



On the unstable relationship between exchange rates and macroeconomic fundamentals[☆]

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ABSTRACT

Survey evidence shows that the relationship between the exchange rate and macro fundamentals is perceived to be highly unstable. We argue that this unstable relationship naturally develops when structural parameters in the economy are unknown. We show that the reduced form relationship between exchange rates and fundamentals is then driven not by the structural parameters themselves, but rather by expectations of these parameters. These expectations can vary significantly over time as a result of perfectly rational “scapegoat” effects. These effects can be expected to hold more broadly in macro and finance beyond the application to exchange rates in this paper.

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1. Introduction

The weight that foreign exchange traders attach to different macro fundamentals as drivers of exchange rates fluctuates considerably over time. This was first documented by [Cheung and Chinn \(2001\)](#) through a survey of U.S. foreign exchange traders. More extensive evidence of these time-varying weights of macro fundamentals was recently reported by [Fratzscher et al. \(2012\)](#) based on data from Consensus Economics for 12 currencies over a period of 9 years. Forty six foreign exchange market participants from large countries are asked on a monthly basis to “rank the current importance of a range of different factors in determining exchange rate movements.” The rankings, on a scale from 0 to 10 for six key macroeconomic indicators, vary significantly over time for all six indicators. The survey

evidence suggests a time-varying relationship between exchange rates and macro fundamentals. [Fratzscher et al. \(2012\)](#) confirm this by regressing changes in exchange rates on the macro fundamentals as well as the fundamentals interacted with the survey weights. The latter are statistically significant, implying that the derivative of the exchange rate with respect to a fundamental depends on its survey weight. The time-varying ranking of the macro fundamentals therefore implies a time-varying relationship between exchange rates and fundamentals.

The main goal of this paper is to show that large and frequent variations in the relationship between the exchange rate and macro fundamentals occur naturally when there is uncertainty about the structural parameters in the economy. We show that with parameter uncertainty, the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of structural parameters. Moreover, we show that these expectations can vary significantly over time, giving rise to a highly unstable reduced form relationship between exchange rates and fundamentals. This happens even though agents are perfectly rational Bayesian learners.

In addition to the usual learning process, whereby parameter expectations converge to their actual values, expectations can exhibit large short-term fluctuations. These are due to a mechanism that we refer to as a “scapegoat” effect. Some information about the nature of

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structural parameters can be derived by analyzing macroeconomic data and exchange rates. But these data are also driven by shocks to *unobserved* fundamentals. Such unobserved fundamentals can generate considerable confusion in the short to medium run. When the exchange rate fluctuates as a result of an unobserved macroeconomic shock, it can be optimal for agents to give more weight to an observed macro fundamental and therefore making it a “scapegoat”.¹ For example, if the dollar depreciates it may be natural to attribute it to a large current account deficit, even when the depreciation is unrelated to this deficit.

When we refer to a variable as becoming a scapegoat, we mean that the expectation of the structural parameter associated with that variable becomes much larger than the structural parameter itself, so that the variable temporarily has a large weight in the reduced form exchange rate equation. As in the example above, this may be caused by an unrelated change to an unobserved fundamental. More generally, we will show that even when the true structural parameters are constant the expectation of structural parameters is affected by changes in both observed and unobserved fundamentals. In that context we also think of the term scapegoat more broadly as referring to a situation where macroeconomic news that is unrelated to changes in structural parameters nonetheless leads to rational changes in beliefs about structural parameters. We show that this leads to a time-varying relationship between the exchange rate and fundamentals.

In illustrating the importance of such scapegoat effects, and their role in the unstable reduced form relationship between exchange rates and fundamentals, we slightly generalize the “canonical” exchange rate model. There is a broad class of exchange rate models that can be reduced to a single stochastic difference equation, derived from an interest rate parity equation and an equation that relates the interest differential to observed fundamentals. The latter can be obtained either from monetary policy specifications or money market equilibrium in a standard monetary model. We assume that the underlying parameters in this second equation, such as monetary policy or money demand parameters, or the relationship between policy targets and observed fundamentals, are not perfectly known. Moreover, we assume that some macro fundamental is not observable.

Our paper is not the first to introduce parameter uncertainty and Bayesian learning in a standard exchange rate model. Lewis (1989) assumes the existence of a one-time change in the constant term of the money demand equation. Kaminsky (1993) assumes that money growth is equal to a drift term that can switch between two values based on a Markov process. In both cases agents learn about the unknown parameters through Bayesian updating. Tabellini (1988) emphasized that such a framework can lead to increased exchange rate volatility relative to the case where parameters are known.² However, these papers do not consider uncertainty about parameters multiplying fundamentals and the role of unobserved fundamentals. Consequently, the scapegoat effect is not present in this prior literature.

To examine the quantitative relevance of the effects we describe, we calibrate the model to data for 5 industrialized countries, matching moments related to interest rates and exchange rates and the explanatory power of observed fundamentals. We find that the derivative of the exchange rate with respect to fundamentals can be very volatile when structural parameters are unknown. However, we show that this instability in the reduced form relationship between exchange rates and

macro fundamentals is not detected by standard parameter instability tests based on regressions of exchange rates on fundamentals.

We show that knowing the time-varying weights improves the explanatory power of fundamentals for exchange rates. While this improvement is not large in a statistical sense, it may be important in an economic sense since small increases in predictive power can generate substantial economic gains. This point is illustrated by the existence of a large carry trade industry that is based on the statistically small predictive power of interest differentials (in terms of R^2 s).

The next section presents the model. It shows that the relationship between the exchange rate and fundamentals depends on expectations of structural parameters. We then go on to derive how these expectations are updated with a Kalman filter. Section 3 calibrates the model based on data on interest rates and exchange rates and presents numerical results for the relationship between exchange rates and fundamentals based on simulations. Section 4 discusses the empirical relevance of the scapegoat effect, time-varying structural parameters and the broader role of the mechanism in macroeconomics and finance. Section 5 concludes.

2. A model with unknown parameters

The underlying framework is a standard exchange rate model with constant parameters. We first describe the model when parameters are known. Then we show how the exchange rate is affected by parameter expectations when parameters are unknown. In deriving parameter expectations, we show that in addition to a standard learning process, there is a mechanism that we call a scapegoat effect. This mechanism leads to an unstable relationship between fundamentals and exchange rates.

2.1. Basic framework

We consider the class of fundamental-based exchange rate models that can be reduced to a single stochastic difference equation. The equilibrium value of the exchange rate in these models depends on the present value of expected future fundamentals. We start with the usual case of known parameters. We follow Engel and West (2005) and slightly rewrite their Eq. (1):

$$s_t = (1-\lambda) \left[F_t + b_t + \sum_{j=1}^{\infty} \lambda^j E_t (F_{t+j} + b_{t+j}) \right] - \lambda \left[\phi_t + \sum_{j=1}^{\infty} \lambda^j E_t \phi_{t+j} \right] \quad (1)$$

where s_t is the log nominal exchange rate (domestic per foreign currency), E_t is the expectation of the representative investor, ϕ_t is the risk premium and $0 < \lambda < 1$. We denote by F_t a linear combination of observed macro fundamentals: $F_t = \mathbf{f}_t' \boldsymbol{\beta}$ where $\mathbf{f}_t = (f_{1t}, f_{2t}, \dots, f_{Nt})'$ is the vector of N observed macroeconomic fundamentals and $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_N)'$ is the vector of associated parameters. Finally, b_t represents unobserved macro fundamentals.

Engel and West (2005) present several models that lead to this equation. A core element of these models is an interest rate parity condition:

$$E_t s_{t+1} - s_t = i_t - i_t^* + \phi_t \quad (2)$$

where i_t and i_t^* represent the domestic and foreign nominal one-period interest rates. The other basic element is an equation that relates the interest rate differential to the exchange rate and to observed and unobserved fundamentals. This equation represents for example the reduced form of a monetary model or the differential in interest rate rules. This equation can be written in the following form:

$$i_t - i_t^* = \mu s_t - \mu (F_t + b_t). \quad (3)$$

Combining Eqs. (2) and (3), integrating forward and assuming no bubble gives Eq. (1), where $\lambda = 1/(1 + \mu)$. Notice that in practice we know that λ is very close to 1 (e.g., see Engel and West, 2005).

¹ In a previous short paper, Bacchetta and van Wincoop (2004), we developed the idea of such a scapegoat effect in the context of a simple static noisy rational expectations model in which some parameters are unknown. However, a static model does not allow us to address the unstable dynamic relationship between exchange rates and fundamentals and its implications. Apart from the dynamic setup, the model in this paper also differs in that there is no private information. Scapegoat effects naturally develop as long as there is incomplete information about parameters.

² There is also a vast literature on learning and exchange rates with deviations from rational expectations (e.g., Goldberg and Frydman, 1996; Gourinchas and Tornell, 2004, or Lewis and Markiewicz, 2009).

Eq. (3) is a source of information for the agents. Agents know the value of the sum $F_t + b_t$ as they can observe the exchange rate and the interest rate. When the parameter β are known, agents can then infer the value of b_t . When the parameters β are unknown, knowing $F_t + b_t$ gives an imperfect signal about β as a result of the unobserved fundamental b_t . However, if the observed and unobserved fundamental followed the same AR process, this lack of knowledge of the parameters β would not matter. We would have $E_t(F_{t+j} + b_{t+j}) = \rho^j(F_t + b_t)$, with ρ as the AR parameter. Ignoring the risk premium term, the exchange rate in Eq. (1) then would become $s_t = [(1-\lambda)/(1-\lambda\rho)](\mathbf{f}'_t\beta + b_t)$. It is only in this knife-edge case, where the processes for the observed and unobserved fundamentals are exactly identical, that the exchange rate depends on the actual parameters β and not on their expectations.

We therefore assume that \mathbf{f}_t and b_t follow different processes. We will assume that the unobserved fundamental b_t follows an AR process:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b \quad (4)$$

where the innovation has mean zero and variance σ_b^2 . Because of extensive evidence of a unit root in the nominal exchange rate, as well as in many macro variables driving the exchange rate, we will assume that the observed fundamentals \mathbf{f}_t follow a random walk.³

Assuming for now that the risk premia ϕ_t are zero, when the parameter β are known these assumptions imply that Eq. (1) becomes

$$s_t = \mathbf{f}'_t\beta + \theta b_t \quad (5)$$

where

$$\theta = \frac{1-\lambda}{1-\rho_b\lambda}. \quad (6)$$

It is immediate in this case that the derivative of the exchange rate with respect to fundamentals is

$$\frac{\partial s_t}{\partial f_{nt}} = \beta_n. \quad (7)$$

The derivative of the exchange rate with respect to fundamentals is therefore equal to a known constant, which is equal to the structural parameter β_n .

2.2. Exchange rate when parameters are unknown

We now turn to the case where the parameters β are unknown.⁴ Specifically, assume that during an initial period 1 all parameters are drawn from a distribution with mean $\bar{\beta}$ and standard deviation σ . Agents can learn over time about the value of the parameters from the observation Eq. (3), which allows them to observe $F_t + b_t = \mathbf{f}'_t\beta + b_t$. Agents use these observations for each period to update their beliefs about the parameters and b_t .

When parameters are unknown, Eq. (1) no longer implies Eq. (5). Instead, we have

$$s_t = (1-\lambda)\mathbf{f}'_t\beta + \lambda\mathbf{f}'_tE_t\beta + (1-\lambda)b_t + \lambda\theta\rho_b E_t b_t. \quad (8)$$

³ The key results do not depend on this assumption. They will also hold when the observed and unobserved fundamentals are all stationary. We only need to assume that F_t and b_t follow a different process, for example in the form of a different AR coefficient.

⁴ The only source of imperfect knowledge is about the parameter vector β . We could have assumed that μ and ρ_b are unknown as well, but that complicates the signal extraction problem in learning about these parameters as the observed signals then involve products of unknown parameters.

Since agents know the value of $\mathbf{f}'_t\beta + b_t$, for a given expectation $E_t\beta$ of the parameters we have

$$E_t b_t = \mathbf{f}'_t\beta + b_t - \mathbf{f}'_tE_t\beta = \mathbf{f}'_t(\beta - E_t\beta) + b_t. \quad (9)$$

The exchange rate can then be written as

$$s_t = \mathbf{f}'_t(\theta\beta + (1-\theta)E_t\beta) + \theta b_t. \quad (10)$$

The only difference in comparison to Eq. (5) is that the coefficients multiplying the fundamentals are not equal to the structural parameters β , but to a weighted average of the structural parameters and of the expectations of these parameters. This significantly changes the relationship between the exchange rate and fundamentals as the weight on the true parameters is small: since λ is close to one, θ is close to zero.⁵ Moreover, as we will see, even though the parameters themselves are constant, the expectations of parameters can change significantly over time. This affects the impact of fundamentals on the exchange rate.

The derivative of the exchange rate with respect to fundamentals is now

$$\frac{\partial s_t}{\partial f_{nt}} = \theta\beta_n + (1-\theta)E_t\beta_n + (1-\theta)\mathbf{f}'_t \frac{\partial E_t\beta}{\partial f_{nt}}. \quad (11)$$

First consider the sum of the first two terms, which is the weighted average of the parameter β_n and its expectation. Again, since θ is close to zero almost all the weights is on the expectation rather than the parameters themselves. As we will see below, changes in fundamentals also change the expectations of parameters. In the last term of Eq. (11) the derivative of the expected parameters with respect to the fundamental interacts with the level of fundamentals.

2.3. Expectation of parameters

We now turn to the computation of the expected parameters. For illustrative purposes we describe the case where $N = 1$, so that there is only one fundamental. After that we briefly describe how the results generalize to multiple fundamentals, leaving the derivation to [Appendix A](#).

In each period agents observe $y_t = f_t\beta + b_t$. In period 1 the parameter β is drawn from a distribution with mean $\bar{\beta}$ and standard deviation σ . We assume that $f_1 = b_1 = 0$, so that in period 1 itself the signal y_1 is simply zero and provides no information about the value of β . Starting in period 2, agents gradually learn through the signals $f_t\beta + b_t$. It is useful to rewrite the new information that comes available at time t , starting at $t = 2$, as

$$\tilde{\Delta}y_t = y_t - \rho_b y_{t-1} = \tilde{\Delta}f_t\beta + \varepsilon_t^b \quad (12)$$

where $\tilde{\Delta}f_t = f_t - \rho_b f_{t-1}$. Written in this way, the signal $\tilde{\Delta}y_t$ depends on the unknown parameter β , multiplied by the fundamental $\tilde{\Delta}f_t$, plus an idiosyncratic noise ε_t^b .

In period 1 the expectation of β is equal to $\bar{\beta}$ and the perceived standard deviation is equal to σ . After that expectations can be updated using the Kalman filter. It is useful to first consider the Kalman updating formulas for uncertainty about the parameter. Let p_t be the variance of the expectation of β at time t .

⁵ The only exception is again the limiting case where the process of the observed and unobserved fundamentals become identical, which here implies $\rho_b \rightarrow 1$, so that $\theta \rightarrow 1$. But this is only relevant when we are very close to the knife-edge case where these processes are identical.

Starting with $p_1 = \sigma^2$, the Kalman updating formulas give (see Eq. (24) in Appendix A):

$$p_t = p_{t-1}\alpha_t \tag{13}$$

where

$$\alpha_t = \frac{\sigma_b^2}{(\tilde{\Delta}f_t)^2 p_{t-1} + \sigma_b^2}. \tag{14}$$

Note that $0 < \alpha_t < 1$ and in fact α_t is close to 1. The difference from one is second-order as $(\tilde{\Delta}f_t)^2 p_{t-1}$ is fourth order. This means that agents learn slowly. While uncertainty about the parameter declines over time, this happens at a slow rate. The logic behind this is that in the signal $\tilde{\Delta}f_t\beta + \varepsilon_t^b$, the parameter β is multiplied by a small first-order variable $\tilde{\Delta}f_t$, especially when ρ_b is close to one.

Using this result we have

$$p_t = g_t\sigma^2 \tag{15}$$

where

$$g_t = \prod_{i=2}^t \alpha_i. \tag{16}$$

Here g_t represents cumulative learning. It is the remaining uncertainty about the parameter as a fraction of the initial uncertainty σ^2 . As α_i is close to 1, g_t declines only slowly with time.

Next consider the updating formula for the parameter β . Starting with $E_1\beta = \bar{\beta}$, for $t > 1$ we have (see Eq. (28) in Appendix A):

$$E_t\beta = \alpha_t E_{t-1}\beta + (1-\alpha_t)\beta + z_t \tilde{\Delta}f_t \varepsilon_t^b \tag{17}$$

where

$$z_t = \frac{p_{t-1}}{(\tilde{\Delta}f_t)^2 p_{t-1} + \sigma_b^2}. \tag{18}$$

The first two terms on the right hand side of Eq. (17) reflect the speed of learning. If the last term is zero, the expectation of β at time t is a weighted average of the expectation at time $t - 1$ and the actual value of the parameter β . We have already seen that agents learn slowly when α_t is close to 1. Here this is reflected in a high weight of the current expectation on the last period's expectation.

The last term on the right hand side of Eq. (17) is the key to our results. It reflects what we call a scapegoat effect. It depends on the product $\tilde{\Delta}f_t$ and ε_t^b . In order to understand this result, recall that through the exchange rate and the interest rate differential agents observe in each period $\tilde{\Delta}f_t\beta + \varepsilon_t^b$. Assume that $\varepsilon_t^b > 0$ and $\tilde{\Delta}f_t > 0$. Agents do not know whether the larger than expected value of the signal is a result of a positive innovation ε_t^b or $\tilde{\Delta}f_t > 0$ combined with a larger than previously expected value of β . They will at least give some weight to the latter by raising their expectation of β . In this case the fundamental f_t becomes the scapegoat when the true shock is in the noise ε_t^b . Making the fundamental the scapegoat for the noise shock is perfectly rational.

Integrating Eq. (17) forward, we have

$$E_t\beta = g_t\bar{\beta} + (1-g_t)\beta + g_t \sum_{i=2}^t \frac{z_i}{g_i} \tilde{\Delta}f_i \varepsilon_i^b. \tag{19}$$

The first two terms are a weighted average of the initial estimate $\bar{\beta}$ and the actual parameter itself. As learning is slow, g_t remains close to 1 for a long time, so that the expectation of β remains more

influenced by $\bar{\beta}$ than by β for a long time. Moreover, the expectation of the parameter remains disconnected from its actual value because of the last term, which reflects the scapegoat effect. It depends on the product of current and past values of $\tilde{\Delta}f_t$ and ε_t^b . While the fundamentals and the noise shocks have nothing to do with the value of the parameter itself, the scapegoat effect that results from their current and past products can have a significant effect on the expectation of the parameter.

Finally, to determine the last term in Eq. (11) we need to compute the derivative of $E_t\beta$ with respect to f_t . From Eq. (17) we see that:

$$\frac{\partial E_t\beta}{\partial f_t} = \frac{\partial \alpha_t}{\partial f_{nt}} (E_{t-1}\beta - \beta) + \left(\frac{\partial z_t}{\partial f_t} \tilde{\Delta}f_t + z_t \right) \varepsilon_t^b. \tag{20}$$

The last term on the right hand side depends on the innovation ε_t^b , which is a transitory shock. This term therefore leads to a transitory component in the derivative of the exchange rate with respect to fundamentals.

Appendix A derives analogous expressions for the case of N fundamentals. For uncertainty, the extension to the multivariate case gives

$$\mathbf{P}_t = \mathbf{G}_t\sigma^2 \tag{21}$$

where \mathbf{P}_t is now the variance of the entire vector β of parameters. \mathbf{G}_t is defined in Appendix A, representing cumulative learning in a way that is analogous to g_t for the single fundamental case.

The generalization of Eq. (19) to the multivariate case is

$$E_t\beta = \mathbf{G}_t\bar{\beta} + (I_N - \mathbf{G}_t)\beta + \mathbf{G}_t \sum_{i=2}^t \mathbf{G}_i^{-1} \mathbf{Z}_i \tilde{\Delta}\mathbf{f}_i \varepsilon_i^b \tag{22}$$

where I_N is an identity matrix of size N and \mathbf{Z}_t is a multivariate generalization of z_t that is defined in Appendix A. Notice that the scapegoat effect is again associated with the interaction between current and past fundamentals $\tilde{\Delta}\mathbf{f}_t$ and noise innovations ε_t^b . In the multivariate case the extent to which a fundamental becomes a scapegoat depends on the product $\tilde{\Delta}f_{nt}\varepsilon_t^b$ for other fundamentals as well, which determines the plausibility of other fundamentals as scapegoats.

Finally, in analogy to Eq. (20), Appendix A derives an expression of the derivative of $E_t\beta$ with respect to fundamentals that depends on $E_{t-1}\beta - \beta$ and a term that is proportional to ε_t^b . The latter again leads to an entirely transitory component in the derivative of the exchange rate with respect to fundamentals.

3. Numerical analysis

To evaluate the potential significance of the scapegoat effect, we calibrate the model to monthly data for exchange rates, interest rates and observed fundamentals. We then compute the derivative of the exchange rate with respect to fundamentals, as well as the expectation of parameters, during a simulation of the model. We show that both can be very volatile, while the learning process about the structural parameters is slow.

3.1. Calibration

We use data on 5 currencies relative to the U.S. dollar: Swiss Franc, British pound, Canadian dollar, Japanese yen and German mark (Euro since 1999) with monthly data from September 1975 to September 2008. The macro fundamentals are the differential of money supply growth, industrial production growth and unemployment rate growth relative to the U.S., the growth in the oil price and the lagged interest rate differential relative to the U.S. A description of the data can be found in Bacchetta et al. (2010). Consistent with the data, we treat one period in the model as one month.

Table 1
Benchmark parameter assumptions.

Parameters	
σ_b	0.022
ρ_b	0.9
σ_v	0.0284
ψ_1	0.1
ψ_2	0.1
N	5
σ_f	0.000252
λ	0.97
$\bar{\beta}$	5
σ	2.5

For calibration purposes we re-introduce time-varying risk premia in order to match the observed properties of exchange rates. Let v_t be the present discounted value of the risk premium in the last term of Eq. (1):

$$v_t = \sum_{k=0}^{\infty} \lambda^k E_t \phi_{t+k}.$$

To match the observed volatility and autocorrelation of Δs_t , we assume that v_t follows the process

$$v_{t+1} - v_t = \psi_1(v_t - v_{t-1}) - \psi_2 v_t + \varepsilon_{t+1}^v \quad (23)$$

where $\varepsilon_{t+1}^v \sim N(0, \sigma_v^2)$.

In calibrating the model, we first generate a history of 10 years. This is more realistic than to start from time 1 as expectations of the parameters, interest rates and exchange rates depend on the history of past innovations in observed and unobserved fundamentals. As we discuss below, a history of 10 years has little impact on uncertainty about the unknown parameters. After the 10 year history we simulate the model for 397 months (just over 33 years), the same sample as in the data. We repeat this 1000 times and compute the average moments across these simulations.

Table 1 reports the parameters adopted for the benchmark parameterization. The first 5 parameters are associated with the process for b_t and v_t . These are set to match the average across the 5 currencies of four moments related to exchange rates and interest rates: the standard deviation of Δs_t , the standard deviation of $i_t - i_t^*$, the first-order autocorrelation of Δs_t and the first-order autocorrelation of $i_t - i_t^*$. The next two parameters relate to the fundamentals. Consistent with the data, we set the number of fundamentals equal to $N = 5$. The standard deviation σ_f of fundamental innovations is set to match the explanatory power of the fundamentals. This is the average across the currencies of the R^2 of a regression of Δs_t on the fundamentals. It is a very low 0.023 in the data, consistent with the well-known limited explanatory power of macro fundamentals for the exchange rate. Finally, we set λ equal to 0.97, consistent with values for λ reported in Engel and West (2005) that are close to 1.

We set $\beta_n = \bar{\beta} = 5$ for all parameters. We set the initial uncertainty at $\sigma = 2.5$ in the benchmark parameterization.⁶ This means that a 2 standard deviation confidence band lies between 0 and 10. We do not calibrate this uncertainty, but will consider the impact of changing the degree of uncertainty about the parameter.

⁶ The important parameter is the ratio $\sigma/\bar{\beta}$. The levels of σ and $\bar{\beta}$ are just a matter of standardization: e.g., doubling them leads to identical results if we half the volatility of fundamentals.

3.2. Results

3.2.1. Learning

Before considering a particular simulation, it should be noted that agents learn very slowly. The uncertainty about parameters, as measured by their standard deviation, is 2.5 in period 1. At the end of the 10-year history that we generate, it is on average 2.3. This is the uncertainty at the start of the sample of 397 months. At the end of the sample the standard deviation is on average 1.9. Overall the reduction in risk is therefore small. While we only simulate the model over 33 years, we find that it takes 145 years for this uncertainty to be reduced by half.

3.2.2. Unstable relationship between exchange rate and fundamentals

Fig. 1 reports the derivative of the exchange rate with respect to the 5 fundamentals for a particular simulation of a 397 month period. Again the simulation starts with a history of 10 years prior to the reported results.

Fig. 1 shows a highly unstable derivative of the exchange rate with respect to each of the fundamentals. If the parameters were known, the derivative would be constant and equal to 5 for each of the parameters. Instead we see substantial fluctuations in the derivative. There are both large high frequency fluctuations and lower frequency fluctuations. The weight attached to fundamental 1 is mostly below 5, while that of fundamental 5 is mostly above 5. For the other fundamentals it fluctuates between levels well above and well below 5. The range over which the weights vary is about 3 to 4 for all fundamentals.

To provide further insight into what is driving this unstable relationship between exchange rates and fundamentals, Fig. 2 shows the expectation of the 5 parameters. Not surprisingly, this expectation is also very volatile, with both low and high frequency fluctuations. One striking difference between Figs. 1 and 2 is that the very sharp high frequency fluctuations in Fig. 1 are absent from Fig. 2. The derivative of the exchange rate with respect to fundamentals does not just depend on the expectation of the parameters, but also on the derivative of expected parameters with respect to the fundamentals. As we saw in Eq. (20), the latter has a transitory component that depends on ε_t^b . This generates the large high frequency fluctuations in Fig. 1 that are absent from Fig. 2.

Fig. 3 further illustrates how this unstable relationship between the exchange rate and the fundamentals is a result of the uncertainty about the level of the parameters. It shows the average standard deviation of $\partial s_t / \partial f_{nt}$ and $E_t \beta_n$ as a function of the standard deviation σ that represents the initial uncertainty about the parameters. The average is computed over 1000 simulations and over the 5 fundamentals. We vary σ from 0 to 5 (twice the value in the benchmark). As expected, the unstable relationship between the exchange rate and fundamentals, as measured by the standard deviation of $\partial s_t / \partial f_{nt}$, is closely related to the initial uncertainty about the parameters. Except for very low levels of uncertainty, the relationship between the volatility of $\partial s_t / \partial f_{nt}$ and σ is almost linear.

3.2.3. Parameter instability test

Even though we have documented the very unstable relationship between exchange rates and fundamentals, this does not mean that the instability can be easily detected using regressions of exchange rates on fundamentals. The econometrician who conducts such tests has far less information than the agents, who know the entire model and therefore know the time-varying weights documented in Fig. 1. For example, if one of the reduced-form weights is high one period and low the next, we would clearly not have sufficient data to detect this based on regressions of exchange rates on fundamentals. The limited explanatory power of macro fundamentals further complicates the detection of time-varying weights on the fundamentals.

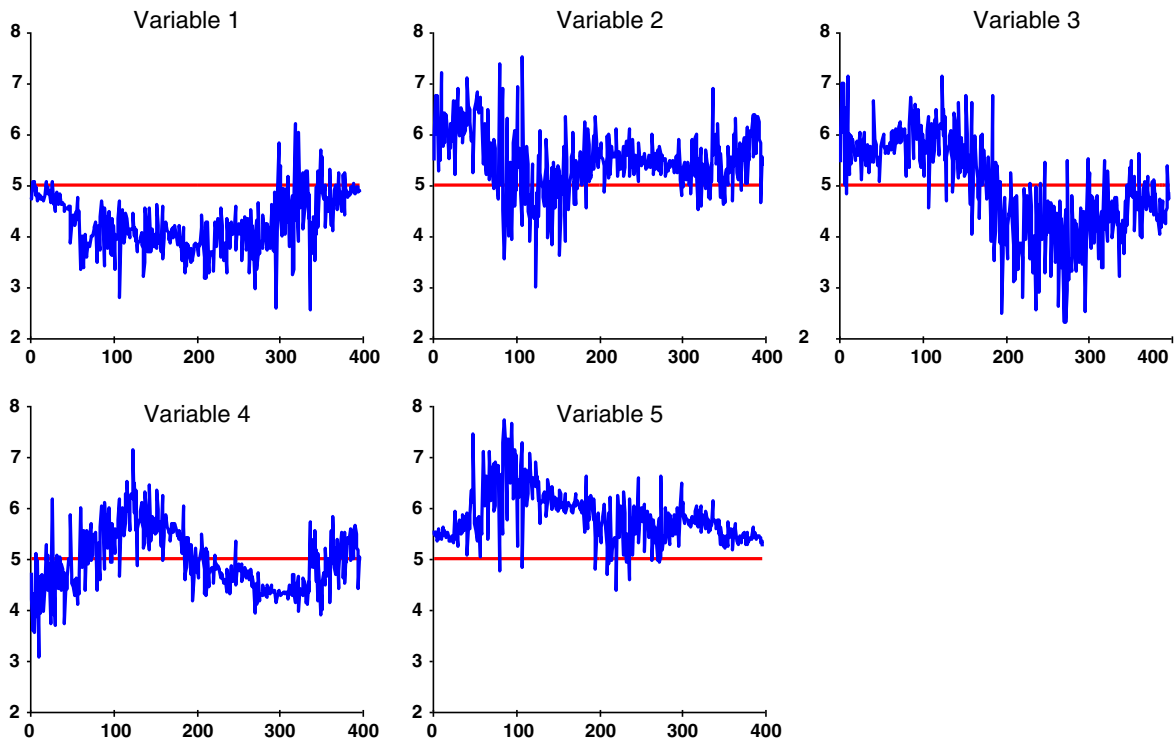


Fig. 1. Derivative Δs_t with respect to Δf_{nt} .

To illustrate this, we conduct a Quandt–Andrews breakpoint test. The test statistic, due to Andrews (1993), is the average F statistic over multiple breakpoint tests. All breakpoints from 60 months to 338 months into the sample are considered. The average F statistic is 1.0 in the model under the benchmark parameterization. This is

well below what is needed to reject the null of a constant relationship between the exchange rate and parameters. The 5% critical value is 6.13 (see Andrews and Ploberger, 1994). When we apply the same test to the data, the average over the 5 currencies for the average F statistics is 1.3, which also cannot reject stability and is close to

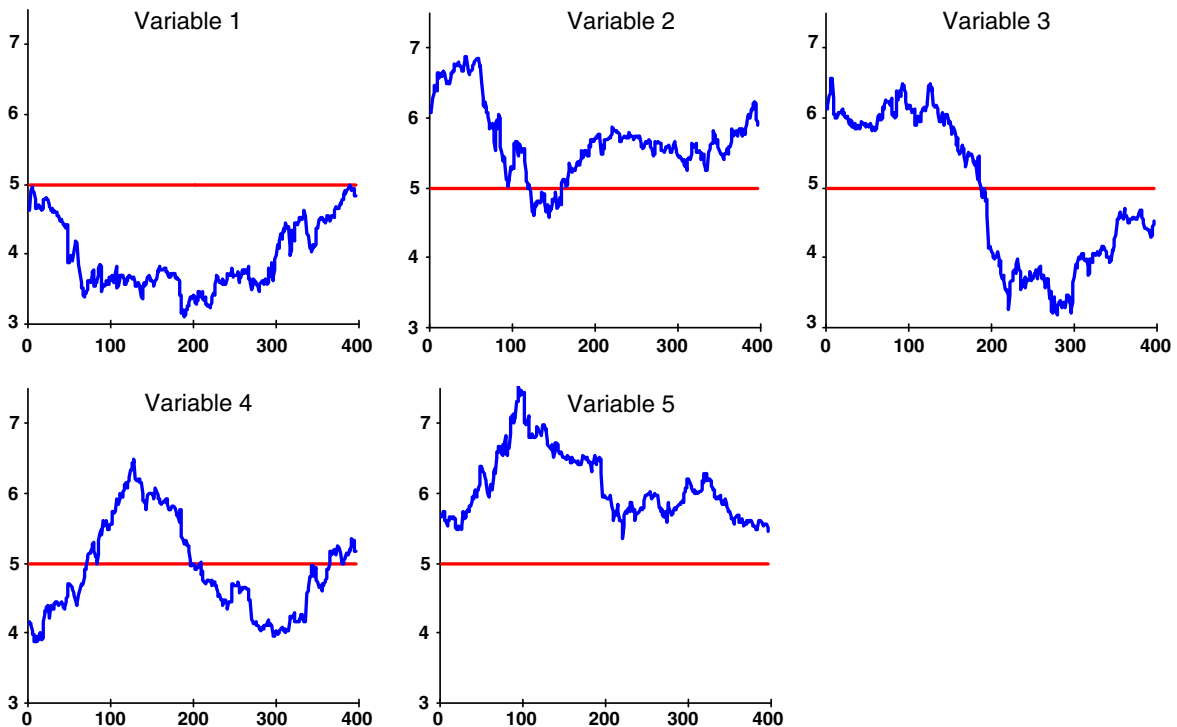


Fig. 2. Expectation of parameters.

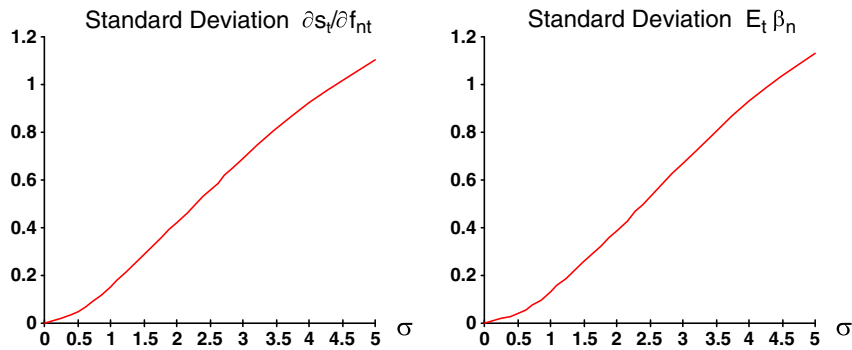


Fig. 3. Role of parameter uncertainty.

what we find in the model. The results do not change if we consider multiple breakpoints at once.⁷

3.2.4. Additional predictive power of fundamentals

The explanatory power of fundamentals should be higher when their weights are known. This is indeed what Fratzscher et al. (2012) find when using survey measures of the weights. They regress Δs_t on the fundamentals Δf_t as well as the survey weights times the fundamentals and find the latter to be highly significant. For our purposes we use $\partial s_t / \partial f_{nt}$ in Eq. (11) as the weights. We use the adjusted R^2 as a measure of explanatory power.

The explanatory power of fundamentals is low, both in the data and the model. Under the benchmark parameterization the R^2 of a regression of Δs_t on the five fundamentals is on average 0.023 (see Table 2). The adjusted R^2 is only 0.010. This low predictive power is behind the well-known Meese–Rogoff puzzle that it is hard to outperform a pure random walk. We find that adding the interaction term of the fundamentals and the weights $\partial s_t / \partial f_{nt}$ raises the adjusted R^2 to 0.014. While this is a 40% increase, it is clearly not impressive as the predictive ability remains small. This is also consistent with Table 2, which reports that the standard deviation of the exchange rate is virtually the same for the benchmark model as for the version of the model where the parameters are known ($\sigma = 0$).

Fratzscher et al. (2012) report larger adjusted R^2 s. This is partially a result of different macro fundamentals, some of which are usually not included in exchange rate models (e.g., equity flows). We can give a somewhat more important role to the macro fundamentals by doubling the innovations ϵ_t^b and ϵ_t^f . Doubling σ_b and σ_f leaves parameter expectations unchanged. We find that in that case the adjusted R^2 is 0.036 when regressing on the fundamentals alone and rises to 0.048 when also regressing on the interaction term of fundamentals and weights $\partial s_t / \partial f_{nt}$. Tripling the standard deviations gives respectively 0.066 and 0.087.

While the improvement in predictive power due to known time-variation in the weights on the fundamentals is small from a statistical point of view, it may well be economically important. For example, Abhyankar et al. (2005) report non-negligible utility gains for investors who manage a portfolio using exchange rate forecasts based on the monetary model of exchange rates as opposed to using a random walk forecast. Moreover, there is a huge industry that manages FX portfolios based on interest differentials, known as the carry trade.

⁷ Rossi (2006) provides some evidence of parameter instability in reduced-form exchange rate models, but the evidence is not very strong. Of the four models she considers, only two provide some limited evidence of instability. Apart from structural break tests, she also considers out-of-sample test where parameters are allowed to vary gradually over time. The latter includes a random walk time-varying parameter (TVP) model, where the parameters are assumed to follow a random walk process. However, for all currencies, the model remains unable to outperform the random walk exchange rate forecast, even when parameters are explicitly allowed to change each period in the TVP specification.

This industry exists even though the R^2 of regressions of Δs_t on lagged interest differentials is on average only about 0.01 for monthly data. As exchange rates are so hard to predict, small improvements in predictive power can be important to investors who actively trade based on such predictive power.

4. Discussion

In this section we address three issues. First, we consider the empirical relevance of the scapegoat theory. Second, we discuss how results are affected when the structural parameters are not just unknown but also time-varying. Finally, we discuss broader implications for parameter instability in macroeconomics.

4.1. Empirical relevance

The survey data used by Fratzscher et al. (2012) may not necessarily be the result of scapegoat effects. First, survey respondents could simply give weights based on the relative volatility that different fundamentals recently displayed. Second, time-varying weights could also result from a known variation in structural parameters. However, Fratzscher et al. (2012) run two types of regressions that are inconsistent with these interpretations. First, they find that, separately from the fundamentals themselves, the interaction between fundamentals and survey weights has a highly statistically significant impact on exchange rate changes. If respondents had simply based their answers on fundamentals' behavior, this would already be captured by the fundamentals themselves in the regression. Second, Fratzscher et al. (2012) regress the survey weights on an interaction term similar to $\Delta f_{nt} \epsilon_t^b$ in our model, which captures the scapegoat effect. They measure the unobserved macro shock using order flow data. They present strong evidence that the survey weight on a fundamental rises with $\Delta f_{nt} \epsilon_t^b$ (Eq. (17)). They interpret this as supporting the scapegoat theory developed in this paper.

4.2. Time-varying structural parameters

We have not taken a stand on what generates parameter uncertainty. One possibility is that structural parameters change slowly

Table 2
Moments: data and model.

	Data	Benchmark $\sigma = 2.5$	$\sigma = 0$	$\sigma = 5$
Standard deviation Δs_t in %	2.91	2.91	2.90	2.93
Corr($\Delta s_t, \Delta s_{t-1}$)	0.04	0.04	0.05	0.04
Standard deviation $i_t - i_t^*$ in %	0.22	0.22	0.22	0.22
Corr($i_t - i_t^*, i_{t-1} - i_{t-1}^*$)	0.92	0.90	0.90	0.90
R^2 regression Δs_t on fundamentals	0.023	0.023	0.022	0.029

over time but can change significantly over a very long period of time. Such a setup is considered in a previous version of this paper, Bacchetta and van Wincoop (2009). This delivers very similar results to those in this paper. The only difference is that even in the very long-run agents will never learn the true values of parameters as they are chasing a moving target.

Parameter uncertainty can also be generated by infrequent very large jumps in parameters, perhaps because of a revolution, a technological breakthrough or a fundamental change in the nature of the government or monetary policy. Our results suggest that even if such changes occur only once in a century, the uncertainty about the magnitude of the parameters may remain very large at all times.

4.3. Parameter instability in macro and finance

While the focus of this paper is on exchange rates, our explanation for the unstable reduced-form relationship could apply similarly to other forward looking financial or macroeconomic variables. As first shown by Stock and Watson (1996), and since then by many others, the phenomenon of reduced form parameter instability in macroeconomic data is widespread.⁸ There has also been great interest in the impact of parameter or model uncertainty on optimal monetary policy.⁹ The same is the case for financial data. In a survey, Pastor and Veronesi (2009) point out that “parameter uncertainty is ubiquitous in finance” and “many facts that appear baffling at first sight seem less puzzling once we recognize that parameters are uncertain and subject to learning”.¹⁰

The application to exchange rates in this paper is just one illustration of how uncertainty about structural parameters can generate significant instability in reduced form parameters. More generally, the mechanism that we have highlighted occurs whenever agents have difficulty distinguishing between unobserved macro fundamentals and unobserved structural parameters. This applies not just to other asset prices, such as equity prices, but to macro models in general in which agents need to learn about the structure of the model or its parameters.

5. Conclusion

Survey evidence suggests that the relationship between the exchange rate and macro fundamentals is highly unstable. In order to explain this, we have developed a model where structural parameters are unknown. We have shown that the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of these structural parameters. These expectations can vary significantly over time due to perfectly rational scapegoat effects as agents have difficulty distinguishing unobserved fundamentals and unobserved structural parameters. Such effects can be expected to hold more broadly in macro and finance models beyond the particular application to exchange rates discussed here.

Appendix A. Kalman filter with N fundamentals

While in Expectation of parameters section we focused on a single fundamental, in this Appendix A we describe the expectation of parameters $E_t\beta$ in the case of N fundamentals. We follow Hamilton (1994), particularly Section 13.8. Since underlying parameters are constant, the state equation is trivial and, using Hamilton’s notation (Section 13.2), we have $\mathbf{Q} = 0$, $\mathbf{v}_t = 0$, and $\mathbf{F} = I_N$, where I_N is the

identity matrix. Constant underlying parameters also imply that $\mathbf{P}_{t+1|t} = \mathbf{P}_{t|t} \equiv \mathbf{P}_t$. The observation equation is the analog of Eq. (12) so that $\mathbf{H}_t = \tilde{\Delta}\mathbf{f}_t$, $\mathbf{w}_t = \varepsilon_t^b$, and $\mathbf{R} = \sigma_b^2$.

The uncertainty is updated according to Eq. (13.8.7) in Hamilton, which gives

$$\mathbf{P}_t = \mathbf{A}_t\mathbf{P}_{t-1} \tag{24}$$

where

$$\mathbf{A}_t \equiv I_N - \mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}'_t \left(\tilde{\Delta}\mathbf{f}'_t\mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}_t + \sigma_b^2 \right)^{-1} \tilde{\Delta}\mathbf{f}'_t. \tag{25}$$

In period 1, we start from $\mathbf{P}_1 = \sigma^2 I_N$. It follows that

$$\mathbf{P}_t = \mathbf{G}_t\sigma^2 \tag{26}$$

where

$$\mathbf{G}_t = \mathbf{A}_t\mathbf{A}_{t-1}\dots\mathbf{A}_2. \tag{27}$$

Applying Eq. (13.8.6) in Hamilton, we have

$$E_t\beta = \mathbf{A}_tE_{t-1}\beta + (I_N - \mathbf{A}_t)\beta + \mathbf{Z}_t\tilde{\Delta}\mathbf{f}_t\varepsilon_t^b \tag{28}$$

where

$$\mathbf{Z}_t = \mathbf{P}_{t-1} \left(\tilde{\Delta}\mathbf{f}'_t\mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}_t + \sigma_b^2 \right)^{-1}. \tag{29}$$

Starting from $E_1\beta = \bar{\beta}$, it follows that

$$E_t\beta = \mathbf{G}_t\bar{\beta} + (I_N - \mathbf{G}_t)\beta + \mathbf{G}_t \sum_{i=2}^t \mathbf{G}_i^{-1} \mathbf{Z}_i \tilde{\Delta}\mathbf{f}_i \varepsilon_i^b. \tag{30}$$

We finally compute $\partial E_t\beta / \partial f_{nt}$. Define η_n as a vector of size N with a 1 in space n and zeros otherwise. Using Eq. (28), we have:

$$\frac{\partial E_t\beta}{\partial f_{nt}} = \frac{\partial \mathbf{A}_t}{\partial f_{nt}} (E_{t-1}\beta - \beta) + \left(\frac{\partial \mathbf{Z}_t}{\partial f_{nt}} \tilde{\Delta}\mathbf{f}_t + \mathbf{Z}_t \eta_n \right) \varepsilon_t^b. \tag{31}$$

Defining the scalar:

$$\pi_t = \tilde{\Delta}\mathbf{f}'_t\mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}_t + \sigma_b^2 \tag{32}$$

we can write:

$$\mathbf{Z}_t = \frac{\mathbf{P}_{t-1}}{\pi_t}. \tag{33}$$

This implies that:

$$\frac{\partial \mathbf{Z}_t}{\partial f_{nt}} = -2 \frac{\tilde{\Delta}\mathbf{f}'_t\mathbf{P}_{t-1}\eta_n\mathbf{P}_{t-1}}{\pi_t^2}. \tag{34}$$

Similarly we can write:

$$\mathbf{A}_t = I_N - \frac{\mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}_t\tilde{\Delta}\mathbf{f}'_t}{\pi_t}. \tag{35}$$

Therefore:

$$\frac{\partial \mathbf{A}_t}{\partial f_{nt}} = - \frac{\mathbf{P}_{t-1} \left(\eta_n \tilde{\Delta}\mathbf{f}'_t + \tilde{\Delta}\mathbf{f}_t \eta'_n \right)}{\pi_t} + 2 \frac{\tilde{\Delta}\mathbf{f}'_t\mathbf{P}_{t-1}\eta_n\mathbf{P}_{t-1}\tilde{\Delta}\mathbf{f}_t\tilde{\Delta}\mathbf{f}'_t}{\pi_t^2}. \tag{36}$$

We can just substitute Eqs. (34) and (36) into Eq. (31).

⁸ Recent contributions include Cogley and Sargent (2005), Del Negro and Otrok (2007), or Primiceri (2005).

⁹ See for example contributions by Hansen and Sargent (2008), Onatski and Williams (2003) or Levin et al. (2006).

¹⁰ For example, Cogley (2005) and Piazzesi and Schneider (2007) introduce uncertainty about time-varying parameters to explain the term spread.

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