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Capital Controls with International Reserve Accumulation: Can this Be Optimal?†

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Motivated by the Chinese experience, we analyze an economy where the central bank has access to international capital markets, but the private sector does not. The central bank is modeled as a Ramsey planner who can choose the domestic interest rate and the level of international reserves. Consumers are credit-constrained as in Woodford (1990). We find that a rapidly growing economy has a higher welfare without capital mobility. In the Chinese context, we argue that the domestic interest rate should be temporarily above the international rate and that there should be more foreign asset accumulation than in an open economy. (*JEL* E58, E62, F32, F41, O19, O24, P33)

China has been a key contributor to global imbalances with both a significant current account surplus and a substantial accumulation of international reserves by the central bank. Figure 1 shows the increase in both variables in recent years. The parallel evolution of these two variables illustrates an interesting feature of the Chinese economy. On the one hand, there are strict restrictions on private capital flows, which characterize a closed economy. On the other hand, there are substantial net capital outflows, through the accumulation of international reserves. This hybrid system differs from the usual open economy or closed economy paradigms and has received little attention in the literature. However, to analyze the macroeconomic behavior of the Chinese economy, it seems fundamental to have a good understanding of this specific structure.

The objective of this paper is to analyze an economy where the central bank has access to international capital markets, but the private sector does not. We call this situation a *semi-open* economy.[1] We want to address two main questions in this

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[1] Jeanne (2012) also analyzes a semi-open economy, but he does not focus on optimal policies.

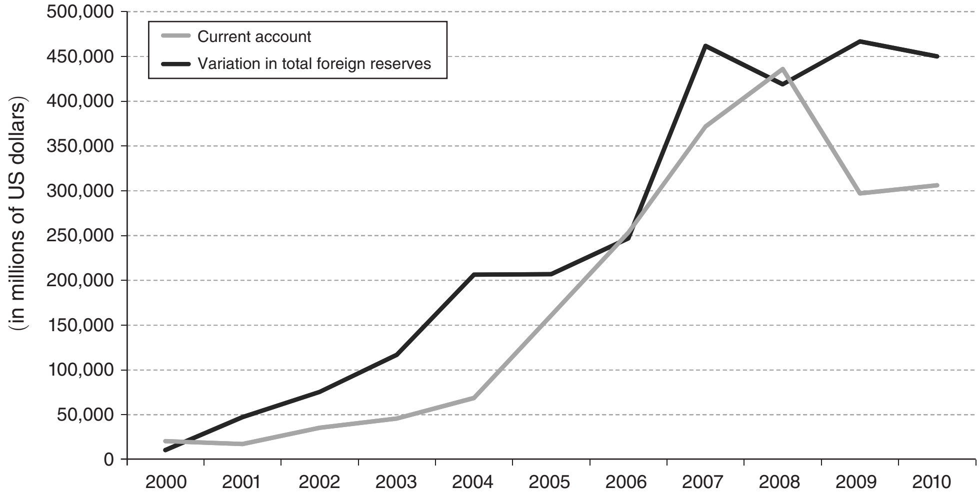

Figure 1. Evolution of Chinese Current Account and Foreign Reserves Accumulation

Source: International Monetary Fund, International Financial Statistics (IMF IFS), and World Economic Outlook (IMF WEO) databases

context. First, what is the optimal policy of the central bank? For this purpose, we model the central bank as a Ramsey planner who has exclusive access to the international capital market, and we examine optimal policies that maximize the average utility of the population. The second question is how does the semi-open economy compare to a small open economy? This question is interesting because we know that an open economy typically produces a higher welfare than a closed economy, in particular because it allows intertemporal trade. But the semi-open economy also enables intertemporal trade. Moreover, in a semi-open economy the central bank can choose a real interest rate different from the world interest rate. Thus, an economy with limited capital mobility may have a higher welfare than an open economy.

In a model where households face a borrowing constraint, we find that the competitive equilibrium of an open economy may not be socially optimal and a combination of capital controls and reserve policy may improve welfare. If the planner is not subject to the same borrowing constraint, it can improve the households' intertemporal allocation of resources. When the set of policy instruments is limited, the best way to improve intertemporal allocation is to manipulate the interest rate, which is equivalent to subsidizing borrowing or saving. But allowing the domestic interest rate to deviate from the world interest rate requires capital controls. And when capital controls are in place, intertemporal trade for the aggregate economy can only be achieved by variations in the level of reserves.

We consider a simple economy with saving emanating from credit-constrained consumers. The model is an extension of the endowment economy presented by Woodford (1990). There are two groups of consumers with endowments fluctuating periodically. In each period, one of the groups has a low endowment and may not be able to smooth consumption due to a credit constraint. This may generate

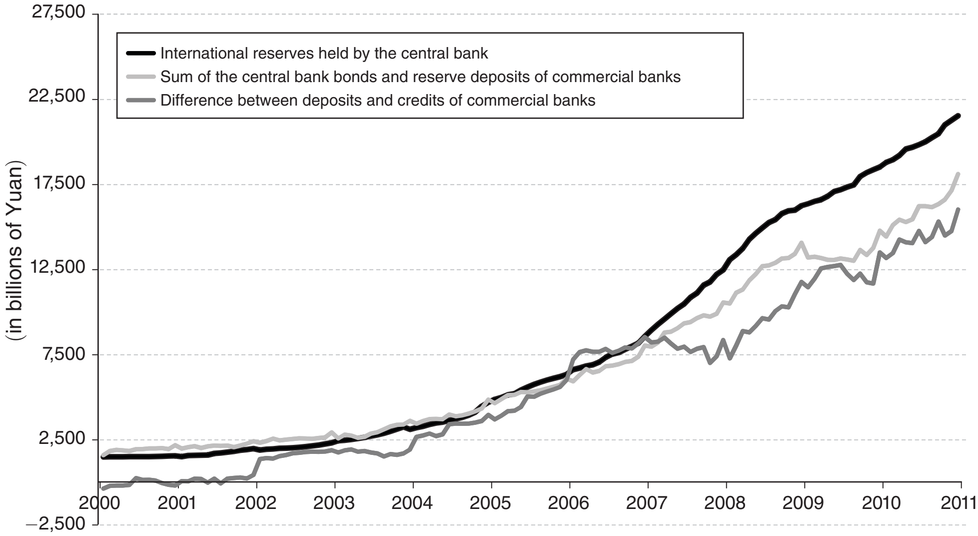

Figure 2. Saving Instruments of the Central Bank and the Excess of Deposits over Lending

Source: IMF IFS database

additional saving in the period of high endowment. In an open economy, this model would imply excess saving based on several mechanisms proposed in the recent literature on global imbalances. First, there is a large potential demand for assets as in Mendoza, Quadrini, and Rios-Rull (2009) and in Bacchetta and Benhima (2012). Second, when credit constraints are tight, domestic assets are scarce in the spirit of Caballero, Farhi, and Gourinchas (2008). In a semi-open economy, the central bank may improve the saving opportunities by providing assets. This can be associated with an increase in international reserves. In practice, the assets provided by a central bank are typically made of commercial banks' reserves and of central bank bonds. Figure 2 shows that in the Chinese economy there is a close relationship between the liabilities of the central bank and international reserves.

In the steady state of this economy, however, there is no need to improve the intertemporal allocation if we assume that the discount rate and the growth rate are the same as in the rest of the world. The reason is that consumers are able to avoid the constraint when they approach the steady state. Consequently, it is optimal to replicate the open economy in the steady state, and thus to set the domestic interest rate equal to the international rate. The optimal amount of international reserves in a semi-open economy is then equal to the amount of foreign assets that would prevail in an open economy. Basically, the central bank provides assets and finances it by the accumulation of international reserves. Therefore, as suggested by Song, Storesletten, and Zilibotti (2010) or Wen (2011), the central bank may simply serve as intermediary between the private sector and international capital markets when the economy has limited capital mobility.

Results are different when we consider growing economies that converge to their steady state. This situation is more relevant in the context of the Chinese economy.

In this case, the open economy is usually not the first best, so that the optimal interest rate differs from the international interest rate. The reason is that credit constraints are binding on the convergence path as consumers are not able to smooth their consumption. Thus, there is an incentive for the planner to relax the credit constraint. If households are constantly borrowing in the transition, it could be optimal to lower the interest rate. However, when households face fluctuating income and tight credit constraints, the opposite is true. For example, assume that no borrowing is allowed, as in Woodford (1990). In that case, households need to draw down their savings when their income is low. It is then optimal to increase the return on saving to subsidize temporarily low-income households. We show that the incentive to subsidize savers with a higher interest rate dominates when borrowing is small, when saving is high, and when current savers are likely to be constrained in the future. Consequently, we argue that it is optimal to temporarily increase the interest rate in an economy with characteristics similar to the Chinese economy, namely: tight credit constraints, which induce low borrowing; substantial fluctuations in individual revenues, which induce high savings; and sustained growth, which induces binding future constraints.

We focus on a planner, the central bank, who has only two instruments: the levels of domestic public debt and of international reserves, which allows it to manipulate the domestic interest rate. When more instruments are available, like lump-sum taxes or consumption taxes, we show that the planner can reach a first best by fully relaxing the credit constraint.[2] In these cases, however, excess saving by the private sector disappears and there is no role for reserve accumulation.

Our analysis shares various features with existing literature. In particular, the motive for saving is similar to the precautionary saving motive of Mendoza, Quadrini, and Rios-Rull (2009). As it is known from the saving literature (see Huggett 1993; Aiyagari 1994; and Carroll 1997), idiosyncratic risk can generate precautionary saving in the presence of credit constraints, even if the utility function does not feature "prudence" (i.e., a positive third derivative). The Woodford (1990) model is a simple way to mimic this precautionary saving motive. In this model, the income stream is deterministic, but it fluctuates, which generates additional saving when agents face financial constraints, even in the absence of risk. We should therefore find similar results in a model with idiosyncratic uncertainty.

Similarly, the optimal provision of public debt in the presence of borrowing constraints is a standard result (e.g., see Woodford 1990; or Aiyagari and McGrattan 1998). Moreover, the desirability of using the international capital market to provide domestic liquidity when taxes are distortionary can be found in Holmström and Tirole (2002, 2011).

On the other hand, our perspective differs from the vast literature on international reserves and on capital flows. Much of the literature on international reserves focuses on its role as an insurance against *aggregate* shocks.[3] In contrast, the

[2] Benigno et al. (2011) also find that the credit constraint stops binding when the planner has a tax on traded or on nontraded goods consumption.

[3] For recent contributions see, for example, Aizenman (2011); Aizenman and Lee (2007); Barnichon (2009); Durdu, Mendoza, and Terrones (2009); or Obstfeld, Shambaugh, and Taylor (2012).

accumulation of reserves in our paper arises from the insurance of *idiosyncratic* shocks.[4] Consequently, the perspective taken in this paper should be seen as complementary to the literature. Actually, Jeanne and Rancière (2011) find that the precautionary motive against aggregate shocks is not sufficient to explain international reserve accumulation in China.

Our analysis also differs from the recent literature on the optimality of capital controls or more generally on limits to borrowing (e.g., Korinek 2010; Jeanne and Korinek 2011; Bianchi 2011; or Bianchi and Mendoza 2010). In that literature, the justification for limits to capital mobility comes from pecuniary externalities. Typically, external borrowing affects a relative price, the exchange rate or an asset price, so that the financial constraint becomes tighter. The private sector does not internalize the effect, which gives a role for government intervention. In our case, however, the justification comes simply from the presence of credit constraints in a growing economy. Consumers would be better off in reallocating resources from the future to the present and government intervention can increase welfare by using its available instruments.

The rest of the paper is organized as follows. In Section I, we describe the model and the various equilibrium concepts. In Section II, we analyze the competitive economy for a given policy and examine the impact of changing the supply of government bonds. In Section III, we examine the Ramsey planner problem with foreign reserves. In Section IV, we examine the case where the planner can also choose optimally the consumption tax rate. Section V evaluates the model's assumptions in the context of the Chinese economy and compares the implied optimal policies with the actual policies conducted in China. Section VI concludes.

I. Model

The economy is inhabited by infinitely-lived households that consume every period, but alternate between low and high endowment periods, as in Woodford (1990, section I). This structure implies that households save in their periods of high endowment and would like to borrow in their periods of low endowment.[5] But the extent of borrowing can be limited by creditors, which leads to a desire for additional saving. Saving is in the form of bonds. There is a gross interest rate r_t on lending and borrowing.

In addition to households there is a Ramsey planner that we call the central bank, which can issue bonds and hold international reserves. When credit constraints are tight, the demand for funds by cash-poor households is small. In a closed economy, this limits the opportunities to save for cash-rich households. In this case, the provision of bonds by the central bank may be desirable.

[4] A similar difference is found in the literature on optimal government debt in contexts where Ricardian equivalence does not hold. When shocks are at the aggregate level, it is optimal for a government to accumulate assets (e.g., see Aiyagari et al. 2002). In contrast, when there are idiosyncratic shocks in the private sector, it is optimal for the government to issue debt. Shin (2006) introduces both motives in a closed economy. It would be interesting to extend such an analysis to a semi-open economy. In general, we can conjecture that there would be motives for holding international reserves coming both from aggregate and from idiosyncratic shocks.

[5] There are three basic differences with Woodford (1990): consumers may be able to borrow, there is a Ramsey planner, and there is no capital stock.

A. *Households*

There are two groups of mass one of households. At time t, a first group of households receives an endowment Y_t, while the second group receives aY_t, with $0 \leq a < 1$. At $t + 1$, the first group receives aY_{t+1}, while the second receives Y_{t+1}, and so on. We refer to the group with Y as cash-rich households, or savers, and the group with aY as cash-poor households, or borrowers. Each household alternates between a cash-rich and a cash-poor state, and each period there is an equally sized population of rich and poor.

Households maximize

$$\sum_{s=0}^{\infty} \beta^s u(c_s). \tag{1}$$

We denote consumption during the cash-rich period as c^D. In this period, households will typically save the amount D. Saving takes the form of one-period contracts, either as direct loans to borrowing cash-poor households or in public debt holdings.[6] Consumption during the cash-poor period is denoted c^L. In this period, households borrow L. At time t, households choose D_{t+1} or L_{t+1} with a (known) gross interest rate r_{t+1}. Consider a household that is cash-rich at time t and cash-poor at date $t + 1$. Its budget constraints at t and $t + 1$ are

$$Y_t - r_t L_t = \tau_t c_t^D + D_{t+1}, \tag{2}$$

$$aY_{t+1} + r_{t+1} D_{t+1} = \tau_{t+1} c_{t+1}^L - L_{t+2}. \tag{3}$$

The income of the household at date t, which is composed of endowment Y_t minus debt repayments $r_t L_t$, is allocated to saving D_{t+1} and consumption c_t^D, including a flat-rate consumption tax $\tau_t - 1$ with $\tau_t > 0$. In the following period, at $t + 1$, its income is composed of the return on saving, $r_{t+1} D_{t+1}$, and of aY_{t+1}. This has to pay for consumption c_{t+1}^L and taxes. Typically the cash-poor household will borrow, so that at the optimum, $L_{t+2} \geq 0$.

The cash-poor household might face a credit constraint when borrowing at date $t + 1$. Due to standard moral hazard arguments, a fraction $0 \leq \phi < 1$ of the endowment is used as collateral for bond repayments:

$$r_{t+2} L_{t+2} \leq \phi Y_{t+2}. \tag{4}$$

The multiplier associated with this constraint is denoted $u'(c_{t+2}^D)\lambda_{t+2}/\tau_{t+2}$.

Cash-rich households at time t satisfy the following Euler equation:

$$u'(c_t^D) = \beta r_{t+1} \frac{\tau_t}{\tau_{t+1}} u'(c_{t+1}^L). \tag{5}$$

[6] Alternatively, we could introduce a banking sector that allocates deposits between loans and public debt.

Similarly, poor households at date t satisfy the following Euler equation:

(6) $$u'(c_t^L) = \beta r_{t+1} \frac{\tau_t}{\tau_{t+1}} u'(c_{t+1}^D)(1 + \lambda_{t+1}).$$

The intertemporal choice of a cash-poor household is distorted when the credit constraint is binding, because $\lambda_{t+1} > 0$. The following slackness condition also has to be satisfied:

(7) $$(\phi Y_{t+1} - r_{t+1} L_{t+1})\lambda_{t+1} = 0.$$

B. *Central Bank Policy*

The central bank issues domestic bonds B_{t+1} at time t that pay an interest rate r_{t+1} and have access to foreign reserves B_{t+1}^* that yield the world interest rate r^*. We assume that the world interest rate is $r^* = 1/\beta$. Private agents cannot buy external bonds directly, so the domestic interest rate is determined in the domestic bond market. Equilibrium in this market is

(8) $$B_{t+1} = D_{t+1} - L_{t+1}.$$

In the presence of capital controls, only the central bank has access to external assets, so it has a monopoly over the supply of bonds to domestic agents. It can therefore manipulate the domestic interest rate r_t by appropriately setting the supply of bonds B. The possibility of accumulating reserves B^* enables the central bank to change the domestic supply of bonds by simply expanding its balance sheet. For example, the central bank can increase the domestic supply of assets by simultaneously buying foreign reserves B^* and issuing public debt B on the internal bond market, which leads to a higher domestic interest rate.[7]

When the central bank policy creates a wedge between r_t and r^*, this generates revenues or losses that have to be financed by the government. To focus on the central bank, we assume that there is neither government consumption nor government debt[8] and the only tax is τ_t. Consequently, the consolidated budget constraint of the central bank and the government gives

(9) $$B_{t+1}^* + r_t B_t = r^* B_t^* + B_{t+1} + (\tau_t - 1)(c_t^D + c_t^L).$$

We impose the usual no-Ponzi condition to the central bank net asset position:

(10) $$\lim_{T\to\infty} \frac{B_T^* - B_T}{(r^*)^T} = 0.$$

[7] Notice that this institutional framework is well illustrated by Figure 2, where the sum of the central bank bonds and reserve deposits of commercial banks (B) closely matches the difference between deposits and credits of commercial banks ($D - L$) and moves together with international reserves (B^*).

[8] Introducing government debt in addition to central bank debt would change little of the analysis.

In the following, we consider two cases. In the first case, fiscal policy is constrained. In each period, the central bank distributes all its profits $(r^* - 1)B_t^* - (r_t - 1)B_t$ to the government and the government balances its budget on a period-by-period basis. Then the tax rate τ_t is given by

$$\tau_t = 1 + \frac{(r_t - 1)B_t - (r^* - 1)B_t^*}{c_t^D + c_t^L}. \tag{11}$$

This implies that the net asset position of the central bank, $B_t^* - B_t$, is constant. The optimal central bank policy in this context has to take into account its consequences in terms of tax burden; a policy that generates revenues (losses) will decrease (increase) the contemporaneous tax rate. This first case with constrained fiscal policy is the most realistic and is examined in Section III.

Section IV examines the second case where fiscal policy is unconstrained. This corresponds to a situation where the Ramsey planner is a consolidated entity made of both the central bank and the government and can freely choose τ_t every period as an additional instrument. In this case, (11) does not need to hold.

Notice that the wedge between r and r^* is akin to a subvention (tax) on saving (loans).[9] Setting a higher domestic interest rate is therefore equivalent to subsidizing saving and taxing loans, while setting a lower interest rate is equivalent to taxing saving and subsidizing loans. When setting the domestic interest rate, the central bank faces a trade-off between savers and borrowers.

When the central bank sets $r = r^*$ it can replicate the open economy. In general, we consider three policy regimes, which correspond to different constraints imposed on the set of policy instruments:

DEFINITION 1 (Policy Regimes): *We define the following policy regimes:*

(*i*) *The closed economy, where* $B^* = 0$ *and* $r \in \Re^+$;

(*ii*) *The open economy, where* $B^* \in \Re$ *and* $r = r^*$;

(*iii*) *The semi-open economy, where* $B^* \in \Re$ *and* $r \in \Re^+$.

In the semi-open economy regime, the central bank uses its exclusive access to foreign bonds to set the domestic supply of bonds and manipulate the domestic interest rate. Since this is the more general case, we consider optimal policy within this regime. Indeed, both the open economy and the closed economy are nested in the semi-open economy. In the closed economy, the central bank's access to reserve accumulation is restricted, so neither the central bank nor the private agents can trade foreign bonds. In the open economy regime, we assume that the central bank provides the desired supply B_t at interest rate r^* to the private

[9] Indeed, the net return $r^*B^* - rB$ of the central bank balance sheet can be rewritten as $r^*(B^* - B) - (r - r^*)B = r^*(B^* - B) - (r - r^*)D + (r - r^*)L$, which is the net return on government assets at the world interest rate, minus subsidies on deposits, plus taxes on loans, if $r > r^*$.

sector. This is equivalent to let the private sector directly buy foreign assets as B_t and B_t^* are perfect substitutes in this case. This is because private agents and the central bank face the same world interest rate. Besides, the central bank does not have a superior capacity to enforce repayment by domestic agents, so it cannot relax their borrowing constraint. As a result, for a given tax rate, private borrowing at rate r^* from the central bank is perfectly equivalent to borrowing directly from foreigners.

The semi-open economy regime can be implemented by the central bank with the use of capital controls and reserve accumulation. As a Ramsey planner, it will choose a policy under that regime to maximize its social objective:

$$\sum_{s=0}^{\infty} \beta^s \left[u\left(c_s^D\right) + u\left(c_s^L\right)\right]. \tag{12}$$

If the optimal interest rate is equal to the world interest rate r^*, then capital controls are unnecessary. But if the optimal r differs from r^*, it means that capital controls are welfare-improving. Notice, however, that optimal policies are not necessarily Pareto optimal, as one of the groups may have a lower welfare. In the next section, we describe the competitive equilibrium, and in Section III we analyze optimal policies.

II. Competitive Equilibrium

In this section, we examine the properties of a competitive equilibrium for a given policy. First, we describe how the credit constraint affects consumption behavior and leads to additional saving. Then, we analyze the steady state and determine the conditions under which the economy is constrained. At the end of the section, as a benchmark case, we analyze a growing open economy without any policy intervention. In order to get analytical results, this section considers the case of a logarithmic utility function $u(c_t) = \log(c_t)$.

We define a competitive equilibrium as follows:

DEFINITION 2 (Competitive Equalibrium): *Given an endowment stream* $\{Y_t\}_{t\geq 0}$ *and initial conditions* $r_0, D_0, L_0, B_0, B_0^*$ *with* $B_0 = D_0 - L_0$, *a competitive equilibrium under a given policy regime is a sequence of prices* $\{r_{t+1}\}_{t\geq 0}$, *Lagrange multipliers* $\{\lambda_{t+1}\}_{t\geq 0}$, *an allocation* $\left\{D_{t+1}, L_{t+1}, c_t^D, c_t^L\right\}_{t\geq 0}$, *and a policy* $\{B_{t+1}, B_{t+1}^*, \tau_t\}_{t\geq 0}$ *such that:*

(*i*) *given the price system and the policy, the allocation and the Lagrange multipliers solve the households' problems* (*equations* (2)–(7) *are satisfied*);

(*ii*) *given the allocation and the price system, the policy satisfies the sequence of consolidated budget constraints* (9), *the no-Ponzi condition* (10), *and a given policy regime;*

(*iii*) *equilibrium in the domestic bond market* (8) *is satisfied.*

In each competitive equilibrium, the only constraints that the policy must satisfy are the consolidated budget constraints, the no-Ponzi condition, and the constraints imposed by the policy regime on r or B^*. The central bank's Ramsey policy will consist of maximizing the social objective over its policy set. As explained in Section I, the policy set consists of $\{B_{t+1}, B^*_{t+1}, \tau_t\}$, with a tax rate τ_t either set to balance the government budget according to equation (11), or set optimally without the constraint (11).

A. *Saving Behavior*

In the absence of credit constraints and with log-utility, households would consume a fixed fraction of their intertemporal wealth:

$$\hat{c}^D_t = \frac{1-\beta}{\tau_t}\left(\sum_{k=0}^{\infty}\frac{Y_{t+2k}}{\prod_{i=1}^{2k} r_{t+i}} + \sum_{k=0}^{\infty}\frac{aY_{t+2k+1}}{\prod_{i=1}^{2k+1} r_{t+i}} - r_t L_t\right), \tag{13}$$

$$\hat{c}^L_t = \frac{1-\beta}{\tau_t}\left(\sum_{k=0}^{\infty}\frac{aY_{t+2k}}{\prod_{i=1}^{2k} r_{t+i}} + \sum_{k=0}^{\infty}\frac{Y_{t+2k+1}}{\prod_{i=1}^{2k+1} r_{t+i}} + r_t D_t\right), \tag{14}$$

where $\prod_{i=1}^{0} r_{t+i} \equiv 1$. Then, the unconstrained (or notional) demand for saving instruments and loans $\hat{D}_{t+1}$ and $\hat{L}_{t+1}$ are given by replacing $\hat{c}^D_t$ and $\hat{c}^L_t$ in the budget constraints (2) and (3).

To understand what happens when cash-rich households are constrained in $t+1$, we use the Euler equation to substitute for consumptions in the budget constraints and derive the expression for saving:

$$D_{t+1} = \frac{1}{1+\beta}\left(\beta(Y_t - r_t L_t) - \frac{aY_{t+1}}{r_{t+1}} - \frac{L_{t+2}}{r_{t+1}}\right). \tag{15}$$

Since saving is used to smooth consumption between the cash-rich period and the cash-poor period, it depends negatively on future borrowings L_{t+2}. As the credit constraint imposes that $L_{t+2} \leq \hat{L}_{t+2}$, we have then $D_{t+1} \geq \hat{D}_{t+1}$. This means that households save more when they anticipate that their borrowing capacity will be limited in the future. More specifically, the level of loans contracted by cash-poor households that are constrained in t is given by

$$L_{t+1} = \frac{\phi Y_{t+1}}{r_{t+1}}. \tag{16}$$

The interest rate r_{t+1} that clears the market for domestic bonds must be such that total saving D_{t+1} equals outside bonds B_{t+1} and borrowing L_{t+1}, as stated by equation (8), for a given level of B_{t+1}.

B. *Bonds Supply in Symmetric Steady States*

Whether the economy is constrained or not depends on the relative supply and demand for bonds. An economy with a tight constraint needs a larger supply of bonds B. This can be analyzed precisely in deterministic symmetric steady states, defined as follows.

DEFINITION 3 (Symmetric Steady State): *Consider a constant endowment stream* $Y_t = Y$ *for* $t \geq 0$. *A symmetric steady state is a constant interest rate* r, *Lagrange multiplier* λ, *allocation* (D, L, c^D, c^L), *and policy* (B, B^*, τ) *that form a competitive equilibrium associated to the endowment stream* Y *and the initial conditions* r, D, L, B, B^*.

In a symmetric steady state, endowments and consumptions of a given individual can still fluctuate through time; but their distributions across agents, respectively, $\{Y, aY\}$ and $\{c^D, c^L\}$, are stationary. Such a steady state is symmetric in the sense that all individuals have the same state-contingent consumption and wealth.

The following proposition characterizes the steady states of the model depending on the amount of bonds B.

PROPOSITION 1: *For all* $(Y, B, B^*) \in \Re^{+*} \times \Re^2$, *there is a unique symmetric steady state.*

If $\frac{B}{Y} < \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$, *the credit constraint is binding, the interest rate* $r < 1/\beta$ *increases with* $\frac{B}{Y}$ *and the ratio of relative consumption is given by* $\frac{c^L}{c^D} = \beta r < 1$.

If $\frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$, *the credit constraint does not bind and* $\beta r = 1$.

PROOF:

See Appendix Section A.

Whether the borrowing constraint binds in the symmetric steady state depends on the ratio B/Y. When this ratio is low, so is the interest rate. A low interest rate (chosen by the planner through the supply of bonds) leads to a binding credit constraint.

Intuitively, when the constraint binds, cash-poor households are not able to supply enough saving instruments to cash-rich households because of their limited collateral. As a result, bonds are overpriced compared to the first best, which corresponds to a depressed interest rate. It also prevents cash-poor households from transferring some of their consumption from the next to the current period, so that consumption is lower in the L-state than in the D-state.

A larger supply of bonds by the central banks provides more saving instruments to cash-rich households, alleviating the limited supply of bonds by cash-poor households and decreasing the price of bonds. As shown in Proposition 1, this results in a higher interest rate and better consumption smoothing. When the interest rate reaches $r = 1/\beta$, the constraint stops binding and the supply of central bank bonds has no more effect on the interest rate and the allocation of resources.[10]

[10] While we restrict the analysis to symmetric steady states, there can also be nonsymmetric steady states. When the borrowing constraint is not binding, there is a continuum of nonsymmetric steady states corresponding to

A direct consequence of Proposition 1 is that borrowing constraints never bind in the steady state of an open economy.

COROLLARY 1 (Open Economy): *Consider an open economy with* $\beta r^* = 1$ *and no taxes* ($\tau = 1$). *The constraint does not bind in the symmetric steady state, and* $\frac{B^*}{Y} = \frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$.

PROOF:

From Proposition 1, a binding constraint in a symmetric steady state implies $\beta r < 1$. As a consequence, $\beta r^* = 1$ in the open economy implies that the constraint does not bind in symmetric steady states. From the consolidated budget constraint (9), taken at the steady state, $\tau = 1$ and $r = r^*$ implies $B = B^*$. Then, from Proposition 1, the nonbinding constraint implies $\frac{B^*}{Y} = \frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$.

Notice that the open economy has positive reserves in a symmetric steady state if $\phi < \frac{1-a}{2(1+\beta)}$. In that case, stringent borrowing constraints prevent private agents from supplying enough saving instruments. Then, savers need a positive supply of bonds by the central bank, and therefore positive reserves, to overcome the constraint.[11] That $B^* > 0$ makes the borrowing constraint unbinding in countries with insufficient supply of saving instruments, which is socially optimal as we will see, depends in particular on the absence of lump-sum taxes. As suggested by Holmström and Tirole (2002, 2011), when taxes are distortionary, the international capital market is the best source for domestic bonds.[12] Thus, it is optimal for the central bank to simply serve as intermediary between the private sector and the international capital market. Equivalently, it would be optimal to liberalize private capital flows when the economy is in the steady state with $\beta r^* = 1$.

The above analysis shows that government debt can improve welfare in the presence of credit constraints. Credit constraints do not bind in steady state, however. Below we show that the situation may be different in growing economies. In this case, even the open economy might face binding borrowing constraints on the convergence path before reaching the steady state. We examine the optimal policy in the next section, but as a benchmark it is useful to examine the dynamics of open economies.

C. *The Convergence of Open Economies*

Consider a growing open economy with zero initial net assets and without planner's intervention, that is $\tau = 1$, $r = r^* = 1/\beta$, and $B = B^*$. The endowment is

different distribution of wealth across groups of households; consumption is constant for individual households but differs across groups. With a binding constraint, the steady state is necessarily symmetric since financial wealth is uniquely determined by the constraint.

[11] Under the same condition on ϕ, the borrowing constraint is binding in the steady state of a closed economy with no taxation and no supply of bonds by the central bank.

[12] Without reserves, issuing domestic bonds (i.e., government debt) would require varying the consumption tax, which would distort the households' Euler equation.

growing at rate g_t, i.e., $Y_{t+1} = (1 + g_{t+1})Y_t$. We consider an economy where g_t is driven by the following process:

$$g_{t+1} = \mu g_t, \tag{17}$$

where $0 \leq \mu < 1$. This is for example the case of an economy that catches up toward the world's productivity frontier. We examine, in particular, whether the economy is constrained on the convergence path and whether it is asymptotically constrained or unconstrained.

In the open economy case, each household faces the world interest rate, so it behaves independently from the others. It is therefore sufficient to examine the behavior of a given household. The open economy is then only the aggregation over the two groups of households.

PROPOSITION 2: *Consider an open economy with $B_0 = B_0^*$, $\beta r^* = 1$, and no taxes ($\tau = 1$). If cash-poor households are constrained in t and $g_0 > 0$, then they are constrained in all their subsequent cash-poor periods. Additionally, if growth is not too large so that $1 + g_0 \leq \frac{1-\phi}{a(1-\phi)+\beta\phi(1-a+\mu g_0)}$, then they are unconstrained in all their subsequent cash-rich periods. Moreover, if g_0 is close to zero, then we can make the following first-order approximation: $\lambda_{t+2k+1} \simeq \mu^{2k}(1 + \mu) \times \frac{1-\phi+\mu\beta a+\mu^2\beta^2\phi}{1-\phi+\beta a+\beta^2\phi} g_t$, $k \geq 0$.*

PROOF:
See Appendix Section B.

The Proposition gives conditions for the credit constraints to stay binding, as well as an approximation for λ. During the transition, the economy can be constrained for any level of $\phi < 1$ because growth generates a strong need for borrowing to smooth consumption. The economy stays constrained on the convergence path but gradually moves toward the edge of the unconstrained region and becomes asymptotically unconstrained ($\lim_{t\to\infty} \lambda_{t+1} = 0$). Moreover, the convergence speed of λ is $1 - \mu$, which is the convergence speed of the growth rate. When growth is sustained (μ is large), the credit constraint remains stringent. In the limit, when $\mu \to 1$, λ remains indefinitely equal to $2g_0$.

If agents are initially unconstrained, they liquidate their assets progressively as g goes to zero. They eventually end up with binding constraints if initial wealth is low, if g converges to zero slowly, and if credit constraints are stringent (ϕ is small).

III. Optimal Policy

The optimal policy crucially depends on the set of instruments available to the planner. As explained in Section I, we first consider the case of a constrained fiscal policy where the level of the distortive tax $\tau_t - 1$ follows the balanced-budget rule (11). In the next section, we consider the case where the planner can freely choose the level of the consumption tax $\tau_t - 1$ every period.

We consider the optimal policy both in the steady state and in an economy converging to its steady state. In both cases, we find that it is optimal to accumulate reserves. Moreover, when the economy is away from its steady state, it is optimal to have r_t diverge from r^*. When r_t is larger (smaller) than r^*, it is then optimal to accumulate more (less) reserves than in the open economy.

A. *The Ramsey Problem*

To analyze optimal policy, we consider the Ramsey planner under the semi-open economy regime. The planner maximizes its objective (12) over the set of competitive equilibria subject to the balanced budget fiscal rule (11). Without loss of generality, we assume zero initial net assets ($B_0^* - B_0 = 0$). This implies $B_t = B_t^*$. The Lagrangian of the Ramsey problem in the semi-open economy can then be defined as follows:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t^D) + u(c_t^L) \\
& + \gamma_t^D \big[Y_t - \tau_t c_t^D - D_{t+1} - r_t L_t\big] \\
& + \gamma_t^L \big[aY_t + r_t D_t + L_{t+1} - \tau_t c_t^L\big] \\
& + \gamma_t^G \big[r^*(D_t - L_t) - (D_{t+1} - L_{t+1}) + (1+a)Y_t - c_t^D - c_t^L\big] \\
& + \kappa_t^D \big[u'(c_t^D)\tau_{t+1} - \beta r_{t+1} u'(c_{t+1}^L)\tau_t\big] \\
& + \kappa_t^L \big[u'(c_t^L)\tau_{t+1} - \beta r_{t+1} u'(c_{t+1}^D)\tau_t(1+\lambda_{t+1})\big] \\
& + \Gamma_t[\phi Y_t - r_t L_t] \\
& + \Delta_t[(\phi Y_t - r_t L_t)\lambda_t]\Big\}.
\end{aligned}
$$

Maximization is carried out with respect to $\{L_{t+1}, D_{t+1}, c_t^D, c_t^L, r_{t+1}, \lambda_{t+1}, \tau_t\}_{t \geq 0}$. The seven constraints are: the household budget constraints (2) and (3), the aggregate resource constraint (corresponding to the multiplier γ_t^G), the first-order conditions (5) and (6), the borrowing constraint (4), and the complementary slackness condition (7). To get the aggregate resource constraint, we have substituted the agents' budget constraints into the consolidated budget constraint (9), and used the fact that $B_t = B_t^* = D_t - L_t$ from the equilibrium on the bond market (8). Notice that the planner takes as constraints both the borrowing constraint (which does not necessarily bind) and the complementary slackness condition, which both enter in the definition of the competitive equilibrium. It is useful to define $\Lambda_t = \Gamma_t + \lambda_t \Delta_t$.

While the full solution to this dynamic optimization has to be solved numerically, some interesting properties can be derived analytically. In particular, one can determine whether the planner wants to deviate from the open economy regime with $r = r^*$. For this purpose, we focus on the first-order condition with respect to r_{t+1}:[13]

$$\gamma_{t+1}^{L} D_{t+1} - (\gamma_{t+1}^{D} + \Lambda_{t+1}) L_{t+1} - \kappa_t^D u'(c_{t+1}^L)\tau_t - \kappa_t^L u'(c_{t+1}^D)\tau_t(1 + \lambda_{t+1}) = 0. \tag{18}$$

The first two terms reflect the direct distributive effects between savers and borrowers (or cash-rich and cash-poor households). The last two terms reflect the effect of the interest rate on the intertemporal choices of households. These terms reflect the potential need for Pigovian taxation.

To examine the optimality of the open economy, we evaluate the above first-order condition in an open economy with no central bank intervention in a constrained transition path as the one studied in the previous section.

B. *Is the Open Economy Optimal?*

To determine whether r_{t+1} should be lower or higher than r^*, we evaluate the left-hand side of (18) at $r_{t+1} = r^*$. Let us denote this expression by I_{t+1}^*. This amounts to considering the optimal policy in the open economy regime. Then, the first-order condition with respect to r_{t+1} is replaced by $r_{t+1} = r^*$ and I_{t+1}^* is not necessarily equal to zero. Since the budget is balanced and $B_0 = B_0^*$, we also have $\tau_t = 1$. In general, any deviation of I_{t+1}^* from zero means that the open economy is suboptimal and that the central bank can improve welfare by setting r_{t+1} different from r^*—i.e., by accumulating more or less reserves than in the open economy. When I_{t+1}^* is positive, social welfare can be increased by raising the interest rate with respect to r^*. Similarly, if I_{t+1}^* is negative, it is optimal to have a lower interest rate.

To determine the sign of I_{t+1}^*, it is useful to note that if the economy is constrained along the convergence path, $\kappa_t^D = \kappa_t^L = 0$, for all $t \geq 0$. This means that there is no Pigovian tax. This is because distorting the households' intertemporal choices is ineffective when credit constraints are binding. As a result, I_{t+1}^* is simply equal to the direct redistributive effect of the interest rate. Using the other FOCs of the planner's program, we have (see Appendix Section C for more details):

$$\begin{aligned} I_{t+1}^* = {} & \left(\sum_{i=1}^{\infty} \Lambda_{t+2i}\right) D_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+1+2i}\right) L_{t+1} \\ & - \left(\frac{c_{t+1}^L}{c_{t+1}^D + c_{t+1}^L} \sum_{i=1}^{\infty} \Lambda_{t+2i} + \frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} \sum_{i=1}^{\infty} \Lambda_{t+1+2i}\right)(D_{t+1} - L_{t+1}). \end{aligned} \tag{19}$$

The first term corresponds to the net effect of the interest rate on savers and is positive, as they benefit from higher returns on saving, which alleviates their future constraints. The second term corresponds to the net effect on borrowers. This term is negative because a high interest rate hurts the borrowing households both through higher interest payment and through a more stringent credit constraint. The

[13] The full set of first-order conditions is available in Appendix Section F.

third term corresponds to the effect of the tax burden. Indeed, if $D > L$ and $r > r^*$, the interest payments on domestic debt are higher than the proceeds from external reserves, so the tax rate needs to increase in order to balance the budget. This tax affects the households proportionally to their consumption and hampers their capacity to smooth consumption over time.

In the steady state, I^*_{t+1} converges to zero as Λ goes to zero. As discussed before, an open economy that is constrained on its transition path is unconstrained in the long run as long as its growth is asymptotically zero. In the steady state the utilities of the two groups of agents converge so that the central bank has no incentive to redistribute wealth by distorting the interest rate. This means that an open economy in the steady state is at the Ramsey optimum. This is because this economy has accumulated a sufficient amount of reserves.

However, in the transition $\Lambda > 0$, so that I^*_{t+1} can be either positive or negative. This means that the central bank has incentives to manipulate the domestic interest rate by setting capital controls. Whether the optimal r_{t+1} should be higher or lower than r^* depends on whether it is better to subsidize current savers or current borrowers. Current borrowers should be subsidized because they are constrained in the current period. But current savers might also be subsidized as they will be constrained in the next period. The trade-off between the two agents depends on their relative exposure to interest rates and on the dynamics of the economy: from (19), the sign of I^*_{t+1} depends on the relative size of D_t and L_t as well as on the evolution of the shadow cost of credit constraints, Λ_t. A special case is when Λ_t follows an autoregressive process. In this case, Lemma 1 gives precise conditions for the sign of I^*_{t+1}:

LEMMA 1: *If* $\Lambda_{t+2} = \xi\Lambda_{t+1}$ *for all* $t \geq 0$, *with* $0 \leq \xi < 1$, *then* I^*_{t+1} *is of the same sign as* $D_{t+1}/L_{t+1} - [(c^D_{t+1} + c^L_{t+1})/\xi c^D_{t+1} + 1]$ *for all* $t \geq 0$.

PROOF:
See Appendix Section D.

The expression $D/L - \left[(c^D + c^L)/\xi c^D + 1\right]$ illustrates the forces at work. First, the incentives for the central bank to raise the interest rate are strong if the amount of saving is large as compared to the amount of loans. Second, these incentives are increasing in the ratio between the shadow costs of the future and current credit constraints ξ. Indeed, if the constraints faced by future borrowers (today's savers) are more stringent than those faced by today's borrowers, then it is better to transfer resources to the former by increasing the return on saving. Third, the incentives to raise the interest rate increase with the share of savers in the tax base $c^D/(c^D + c^L)$. This share reduces the tax costs of increasing the interest rate since the borrowers, who suffer from the interest rate increase, face a lower tax burden.

Results are more explicit when we consider the specific process for converging endowments described in (17). In this case, we can derive the following proposition.

PROPOSITION 3: *Consider an open economy with* $B^*_0 = B_0$, $\beta r^* = 1$, *and no taxes* $(\tau = 1)$ *in which cash-poor agents are constrained in* t *and* $t - 1$, *with* $g_0 > 0$ *and* g_0 *close to zero. Then* I^*_{t+1} *is of the same sign as* $\frac{1 - a}{2(1 + 1/\mu)(1 + \beta)} - \phi$.

PROOF:
See Appendix Section F.

In this case, the sign of I^*_{t+1} depends on three parameters (a, μ, ϕ) related to the effects described above (abstracting from the discount rate). First, a higher ϕ implies a higher L, and therefore an incentive for a lower interest rate (lower I^*_{t+1}). But when ϕ tends to zero, it is optimal to subsidize saving and have a higher interest rate. Second, a lower a implies higher saving and an incentive for a higher interest rate (higher I^*_{t+1}). Finally, when μ is small, the convergence speed of g is high and future constraints are less stringent than the current ones. In that case, it is optimal for the central bank to decrease the domestic interest rate in order to subsidize the currently constrained agents. When μ is large, growth is sustained, and future constraints are stringent. This increases the incentive to raise the interest rate to subsidize future borrowers.

The trade-off between savers and borrowers naturally disappears when the economy tends toward a representative agent setup. Consider the limit $a \rightarrow 1$, where all households receive the same endowment. In that case, I^*_{t+1} is always negative so that r_{t+1} should unambiguously be lower than r^*. The representative agent is prevented by the binding borrowing constraint to consume early on its growing future endowment. The planner can help by decreasing the interest rate, which amounts to subsidizing borrowing. A similar result arises when $\mu \rightarrow 0$. In that case, the constraint lasts only one period. The objective of the planner is simply to alleviate today's constraint, and it is then optimal to decrease the interest rate.

To summarize, we have shown that outside the steady state it is generally optimal to set $r_t \neq r^*$, i.e., to deviate from the open economy regime. The optimal deviation is complex given the trade-off between borrowers and lenders and the need to balance the government budget.[14] In particular, we have found circumstances under which it is optimal to have $r_t > r^*$. The intuition given at the beginning of this paper is that when credit constraints are tight, it is more efficient to subsidize saving. However, this case implies a positive tax rate on consumption $(\tau - 1 > 0)$, which has to be taken into account.

To understand in more details why this can be an optimal policy, assume that ϕ is small and that the endowment starts increasing at time t $(g_t > 0)$. First, consider savers at time t who will face a low endowment at $t + 1$. They clearly gain from a higher interest rate as they receive it in their low-endowment period $t + 1$ and as the tax burden is shared among borrowers and savers. In contrast, consumers who are borrowers at time t suffer at $t + 1$ both because of a high interest rate and a higher tax rate. However, these consumers have a high endowment at $t + 1$. Thus, the loss of borrowers has a lower weight because of limited borrowing and a lower marginal utility. Overall, the high interest rate policy is a transfer from the high-endowment to the low-endowment, credit-constrained consumers. Then, from $t + 2$, the initial borrower will also benefit from the high interest rate. The initial borrower has a

[14] Notice that the trade-off between lenders and borrowers would disappear if the central bank could discriminate between cash-rich and cash-poor households and make conditional transfers.

lower utility in period $t + 1$ and may have a lower lifetime utility, which implies the optimal policy is not necessarily Pareto improving.[15]

C. *Numerical Simulations*

In order to derive optimal policies, we simulate the semi-open economy under the Ramsey policy. We consider growth episodes starting from the steady state. At $t = 0$, the economy is supposed to be in a steady state, where $\frac{B^*}{Y} = \frac{B}{Y} = \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$, so that the borrowing constraint is (marginally) nonbinding. At $t = 1$, the economy starts growing at an initial rate of 10 percent and the central bank implements the corresponding Ramsey policy. We set $r^* = 1/\beta = 1.05$. We consider a baseline case of sustained growth, strong borrowing needs, and stringent borrowing constraint: $\mu = 0.9$, $a = 0$, and $\phi = 0.1$. To simulate the optimal dynamics, we assume that the borrowing constraint is always binding during the transition and only becomes (asymptotically) nonbinding in the new steady state, as in the converging open economies described in Section IIC. For each simulation, we check that indeed $\lambda_{t+1} > 0$ for all $t \geq 1$.[16]

Figure 3 shows the result of the simulation in the baseline case. We see that it is optimal to increase the level of reserves and that saving, borrowing, and consumption all increase. An interesting feature is that the optimal interest rate is higher than the world rate in the transition to the new steady state. Correspondingly, the tax level is positive.

However, we saw in the previous section that the optimal interest rate depends on parameters. To illustrate the different outcomes, Figure 4 shows the optimal interest rates for three other parameter specifications that are more favorable to a lower interest rate according to Proposition 3: we lower μ to 0.3; we raise ϕ to 0.15; and we raise a to 0.5.

To better understand optimal policies we examine more closely two cases with different optimal interest rates: the baseline case with a higher interest rate, and the case where $\phi = 0.15$ with a lower interest rate. Figure 5 compares the dynamics of these two cases with the open economy.

In the baseline case (solid line), the central bank increases the supply of public bonds B above its open economy level and accumulates more reserves in order to increase the interest rate above its world level. Taxes increase to pay for the interest rate differential between reserves and public bonds. In this case, agents who borrow at the beginning of the growth episode initially suffer because the high interest rate makes their constraint more stringent and increases their debt repayment next period (c^L falls below its open-economy value in the first period). But later, they benefit from the larger return of their assets (c^L is above its open-economy value for $t \geq 2$). Agents consume less in their cash-rich periods than in the open economy because they save more (c^D is always lower than in the open economy). With a

[15] This is also true when it is optimal to have $r_t < r^*$. In that case the initial savers may have a lower lifetime utility.

[16] The model is simulated with DYNARE (Adjemian et al. 2011). The first-order conditions of the Ramsey problem are given in Appendix Section F.

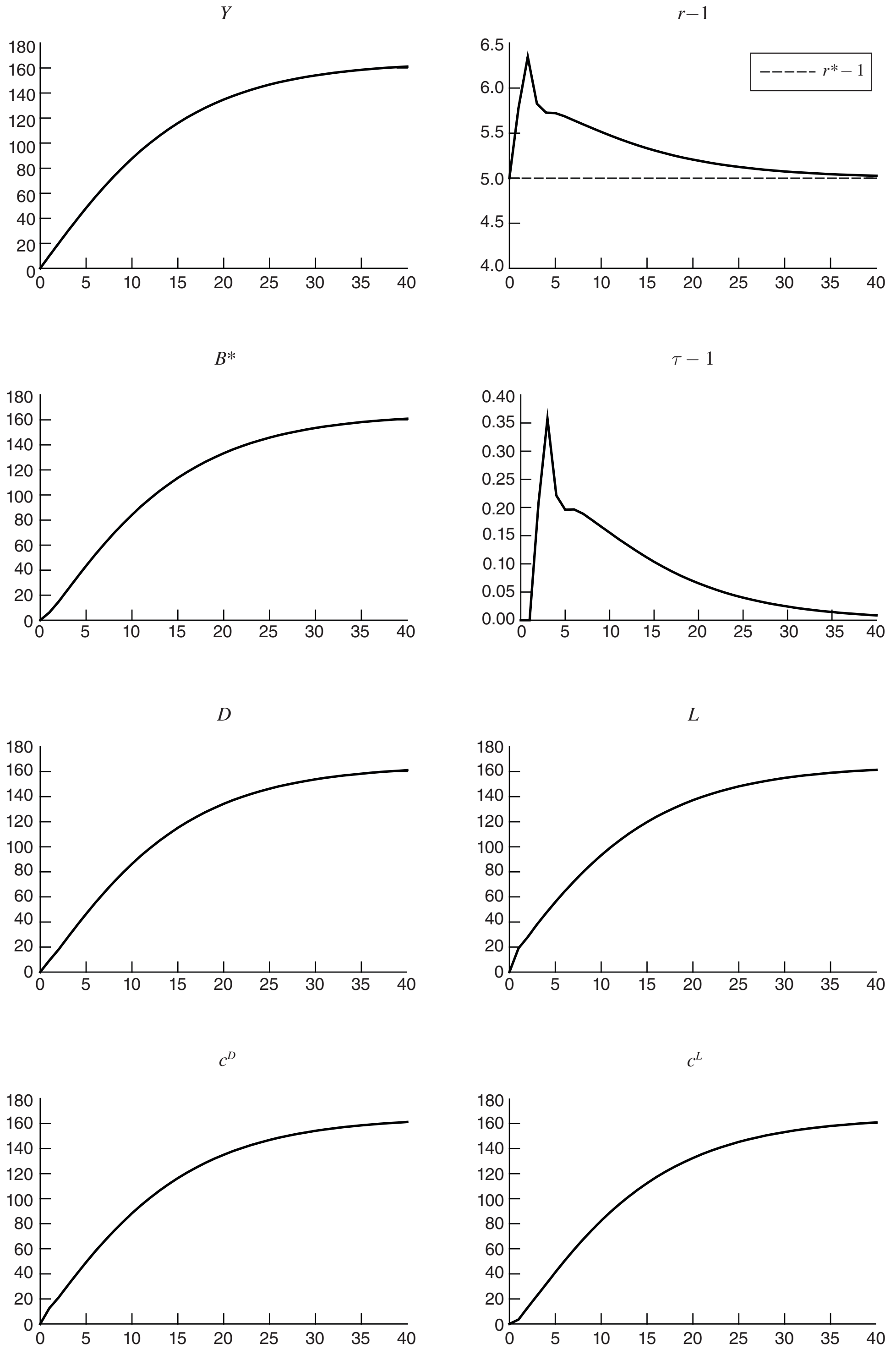

FIGURE 3. BASELINE SIMULATION

Note: Variables are displayed as percentage changes from their initial value, except $r - 1$ and $\tau - 1$, which are in percentage points.

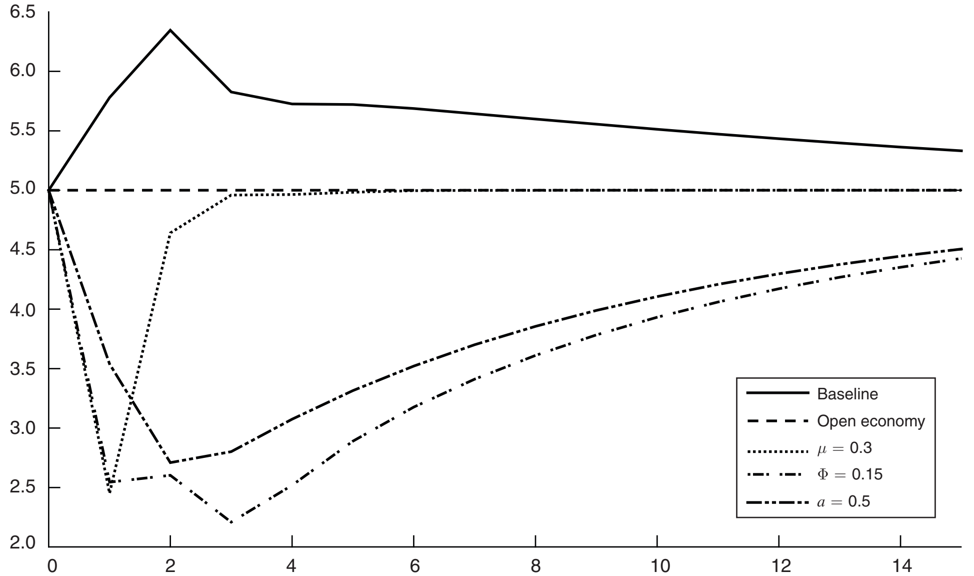

Figure 4. Optimal Interest Rate $r - 1$, in Percentage Points

higher interest rate, borrowing is lower and lending higher. This policy is welfare-improving because of the strong borrowing constraint; as the level of borrowing is limited, the negative impact of the higher interest rate on borrowers stays small. Consistently, we find that total welfare increases due to the increase in the welfare of agents who are savers at the date of the shock, even though it hurts agents who are initially borrowers.

The case of less stringent constraints $(\phi = 0.15)$ is plotted with a dashed line. To decrease the domestic interest rate, the central bank decreases the supply of public bonds below its open economy level and accumulates fewer reserves. The tax rate decreases to redistribute the fiscal resources created by the lower interest rate on central bank debt. In this case, agents who are borrowers at the time of the shock initially benefit from the lower interest rate as they can borrow and consume more in the first period. But later, they suffer from the lower return on their assets (c^L is above its open-economy level at $t = 1$ but below it later on). Savers consume more because they pay a smaller interest on their debt and receive a subsidy from the government. This policy is welfare-improving because loans are larger and saving lower than in the baseline. Then, a lower rate has a large effect on initial borrowers without hurting savers too much. Consistently, we find that total welfare increases, due to the increase in the welfare of agents who are borrowers at the date of the shock, even though the low-interest rate hurts agents who are initially savers.

IV. Optimal Policy with Unconstrained Fiscal Policy

In this section, we consider the case where fiscal policy is unconstrained and the planner can freely choose the optimal tax rate of consumption $\tau_t - 1$. We find that on a transition path, it is still optimal to have $r_t \neq r^*$, but it is no longer optimal to

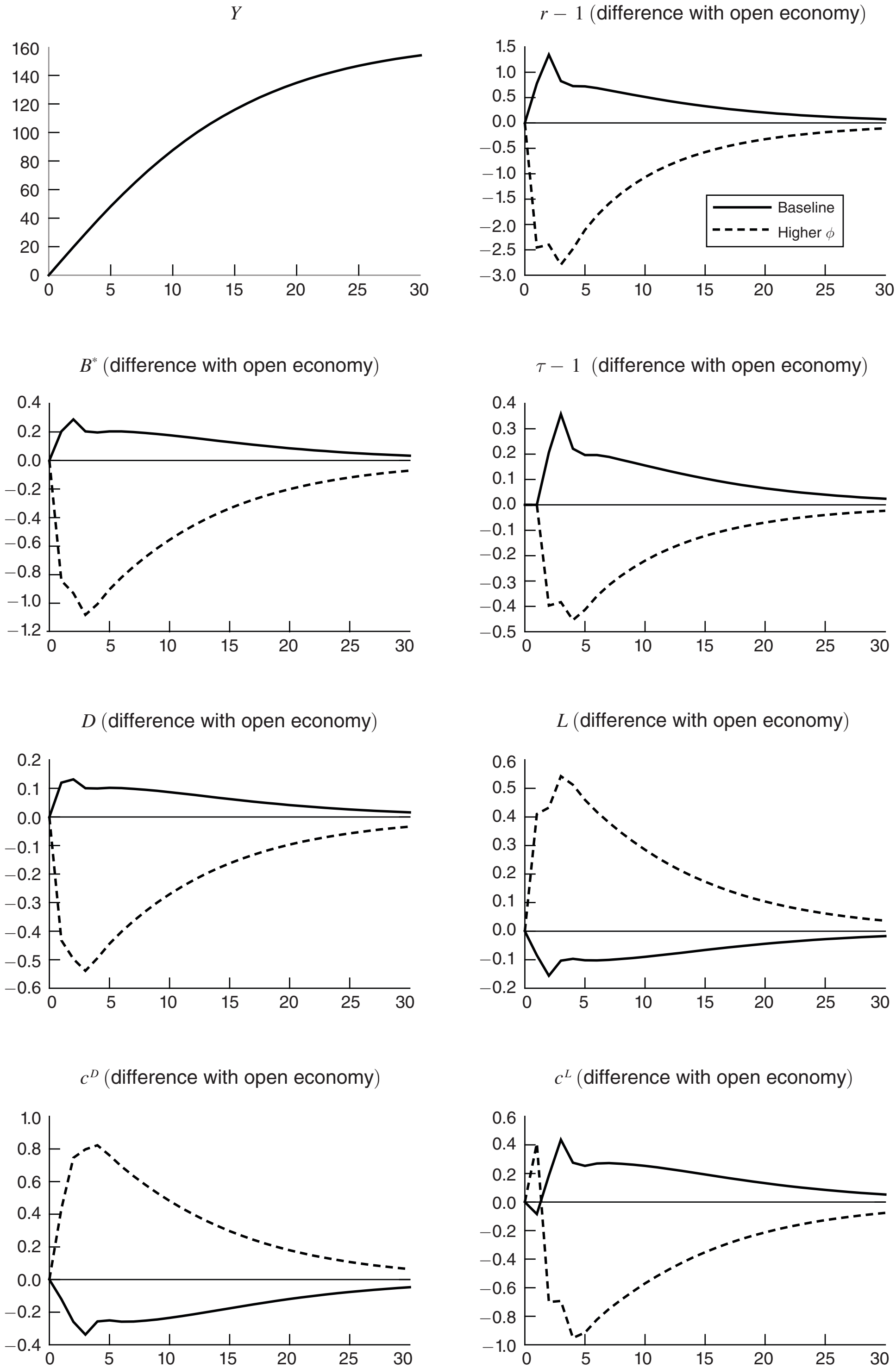

FIGURE 5. (ABSOLUTE) DIFFERENCE BETWEEN OPTIMAL POLICY AND OPEN ECONOMY, IN THE BASELINE AND WHEN $\phi = 0.15$

Note: Variables are multiplied by 100 (hence, $r - 1$ and $\tau - 1$ are in percentage points).

accumulate reserves. This is because the set of taxes is complete, but not redundant. The domestic interest rate and the consumption tax can be sufficient to overcome the borrowing constraint and transfer consumptions both across periods and across households. Reserve accumulation is not needed anymore; on the contrary, a growing economy actually borrows abroad (or depletes foreign reserves), as it would in a frictionless environment. To achieve this first-best allocation, however, the planner is compelled to use the domestic interest rate, which has to deviate from the world interest rate, in the absence of an alternative tax.

The following proposition describes how the planner can achieve a first-best allocation where borrowing constraints never bind and the marginal utility of consumption is constant across periods and households.

PROPOSITION 4: *Consider an endowment stream* $\{Y_t\}_{t\geq 0}$. *Assume initial conditions* $r_0 = r^*$, D_0, L_0, *and* $B_0^* = B_0 = D_0 - L_0$ *corresponding to the symmetric steady state of an open economy at the edge of the unconstrained region. Then, there exists a sequence of policy instruments* $\{B_{t+1}, B_{t+1}^*, \tau_t\}_{t\geq 0}$, *such that in the corresponding competitive equilibrium borrowing, constraints never bind and consumption is equal across time and across households.*

PROOF:
See Appendix Section G.

The proof of the proposition consists in explicitly constructing a policy that leads to constant consumptions and lax constraints. Intuitively, the planner can tax or subsidize consumption to transfer wealth across agents and periods. Subsidies are financed by borrowing abroad or depleting reserves, and the product of taxes can be saved by buying foreign reserves. However, as can be seen from equations (5) and (6), changes in the tax rate distort the Euler equation of households. To make sure that agents choose a constant consumption stream, the planner has to move the domestic real interest rate away from the world interest rate so that the tax-adjusted interest rate $r_{t+1}\tau_t/\tau_{t+1}$ stays equal to the world rate. Thus, in general, the optimal policy requires the economy to deviate from the open economy, as shown in the following corollary.

COROLLARY 2: *Consider an endowment stream* $\{Y_t\}_{t\geq 0}$ *and initial conditions satisfying the assumptions of Proposition* 4, *such that the borrowing constraint binds at some date* $t \geq 0$ *in the open economy with zero taxes* (*i.e., with* $\tau_t = 1, \forall t \geq 0$). *Then, along the optimal policy,* r_t *deviates from* r^* *for some date* t.

PROOF:
From the first-order conditions (5) and (6), we must have $r_{t+1}\tau_t = r^*\tau_{t+1}$ along the optimal policy. Suppose the domestic interest rate stays equal to r^*. This entails $\tau_{t+1} = \tau_t$, so that the tax rate can only change once and for all at $t = 0$. But if the tax rate deviates from $\tau = 1$ for $t \geq 0$ on, the consolidated budget constraint (9) implies that B_t^* will diverge either to $+\infty$ or $-\infty$, and violate the no-Ponzi condition (10). We conclude that the domestic interest rate has to deviate from the world interest rate in order to implement the optimal policy.

This case provides another justification for imposing capital controls during the convergence to the steady state. In the previous section, it was the binding constraint and absence of tax instruments that justified a different interest rate. In this section, the justification comes from distortionary taxes. The planner needs the domestic interest rate to deviate from the world interest rate to offset the distortionary effect of the tax. But if the planner had access to an alternative tax, like a wealth tax, distorting the domestic interest rate through capital controls would not be necessary. Notice that both reasons disappear once we reach a steady state (assuming $\beta r^* = 1$).

Figure 6 shows an example of optimal policy when taxes are flexible, and compares it to the baseline of the constrained fiscal policy case ($\mu = 0.9$, $a = 0$, $\phi = 0.1$). When taxes are flexible, the planner sets a large subsidy in the first period so that consumption jumps to its constant first-best level, and taxes consumption later on. It also issues bonds, allowing agents to save part of their subsidy to overcome the borrowing constraints in the subsequent periods. This raises the domestic interest rate and exactly offsets the future increase in tax rates in households' Euler equations. As aggregate consumption increases immediately to its higher optimal level, this policy requires running a current account deficit during the transition, and thus, decreasing foreign reserves. Notice that the initial increase in interest rate favors the initial savers. This explains the subsequent fluctuations in D, L, and the optimal interest rate.

V. Accumulation of Reserves and Capital Controls in China

Since the paper is motivated by developments in the Chinese economy in the last decade, it is interesting to compare the implications of the model with these developments. First, consider our basic assumptions. China has strict limits to capital mobility, in particular on outflows, while the central bank is very active on international capital markets. This corresponds well to the semi-open economy. Moreover, most households and firms have very limited access to finance. It is well-known that state-owned enterprises receive most of the banks' credit. In addition, the potential need for funds is high, due, for example, to a lack of a social safety net or to various life-cycle considerations in a fast-growing economy, e.g., funding for education, starting own business projects (e.g., see Yang, Zhang, and Zhou 2011). In the empirical literature, income uncertainty and poor access to finance in China are actually considered as the main drivers of the high private saving rate in that country and of global imbalances (IMF 2009; Chamon and Prasad 2010; Forbes 2010). Another important assumption is the high and sustained growth rate. China experienced an average growth rate of 10 percent from 2001 to 2010.

The first implication of our model is that it is optimal for the central bank to issue debt to the private sector and to accumulate international reserves. This is exactly what Figures 1 and 2 show. Thus, the accumulation of reserves of the Chinese central bank can be part of an optimal plan, even though our model is too stylized to determine quantitatively whether the magnitude of reserves accumulation is optimal.

The other implication is that it is optimal to impose capital controls so that the domestic interest rate can differ from the foreign rate. Our model suggests that in the Chinese context it is optimal to keep the domestic interest rate temporarily

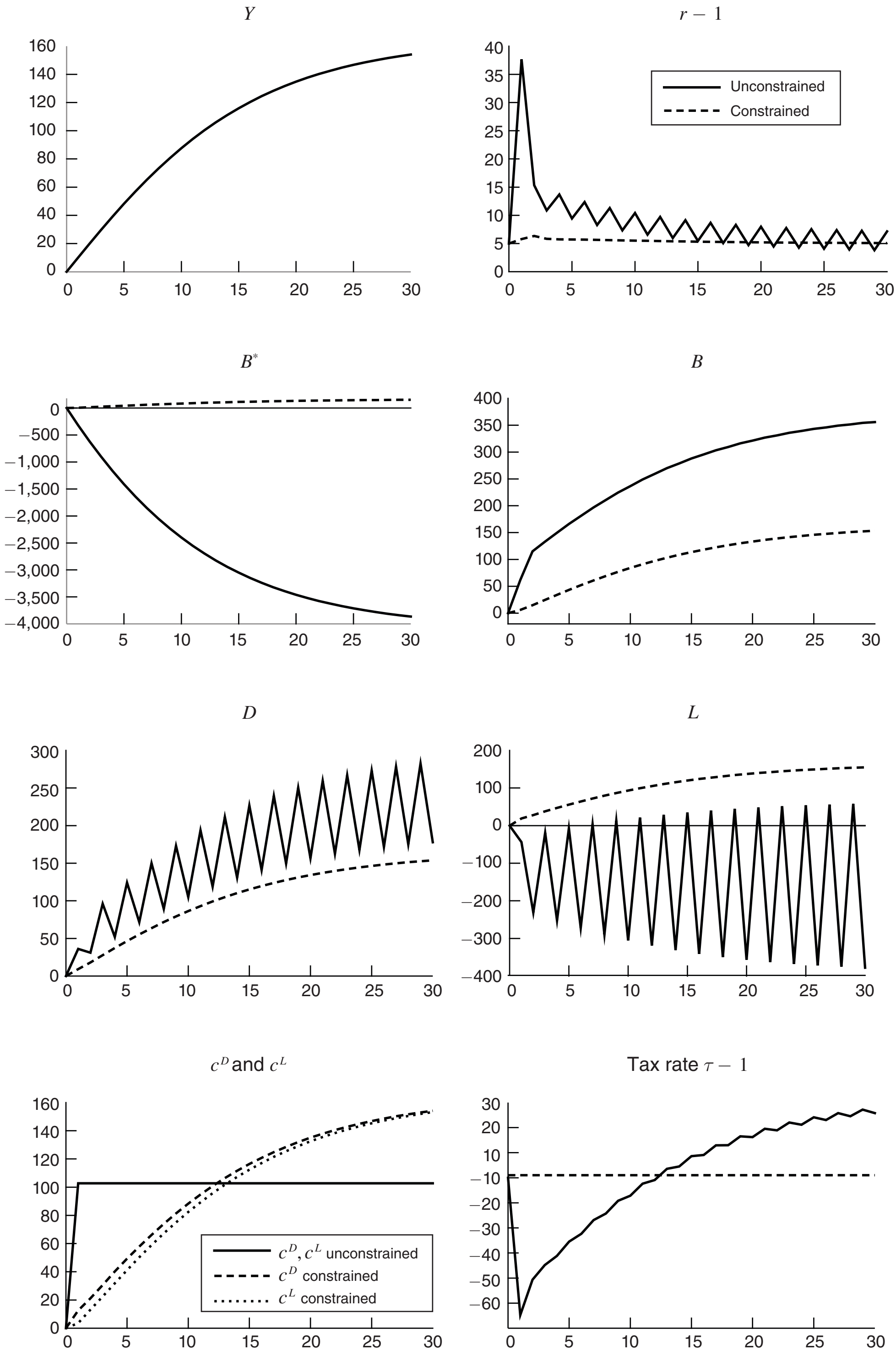

FIGURE 6. BASELINE SIMULATION WITH BOTH UNCONSTRAINED FISCAL POLICY AND CONSTRAINED FISCAL POLICY

Note: Variables are displayed as percentage changes from their initial value, except $r - 1$ and $\tau - 1$, which are in percentage points.

higher than the international one. Indeed, it appears reasonable to characterize the Chinese economy with a low value of ϕ, reflecting tight credit constraints; a low value of a, reflecting a high level of income uncertainty and heterogeneity; and a high value for μ, reflecting sustained growth. If we consider Proposition 3, the combination of low ϕ and a and high μ imply that a high interest rate is desirable. This implies an accumulation of reserves that is larger than what would happen in an open economy. Intuitively, with growth expected to stay high in the coming years, China should worry that the private sector might face strong funding constraints for consumption or investment needs for a long time. Increasing the interest rate on saving instruments should then help the private sector accumulate assets to fund those future needs.

Comparing the models' implications with Chinese interest rate policy is tricky because our theory abstracts from various aspects, such as inflation, nominal exchange rates, spreads between loan and deposit rates, term spreads, etc. However, it is safe to argue that the policy observed in China in the last decade does not correspond to the optimal policy produced by our model. If anything, Chinese authorities are notorious for trying to keep interest rates on saving instruments (such as deposits) at a low level through several channels, including moral suasion (Green 2005). Our results suggests that this policy is not socially optimal. It helps current borrowers at the expense of future borrowers, who should be favored instead. Actually, this policy aims at subsidizing loans to state-owned enterprises (Fung, Ho, and Zhou 2000; Laurens and Maino 2007), which are less credit-constrained than private firms. This means that not only does this policy fail at transferring resources to savers, but it also fails at alleviating the current credit constraints of borrowers.

Our theory does not feature a spread between loan and deposit rates, but it suggests that this spread should be kept to a minimum. However, the four state-owned commercial banks have a quasi-monopoly in China's financial market (Wong and Wong 2001; Berger, Hasan, and Zhou 2009). This situation is likely to maintain monopoly rents that inflate the spread between loan and deposit rates. Besides, despite the substantial interest rate liberalization that took place in October 2004, the authorities still impose a ceiling on deposit rates and a floor on lending rates as a means to preserve intermediation margins (Laurens and Maino 2007). This policy maintains a lower bound on the spread, which is suboptimal because it generates rents, but also because it further hinders the optimal allocation of resources according to our model, even though it might be partly justified by the mitigation of credit risk. In practice, it seems that these measures are effective in maintaining low interest rates on saving. This is evident from the fact that the People's Bank of China has managed to contain the cost of sterilization operations associated with international reserves accumulation, and even to make profits (Green 2005; Laurens and Maino 2007).

VI. Conclusion

This paper has analyzed optimal policy in a semi-open economy, where the Ramsey planner is a central bank that has access to international capital markets. We found that the accumulation of reserves combined with capital controls gives a higher welfare than full capital mobility in a rapidly growing economy. This characterization clearly corresponds to the Chinese economy. On the other hand, with tight credit constraints and fluctuating income, we found that it is optimal to have high interest rates on saving instruments. This is different from what is observed in China.

The accumulation of assets by a Ramsey planner when consumers are credit constrained may be surprising. Indeed it would seem more intuitive if the planner would borrow in the international market as an intermediary for the consumers. This would actually correspond to the first-best allocation in a converging phase, as shown in Section IV. But with a limited set of instruments, the central bank is not able to relax the credit constraint, and the optimal policy is lend to, rather than borrow from, the rest of the world.

Changes in foreign exchange reserves are typically associated with exchange rate policy. In our context, the exchange rate does not play a role as the analysis is conducted in a one-good economy. A natural extension is to add traded and nontraded goods. In Bacchetta, Benhima, and Kalantzis (2012), we show that international reserve accumulation may go parallel with a real depreciation (see also Jeanne 2012). Consequently, a real depreciation may be optimal if reserve accumulation is optimal. However, this is only the case in the initial transition phase. As the economy approaches its steady state, a higher level of reserves leads to an appreciated currency.

Another interesting extension would be to model explicitly the domestic financial sector. This would give more instruments to the central bank. For example, central bank liabilities are made of commercial banks' reserves and of central bank bonds. The central bank can typically adjust the interest rate on reserves as well as the reserve requirements. In the literature with pecuniary externalities (e.g., Bianchi 2011), reserve requirements are desirable as they limit excessive borrowing. However, reserve requirements bearing a low interest rate act like a tax on deposit interest rates (e.g., see Bacchetta and Caminal 1994). Such a tax is undesirable in our context, as it is optimal to have a high interest rate on saving.

Appendix

A. *Proof of Proposition* 1

We start by looking for a symmetric steady state where the borrowing constraint is binding. Using the expression for saving (15) and loans (16) in the steady state, the bond market equilibrium (8) can be written $P(1/r) = 0$, with

$$P(X) = \phi X^2 + [a + \phi(1 + \beta)]X - \left[\beta(1 - \phi) - (1 + \beta)\frac{B}{Y}\right].$$

It is easy to see that $P(\beta) < 0$ if and only if $\frac{B}{Y} < \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$. In that case, there is a unique, positive equilibrium interest rate $r < 1/\beta$. This equilibrium interest rate is an increasing function of $\frac{B}{Y}$. Saving, loans and after-tax consumption expenditures D, L, τc^D, τc^L are given by equations (15) and (16) and individual budget constraints (2) and (3). The Lagrange multiplier follows from the first-order condition (6); it is strictly positive since $r\beta < 1$, which ensures that the borrowing constraint is indeed binding in this steady state. Given B^*, and knowing the after-tax consumption expenditures, the tax rate τ and the (pretax) consumption levels then follow from (9). This uniquely determines the symmetric steady state. In this steady state, the ratio of relative consumption is given by the first-order condition (5) and is an increasing function of r.

We now consider the case $\frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$ and look for a symmetric state where the borrowing constraint does not bind, i.e., $\lambda = 0$. From the first-order conditions (5) and (6), it is clear that we must have $\beta r = 1$, so that consumption will be constant for both types of households. Then, $c^L = c^D$. Taking the difference of budget constraints (2) and (3) together with the bond market equilibrium (8), we get $2L = \frac{\beta}{1+\beta}\,(1-a)\,Y - B$. It is easy to see that $\frac{B}{Y} \geq \beta\left(\frac{1-a}{1+\beta} - 2\phi\right)$ implies $L \leq \beta\phi Y$, so that the borrowing constraint indeed does not bind. Saving D follows from (15), and, given B^*, we can recover c^D, c^L, and τ from (13), (14), and (9).

B. *Proof of Proposition* 2

We denote by $\tilde{X} = X/Y$ the normalized variables. Take an agent who is constrained in $t-2$, her cash-poor period. We look for conditions under which the agent is constrained for all $t+2k$ and unconstrained for all $t+2k+1$, $k \geq 0$. We assume that such conditions exist and derive an expression for λ_{t+2k+1}. We then derive conditions under which $\lambda_{t+2k+1} > 0$ for all $k \geq 0$ and $\tilde{D}_{t+2k+1} \geq -\beta\phi a$.

If the agent is constrained for all $t+2k$ and unconstrained for all $t+2k+1$, $k \geq 0$, then

$$\text{(A1)} \qquad \tilde{c}^L_{t+2k} = \frac{1}{1+\beta}\left[\frac{1-\phi}{1+g_{t+2k}} + \beta a + \beta^2\phi(1+g_{t+2k+1})\right],$$

$$\text{(A2)} \qquad \tilde{c}^D_{t+2k+1} = \frac{1}{1+\beta}\left[(1-\phi) + \beta a(1+g_{t+2k+2}) + \beta^2\phi(1+g_{t+2k+2})(1+g_{t+2k+3})\right].$$

Equations (A1) and (A2) are found by combining the budget constraint (2) and (3) with (15) and (16).

We then use the Euler equation (6) and $\beta r^* = 1$ to derive the multiplier of the credit constraint:

$$1+\lambda_{t+2k+1} = (1+g_{t+2k+1})\frac{\tilde{c}^D_{t+2k+1}}{\tilde{c}^L_{t+2k}}$$

$$= (1+g_{t+2k})(1+g_{t+2k+1})\,\frac{1-\phi+(1+g_{t+2k+2})\beta a+(1+g_{t+2k+2})(1+g_{t+2k+3})\beta^2\phi}{1-\phi+(1+g_{t+2k})\beta a+(1+g_{t+2k})(1+g_{t+2k+1})\beta^2\phi}.$$

Since $\phi < 1$, $\lambda_{t+2k+1} > 0$ for all $k \geq 0$. This implies that agents are constrained in all their cash-poor periods.

We check now whether the agents are unconstrained in all their cash-rich periods. We have

$$\tilde{D}_t = \frac{1}{1 + \beta}\left[\beta \frac{1 - \phi}{1 + g_t} - \beta a - \beta^2 \phi(1 + g_{t+1})\right].$$

For the agents not to be constrained in their cash-rich periods, we must have $\tilde{D}_t \geq -\beta\phi a$. We can show that this condition is equivalent to

$$1 + g_t \leq \frac{1 - \phi}{a(1 - \phi) + \beta\phi(1 - a + g_{t+1})}.$$

Since $g_{t+1} = \mu g_t$, with $0 < \mu < 1$, $\phi < 1$, and $a < 1$, a sufficient condition for this inequality to be satisfied is the following:

$$1 + g_0 \leq \frac{1 - \phi}{a(1 - \phi) + \beta\phi(1 - a + \mu g_0)}.$$

Finally, using the fact that $g_{t+k} = \mu^k g_t \simeq 0$, we approximate

$$\lambda_{t+2k+1} \simeq g_{t+2k}(1 + \mu)\,\frac{1 - \phi + \mu\beta a + \mu^2\beta^2\phi}{1 - \phi + \beta a + \beta^2\phi}.$$

C. *Derivation of Equation* (19)

For $t \geq 0$, by taking the derivative of the Lagrangian $\mathcal{L}$ with respect to λ_{t+1}, we find that $\kappa_t^L = \Delta_{t+1}[\phi Y_{t+1} - r^* L_{t+1}]/[r^* u'(c_{t+1}^D)]$. If the constraint binds during the transition, we have $\kappa_t^L = 0$. Using the planner's FOCs, we now show that this implies $\kappa_t^D = 0$. Differentiating with respect to D and L are, with $r = r^*$:

$$\gamma_t^D + \gamma_t^G = \gamma_{t+1}^L + \gamma_{t+1}^G, \tag{A3}$$

$$\gamma_t^L + \gamma_t^G = \gamma_{t+1}^D + \gamma_{t+1}^G + \Lambda_{t+1}. \tag{A4}$$

Using $r_t = r^*$, $\tau_t = 1$ and $\kappa_t^L = 0$ for all $t \geq 0$, we can show that for $t \geq 0$, the planner FOC with respect to c_t^D and c_{t+1}^L are as follows:

$$u'(c_t^D) - \gamma_t^D + \kappa_t^D u''(c_t^D) - \gamma_t^G = 0, \tag{A5}$$

$$u'(c_{t+1}^L) - \gamma_{t+1}^L - \kappa_t^D r^* u''(c_{t+1}^L) - \gamma_{t+1}^G = 0. \tag{A6}$$

From equation (5), we know that $u'(c_t^D) = u'(c_{t+1}^L)$. Combining with the above equations, we obtain

$$\kappa_t^D u''(c_t^D)[1 + r^*] = \gamma_t^D + \gamma_t^G - \gamma_{t+1}^L - \gamma_{t+1}^G.$$

Using the planner FOCs with respect to D_{t+1} and L_{t+1}, we get

$$\kappa_t^D u''(c_t^D)[1 + r^*] = 0.$$

Since $u'' < 0$, then $\kappa_t^D = 0$ for all $t \geq 0$.

This yields, for $t \geq 0$,

$$I_{t+1}^* = \gamma_{t+1}^L D_{t+1} - (\gamma_{t+1}^D + \Lambda_{t+1}) L_{t+1}.$$

Using the planner FOCs with respect to D and L, we solve this equation forward and obtain

$$I_{t+1}^* = \left(\sum_{i=1}^{\infty} \Lambda_{t+2i} - \gamma_{t+1}^G + \gamma_\infty^G\right) D_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+1+2i} - \gamma_{t+1}^G + \gamma_\infty^G\right) L_{t+1},$$

where we use the fact that $\gamma_\infty^D = \gamma_\infty^L = 0$. Indeed, since the economy is asymptotically unconstrained, as implied by Proposition 2, we have $u'(c_\infty^D) = u'(c_\infty^L)$, which, combined with equations (A5) and (A6), yields $\gamma_\infty^D = \gamma_\infty^L$. Then, the FOC with respect to τ_t, gives $\gamma_t^D c_t^D + \gamma_t^L c_t^L = 0$, from which we conclude $\gamma_\infty^D = \gamma_\infty^L = 0$.

From the FOC with respect to τ_{t+1}, we can get

$$\gamma_{t+1}^G = \frac{(\gamma_{t+1}^D + \gamma_{t+1}^G) c_{t+1}^D + (\gamma_{t+1}^L + \gamma_{t+1}^G) c_{t+1}^L}{c_{t+1}^D + c_{t+1}^L}.$$

Using equations (A3) and (A4), and iterating forward, we obtain

$$\gamma_{t+1}^G - \gamma_\infty^G = \frac{c_{t+1}^L}{c_{t+1}^D + c_{t+1}^L} \sum_{i=1}^{\infty} \Lambda_{t+2i} + \frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} \sum_{i=1}^{\infty} \Lambda_{t+1+2i}.$$

Replacing in I_{t+1}^*, we obtain expression (19).

D. *Proof of Lemma* 1

First, we rewrite equation (19) as follows:

$$\begin{aligned} I_{t+1}^* = &\left(\frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} \sum_{i=1}^{\infty} (\Lambda_{t+2i} - \Lambda_{t+1+2i})\right) D_{t+1} \\ &- \left(\frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} \Lambda_{t+1} + \frac{c_{t+1}^L}{c_{t+1}^D + c_{t+1}^L} \sum_{i=0}^{\infty} (\Lambda_{t+1+2i} - \Lambda_{t+2+2i})\right) L_{t+1}. \end{aligned}$$

Suppose $\Lambda_{t+2} = \xi \Lambda_{t+1}$ for all $t \geq 0$, with $0 \leq \xi < 1$. We can use this to rewrite (19) as follows:

$$I_{t+1}^* = \Lambda_t \left[\frac{\xi^2}{1+\xi} \frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} D_{t+1} - \xi \left(\frac{c_{t+1}^D}{c_{t+1}^D + c_{t+1}^L} + \frac{1}{1+\xi} \frac{c_{t+1}^L}{c_{t+1}^D + c_{t+1}^L}\right) L_{t+1}\right].$$

From this we derive that $I^*_{t+1} > 0$, if and only if $\frac{D_{t+1}}{L_{t+1}} > \frac{c^D_{t+1} + c^L_{t+1}}{\xi c^D_{t+1}} + 1$.

E. *Proof of Proposition* 3

According to Proposition 2, an economy with binding credit constraint in t and $t-1$ and $g_{t+1} = \mu g_t$ for all $t \geq 0$, with $g_0 > 0$ and g_0 small, satisfies $\lambda_{t+k+1} \simeq \mu^k (1+\mu) \frac{1-\phi+\mu\beta a+\mu^2\beta^2\phi}{1-\phi+\beta a+\beta^2\phi} g_t$ for all $k \geq 0$. To apply Lemma 1, we need to relate Λ to λ. We use equations (A3)–(A6) and the households' Euler equations to achieve that.

Using equation (A5) expressed in period $t+1$ and equation (A6) expressed in period t, $t \geq 1$, and using $\kappa^D_t = \kappa^L_t = 0$, we derive

$$u'(c^L_t) - u'(c^D_{t+1}) - \gamma^L_t + \gamma^D_{t+1} - \gamma^G_t + \gamma^G_{t+1} = 0.$$

Using equations (A3) and (A4), and the Euler equation for the cash-poor agent, we obtain

$$u'(c^D_{t+1})\lambda_{t+1} = \Lambda_{t+1}.$$

With log-utility, we have

$$\frac{\Lambda_{t+1}}{\Lambda_{t+2}} = \frac{\lambda_{t+1}}{\lambda_{t+2}} \frac{(1+g_{t+2})\tilde{c}^D_{t+2}}{\tilde{c}^D_{t+1}}.$$

When g is small, this can be approximated as follows:

$$\frac{\Lambda_{t+1}}{\Lambda_{t+2}} \simeq \frac{1}{\mu}.$$

This means that $\Lambda_{t+2} = \mu\Lambda_{t+1}$ for all $t \geq 0$, which, according to Lemma 1, implies that I^*_{t+1} is of the same sign as $D_{t+1}/L_{t+1} - \left(\frac{c^D_{t+1} + c^L_{t+1}}{\mu c^D_{t+1}} + 1\right)$.

Finally, we can approximate

$$\frac{D_{t+1}}{L_{t+1}} = \frac{\frac{1}{1+\beta}\left[\beta\frac{1-\phi}{1+g_{t+1}} - \beta a - \beta^2\phi(1+g_{t+2})\right]}{\beta\phi} \simeq \frac{1-a}{(1+\beta)\phi} - 1$$

and

$$\frac{c^D_{t+1}}{c^L_{t+1}} = \frac{1-\phi+\beta a(1+g_{t+2}) + \beta^2\phi(1+g_{t+2})(1+g_{t+3})}{(1-\phi)/(1+g_{t+1}) + \beta a + \beta^2\phi(1+g_{t+2})} \simeq 1.$$

This implies that I^*_{t+1} is of the same sign as $\frac{1-a}{(1+\beta)\phi} - 2\frac{1+\mu}{\mu}$. Hence, the result.

F. *First-Order Conditions of the Ramsey Problem*

To simulate the optimal policy, we solve for the dynamics that simultaneously satisfy the conditions of a competitive equilibrium (see Definition 2) and the first-order conditions of the Ramsey problem, under the assumption that the borrowing constraint is always (weakly) binding. Those first-order conditions are

$$\text{FOC}(D_{t+1}): \quad \gamma_t^D + \gamma_t^G = \beta r_{t+1}\gamma_{t+1}^L + \beta r^* \gamma_{t+1}^G$$

$$\text{FOC}(L_{t+1}): \quad \gamma_t^L + \gamma_t^G = \beta r_{t+1}\left(\gamma_{t+1}^D + \Lambda_{t+1}\right) + \beta r^* \gamma_{t+1}^G$$

$$\text{FOC}(\tau_t): \quad \gamma_t^D c_t^D + \gamma_t^L c_t^L + \kappa_t^D \frac{\beta r_{t+1}}{c_{t+1}^L} = \frac{\kappa_{t-1}^D}{\beta c_{t-1}^D}$$

$$\text{FOC}(r_{t+1}): \quad \gamma_{t+1}^L D_{t+1} - \left(\gamma_{t+1}^D + \Lambda_{t+1}\right) L_{t+1} = \frac{\kappa_t^D \tau_t}{c_{t+1}^L}$$

$$\text{FOC}\left(c_t^D\right): \quad \frac{1}{c_t^D} = \tau_t \gamma_t^D + \gamma_t^G + \frac{\kappa_t^D \tau_{t+1}}{\left(c_t^D\right)^2}$$

$$\text{FOC}\left(c_t^L\right): \quad \frac{1}{c_t^L} = \tau_t \gamma_t^L + \gamma_t^G - \frac{\kappa_{t-1}^D r_t \tau_{t-1}}{\left(c_t^L\right)^2},$$

where we have used the first-order condition with respect to λ_{t+1}, which gives $\kappa_t^L = 0$. Competitive equilibria satisfying those first-order conditions are solutions of the Ramsey problem if $\lambda_t \geq 0$.

G. *Proof of Proposition* 4

The proof consists in explicitly constructing the policy. We do this in four steps.

Step 1: Denote $\overline{c}$ the constant level of consumption of individual households. The sequence of consolidated budget constraints (9) and the no-Ponzi condition (10), together with the budget constraints of individual households, implies the following intertemporal aggregate resource constraint:

$$\overline{c} = \frac{1-\beta}{2} \sum_{t \geq 0} \beta^t [(1+a)Y_t] + r^* B_0^*. \tag{A7}$$

This sets the value of $\overline{c}$.

Step 2: Given the FOCs of individual households, (5) and (6), together with the assumption that borrowing constraints do not bind, a constant consumption imposes that $\frac{r_{t+1}\tau_t}{\tau_{t+1}} = 1/\beta = r^*$. Given the sequence of taxes, the planner then chooses $\{B_{t+1}\}_{t \geq 0}$ to set the interest rates accordingly and $\{B_{t+1}^*\}_{t \geq 0}$ to satisfy the sequence of consolidated budget constraints (9). To describe a policy, it is therefore enough to determine the sequence of taxes $\{\tau_t\}_{t\geq 0}$. Denote $y_t = Y_t/\tau_t$. It is equivalent for the planner to choose $\{\tau_t\}_{t\geq 0}$ or $\{y_t\}_{t\geq 0}$.

Step 3: Consider a policy such that $y_{2t} = y_2$ for $t \geq 2$ and $y_{2t+1} = y_3$ for $t \geq 2$. Such a policy is completely characterized by y_0, y_1, y_2, and y_3. To go further, remark that the optimal consumption plan of a household with initial wealth $-r_0 L_0$ (or $r_0 D_0$), facing an endowment stream $\{Y_t\}_{t\geq 0}$, a sequence of interest rates $\{r_{t+1}\}_{t\geq 0}$, and taxes $\{\tau_t\}_{t\geq 0}$ is also the optimal consumption plan of a household with initial wealth $-\frac{r_0 L_0}{\tau_0}$ $\left(\text{or } \frac{r_0 D_0}{\tau_0}\right)$, facing the endowment stream $\{y_t\}_{t\geq 0}$, the sequence of interest rates $\left\{\frac{r_{t+1}\tau_t}{\tau_{t+1}} = \frac{1}{\beta}\right\}_{t\geq 0}$, and with zero taxes ($\tau_t = 1$).[17] We can use this *dual problem* to solve for the (constant) consumption of both groups of households (assuming that the borrowing constraint never binds):

$$\overline{c} = (1-\beta)\left(-r_0\tilde{L}_0 y_0 + y_0 + \beta a y_1 + \frac{\beta^2}{1-\beta^2}(y_2 + \beta a y_3)\right),$$

$$\overline{c} = (1-\beta)\left(r_0\tilde{D}_0 y_0 + a y_0 + \beta y_1 + \frac{\beta^2}{1-\beta^2}(a y_2 + \beta y_3)\right),$$

where we have used the notation from Appendix Section B: $\tilde{X} = X/Y$. Those two equations yield the following expressions for y_0 and y_1:

$$\text{(A8)} \quad y_0 = \frac{\frac{\overline{c}}{1-\beta} - \frac{\beta^2}{1-\beta^2}(1+a)y_2}{1 + a - \frac{a r_0\tilde{D}_0 + r_0\tilde{L}_0}{1-a}},$$

$$\text{(A9)} \quad y_1 = \frac{1}{\beta(1-a)}\left[(1 - a - r_0\tilde{D}_0 - r_0\tilde{L}_0)x_0 + \frac{\beta^2}{1-\beta^2}(1-a)(y_2 - \beta y_3)\right].$$

If $Y_0 - r_0 L_0 > aY_0 + r_0 D_0 > 0$, then $y_0,\ y_1 \geq 0$ for y_2 and y_3 close enough to zero. This restriction on initial conditions is satisfied by the symmetric steady state of an open economy at the edge of the unconstrained region.

Step 4: Choose y_2 and y_3 lower than ay_1. Then, in the dual problem, households get their maximum endowment either in $t = 0$ or in $t = 1$. As a consequence, borrowing constraints do not bind for $t \geq 1$. Borrowing constraints at $t = 1$ can be written as

$$c \leq a y_0 + r_0\tilde{D}_0 y_0 + \beta\phi y_1,$$

$$c \leq y_0 - r_0\tilde{L}_0 y_0 + \beta\phi a y_1.$$

[17] To see this, simply divide equations (2) to (7) by the tax rate.

Using the expressions (A8) and (A9), and taking the limit y_2, $y_3 \to 0$, we check whether these two inequalities hold. When $\beta \geq (1 - \phi)$, they can be shown to hold for any initial conditions. When $\beta < (1 - \phi)$, they hold provided $\frac{Y_0 - r_0 L_0}{aY_0 + r_0 D_0} \leq \frac{1 - \phi - \beta a}{1 - \phi - \beta}$. This condition is satisfied by the symmetric steady state of an open economy at the edge of the unconstrained region.

We conclude that policies described by y_0, y_1, y_2, y_3, where y_0, y_1 are given by (A8) and (A9), and y_2, y_3 are chosen close enough to zero, yield a competitive equilibrium where all households consume $\overline{c}$ given by (A7) and the borrowing constraint never binds.

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