore mostly identified in the cross-section of firms. In this sense, it tests the graphical intuition presented in Figure 2. Mean absolute sales growth residuals are smaller by about 3.5 percentage points for nonlisted firms than for listed firms. This estimate is not only significant statistically, but it is

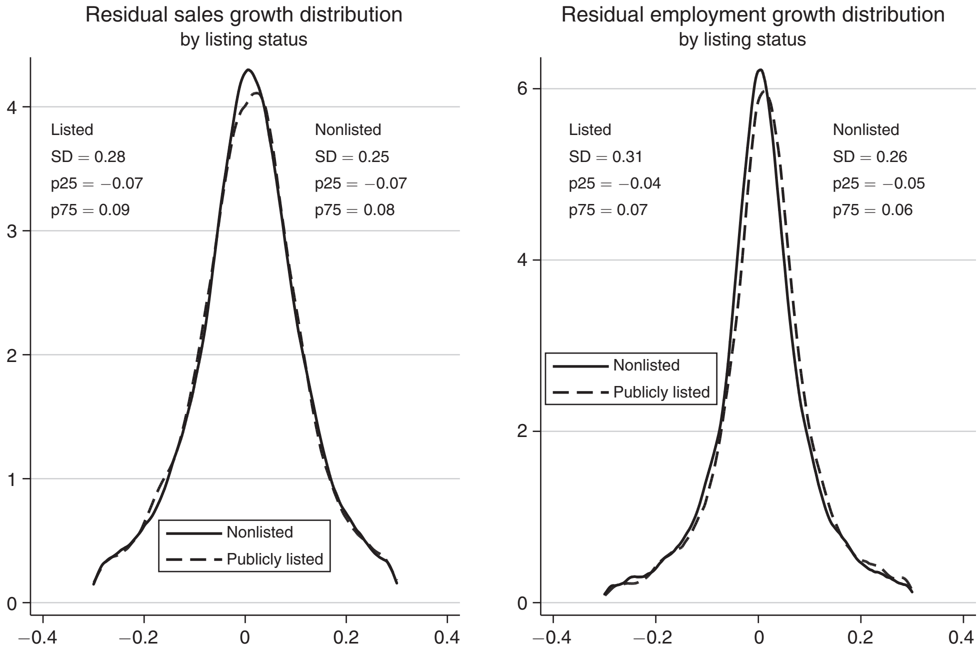

Figure 3. Sales Growth Dispersion for Listed and Nonlisted Firms

also economically large. The sample mean stands at around 14 percent, and the sample standard deviation is about 20 percent. For employment the difference is a bit smaller but still statistically significant—1.4 percent compared to a sample mean of 12 percent and standard deviation of 24 percent. Column 2 controls by firm size (as measured by $\log(sales_{it-1})$) to filter out some of the heterogeneity, but this does not affect our estimates. Column 3 adds firm fixed effects. In this new specification, ownership concentration effect is identified for firms that transit between the state *INDI* $V_{it} = 1$ (being controlled by individuals) and the state $INDIV_{it} = 0$. On the cross-
AQ 3 sectional sample of 2,254 firms (60,120 observations), only 350 firms change of $INDIV_{it}$ over the period. In spite of this demanding identification strategy, these panel estimates are statistically significant at 1 percent. The FE estimates have an order of magnitude of 2 percentage points, or about 10 percent of the sample standard deviation of employment or sales growth.

Panels C and D of Table 1 replicate the exercise using the *LISTED* dummy as explanatory variable. Our findings confirm the pattern observed in Figure 3 as we estimate that listed firms have more volatile sales and employment growth. The difference is about 2 percentage points in the cross section and statistically significant at the 1 percent level. However, the magnitude is smaller than in the specifications of panels A and B, and, once we include fixed effects, the pattern vanishes.

All in all, we believe these estimates offer supporting, though preliminary, evidence to our core theoretical assumption, namely that risk-taking is a decision variable at the firm level and depends on the degree of ownership concentration.

TABLE 1—OWNERSHIP CONCENTRATION AND RISK-TAKING

	Model 1	Model 2	Model 3
Panel A. Dependent variable = sales growth residual ($\times$ *100*)			
Direct ownership of known individuals > 50 percent	−3.5***	−3.8***	−2.4**
	(11.4)	(12.1)	(4.6)
log(sales)	—	−1.4***	5.5***
		(9.1)	(11.4)
Year FE	Yes	Yes	Yes
Firm FE	No	No	Yes
Observations	59,119	59,119	59,119
Panel B. Dependent variable = employment residual ($\times$ *100*)			
Direct ownership of known individuals > 50 percent	−1.3***	−1.5***	−2.4***
	(3.1)	(3.5)	(3.3)
log(sales)	—	1.0***	−4.6***
		(5.6)	(8.5)
Year FE	Yes	Yes	Yes
Firm FE	No	No	Yes
Observations	58,067	58,067	58,067
Panel C. Dependent variable = sales growth residual ($\times$ *100*)			
Publicly listed	1.5***	1.8***	0.4
	(4.5)	(5.4)	(0.8)
log(sales)	—	−1.4***	−5.4***
		(9.1)	(11.4)
Year FE	Yes	Yes	Yes
Firm FE	No	No	Yes
Observations	59,119	59,119	59,119
Panel D. Dependent variable = employment residual ($\times$ *100*)			
Publicly listed	1.6***	1.9***	0.6
	(4.3)	(4.9)	(1.0)
log(sales)	—	−1.0***	−4.6***
		(5.8)	(8.5)
Year FE	Yes	Yes	Yes
Firm FE	No	No	Yes
Observations	58,067	58,067	58,067

Notes: The dependent variable is obtained through the following procedure. First, we run

$$g_{it} = \alpha_s + \delta_t + LISTED_{it} + INDIV_{it} + \log(SALES_{it-1}) + \varepsilon_{it}.$$

α_s and δ_t are industry and year dummies, and $INDIV_{it} = 1$ if the firm has more than 50 percent of known shareholders. Then, we take $\nu_{it} = |\varepsilon_{it}|$ as our measure of volatility, which we regress either on $INDIV_{it}$ (panels A and B) or on the listed dummy $LISTED_{it}$ (panels C or D). In panels A and C, g_{it} is the annual sales growth. In panels B and D, g_{it} is annual employment growth. Error terms are clustered at the firm level. *T*-stats are reported between parentheses.

***Significant at the 1 percent level.

**Significant at the 5 percent level.

Source: Tax files and financial relation survey (INSEE) on 1984–2004.

II. The Baseline Model

In this section, we present the baseline model and derive some general properties.

AQ4

A. *Set Up*

We consider a static economy populated by $\tilde{L}$ workers, each of whom supply one unit of labor. A measure N of firms compete imperfectly on the product market. N_L

firms are listed on the stock market, while each of the remaining $N_P \equiv N - N_L$ firms are held by a single *entrepreneur*, who does not supply labor. Among the $\tilde{L}$ workers, there are *I investors* who trade the stocks of listed firms with each other.[11] There are two sources of risk: an idiosyncratic shock to firm-level demand, and an aggregate supply shock on labor supply. There are three periods.[12] At date 1, each firm i implements a strategy s_i that indirectly affects the mean and variance of future profits. At date 2, investors trade stocks and the financial market clears. At date 3, uncertainty is revealed; the product and the labor markets clear; the workers receive wages; and firm owners (investors or entrepreneurs) receive the profits.

Preferences and Technology.—Each agent k in the economy has a utility $U(C_k)$ with constant[13] relative risk aversion $\gamma > 0$. The consumption index C_k is a composite of the consumptions $y_{k,i}$ of goods supplied under monopolistic competition by firms $i \in [0;N]$: $C_k = \left(\int_0^N \tilde{\delta}_i^{1/\sigma} y_{k,i}^{(\sigma-1)/\sigma} di\right)^{\sigma/(\sigma-1)}$, where $\sigma \geq 2$ by assumption.[14] The random coefficients $\tilde{\delta}_i > 0$ correspond to the good i specific demand shifter. A convenient feature of this Dixit-Stiglitz index is that aggregating individuals consumption is simple in spite of heterogeneity in individual incomes between the three groups of agents populating this economy (pure workers, entrepreneurs, investors). Indeed, the total demand $\tilde{y}_i$ addressed to each monopoly i is obtained by aggregating consumptions $y_{k,i}$ over the whole population, and standard computations lead to $\tilde{y}_i = \tilde{\delta}_i E P^{\sigma-1}/p_i^{\sigma}$, where p_i is the monopoly price charged by firm i; $E \equiv \int_0^{\tilde{L}+N_P} E_k dk$ is the aggregate nominal expenditure; and P is a price index equal to $P \equiv \left(\int_0^N \tilde{\delta}_j \, p_j^{1-\sigma} dj\right)^{1/1-\sigma}$.

On the supply side, we assume that the total number of workers is random and equal to $\tilde{L} = \tilde{A} L$, where $\tilde{A}$ is positive with mean 1 and variance σ_A^2. We interpret $\tilde{A}$ as an aggregate supply shock.[15] Each firm i hires l_i workers at nominal wage w, and produces according to the constant returns to scale technology $y_i = l_i$. We implicitly rule out entry on the product market by fixing exogenously the number of firms at N.

Strategies of Firms.—At date 1, each firm i implements a strategy $s_i \geq 0$ at a cost $C(s_i)$ in real terms, which affects its demand shifter $\tilde{\delta}_i$ in the following way:[16]

$$\tilde{\delta}_i = 1 + s_i \tilde{d}_i, \tag{4}$$

[11] For expositional simplicity, we assume that entrepreneurs do not supply labor. Relaxing this assumption does not affect our results.

[12] Inverting period 1 and period 2 would not change the result.

[13] Most of our results are robust to assuming a DRRA utility.

[14] This assumption is fully consistent with the empirical estimates of the elasticity of substitution based on micro-data (Keith Head and John Ries 2001; Christian Broda and David E. Weinstein 2006) and with recent estimates based on macro data (Jean Imbs and Isabelle Méjean 2008). Moreover, it is now standard in the macro literature with heterogenous industries or firms (as it is the case in our model) to calibrate σ with values above 4 (for recent examples, see Andrew Atkeson and Ariel Burstein 2007, Giancarlo Corsetti, Luca Dedola, and Sylvain Leduc 2008).

[15] An alternative modelization would be to take $\tilde{A}$ as a productivity shock. This does not change the heart of the analysis but makes exposition more complex. Moreover, we assume that for all $\tilde{A}$ we have $\tilde{A}L > I$. Hence, the number of investors is not affected by aggregate uncertainty.

[16] All our results can easily be generalized to any functional form $\tilde{\delta}_i = G(s_i, \tilde{d}_i)$ such as $G(.,.)$ is positive, increasing, and quasi concave in both arguments with the following log-supermodularity condition: $\partial^2 \log G / \partial s_i \partial \tilde{d}_i > 0$.

where $\tilde{d}_i$ are independently and identically distributed positively distributed shocks with mean 1 and variance σ_d^2. The cost function $C(\cdot)$ is increasing and convex with $C(0) = C'(0) = 0$ and $C'(\infty) = \infty$. For private firms, the optimal strategy s_i maximizes the entrepreneur's expected utility. For listed firms, it maximizes the date 2 stock price.

Assumption (4) is motivated by the literature on growth and finance (see, for instance, Saint-Paul 1992, Obstfeld 1994, Acemoglu and Zilibotti 1997). In existing theories, enhanced risk sharing reduces the costs of risk-taking for the representative firm, which has the effect of increasing the volatility of aggregate output. Hence, financially developed and integrated economies should be more volatile, which is a counterfactual prediction (Koren and Tenreyro 2007). In our model, this does not happen (see Section IIB). Moreover, our view of risk-taking is precisely microfounded with firm-level demands. Here, the strategies s_i can be interpreted as a choice of *customization* by the firm. Customized goods can be highly valuable, but their demand is difficult to predict because of erratic preferences. This interpretation is close to the view developed in standard models of advertising (Richard Schmalensee 1974), in models with consumer inertia (Arthur Fishman and Rafael Rob 2003), and in Michael J. Piore and Charles F. Sabel's (1984) vision of flexible manufacturing.

B. *Solving the Model*

We solve this model backward, starting with labor and product market clearing conditions in period 3, stock trading in period 2, and strategy choice in period 1.

Product and Labor Market Equilibria.—At date 3, the aggregate labor supply shock $\tilde{A}$ and final demand shocks $\tilde{d}_i$ are revealed.[17] Each firm i charges an optimal monopoly price p_i in order to maximize its profits in real terms $\tilde{\pi}_i = \tilde{y}_i(p_i - w)/P - C(s_i)$. As it is standard in this monopolistic competition setting, N is assumed to be sufficiently large such that the marginal effect of p_i on the price index P is negligible. Solving this standard maximization problem leads to a constant mark-up over marginal cost $p_i = w/(1 - 1/\sigma)$. By aggregating labor demands across firms and using the definition of P, we get the labor market clearing condition $\tilde{A}L = (1 - 1/\sigma)E/w$. This yields the real wage $w/P = (1 - 1/\sigma) \times \left(\int_0^N \tilde{\delta}_j dj\right)^{1/(\sigma-1)}$ and the real spending $E/P = \tilde{A}L\left(\int_0^N \tilde{\delta}_j dj\right)^{1/(\sigma-1)}$. Together with (4), we then obtain real profits at equilibrium

$$\tilde{\pi}_i = \frac{\tilde{A}}{M}(1 + s_i\tilde{d}_i) - C(s_i), \tag{5}$$

AQ5 where $M \equiv (\sigma/L)\left(\int_0^N \tilde{\delta}_j dj\right)^{(\sigma-2)/(\sigma-1)}$. We label M the degree of *market pressure* because it measures the extent to which labor market pressure feedbacks on real

[17] The shocks are assumed to be independent. This drastically simplifies the analysis. In reality, however, shocks are correlated in nontrivial ways. Schumpeterian dynamics leads some firms to win at the same time as others lose. Francesco Franco and Philippon (2007) find evidence on this. This negative correlation of shocks is also at the root of the mechanisms that Comin and Mulani (2005) use to link firm volatility with aggregate volatility.

wage and firm profits. Indeed, using the definition of M, we can rewrite the real wage as an increasing function of M:

$$w/P = \omega M^{\frac{1}{\sigma-2}}, \tag{6}$$

where ω is a combination of exogenous parameters equal to $\omega \equiv (1 - 1/\sigma) \times (L/\sigma)^{1/\sigma-2}$.

The interpretation of the negative impact of M on profits in equation (5) is the following: when all firms choose a very high value of s_i, aggregate demand for labor is high. This props up real wage and reduces profits.[18]

Noticing that the shocks $\tilde{d}_j$ are independently and identically distributed, the law of large numbers leads to the following formula:

$$M = \frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} \left[1 + \frac{N_P}{N} s_P + \frac{N_L}{N} s_L\right]^{\frac{\sigma-2}{\sigma-1}}, \tag{7}$$

where (s_P, s_L) correspond to the risky strategies for private and listed firms, respectively. Note that all firms within each group are identical and therefore adopt identical strategies.

From the previous computations, we get immediately that (s_P, s_L) positively impact the volatility of sales at the firm level. This allows us to interpret s_i as a measure of risk-taking at the firm level. With respect to aggregate volatility, we look at the volatility of real spending E/P. Simple computations show that this does not depend on (s_P, s_L) and is always equal to σ_A, hence, our model makes no prediction on the evolution of aggregate volatility.

Stock Trading and Prices.—At date 2, each investor initially owns a potentially unbalanced portfolio of N_L/I shares of listed firms. Stock trading allows her to optimally rebalance her portfolio.[19] All listed firms being ex ante symmetric, their equilibrium holding of each stock is exactly $1/I$. The Euler condition of the underlying portfolio choice problem leads to the following standard expression for the stock price of firm i:

$$\rho_i = E\left[U'(\tilde{R}_I)\,\tilde{\pi}_i\right]/E\left[U'(\tilde{R}_I)\right], \tag{8}$$

where $\tilde{R}_I$ is the date 3 real income of the representative investor (all investors are identical) and is composed of the real wage and the dividends paid by stocks owned, $\tilde{R}_I = w/P + (1/I)\int_0^{N_L} \tilde{\pi}_j\,dj$. Taking (5) and (6), and N_L being large, we get

$$\tilde{R}_I = \omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\left[\tilde{A}(1 + s_L)/M - C(s_L)\right]. \tag{9}$$

[18] In fact, this effect competes with another force; due to Dixit-Stiglitz preferences, larger $\tilde{\delta}_j$ by competitors increases real spending E/P and profits for all firms. This second effect is, however, dominated under our assumption that the elasticity of substitution is large enough (i.e., $\sigma > 2$).

[19] The safe investment is assumed nontradable to simplify exposition.

The investor real income can be decomposed into labor income and financial income. A larger aggregate supply shock $\tilde{A}$ increases financial income, while market pressure M increases the labor component but decreases the financial component. The financial income is unaffected by idiosyncratic demand shocks $\tilde{d}_i$ because of efficient risk diversification through the stock market.

Strategy Choice of Privately Held Firms.—We first analyze the strategy choice of private firms. The date 3 real income of the entrepreneur owning private firm i is equal to

$$\tilde{R}_{E,i} = \tilde{A}(1 + s_i \tilde{d}_i)/M - C(s_i). \tag{10}$$

Entrepreneurs do not supply labor, and their income $\tilde{R}_{E,i}$ corresponds to the profits of their firm. $\tilde{R}_{E,i}$ is affected by the idiosyncratic shock $\tilde{d}_i$ because the entrepreneur is underdiversified.[20] At date 1, the entrepreneur chooses her optimal strategy s_p, taking the expected market pressure M as given, so as to maximize her expected utility $s_p = \arg\max_{s_i} E\left[U(\tilde{R}_{E,i})\right]$. Omitting the index i, the first-order condition of this problem can be rewritten as:

$$C'(s_p) = \frac{1}{M} + \operatorname{cov}\left[\frac{U'(\tilde{R}_E)}{EU'(\tilde{R}_E)}, \frac{\tilde{A}\,\tilde{d}}{M}\right]. \tag{11}$$

The optimal s_P equalizes the LHS marginal cost of risk-taking to the RHS marginal benefit of risk-taking. Benefit is composed of the marginal increase in expected income, i.e., $E(\tilde{A}\,\tilde{d}/M) = 1/M$, corrected for the marginal increase in risk exposure. This risk correction term is negative because the marginal increase in revenue, $\tilde{A}\,\tilde{d}/M$, is negatively correlated with marginal utility, $U'(\tilde{R}_E)$. A marginal increase in s is less desirable for a risk-averse entrepreneur than for a risk-neutral one because it generates the most income when it is the least needed. We show in Appendix A equation (A1) that the LHS (resp. RHS) is increasing (resp. decreasing) in s, such that there is one and only one interior solution to equation (11). In general, it is not possible to find a closed-form solution for s_P except in a CARA-Gaussian framework.[21] Yet, we can infer the following partial equilibrium property (for a Proof see Appendix A equation (A1)):

LEMMA 1: *Risk-taking by private firms*, s_P, *is a decreasing function of Market Pressure*, M.

The RHS of (11) highlights how market pressure M impacts the marginal benefit of risk-taking and consequently the optimal s_P. The overall effect results from three

[20] For instance, Tobias J. Moskowitz and Annette Vissing-Jorgensen (2002, 751, table 2) provide evidence of entrepreneurial underdiversification. They document that owners of private stock in the Survey of Consumer Finances place, on average, half of their wealth in private stocks, most of it corresponding to a single, actively managed firm.

[21] For the analytical results derived in such a CARA-gaussian setup, see Thesmar and Thoenig (2004) and the online Appendix. However, existing evidence from the experimental and empirical literature shows that the assumption of constant absolute risk aversion is not empirically relevant.

counterbalancing forces. First, an increase in M reduces profits, and therefore the expected marginal benefit of raising s_P. This tends to decrease s_p. The second effect goes in the opposite direction. Since entrepreneurial income is proportional to $\tilde{A}/M$, an increase in M makes profits more equal across states of nature. The entrepreneur becomes, in effect, less risk averse and more willing to increase s_P. Last, a larger M reduces real income. Because relative risk aversion is constant, risk aversion increases as the entrepreneur's income decreases. Those effects shape entrepreneurs' willingness to take risks. Lemma 1 shows that the first and third effects dominate the second. Interestingly, it is also possible to show that when the third effect is absent (i.e., assuming CARA utility), the second effect dominates the first effect, such that an increase in M tends to increase s_P. In this particular case, there is no reasonable comparative static exercise that is able to generate a pattern of diverging trends in volatility for private and listed firms. Hence, the assumption that absolute risk aversion is decreasing is central. Without it, our model could not replicate the opposite volatility trends.

Strategy Choice of Listed Firms.—At date 1, the shareholders of listed firms choose an optimal strategy s_L, so as to maximize the date 2 stock price, $s_L \equiv \arg\max_{s_i} \rho_i$, taking the expected market pressure M as given. Omitting the index i and considering (8) and (9), the FOC of this problem is given by

$$C'(s_L) = \frac{1}{M} + \operatorname{cov}\left[\frac{U'(\tilde{R}_I)}{EU'(\tilde{R}_I)}, \frac{\tilde{A}}{M}\right]. \tag{12}$$

At the optimum, the marginal cost equals the marginal gain of risk-taking, which can be decomposed into an expected marginal gain corrected for the marginal increase in risk exposure. The interpretation is thus similar to the private firm case (11), except that now idiosyncratic risk $\tilde{d}$ is completely diversified. Combined with the definition of investors income $\tilde{R}_I$, equation (12) implicitly defines s_L as a function of market pressure M and number of investors I. In Appendix A, equation (A1), we show that this problem admits a unique interior solution, and we establish the following partial equilibrium property (for a Proof see Appendix A equation (A1)):

LEMMA 2: *Risk-taking by listed firms s_L is an increasing function of stock market participation I. There exists I_0, such that, for all $I > I_0$ risk-taking by listed firms s_L is unambiguously a decreasing function of market pressure M.*

In contrast to the case of private firms, it is not possible to show that s_L is always decreasing in M, unless I is large enough. The same three forces present in the private firm case are at work. First, a larger M reduces expected marginal benefit of risk-taking for investors. There is less to gain by taking risks. Second, it tends to make profits more equal across states of nature, which encourages risk-taking. Third, the increase in M also has a direct effect on investors' risk aversions. But while the effect was unambiguous in the case of privately held firms (it made entrepreneurs more risk averse), the impact on investors' effect risk aversion is ambiguous for a listed firm. This is because investors are also workers, and therefore receive labor income. When M increases, real wages go up (firms compete on the labor market), while profits go down. The overall impact on their income is ambiguous, and so is the effect on risk aversion. As noticed in the nonlisted entrepreneur case, this third effect is crucial to establish that s is decreasing in M. The overall impact is therefore ambiguous. However, for a large number of investors I, the investor real income depends less on financial income, and is thus less exposed to aggregate risk. The second and third effects vanish to zero and only the first one remains. An increase in M reduces incentives to take risks, hence it reduces s_L. In the log-linearized version of the model in Section III, we will be able to drop the assumption $I > I_0$.

The impact of I on s_L follows from the previous discussion. A larger I reduces risk exposure by decreasing the share of financial income in investor total income.

C. *Equilibrium and Comparative Statics*

The general equilibrium is the solution of a system of three equations.

DEFINITION 1: *The equilibrium consists of* (M, s_P, s_L) *such that:*

(*i*) *M depends positively on s_L and s_P through equation* (*7*);

(*ii*) *s_P depends negatively on M through equation* (*11*); *and*

(*iii*) *s_L depends negatively on M through equation* (*12*).

This is a rational expectation equilibrium. In period 1, each firm takes the expected M as given, and sets up its optimal strategy choice. Given the above definition, the equilibrium value of market pressure can be written as the solution to the following fixed point problem:

$$M = \frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} \left[1 + \frac{N_L}{N} s_L(\underset{(-)}{M}, \underset{(+)}{I}) + \frac{N_P}{N} s_P(\underset{(-)}{M}) \right]^{\frac{\sigma-2}{\sigma-1}}, \tag{13}$$

where s_L and s_P both decrease in M by virtue of Lemmas 1 and 2. The LHS of this equation corresponds to the 45° line, while the RHS is positive and decreasing in M as long as $I > I_0$. Hence, the equilibrium exists and is unique. From this simple expression, we easily get the following comparative static properties (for a Proof see Appendix A equation (A1)).

AQ 6

PROPOSITION 1: *Assume $I > I_0$. Then the equilibrium (M, s_L, s_P) exists and is unique. And:*

(*i*) *an increase in stock market participation I leads to an increase in market pressure M, an increase in risk-taking for listed firms, s_L, and a decrease in risk-taking for private firms, s_P;*

(*ii*) *an increase in the number of firms N leads to an increase in M and a decrease both in s_L and s_P; and* (*iii*) *an increase in market size L leads to a decrease in M and an increase both in s_L and s_P.*

Let us begin with parts ii and iii. An increase in the number of firms, N, increases market pressure M, which in turn tends to decrease risk-taking for all firms. An increase in market size, L, leads to increased profits. This decreases market pressure M and increases risk-taking for all firms. In the two cases, risk-taking for both categories of firms evolve in the same direction. The general intuition is straightforward—competition and market size affect both types of firm in the same fashion. They should, in equilibrium, behave in the same way.

Part i of Proposition 1 focuses on the effect of stock market participation interpreted in our model as an increase in the number of investors I. As I increases, this directly induces listed firms to increase risk-taking s_L. This in turn tends to increase competitive pressure M. Listed firms now face, on average, higher demand. They exert more pressure on wages, and reduce profits for other firms. As an indirect consequence, private firms reduce their level of risk-taking s_P. This effect is also there for listed firms, but it is dominated by the direct effect.

The above proposition thus suggests that increased stock market participation, more than competition or market size expansion, is a natural candidate to explain the divergence in volatility trends. As it turns out, participation in the United States has increased dramatically over the past decade, in large part because of the development of mutual funds and individual retirement accounts, such as the 401k. For instance, Jack Favilukis (2008) reports that the fraction of households owning stocks directly or indirectly has risen from 33 percent to 43 percent between 1982 and 2004 (see also evidence from Guiso, Haliassos, and Jappelli 2003). In France, the number of shareholders has gone up from less than 2 million in 1978 to more than 6 million in 2006, with most of the rise taking place during the mass privatization of the 1980s (NYSE Euronext—SOFRES 2007).

The above results are, however, obtained under the parameter restriction that $I > I_0$. In addition, it is not possible to obtain closed-form solutions that would allow us to fully characterize the equilibrium and derive more comparative
AQ 7 static properties. This is why we approximate the model in the Section III.

III. A Closed-Form Version of the Model with Small Shocks

In this section, we derive an approximated version of the model by assuming small aggregate and idiosyncratic shocks. Beyond providing closed-form solutions for the main endogenous variables (M, s_L, s_P), the benefit of this approach is that it allows us to derive additional results, in particular on international risk sharing.

A. *Log-linearization*

Assuming small variations of $(\tilde{A}, \tilde{d}_i)$ around their means, we log-linearize the system around the deterministic equilibrium which corresponds to the special case where shocks take their mean value, $\tilde{A} = \tilde{d}_i = 1$. In the deterministic equilibrium, private and listed firms face similar incentives because the risk adjustment terms in

(11) and (12) are both equal to zero. As a result, they choose the same strategy s_0, which satisfies

$$C'(s_0) = 1/M_0, \tag{14}$$

where market pressure M_0 is given by equation (13). This yields

$$M_0 = \frac{\sigma}{L} N^{(\sigma-2)/(\sigma-1)} (1 + s_0)^{(\sigma-2)/(\sigma-1)}. \tag{15}$$

Since C is convex, it is clear that there exists one and only one deterministic equilibrium (M_0, s_0), which solves (14) and (15).

For each variable x in the stochastic equilibrium set up, we denote by x_0 the value of x in the deterministic equilibrium, and by $\hat{x} \equiv (x - x_0)/x_0$ the percentage deviation from x_0. In Appendix A, equation (A2), we derive the first-order Taylor expansion of the equilibrium (s_L, s_P, M) as defined by equations (11), (12), and (13), where incomes $(\tilde{R}_E, \tilde{R}_I)$ are given by (9) and (10). We obtain

$$\begin{aligned} \varepsilon_C \hat{s}_P &= -\hat{M} - \gamma \Omega_P \\ \varepsilon_C \hat{s}_L &= -\hat{M} - \gamma \Omega_L \\ \hat{M} &= m_0 \left(\frac{N_L}{N} \hat{s}_L + \frac{N_P}{N} \hat{s}_P \right), \end{aligned} \tag{16}$$

where γ is relative risk aversion; ε_C is the elasticity of marginal cost C' to s; $m_0 \equiv s_0(\sigma - 2)/(\sigma - 1)$ is a parameter capturing the elasticity of market pressure to risk-taking; and (Ω_P, Ω_L) are the *risk exposures* of owners of private and listed firms

$$\begin{aligned} \Omega_P &= \theta_{E,A}\, \sigma_A^2 + \theta_{E,d}\, \sigma_d^2 \\ \Omega_L &= \theta_{I,A}\, \sigma_A^2, \end{aligned} \tag{17}$$

where $\theta_{E,A} \equiv (1 + s_0)/R_{E0} M_0$; $\theta_{E,d} \equiv s_0/R_{E0} M_0$; and $\theta_{I,A} \equiv (N_L/I)(1 + s_0)/R_{I0} M_0$.

The weights $(\theta_{E,A}, \theta_{E,d})$ correspond to the shares of entrepreneur income, which are affected by the aggregate and the idiosyncratic shocks. The weight $\theta_{I,A}$ is the share of investor income, which is affected by the aggregate shocks. Implicitly, we have $\theta_{I,d} = 0$ because investors are fully sheltered from idiosyncratic risk. We also see that $\theta_{E,A} > \theta_{I,A}$: investors, who earn a deterministic labor income, have a smaller exposure to aggregate shocks than entrepreneurs.

Looking at equations (16) and (17), a few interesting features emerge. First, the elasticities of $\hat{s}_L$ and $\hat{s}_P$ to $\hat{M}$ are similar. In the approximated equilibrium, the first-order impact of market pressure on risk-taking transits only through the expected
AQ 8 marginal benefit channel (see the discussion in Section IIB and IIB). Second, m_0 can be interpreted as the strength of the general equilibrium feedback. Third, both $\hat{s}_P$ and $\hat{s}_L$ are decreasing functions of risk exposures (Ω_P, Ω_L); a reduction in the share of incomes that is exposed to risk generates a direct increase in risk-taking.

Inverting the equilibrium system (16), we get the following closed form solutions:

$$\hat{s}_P = \frac{\gamma}{m_0 + \varepsilon_C}\left[-\Omega_P + \frac{m_0}{\varepsilon_C}\frac{N_L}{N}(\Omega_L - \Omega_P)\right] \tag{18}$$

$$\hat{s}_L = \frac{\gamma}{m_0 + \varepsilon_C}\left[-\Omega_L + \frac{m_0}{\varepsilon_C}\frac{N_P}{N}(\Omega_P - \Omega_L)\right] \tag{19}$$

$$\hat{M} = -\frac{\gamma m_0}{m_0 + \varepsilon_C}\left[\frac{N_P}{N}\Omega_P + \frac{N_L}{N}\Omega_L\right], \tag{20}$$

where it appears that risk-taking by private firms is a decreasing function of entrepreneurs' risk exposures, but an increasing function of investors' risk exposures. This second effect is channelized by the feedback effect of market pressure on risk-taking, and is how our model generates the pattern of opposite trends in volatility for listed and private firms.

B. *Stock Market Participation*

This first proposition summarizes the impact of an increase in stock market participation (for a Proof see Appendix A, equation (A2)).

PROPOSITION 2: *A positive shock on stock market participation I:*

(*i*) *increases market pressure, M, and risk-taking by listed firms s_L but decreases risk-taking by private firms, s_P;*

(*ii*) *increases the real wage and the utility of workers; and*

(*iii*) *decreases expected real profits for all firms and the utilities of investors and entrepreneurs.*

The first point of Proposition 2 is already established in Proposition 1 except that now the result does not hinge on a condition on *I*. Regarding points ii and iii, we know from equation (6) that the real wage is an increasing function of $\hat{M}$ and that the closed-form solution for expected profits is obtained by linearizing expression (5),

$$E\tilde{\pi}_L = E\tilde{\pi}_P = \pi_0 - \frac{1 + s_0}{M_0}\hat{M},$$

where π_0 is profit in the deterministic equilibrium. Since we compute a first-order approximation, the envelope theorem applies and wipes out the terms in $(\hat{s}_P, \hat{s}_L)$. Hence, the only equilibrium variable that affects expected profits is market pressure $\hat{M}$. Stock market participation (or anything that increases market pressure in our model) reduces real profits of both types of firms to the same extent. Using our French data presented in Section I, we draw in Figure 4 the evolution of the return

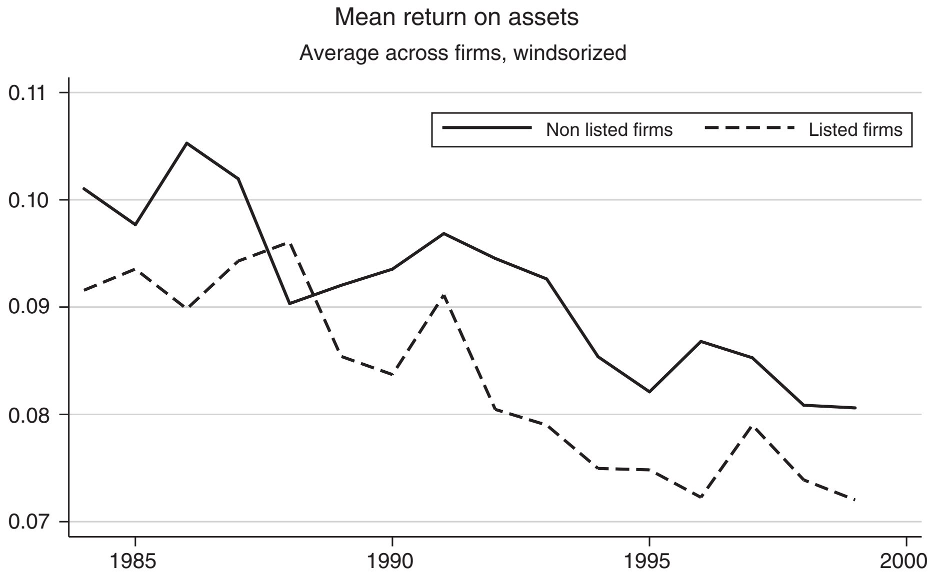

Figure 4. Mean Profitability: Comparing Listed and Nonlisted Firms

on assets of the two categories of firms.[22] Both groups exhibit the same downward trend, even though we saw earlier that patterns of volatility diverge sharply. This confirms that intuition based on partial equilibrium reasoning may be misleading. Even though listed firms take on more risk to increase their profitability, in general equilibrium, their ROA decreases.

The last outcome of interest is the equity premium. To this purpose, we assume that there exists a risk-free security with a positive but very small exogenous return r such that investors real income, $\tilde{R}_I$, is still given by (9). In Appendix A, equation (A2), we log-linearize the stock price (8) and find the following expression for the equity premium,

$$EP \;=\; r\gamma\, \theta_{I,A}\, \theta_{E,A}\, \sigma_A^2, \tag{21}$$

which is, through $\theta_{I,A}$, a decreasing function of stock market participation I. As more and more investors can share the aggregate risk, the premium that stocks demand tends toward zero. Thus, besides predicting a diverging pattern in firm volatility, our theory predicts that the equity premium should have declined over the past 30 years so as to compel listed firms to take on more aggregate and idiosyncratic risk. There exists such evidence, at least for the United States. Using very different methodologies, Lubos Pastor and Robert F. Stambaugh (2001) and Fama and French (2002) both conclude that the US equity premium has declined by about 100 bp over the past decades.

[22] We compute ROA by dividing operating income through net assets. Such accounting variables are available from our French dataset because the unit of observation is a firm with its own financial statement. This contrasts with the data exploited by Davis, Haltiwanger, and Jarmin (2007), which are also firm-level but lack financial statements and asset-return measures for nonlisted firms.

C. *The Great Moderation*

Over the past three decades preceding the onset of the financial crisis in 2008, aggregate volatility has declined significantly in most developed countries (see Jordi Gali and Luca Gambetti 2009 for a recent survey). The determinants of this "Great Moderation" are still not clear. It could be luck, stabilizing monetary policy (Richard Clarida, Gali, and Mark Gertler 2000) or improvements in inventory management (James A. Kahn, Margaret M. McConnell, and Gabriel Perez-Quiros 2002). As mentioned in Section IIB, our theory has no predictive power about size adjusted GDP volatility. Hence, we assume hereafter that the decrease in aggregate volatility is exogenous and structural. In our model, this corresponds to an exogenous decline in σ_A^2 (for details see Appendix A, equation (A2)).

PROPOSITION 3: *A negative shock on aggregate volatility* σ_A^2*:*

(*i*) *increases risk-taking by private firms,* s_P*, and market pressure,* M*;*

(*ii*) *increases* (*resp. decreases*) *risk-taking by listed firms,* s_L*, if* $\omega M_0^{1/(\sigma-2)}$ *is sufficiently large* (*resp. small*)*; and*

(*iii*) *decreases real profits and increases real wage.*

The predicted effect of a dampening of macroeconomic shocks differs from the impact of stock market participation. In our model, this tends to systematically *increase* risk-taking by entrepreneurs, which goes against the evidence presented in this paper and by Davis, Haltiwanger, and Jarmin (2007). The effect on listed firms is, however, ambiguous. This comes from two conflicting forces. First, equation (16) makes clear that the direct effect of a reduction in σ_A^2 is to increase risk-taking by both listed and private firms. Yet, because entrepreneurs are more exposed to aggregate risk than investors ($\theta_{E,A} > \theta_{I,A}$), private firms increase their level of risk by more than listed firms.[23] Second, since all firms tend to increase risk-taking, the resulting increase in market pressure M results in higher wages and lower expected profits. This reduces incentives to take risk by the same amount for listed and private held firms. On balance, the first effect always dominates for private firms, but not always for listed ones.

IV. The Globalization of Capital Markets

In the preceding analysis, we show that the contrasting trends in firm volatility are caused by an improvement in risk sharing which follows, within our model, from an increase in stock market participation. Yet, an increase in stock market participation neither increases risk sharing nor reduces the equity premium in any of the models. In practice, Guiso, Haliassos, and Jappelli (2003) recall that, since wealth is

[23] This is a consequence of the assumption that $N_L < I$. Because there are more shareholders than listed firms, the share of risky income for entrepreneurs is larger than what it is for investors.

concentrated, marginal shareholders tend to hold very small portfolios, and do therefore not contribute much to global risk sharing.[24] They even note that participation may *increase* the equity premium. Lowering participation costs, for example, will mechanically add investors that are less wealthy and more risk averse than existing stockholders. Since the equity premium is determined by the preferences of the marginal investors, it may very well increase.

In this section, we consider a less controversial mechanism at the root of the improvement in risk sharing. Capital markets integration. In a recent paper, Chari and Henry (2004) look at the effect of stock market liberalizations around the world. They find that, on average, stock prices increase by about 15 percent. Part of this increase can be explained by a straightforward decline in the risk free rate, reflecting more abundant capital in liberalized stock markets, although approximately $2/5$ of it (6.8 percentage points) is explained by the fact that domestic firms' stocks covariate much less with the world market portfolio than with the domestic one (by a factor of 200).

A. *The Model with Two Countries*

We therefore expand the baseline model of Section II by considering a second, identical country ("foreign"). The world now has $2I$ investors, $2\tilde{L}$ workers, $2N_L$ listed firms (half of them domestic, half of them foreign), and $2N_P$ privately held firms. To clarify exposition, we assume that investors do not supply labor.[25] The number of goods is now $2N = 2N_P + 2N_L$, and we allow both countries to trade goods; by assumption there are no trading costs. The only ex post difference between the two countries comes from the realization of their aggregate shock on labor supply. Labor supply is $\tilde{L}_D = \tilde{A}_D L$ in the domestic country, and $\tilde{L}_F = \tilde{A}_F L$ in the foreign country. The shocks $\tilde{A}_D$ and $\tilde{A}_F$ are identically distributed and assumed to be uncorrelated to simplify exposition (although this assumption is not necessary). All the computational details are given in Appendix A, equation (A3).

Let us start with period 3. Product and labor markets clear, and the profit of a domestic firm operating on the world market is now given by

$$\tilde{\pi}_{D,i} = \frac{\tilde{B}_D}{M}(1 + s_i \tilde{d}_i) - C(s_i), \tag{22}$$

where the subscript D is an index for domestic firms, and

$$\tilde{B}_D \equiv \tilde{A}_D \left[\frac{1}{2} + \frac{1}{2}(\tilde{A}_F/\tilde{A}_D)^{(\sigma-1)/\sigma}\right]^{1/(\sigma-1)}.$$

[24] For instance, Wojciech Kopczuk and Emmanuel Saez (2004) find, using estate tax files, that in 2000 the top 2 percent of the wealth distribution in the United States owns more than 25 percent of aggregate wealth, and probably a larger fraction of outstanding corporate equity. Given that, according to Guiso, Haliassos, and Jappelli (2003), 93 percent of the households present in the top quartile of the wealth distribution already own some equity, additional participation has to come from relatively poor households.

[25] In this model with trade, real wages become random because of the terms-of-trade effect. This makes computations somewhat more cumbersome, although still feasible thanks to the linearization. In particular, portfolio composition is not symmetric anymore between domestic and foreign stocks because domestic workers seek to hedge the terms-of-trade effect. We have verified that including labor income in investors' revenues does not affect our results (computations are available from the authors upon request).

This is very similar to the expression in a closed economy (equation (5)), the only difference being the term of trade effect $\tilde{A}_F/\tilde{A}_D$, which affects domestic profits (see Nicolas Coeurdacier 2009, for a discussion). Following a positive relative supply shock in the domestic economy, domestic production and income expand and this increases domestic profits; but since foreign and domestic goods are imperfect substitutes, domestic demand for foreign goods increases relatively more than foreign production. As a result, the domestic terms of trade deteriorate and, *everything else equal*, this negatively impacts the domestic profits.

The aggregate shocks $(\tilde{A}_D, \tilde{A}_F)$ are identically distributed, which means that the ex ante strategy choice s_L (resp. s_P) is similar for listed firms (resp. private) in both countries. As a consequence the equilibrium level of market pressure is now given by

$$M = \frac{\sigma}{2L}(2N)^{\frac{\sigma-2}{\sigma-1}}\left[1 + \frac{N_L}{N}\, s_L + \frac{N_P}{N}\, s_P\right]^{\frac{\sigma-2}{\sigma-1}}.$$

At date 1, the optimal strategy choices are very close to those in the closed economy (see Sections IIB and IIB). Entrepreneurs choose s_P so as to maximize their expected utility,

$$C'(s_P) = \frac{1}{M} + \operatorname{cov}\left[\frac{U'(\tilde{R}_{D,E})}{EU'(\tilde{R}_{D,E})}, \frac{\tilde{B}_D\,\tilde{d}_i}{M}\right],$$

where $\tilde{R}_{D,E} = \tilde{\pi}_{D,i}$ is the income of domestic entrepreneurs at date 3. Investors choose s_L so as to maximize the stock price,

$$C'(s_L) = \frac{1}{M} + \operatorname{cov}\left[\frac{U'(\tilde{R}_I)}{EU'(\tilde{R}_I)}, \frac{\tilde{B}_D}{M}\right], \tag{23}$$

where $\tilde{R}_I$ is the income of investors in period 3. $\tilde{R}_I$ depends on investors' trading opportunities at date 2, which in turn depends on whether capital markets in the two countries are integrated or not.

B. *Trade in Goods, No Asset Trade*

As a benchmark, we look at the case where there is no trade in assets. Only domestic investors can purchase domestic stocks, and domestic investors cannot purchase foreign stocks. The solution to equation (23) is the same as in Section IIB, except for the difference that now the aggregate shock is $\tilde{B}_D$ instead of $\tilde{A}_D$. After linearization around the deterministic equilibrium, the equilibrium equations in $\hat{s}_L, \hat{s}_P, M$ are identical to (16) except that risk exposures Ω_P and Ω_L are given by

$$\Omega_P = \theta_{E,A}\,\sigma_B^2 + \theta_{E,d}\,\sigma_d^2 \tag{24}$$

$$\Omega_L = \theta_{I,A}\,\sigma_B^2,$$

where $\sigma_B^2 = [1 - (2\sigma - 1)/2\sigma^2]\sigma_A^2 < \sigma_A^2$. Compared to the closed economy, international trade provides some diversification via the terms of trade effect, since the terms of trade tend to appreciate when domestic productivity is low.

C. *Trade in Goods and Assets*

We first compute the stock price at which investors can freely buy foreign stocks. Because aggregate shocks are independently and identically distributed, in equilibrium the representative investor holds half of her wealth in domestic stocks, and half in foreign stocks. Hence, the investor's income is given by

$$\tilde{R}_I = \frac{N_L}{I}\left[\frac{\tilde{B}_D + \tilde{B}_F}{2}\frac{(1 + s_L)}{M} - C(s_L)\right].$$

Combined with equation (23) and linearizing around the deterministic equilibrium, the equilibrium equations in $\hat{s}_L, \hat{s}_P, M$ are identical to (16) except that now risk exposures Ω_P and Ω_L are given by

$$\Omega_P = \theta_{E,A}\,\sigma_B^2 + \theta_{E,d}\,\sigma_d^2 \tag{25}$$
$$\Omega_L = \theta_{I,A}\,\frac{\sigma_B^2 + \sigma_{D,F}}{2},$$

where $\sigma_{D,F}$ is the covariance between trade adjusted country shocks $\tilde{B}_D$ and $\tilde{B}_F$. Since these two shocks are not perfectly correlated, it is easy to deduce that $(\sigma_B^2 + \sigma_{D,F})/2 < \sigma_B^2$.

PROPOSITION 4: *Capital market integration:*

(*i*) *increases risk-taking by listed firms, s_L, and market pressure, M;*

(*ii*) *decreases risk-taking by private firms, s_P;*

(*iii*) *increases the real wage and the utility of workers; and*

(*iv*) *decreases real profits and the utilities of investors and entrepreneurs.*

The effects at work are the same as for stock market participation. Capital market integration enhances risk sharing among investors, which increases listed firms risk-taking. Market pressure increases and this reduces profits, which in turn forces nonlisted firms to scale back on their risk-taking.

D. *Illustrative Calculations*

We have argued that risk sharing opportunities providing financial globalization can qualitatively explain the volatility divergence of listed and nonlisted firms. This section checks whether the mechanism is empirically plausible, i.e., whether risk sharing opportunities between countries are large enough so as to induce firms to assume the level of risk observed in the data. The change in sales variances for private and listed firms, v_P and v_L, that results from capital markets integration can easily be computed from expression (25). We obtain

$$\Delta v_L = \lambda \gamma v_0 \frac{1}{m_0 + \varepsilon_C} \left(1 + \frac{N_P}{N} \frac{m_0}{\varepsilon_C}\right) \theta_{I,A} \frac{\sigma_B^2}{2} (1 - \rho) \tag{26}$$

$$\Delta v_P = -\lambda \gamma v_0 \frac{1}{m_0 + \varepsilon_C} \left(1 + \frac{N_L}{N} \frac{m_0}{\varepsilon_C}\right) \theta_{I,A} \frac{\sigma_B^2}{2} (1 - \rho),$$

where ρ is the correlation between $\tilde{B}_D$ and $\tilde{B}_F$; the parameter v_0 corresponds to sales variance for $s \equiv s_0$; and $\lambda = [s_0/(1 + s_0)] \times [1 - (\sigma_A^2/v_0^2)]$.

From equation (26), we obtain that the differential change in volatility is given by

$$\Delta v_L - \Delta v_P = \frac{\lambda}{\varepsilon_C} v_0 \gamma \theta_{I,A} \frac{\sigma_B^2}{2} (1 - \rho).$$

From the above expression, it appears clearly that ρ needs to be small enough (i.e., diversification opportunities for the owners of listed firms are large enough). We need to have a measure of the sensitivity of size to risk-taking: λ/ε_C. To obtain it, we use the difference in volatility levels between listed and nonlisted firms that we observe in the data. This volatility difference is given by

$$\begin{aligned} v_L - v_P &= \lambda v_0 (\hat{s}_L - \hat{s}_P) \\ &= \frac{\lambda v_0}{\varepsilon_C} \gamma [\theta_{E,d} \sigma_d^2 + (\theta_{E,A} - \theta_{I,A}) \sigma_A^2]. \end{aligned} \tag{27}$$

Dividing one equation by the other, we obtain

$$\frac{\Delta v_L - \Delta v_P}{v_L - v_P} = \frac{\theta_{I,A} \frac{\sigma_B^2}{2} (1 - \rho)}{[\theta_{E,d} \sigma_d^2 + (\theta_{E,A} - \theta_{I,A}) \sigma_A^2]}.$$

We take $\rho = 0.2$, which is the mean correlation across GDP growth reported in Imbs (2004). We take $\sigma_B^2 = 0.0015$. In the model, $\tilde{B}$ is the shock to GDP, so we use French 1978–2004 data on GDP growth to compute aggregate variance. For simplicity, we assume that domestic productivity shocks $\tilde{A}$ have the same volatility as GDP shocks $\tilde{B}$ (in the model, domestic GDP is also affected by foreign shocks through the terms of trade effect so the volatilities of domestic and foreign shocks differ somewhat). As a measure of idiosyncratic risk, we take $\sigma_d^2 = 0.04$, which is the square of 20 percent, the mean sales growth volatility in our French data (variance of idiosyncratic risk is therefore 30 times bigger than the variance of aggregate shocks). We then move to wealth shares. From the Federal Reserve's Flow of Funds data, we obtain that directly and indirectly held equities are approximately
AQ 9 30 percent of US households' net worth (see Table B100e), so $\theta_{I,A} = 30$ percent.

From Moskowitz and Vissing-Jorgensen (2002), we take $\theta_{E,d} = \theta_{E,A} = 50$ percent. Combining these values leads to

$$\frac{\Delta v_L - \Delta v_P}{v_L - v_P} = 0.01.$$

To calibrate the induced differential change in variances, we need $v_L - v_P$. In our French sample, v_L is on average equal to $0.067 = (26 \text{ percent})^2$, while $v_P = 0.04$. This suggests that the differential change in volatility induced by radical financial globalization (from zero to full asset trading) is equal to $0.027 \times 0.01 = 0.0003$. This is nonnegligible. In our French sample, whether we take employment or sales, weighted or unweighted averages, the variance divergence over the period is about 0.0016 (i.e., the square of 4 percent). So the drastic "financial globalization" comparative static we perform here can generate, in our model and for reasonable parameter values, about 20 percent of the volatility divergence that we see in our French sample. Although this result should not be taken too literally, our very crude calculation suggests that there is enough scope for risk sharing across countries to explain a sizable fraction of the divergence in corporate risk-taking.

V. Conclusion

This paper is motivated by the fact that listed and nonlisted firms have experienced opposite trends in their volatility level over the past decades. Our starting point, econometric analysis on panel data of French firms, relies on an insight from the development literature. Risk sharing among investors should promote corporate risk-taking. We then extend existing models by (1) including a class of firms that do not benefit from risk sharing, and (2) modelling product market competition. We find that an increase in risk sharing, through capital market integration or rising stock market participation, can generate opposite trends in volatility for private and listed firms. The model is also used to investigate the impact of alternative determinants of firm volatility, such as an increase in product market size, an increase in the number of firms, or a decrease in aggregate volatility. All these alternative comparative statics generate counterfactual trends in firm volatility. An interesting by-product of our analysis is that, in spite of relying on CRRA utility functions, we are able to derive closed-form solutions. This allows us to investigate, in particular, the impact of financial globalization. In the future, the diverging trends that this paper seeks to explain could very well be reversed, with the volatility of privately held firms going up, and listed firm volatility going down. Indeed, against the backdrop of a thriving private equity market, many firms that used to be family controlled are now owned by funds whose investors (institutions such as pension funds or insurance companies) are well diversified. As a result from this change in their ownership structure, privately held firms may be induced to take more risk. If this happens, our model predicts that listed firms would be induced to reduce risk-taking, as competitive pressures on labor and product markets would make it less attractive for them to do so. The competitive advantage of listed firms (having diversified owners) would diminish, and volatilities should reconverge. We plan on studying these developments in future research.

Mathematical Appendix

A. *Proofs in the Benchmark Model*

A Useful Lemma.—We first demonstrate a Lemma that will be used in most of the proofs of the baseline model.

LEMMA 3: *Assume $h(\tilde{A},\tilde{d})$ is positive and strictly increasing in $\tilde{A}$ and weakly increasing in $\tilde{d}$.*

Furthermore, assume that either (1) $\partial f(\tilde{A},\tilde{d})/\partial d = 0$ or (2) $f(\tilde{A},\tilde{d})$ is such that there exists a unique $d^*(\tilde{A})$, such that $0 = f(\tilde{A}, d^*(\tilde{A}))$.

Last assume that $\partial f(\tilde{A},d)/\partial\tilde{A} = -f(A^*,\tilde{d})h(A^*,d)/A^*$.

Then, $E[f(\tilde{A},\tilde{d})] = 0 \Rightarrow E[h(\tilde{A},\tilde{d}) \times f(\tilde{A},\tilde{d})] > 0$.

Assume first that $\partial f(\tilde{A},\tilde{d})/\partial d \neq 0$. By assumption, for each $\tilde{A}$

$$\begin{cases} \tilde{d} \leq d^*(\tilde{A}) \Rightarrow f(\tilde{A},\tilde{d}) \leq 0 \\ \tilde{d} > d^*(\tilde{A}) \Rightarrow f(\tilde{A},\tilde{d}) > 0, \end{cases}$$

and since $h(\tilde{A},\tilde{d})$ is decreasing in $\tilde{d}$, we can deduce that, for every $\tilde{A}$,

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$$\begin{cases} \tilde{d} \leq d^*(\tilde{A}) \Rightarrow f(\tilde{A},\tilde{d})h(\tilde{A},\tilde{d}) \leq f(\tilde{A},\tilde{d})h(\tilde{A},d^*(\tilde{A})) \\ \tilde{d} > d^*(\tilde{A}) \Rightarrow f(\tilde{A},\tilde{d})h(\tilde{A},\tilde{d}) \leq f(\tilde{A},\tilde{d})h(\tilde{A},d^*(\tilde{A})). \end{cases}$$

As a consequence it is easy to see that

$$E\left[f(\tilde{A},\tilde{d})h(\tilde{A},\tilde{d})\,|\,\tilde{A}\right] \leq \hat{f}(\tilde{A})h\left(\tilde{A},d^*(\tilde{A})\right), \tag{A1}$$

where $\hat{f}(\tilde{A}) \equiv E\left[f(\tilde{A},\tilde{d})\,|\,\tilde{A}\right]$.

At this stage, we need to show that there exists a unique A^*, such that

$$\tilde{A} \leq A^* \Leftrightarrow \hat{f}(\tilde{A}) \leq 0 = \hat{f}(A^*).$$

At least one A^* exists, by virtue of the intermediate value theorem and the fact that, by assumption, $E[\hat{f}(\tilde{A})] = 0$. This A^* is unique because $\hat{f}$ is locally increasing in $\tilde{A}$ in A^*. To see why, we compute the first derivative of $\hat{f}$: evaluated in A^*,

$$\frac{d\hat{f}}{dA}(A^*) = \int \frac{\partial f(A^*,d)}{\partial A}\,dG_d = -\frac{1}{A^*}\int f(A^*,d)h(A^*,d)\,dG_d,$$

by assumption. Then,

$$\frac{d\hat{f}}{dA}(A^*) \;\geq\; -\frac{1}{A^*}\left[\int_{d \leq d^*(A^*)} f(A^*, d)h(A^*, d)dG_d \right.$$

$$\left. + \int_{\tilde{d} > d^*(A^*)} f(A^*, d)h(A^*, d)dG_d \right]$$

$$\geq\; -\frac{h^*}{A^*}\int f\big(A^*, \tilde{d}\big)\, dG_d \;=\; -\frac{h^*}{A^*}\hat{f}(A^*) \;=\; 0,$$

where $h^* \equiv h(A^*, d^*(A^*))$. As a consequence, $\hat{f}$ is locally increasing function around A^*, thus A^* is unique.

Coming back to (A1), the existence and unicity of A^* implies that

$$\begin{cases} \tilde{A} \;\leq\; A^* \;\Rightarrow\; \hat{f}\big(\tilde{A}\big) \;\leq\; 0 \;\Rightarrow\; \hat{f}\big(\tilde{A}\big)h\big(\tilde{A}, d^*\big(\tilde{A}\big)\big) \;\leq\; \hat{f}\big(\tilde{A}\big)h^* \\ \tilde{A} \;\geq\; A^* \;\Rightarrow\; \hat{f}\big(\tilde{A}\big) \;\geq\; 0 \;\Rightarrow\; \hat{f}\big(\tilde{A}\big)h\big(\tilde{A}, d^*\big(\tilde{A}\big)\big) \;\leq\; \hat{f}\big(\tilde{A}\big)h^*. \end{cases}$$

Hence, we have an upper bound for $E\big[f(\tilde{A}, \tilde{d})h(\tilde{A}, \tilde{d})\big]$,

$$E\big[f\big(\tilde{A}, \tilde{d}\big)h\big(\tilde{A}, \tilde{d}\big)\big] \;\leq\; h^* E\big[\hat{f}\big(\tilde{A}\big)\big] \;=\; h^* E\big[f\big(\tilde{A}, \tilde{d}\big)\big] \;=\; 0,$$

which proves the Lemma if $\partial f(\tilde{A}, \tilde{d})/\partial d \neq 0$. If $\partial f(\tilde{A}, \tilde{d})/\partial d = 0$. The proof is the same, except that $\hat{f} = f$, so the first part is irrelevant (until equation (A1)).

PROOF OF LEMMA 1:

The FOC (11) may be rewritten as

$$\text{(A2)} \qquad 0 \;=\; E\left[U'\big(\tilde{R}_E\big)\left(\frac{\tilde{A}\,\tilde{d}_i}{M} \;-\; C'(s_P)\right)\right].$$

This maximization problem is well defined because the SOC of this problem is negative:

$$\text{(A3)} \qquad \text{SOC} \;\equiv\; E\left[-C''(s_P)U'\big(\tilde{R}_E\big) \;+\; \left(\frac{\tilde{A}\,\tilde{d}_i}{M} \;-\; C'(s_P)\right)^2 U''\big(\tilde{R}_E\big)\right] \;<\; 0.$$

Since $C'(0) = 0$, the first-order derivative is strictly positive in $s_P = 0$ and negative for large s_P. This ensures the existence of an interior solution.

Given the SOC, the first derivative of s_P w.r.t. M has the same sign as

$$\Delta \;\equiv\; \underbrace{-\frac{C'(s_P)}{M}EU'\big(\tilde{R}_E\big)}_{<\,0} \;-\; \frac{\gamma}{M}E\left[\underbrace{U'\big(\tilde{R}_E\big)\left(\frac{\tilde{A}\,.\,\tilde{d}_i}{M} \;-\; C'(s_P)\right)}_{\equiv f(\tilde{A}, \tilde{d})} \underbrace{\left(\frac{1}{\tilde{R}_E}\frac{d\tilde{R}_E}{dM}\right)}_{\equiv I(\tilde{A}, \tilde{d})}\right].$$

The first component in Δ is negative. It is straightforward to show that f and I satisfy the conditions required by Lemma 3. This proves the result.

PROOF OF LEMMA 2:

Given that $\tilde{R}_I$ is not affected by idiosyncratic risk, the FOC (12) may be rewritten as

$$0 = E\left[U'(\tilde{R}_I)\left(\frac{\tilde{A}\tilde{d}_i}{M} - C'(s_L)\right)\right]. \tag{A4}$$

The SOC of this problem is satisfied since $C'' > 0$. Since $C'(0) = 0$, the first-order derivative is strictly positive in $s_L = 0$ and negative for large s_L. This ensures the existence of an interior solution. It also ensures that profits are never negative. If they are in some states of nature, expected utility is equal to $-\infty$. What ensures that it can be greater than $-\infty$ is that it is positive for $s_L = 0$.

Step 1.—We first show that s_L is increasing in I. Given the second-order condition of this problem, it is the case if and only if

$$\Omega \equiv \frac{\partial}{\partial I}\left\{E\left[U'(\tilde{R}_I)\left(\frac{\tilde{A}\tilde{d}_i}{M} - C'(s_L)\right)\right]\right\} > 0;$$

rewriting Ω, we find

$$\Omega = \gamma E\left[\underbrace{U'(\tilde{R}_I)\left(\frac{\tilde{A}\tilde{d}_i}{M} - C'(s_L)\right)}_{\equiv f(\tilde{A})}\underbrace{\left(-\frac{1}{\tilde{R}_I}\frac{d\tilde{R}_I}{dI}\right)}_{\equiv I(\tilde{A})}\right].$$

It is easy to see that $I(\tilde{A})$ is positive and increasing in $\tilde{A}$. It is easy to see that f satsifies the properties required by Lemma 3. From the first-order condition, we know that $Ef(\tilde{A}) = 0$, which implies that $\Omega > 0$. This proves the first point of Lemma 2.

Step 2.—We then look at the conditions under which s_L is decreasing in M. Given the SOC of the problem, it is the case if and only if

$$\Omega \equiv -\frac{1}{M^2}E\left[U'(\tilde{R}_I)\,\tilde{A}\right] + \gamma E\left[U'(\tilde{R}_I)\left(\frac{\tilde{A}\tilde{d}_i}{M} - C'(s_L)\right)f(\tilde{A})\right] < 0$$

is negative. Notice that

$$f(\tilde{A}) = \frac{1}{M}\frac{N_L}{I}\left(\frac{\tilde{A}(1 + s\tilde{d}_i)}{M}\right)\left[\omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\left(\frac{\tilde{A}(1 + s_i\tilde{d}_i)}{M} - C(s_L)\right)\right]^{-1}$$
$$- \frac{1}{\sigma - 2}\frac{1}{M}\omega M^{\frac{1}{\sigma-2}}\left[\omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\left(\frac{\tilde{A}(1 + s_i\tilde{d}_i)}{M} - C(s_L)\right)\right]^{-1},$$

as I goes to infinity

$$f(\tilde{A}) = -\frac{1}{\sigma - 2}\frac{1}{M};$$

thus

$$\begin{aligned}\Omega &= -\frac{1}{M^2}E\left[U'\left(\tilde{R}_I\right)\tilde{A}\right] - \frac{1}{\sigma - 2}\frac{1}{M}E\left[I\left(\tilde{A}\right)\right] \\ &= -\frac{1}{M^2}E\left[U'\left(\tilde{R}_I\right)\tilde{A}\right] < 0,\end{aligned}$$

when I goes to infinity. By continuity, there exists an I_0 such that $I > I_0 \Rightarrow \Omega < 0$, which proves the proposition.

PROOF OF PROPOSITION 1:

First, notice that M is the solution of the fixed point problem (13). Given Lemmas 2 and 1 hold, the right-hand side of (13) is decreasing in M. Hence, the equilibrium M is unique and can be thought of as the intersection of the 45° line and the RHS of (13). We now prove the three points of Proposition 1.

(i) When ϕ increases, the RHS of equation (13) shifts up. This ensures that M increases. s_P depends on M only, so it decreases. If s_L was also decreasing, then M would also increase, which leads to a contradiction—s_L therefore increases.

(ii), (iii) Assume that N increases of L decreases. The RHS of (13) shifts upward. The equilibrium M increases, which reduces both s_P and s_L.

B. *Linearizing the Closed-Economy Model*

We log-linearize around their deterministic values the two Euler conditions (11)–(12) and the general equilibrium equation (13).

Log-Linearizing Euler Equations: The General Case.—Both Euler conditions take the following form $E[F(\tilde{\boldsymbol{\theta}}, \mathbf{x})] = 0$, where $F(\cdot)$ is differentiable; $\mathbf{x}$ is the vector of endogenous variables (s_P, s_L, M); $\tilde{\boldsymbol{\theta}}$ is the stochastic vector $(\tilde{A}, \tilde{d})$, which is distributed in the neighborhood of its mean $\boldsymbol{\theta}_0 = (1,1)$. A second-order Taylor expansion of the Euler condition in $\tilde{\boldsymbol{\theta}}$ around $\boldsymbol{\theta}_0$ leads to

$$\begin{aligned}0 &= E\left[F\left(\tilde{\boldsymbol{\theta}}, \mathbf{x}\right)\right] \\ &\simeq E\left[F(\boldsymbol{\theta}_0, \mathbf{x}) + \sum_i\left(\tilde{\theta}_i - \theta_{i,0}\right)\frac{\partial F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial\tilde{\theta}_i} + \frac{1}{2}\sum_{i,j}\left(\tilde{\theta}_i - \theta_{i,0}\right)\left(\tilde{\theta}_j - \theta_{j,0}\right)\frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial\tilde{\theta}_i\partial\tilde{\theta}_j}\right] \\ &= F(\boldsymbol{\theta}_0, \mathbf{x}) + \sum_{i,j}\frac{\sigma_{i,j}}{2}\frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x})}{\partial\tilde{\theta}_i\partial\tilde{\theta}_j},\end{aligned}$$

where $\sigma_{i,j}$ corresponds at the variance-covariance terms. Then we develop at the *first-order only* this equation in $\mathbf{x}$ around $\mathbf{x}_0$, and we find

$$0 = F(\boldsymbol{\theta}_0, \mathbf{x}_0) + \sum_k (x_k - x_{k,0}) \frac{\partial F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial x_k} + \sum_{i,j} \frac{\sigma_{i,j}}{2} \frac{\partial^2 F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j}$$

$$+ \sum_k (x_k - x_{k,0}) \frac{\sigma_{i,j}}{2} \frac{\partial^3 F(\boldsymbol{\theta}_0, \mathbf{x}_0)}{\partial \tilde{\theta}_i \partial \tilde{\theta}_j \partial x_k}.$$

In the previous equation, the terms $(x_k - x_{k,0})\sigma_{i,j}$ are dominated by the terms in $(x_k - x_{k,0})$ and $\sigma_{i,j}$, so we can ignore them. This also justifies why a first-order expansion in $\mathbf{x}$ is sufficient, while a second-order expansion in $\tilde{\boldsymbol{\theta}}$ is necessary (Cedric Tille and Eric Van Wincoop 2010). Moreover from the deterministic equilibrium FOC, we get $F(\boldsymbol{\theta}_0, \mathbf{x}_0) = 0$. This leads to the following approximated Euler equation

$$\text{(A5)} \qquad \sum_k x_{k,0}\, \hat{x}_k\, F'_{x_k}(\boldsymbol{\theta}_0, \mathbf{x}_0) = -\frac{1}{2} \sum_{i,j} \sigma_{i,j} F''_{\tilde{\theta}_i \tilde{\theta}_j}(\boldsymbol{\theta}_0, \mathbf{x}_0),$$

which also shows why the log deviation in equilibrium variables $\hat{x}_k$ are of the order of the variances $\sigma_{i,j}$ (hence, second-order in the log deviation of $\tilde{\theta}$).

Linearizing Equilibrium Conditions (11)–(12)–(13).—We start with the Euler equation of privately held firms:

$$F(\theta, \mathbf{x}) \equiv U'\left(\frac{\tilde{A}(1 + s_P \tilde{d}_i)}{M} - C(s_P)\right)\left(\frac{\tilde{A}.\tilde{d}_i}{M} - C'(s_P)\right).$$

By definition, $1/M_0 - C'(s_0) = 0$, and $R_{E,0} = (1 + s_0)/M_0 - C(s_0)$. Hence, the derivatives of F simplify into

$$F'_s(\boldsymbol{\theta}_0, \mathbf{x}_0) = -C''(s_0)\, U'(R_{E,0})$$

$$F'_M(\boldsymbol{\theta}_0, \mathbf{x}_0) = -\frac{1}{M_0^2} U'(R_{E,0});$$

and

$$F''_{AA}(\boldsymbol{\theta}_0, \mathbf{x}_0) = 2\frac{1 + s_0}{M_0^2} U''(R_{E,0})$$

$$F''_{dd}(\boldsymbol{\theta}_0, \mathbf{x}_0) = 2\frac{s_0}{M_0^2} U''(R_{E,0}).$$

Using formula (A5) and the fact that $\sigma_{Ad} = 0$, we find that

$$U'(R_{E,0})\left(C''(s_0)\, s_0\, \hat{s} + \frac{\hat{M}}{M_0}\right) = \frac{U''(R_{E,0})}{M_0}\left(\frac{1 + s_0}{M_0}\sigma_A^2 + \frac{s_0}{M_0}\sigma_d^2\right),$$

given that $C'(s_0) = 1/M_0$; and rearranging we find

$$\left(\varepsilon_C \hat{s} + \hat{M}\right) = -\gamma \underbrace{\left(\frac{1}{R_{E,0}} \frac{1 + s_0}{M_0} \sigma_A^2 + \frac{1}{R_{E,0}} \frac{s_0}{M_0} \sigma_d^2\right)}_{\equiv \Omega_P}$$

with $\varepsilon_C = C''(s_0) s_0 / C'(s_0)$.

The derivation of $\hat{s}_L$ is similar, and we skip it to save space. And the equilibrium on the labor market is given by (13):

$$M_0(1 + \hat{M}) = \frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} \left[1 + s_0 + s_0 \frac{N_L}{N} \hat{s}_L + s_0 \frac{N_P}{N} \hat{s}_P\right]^{\frac{\sigma-2}{\sigma-1}}$$

$$\simeq \underbrace{\frac{\sigma}{L} N^{\frac{\sigma-2}{\sigma-1}} (1 + s_0)^{\frac{\sigma-2}{\sigma-1}}}_{\equiv M_0} \left[1 + \underbrace{\frac{\sigma - 2}{\sigma - 1} \frac{s_0}{1 + s_0}}_{\equiv m_0} \left(\frac{N_L}{N} \hat{s}_L + \frac{N_P}{N} \hat{s}_P\right)\right],$$

which leads to the expression of $\hat{M}$ in the text.

Additional Equilibrium Variables.—Expected profits of entrepreneurs are given by

$$E\tilde{\pi}_P = \frac{1}{M}(1 + s_P) - C(s_P)$$

$$\simeq \frac{1 + s_0}{M_0} \left(1 - \hat{M}\right)\left(1 + \frac{s_0}{1 + s_0} \hat{s}_P\right) - C(s_0) - C'(s_0) s_0 \hat{s}_P$$

$$\simeq \pi_0 + \hat{s}_P s_0 \underbrace{\left(\frac{1}{M_0} - C'(s_0)\right)}_{=0 \text{ by definition}} - \frac{1 + s_0}{M_0} \hat{M},$$

which proves the result. The computation for $E\tilde{\pi}_L$ is identical.

The expected utility of entrepreneurs is given by

$$EU(\tilde{\pi}_P) \simeq U(E\tilde{\pi}_P) + \frac{1}{2} U''(E\tilde{\pi}_P) V\tilde{\pi}_P,$$

with

$$\tilde{\pi}_P \simeq \pi_0 - \frac{1 + s_0}{M_0} \hat{M} + \frac{1 + s_0}{M_0} \hat{A} + \frac{s_0}{M_0} \hat{d} + \frac{s_0}{M_0} \hat{A} \hat{d};$$

hence,

$$V\tilde{\pi}_P = \left(\frac{1 + s_0}{M_0}\right)^2 \sigma_A^2 + \left(\frac{s_0}{M_0}\right)^2 \sigma_d^2.$$

Plugging $V\tilde{\pi}_P$ and $E\tilde{\pi}_P$ back into the utility formula we get:

$$EU(\tilde{\pi}_P) \simeq U(\pi_0) - U'(\pi_0)\frac{1 + s_0}{M_0}\hat{M}$$

$$- \frac{1}{2}U''(E\tilde{\pi}_P)\left[\left(\frac{1 + s_0}{M_0}\right)^2 \sigma_A^2 + \left(\frac{s_0}{M_0}\right)^2 \sigma_d^2\right]$$

$$\simeq U(\pi_0)\begin{bmatrix} 1 - (1 - \gamma)\frac{1}{R_{E,0}}\frac{1 + s_0}{M_0}\hat{M} \\ -\frac{1}{2}\gamma(1 - \gamma)\left[\left(\frac{1}{R_{E,0}}\frac{1 + s_0}{M_0}\right)^2 \sigma_A^2 + \left(\frac{1}{R_{E,0}}\frac{s_0}{M_0}\right)^2 \sigma_d^2\right]\end{bmatrix},$$

the differentiation of the other expected utilities follows similar lines and we do not report them to save space.

The equity premium is derived from the asset pricing condition (8):

$$\lambda\rho = E\big[U'(\tilde{R}_I)\,\tilde{\pi}_L\big].$$

To fix λ, we use the demand for safe asset:

$$\lambda = E\big[U'(\tilde{R}_I)r\big].$$

Hence, the equilibrium price is given by

$$E\big[U'(\tilde{R}_I)(\tilde{\pi}_L - r\rho)\big] = 0.$$

Given that the supply of safe asset is negligible, investor's income is still given by

$$\tilde{R}_I = \omega M^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\left(\frac{\tilde{A}(1 + s_L)}{M} - C(s_L)\right)$$

$$\simeq \omega M_0^{\frac{1}{\sigma-2}} + \frac{\omega}{\sigma - 2}M_0^{\frac{1}{\sigma-2}}\hat{M} + \frac{N_L}{I}\left(\pi_0 - \frac{1 + s_0}{M_0}\hat{M} + \frac{1 + s_0}{M_0}\hat{A}\right)$$

$$\simeq \underbrace{\left(\omega M_0^{\frac{1}{\sigma-2}} + \frac{N_L}{I}\pi_0\right)}_{\equiv R_{I,0}}\left[1 - \underbrace{\frac{1}{R_{I,0}}\left(\frac{\omega}{\sigma - 2}M_0^{\frac{1}{\sigma-2}} - \frac{N_L}{I}\frac{1 + s_0}{M_0}\right)}_{\equiv\frac{1}{\sigma-2}\theta_{I,W}-\theta_{I,A}}\hat{M}\right.$$

$$\left. + \frac{1}{R_{I,0}}\frac{N_L}{I}\frac{1 + s_0}{M_0}\hat{A}\right].$$

Differentiating the asset pricing condition,

$$\begin{aligned}
0 &\simeq E\Big[\big(U'(R_{I,0}) + U''(R_{I,0})\,(R_I - R_{I,0})\big)\,(\pi_0\,\hat{\pi}_L - r\,\rho_0\hat{\rho})\Big] \\
&\simeq E\left[\left(1 + \gamma\left(\frac{1}{\sigma - 2}\theta_{I,W} - \theta_{I,A}\right)\hat{M} - \gamma\,\theta_{I,A}\,\hat{A}\right)(\pi_0\,\hat{\pi}_L - r\,\rho_0\,\hat{\rho})\right] \\
&\simeq E\left[\pi_0\,\hat{\pi}_L - r\,\rho_0\,\hat{\rho} - \gamma\theta_{I,A}\frac{1 + s_0}{M_0}\,\hat{A}^2\right] \\
&\simeq -\frac{1 + s_0}{M_0}\,\hat{M} - r\,\rho_0\,\hat{\rho} - \gamma\theta_{I,A}\frac{1 + s_0}{M_0}\sigma_A^2.
\end{aligned}$$

Hence,

$$\begin{aligned}
EP &= \frac{E\tilde{\pi}_L - r\rho}{\rho} \\
&\simeq \frac{\pi_0}{\rho_0}\,\hat{\pi}_L - r\hat{\rho} \\
&\simeq r\gamma\theta_{I,A}\frac{1}{r\rho_0}\frac{1 + s_0}{M_0}\,\sigma_A^2,
\end{aligned}$$

since $\pi_0 = r\rho_0$. This proves the result reported in the text.

Comparative Statics in the Closed Economy.—

Step 1.—PROOF OF PROPOSITION 2:

This comparative static is sensible because the deterministic equilibrium $(s_{0,}s_0, M_0)$ is unaffected by ϕ. Ω_L is a decreasing function of I, since

$$\theta_{I,A} = \frac{1}{r + \frac{N_L}{I}\left(\frac{1 + s_0}{M_0} - C(s_0)\right)}\left(\frac{N_L}{I}\frac{1 + s_0}{M_0}\right)$$

is decreasing in I, while Ω_P is unaffected by changes in I. The first two points of Proposition 2 derive from equations (19)–(18). From (20), market pressure increases, which raises the real hourly wage given by (6). Results on profit and utilities follow.

Step 2.—PROOF OF PROPOSITION 3:

The deterministic equilibrium is unaffected by shifts in σ_A^2. Since $\theta_{I,A} < \theta_{E,A}$, $\Omega_P - \Omega_L$ is an increasing function of σ_A. Hence, given that $\hat{s}_P$ depends on $-\Omega_P$ and $-(\Omega_P - \Omega_L)$, $\hat{s}_P$ is a decreasing function of σ_A^2. Written in terms of σ_A^2, equation (19) rewrites as

$$\hat{s}_L = \gamma\frac{m_0}{m_0 + \varepsilon_C}\left[\left(\theta_{E,A}\frac{m_0}{\varepsilon_C}\frac{N_P}{N} - \left(\frac{m_0}{\varepsilon_C}\frac{N_P}{N} + 1\right)\theta_{I,A}\right)\sigma_A^2 + \frac{m_0}{\varepsilon_C}\frac{N_P}{N}\theta_{E,d}\,\sigma_d^2\right],$$

the condition stated in the proposition ensures that the term in front of σ_A^2 is positive. Last, since $\hat{M}$ is a decreasing function of Ω_P and Ω_L, it is also decreasing in σ_A^2.

C. *Resolution of the Open Economy Model*

Computing Profits.—In period 3, consumers can consume both domestic and foreign goods, without restriction. Let set

$$\Delta_c \equiv \int \tilde{\delta}_j^c \, dj$$

$$\Theta \equiv \left[\Delta_D^{1/\sigma} \tilde{L}_D^{(\sigma-1)/\sigma} + \Delta_F^{1/\sigma} \tilde{L}_F^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

for $c = D, F$. Then, optimizing profits and writing down the labor market equilibrium, we find that

$$\frac{p_c}{P} = \frac{\sigma}{\sigma - 1} \frac{w_c}{P}$$

$$\frac{E_D + E_F}{P} = \Theta$$

$$\frac{w_c}{P} = \frac{\sigma - 1}{\sigma} \left(\frac{\Delta_c}{\tilde{L}_c}\right)^{1/\sigma} \Theta^{1/\sigma}.$$

We solve the equilbrium by assuming that all listed firms in the domestic and foreign country choose the same strategies s_L for listed firms, and s_P for nonlisted firms. Implicitly, we are restricting our analysis to symmetric equilibria. Under these conditions, the profit of firm i is given by equation (22).

Financial Autarky.—Under financial autarky, s_P and s_L are given by the new first-order conditions:

$$0 = E\left[U'\left(\frac{\tilde{B}_D(1 + s_P \tilde{d}_i)}{M} - C(s_P)\right)\left(\frac{\tilde{B}_D \,.\, \tilde{d}_i}{M} - C'(s_P)\right)\right]$$

$$0 = E\left[U'\left(\frac{N_L}{I}\left(\frac{\tilde{B}_D(1 + s_L)}{M} - C(s_L)\right)\right)\left(\frac{\tilde{B}_D}{M} - C'(s_L)\right)\right],$$

which is formally identical to the Euler conditions in the closed economy, except that the aggregate shock is now $\tilde{B}_D$ instead of $\tilde{A}_D$. Hence, the linearization of these conditions is identical. The only difference is that $E\tilde{B}_D = 1 + (1/2 - 1/4\sigma).\sigma_A^2 > 1$ even though $E\tilde{A}_D = 1$, but the first-order term vanishes in the Taylor expansion.

International Trade.—The problem only changes for investors. In period 2, domestic investors now solve the following optimization problem:

$$\max EU\left(\int x_i \pi_{D,i}\, di \;+\; \int x_j \pi_{F,j}\, dj\right)$$

$$\text{s.t.} \int x_i \rho_{D,i}\, di \;+\; \int x_j \rho_{F,j}\, dj \;<\; E,$$

where E is their endowment; $E = \rho_D N_L/I$. Each domestic firm's stock solves

$$E\left[U'\left(\tilde{R}_I\right)\pi_{D,i}\right] \;=\; \lambda \rho_i;$$

the equilibrium will be symmetric, so that all investors (domestic or foreign) will hold a fraction $1/2I$ of each firm. Domestic and foreign firms will choose the same strategies. Hence,

$$\tilde{R}_I \;=\; \frac{N_L}{I}\left(\left(\frac{\tilde{B}_D \;+\; \tilde{B}_F}{2}\right)\frac{(1 \;+\; s_L)}{M} \;-\; C(s_L)\right).$$

Maximizing ρ_i with respect to s_i amounts to solving

$$E\left[U'\left(\frac{N_L}{I}\left(\left(\frac{\tilde{B}_D \;+\; \tilde{B}_F}{2}\right)\frac{(1 \;+\; s_L)}{M} \;-\; C(s_L)\right)\right)\left(\frac{\tilde{B}_D}{M} \;-\; C'(s_L)\right)\right] \;=\; 0;$$

using formula (A5) requires to compute the various derivatives

$$F'_s(\boldsymbol{\theta}_0, \mathbf{x}_0) \;=\; -C''(s_0)\, U'(R_{E,0})$$

$$F'_M(\boldsymbol{\theta}_0, \mathbf{x}_0) \;=\; -\frac{1}{M_0^2}\, U'(R_{E,0})$$

and

$$F''_{DD}(\boldsymbol{\theta}_0, \mathbf{x}_0) \;=\; 0$$

$$F''_{DB}(\boldsymbol{\theta}_0, \mathbf{x}_0) \;=\; 2\frac{1 \;+\; s_0}{M_0^2}\, U''(R_{E,0})$$

$$F''_{BB}(\boldsymbol{\theta}_0, \mathbf{x}_0) \;=\; 0.$$

Hence the linearized FOC is given by

$$U'(R_{E,0})\left(C''(s_0)s_0\, \hat{s}_L \;+\; \frac{\hat{M}}{M_0}\right) \;=\; \frac{U''(R_{E,0})}{M_0}\left(\frac{1 \;+\; s_0}{M_0}\, \sigma_{DB}\right),$$

which leads to the expression in the text.

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