Journal of International Money and Finance (1990), **9**, 246–257

Temporary capital controls in a balance-of-payments crisis

PHILIPPE BACCHETTA*

ESADE, 08034 Barcelona, Spain

This paper analyzes the effect of anticipated temporary capital controls in a balance-of-payments crisis, using a model based on intertemporal optimization. The anticipation of the controls affects the economy from the beginning of the crisis, usually creating a current account deficit. A speculative attack can occur just before the controls are imposed, but a transitory period of capital inflows is also possible. A speculative attack is less likely when the fiscal deficit is small or when the level of reserves is high at the time the controls are imposed.

A common feature in a balance-of-payments crisis is the imposition of restrictions on capital outflows to limit losses of foreign exchange reserves. These restrictions can be imposed either permanently or on a temporary basis after significant losses of foreign reserves. Examples of temporary controls abound. Edwards (1987) documents the use of controls in several Latin American countries. Many other countries often resort to temporary controls when the domestic currency comes under pressure. Moreover, the incentive to use temporary controls is likely to be increased in the future, as permanent restrictions are progressively removed. For example, the recent capital liberalization directive of the European Community allows explicitly for temporary controls when a country faces balance-of-payments difficulties.

The repeated use of temporary restrictions may affect their effectiveness, as economic agents are likely to anticipate them when a balance-of-payments crisis develops. The question might then be asked whether temporary controls are useful to limit the loss of foreign reserves and how the dynamics of the crisis are affected by the anticipation of future controls. This paper sheds some light on these issues by using a model based on intertemporal optimization. It analyzes the particular case of a temporary, but complete, prohibition of net capital outflows. The controls are imposed during a balance-of-payments crisis when the reserves reach a given level, known by the private sector.

The model used is a small open economy with a fixed exchange rate inhabited by Sidrauski-type families. The country faces balance-of-payments difficulties caused by an unsustainable fiscal deficit. The effect of permanent controls on both capital outflows and inflows in a balance-of-payments crisis in a similar framework has

* This paper is based on Chapter 3 of my PhD thesis. I would like to thank Jeffrey Sachs and two anonymous referees for helpful suggestions. All remaining errors are mine.

been described in Auernheimer (1987) and in Bacchetta (1988b). It is shown that when permanent controls are imposed, a current account deficit develops as capital outflows are substituted by increased imports. The deficit depletes reserves and the government must abandon the fixed exchange rate. To eliminate the current account deficit a devaluation must occur when the policy is abandoned. Temporary controls will obviously have similar effects once they are imposed. This study investigates how the economy reacts before the temporary controls are imposed. In addition, it looks at the case where the restrictions are on outflows only.

The experiment conducted in this paper is the following. First, at time 0, the government incurs a budget deficit leading to capital outflows and a loss of foreign reserves. When reserves reach a given level, say k_1, temporary controls on capital outflows are imposed at time T_1. Foreign reserves, however, continue to decline, due to a trade deficit, and the government abandons the fixed exchange rate at time T_2, when foreign reserves are depleted.[1]

The analysis shows that the anticipation of controls affects the behavior of the economy from time 0. In particular, it leads to a current account deficit. Interestingly enough, two types of behavior are possible just before the controls are imposed, at T_1: there can be either a speculative attack or a repatriation of foreign assets. A speculative attack is more likely the larger the fiscal deficit and the smaller the level of reserves k_1 at which the controls are imposed. It is also shown that when inflows are desired, restrictions on outflows become binding only at a later date $T' > T_1$.

The rest of the paper is organized as follows. Section I describes precisely the model. Section II presents the optimal behavior, formally derived in the appendix. Section III presents the dynamics of the crisis and shows that either a speculative attack or capital inflows can occur when the controls are imposed. Section IV determines the main factors affecting the dynamics of the crisis and Section V offers some concluding remarks.

I. The model

The model is similar to Obstfeld (1986a), with infinitely lived, intertemporally optimizing individuals. They live in a small open economy producing a single commodity in constant quantity y.[2] The world price level is assumed constant and equal to one. Purchasing power parity is assumed to hold. The domestic price level P is therefore equal to the exchange rate E and the inflation rate π is equal to the rate of depreciation of the domestic currency. The exchange rate is assumed pegged with either a fixed rate or a constant preannounced devaluation rate. It follows that the government can also control the inflation rate.

The representative individual consumes c of the good and holds m of money for transactions purposes. He optimizes his lifetime utility function, which is of the form:

$$\langle 1 \rangle \qquad V = \int_0^{\infty} e^{-rt} \cdot U(c, m) dt$$

where $U(c, m)$ has the usual properties (*i.e.*, is increasing, strictly concave, and twice differentiable with respect to both its arguments. Moreover, the Inada conditions are assumed to be satisfied). The rate of time preference, r, is assumed to be equal to the world interest rate.

Besides money, the individual holds short-term domestic bonds b and foreign assets f. Domestic bonds yield a nominal interest i and foreign bonds yield the interest r. The initial level of foreign assets is assumed to be positive, while the net supply of domestic bonds is assumed to be zero for the aggregate economy. Total real portfolio wealth, a, is therefore defined as $a=m+b+f$ and its change over time is described by:

$$\langle 2a \rangle \qquad \dot{a} = (i-\pi)b + rf + y + \tau - c - \pi \cdot m,$$

where τ is a real monetary transfer from the government. As the individual cannot borrow forever, the usual solvency constraint is imposed:

$$\langle 2b \rangle \qquad \lim_{t \to \infty} a(t)e^{-rt} \geqslant 0.$$

The individual maximizes $\langle 1 \rangle$ subject to $\langle 2 \rangle$ and additional constraints imposed by the particular policies described below.

The government gives a constant real transfer τ to the individual, consumes a quantity g of the good, and finances the accumulation of foreign exchange reserves k held at the central bank. The government's resources are composed of money creation and of the interest on the central bank's foreign reserves. The government budget constraint is then:

$$\langle 3 \rangle \qquad g + \tau + \dot{k} = \dot{m} + \pi m + rk.$$

Equation $\langle 3 \rangle$ can be rewritten to give the evolution of foreign exchange reserves over time:

$$\langle 4 \rangle \qquad \dot{k} = rk + \dot{m} + \pi m - \tau - g.$$

When the government runs a fiscal deficit, previous studies showed that the fixed exchange rate could not be sustained. Under full capital mobility, capital outflows deplete the central bank reserves. Just before the exchange rate is abandoned, a speculative attack occurs. Under permanent controls a current account deficit develops, also depleting foreign reserves. When the exchange rate is abandoned, a devaluation occurs. Nevertheless, the abandonment of the exchange rate is postponed by permanent capital controls.[3] This is due to the speculative attack occurring with capital mobility but not with capital controls. Before the attack occurs, however, foreign reserves are usually depleted at a faster rate under capital controls than under capital mobility. To postpone the collapse of the fixed exchange rate, the government could allow free capital mobility and impose capital controls just before the speculative attack occurs. Therefore, *the government has an incentive to impose unanticipated temporary controls if it wants to postpone the collapse of the fixed exchange rate.*

While this strategy may work once, a repeated use will not work as the private sector anticipates it, and would attack the reserves before the controls are imposed. Thus, there is a problem of time inconsistency and only permanent controls can be used. An equilibrium with temporary restrictions can exist if they are imposed according to an announced rule. An example of such a rule is to impose controls when foreign exchange reserves reach a given level. The next sections analyze the behavior of the economy under such a rule.

II. The solution with anticipated temporary controls

The experiment analyzed is the following. From an initial steady-state position, the government starts to run a deficit at time 0. This deficit can be caused by an increase in government expenditures g, an increase in transfers τ, or a decrease in the devaluation rate π. The latter policy reduces the inflation tax and creates the need to finance the deficit through other means. Each of these policies leads to capital outflows and to a loss of foreign reserves. When reserves reach a given level k_1, at time T_1, complete controls on capital outflows are imposed. More formally, the rule is:

$$\langle R \rangle \qquad \dot{f} \leqslant 0, \qquad \text{when } k(t) \leqslant k_1 .$$

If reserves continue to be depleted, the central bank abandons the fixed exchange rate at T_2, where $k(T_2)=0$. After T_2, the exchange rate floats and actually depreciates as the inflation tax finances the fiscal deficit. The previous analyses show that at time T_2 a devaluation occurs as the individual attempts to reduce his money holdings because of a higher inflation rate.

Appendix A derives the optimal individual behavior given rule ⟨R⟩ when the fiscal deficit is caused by an increase in transfers to the individual. Basically, the individual maximizes ⟨1⟩ subject to ⟨2⟩ taking the imposition of controls at T_1 and the devaluation at T_2 into account. This devaluation leads to a jump in c at time T_2.[4] The optimal jump is given by:

$$\langle 5 \rangle \qquad U_c(T_2^-) = (1-\kappa)\cdot U_c(T_2^+),$$

where κ is the size of the devaluation, T_2^- is the instant just before the devaluation and T_2^+ is the instant just after. At each point in time, the relationship between consumption and money holdings is given by:

$$\langle 6 \rangle \qquad U_m = i\cdot U_c .$$

In periods with full capital mobility, the interest parity condition prevails with $i=r+\pi$ and c and m are in a constant ratio. From the first-order conditions, the behavior of consumption and real money holdings can be characterized in four different stages (where c_0, c_1, c_2, m_0, m_1, m_2 are constants).

(i) Before $t=0$, *i.e.*, before the beginning of the crisis and with full capital mobility. Reserves k are constant.

$$\langle 7a \rangle \qquad c = c_0 = r\cdot(k_0+f_0)+y,$$

$$\langle 7b \rangle \qquad m = m_0 .$$

(ii) From $t=0$ to $t=T_1$, *i.e.*, during the crisis but before the controls are imposed. k is declining.

$$\langle 8a \rangle \qquad c = c_1 > c_0,$$

$$\langle 8b \rangle \qquad m = m_1 > m_0 .$$

(iii) From $t=T_1$ to $t=T_2$, *i.e.*, during the period over which the controls are binding. k is declining.

$$\langle 9 \rangle \qquad \dot{c} = (r\cdot U_c - U_m - \dot{m}\cdot U_{mc})/U_{cc},$$

$$\langle 10 \rangle \qquad \dot{m} = r\bar{f}+y+\tau-c,$$

$$\langle 11 \rangle \qquad U_c(T_2^-) = (1-\kappa)\cdot U_c(T_2^+),$$

$$\langle 12 \rangle \qquad \int_{T'}^{T_2} e^{\int_t^{T_2} i_s \cdot ds} \cdot (i_t - r)dt = \kappa/(1-\kappa).$$

(iv) After $t = T_2$, *i.e.*, after the crisis and with full capital mobility. $k = 0$.

$$\langle 13a \rangle \qquad c = c_2 = r\cdot\bar{f} + y < c_0,$$

$$\langle 13b \rangle \qquad m = m_2 \qquad < m_0.$$

Consumption and money holdings are constant over the various intervals, except between T_1 and T_2.[5] The first-order conditions also show how the variables are related from one interval to the other:

1. At time 0, both c and m jump up.
2. At time T_1, c does not jump ($c(T_1) = c_1$) but m can jump.
3. At time T_2, both c and m jump down.

The interesting behavior is at time T_1, *i.e.*, just before the controls are imposed. The complete system is complex to solve as the various intervals must be solved simultaneously[6]. Moreover, the length of the intervals is endogenous.[7] The behavior at $t = 0$ and at $t = T_2$, however, is unambiguous. What remains to be determined is the behavior of m at time T_1: does m go up or down when $t = T_1$? If m jumps down at T_1 (*i.e.*, $m(T_1) < m_1$), this can be done only through the purchase of foreign assets. As the exchange rate is fixed, a downward jump in m means a speculative attack on the central bank's foreign reserves. The magnitude of the attack is equal to the size of the reduction in m. On the other hand, if m increases at T_1, foreign assets are repatriated.

The next section examines the behavior of consumption and money holdings in more detail. Section IV determines the factors leading to a speculative attack or to a capital inflow.

III. The dynamics of a balance-of-payments crisis with temporary controls

Consumption and real money holdings are constant when the controls are not binding. When they are binding, between T_1 and T_2, the behavior of c and m is described by the system ⟨9⟩ to ⟨12⟩ and can be depicted in a phase diagram. There are actually two possible types of behavior at time T_1: there is either a speculative attack or a capital inflow. Figures 1 and 2 illustrate each of these cases. They show that when a speculative attack is optimal, there is a discrete capital outflow at $t = T_1$. When a capital inflow is desired, no discrete capital inflow occurs at time T_1. Instead capital inflows occur for a while, from T_1 to T', where T' is chosen by the individual so that his desired level of money holdings is reached at this time. In this latter case, even though the controls are imposed at time T_1, they become binding only from T'.

III.A. A speculative attack at T_1

Figure 1 describes the case of a speculative attack when $U_{cm} = 0$.[8] The laws of motion in the phase diagram are given by ⟨9⟩ and ⟨10⟩. Before $t = 0$, c and m are at point E, where the initial $\dot{c} = 0$ and $\dot{m} = 0$ schedules cross. At time 0, τ is increased

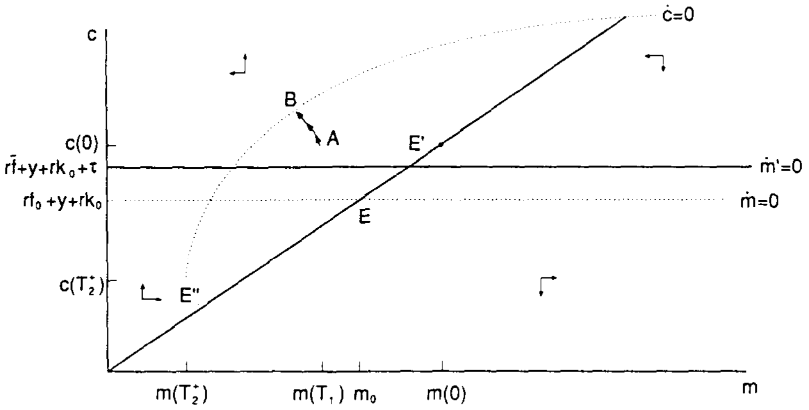

FIGURE 1. Phase diagram for c and m when a speculative attack occurs.

and the $\dot{m}=0$ schedule drifts up. Equation ⟨11⟩ is described by the broken line going through E''. At time T_2, c and m must be on this line. From T_1 to T_2, c and m behave according to ⟨9⟩ and ⟨10⟩, but the optimal path is determined by $m(T_1)$, which is given by ⟨12⟩. This path could be AB. Point A represents the optimal m and c at T_1^+ (after the controls are imposed). As c does not jump at T_1, this determines $c(0)$. $m(0)$ is then determined by ⟨6⟩ and is on the $\dot{c}=0$ schedule, in E'.

To summarize, before the crisis the system is at point E. When the fiscal deficit is increased, at time 0, the system jumps to E' with higher c and m. When the controls are imposed, at time T_1, the system jumps to A, *i.e.*, m jumps down and there is a speculative attack. Between T_1 and T_2, the system moves from A to B with m decreasing and c increasing. After T_2, the system is at point E'' with lower c and m.

III.B. Capital inflows from T_1 to T'

Figure 2 shows the case where the individual would prefer a capital inflow. There is an asymmetry of behavior between Figure 1 and Figure 2. The rule ⟨R⟩ allows a discrete capital outflow at T_1, but is inconsistent with a discrete capital inflow. With a reasoning similar to Figure 1, the optimal path would be AB on Figure 2. To move on this path, the system should jump from E' to A at time T_1. The upward jump in m at T_1 represents a discrete capital inflow and therefore an increase in foreign reserves. This behavior, however, is inconsistent with the rule ⟨R⟩: a discrete capital inflow at T_1 means that $k(T_1)>k_1$ and the controls are removed immediately. Therefore, *there is no equilibrium with a discrete capital inflow at time* T_1 and the path AB must be ruled out, as well as any path starting on the right of the $\dot{c}=0$ schedule.

When a capital inflow is desired there is no jump in m at time T_1 and the optimal path could be like $E'D$. This path is actually shorter than AB and would start at a date later than T_1, at $t=T'$. Between T_1 and T' there are capital inflows and the controls are not binding. In this case, the individual does not choose his initial money holdings, but chooses the time $T'>T_1$ where he starts to increase his consumption. The optimal T' is determined by ⟨12⟩. When a speculative attack is desired, $T'=T_1$.

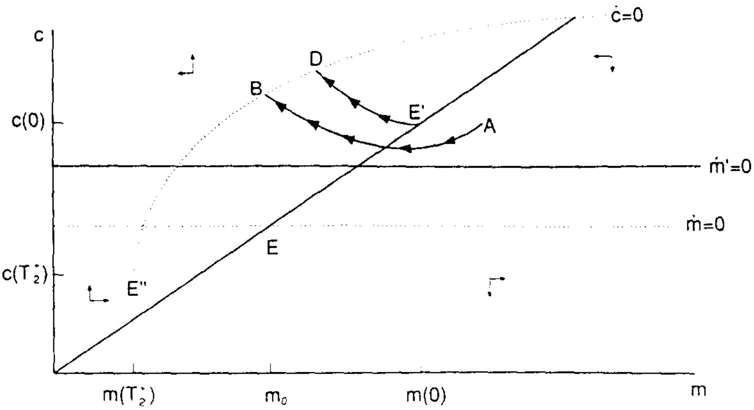

FIGURE 2. Phase diagram for c and m when capital inflows occur.

To summarize, the system jumps from E to E' at time 0 and stays at point E' until time T': at T_1, the controls are imposed but as the individual desires capital inflows, the restrictions are not binding until T'. From T', the controls start to bite and c increases and m decreases. At time T_2, both c and m jump down. It is worth noticing that *while anticipated controls have an effect from time 0, they may have no effect at the moment they are imposed, at time T_1.*

IV. A speculative attack or capital inflows

A speculative attack occurring just before the temporary controls are imposed is not surprising: a loss on nominal domestic assets through a devaluation is anticipated and controls on capital outflows are known to be binding before the devaluation. On the other hand, how can desired capital inflows be explained under these conditions? This paradox is explained by the dual role played by money in this framework: it is the only domestic alterable store of value, but it is also used for transactions purposes. The first role leads to capital outflows and explains the speculative attack when a devaluation is anticipated. The second role leads to larger desired money holdings as consumption is increasing when the controls are binding. When the transactions role of money dominates its role as a store of value, capital inflows will result.

This section examines the main factors determining the behavior of money holdings at time T_1. The analysis is done in two steps. First, it is shown that the two main elements influencing the behavior at T_1 are the length of the interval over which the controls are imposed and the size of the devaluation. Second, it is shown that these two elements are primarily affected by the size of the fiscal deficit and by the level of foreign reserves at time T_1.

Whether capital inflows or a speculative attack occur at time T_1 depends on the optimal level of money holdings between T_1 and T_2. This optimal level is in general ambiguous as it influences total utility through various channels. The three main effects of a decrease in $m(T_1)$ are:

1. A lower level of money holdings allows for fewer transactions between T_1 and T_2, and hence gives a lower utility between T_1 and T_2.

2. Lower money holdings means a higher level of foreign assets after the devaluation, thus a higher consumption and a higher utility after T_2.
3. Lower money holdings before the devaluation mean a smaller loss on real money holdings.

A trade-off of utility before and after T_2 arises from the first two effects. The result from this trade-off depends critically on the length of the period where the controls are binding, *i.e.*, on $T_2 - T'$. For example, if this period is short the first effect is smaller. The third effect means that the size of the devaluation will affect the optimal level of money holdings.

The two main elements influencing the optimal level of money holdings are therefore the size of the devaluation κ and the length of the interval $T^* = T_2 - T'$ over which the controls are binding. Appendix B formally shows, in the particular case of the log utility function, that lower money holdings are implied either by a larger κ or by a smaller T^*. In other terms, *a speculative attack at time T_1 is more likely when the devaluation is large or when the period over which the controls are imposed is small.*

In the second step, it is easily seen that the main factors affecting κ and T^* are the level of foreign exchange reserves at time T_1, k_1, and the size of the fiscal deficit, τ.[9] First, an increase in the level of reserves k_1 increases T^*. From equation ⟨B1⟩, an increase in k_1 implies an increase in the present value of c on the interval T^*. This implies an increase in T^*. As a larger T^* leads to larger money holdings, *a higher level of foreign reserves at time T_1 makes a speculative attack less likely.*

A higher fiscal deficit leads to a larger devaluation and a smaller interval T^*. The reason is that an increase in τ leads to an increase in consumption. From equation ⟨11⟩, this means a larger jump in c as $c(T_2^-)$ is larger and therefore a larger κ. Furthermore, the interval T^* is decreased: from equation ⟨B1⟩ the present value of c must remain constant. If c increases, then T^* must decrease. As both a larger devaluation and a smaller interval T^* lead to lower money holdings from T_1, *a larger fiscal deficit makes a speculative attack at T_1 more likely.*

Finally notice that the absolute size of the fiscal deficit matters for the timing T^*, but the *relative* size matters for the magnitude of the devaluation. As condition ⟨11⟩ involves the jump in total consumption, it is the size of τ relative to income $y + r \cdot f$ that is relevant.

V. Concluding remarks

This paper has analyzed the effect of anticipated temporary controls in a balance-of-payments crisis, when the controls are imposed contingently on the level of foreign exchange reserves. The analysis has shown that the anticipation of these controls modifies the dynamics of the crisis and in particular leads to a current account deficit. When the controls are imposed there may be a speculative attack just before the imposition or there may be capital inflows. The possible occurrence of capital inflows is surprising, but is explained by the use of money for transactions purposes: as transactions usually increase when capital controls are binding, larger money holdings may be desired.

The framework used in this analysis is highly simplified and the role of temporary controls deserves further investigation. One direction is to look at other types of shocks. This paper has analyzed one particular type of disturbance, namely a permanent increase in the fiscal deficit. Other disturbances and especially temporary ones certainly deserve attention.

The issue of uncertainty is clearly of prime importance and understanding the effect of the various sources of uncertainty seems desirable. Dellas and Stockman (1988) make a first step in this direction. There may also be an issue of multiple equilibria, as mentioned for example by Obstfeld (1986b, 1988). We can imagine an equilibrium where everybody speculates and this actually leads to the collapse of the exchange rate. Alternatively, we can imagine another equilibrium where nobody speculates and where the exchange rate does not collapse. The mere presence of anticipated controls may rule out the speculative equilibrium. A plausible setup for this story is however difficult to develop.

Finally, in the analysis presented capital controls cannot avoid a collapse of the exchange rate. There are instances, however, where the imposition of temporary controls allows the avoidance of such a collapse. Understanding when these instances arise is of considerable interest.

Appendix A

This appendix derives the optimal behavior of an economy with anticipated temporary controls by using optimal control techniques (*e.g.*, see Bryson and Ho, 1975). They are more convenient than the Lagrangian method as the controls are on capital outflows only and represent an inequality constraint.[10] To impose this constraint, it is useful to define the variable $z=\dot{f}$. With the rule ⟨R⟩, we have $z \leqslant 0$ from T_1 to T_2. From T_1, if capital inflows are desired the constraint is not binding, but it could be effective at a later date, say T'. Once the constraint is binding, it can be shown that it remains binding until T_2, *i.e.*, no capital inflows occur. If $T'=T_2$, the constraint is never binding and if $T'=T_1$ it is binding all along. Thus, we have $z<0$ from T_1 to T' and $z=0$ from T' to T_2 and the individual chooses the optimal T' until which he repatriates foreign assets. Overall, there are four possible periods from $t=0$:

1. Between 0 and T_1, with capital mobility, c and m are constant and the interest parity holds, *i.e.*, $i=r+\pi$.
2. Between T_1 and T' with capital inflows: the controls are not binding and foreign assets can be modified.
3. Between T' and T_2, the controls are binding.
4. From T_2, capital mobility as in (1) but with a different inflation rate.

T' is determined by the individual, while T_1 and T_2 are taken as given in the optimization problem. When a speculative attack is desired, $T'=T_1$ and there are only three periods. Assuming for simplicity that $\pi=0$ from 0 to T_2, the full problem can be written:

⟨A1⟩ $$\max V = \int_0^\infty e^{-rt}\cdot U(c,m)\cdot dt \qquad s.t.$$

⟨A2a⟩ $$\dot{a} = r\cdot a+(i-r)\cdot b+y+\tau-c-r\cdot m, \qquad 0 \leqslant t \leqslant T_1^-,$$

⟨A2b⟩ $$\dot{a} = i\cdot a+(r-i)\cdot f+y+\tau-c-i\cdot m, \qquad T_1^+ \leqslant t < T',$$

⟨A2c⟩ $$\dot{a} = i\cdot a+(r-i)\cdot \bar{f}+y+\tau-c-i\cdot m, \qquad T' \leqslant t \leqslant T_2^-,$$

⟨A2d⟩ $$\dot{a} = r\cdot a+(i-\pi-r)\cdot b+y+\tau-c-(r+\pi)\cdot m, \qquad T_2^+ \leqslant t,$$

⟨A3⟩ $$a(T_2^+) = (1-\kappa)\cdot a(T_2^-)+\kappa\cdot\bar{f},$$

⟨A4⟩ $$f(t) = f(T_1^+)+\int_{T_1^+}^{t} z\cdot dt, \qquad T_1^+ \leqslant t < T',$$

⟨A5⟩ $$\bar{f} = f(T_1^+) + \int_{T_1^-}^{T'} z \cdot dt,$$

⟨A6⟩ $$z(t) < 0, \qquad T_1^+ \leqslant t < T',$$

⟨A7⟩ $$a(0) = a_0,$$

⟨A8⟩ $$\lim_{t \to \infty} a(t) e^{-rt} \geqslant 0,$$

where T_1^- and T_1^+ indicate the moment just before and after the controls are imposed and T_2^- and T_2^+ are the moments just before and after the devaluation. ⟨A3⟩ represents the loss in domestic assets from the devaluation. ⟨A4⟩ describes the evolution of foreign assets between T_1^+ and T' and ⟨A5⟩ represents the level of foreign assets once the controls are imposed.

The first-order conditions for this problem are (where H is the Hamiltonian, λ is the costate variable, and v is a scalar multiplier associated with the constraint ⟨A3⟩):

⟨A9⟩ $$\frac{\partial H}{\partial c} = 0, \qquad \forall t,$$

⟨A10⟩ $$\frac{\partial H}{\partial m} = 0, \qquad \forall t,$$

⟨A11⟩ $$\frac{\partial H}{\partial b} = 0, \qquad t \leqslant T_1^-, t \geqslant T_2^+,$$

⟨A12⟩ $$\frac{\partial H}{\partial a} = r \cdot \lambda - \dot{\lambda}, \qquad \forall t,$$

⟨A13⟩ $$\lambda(T_2^-) = (1 - \kappa) \cdot \lambda(T_2^+),$$

⟨A14⟩ $$\int_{t}^{T_2^-} e^{-rs} \cdot \frac{\partial H(s)}{\partial z(t)} \cdot ds = v \cdot \kappa, \qquad T_1^+ \leqslant t < T',$$

⟨A15⟩ $$\int_{T_1^-}^{T_2^-} e^{-rt} \cdot \frac{\partial H(t)}{\partial f(T_1^+)} \cdot dt = v \cdot \kappa,$$

⟨A16⟩ $$\int_{T'}^{T_2^-} e^{-rt} \cdot \frac{\partial H(t)}{\partial T'} \cdot dt = v \cdot \kappa \cdot z(T').$$

Conditions ⟨A9⟩ to ⟨A12⟩ are standard. ⟨A11⟩ does not hold between T_1 and T_2 because the controls are binding: as the individual can choose his money holdings, the level of domestic bonds is determined. Condition ⟨A13⟩ is implied by the devaluation at T_2. In ⟨A14⟩ the individual chooses his optimal capital inflows from T_1 to T', given the loss κ on money holdings at T_2. ⟨A15⟩ determines the level of foreign assets at T_1 given the future devaluation and ⟨A16⟩ determines the timing T'. The combination of ⟨A14⟩ to ⟨A16⟩ determines whether there is a speculative attack, *i.e.*, whether $f(T_1^+) > f(T_1^-)$ or whether there are capital inflows, *i.e.*, whether $T' > T_1$. Using the definition of the Hamiltonian and simplifying, the first-order conditions give:

⟨A17⟩ $$U_m = i \cdot U_c, \qquad \forall t,$$

⟨A18⟩ $$i = r + \pi, \qquad t \leqslant T', t \geqslant T_2^+,$$

⟨A19⟩ $$U_c = \text{constant}, \qquad t \leqslant T', t \geqslant T_2^+.$$

The result ⟨A18⟩ between T_1^+ and T' can be found by using ⟨A14⟩ to ⟨A16⟩. This is intuitive as it means that there is arbitrage when the controls are not binding. As c is constant between 0 and T', ⟨A17⟩ and ⟨A18⟩ imply that m is also constant. Finally, between T' and T_2^-, we get the system ⟨9⟩ to ⟨12⟩. Equation ⟨10⟩ is derived from ⟨A2c⟩, realizing that $\dot{f}=\dot{b}=0$ and setting $b=0$. ⟨12⟩ is derived from ⟨A14⟩ to ⟨A16⟩ by using ⟨A12⟩ and the condition setting the derivative with respect to $da(T_2^-)$ equal to 0. ⟨12⟩ means that the domestic rate is on average higher than the foreign rate when the controls are imposed.

Appendix B

This appendix shows that a larger κ and a smaller $T^*=T_2-T'$ lead to lower money holdings between T' and T_2. To be more specific, the log utility function is used: $U(c,m)=\ln c+\beta\ln m$. As a preliminary, it is useful to derive the present value of consumption between T' and T_2. Using ⟨4⟩ and ⟨10⟩ and integrating we have:

$$\langle \text{B1} \rangle \qquad \int_0^{T^*} e^{-rt}\cdot c(T'+t)\cdot dt = k_1+\left(\frac{y}{r}+\bar{f}\right)\cdot(1-e^{-rT^*}).$$

Equation ⟨B1⟩ means that the present value of consumption over the interval T^* is given. In particular it does not depend on κ or on τ.

The optimal level of money holdings between T' and T_2 is determined by equation ⟨12⟩. By using equations ⟨A14⟩ to ⟨A16⟩ in Appendix A, equation ⟨12⟩ can be rewritten as:

$$\langle \text{B2} \rangle \qquad \int_{T'}^{T_2} e^{-rt}\cdot\lambda\cdot(r-i)\cdot dt = v\cdot\kappa,$$

where v is a negative constant and λ is the costate variable and $\lambda=U_c=U_m/i$ (from ⟨A9⟩ and ⟨A10⟩). With the log utility function, we have $\lambda=1/c=\beta/(m\cdot i)$. Thus, ⟨B2⟩ can be rewritten as:

$$\langle \text{B3} \rangle \qquad r\cdot\int_0^{T^*} e^{-rt}\cdot\frac{1}{c(T'+t)}\cdot dt-\beta\cdot\int_0^{T^*} e^{-rt}\cdot\frac{1}{m(T'+t)}\cdot dt = v\cdot\kappa\cdot e^{-rT'}.$$

As $v<0$, an increase in the size of the devaluation κ means a decrease in the RHS of ⟨B3⟩. On the LHS, as the first term is constant (from ⟨B1⟩) the second term must increase. Hence the level of m must decrease. Therefore, *an increase in the size of the devaluation leads to lower money holdings.*

When T^* increases, the RHS remains constant and the first term on the LHS increases. The level of m must then clearly increase. Therefore, *an increase in the period where capital controls are binding leads to higher money holdings.*

Notes

1. See Dellas and Stockman (1988) for a similar experiment in a different framework.
2. Lower case letters represent real variables at time t.
3. See Park and Sachs (1987) for a proof.
4. See Calvo (1988) for a careful analysis of the effect of an anticipated devaluation in a similar model.
5. The differential equations ⟨9⟩ and ⟨10⟩ describing the changes in c and m between T_1 and T_2 are typical of Sidrauski-type models with money in the utility function (*e.g.*, see Calvo, 1981). In these models, the initial level of money holdings, here $m(T_1)$, is predetermined and both c and m converge to their steady state. These two conditions are boundary conditions and determine the *level* of c and m. In the system ⟨9⟩ to ⟨12⟩, however, the boundary conditions are different: $m(T_1)$ is not predetermined and its optimal level is given by equation ⟨12⟩. Furthermore, c and m do not converge to their steady state as both jump at time T_2. The system is represented graphically in Section III.

6. For example, $m(T_1)$ is determined by the behaviour of c and m between T_1 and T_2. This behavior depends on c_2 (from $\langle 11 \rangle$). But c_2 depends on $\bar{f}$ which is in turn determined by $m(T_1)$.
7. For a complete solution, simulations must be used. See Bacchetta (1988a).
8. The phase diagram in Figure 1 is typical of systems with money in the utility function. See for example Auernheimer (1987), Calvo (1981, 1988), and Obstfeld (1986a).
9. While τ represents total transfers, in the experiment analyzed it also represents the total fiscal deficit from T_2.
10. See Bacchetta (1988b) and Calvo (1988) for use of Lagrangians with permanent controls both on outflows and inflows.

References

AUERNHEIMER, L., 'On the Outcome of Inconsistent Programs under Exchange Rate and Monetary Rules,' *Journal of Monetary Economics*, March 1987, **19**: 279–305.

BACCHETTA, P., 'Restrictions on International Capital Flows,' PhD Thesis, Harvard University, May 1988. (1988a)

BACCHETTA, P., 'Fiscal Deficits, Capital Controls, and the Dual Exchange Rate,' Working Paper No. 214, Brandeis University, September 1988. (1988b)

BRYSON, A. E., AND Y. HO, *Applied Optimal Control*, Washington: Hemisphere Publishing, 1975.

CALVO, G. A., 'Devaluation: Levels versus Rates,' *Journal of International Economics*, 1981, **11**: 165–172.

CALVO, G. A., 'Anticipated Devaluations,' mimeo, University of Pennsylvania, February 1988.

DELLAS, H., AND A.C. STOCKMAN, 'Self-Fulfilling Expectations, Speculative Attacks and Capital Controls,' NBER Working Paper, No. 2625, June 1988.

EDWARDS, S., 'Exchange Controls, Devaluations and Real Exchange Rates: The Latin American Experience,' NBER Working Paper, No. 2348, August 1987.

OBSTFELD, M., 'Capital Controls, the Dual Exchange Rate, and Devaluation,' *Journal of International Economics*, February 1986, **20**: 1–20. (1986a)

OBSTFELD, M., 'Rational and Self-Fulfilling Balance-of-Payments Crises,' *American Economic Review*, March 1986, **76**: 72–81. (1986b)

OBSTFELD, M., 'Competitiveness, Realignment, and Speculation: The Role of Financial Markets,' in F. Giavazzi, S. Micossi, and M. Miller, eds, *The European Monetary System*, Cambridge: Cambridge University Press, 1988.

PARK, D., AND J. SACHS, 'Capital Controls and the Timing of the Exchange Regime Collapse,' NBER Working Paper, No. 2250, May 1987.