Online Appendix to "Valuing Life as an Asset, as a Statistic, and at Gunpoint"[1]

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[1]This appendix is made available through the *Economic Journal*'s webpage. It complements the paper with a more extensive literature review, proofs, additional theoretical results, as well as empirical details and robustness tests.

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A Contributions to the life valuation literature

This section reviews our paper's contributions to the Human Capital, Statistical Life and Gunpoint life values, as well as to the theoretical approaches in life valuation. For additional perspective, we provide a summary of the main features of the HK, VSL and GPV models in Table A.1.

Table A.1: HK, VSL and GPV life valuations

	Human Capital (HK)	**Value Statistical Life (VSL)**	**Gunpoint (GPV)**
Theory	Asset pricing	Marginal rate of substitution Marginal willingness to pay	Hicksian variation Willingness to pay
Method	$v_{ht}^j = \mathrm{E}_t \left\{ \sum_{s=0}^{T_m} m_{t,t+s} D(H_{t+s}^j) \right\}$ $D(H_t^j) = Y(H_t^j) - I_t^j$	$\left\{ v^j(\Delta) \right\}_{j=1}^n$ for $\Delta = 1/n$ $v_s^e(\Delta) = \sum_{j=1}^n v^j(\Delta) \approx \frac{v(\Delta)}{\Delta}$	$\mathcal{P} = \Pr(\text{Death})$ $V(W - v_g, H, \mathcal{P}) = V(W, H, 1)$
Valued life	Identified	Unidentified (statistical)	Identified
Proxies	- Labour income	- Responses to fines - Wage-fatality nexus, ...	**This paper**: - Consumption, portfolios, - Health spending, insurance
Applications	Fatality risk pricing/litigation - Occupational - End users	Public safety, health - Transportation - Pollution control	End of life -Terminal care Insur. irrepl. losses Hedonic damages
Values	300K\$–900K\$ (Huggett and Kaplan, 2016) **This paper**: 300 K\$	4.2M\$–13.7M\$ (Robinson and Hammitt, 2016) **This paper**: 4.98 M\$	**This paper**: 251 K\$
Issues	- Non-workers Y^j - Rate of discounting $m_{t,t+s}$ - Endogeneity div./surv. D^j, T_m	- Exogeneity death risk Δ - Agency, non-payers v^j - Linearity/aggreg. prefs. - Stat. vs identified life	- computation v_g - finite values v_g - linearity/concavity v

A.1 Human Capital values of life

As illustrated in the first column of Table A.1, the HK model draws from asset pricing theory to compute the economic value of an identified person j by pricing his expected discounted lifetime net cash-flow stream.[1] That dividend $D(H^j)$ is the agent's income $Y(H^j)$, net of investment expenses $I(H^j)$ to maintain his human capital H^j. Well-known issues include accounting for the distribution of stochastic dividends, defining the appropriate discount factor $m_{t,t+s}$ which is compatible with the investment opportunity

[1] See Kiker (1966) for historical perspective on HK valuation.

set, as well as the endogeneity of the agent's income and investment. Moreover, the endogeneity of the duration T^m of the dividends flow is an issue, with pricing non-income activities (e.g. leisure among the elderly) also associated with HK challenges.

Huggett and Kaplan (2013, 2016) abstract from capital investment I^j entirely and calculate a HK value by discounting an exogenous income stream $D(H^j) = Y(H^j)$ using an agent-specific stochastic discount factor $m_{t,t+s}$ induced by the agent's optimal consumption and portfolio decisions, i.e. the agent's IMRS evaluated at the optimal plan. Using estimated distributional parameters for income, and calibrated preferences parameters, they find that the HK value is hump-shaped in the life cycle, peaking at mid-life, and much lower than that implied by (naive) discounting at the risk-free rate $m_{t,t+s} = (1+r)^{-s}$.[2] They attribute the differences to correlation between the agent's SDF and the income processes, i.e. $\text{Cov}(m_{t,t+s}, Y_{t+s}) < 0$, and to corner solutions at the risk-free rate for younger households' portfolio decisions, that both induce heavier discounting of the dividends flow.

As for Huggett and Kaplan (2013, 2016), we compute the capital value of an income stream. Furthermore, we also rely on recursive preferences to compute optimal consumption and portfolio decisions. However, we focus on the endogenous net dividends stream, where neither income nor investment expenses are exogenously set, but where both are solved in closed form. Moreover, we follow Asset Pricing theory by valuing the human capital dividends stream using the *market-based* and not agent-specific stochastic discount factor, and where the SDF is stemming from the investment opportunity set that is considered in the model in order to guarantee full theoretical consistency. Consequently, the subtraction of investment in our case lowers the capitalized value of the dividends flow whereas the market SDF being orthogonal to the agent's idiosyncratic net income flow will increase the HK value. Furthermore, Huggett and Kaplan (2013, 2016) also rely on PSID data, but do so to estimate the income forcing process parameters only; preferences parameters are calibrated ex-post to compute the HK value and are not confronted with other variables. In contrast, our empirical approach is much more structurally-oriented; we use PSID data on consumption, portfolio, income, health investment and insurance,

[2] For high-school workers with low risk aversion ($\gamma = 4$), Huggett and Kaplan (2016, Fig. 4, p. 34) find a HK value of 300K\$ at age 20, 1.1M\$ at age 40 and 500K\$ at age 60. Those values increase to respectively 700K\$, 1.8M\$, and 800K\$ for college graduates. In comparison, the constant risk-free discounting continuously falls from a peak of 2.2M\$ at age 20 to 700K\$ at age 60 for high-school graduates, and from 40M\$ to 1.2M\$ for college graduates.

combined with health and wealth statuses data to structurally estimate the exogenous forcing processes, human capital technology, distributional and preferences parameters. Finally, our parametrized model is fully adaptable to non-labour valuation since the flow of marketed income related to human capital can also be equivalently recast as non-marketed utilitarian services (see Appendix C.3). We also show how the model can be adapted for explicit modelling of endogenous work/leisure decisions (see Section C.1). Both elements are important to calculate HK-inspired valuations for non-working agents.

A.2 Value of a Statistical Life

Empirical VSL The vast VSL literature was initiated by Drèze (1962) and Schelling (1968). In column 2 of Table A.1, the Value of a Statistical Life measures a societal marginal rate of substitution between additional life and wealth, also corresponding to its marginal willingness to pay for additional longevity. As a canonical example (e.g. Aldy and Viscusi, 2007), suppose agents $j = 1, 2, \dots n$ are individually willing to pay $v^j = v(\Delta)$ to attain (avert) a small beneficial (detrimental) change $\Delta = 1/n$ in death risk exposure and satisfying $v(0) = 0$. The empirical VSL is the collective willingness to pay $v_s^e(\Delta) = nv(\Delta) = v(\Delta)/\Delta$, corresponding to the slope of the WTP function and approximating the MWTP $v'(\Delta) = \lim_{\Delta \to 0} v(\Delta)/\Delta$.

The empirical VSL alternative relies on explicit and implicit evaluations of the Hicksian WTP v^j for a small reduction Δ in fatality risk which is then linearly extrapolated to obtain the value of life. Explicit VSL uses stated preferences for mortality risk reductions obtained through surveys or lab experiments, whereas implicit VSL employs a revealed preference perspective in using decisions and outcomes involving fatality risks to indirectly elicit the Hicksian compensation.[3] Examples of the latter include responses to prices and fines in the use of life-saving measures such as smoke detectors, speed limitations, or seat belt regulations. The Hedonic Wage (HW) variant of the implicit VSL evaluates the equilibrium willingness to accept (WTA) compensation in wages for given increases in work dangerousness. Controlling for job/worker characteristics, the wage elasticity with respect to job fatality risk can be estimated and again extrapolated linearly to obtain the VSL (e.g. Aldy and Viscusi, 2008; Shogren and Stamland, 2002).

[3] A special issue directed by Viscusi (2010) reviews recent findings on VSL heterogeneity. A meta analysis of the implicit VSL is presented in Bellavance et al. (2009). See also Doucouliagos et al. (2014) for a *meta*-meta analysis of the stated- and revealed-preferences valuations of life.

Ashenfelter (2006) provides a critical assessment of the VSL's theoretical and empirical underpinnings. He argues that the assumed exogeneity of the change in fatality risk Δ can be problematic. For instance, safer roads will likely result in faster driving, which will in turn increase the number of fatalities. He also argues that agency problems might arise and lead to overvaluation in cost-benefit analysis when the costs of safety measures are borne by groups other than those who benefit (see also Sunstein, 2013; Hammitt and Treich, 2007, for agency issues). Ashenfelter further contends that it is unclear whose preferences are involved in the risk/income tradeoff and how well these arbitrage are understood. For example, if high fatality risk employment attracts workers with low risk aversion and/or high discount rates, then generalizing the wages risk gradient to the entire population could understate the true value of life. An argument related to Ashenfelter's preferences indeterminacy can be made for the HW variant of the VSL. Because wages are an equilibrium outcome, they encompass both labour demand and supply considerations with respect to mortality risk. Hence, a high death risk gradient in wages could reflect high employer aversion to the public image costs of employee deaths, as much as a high aversion of workers to their own death.

Our approach addresses many of the issues raised by Ashenfelter (2006). First, we fully allow for endogenous adjustments in the optimal allocations resulting from changes in death risk exposure when we compute the willingness to pay and the VSL. Second, agency issues are absent as the agent bears the entire costs and benefits of changes in mortality. Third, whose preferences are at stake is not an issue as the latter are jointly estimated with the WTP and life valuations by resorting to a widely-used panel of households (PSID). Consequently, these values can safely be considered as representative of the general population. Fourth, labour demand considerations are absent as our partial equilibrium approach takes the return on investment as mortality-risk independent in characterizing the agent's optimal human capital allocations. More fundamentally, we neither rely on the wage/fatality nexus, nor on any other proxy and we make no assumption on the shape of the WTP function but rather derive its properties from the indirect utility function induced by the optimal allocation.

Our results also confirm early conjectures on the pitfalls associated with personalizing unidentified VSL life valuations. Indeed, Pratt and Zeckhauser (1996) argue that concentrating the costs and benefits of death risk reduction leads to two opposing effects on valuation. On the one hand, the *dead anyway* effect leads to higher payments on identified (i.e. small groups facing large risks), rather than statistical (i.e. large groups facing small risks) lives. In the limit, they contend that an individual might be willing to pay infinite amounts to save his own life from certain death. On the other hand, the wealth or *high payment* effect has an opposite impact. Since resources are limited, the marginal utility of wealth increases with each subsequent payment, thereby reducing the marginal WTP as mortality exposure increases.[4] Although the net effect remains uncertain, Pratt and Zeckhauser (1996, Fig. 2, p. 754) argue that the wealth effect is dominant for larger changes in death risk, i.e. for those cases that naturally extend to our Gunpoint threat. Their conjecture is warranted in our calculations. We show that the willingness to pay is finite and bounded above by the Gunpoint Value. Diminishing MWTP entails that the latter is much lower than what can be inferred from the VSL.

Theoretical models of VSL Hall and Jones (2007) propose a semi-structural measure of life value akin to the Value of a Statistical Life. They adopt a marginal value perspective by equating the VSL to the marginal cost of saving a human life. In their setting, the cost of reducing mortality risk can be imputed by estimating a health production function and by linking health status to death risks. Dividing this marginal cost by the change in death risk yields a VSL-inspired life value. Unlike Hall and Jones (2007) we do not measure the health production function through its effects on mortality, but estimate the technology through the measurable effects of investment on future health status. Indeed, mortality is treated exogenously in our baseline model. Moreover, our fully structural approach does not indirectly evaluate the marginal value of life via its marginal cost, but rather directly through the individual willingness to pay to avoid changes in death risks.

Finally, we share similarities with Murphy and Topel (2006) who resort to a life cycle model with direct utilitarian services of health to study life valuations. In particular, both continuous-time approaches study permanent changes in Poisson death intensity, under perfect markets assumption, and both identify the VSL as a marginal rate of substitution between longevity and wealth. Moreover, both emphasize the key role of the elasticity of inter-temporal substitution in generating diminishing marginal values. However, contrary

[4]Pratt and Zeckhauser (1996, p. 753) point out that whereas a community close to a toxic waste dump could collectively pay $1 million to reduce the associated mortality risk by 10%, it is unlikely that a single person would be willing to pay that same amount when confronted with that entire risk.

to Murphy and Topel (2006), our human capital (i.e. health) is endogenously determined in a stochastic environment, whereas we abstract from leisure (see however Appendix C.1 for an extension). The associated VSL, as well as other life measures, are all increasing in health, rather than health-independent. Importantly, whereas Murphy and Topel (2006) posit an arbitrary process for consumption (see eq. (19), p. 885) and restrict their analysis to hand-to-mouth in their calibration, we solve for optimal consumption, portfolio, insurance, and health expenditures. This allows us to analyse and structurally estimate all life valuations – including the HK, WTP and GPV that are abstracted from in Murphy and Topel (2006) – through the prism of the indirect utility function.

A.3 Gunpoint value of life

In column 3 of Table A.1, a Gunpoint value measures the maximal amount v^g an agent is willing to pay to remain at current death probability $\mathcal{P} \in (0, 1)$, rather than face instantaneous and certain death, i.e $\mathcal{P} \equiv 1$. Early references to a Gunpoint value include Jones-Lee (1974) who analyses the Hicksian Compensating Variation (CV) for changes in the probability of dying in a static setting. The extreme case where the latter tends to one corresponds to a willingness to accept compensation for imminent death. Jones-Lee (1974) shows that this WTA exists and is finite when the least upper bound on the utility at death (e.g. from bequeathed wealth) is large relative to reference expected utility. Our analysis abstracts from bequests and normalizes utility at death to zero, so that the Hicksian Equivalent Variation (EV), i.e. the WTP to avoid death is the appropriate Gunpoint measure and we show formally that it corresponds to the least upper bound on the WTP.

Other early references include Cook and Graham (1977) who study the demand for insurance against irreplaceable losses, defined as one where personal valuation considerations dominate market ones, i.e as having no readily identifiable market-provided replacement in the case of loss (e.g. a family pet, health, a spouse's, or a child's life). The willingness to pay to avoid this loss is defined as the *Ransom value*. If the ransom is a normal good (i.e. is increasing in wealth), Cook and Graham (1977) show that the state-dependent marginal utility of wealth, conditional on loss, is less than that of wealth minus ransom, conditional on no loss. The agent consequently optimally under-insures at actuarially fair contracts. Under sufficiently large wealth effects on ransom, the agent does not insure against the loss of the irreplaceable good, but against the associated wealth loss. For example, he then selects a life insurance against a spouse's death corresponding to foregone income (plus funeral expenses) that has clear analogs to the HK value. Finally, they show that the MRS between wealth and death (corresponding to the VSL) is necessarily larger than the Ransom value.

Eeckhoudt and Hammitt (2004) rely on this framework to focus on the impact of risk aversion on four measures of life value: the VSL, the WTP to fully eliminate death risk (i.e. $\mathcal{P} > 0 \rightarrow \mathcal{P}^* = 0$), or to partially lower it (i.e. $\mathcal{P} > 0 \rightarrow \mathcal{P}^* < \mathcal{P}_0$) and the WTP to eliminate the certainty of death (i.e. $\mathcal{P} = 1 \rightarrow \mathcal{P}^* = 0$). The latter corresponds to Cook and Graham (1977)'s Ransom value where the irreplaceable good is one's own life. In the special case where both the utility and marginal utility of wealth at death are zero (e.g. in the absence of bequest value), they confirm that the Ransom value is the agent's wealth and is independent of attitudes toward risk.

The Ransom value of Cook and Graham (1977); Eeckhoudt and Hammitt (2004) is clearly related to the Gunpoint value as both depend on Hicksian WTP to avoid certain death in gauging a person's own value. The main difference is that we do not rely on a generic utility, but instead we base our analysis on the indirect utility associated to a dynamic human capital problem to characterize the WTP, VSL and GPV. This approach allows us to encompass the HK value as well, to link the different measures and to fully identify the role of preferences, distributional and technological parameters on life valuation.

Implicit references to a GPV are also found in the context of end-of-life care. For example,

> "[the VSL] is often prefaced with claiming that it is not how much people are willing to pay *to avoid having a gun put to their head* (presumably one's wealth). However, terminal care decisions are often exactly of that nature"
>
> (Philipson et al., 2010, p. 2, emphasis added)

We confirm their conjecture that financial wealth is entirely pledged in a highwaymen threat, however we show that so is the agent's human wealth. Since our application associates the latter to health, we thus provide explicit adjustment for an agent's health status in his life valuations in the spirit of the Quality-Adjusted Life Years (QALY, e.g. Round, 2012). One could also argue that HK measures are inappropriate in terminal care situations where agents are unable to work. The GPV we propose handles such case by equivalently associating the value of health capital to the utilitarian services it can provide.

Murphy and Topel (2006) also implicitly refer to a Gunpoint value in their parametrized analysis of the value of a life year (i.e. utility and net savings at given age). Indeed, commenting on a key variable in their Value of Statistical Life Year (VSLY) analysis, they write that "[t]he ratio z_0/z asks how much of current composite consumption individuals would sacrifice before they would rather be dead" (p. 885). However, a closer analysis reveals that this ratio, which they calibrate between 5-20% of composite consumption, rather corresponds to a minimal consumption ratio in their non-homothetic VNM preferences. Whereas we show that the Gunpoint value i.e. the total wealth that leaves the agent indifferent between life and death corresponds to the expected discounted value of the *lifetime* consumption stream, and is therefore much larger than minimal consumption.

Finally, in addition to tangible costs, such as the HK values of lost net earnings, or the deceased's medical and funeral expenses, wrongful death litigation courts can also award compensation for intangible losses. The latter include survivors' pain and suffering from loss of the deceased's companionship (e.g. Peeples and Harris, 2015; Lewbel, 2003), as well as compensation for Hedonic Damages representing the value of the deceased's 'lost life pleasures' (see Posner and Sunstein, 2005; Karns, 1990; Smith, 1988, for discussions of legal aspects). Viscusi (2007, 2000); Raymond (1999) provide critical assessments of the erroneous association of Hedonic Damages with the VSL measures. Indeed, the latter better gauges a societal willingness to pay to save someone rather than a person's own valuation of his life. The HK life value is also inadequate as an utilitarian flow valuation in that it computes the market value of an agent's net income stream. In contrast, our GPV assesses the WTP that leaves the agent indifferent between life and death and is thus a direct measure of the monetary equivalent of life's continuation utility. We innovate by computing an integrated Gunpoint value that has so far proved elusive, and that accords with and complement the VSL and HK measures.

A.4 Theoretical models of life value

Integrated models Other researchers have offered encompassing approaches to life valuations. Jones-Lee (1974) proposes a static VNM framework, albeit without human capital considerations, and which focuses on the utility of wealth when alive and at death to analyse the WTP's properties. Marginal WTP for small changes in death risk yield the VSL whereas a Gunpoint-equivalent life value is studied through the willingness to accept compensation for certain death. Conley (1976); Shepard and Zeckhauser (1984); Rosen (1988) analyse Human Capital and Statistical Life values in a life cycle model with perfect and imperfect capital markets. These studies emphasize the role of the EIS and conclude that the VSL is much larger than the HK under reasonable assumptions. Our main contribution to these analyses are that we calculate and structurally estimate closed-form solutions to a much richer parametrized encompassing framework. In particular, we provide WTP, HK, VSL and GPV solutions under non-expected utility settings with endogenous stochastic human capital accumulation. These formulas are estimated under the full set of theoretical restrictions with a common data base.

Role of preferences Córdoba and Ripoll (2017) concur with us on the relevance of recursive preferences for life valuation. In particular, they emphasize the importance of disentangling attitudes towards risk, from those towards time. This separation allows for non-indifference with respect to the timing of the resolution of survival uncertainty, and guarantees preference for life over death, even at high risk aversion levels. They also contend that more realistic curvature of the willingness to pay for survival can only be attained by allowing non-linear effects of death probabilities on utility that are abstracted from under VNM preferences. Both their discussion and their calibration emphasize a preference for late, rather than early, resolution of death uncertainty, as well as a diminishing marginal willingness to pay for additional longevity (Córdoba and Ripoll, 2017, Sec. 2.2, and Tab. 1).

Despite these similarities, the parametrized model of Córdoba and Ripoll (2017) remains different from ours. Their closest analog in their Section 3.2 is set in discrete (rather than continuous) time, and lets the agent select consumption only. It fully abstracts from our analysis of endogenous human capital accumulation, stochastic capital depreciation, risky portfolio and insurance choices and their main solutions for life values are characterized for hand-to-mouth consumers only. Moreover they emphasize mortality risk aversion as key determinant of life values in an homethetic recursive preferences specification. In our setting, the agent is risk-neutral with respect to mortality risk, so that the elasticity of inter-temporal substitution is the main driver of mortality preferences, and we allow for non-homotheticity in a recursive utility setting by introducing minimal consumption requirements. Finally, whereas they obtain closed-form solutions for the VSL, they do not explicitly compute the WTP,[5] and fully abstract from both HK and GPV.

Bommier et al. (2019) also analyse the implications for life valuation of life cycle models of consumption and portfolio choices with recursive preferences. However, important differences remain between the two approaches. Indeed, Bommier et al. (2019) neither allow for human capital and insurance decisions, nor do they analytically solve their model, and therefore do not formally characterize how structural parameters and state variables affect a broad set of life valuations. More specifically, whereas we rely on the explicit solutions for the optimal human capital dynamics and the indirect utility to analyse the HK, WTP, VSL and GPV, Bommier et al. (2019) use the (unsolved) marginal utility of consumption and of death risk to discuss the implications for the VSL only. Moreover, Bommier et al. (2019) calibrate their model to fit the empirical VSL estimates, and *ex-post* assess the resulting life cycle paths of consumption, financial market participation and portfolio. Conversely, we structurally estimate the model by relying on a wide set of cross-equations theoretical restrictions in a multivariate econometric setting to fit the observed financial and human capital decisions, and *then* proceed to gauge the empirical implications for the life valuations.

We borrow from Hugonnier et al. (2013) for specifying and solving our human capital model. We consider a restricted case of that setup along two dimensions. First, while retaining their non-expected utility setting, we simplify our specification of preferences by abstracting from source-dependent risk aversion. Second, while maintaining stochastic sickness and death shocks, we abstract from self-insurance against these risks. Whereas our model is admittedly less general, one benefit is that our optimal rules are characterized in closed form, rather than as approximate solutions. Furthermore, our solutions are much

[5] More precisely, the WTP in their setup is simply the VSL times the change in death probability (see the equation before eq. (16)). Instead, we compute the WTP from Hicksian variational analysis and rely on its marginal and limiting properties to characterize the VSL and Gunpoint values.

more tractable, allowing us to pinpoint more clearly how distributional, preferences and technological parameters affect the values of life. More fundamentally, the focus of the two papers is much different. Indeed, the main emphasis of Hugonnier et al. (2013) is on the separation between financial and health-related choices, rather than on the value of life. Whereas they do consider the value of an additional year of longevity (Hugonnier et al., 2013, Tab. 6), they completely abstract from the HK, VSL, WTP, and Gunpoint values for which we provide and estimate analytical solutions. Finally, in a separate technical appendix (Hugonnier et al., 2021) we show that the main theoretical conclusions for that generalized model are maintained and that the empirical life values are of the same order of magnitude.

B Proofs

B.1 Theorem 1

The benchmark human capital model of Section 2 is a special case of the one considered in Hugonnier et al. (2013). In particular, the death and depreciation intensities are constant at λ_m, λ_s (corresponding to their order-0 solutions) and the source-dependent risk aversion is abstracted from (i.e. $\gamma_s = \gamma_m = 0$). Imposing these restrictions in Hugonnier et al. (2013, Proposition 1,Theorem 1) yields the the optimal solution in (14).

■

B.2 Proposition 1

The proof follows from Hugonnier et al. (2013, Prop. 1) which computes the value of the human capital $P(H)$ from

$$\begin{aligned} P(H) &= E_t \int_t^{\infty} \frac{m_\tau}{m_t} \left[\beta H_\tau^* - I_\tau^*\right] \mathrm{d}\tau, \\ &= BH. \end{aligned}$$

Straightforward calculations adapt this result to a stochastic horizon T^m and include the fixed income component y in income (4).

■

B.3 Proposition 2

Combining the Hicksian EV (19) with the indirect utility (13a) and using the linearity of the net total wealth in (12) reveals that the WTP v solves:

$$\Theta(\lambda_m^*)N(W,H) = \Theta(\lambda_m)N(W-v,H),$$
$$= \Theta(\lambda_m)\left[N(W,H)-v\right],$$

where we have set $\lambda_m^* = \lambda_m + \Delta$. The WTP $v = v(W,H,\lambda_m,\Delta)$ is solved directly as in (21).

Next, by the properties of the marginal value of net total wealth, $\Theta(\lambda_m^*)$ in (15) is monotone decreasing and convex in Δ. It follows directly from (20) that the WTP

$$v(W,H,\lambda_m,\Delta) = \left[1-\frac{\Theta(\lambda_m^*)}{\Theta(\lambda_m)}\right]N(W,H)$$

is monotone increasing and concave in Δ.

The lower bound follows directly from evaluating finite and admissible $A(\lambda_m^*),\Theta(\lambda_m^*)$ at $\lambda_m^* = 0$ in (21). To compute the upper bound, two cases must be considered:

1. For $0 < \varepsilon < 1$, the MPC in (10) is monotone decreasing and is no longer positive beyond an upper bound given by:

$$\lambda_m^* = \lambda_m + \Delta < \bar{\lambda}_m = \left(\frac{\varepsilon}{1-\varepsilon}\right)\rho + \left(r+\frac{\theta^2}{2\gamma}\right).$$

   Admissibility $\mathcal{A}$ therefore requires $\Delta < \bar{\Delta} = \bar{\lambda}_m - \lambda_m$ for the transversality conditions (10) to be verified. The supremum of the WTP is then $v(W,H,\lambda_m,\bar{\Delta}) = N(W,H)$.

2. For $\varepsilon > 1$, the MPC is monotone increasing and transversality is always verified. Consequently, the WTP is well-defined over the domain $\Delta \geq -\lambda_m$. It follows that:

$$\lim_{\Delta\to\infty}\Theta(\lambda_m+\Delta) = 0,$$
$$\lim_{\Delta\to\infty} v(W,H,\lambda_m,\Delta) = N(W,H),$$

i.e. the willingness to pay asymptotically converges to net total wealth as stated in (22b).

■

B.4 Proposition 3

By the VSL definition (23) and the properties of the Poisson death process (11):

$$v_s = \frac{-V_{\lambda_m}(W, H, \lambda_m)}{V_W(W, H, \lambda_m)}.$$

From the properties of the welfare function (13a), we have that $V_{\lambda_m} = \Theta'(\lambda_m)N(W, H)$, whereas $V_W = \Theta(\lambda_m)$. Substituting for Θ in (13b) yields the VSL in (24). ■

B.5 Proposition 4

Combining the Hicksian EV (27) with the indirect utility (13a) and the net total wealth in (12) reveals that the WTP v solves:

$$\begin{aligned} V^m \equiv 0 &= \Theta(\lambda_m)N(W - v_g, H), \\ &= \Theta(\lambda_m)\left[N(W, H) - v_g\right]. \end{aligned}$$

Solving for v_g reveals that it is as stated in (28). Because net total wealth is independent of the preference parameters $(\varepsilon, \gamma, \rho)$, so is the Gunpoint Value. ■

C Other theoretical results

C.1 Labour-leisure choices

Our model abstracts from work-leisure decisions. Appending the latter does not modify our main framework which can be interpreted as a reduced-form version with embedded optimal work-leisure choices. To see why, consider a modification along standard practices where the agent allocates a unit time endowment between paid work and valuable leisure,

$\ell \in [0,1]$, and replace income (4) and preferences (6c) with:

$$Y_t = y + \beta H_t + w(1-\ell), \tag{C.1a}$$

$$f(c,u) = \frac{\rho u}{1-1/\varepsilon}\left(\left(\frac{c-a+b\ln(\ell)}{u}\right)^{1-\frac{1}{\varepsilon}} - 1\right), \tag{C.1b}$$

where w is a wage and $b \in [0,w]$ denotes the strength of the preference for leisure.

Next, denote by $V(W,H)$ the value function and by V_i its derivatives with respect to $i = H, W$. The Hamilton-Jacobi-Bellman corresponding to the Human Capital model of Section 2.1 can be modified to allow for optimal work-leisure choices as follows:

$$\begin{aligned} 0 = \max_{\{c,\pi,x,I,\ell\}} & \frac{(\pi\sigma_S)^2}{2} V_{WW} + H\left[(I/H)^2 - \delta\right] V_H \\ & + \left[rW + \pi\sigma_S\theta - c + y + \beta H + w(1-\ell) - I - x\lambda_s\right] V_W \\ & + \frac{\rho V(W,H)}{1-\frac{1}{\varepsilon}}\left[\left(\frac{c-a+b\ln(\ell)}{V(W,H)}\right)^{1-\frac{1}{\varepsilon}} - 1\right] \\ & - \frac{\gamma\left(\pi\sigma_S V_W\right)^2}{2V(W,H)} - \lambda_m V(W,H) - \lambda_s V(W,H)\left[1 - \frac{V(W+x, H(1-\phi))}{V(W,H)}\right]. \end{aligned} \tag{C.2}$$

Under general separation principles (e.g. Basak, 1999), we can solve for optimal leisure ℓ^* in a first step, substitute back into the HJB, and solve for the other optimal controls in a second step. In particular, the first-step FOC's for consumption and leisure are respectively given as:

$$\begin{aligned} V_W &= \rho V(W,H)^{\frac{1}{\varepsilon}}\left(c-a+b\ln(\ell)\right)^{\frac{-1}{\varepsilon}}, \\ V_W w &= \rho V(W,H)^{\frac{1}{\varepsilon}}\left(c-a+b\ln(\ell)\right)^{\frac{-1}{\varepsilon}}\frac{b}{\ell}, \end{aligned}$$

dividing one by the other solves for optimal leisure as a constant share of the time endowment given by:

$$\ell^* = \frac{b}{w} \in [0,1]$$

under the restriction that $0 \leq b \leq w$. For the second step, substituting back ℓ^* into the HJB (C.2) reveals that the latter then becomes:

$$
\begin{aligned}
0 = & \max_{\{c,\pi,x,I\}} \frac{(\pi\sigma_S)^2}{2} V_{WW} + H\left[(I/H)^2 - \delta\right] V_H \\
& + \left[rW + \pi\sigma_S\theta - c + y^* + \beta H - I - x\lambda_s\right] V_W \\
& + \frac{\rho V(W,H)}{1-\frac{1}{\varepsilon}} \left[\left(\frac{c-a^*}{V(W,H)}\right)^{1-\frac{1}{\varepsilon}} - 1\right] \\
& - \frac{\gamma\left(\pi\sigma_S V_W\right)^2}{2V(W,H)} - \lambda_m V(W,H) - \lambda_s V(W,H)\left[1 - \frac{V(W+x, H(1-\phi))}{V(W,H)}\right],
\end{aligned}
$$

where

$$
y^* \equiv y + (w-b) \geq y, \quad a^* \equiv a - b\ln(b/w) \geq a,
$$

which is isomorphic to the HJB for the original problem. Consequently, the solutions in Theorem 1 remain valid, with (y, a) replaced by (y^*, a^*).

■

C.2 Health investment and out-of-pocket expenses

Our empirical strategy assumes a one-to-one relationship between investment I_t and out-of-pocket medical spending in the PSID data-set. Two reasons suggest why this might not be the case. First, the individual co-payments are only a share of total medical expenses for health-insured agents. Second, this assumption entails that all of out-of-pocket expenditures have beneficial effects on H_t. However, one may argue that at least part of the uninsured health expenditures, especially with respect to dental or home care is more attributable to consumption, than to actual investment in one's health.

We can consider the case of discrepancies between OOP, O_t and investment, $I_t = \psi O_t$, where $\psi < 1$ captures non-investment (e.g. consumption) components in health expenses, $\psi > 1$ captures co-payment rates for insured agents, and our benchmark model imposes $\psi = 1$. For the modified human capital dynamics:

$$
\mathrm{d}H_t = \left[\Psi O_t^\alpha H_t^{1-\alpha} - \delta H_t\right] \mathrm{d}t - \phi H_t \mathrm{d}Q_{st}, \quad \Psi = \psi^\alpha.
$$

The Hamilton-Jacobian-Bellman (HJB) corresponding to our problem is:

$$r\tilde{P} = \beta H + \lambda_s \left[\tilde{P}_H(1-\phi)H - \tilde{P}(H)\right] + \max_{\{O\}} \left\{ \left[\Psi\left(\frac{O}{H}\right)^{\alpha} - \delta\right] H\tilde{P}_H - O \right\}.$$

Solving the FOC, and using candidate solution for the human wealth $\tilde{P}(H) = \tilde{B}H$ reveals that out-of-pocket expenditures O are proportional to health:

$$O = \left(\Psi\alpha\tilde{B}\right)^{\frac{1}{1-\alpha}} H.$$

Substituting back into the HJB shows that the shadow price $\tilde{B}$ must satisfy:

$$(r+\delta+\phi\lambda_s)^{\frac{1}{\alpha}} > \beta, \tag{C.3a}$$

$$\beta - (r+\delta+\phi\lambda_s)\tilde{B} - (1-1/\alpha)(\Psi\alpha\tilde{B})^{\frac{1}{1-\alpha}} = 0, \tag{C.3b}$$

$$r+\delta+\phi\lambda_s > \Psi^{\frac{1}{1-\alpha}}(\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}}. \tag{C.3c}$$

As for the benchmark model, the expressions for net total wealth are obtained with the modified human wealth:

$$\tilde{N}(W,H) = W + \frac{y-a}{r} + \tilde{P}(H),$$

with the expression for the valuations remaining valid and using $\tilde{N}, \tilde{P}$.

When evaluated at our benchmark parameter estimates and at the mean health and wealth levels, we find minimal effects on the Tobin's-q, human wealth and net total wealth in Table C.1. This allows us to conclude that the effects on the main determinants of life valuations are limited at best.

C.3 Other health services

The model assumes that the sole motivation for investing in H_t relates to its positive effects on marketed income in (4). However, the valuation of human capital can also be made with respect to its non-marketed utilitarian services. Indeed, the model can be adapted for non-workers by first defining $\tilde{c}_t \equiv c_t - \beta H_t$, and rewriting the budget

Table C.1: Health investment and OOP

Ψ	$\tilde{B}$	$\tilde{P}(H)$	$\tilde{N}(W,H)$
0.5000	0.0708	201.8595	250.4472
0.7500	0.0709	202.2337	250.8215
1.0000	0.0709	202.3212	250.9089
1.2500	0.0710	202.4957	251.0834
1.5000	0.0711	202.7688	251.3566

Notes: At estimated parameters for benchmark model in Table 3, column 1, at mean health and wealth levels. Benchmark model assumes $\Psi = 1$.

constraint (5) and aggregator (6c) as:

$$dW_t = (rW_t + y - \tilde{c}_t - I_t)\,\mathrm{d}t + \pi_t\sigma_S\,(\mathrm{d}Z_t + \theta\mathrm{d}t) + x_t\,(\mathrm{d}Q_{st} - \lambda_s\mathrm{d}t)\,, \tag{C.4a}$$

$$f(\tilde{c}, u, H) = \frac{\rho\, u}{1 - 1/\varepsilon}\left(\left(\frac{\tilde{c} - a + \beta H}{u}\right)^{1-\frac{1}{\varepsilon}} - 1\right). \tag{C.4b}$$

The agent then selects $\tilde{c}_t$ and the other controls where income is fixed at y in (C.4a), and taking into account the utilitarian benefits of human capital βH in (C.4b). As shown in Hugonnier et al. (2013, Remark 3), the theoretical results are unaffected under this alternative interpretation. This property is especially useful when applying the model to agents who, for reasons of age, illness, or choice are unable or unwilling to work, e.g. in end-of-life analysis (e.g. Philipson et al., 2010; Hugonnier et al., 2020). In the equivalent setup in (C.4), y refers to a fixed (e.g. pension) income flow, while βH captures implicit services (e.g. health marginal benefits associated with consumption and/or leisure).

C.4 Health-dependent and aversion for mortality and morbidity risks

A second source of valuable services of health capital concerns its capacity to lower sickness and death risks exposure for healthier agents. These effects can be captured by replacing the constant arrival rates λ_m, λ_s in (1) and (3) by health-decreasing Poisson intensities:

$$\lambda_m(H_{t-}) = \lambda_{m0} + \lambda_{m1} H_{t-}^{-\xi_m},$$
$$\lambda_s(H_{t-}) = \eta + \frac{\lambda_{s0} - \eta}{1 + \lambda_{s1} H_{t-}^{-\xi_s}},$$

where $H_{t-} = \lim_{s \uparrow t} H_s$ is health prior to occurrence of the sickness shock. It may be further argued that the agent is not indifferent to exposure to these risks, but displays separate risk aversions towards mortality (γ_m) and morbidity (γ_s). This model is analysed in further details in Hugonnier et al. (2013). Our model is a restricted case where both endogeneity and source-dependent aversion are abstracted from, i.e. $\lambda_{k1} = 0$, and $\gamma_k = 0$ for $k = m, s$.

In a separate technical appendix (Hugonnier et al., 2021), we show how approximate closed-form solution to the agent's optimal rules can be obtained for this more general case. Overall, our main theoretical conclusions remain valid when adjusted for endogenous death and sickness risk exposures, as well as non-indifference to the source of those risks. Somewhat unsurprisingly, a structural estimation reveals that adding these additional services from health capital raises life valuations. Indeed, using 2013-PSID data set for our benchmark (reported in Table E.2, column 4) and Hugonnier et al. (2013) reported in Hugonnier et al. (2021, Tab. 2) models yields the following average life values:

Table C.2: Life values endogenous mortality and morbidity risks

Model Year	Benchm. 2013	Hugonnier et al. (2013) 2013
v_h	377.66	493.63
v_s	5536.52	8142.57
v_g	282.34	460.09

We conclude that our key findings remain qualitatively robust when accounting for positive health effects on morbidity and mortality as well as source-dependent risk aversion.

C.5 Ageing

Our closed-form expressions for the willingness to pay and the three life valuations have thus far abstracted from ageing processes. The latter can be incorporated although at some computational cost. In particular, Hugonnier et al. (2013, Appendix B) show that any admissible time variation in $\lambda_{mt}, \lambda_{st}, \phi_t, \delta_t$, or β_t results in age-dependent MPC and Tobin's-Q that solve the system of ordinary differential equations:

$$\dot{A}_t = A_t^2 - \left(\varepsilon\rho + (1-\varepsilon)\left(r - \lambda_{mt} + \theta^2/(2\gamma)\right)\right) A_t, \tag{C.5a}$$

$$\dot{B}_t = (r + \delta_t + \phi_t\lambda_{st})B_t + (1 - 1/\alpha)(\alpha B_t)^{\frac{1}{1-\alpha}} - \beta_t, \tag{C.5b}$$

subject to appropriate boundary conditions.

$$\lim_{t\to\infty}(r - \lambda_{mt} + \theta^2/(2\gamma) - A_t) < 0,$$

$$\lim_{t\to\infty}((\alpha B_t)^{\frac{\alpha}{a-\alpha}} - r - \delta_t - \phi_t\lambda_{st}) < 0.$$

Allowing for ageing and solving these differential equations for A_t, B_t implies that the solutions for C_{0t}, C_{1t}, the marginal value $\Theta_t(\lambda_{mt})$, as well as the human and total wealth $P_t(H), N_t(W,H)$ are also age-dependent. All the previous results remain applicable with these time-varying expressions. Such ageing processes are particularly suitable for elders who face age-increasing exposures to sickness $\dot{\lambda}_{st} > 0$, and death $\dot{\lambda}_{mt} > 0$. Appending these processes along the lines suggested by equations (C.5) is useful to produce realistic life cycle paths for wealth and health (e.g. see St-Amour, 2018, for a survey).

C.6 Life valuations for the GEC model

Our benchmark model nests other well-known life cycle models of health demand. In particular, the widely-used Grossman (1972); Ehrlich and Chuma (1990) (GEC) framework abstracts from morbidity ($\phi, \lambda_s = 0$) and associated insurance ($x = 0$). It also simplifies preferences by imposing VNM utility ($\gamma = 1/\varepsilon$), without minimal consumption requirements ($a = 0$). In this case, the agent's problem simplifies to:

$$V(W_t, H_t) = \sup_{(c,\pi,I)} U_t,$$
$$U_t = 1_{\{T_m > t\}} E_t \int_t^{T_m} e^{-\rho\tau} \left(\frac{c_\tau^{1-\gamma}}{1-\gamma} \right) \mathrm{d}\tau, \tag{C.6a}$$

subject to:

$$\mathrm{d}H_t = \left[I_t^\alpha H_t^{1-\alpha} - \delta H_t \right] \mathrm{d}t,$$
$$dW_t = [rW_t + Y_t - c_t - I_t]\,\mathrm{d}t + \pi_t \sigma_S \left[\mathrm{d}Z_t + \theta \mathrm{d}t \right], \tag{C.6b}$$
$$Y_t = y + \beta H_t.$$

Adapting our results reveals the following result.

Corollary 1 *Assume that the parameters of the model are such that*

$$\beta < (r + \delta)^{\frac{1}{\alpha}}, \tag{C.7a}$$

and denote the Tobin's-q of human capital by $\tilde{B} > 0$, the unique solution to:

$$\beta - (r + \delta)\tilde{B} - (1 - 1/\alpha)(\alpha\tilde{B})^{\frac{1}{1-\alpha}} = 0, \tag{C.7b}$$

subject to

$$(\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} < r + \delta. \tag{C.7c}$$

Assume further that the marginal propensity to consume out of net total wealth, $\tilde{A} > 0$ satisfies:

$$\tilde{A}(\lambda_m) = \frac{\rho}{\gamma} + \left(\frac{\gamma - 1}{\gamma} \right) \left(r - \lambda_m + 0.5 \frac{\theta^2}{\gamma} \right), \tag{C.8a}$$
$$> \max \left(0, r - \lambda_m + \frac{\theta^2}{\gamma} \right). \tag{C.8b}$$

Then,

1. *the human wealth and net total wealth are given as:*

$$\tilde{P}(H_t) = \tilde{B}H_t \geq 0,$$
$$\tilde{B}(W_t, H_t) = W_t + \frac{y}{r} + \tilde{B}(H_t) \geq 0,$$

2. *the indirect utility for the agent's problem is:*

$$V_t = V(W_t, H_t, \lambda_m) = \tilde{\Theta}(\lambda_m) N(W_t, H_t) \geq 0, \tag{C.9a}$$
$$\tilde{\Theta}(\lambda_m) = \tilde{\rho}\tilde{A}(\lambda_m)^{\frac{\gamma}{\gamma-1}} \geq 0, \quad \tilde{\rho} = \rho^{\frac{1}{1-\gamma}} \tag{C.9b}$$

and generates the optimal rules:

$$\tilde{c}_t = \tilde{A}(\lambda_m)\tilde{N}(W_t, H_t) \geq 0,$$
$$\tilde{\pi}_t(\theta/(\gamma\sigma_S))\tilde{N}(W_t, H_t),$$
$$\tilde{I}_t = \left(\alpha^{\frac{1}{1-\alpha}} \tilde{B}^{\frac{\alpha}{1-\alpha}}\right) \tilde{P}(H_t) \geq 0,$$

where any dependence on death intensity λ_m is explicitly stated.

Corollary 2 (Life valuation for restricted model) *The HK, WTP, VSL, and GPV corresponding to the Grossman (1972); Ehrlich and Chuma (1990) model are given by:*

$$\tilde{v}_h(H) = C_0 y + \tilde{C}_1 \tilde{P}(H),$$
$$\tilde{v}(W, H, \lambda_m, \Delta) = \left[1 - \frac{\tilde{\Theta}(\lambda_m^*)}{\tilde{\Theta}(\lambda_m)}\right] \tilde{N}(W, H),$$
$$\tilde{v}_s(W, H, \lambda_m) = \frac{1}{\tilde{A}(\lambda_m)} \tilde{N}(W, H),$$
$$\tilde{v}_g(W, H) = \tilde{N}(W, H),$$

with the constants:

$$C_0 = \frac{1}{r + \lambda_m},$$
$$\tilde{C}_1 = \frac{r - (\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} + \delta}{r + \lambda_m - (\alpha\tilde{B})^{\frac{\alpha}{1-\alpha}} + \delta},$$

and where the modified expressions for marginal value $\tilde{\Theta}(\lambda_m)$*, and human* $\tilde{P}(H)$*, and net total wealth* $\tilde{N}(W,H)$ *are given in Corollary 1.*

The proofs of Corollaries 1 and 2 follow directly from imposing the restrictions ($\phi, \lambda_s, x, a = 0$ and $\gamma = 1/\varepsilon$) in the closed-form solutions for our benchmark model and are therefore omitted.

Corollaries 1 and 2 reveal that, *ceteris paribus*, both the relevant human, and net wealth measures $\tilde{P}(H)$ and $\tilde{N}(W,H)$ are increased by the absence of exogenous morbidity $\lambda_s = 0$, which raises the Tobin's-Q to $\tilde{B} \geq B$. Moreover, the absence of survival consumption ($a = 0$) further raises $\tilde{N}(W,H) \geq N(W,H)$. Notwithstanding these quantitative differences, the restricted valuations are qualitatively similar to those for the more general model. The HK value $\tilde{v}_h$ remains an affine function of health only. The marginal value $\tilde{\Theta}(\lambda_m)$ remains a decreasing, and convex function such that the WTP $\tilde{v}$ is again an increasing, and concave function in the death risk increment Δ and converges to the latter as exposure to death risk increases. It follows that the linear projection bias of the VSL discussed earlier is unconditionally present for the restricted model. The Value of a Statistical Life $\tilde{v}_s$ is also increasing in net total wealth, whereas the Gunpoint Value $\tilde{v}_g$ confirms that all available net worth is spent to survive a highwaymen threat. From a theoretical perspective, we conclude that our main conclusions regarding life valuations remain valid when we consider an alternative model for human capital.

C.7 Hicksian Compensating Variation

For those instances where appropriate, we can also rely on a similar reasoning to define the Hicksian Compensating Variation as follows:

$$V\left(W - v^c, H; \lambda_m^*\right) = V\left(W, H; \lambda_m\right)$$

which can be solved as

$$\begin{aligned} v^c(W,H,\lambda_m,\Delta) &= \left[1 - \frac{\Theta(\lambda_m)}{\Theta(\lambda_m^*)}\right] N(W,H), \\ &= \frac{-\Theta(\lambda_m)}{\Theta(\lambda_m^*)} v(W,H,\lambda_m,\Delta). \end{aligned}$$

Since $\Theta'(\lambda_m) < 0$, it follows that $0 < v^c < -v$ for $\Delta < 0$ and $0 < v < -v^c$ for admissible $\Delta > 0$, i.e. the WTP to attain a beneficial or avert a detrimental change in death risk is always less that the corresponding WTA to forego a favorable or accept an unfavorable change in mortality, consistent with standard Hicksian variational analysis (e.g. Smith and Keeney, 2005; Hammitt, 2008).

D Estimation details

D.1 Cross-sectional identification

The econometric model (32) reveals that there exists a subset of cross-equation restrictions that prevent using a (single) reduced-form estimation of $\mathbf{B}(\boldsymbol{\Theta})$, followed by a just-identified contrast estimator (e.g. minimum distance estimator) of $\boldsymbol{\Theta}$. In addition, the structural parameters are further constrained by the non-linear inequalities in our model (Discussed in Appendix D.3 below).

A large subset of the deep parameters $\boldsymbol{\Theta}^e \subset \boldsymbol{\Theta}$ are thus theoretically identified from the cross-equation restrictions governing $\mathbf{B}(\boldsymbol{\Theta})$ in Table 1, combined with the non-linear implicit equality constraint defining the Tobin's-q in (9b) and the non-linear inequality constraints (9a), (9c) as well as (10b). Towards that purpose, we first follow standard practices in the Asset Pricing and Life Cycle literature by calibrating the returns parameters $(\mu, r, \sigma_S, \theta)$ and discount rate (ρ) at usual values. Finally, we also calibrate the capital shock parameter ϕ following a thorough search procedure, such that the remaining estimable parameters are:

$$\boldsymbol{\Theta}^e = (y, \beta, \delta, \alpha, \lambda_s, \lambda_m, a, \gamma, \varepsilon)\,.$$

With these elements in mind, the theoretical restrictions (9) and (10) imply that the composite parameters are linked to $\boldsymbol{\Theta}^e$ as follows:

$$\begin{aligned} B(\boldsymbol{\Theta}^e) &= B(\beta, \delta, \alpha, \lambda_s), \\ A(\boldsymbol{\Theta}^e) &= A(\varepsilon, \gamma, \lambda_m). \end{aligned} \tag{D.1}$$

Next, the ten non-zero reduced-form parameters $\mathbf{B}(\boldsymbol{\Theta})$ in (32), combined with composite restrictions (D.1) show that the parameters in $\boldsymbol{\Theta}^e$ are theoretically identifiable from the RFP's as follows:

Estim. struct. param. $\mathbf{\Theta^e}$	Identif. from RFP $\mathbf{B}$
y, a	$\mathrm{B}_0^Y, \mathrm{B}_0^c, \mathrm{B}_0^\pi$
$\beta, \delta, \alpha, \lambda_s$	$\mathrm{B}_H^Y, \mathrm{B}_H^c, \mathrm{B}_H^\pi, \mathrm{B}_H^x, \mathrm{B}_H^I$
γ	$\mathrm{B}_0^c, \mathrm{B}_W^c, \mathrm{B}_H^c, \mathrm{B}_0^\pi, \mathrm{B}_W^\pi, \mathrm{B}_H^\pi$
ε, λ_m	$\mathrm{B}_0^c, \mathrm{B}_W^c, \mathrm{B}_H^c$

(D.2)

Indeed, contrasting the number of $\mathbf{B}$ terms and $\mathbf{\Theta^e}$ in (D.2) shows that the rank condition is satisfied and there might exist (at least) one solution to the non-linear estimation method. As a heuristic argument (i.e., without cross-equation restrictions and non-linear constraints), using the mapping between the structural parameters and the $\mathbf{B}$ terms in Table 1, it follows that the income equation Y_j identifies y and β, the consumption equation c_j identifies ε, λ_m and γ, the non-linear parameter functions of the health variable in the insurance x_j, portfolio π_j, and investment I_j equations identify δ, λ_s, α, and the constant term of the portfolio equation identifies a. Nevertheless, as to be expected, it does not guarantee the global identification of $\mathbf{\Theta^e}$. To circumvent this issue, we first assess the flatness of the likelihood function in each dimensions of the parametric space, and then rely on Neural Network methods to select starting values by putting more weight in those regions of $\mathbb{R}_+^k$ with steeper gradients.

D.2 Panel with fixed effects alternative

Instead of identifying and estimating the structural parameters of interest using a cross-sectional perspective, an alternative might be to combine both the cross-section and time dimension, and thus consider a panel regression. Notably, the non-linear multivariate econometric model can be appended to include (unobserved) individual heterogeneity, and especially individual fixed effects. However, taking the presence of (non-linear) intercepts in the consumption, portfolio and income equations, one key issue is the standard dummy variable trap or perfect multicollinearity engendered by the Within transformation. Indeed, exploiting the (individual) Within variability would lead to drop out $\mathbf{B_0}(\mathbf{\Theta})$ and thus results in a loss of identification and information for the structural parameters (e.g., the base income or the minimal consumption level) that belongs to $\mathbf{B_0}(\mathbf{\Theta})$. Furthermore, given the non-separability of the vector of structural parameters, standard adding-up constraints (e.g., the sum of the individual effects for consumption is zero) would not solve the identification issue.[6] In this respect, our estimation strategy only exploits the cross-sectional predictions of the optimal rules (and the income equation) and still remains fully consistent with our theory.

D.3 Non-linear inequality constraints

Our econometric model (32) can be written as a constrained regression problem with non-linear equality and inequality constraints. Define $\mathbf{\Theta}^e$ (resp. $\mathbf{\Theta}^c$) as the vector of estimated (resp. calibrated) structural parameters in $\mathbf{\Theta} = (\mathbf{\Theta}^e, \mathbf{\Theta}^c)$ and let B and A be the composite Tobin's-q and MPC parameters characterized by (9), and (10). For any objective function S_n associated with a sample of size n, the estimation procedure is:

$$\begin{aligned} \max_{B,A;\, \mathbf{\Theta}^e \subset \mathbf{\Theta}} \; & S_n(\mathbf{\Theta}) \quad \text{s.t.} \\ g_1(\mathbf{\Theta}) &\geq 0, \\ g_2(B, A, \mathbf{\Theta}) &= 0, \\ g_3(B, A, \mathbf{\Theta}) &\geq 0, \end{aligned} \tag{D.3}$$

where g_1 is a vector of non-linear inequality constraints capturing sign restrictions on $\mathbf{\Theta}$, g_2 is a vector of non-linear equality constraint(s) associated with (9b) and (10a), and g_3 is a vector of non-linear inequality constraints (9c) and (10b). It is worth noting that S_n can be the objective function corresponding to Maximum Likelihood stimation, asymptotic Least Squares estimation, M-estimation or the Generalized Method of Moments estimations (see Gourieroux and Monfort (1995a, ch. 10) and Gourieroux and Monfort (1995b, ch. 21)). Since g_2 implicitly defines $B = B(\mathbf{\Theta})$ and $A = A(\mathbf{\Theta})$, the estimation problem (D.3) can equivalently be recast as:

$$\begin{aligned} \max_{\mathbf{\Theta}^e \subset \mathbf{\Theta}} \; & \tilde{S}_n(\mathbf{\Theta}) \quad \text{s.t.} \\ m(\mathbf{\Theta}) &\geq 0, \end{aligned} \tag{D.4}$$

[6] At the same time, a time-varying specification (through age-varying structural parameters) will allow for identification and estimation without resorting to further (arbitrary) identifying restrictions. We leave this issue for future research.

where $m(\mathbf{\Theta}) = [g_1(\mathbf{\Theta}), g_3(B(\mathbf{\Theta}), A(\mathbf{\Theta}), \mathbf{\Theta})]' \in \mathbb{R}^r$.

Because of the presence of the (non-linear) inequality constraints $m(\mathbf{\Theta})$, one key issue is whether or not $n^{1/2}\left(\widehat{\mathbf{\Theta}}^e_n - \mathbf{\Theta}^e_0\right)$ and consequently $n^{1/2}\left(\widehat{B}_n - B_0\right)$ and $n^{1/2}\left(\widehat{A}_n - A_0\right)$, where $\mathbf{\Theta}^e_0$ are the true unknown parameters, are asymptotically normal. More generally, one cannot expect to get an explicit expression of the distribution of the estimator (e.g. Wang, 1996; Gourieroux and Monfort, 1995a,b). At the same time, the existence and strong convergence of the estimator does not depend on the presence of the non-linear inequality (and equality) constraints. More specifically, it requires that the observations are independent (in our context), the parameter space being compact, the true parameters $\mathbf{\Theta}^e_0$ being identifiable, the log-likelihood function being continuous w.r.t. $\mathbf{\Theta}^e$, the existence of $\mathbb{E}_0[\tilde{S}_n(\mathbf{\Theta})]$ under the null of $\mathbf{\Theta}^e_0$, the uniform convergence of S_n (see Gourieroux and Monfort, 1995b, ch. 7), and that the Jacobian associated to the non-linear constraints be of full row rank.[7]

The Lagrangean associated with the constrained problem (D.4) is then given by:

$$\max_{\mathbf{\Theta}^e \subset \mathbf{\Theta}} \mathcal{L}_n(\mathbf{\Theta}, \lambda) = \tilde{S}_n(\mathbf{\Theta}) + \sum_{j=1}^{r} \lambda_j m_j(\mathbf{\Theta}), \tag{D.5}$$

where the λ_j's terms denote the Kuhn-Tucker multipliers. The solutions $\left(\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n\right)$ to the Lagrangean problem (D.5) must satisfy the following first-order, sign, and exclusion restrictions:

$$\begin{aligned} \left.\frac{\partial \tilde{S}_n(\mathbf{\Theta})}{\partial \mathbf{\Theta}^e}\right|_{\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n} + \sum_{j=1}^{r} \lambda_j \left.\frac{\partial m_j(\mathbf{\Theta})}{\partial \mathbf{\Theta}^e}\right|_{\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n} &= 0, \\ m_j(\mathbf{\Theta}), \text{ and } \lambda_j|_{\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n} &\geq 0, \\ \lambda_j m_j(\mathbf{\Theta})|_{\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n} &= 0, \end{aligned}$$

for $j = 1, 2, \ldots, r$. The solutions $\left(\widehat{\mathbf{\Theta}}^e_n, \widehat{\lambda}_n\right)$ are associated with the restriction on the composite parameters, $\widehat{B}_n = B(\widehat{\mathbf{\Theta}}^e_n, \mathbf{\Theta}^c)$, and $\widehat{A}_n = A(\widehat{\mathbf{\Theta}}^e_n, \mathbf{\Theta}^c)$. Given these elements, two situations might arise:

[7] Notably uniform convergence is insured if the interior of $\mathbf{\Theta}^e$ is non-empty and $\mathbf{\Theta}^e_0$ belongs to the interior of $\mathbf{\Theta}^e$. In addition, the Jacobian condition is insured by evaluating the rank of this matrix at the ML estimate of $\mathbf{\Theta}^e$. Finally, due to the non-linear constraints and model, there are no general conditions for global identification.

- If $\boldsymbol{\Theta}_0^e$ belongs to the interior, i.e. is such that $m_j(\boldsymbol{\Theta})|_{\boldsymbol{\Theta}_0^e} > 0$, for $j = 1, \cdots, r$, then $\widehat{\boldsymbol{\Theta}}_n^e$ is asymptotically equivalent to the unconstrained estimator (in the absence of inequality constraints) defined by:

$$\max_{\boldsymbol{\Theta}^e \subset \boldsymbol{\Theta}} \tilde{S}_n(\boldsymbol{\Theta}),$$

  with associated composite parameters $\widehat{B}_n = B(\widehat{\boldsymbol{\Theta}}_n^e, \boldsymbol{\Theta}^c)$, and $\widehat{A}_n = A(\widehat{\boldsymbol{\Theta}}_n^e, \boldsymbol{\Theta}^c)$. Consequently $\widehat{\boldsymbol{\Theta}}_n^e, \widehat{B}_n, \widehat{A}_n$ are *asymptotically normally distributed*.

- If $\boldsymbol{\Theta}_0^e$ belongs to the boundary, i.e. is such that $m_j(\boldsymbol{\Theta})|_{\boldsymbol{\Theta}_0^e} = 0$, for $j = 1, \cdots, r$, then the asymptotic distribution does not have a closed-form solution.[8]

In practice, we proceed with an ex-post verification, i.e.

1. Estimate $\widehat{\boldsymbol{\Theta}}^e$ in the unconstrained equation:

$$\max_{\boldsymbol{\Theta}^e \subset \boldsymbol{\Theta}} \mathcal{L}_n(\boldsymbol{\Theta}, \lambda) = \tilde{S}_n(\boldsymbol{\Theta}).$$

2. Check that the inequality restrictions $m(\boldsymbol{\Theta})|_{\widehat{\boldsymbol{\Theta}}^e} \geq 0$ are verified at the unconstrained estimate.

E Robustness

We now discuss the empirical robustness of our results to key assumptions. Table E.1 reports the structural parameters, and Table E.2 the implied life valuation measures. For ease of comparison, we rewrite the benchmark estimation in the first columns, and the re-estimated values for the various alternatives in columns 2 to 8.

E.1 Stratification by age

An estimation of the model with age-dependent parameters $\boldsymbol{\Theta}_t$ as discussed in Appendix C.5 is beyond the scope of this paper. We can nonetheless verify the realism of the constant parameters assumption and how it affects our valuations by stratifying across the old (65 and over) and the young (less than 65) agents, re-estimating the

[8]As an application, see Wang (1996)

Table E.1: Robustness: Estimated and Calibrated Structural Parameters

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Model	Benchm.	Benchm.	Benchm.	Benchm.	Benchm.	GEC	Benchm.	Benchm.
Year	2017	2017	2017	2013	2009	2017	2017	2017
Subset		$t \geq 65$	$t < 65$					
Scaling	per-cap	per-cap	per-cap	per-cap	per-cap	per-cap	root	OECD
	a. Law of motion health (3)							
α	0.7413	0.7537	0.7263	0.6913	0.6964	0.6787	0.7592	0.7643
	(0.0155)	(0.0355)	(0.0186)	(0.0053)	(0.0053)	(0.0312)	(0.0201)	(0.0243)
δ	0.0370	0.0670	0.0270	0.0437	0.0442	0.0495	0.0384	0.0390
	(0.0011)	(0.0017)	(0.0016)	(0.0006)	(0.0039)	(0.0025)	(0.0023)	(0.0011)
ϕ^c	0.0136	0.0136	0.0136	0.0136	0.0136		0.0136	0.0136
	b. Sickness (3) and death (1) intensities							
λ_s	0.1000	0.1250	0.0800	0.0812	0.0861		0.0980	0.0966
	(0.0112)	(0.0159)	(0.0135)	(0.0069)	(0.0259)		(0.0185)	(0.0213)
λ_m	0.0342	0.1053	0.0282	0.0257	0.0237	0.0379	0.0373	0.0365
	(0.0001)	(0.0033)	(0.0000)	(0.0002)	(0.0005)	(0.0008)	(0.0098)	(0.0001)
	c. Income (4) and wealth (5)							
y	0.0127	0.0108	0.0132	0.0134	0.0132	0.0058	0.0122	0.0123
	(0.0004)	(0.0003)	(0.0003)	(0.0003)	(0.0010)	(0.0015)	(0.0004)	(0.0004)
β	0.0061	0.0087	0.0054	0.0082	0.0091	0.0064	0.0092	0.0088
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0006)	(0.0001)	(0.0001)
μ^c	0.1080	0.1080	0.1080	0.1080	0.1080	0.1080	0.1080	0.1080
r^c	0.0480	0.0480	0.0480	0.0480	0.0480	0.0480	0.0480	0.0480
σ_S^c	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
	d. Preferences (6)							
γ	2.4579	2.3758	2.7579	3.1400	3.2008	3.4312	2.2579	2.2579
	(0.0542)	(0.0495)	(0.0397)	(0.0296)	(0.0694)	(0.0012)	(0.0210)	(0.0214)
ε	1.0212	0.8747	1.1779	1.0747	1.2032		1.0264	1.0712
	(0.0004)	(0.0049)	(0.0021)	(0.0015)	(0.0009)		(0.0002)	(0.0006)
a	0.0134	0.0118	0.0138	0.0138	0.0147		0.0139	0.0140
	(0.0007)	(0.0003)	(0.0004)	(0.0004)	(0.0007)		(0.0006)	(0.0006)
ρ^c	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500
	e. MPC and Tobin's q (10), (9)							
A	0.0504	0.0389	0.0525	0.0510	0.0524	0.0310	0.0505	0.0513
	(0.0057)	(0.0067)	(0.0071)	(0.0019)	(0.0011)	(0.0016)	(0.0008)	(0.0004)
B	0.0709	0.0748	0.0717	0.0884	0.0982	0.0659	0.1053	0.0998
	(0.0084)	(0.0015)	(0.0027)	(0.0086)	(0.0077)	(0.0076)	(0.0019)	(0.0006)

Notes: Estimated (standard error in parentheses) and calibrated (c) structural parameters. Column (1): Econometric model (32), estimated by ML, subject to the parametric restrictions in panel (a) of Table 1 for 2017 data, and using per-capita scaling for household level variables. Columns (2) and (3): Estimated by age sub-groups. Columns (4), (5): Benchmark model for 2013, 2009. Column (6): GEC model (Grossman, 1972; Ehrlich and Chuma, 1990), same econometric model subject to the parametric restrictions in panel (b) of Table 1. Columns (7): Benchmark model with square root on household size equivalence scaling for 2017. Columns (8): Benchmark model with modified OECD rule on household size equivalence scaling for 2017.

Table E.2: Robustness: Estimated Life Values (in K$)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Model	Benchm.	Benchm.	Benchm.	Benchm.	Benchm.	GEC	Benchm.	Benchm.
Year	2017	2017	2017	2013	2009	2017	2017	2017
Subset		$t \geq 65$	$t < 65$					
Scaling	per-cap	per-cap	per-cap	per-cap	per-cap	per-cap	root	OECD
				a. HK $v_h(W, H, \lambda_m)$ in (17)				
Poor	205.82	109.76	225.67	250.21	261.24	114.52	217.23	216.54
	(0.63)	(1.02)	(1.90)	(0.68)	(0.61)	(1.30)	(1.26)	(2.37)
Fair	243.89	139.25	264.88	301.91	319.65	149.96	272.55	269.41
	(1.09)	(1.57)	(2.78)	(1.18)	(1.07)	(1.29)	(2.21)	(4.15)
Good	281.96	168.74	304.09	353.60	378.06	185.41	327.88	322.29
	(1.56)	(2.21)	(3.78)	(1.69)	(1.53)	(1.42)	(3.16)	(5.93)
Very Good	320.03	198.22	343.30	405.30	436.48	220.86	383.20	375.17
	(2.03)	(2.86)	(4.83)	(2.20)	(1.99)	(1.64)	(4.11)	(7.71)
Excellent	358.10	227.71	382.50	456.99	494.89	256.30	438.52	428.04
	(2.50)	(3.53)	(5.91)	(2.70)	(2.45)	(1.94)	(5.06)	(9.48)
All	299.52	172.13	324.57	377.66	408.56	201.76	353.92	347.18
	(1.91)	(2.37)	(4.18)	(1.85)	(1.77)	(1.59)	(3.83)	(3.66)
				b. VSL $v_s(W, H, \lambda_m)$ in (24)				
Poor	2178.13	4365.29	1165.79	1859.98	1691.93	7717.35	2701.62	2501.34
	(32.53)	(366.53)	(106.58)	(18.80)	(15.56)	(91.03)	(102.46)	(186.38)
Fair	2720.43	4608.38	2191.78	3200.64	3093.49	8477.67	3619.34	3352.69
	(39.56)	(304.28)	(149.10)	(53.29)	(35.74)	(94.84)	(61.17)	(110.80)
Good	4206.53	7992.17	3220.18	4826.12	4794.26	10771.64	5847.01	5428.31
	(42.94)	(340.91)	(156.93)	(48.49)	(32.76)	(88.60)	(98.01)	(178.58)
Very Good	5802.46	14735.87	4245.13	6381.26	6459.49	13244.07	8082.04	7521.75
	(42.56)	(240.82)	(150.10)	(69.79)	(46.45)	(94.41)	(120.04)	(216.30)
Excellent	7189.48	16810.38	5272.31	7880.11	7868.81	15377.03	10273.67	9554.52
	(40.47)	(179.31)	(144.59)	(74.87)	(56.49)	(153.48)	(185.48)	(336.24)
All	4980.38	9972.32	4474.47	5536.52	5620.35	11972.21	6940.49	6451.44
	(49.08)	(310.43)	(172.04)	(38.02)	(42.19)	(94.65)	(118.69)	(109.50)
				c. GPV $v_g(W, H)$ in (28)				
Poor	109.73	169.96	84.56	94.85	88.61	239.27	136.46	128.38
	(1.59)	(6.39)	(5.82)	(0.88)	(0.77)	(2.47)	(5.07)	(9.40)
Fair	137.05	179.42	129.85	163.22	162.02	262.84	182.82	172.08
	(1.93)	(6.13)	(8.14)	(2.53)	(1.76)	(2.58)	(3.03)	(5.59)
Good	211.92	311.16	201.15	246.11	251.10	333.96	295.34	278.61
	(2.09)	(7.87)	(8.56)	(2.26)	(1.61)	(2.41)	(4.85)	(9.01)
Very Good	292.33	573.72	261.78	325.42	338.31	262.34	408.24	386.05
	(2.07)	(4.11)	(8.18)	(3.25)	(2.29)	(2.56)	(5.94)	(10.91)
Excellent	362.20	654.49	340.75	401.86	412.13	476.74	518.94	490.39
	(1.97)	(3.05)	(7.89)	(3.48)	(2.78)	(4.17)	(9.18)	(16.96)
All	250.91	388.26	234.75	282.34	294.36	371.18	350.58	331.12
	(2.39)	(6.13)	(9.37)	(1.75)	(2.08)	(2.57)	(5.87)	(5.52)

Notes: Computed at corresponding estimated parameter values in Table E.1, columns (1–8). Bootstrapped standard errors in parentheses (500 replications).

econometric model (32) and re-calculating the HK, VSL and GPV life values across the two subgroups.

Table E.1, presents the estimated deep parameters (with calibrated parameters unchanged) for old (column 2) and young (column 3) sub-samples (the calibrated parameters remain set to the values in Table 3). Somewhat unsurprisingly, being older is associated with faster depreciation in the absence of investment to maintain the health capital (δ), as well as increased exposure to sickness (λ_s) and mortality (λ_m) risks. The technological (α), income (y, β) and preferences (a, γ) remain generally unaffected by ageing, except for the elasticity of inter-temporal substitution (ε) which is lower and less than one, suggesting less consumption responsiveness to movements in interest rates and in death risk exposure for elders. In panel e, the shadow value B is slightly increased, whereas the MPC A is lower. We conclude that our assumption of age-invariant deep parameters is not at odds with the data. With the exception of predictable increased exposure to morbidity and mortality risks, and decreased responsiveness to interest rates, elders and young agents share similar parameters.

Table E.2 gauges the effects of ageing on life valuation.[9] In panel a, column 2, the HK value is unsurprisingly lower for elders, a direct consequence of a higher estimated death intensity λ_m lowering the expected duration of the net income flow parameters C_0, C_1 in (18). Conversely, the VSL (panel b) is higher for elders, due to a lower MPC A, as well as a higher shadow value of health B that raises the net total wealth $N(W, H)$; the latter also explains why the GPV is higher for elders.

We conclude that our key assumption of age-invariant parameters is not invalidated. The estimated preference, income and technological parameters remain generally comparable across age groups. Other distributional parameters vary with age in a predictable fashion, consistent with higher death and sickness exposure for elders. Whereas these sub-group results are reassuring for our age-invariance assumption, a full treatment of ageing along the lines of the dynamic processes (C.5), or allowing for cohort effects would be required for more definitive answers on the impact of age. We leave such analysis on the research agenda.

[9] We restrict the presentation of life values by health sub-groups. The full results stratified by wealth quintiles can be obtained upon request.

E.2 Human capital shocks insurance

Our model's solutions are obtained assuming that actuarially-fair insurance against human capital shocks is available. This assumption is essential to compute the net present value of the returns to investment, and consequently the net total wealth that is central to life valuations.

Our empirical implementation associates human capital H_t to health and insurance premia x_t to medical insurance coverage. Since our data set is 2017. i.e. after Affordable Care Act (ACA, aka Obamacare) became operational in 2014, the health insurance coverage assumption appears reasonable.[10] Whereas the model cannot be generalized to allow for imperfect insurance markets,[11] we can partially gauge the effects of incomplete coverage via a time variation assessment. In particular, we re-estimate our benchmark model using PSID data for two pre-ACA years, 2013 and 2009, that are associated with higher health non-insurance rate (see footnote 10).

Tables E.1 reports the parameter estimates for 2013 (column 4) and 2009 (column 5). Again, our results are generally similar, with some exceptions. We estimate lower values for the Cobb-Douglas α, and for sickness and death intensities λ_s, λ_m. Conversely, depreciation δ, income y, β, risk aversion and EIS γ, ε parameter estimates increase. Whereas the shadow price B is higher, the MPC A is unaffected in panel e, indicating that a lower insurance coverage in pre-ACA years is not associated with an increase in precautionary savings. The combination of higher Tobin's-q and lower death intensity results in higher HK, VSL and GPV values in Table E.2, columns 4 and 5.

The presence of other confounding factors (e.g. the aftermath of the financial crisis of 2008) imply that such time variation exercises should be taken with caution. Notwithstanding this caveat, we conclude that our key results remain generally stable and/or vary predictably across time periods.

[10] The uninsured rate for 2017 was 8.7% for all, and 10.2% for individuals under 65, i.e. before Medicare coverage. In comparison, the pre-ACA uninsured rates were 14.5% (all) and 16.7% (less than 65) in 2013, and 15.1% (all) and 17.2% (less than 65) in 2009 (U.S. Census Bureau, 2021, Tab. HIC-6 and 9_ACS).

[11] Allowing strictly positive exposure to capital depreciation $\lambda^s, \phi > 0$ with incomplete coverage $x_t < \phi P(H_t)$ is tantamount to undiversifiable risks in $Y(H_t)$ for which optimal strategies are notoriously difficult to compute in closed-form and require numerical approaches.

E.3 GEC model

The econometric model for the restricted Grossman (1972); Ehrlich and Chuma (1990) framework in Appendix C.6 can be adapted from our benchmark with the following estimation:

$$\begin{aligned}\mathbf{Y}_j &= [Y_j, c_j, \pi_j, I_j]' \\ &= \mathbf{B}_0(\mathbf{\Theta}) + \mathbf{B}_W(\mathbf{\Theta})W_j + \mathbf{B}_H(\mathbf{\Theta})H_j + \mathbf{u}_j, \quad \mathbf{u}_j \sim \text{NID}(\mathbf{0}, \mathbf{\Sigma}).\end{aligned} \tag{E.1a}$$

The corresponding parametric restrictions are outlined as follows:

(E.1b)

Eq. i	$\mathbf{B}_0^i(\mathbf{\Theta})$	$\mathbf{B}_W^i(\mathbf{\Theta})$	$\mathbf{B}_H^i(\mathbf{\Theta})$
Y_j	y	0	β
c_j	$\tilde{A}\left(\frac{y}{r}\right)$	$\tilde{A}$	$\tilde{A}\tilde{B}$
π_j	$\frac{\theta}{\gamma\sigma_S}\left(\frac{y}{r}\right)$	$\frac{\theta}{\gamma\sigma_S}$	$\frac{\theta}{\gamma\sigma_S}\tilde{B}$
I_j	0	0	$(\alpha\tilde{B})^{1/(1-\alpha)}$

With the exception of insurance x_j which is abstracted from, the empirical strategy for the GEC model is therefore isomorphic to our benchmark (32). Moreover, it shares most of the theoretical predictions with respect to the values of life and therefore constitutes a natural alternative to our benchmark model. Finally, since this econometric model is a nested case of (32), the identification arguments in Section 4 also apply. We consequently proceed with its estimation, using the same ML estimator and same data set.

The estimated parameters for the restricted model (with calibrated parameters unchanged) are reported in column 6 of Table E.1. Overall, the deep parameters remain similar, with some exceptions. First, we estimate a lower Cobb-Douglas parameter α. as well as a higher depreciation rate δ which tends to over-compensates the absence of morbidity risk.[12] We also estimate a higher mortality rate λ_m and risk aversion γ, as well as a lower EIS which is restricted to be the inverse of the risk aversion $\varepsilon = 1/\gamma$ under VNM preferences. The composite parameters in panel e indicate a significant reduction in the MPC A and a less pronounced one for the Tobin's-q B.

[12] In particular, the depreciation rate for the restricted model $\delta = 0.0495$ is 30% larger than the deterministic plus expected stochastic depreciation for the benchmark: $\delta + \phi\lambda_s = 0.0383$.

The valuations $\tilde{v}_h, \tilde{v}_s$ and $\tilde{v}_g$ from the GEC model are reported in column 6 of Table E.2. First, the higher depreciation δ, as well as higher mortality rate result in a lower HK value (202 K\$ vs 300 K\$). Conversely, both the VSL (11.97 K\$ vs 4.98 M\$) and GPV (371 K\$ vs 251 K\$) values are higher. These results confirm our discussion in Appendix C.6. Indeed, abstracting from sickness risks λ_s, and from minimal consumption a results in higher net total wealth $\tilde{N}(W, H) > N(W, H)$ justifying a higher GPV. In addition, our estimation reveals a lower MPC for the GEC model; a larger net total wealth divided by a lower MPC, justifies why we obtain a much larger VSL for the restricted model.

We conclude that while the theoretical valuations are qualitatively similar, abstracting from occurrence and insurance against sickness risk, as well as from minimal consumption requirements results in quantitative adjustments for the restricted model that do not overturn our main conclusions. Importantly, our discussion of the estimated parameters in Table 3 revealed that the theoretical restrictions associated with the Grossman (1972); Ehrlich and Chuma (1990) model were individually rejected, thereby validating our benchmark model over the restricted one.

E.4 Effects of equivalence scaling

Our PSID data procedure described in Section 4.3 of the paper scales the resources (financial wealth W_t, income Y_t) and dependent variables (consumption C_t, health investment I_t and insurance x_t, risky asset holdings π_t) by the number of household members to obtain per-capita variables. The respondent's self-reported health status H_t is agent-specific, and does not require scaling.

Other equivalence scaling (ES) approaches, such as square root of household size, OECD and modified OECD ES are also available. Their main purpose is to correct for potential economies of scale in household, especially for determining available resources (e.g. an additional child does not necessarily entail proportional expenses). The literature reveals that there is absence of consensus as to which ES measure to use (e.g. OECD, 2013, ch. 8 for discussion). In particular, ES that are appropriate for stock (e.g. wealth) may not be adequate for flow (e.g. income) and those for resources are not necessarily applicable for expenses. Moreover, ES that are relevant for richer households are not necessarily useful for poorer ones.

In the absence of clear consensus, and because the scale economies arguments are less apparent for health-related expenses (out-of-pocket, insurance) in our estimation, we have selected our simpler per-capita scaling instead of alternative ES approaches. For completeness, we have nonetheless re-estimated our benchmark model, for the 2017 PSID sample, using the square-root and modified OECD ES methods, where the differences in scaling are illustrated in Table E.3.

Table E.3: Alternative equivalence scaling approaches

Nb. Adults	Nb. Child(ren)	Per-capita	Square root	Modified OECD
1	0	1.0	1.0	1.0
2	0	2.0	1.4	1.5
2	1	3.0	1.7	1.8
2	2	4.0	2.0	2.1
2	3	5.0	2.2	2.4
5	0	5.0	2.2	3.0

Notes: Source OECD (2013, Tab. 8.1).

Overall, our estimated parameters in Table E.1 remain very robust to the choice of ES. Indeed, contrasting our benchmark (column 1) with the square-root (column 7) or modified OECD (column 8) reveals minimal effects of scaling in almost all instances. In particular, the structural parameters in (10) are unaffected by the choice of ES, such that the MPC $A(\lambda_m)$ (panel e), and importantly the marginal value $\Theta(\lambda_m)$ in (13b) remain unchanged. One exception is the larger health loading β in the income equation (4). This is unsurprising since household income is scaled by a lower factor under alternative ES, while the health variable is unscaled. A direct consequence of a higher β in (9) is to raise the Tobin's q, B in panel e. Consequently, so are human wealth $P(H) = BH$ and net total wealth $N(W, H) = W + (y - a)/r + P(H)$, where the latter also increases due to a higher financial wealth W from the alternative scaling. The net effects are to raise the life values in Table E.2, where, as expected, the impact is modest for agents in poor health/wealth and more potent for others. Overall, we conclude that the effects of alternative ES measure is predictable, and that in the absence of clear consensus, our per-capita scaling remains warranted.

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