Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?

By Philippe Bacchetta and Eric van Wincoop*

Empirical evidence shows that most exchange rate volatility at short to medium horizons is related to order flow and not to macroeconomic variables. We introduce symmetric information dispersion about future macroeconomic fundamentals in a dynamic rational expectations model in order to explain these stylized facts. Consistent with the evidence, the model implies that (a) observed fundamentals account for little of exchange rate volatility in the short to medium run, (b) over long horizons, the exchange rate is closely related to observed fundamentals, (c) exchange rate changes are a weak predictor of future fundamentals, and (d) the exchange rate is closely related to order flow. (*JEL* F3, F4, G0, G1, E0)

The poor explanatory power of existing theories of the nominal exchange rate is most likely the major weakness of international macroeconomics. Richard A. Meese and Kenneth Rogoff (1983) and the subsequent literature have found that a random walk predicts exchange rates better than macroeconomic models in the short run. Richard Lyons (2001) refers to the weak explanatory power of macroeconomic fundamentals as the "exchange rate determination puzzle."

This puzzle is less acute for long-run exchange rate movements, since there is extensive evidence of a much closer relationship between exchange rates and fundamentals at horizons of two to four years (e.g., see Nelson C. Mark, 1995). Recent evidence from the microstructure approach to exchange rates suggests that investor heterogeneity might play a key role in explaining exchange rate fluctuations. In particular, Martin D. D. Evans and Lyons (2002b) show that most short-run exchange rate volatility is related to order flow, which in turn is associated with investor heterogeneity.[1] Since these features are not present in existing theories, a natural suspect for the failure of current models to explain exchange rate movements is the standard hypothesis of a representative agent.

The goal of this paper is to present an alternative to the representative agent model that can explain the exchange rate determination puzzle and the evidence on order flow. We introduce heterogeneous information into a standard dynamic monetary model of exchange rate determination. There is a continuum of investors who differ in two respects. First, they have symmetrically dispersed information about future macroeconomic fundamentals.[2] Second, they face different exchange rate risk exposure associated with nonasset income. This exposure is private information and leads to hedge trades whose aggregate is unobservable. Our main

* Bacchetta: Study Center Gerzensee, 3115 Gerzensee, Switzerland, University of Lausanne, and Swiss Finance Institute (e-mail: philippe.bacchetta@szgerzensee.ch); van Wincoop: Department of Economics, University of Virginia, Charlottesville, VA 22904 and NBER (e-mail: vanwincoop@virginia.edu). We would like to thank Bruno Biais, Gianluca Benigno, Christophe Chamley, Margarida Duarte, Ken Froot, Harald Hau, Olivier Jeanne, Richard Lyons, Elmar Mertens, two anonymous referees, and many seminar and conference participants for helpful comments. Financial support from the Bankard Fund for Political Economy and the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK) is gratefully acknowledged. The NCCR FINRISK is a research program supported by the Swiss National Science Foundation.

[1] See also Evans and Lyons (2002a), Harald Hau et al. (2002), Geir Bjønnes et al. (2005), and Kenneth A. Froot and Tarun Ramadorai (2005).

[2] We know from extensive survey evidence that investors have different views about the macroeconomic outlook. There is also evidence that exchange rate expectations differ substantially across investors. See Takatoshi Ito (1990), Ronald MacDonald and Ian W. Marsh (1996), Graham Elliott and Ito (1999), and Dionysios Chionis and MacDonald (2002).

finding is that information heterogeneity disconnects the exchange rate from observed macroeconomic fundamentals in the short run, while there is a close relationship in the long run. At the same time there is a close link between the exchange rate and order flow over all horizons.

Our modeling approach integrates several strands of literature. First, it has in common with most of the existing (open economy) macro literature that we adopt a fully dynamic general equilibrium model, leading to time-invariant second moments. Second, it has in common with the noisy rational expectations literature in finance that the asset price (exchange rate) aggregates private information of individual investors, with unobserved shocks preventing average private signals from being fully revealed by the price. The latter are modeled endogenously as hedge trades in our model.[3] Third, it has in common with the microstructure literature of the foreign exchange market that private information is transmitted to the market through order flow.[4]

Most models in the noisy rational expectations literature and microstructure literature are static or two-period models.[5] This makes them ill-suited to address the disconnect between asset prices and fundamentals, which has a dynamic dimension since the disconnect is much stronger at short horizons. Even the few dynamic rational expectation models in the finance literature cannot be applied in our context. Jiang Wang (1993, 1994) develops an infinite horizon, noisy rational expectations model with a hierarchical information structure. There are only two types of investors, one of which can fully observe the variables affecting the equilibrium asset price. We believe that it is more appropriate to consider cases where no class of investors has superior information and where there is broader dispersion of information. Several papers make a step in this direction by examining symmetrically dispersed information in a multiperiod model, but they examine only an asset with a single payoff at a terminal date.[6]

For the dynamic dimension of our paper, we rely on the important paper by Robert M. Townsend (1983). Townsend analyzed a business cycle model with symmetrically dispersed information. As is the case in our model, the solution exhibits infinitely higher-order expectations (expectations of other agents' expectations).[7] We adapt Townsend's solution procedure to our model. The only application to asset pricing we are aware of is Kenneth J. Singleton (1987), who applies Townsend's method to a model for government bonds with a symmetric information structure.[8]

Another feature of our paper is the explicit modeling of order flow in a general equilibrium model. This gives a theoretical framework to guide empirical work on order flow. We show, for example, how order flow precedes prices and thus conveys information. To derive order flow, we take a different perspective on the equilibrium mechanism. Typically, the equilibrium price of a competitive noisy rational ex-

[3] Some recent papers in the exchange rate literature have introduced exogenous noise in the foreign exchange market. They do not, however, consider information dispersion about future macro fundamentals. Examples are Hau (1998), Mark and Yangru Wu (1998), Michael B. Devereux and Charles Engel (2002), Olivier Jeanne and Andrew K. Rose (2002), and Robert Kollman (2002).

[4] See Lyons (2001) for an overview of this literature.

[5] See Markus K. Brunnermeier (2001) for an overview.

[6] See Hua He and Wang (1995), Xavier Vives (1995), F. Douglas Foster and S. Viswanathan (1996), Michael J. Brennan and H. Henry Cao (1997), or Franklin Allen et al. (forthcoming). Brennan and Cao assume that private information is symmetrically dispersed among agents within a country, while there is also asymmetric information between countries.

[7] Subsequent contributions have been mostly technical, solving the same model as in Townsend (1983) with alternative methods. See Thomas J. Sargent (1991) and Kenneth Kasa (2000). Probably as a result of the technical difficulty in solving these models, the macroeconomics literature has devoted relatively little attention to heterogeneous information in the last two decades. This contrasts with the 1970s where, following Robert E. Lucas (1972), there had been active research on rational expectations and heterogeneous information (e.g., see Robert G. King, 1982). Recently, information issues in the context of price rigidity have again been brought to the forefront in contributions by N. Gregory Mankiw and Ricardo Reis (2002) and Michael Woodford (2003).

[8] In Singleton's model there is no information dispersion about the payoff structure on the assets (in this case, coupons on government bonds), but there is private information about whether noise trade is transitory or persistent. The uncertainty is resolved after two periods. John P. Hussman (1992) and Kasa et al. (2004) also study dynamic asset pricing models with infinitely higher-order expectations, but do not adopt a symmetrically dispersed information structure.

pectation model is seen as determined by a Walrasian auctioneer. The equilibrium can also be interpreted, however, as the outcome of an order-driven auction market, whereby market orders based on private information hit an outstanding limit order book. This characterization resembles the electronic trading system that nowadays dominates the interbank foreign exchange market. As is common in the theoretical literature, we define limit orders as orders that are conditional on public information and the (yet unknown) exchange rate. Limit orders provide liquidity to the market. Market orders take liquidity from the market and are associated with private information. Order flow is equal to net market orders. Not surprisingly, the weak relationship in the model between short-run exchange rate fluctuations and publicly observed fundamentals is closely mirrored by the close relationship between exchange rate fluctuations and order flow.[9]

The dynamic implications of the model for the relationship between the exchange rate, observed fundamentals, and order flow can be understood as follows. In the short run, rational confusion plays an important role in disconnecting the exchange rate from observed fundamentals. Investors do not know whether an increase in the exchange rate is driven by an improvement in average private signals about future fundamentals or an increase in unobserved hedge trades. This implies that unobserved hedge trades have an amplified effect on the exchange rate since they are confused with changes in average private signals about future fundamentals.[10] We show that a small amount of hedge trades can become the dominant source of exchange rate volatility when information is heterogeneous, while it has practically no effect on the exchange rate when investors have common information. Moreover, our numerical simulations show that these effects are quantitatively consistent with empirical evidence.

In the long run there is a close relationship between the exchange rate, observed fundamentals, and cumulative order flow. First, rational confusion gradually dissipates as investors learn more about future fundamentals.[11] The impact of unobserved hedge trades on the equilibrium price therefore gradually weakens, leading to a closer long-run relationship between the exchange rate and observed fundamentals. Second, when the fundamental has a permanent component, the exchange rate and cumulative order flow are closely linked in the long run. Private information about permanent future changes in the fundamental is transmitted to the market through order flow, so that order flow has a permanent effect on the exchange rate.

The remainder of the paper is organized as follows. Section I describes the model and solution method. Section II considers a special case of the model in order to develop intuition for our key results. Section III discusses the implications of the dynamic features of the model. Section IV presents numerical results based on the general dynamic model and Section V concludes.

I. A Monetary Model with Information Dispersion

A. *Basic Setup*

Our model contains the three basic building blocks of the standard monetary model of exchange rate determination: (a) money market equilibrium, (b) purchasing power parity, and (c) interest rate arbitrage. We modify the standard monetary model by assuming incomplete and dispersed information across investors. Before describing the precise information structure, we first derive a general solution to

[9] In recent work closely related to ours, Evans and Lyons (2004) also introduce microstructure features in a dynamic general equilibrium model in order to shed light on exchange rate puzzles. There are three important differences in comparison to our approach. First, they adopt a quote-driven market, while we model an order-driven auction market. Second, they assume that all investors within one country have the same information, while there is asymmetric information across countries. Third, their model is not in the noisy rational expectations tradition.

[10] The basic idea of rational confusion can already be found in the noisy rational expectation literature. For example, Gérard Gennotte and Hayne Leland (1990) and David Romer (1993) argued that such rational confusion played a critical role in amplifying noninformational trade during the stock-market crash of October 19, 1987.

[11] Another recent paper on exchange rate dynamics where learning plays an important role is Pierre-Olivier Gourinchas and Aaron Tornell (2004). In that paper, in which there is no investor heterogeneity, agents learn about the nature of interest rate shocks (transitory or persistent), but there is an irrational misperception about the second moments in interest rate forecasts that never goes away.

the exchange rate under heterogeneous information, in which the exchange rate depends on higher-order expectations of future macroeconomic fundamentals. This generalizes the standard equilibrium exchange rate equation that depends on common expectations of future fundamentals.

Both observable and unobservable fundamentals affect the exchange rate. The observable fundamental is the ratio of money supplies. We assume that investors have heterogeneous information about future money supplies. The unobservable fundamental takes the form of an aggregate hedge against nonasset income in the demand for foreign exchange. This unobservable element introduces noise in the foreign exchange market in the sense that it prevents investors from inferring average expectations about future money supplies from the price.[12] This trade also affects the risk premium in the interest rate arbitrage condition. Notice that the unobserved hedge trades are true aggregate fundamentals that drive the equilibrium exchange rate, but they are typically not called fundamentals by macroeconomists because they cannot be directly observed.

There are two economies. They produce the same good, so that purchasing power parity holds:

$$p_t = p_t^* + s_t. \tag{1}$$

Local currency prices are in logs and s_t is the log of the nominal exchange rate (home per foreign currency).

There is a continuum of investors in both countries on the interval [0, 1]. We assume that there are overlapping generations of agents who live for two periods and make only one investment decision. Before dying, investor i passes on his or her private information to the next investor i born the following period. This myopic agent setup significantly simplifies the presentation, helps in providing intuition, and allows us to obtain an exact solution to the model.[13]

Investors in both economies can invest in four assets: money of their own country, nominal bonds of both countries with interest rates i_t and i_t^*, and a technology with fixed real return r that is in infinite supply. We assume a small open-economy setting. The Home country is large and the Foreign country infinitesimally small; variables from the latter are starred. Bond market equilibrium is therefore entirely determined by investors in the large Home country, on which we will focus. We also assume that money supply in the large country is constant. It is easy to show that this implies a constant price level p_t in equilibrium, so that $i_t = r$. For ease of notation, we just assume a constant p_t. Money supply in the small country is stochastic.

The wealth w_t^i of investors born at time t is given by a fixed endowment. At time $t + 1$ these investors receive the return on their investments plus income y_{t+1}^i from time $t + 1$ production. We assume that production depends both on the exchange rate and on real money holdings $\tilde{m}_t^i$ through the function $y_{t+1}^i = \lambda_t^i s_{t+1} - \tilde{m}_t^i(ln(\tilde{m}_t^i) - 1)/\alpha$, with $\alpha > 0$.[14] The coefficient λ_t^i measures the exchange rate exposure of the nonasset income of investor i. We assume that λ_t^i is time varying and known only to investor i. This will generate an idiosyncratic hedging term. Agent i maximizes

$$-E_t^i e^{-\gamma c_{t+1}^i}$$

subject to

$$c_{t+1}^i = (1 + i_t)w_t^i + (s_{t+1} - s_t + i_t^* - i_t)b_{Ft}^i - i_t\tilde{m}_t^i + y_{t+1}^i,$$

where b_{Ft}^i is invested in foreign bonds and $s_{t+1} - s_t + i_t^* - i_t$ is the log-linearized excess return on investing abroad.

Combining the first-order condition for money holdings with money market equilibrium in both countries, we get

[12] For alternative modeling of "noise" from rational behavior, see Matthew Spiegel and Avanidhar Subrahmanyam (1992), Wang (1994), and James Dow and Gary Gorton (1995).

[13] See Singleton (1987) for the same setup. In an earlier version of the paper, Bacchetta and van Wincoop (2003), we also consider an infinite-horizon version. While this significantly complicates the solution method, numerical results are almost identical.

[14] By introducing money through production rather than utility, we avoid making money demand a function of consumption, which would complicate the solution.

$$m_t - p_t = -\alpha i_t \tag{2}$$

$$m_t^* - p_t^* = -\alpha i_t^*, \tag{3}$$

where m_t and m_t^* are the logs of domestic and foreign nominal money supply.

The demand for foreign bonds by investor i is:[15]

$$b_{Ft}^i = \frac{E_t^i(s_{t+1}) - s_t + i_t^* - i_t}{\gamma\sigma_t^2} - b_t^i, \tag{4}$$

where the conditional variance of next period's exchange rate is σ_t^2, which is the same for all investors in equilibrium. We focus on equilibria where the conditional variance of next period's exchange rate is time-invariant. The hedge against nonasset income is represented by $b_t^i = \lambda_t^i$.

We assume that the exchange rate exposure is equal to the average exposure plus an idiosyncratic term, so that $b_t^i = b_t + \varepsilon_t^i$. We consider the limiting case where the variance of ε_t^i approaches infinity, so that knowing one's own exchange rate exposure provides no information about the average exposure. This assumption is made only for convenience and our results do not qualitatively change when we assume a finite, but positive, variance of ε_t^i. The key assumption is that the aggregate hedge component b_t is unobservable. We assume that b_t follows an AR(1) process:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b, \tag{5}$$

where $\varepsilon_t^b \sim \mathcal{N}(0, \sigma_b^2)$. While b_t is an unobserved fundamental, the assumed autoregressive process is known by all agents.

B. *Market Equilibrium and Higher-Order Expectations*

Since bonds are in zero net supply, market equilibrium is given by $\int_0^1 b_{Ft}^i \, di = 0$. One way to reach equilibrium is to have a Walrasian auctioneer to whom investors submit their demand schedule b_{Ft}^i. We show below that the same equilibrium can also be implemented by introducing a richer microstructure in the form of an order-driven auction market.

Market equilibrium yields the following interest rate arbitrage condition:

$$\bar{E}_t(s_{t+1}) - s_t = i_t - i_t^* + \gamma\sigma_t^2 b_t, \tag{6}$$

where $\bar{E}_t$ is the average rational expectation across all investors. The model is summarized by (1), (2), (3), and (6). Other than the risk premium in the interest rate arbitrage condition, associated with nonobservable trade, these equations are the standard building blocks of the monetary model of exchange rate determination.

Defining the *observable* fundamental as $f_t = (m_t - m_t^*)$, in Appendix A we derive the following equilibrium exchange rate:

$$s_t = \frac{1}{1+\alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^k \times \bar{E}_t^k(f_{t+k} - \alpha\gamma\sigma_{t+k}^2 b_{t+k}), \tag{7}$$

where $\bar{E}_t^0(x_t) = x_t$, $\bar{E}_t^1(x_{t+1}) = \bar{E}_t(x_{t+1})$ and higher-order expectations are defined as

$$\bar{E}_t^k(x_{t+k}) = \bar{E}_t\bar{E}_{t+1} \cdots \bar{E}_{t+k-1}(x_{t+k}). \tag{8}$$

Thus, the exchange rate at time t depends on the fundamental at time t, the average expectation at t of the fundamental at time $t + 1$, the average expectation at t of the average expectation at $t + 1$ of the fundamental at $t + 2$, etc. The law of iterated expectations does not apply to average expectations. For example, $\bar{E}_t\bar{E}_{t+1}(s_{t+2}) \neq \bar{E}_t(s_{t+2})$.[16] This is a basic feature of asset pricing under heterogeneous expectations: the expectation of other investors' expectations matters.[17] In a dynamic system, this leads to the infinite regress

[15] Here we implicitly assume that s_{t+1} is normally distributed. We will see in Section ID that the equilibrium exchange rate indeed has a normal distribution.

[16] See Bacchetta and van Wincoop (2004a) and Allen et al. (forthcoming).

[17] Notice that the higher-order expectations are of a dynamic nature, i.e., today's expectations of tomorrow's expectations. This contrasts with most of the literature that considers higher-order expectations in a static context with strategic externalities, e.g., Stephen Morris and Hyun Song Shin (2002) or Woodford (2003).

problem, as analyzed in Townsend (1983): as the horizon goes to infinity, the dimensionality of the expectation term goes to infinity.

C. *The Information Structure*

We assume that at time t investors observe all past and current f_t, while they receive private signals about $f_{t+1}, \ldots, f_{t+T}$. More precisely, we assume that investors receive one signal each period about the observable fundamental T periods ahead. For example, at time t investor i receives a signal

$$(9) \quad v_t^i = f_{t+T} + \varepsilon_t^{vi}, \qquad \varepsilon_t^{vi} \sim \mathcal{N}(0, \sigma_v^2),$$

where ε_t^{vi} is independent from f_{t+T} and other agents' signals.[18] As usual in this context, we assume that the average signal received by investors is f_{t+T}, i.e., $\int_0^1 v_t^i \, di = f_{t+T}$.[19]

We also assume that the observable fundamental's process is known by all agents and consider a general process:

$$(10) \quad f_t = D(L)\varepsilon_t^f, \qquad \varepsilon_t^f \sim \mathcal{N}(0, \sigma_f^2),$$

where $D(L) = d_1 + d_2L + d_3L + \cdots$ and L is the lag operator. Since investors observe current and lagged values of the fundamental, knowing the process provides information about the fundamental at future dates.

D. *Solution Method*

In order to solve the equilibrium exchange rate, there is no need to compute all the higher-order expectations it depends on. The key equation used in the solution method is the interest rate arbitrage condition (6), which captures foreign exchange market equilibrium. It involves only a first-order average market expectation. We adopt a method of undetermined coefficients, conjecturing an equilibrium exchange rate equation and then verifying that it satisfies the equilibrium condition (6). Townsend (1983) adopts a similar method for solving a business cycle model with higher-order expectations.[20] Here we provide a brief description of the solution method, leaving details to Appendix B.

We conjecture the following equilibrium exchange rate equation that depends on shocks to observable and unobservable fundamentals:

$$(11) \qquad s_t = A(L)\varepsilon_{t+T}^f + B(L)\varepsilon_t^b,$$

where $A(L)$ and $B(L)$ are infinite order polynomials in the lag operator L. The errors ε_t^{iv} do not enter the exchange rate equation as they average to zero across investors. Since at time t investors observe the fundamental f_t, only the innovations ε^f between $t + 1$ and $t + T$ are unknown. Similarly shocks ε^b between $t - T$ and t are unknown. Exchange rates at time $t - T$ and earlier, together with knowledge of ε^f at time t and earlier, reveal the shocks ε^b at time $t - T$ and earlier.[21]

Investors solve a signal extraction problem for the finite number of unknown innovations. Both private signals and exchange rates from time $t - T + 1$ to t provide information about the unknown innovations. The solution to the signal extraction problem leads to expectations at time t of the unknowns as a function of observables, which in turn can be written as a function of the innovations themselves. One can then compute the average expectation of s_{t+1}. Substituting the result into the interest rate arbitrage condition (6) leads to a new exchange rate equation. The coefficients of the polynomials $A(L)$ and $B(L)$ can then be derived by solving a fixed point problem, equating the coefficients of the conjectured exchange rate equation to those in the equilibrium exchange rate equation. Although the lag polynomials are of infinite

[18] This implies that, each period, investors have T signals that are informative about future observed fundamentals. Note that the analysis could be easily extended to the case where investors receive a vector of signals each period.

[19] See Anat R. Admati (1985) for a discussion.

[20] The solution method described in Townsend (1983) applies to the model in section 8 of that paper where the economy-wide average price is observed with noise. Townsend (1983) mistakenly believed that higher-order expectations are also relevant in a two-sector version of the model where firms observe each other's prices without noise. Joseph G. Pearlman and Sargent (2005) show that the equilibrium fully reveals private information in that case.

[21] Here we implicitly assume that the $B(L)$ polynomial is invertible, which is the case when the roots of $B(L) = 0$ are outside the unit circle. This assumption holds for all the parameterizations of the model considered below. See Appendix B.3 for a discussion.

order, for lags longer than T periods the information dispersion plays no role and an analytical solution to the coefficients is feasible.[22]

A couple of comments about multiplicity of equilibria are in order. Models with heterogeneous information do not necessarily lead to multiple equilibria.[23] Multiple equilibria can arise when the conditional variance of next period's asset price is endogenous, as shown by Stephen McCafferty and Robert Driskill (1980). But that applies to both common knowledge and heterogeneous information models. In the context of our model, the intuition is that a higher conditional variance of next period's exchange rate leads to a bigger impact of hedge trades on the exchange rate through the risk-premium channel, which indeed justifies the higher conditional variance. For the special case $T = 1$ we discuss below, analytical results can be obtained. It is easy to check in that case that for a given σ^2 there is a unique solution to the exchange rate equation. But when allowing for the endogeneity of σ^2 we find that there are always two equilibria, a low and a high σ^2 equilibrium.[24] For the more general case where $T > 1$, we confirm numerically that there are two equilibria.[25] In Bacchetta and van Wincoop (2003) we show that the high variance equilibrium is unstable. Our numerical analysis in the paper, therefore, always focuses on the low variance equilibrium.

E. *Order Flow*

Evans and Lyons (2002b) define order flow as "the net of buyer-initiated and seller-initiated orders." While each transaction involves a buyer and a seller, the sign of the transaction is determined by the initiator of the transaction. The initiator of a transaction is the trader (either buyer or seller) who acts based on new private information. In our setup, this includes both private information about the future fundamental and private information that leads to hedge trades. The passive side of trade varies across models. In a quote-driven dealer market, such as modeled by Evans and Lyons (2002b), the quoting dealer is on the passive side. The foreign exchange market has traditionally been characterized as a quote-driven multi-dealer market, but the recent increase in electronic trading (e.g., EBS) implies that a majority of interbank trade is done through an auction market. In that case the limit orders are the passive side of transactions and provide liquidity to the market. The initiated orders are referred to as market orders that are confronted with the passive outstanding limit order book.

In the standard noisy rational expectations literature, the order flow plays no role, while the asset price conveys information. But how can the price convey information when the price is unknown at the time asset demand orders are placed? This is possible only when investors submit demand *functions* that are conditional on the price. One can think of those demand functions being submitted to an implicit auctioneer, who then finds the equilibrium price.

There is an alternative interpretation, however, of how the equilibrium price is set in such models, which connects more closely to the explicit auction market nature of the present foreign exchange market. Investors submit their demand functions for foreign bonds in two components, market orders (order flow) and limit orders. Limit orders depend on available public information and are conditioned on the exchange rate itself. These are passive orders that are executed only when confronted with market orders. Market orders are associated with the private information component of asset demand.[26]

[22] In Bacchetta and van Wincoop (2003), we solve the model for the case where investors have infinite horizons. The solution is then complicated by the fact that investors also need to hedge against changes in expected future returns. This hedge term depends on the infinite state space, which is truncated to obtain an approximate solution. Numerical results are almost identical to the case of overlapping generations.

[23] Peter DeMarzo and Costis Skiadas (1998) show that the well-known heterogeneous information model of Sanford J. Grossman (1976) has a unique equilibrium.

[24] A technical appendix that is available from the authors on request proves these points for $T = 1$.

[25] We check this by searching over a very wide space of possible σ^2. There is an equilibrium only when the conjectured σ^2 is equal to the conditional variance implied by the resulting exchange rate equation.

[26] One way to formalize this separation into limit and market orders is to introduce foreign exchange dealers to whom investors delegate price discovery. Dealers are simply a veil, passing on customer orders to the interdealer market, where price discovery takes place. Customers submit their demand functions to dealers through a combination of limit and market orders. Dealers can place both types of

To be more precise, let I_t^i be the private information set available to agent i at time t, and I_t^p the public information set available to all investors at the time market orders are submitted. The exchange rate s_t is not part of the information set at the time orders are placed, but investors can submit limit orders that are conditional on the exchange rate. After computing the expected exchange rate next period as a function of the information set and of s_t, it is easy to show that there are parameters α_1, α_2, and α_3 such that the demand for foreign bonds can be rewritten as

$$b_{Ft}^i = \alpha_1 I_t^p + \alpha_2 s_t + \alpha_3 I_t^i. \tag{12}$$

Market orders are defined as the pure private information component of asset demand, which is equal to

$$\Delta x_t^i = \alpha_3 I_t^i - E(\alpha_3 I_t^i | I_t^p). \tag{13}$$

Note that we do not condition on the exchange rate s_t since it is not known at the time the market orders are placed; only limit orders can be conditioned on the exchange rate. Limit orders consist of the remaining component of asset demand, which depends on the exchange rate and public information. Defining $E(\alpha_3 I_t^i | I_t^p) = \alpha_4 I_t^p$, limit orders are

$$(\alpha_1 + \alpha_4) I_t^p + \alpha_2 s_t. \tag{14}$$

The aggregate order flow is $\Delta x_t = \int_0^1 \Delta x_t^i \, di$. Imposing market equilibrium $\int_0^1 b_{Ft}^i \, di = 0$, which is equivalent to the sum of aggregate order flow and limit orders being zero, the equilibrium exchange rate is

$$s_t = -\frac{1}{\alpha_2}(\alpha_1 + \alpha_2) I_t^p - \frac{1}{\alpha_2} \Delta x_t. \tag{15}$$

orders in the interdealer electronic auction market, but need to place the limit orders before customer orders are known. If we introduce an infinitesimal trading cost in the interdealer market that is proportional to the volume of executed trades, dealers will submit limit orders that are equal to the expected customer orders based on public information. The unexpected customer orders are associated with private information and are submitted as market orders to the interdealer market. This formalization also connects well to the existing data, which is for interdealer order flow.

When demand shifts are due only to public information arrival, the order flow term is zero and executed limit orders will be zero as well. A shift in demand can therefore bring about a change in the exchange rate without any actual trade. Only shifts in demand due to private information lead to trade.

Since s_{t-1} is part of I_t^p, it follows that there are parameters η_1 and η_2 such that

$$\Delta s_t = \eta_1 I_t^p + \eta_2 \Delta x_t. \tag{16}$$

Equation (16) is important. It breaks down changes in exchange rates associated with public information (the first term) and private information (the second term). The two terms are orthogonal since order flow is defined to be orthogonal to public information. This also implies that a regression of the change in the exchange rate on order flow will lead to an unbiased estimate of η_2 and an unbiased measure of the contribution of order flow to exchange rate volatility. There is no simultaneity bias in such a regression. Causality runs from quantity (order flow) to price (the exchange rate), not the other way around. Order flow decisions are made before the equilibrium exchange rate is known. This differs from the implicit auctioneer interpretation, where quantities and prices are set simultaneously by the auctioneer. We want to emphasize though that the equilibrium exchange rate is the same under these two interpretations of price setting. The explicit auction market interpretation simply has the advantage to connect more closely to existing institutions and to evidence on the relationship between order flow and exchange rate.

II. Model Implications: A Special Case

In this section we examine the special case where $T = 1$, which has a relatively simple solution. This example is used to illustrate how information heterogeneity disconnects the exchange rate from observed macroeconomic fundamentals, while establishing a close relationship between the exchange rate and order flow.

One aspect that simplifies the solution for $T = 1$ is that higher-order expectations are the same as first-order expectations. This can be seen as follows. Bacchetta and van Wincoop (2004a) show that higher-order expectations are equal to first-order expectations plus average expectations of future market expectational errors. For example, the second-order expectation of f_{t+2} can be written as $\bar{E}_t^2 f_{t+2} = \bar{E}_t f_{t+2} + \bar{E}_t(\bar{E}_{t+1} f_{t+2} - f_{t+2})$. When $T = 1$, investors do not expect the market to make expectational errors in the next period. An investor may believe at time t that he has different private information about f_{t+1} than others. That information is no longer relevant next period, however, since f_{t+1} is observed at $t + 1$.[27]

While not critical, we make the further simplifying assumptions in this section that b_t and f_t are i.i.d., i.e., $\rho_b = 0$ and $f_t = \varepsilon_t^f$. Replacing higher-order with first-order expectations, equation (7) then becomes

$$s_t = \frac{1}{1+\alpha}\left[f_t + \frac{\alpha}{1+\alpha}\bar{E}_t f_{t+1}\right] - \frac{\alpha}{1+\alpha}\gamma\sigma^2 b_t. \tag{17}$$

Only the average expectation of f_{t+1} appears. We have replaced σ_t^2 with σ^2 since we will focus on the stochastic steady state where second-order moments are time-invariant.

A. *Solving the Model with Heterogenous Information*

When $T = 1$, investors receive private signals v_t^i about f_{t+1}, as in (9). Therefore, the average expectation $\bar{E}_t f_{t+1}$ in (17) depends on the average of private signals, which is equal to f_{t+1} itself. This implies that the exchange rate s_t depends on f_{t+1}, so that the exchange rate becomes itself a source of information about f_{t+1}. The exchange rate is not fully revealing, however, as it also depends on unobserved aggregate hedge trades b_t. To determine the information signal about f_{t+1} provided by the exchange rate, we need to know the equilibrium exchange rate equation. We conjecture that

$$s_t = \frac{1}{1+\alpha} f_t + \lambda_f f_{t+1} + \lambda_b b_t. \tag{18}$$

Since an investor observes f_t, the signal he gets from the exchange rate can be written

$$\frac{\tilde{s}_t}{\lambda_f} = f_{t+1} + \frac{\lambda_b}{\lambda_f} b_t, \tag{19}$$

where $\tilde{s}_t = s_t - (1/(1+\alpha))f_t$ is the "adjusted" exchange rate. The variance of the error of this signal is $(\lambda_b/\lambda_f)^2\sigma_b^2$. Consequently, investor i infers $E_t^i f_{t+1}$ from three sources of information: (a) the distribution of f_{t+1}; (b) the signal v_t^i; and (c) the adjusted exchange rate (i.e., (19)). Since errors in each of these signals have a normal distribution, the projection theorem implies that $E_t^i f_{t+1}$ is given by a weighted average of the three signals, with the weights determined by the precision of each signal. We have

$$E_t^i f_{t+1} = \frac{\beta^v v_t^i + \beta^s \tilde{s}_t/\lambda_f}{D}, \tag{20}$$

where $\beta^v = 1/\sigma_v^2$, $\beta^s = 1/(\lambda_b/\lambda_f)^2\sigma_b^2$, $\beta^f = 1/\sigma_f^2$, and $D = 1/var(f_{t+1}) = \beta^v + \beta^f + \beta^s$. For the exchange rate signal, the precision is complex and depends both on σ_b^2 and λ_b/λ_f, the latter being endogenous. By substituting (20) into (17) and using the fact that $\int_0^1 v_t^i\, di = f_{t+1}$ in computing $\bar{E}_t f_{t+1}$, we get:

$$s_t = \frac{1}{1+\alpha} f_t + z\frac{\alpha}{(1+\alpha)^2}\frac{\beta^v}{D} f_{t+1} - z\frac{\alpha}{1+\alpha}\gamma\sigma^2 b_t, \tag{21}$$

where $z = 1/(1 - (\alpha/(1+\alpha)^2)(\beta^s/\lambda_f D)) > 1$. Equation (21) confirms the conjecture (18). Equating the coefficients on f_{t+1} and b_t in (21) to λ_f and λ_b, respectively, yields implicit solutions to these parameters.

We will call z the *magnification factor:* the equilibrium coefficient of b_t in (21) is the direct effect of b_t in (17) multiplied by z. This magnification can be explained by rational confusion. When the exchange rate changes, investors do not know whether this is driven by hedge trades or by information about future macroeconomic fundamentals by other investors. Therefore, they always revise their expectations of fundamentals when the exchange rate changes (equation (20)). This rational confusion magnifies the impact of the unobserved hedge trades on the exchange rate. More specifically, from (17) and (20), we can see that a change in b_t has two effects on s_t. First, it affects s_t directly in (17) through the risk-premium channel. Second,

[27] See Bacchetta and van Wincoop (2004a) for a more detailed discussion of this point.

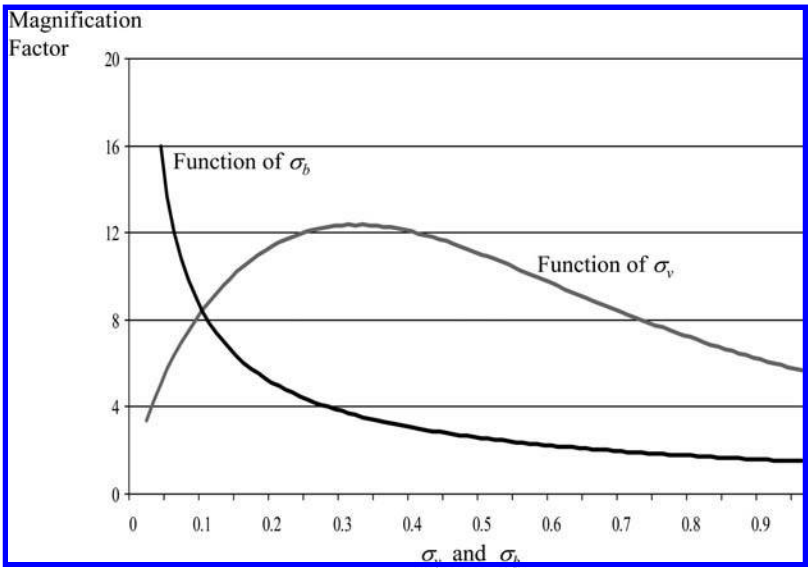

FIGURE 1. MAGNIFICATION FACTOR IN MODEL WITH $T = 1$

Notes: This figure is based on the simulation of the model for $T = 1$, with both b_t and f_t i.i.d. The qualitative results do not depend on other model parameters. We set $\alpha = 10$, $\gamma = 50$, and all standard deviations of the shocks equal to 0.1, unless varied within the figure.

this direct effect is magnified by an increase in $\bar{E}_t f_{t+1}$ from (20).

The magnification factor can be written as[28]

$$z = 1 + \frac{\beta^s}{\beta^v}. \tag{22}$$

The magnification factor, therefore, depends on the precision of the exchange rate signal relative to the precision of the private signal. The better the quality of the exchange rate signal, the more weight is given to the exchange rate in forming expectations of f_{t+1}, and therefore the larger the magnification of the unobserved hedge trades.

Figure 1 shows the impact of two key parameters on magnification. A rise in the private signal variance σ_v^2 at first raises magnification and then lowers it. Two opposite forces are at work. First, an increase in σ_v^2 reduces the precision β^v of the private signal. Investors therefore give more weight to the exchange rate signal, which enhances the magnification factor. Second, a rise in σ_v^2 implies less information about next period's fundamental and therefore a lower weight of f_{t+1} in the exchange rate. This reduces the precision β^s of the exchange rate signal, which reduces the magnification factor. For large enough σ_v^2 this second factor dominates. The magnification factor is therefore largest for intermediate values of the quality of private signals. Figure 1 also shows that a higher variance σ_b^2 of hedging shocks always reduces magnification. It reduces the precision β^s of the exchange rate signal.

B. *Disconnect from Observed Fundamentals*

In order to precisely identify the channels through which information heterogeneity disconnects the exchange rate from observed fundamentals, we now compare the model to a benchmark with identically informed investors. The benchmark we consider is the case where investors receive the same signal on future f_t's, i.e., they have incomplete but common knowledge on future fundamentals. With common knowledge, all investors receive the signal

$$v_t = f_{t+T} + \varepsilon_t^v, \qquad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_{v,c}^2), \tag{23}$$

where ε_t^v is independent of f_{t+T}.

[28] Substitute $\lambda_f = z(\alpha/(1+\alpha)^2)(\beta^v/D)$ into $z = 1/(1 - (\alpha/(1+\alpha)^2)(\beta^s/\lambda_f D))$ and solve for z.

Defining the precision of this signal as $\beta^{v,c} \equiv 1/\sigma^2_{v,c}$, the conditional expectation of f_{t+1} is

$$E^i_t f_{t+1} = \bar{E}_t f_{t+1} = \frac{\beta^{v,c} v_t}{d}, \tag{24}$$

where $d \equiv 1/var_t(f_{t+1}) = \beta^{v,c} + \beta^f$. Substitution into (17) yields the equilibrium exchange rate:

$$s_t = \frac{1}{1+\alpha} f_t + \lambda_v v_t + \lambda^c_b b_t, \tag{25}$$

where $\lambda_v = (\alpha/(1+\alpha)^2)\beta^{v,c}/d$, and $\lambda^c_b = -(\alpha/(1+\alpha))\gamma\sigma^2_c$. Here, σ^2_c is the conditional variance of next period's exchange rate in the common knowledge model. In this case the exchange rate is fully revealing, since by observing s_t investors can perfectly deduce b_t. Thus, λ^c_b is equal to the direct risk-premium effect of b_t given in (17).

We can now compare the connection between the exchange rate and observed fundamentals in the two models. In the heterogeneous information model, the observed fundamental is f_t, while in the common knowledge model it also includes v_t. We compare the R^2 of a regression of the exchange rate on observed fundamentals in the two models. From (18), the R^2 in the heterogeneous information model is defined by

$$\frac{R^2}{1-R^2} = \frac{\frac{1}{(1+\alpha)^2}\sigma^2_f}{\lambda^2_f \sigma^2_f + z^2\left(\frac{\alpha}{1+\alpha}\right)^2 \gamma^2\sigma^4\sigma^2_b}. \tag{26}$$

From (25) the R^2 in the common knowledge model is defined by

$$\frac{R^2}{1-R^2} = \frac{\frac{1}{(1+\alpha)^2}\sigma^2_f + \lambda^2_v(\sigma^2_f + \sigma^2_{v,c})}{\left(\frac{\alpha}{1+\alpha}\right)^2 \gamma^2\sigma^4_c\sigma^2_b}. \tag{27}$$

If the conditional variance of the exchange rate is the same in both models, the R^2 is clearly lower in the heterogeneous information model. Two factors contribute to this. First, the contribution of unobserved fundamentals to exchange rate volatility is amplified, as measured by the magnification factor z in the denominator of (26). Second, the average signal in the heterogeneous information model, which is equal to the future fundamental, is unobserved and therefore contributes to reducing the R^2. It also appears in the denominator of (26). In contrast, the signal about future fundamentals is observed in the common knowledge model, and therefore contributes to raising the R^2. The variance of this signal, $\sigma^2_f + \sigma^2_{v,c}$, appears in the numerator of (27). The conditional variance of the exchange rate also contributes to the R^2. It can be higher in either model, dependent on assumptions about parameter values and quality of the public and private signals.[29]

C. *Order Flow*

It is straightforward to implement for this special case the general definition of order flow and limit orders discussed in Section I. Using (4), (18), and (20), we can write demand for foreign bonds as

$$b^i_{F,t} = \frac{1+\alpha}{\alpha\gamma\sigma^2 z}\left(\frac{1}{1+\alpha} f_t - s_t\right) + \frac{\beta^v}{(1+\alpha)\gamma\sigma^2 D} v^i_t - b^i_t. \tag{28}$$

Limit orders are captured by the first term, while order flow is captured by the sum of the last two terms. Note that the variables v^i_t and b^i_t in the private information set are unpredictable by public information at the time market orders are placed.[30]

Aggregate order flow is then

$$\Delta x_t = \frac{\beta^v}{(1+\alpha)\gamma\sigma^2 D} f_{t+1} - b_t. \tag{29}$$

[29] While we focus here on the exchange rate determination puzzle, which is about the disconnect between exchange rates and *observed* fundamentals, it is easy to show that in the heterogeneous information model the exchange rate is more disconnected from fundamentals "f" generally (both observed and future fundamentals) than in the common knowledge model. In that case the term $\lambda_f\sigma^2_f$ moves from the denominator to the numerator of (26). When the conditional variance of next period's exchange rate is the same in both models, the R^2 remains lower in the heterogeneous information model due to the amplification of unobserved hedge trades.

[30] In terms of the notation introduced in Section I, $E(I^i_t | I^p_t) = 0$.

Taking the aggregate of (28), imposing market equilibrium, we get

$$s_t = \frac{1}{1+\alpha} f_t + z \frac{\alpha}{1+\alpha} \gamma\sigma^2 \Delta x_t. \tag{30}$$

Equation (30) shows that the exchange rate is related in a simple way to a commonly observed fundamental and order flow. The order flow term captures the extent to which the exchange rate changes due to the aggregation of private information. The impact of order flow is larger the bigger the magnification factor z. A higher level of z implies that the order flow is more informative about the future fundamental.

It is easily verified that in the common knowledge model

$$s_t = \frac{1}{1+\alpha} f_t + \lambda_v v_t + \frac{\alpha}{1+\alpha} \gamma\sigma^2 \Delta x_t. \tag{31}$$

In that case order flow is driven only by hedge trades.[31] Since these trades have no information content about future fundamentals, the impact of order flow on the exchange rate is smaller (not multiplied by the magnification factor z). A comparison between (30) and (31) clearly shows that the exchange rate is more closely connected to order flow in the heterogeneous information model and more closely connected to public information in the common knowledge model.

III. Model Implications: Dynamics

In this section, we examine the more complex dynamic properties of the model when $T > 1$. There are two important implications. First, it creates endogenous persistence of the impact of nonobservable shocks on the exchange rate. Second, higher-order expectations differ from first-order expectations when $T > 1$. Even for $T = 2$ expectations of infinite order affect the exchange rate. We show that higher-order expectations tend to increase the magnification effect, but have an ambiguous impact on the disconnect. We now examine these two aspects in turn.

A. *Persistence*

When $T > 1$, even transitory nonobservable shocks have a persistent effect on the exchange rate. This is due to the learning of investors who gradually realize that the change in the exchange is not caused by a shock to future fundamentals.[32] The exchange rate at time t depends on future fundamentals $f_{t+1}, f_{t+2}, \dots, f_{t+T}$, and therefore provides information about each of these future fundamentals. A transitory unobservable shock to b_t affects the exchange rate at time t and therefore affects the expectations of all future fundamentals up to time $t + T$. This rational confusion will last for T periods, until the final one of these fundamentals, f_{t+T}, is observed. Until that time, investors will continue to give weight to s_t in forming their expectations of future fundamentals, so that b_t continues to affect the exchange rate.[33] As investors gradually learn more about $f_{t+1}, f_{t+2}, \dots, f_{t+T}$, both by observing them and through new private signals and exchange rate signals, the impact on the exchange rate of the shock to b_t gradually dissipates.

The persistence of the impact of b-shocks on the exchange rate is also affected by the persistence of the shock itself. When the b-shock itself becomes more persistent, it is more difficult for investors to learn about fundamentals up to time $t + T$ from exchange rates subsequent to time t. The rational confusion is therefore more persistent, and so is the impact of b-shocks on the exchange rate.

B. *Higher-Order Expectations*

The topic of higher-order expectations is a difficult one, but it has potentially important implications for asset pricing. Since a detailed analysis falls outside the scope of this paper, we

[31] Note that aggregate hedge trade b_t is not in the public information set at the time orders are submitted. It is revealed only after the price is known.

[32] Persistence can also arise in models with incomplete but common knowledge, such as Michael Mussa (1976). When agents do not know whether an increase in an observed fundamental is transitory or persistent, a transitory shock will have a larger and more persistent effect because of gradual learning.

[33] This result is related to findings by David P. Brown and Robert H. Jennings (1989) and Bruce D. Grundy and Maureen McNichols (1989), who show in the context of two-period noisy rational expectations models that the asset price in the second period is affected by the asset price in the first period.

limit ourselves to a brief discussion regarding the impact of higher-order expectations on the connection between the exchange rate and observed fundamentals. We apply the results of Bacchetta and van Wincoop (2004a), where we provide a general analysis of the impact of higher-order expectations on asset prices.[34] We still assume that $\rho_b = 0$.

Let $\bar{s}_t$ denote the exchange rate that would prevail if the higher-order expectations in (7) are replaced by first-order expectations.[35] In Bacchetta and van Wincoop (2004a), we show that the present value of the difference between higher- and first-order expectations depends on average first-order expectational errors about average private signals. In Appendix C we show that in our context this leads to

$$s_t = \bar{s}_t + \frac{1}{1+\alpha} \sum_{k=2}^{T} \pi_k (\bar{E}_t f_{t+k} - f_{t+k}). \tag{32}$$

The parameters π_k are defined in the Appendix and are positive in all numerical applications. Higher-order expectations therefore introduce a new asset price component, which depends on average first-order expectational errors about future fundamentals.

Moreover, the expectational errors $\bar{E}_t f_{t+k} - f_{t+k}$ depend on errors in public signals, i.e., observed fundamentals and exchange rates; based on private information alone, these average expectational errors would be zero. There are two types of errors in public signals. First, there are errors in the exchange rate signals that are caused by the unobserved hedge trades at time t and earlier. This implies that unobserved hedge trades receive a larger weight in the equilibrium exchange rate. The other type of errors in public signals are errors in the signals based on the process of f_t. These errors depend negatively on future innovations in the fundamental, which implies that the exchange rate depends less on unobserved future fundamentals. To summarize, hedge shocks are further magnified by the presence of higher-order expectations, while the overall impact on the connection between the exchange rate and observed fundamentals is ambiguous.[36]

Table 1—Benchmark Parameterization

Parameter	Value
σ_f	0.01
σ_b	0.01
σ_v	0.08
ρ_b	0.8
α	10
γ	500
T	8

IV. Model Implications: Numerical Analysis

We now solve the model numerically to illustrate the various implications of the model discussed above. We first consider a benchmark parameterization and then discuss the sensitivity of the results to changing some key parameters.

A. *A Benchmark Parameterization*

The parameters of the benchmark case are reported in Table 1. We assume that the observable fundamental f follows a random walk, whose innovations have a standard deviation of $\sigma_f = 0.01$. We assume a high standard deviation of the private signal error of $\sigma_v = 0.08$. The unobservable fundamental b follows an AR process with autoregressive coefficient of $\rho_b = 0.8$ and a standard deviation $\sigma_b = 0.01$ of innovations. Although we have made assumptions about both σ_b and risk-aversion γ, they enter multiplicatively in the model, so only their product matters. Finally, we assume $T = 8$, so that agents obtain private signals about fundamentals eight periods before they are realized.

Figure 2 shows some of the key results from the benchmark parameterization. Panels A and B show the dynamic impact on the exchange rate in response to one-standard-deviation shocks in the private and common-knowledge models. In the heterogeneous-agent model, there are two shocks: a shock ε^f_{t+T} (f-shock),

[34] Allen et al. (forthcoming) provide an insightful analysis of higher-order expectations with an asset price, but they do not consider an infinite horizon model.

[35] That is, $\bar{s}_t = (1/(1+\alpha)) \sum_{k=0}^{\infty} (\alpha/(1+\alpha))^k \bar{E}_t(f_{t+k} - \alpha\gamma\sigma^2_{t+k} b_{t+k})$.

[36] In Bacchetta and van Wincoop (2004a), we show that the main impact of higher-order expectations is to disconnect the price from the present value of future observable fundamentals.

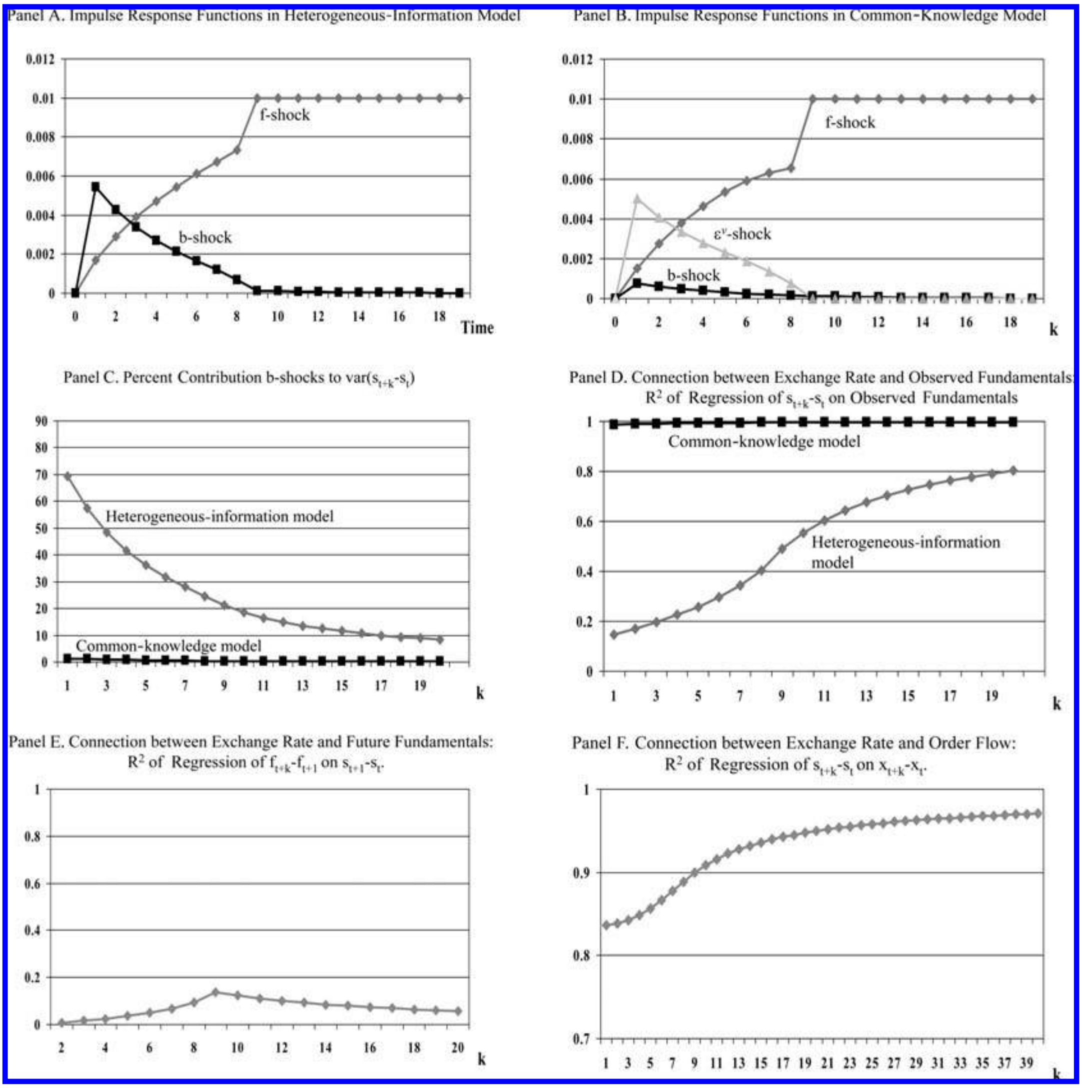

FIGURE 2. RESULTS FOR THE BENCHMARK PARAMETERIZATION

Note: See Table 1 for parameter assumptions.

which first affects the exchange rate at time t, and a shock ε_t^b (b-shock). In the common-knowledge model there are also shocks ε_t^v, which affect the exchange rate through the commonly observable fundamental v_t. In order to facilitate comparison, we set the precision of the public signal such that the conditional variance of next period's exchange rate is the same as in the heterogeneous-information model. This implies that the unobservable hedge trades have the same risk-premium effect in the two models. We will show below that our key results do not depend on the assumed precision of the public signal.

Magnification.—The magnification factor in the benchmark parameterization turns out to be substantial: 7.2. This is visualized in Figure 2 by comparing the instantaneous response of the exchange rate to the b-shocks in the two models in panels A and B. The only reason the impact of a b-shock is so much bigger in the heterogeneous information model is the magnification factor associated with information dispersion.

Persistence.—We can see from panel A that after the initial shock the impact of the b-shocks dies down almost as a linear function of time. The half-life of the impact of the shock is three periods. After eight periods the rational confusion is resolved and the impact is the same as in the public information model, which is close to zero.

The meaning of a three-period half-life depends of course on what we mean by a period in the model. What is critical is not the length of a period, but the length of time it takes for uncertainty about future macro variables to be resolved. For example, assume that this length of time is eight months. If a period in our model is a month, then $T = 8$. If a period is three days, then $T = 80$. We find that the half-life of the impact of the unobservable hedge shocks on the exchange rate that can be generated by the model remains virtually unchanged as we change the length of a period. For $T = 8$, the half-life is about three, while for $T = 80$ it is about 30.[37] In both cases the half-life is three months. Persistence is therefore driven critically by the length of time it takes for uncertainty to resolve itself. Deviations of the exchange rate from observed fundamentals can therefore be very long-lasting when it takes a long time before expectations about future fundamentals can be validated, such as expectations about the long-term technology growth rate of the economy.

Exchange Rate Disconnect in the Short and the Long Run.—Panel C reports the contribution of unobserved hedge trades to the variance of $s_{t+k} - s_t$ at different horizons. In the heterogeneous information model, 70 percent of the variance of a one-period change in the exchange rate is driven by the unobservable hedge trades, while in the common-knowledge model it is a negligible 1.3 percent (such a small effect is typical of standard portfolio-balance models). While in the short run unobservable fundamentals dominate exchange rate volatility, in the long run observable fundamentals dominate. For example, the contribution of hedge trades to the variance of exchange rate changes over a ten-period interval is less than 20 percent. As seen in panel A, the impact of hedge trades on the exchange rate gradually dies down as rational confusion dissipates over time.

In order to determine the relationship between exchange rates and observed fundamentals, panel D reports the R^2 of a regression of $s_{t+k} - s_t$ on all current and lagged *observed* fundamentals. In the heterogeneous-information model, this includes all one-period changes in the fundamental f that are known at time $t + k$: $f_{t+s} - f_{t+s-1}$, for $s \leq k$. In the common-knowledge model, it also includes the corresponding one-period changes in the public signal v. The R^2 is close to one for all horizons in the common-knowledge model, while it is much lower in the heterogeneous-information model. At the one-period horizon it is only 0.14; it then rises as the horizon increases to 0.8 for a 20-period horizon. This is consistent with extensive findings that macroeconomic fundamentals have weak explanatory power for exchange rates in the short to medium run, starting with Meese and Rogoff (1983), and findings of a closer relationship over longer horizons.[38]

Two factors account for the results in panel D. The first is that the relative contribution of unobservable hedge shocks to exchange rate volatility is large in the short run and small in the long run, as illustrated in panel C. The second factor is that, through private signals, the exchange rate at time t is also affected by innovations $\varepsilon^f_{t+1}, \ldots, \varepsilon^f_{t+T}$ in future fundamentals that are not yet observed today. In the long run these become observable, again contributing to a closer relationship between the exchange rate and observed fundamentals in the long run.

Exchange Rate and Future Fundamentals.—Recently Engel and Kenneth D. West (2005) and Froot and Ramadorai (2005) have reported

[37] When we change the length of a period, we also need to change other model parameters, such as the standard deviations of the shocks. In doing so we restrict parameters such that (a) the contribution of b-shocks to $var(s_{t+1} - s_t)$ is the same as in the benchmark parameterization, and (b) the impact of b-shocks on exchange rate volatility remains largely driven by information dispersion (large magnification factor). For example, when we change the benchmark parameterization such that $T = 80$, $\sigma_v = 0.26$, $\sigma_f = 0.0016$, and $\alpha = 44$, the half-life is 28 periods. The magnification factor is 48.

[38] See MacDonald and Mark P. Taylor (1993), Menzie D. Chinn and Meese (1995), Mark (1995), Mark and Donggyu Sul (2001), Froot and Ramadorai (2005), and Gourinchas and Hélène Rey (2005).

evidence that exchange rate changes predict future fundamentals, but only weakly so. Our model is consistent with these findings. Panel E of Figure 2 reports the R^2 of a regression $f_{t+k} - f_{t+1}$ on $s_{t+1} - s_t$ for $k \geq 2$. The R^2 is positive, but is never above 0.14. The exchange rate is affected by the private signals of future fundamentals, which aggregate to the future fundamentals. Most of the short-run volatility of exchange rates, however, is associated with unobservable hedge trades, which do not predict future fundamentals.

Exchange Rate and Order Flow.—Order flow is computed as discussed in Section IE. Appendix D discusses further details for the case where the fundamental f is a random walk. With x_t defined as cumulative order flow, panel F reports the R^2 of a regression of $s_{t+k} - s_t$ on $x_{t+k} - x_t$. The R^2 is large. At a one-period horizon it is 0.84, so that 84 percent of the variance of one-period exchange rate changes can be accounted for by order flow as opposed to public information. The relationship between cumulative order flow and exchange rates gets even stronger as the horizon k increases, with the R^2 rising to 0.97 for $k = 40$. As k approaches infinity, the R^2 approaches a level near 0.99, so that there is a very close long-run relationship between cumulative order flow and exchange rates.[39]

It is important to point out that the close relationship between the exchange rate and order flow in the long run is not inconsistent with the close relationship between the exchange rate and observed fundamentals in the long run. When the exchange rate rises due to private information about permanently higher future fundamentals, the information reaches the market through order flow. Eventually the future fundamentals will be observed, so that there is a link between the exchange rate and the observed fundamentals. But most of the information about higher future fundamentals is aggregated into the price through order flow. Order flow associated with information about future fundamentals has a permanent effect on the exchange rate.

Our results can be compared to similar regressions that have been conducted based on the data. Evans and Lyons (2002b) estimate regressions of one-day exchange rate changes on daily order flow. They find an R^2 of 0.63 and 0.40 for, respectively, the DM-\$ and the yen-\$ exchange rate, based on four months of daily data in 1996. Evans and Lyons (2002a) report results for nine currencies. They point out that exchange rate changes for any currency pair can also be affected by order flow for other currency pairs. Regressing exchange rate changes on order flow for all currency pairs, they find an average R^2 of 0.67 for their nine currencies.

The pictures for the exchange rate and cumulative order flow reported in Evans and Lyons (2002b) for the DM-\$ and yen-\$ exchange rate suggest that the link is even stronger over horizons longer than one day, although their dataset is too short for formal regression analysis. These pictures look very similar to their theoretical counterparts, which are reported in Figure 3 for four simulations of the model over 40 periods.[40] The simulations confirm a close link between the exchange rate and cumulative order flow at both short and long horizons.

While not reported in panel F, the R^2 of regressions of exchange rate changes on order flow in the public-information model is close to zero. Two factors contribute to the much closer link between order flow and exchange rates in the heterogeneous-information model. First, in the heterogeneous-information model both private information about future fundamentals and hedge trades contribute to order flow, while in the public-information model only hedge trades contribute to order flow. Second, the impact on the exchange rate of the order flow due to hedge trades is much larger in the heterogeneous information model. The reason is that order flow is informative about future fundamentals in the

[39] The relationship between $s_{t+k} - s_t$ and $x_{t+k} - x_t$ does not always get stronger for longer horizons. For low values of T, the R^2 declines with k and then converges asymptotically to a positive level. Appendix D shows that cumulative order flow and exchange rates are not cointegrated, which explains why the R^2 never approaches one as k approaches infinity. The Appendix shows that there is a cointegrating relationship between s_t, x_t, and $\hat{b}_t = \sum_{s=0}^{\infty} \varepsilon^b_{t-T-s}$. Shocks to the fundamental f have a permanent affect on both the exchange rate and cumulative order flow. Hedge trade innovations affect cumulative order flow permanently, but their effect on the exchange rate dies out when hedge trade shocks are temporary ($\rho_b < 1$).

[40] Both the log of the exchange rate and cumulative order flow are set at zero at the start of the simulation.

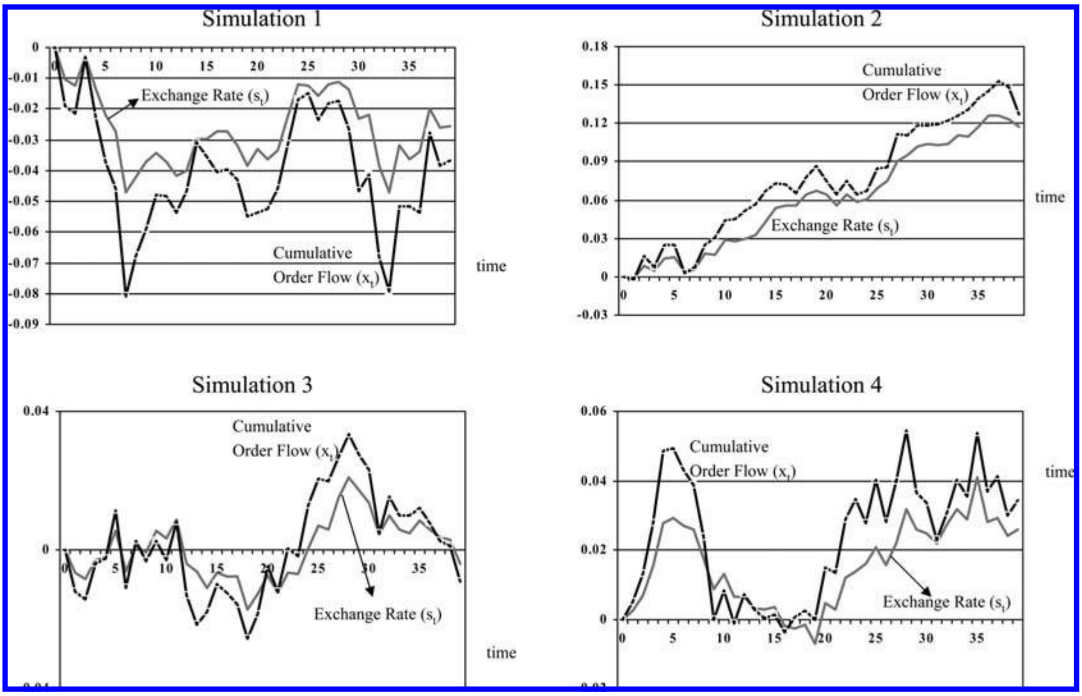

FIGURE 3. FOUR MODEL SIMULATIONS EXCHANGE RATE AND CUMULATIVE ORDER FLOW

heterogeneous-information model. As illustrated in Section IIC, the magnification factor z applied to the impact of b-shocks on the exchange rate also applies to the impact of order flow on the exchange rate.

B. *Sensitivity to Model Parameters*

In this subsection, we consider the parameter sensitivity of two key moments: the R^2 of a regression of $s_{t+1} - s_t$ on observed fundamentals at $t + 1$ and earlier, and the R^2 of a regression of $s_{t+1} - s_t$ on order flow $x_{t+1} - x_t$. These are the moments reported for $k = 1$ in panels D and F of Figure 2.

A first issue is that the precision of the public signal in the common knowledge model does not play an important role in the comparison with the heterogeneous-information model. In particular, it has little influence on the stark difference between the two models regarding the connection between the exchange rate and observed fundamentals. Consider the R^2 of a regression of a one-period change in the exchange rate on all current and past observed fundamentals, as reported in Figure 2D. In the heterogeneous-information model it is 0.14, while in the public-information model it varies from 0.97 to 0.99 as we change the variance of the noise in the public signal from infinity to zero.[41]

We now consider sensitivity analysis to four key model parameters in the heterogeneous-information model: σ_v, σ_b, ρ_b, and T. The results are reported in Figure 4. Not surprisingly, the two R^2's are almost inversely related as we vary parameters. The larger the impact of order flow as a channel through which information is transmitted to the market, the smaller is the explanatory power of commonly observed macro fundamentals.[42]

[41] In Figure 2, we have assumed that the precision of the public signal is such that the conditional variance of the exchange rate is the same in the two models. This implies a standard deviation of the error in the public signal of 0.033.

[42] The two lines do not add to one. The reason is that some variables that are common knowledge are not included in the regression on observed fundamentals. These are past exchange rates and hedge demand T periods ago. Past exchange rates are not included since they are not traditional fundamentals. Hedge demand T periods ago can be indirectly derived from exchange rates T periods ago and earlier, but is not a directly observable fundamental.

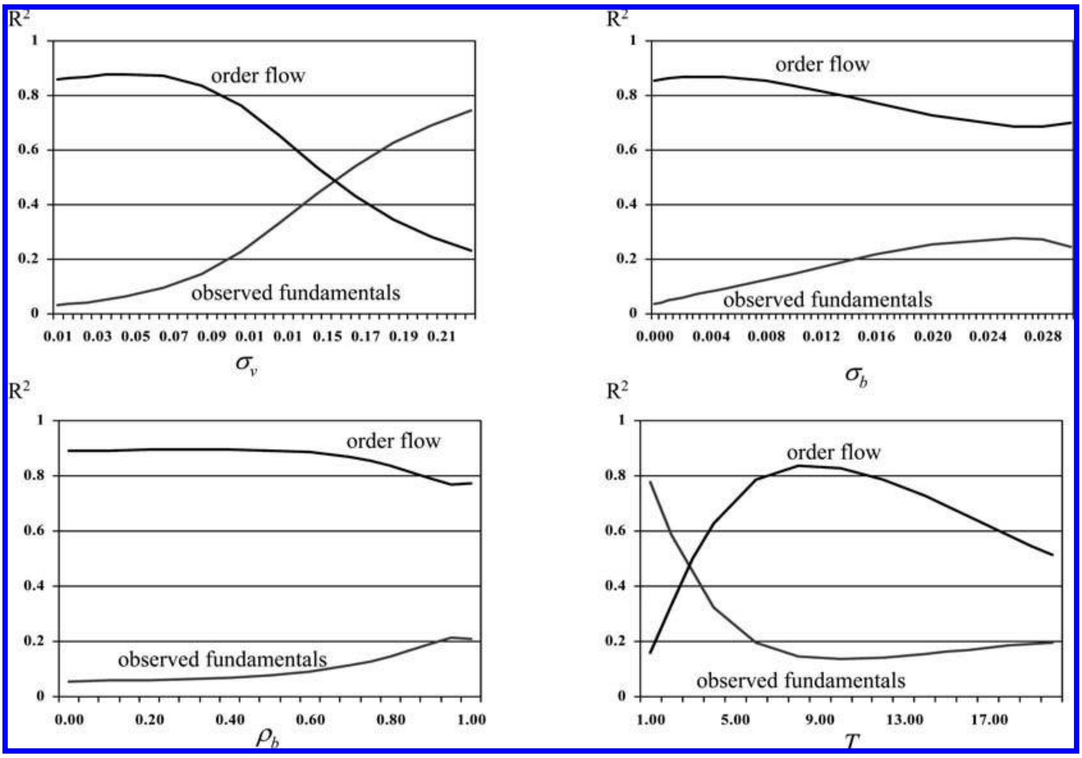

FIGURE 4. R^2 OF REGRESSION OF $s_{t+1} - s_t$ ON (A) OBSERVED FUNDAMENTALS AND (B) ORDER FLOW: SENSITIVITY ANALYSIS

Notes: Order flow: R^2 of regression of $s_{t+1} - s_t$ on $x_{t+1} - x_t$ (same as Figure 2F for $k = 1$). Observed fundamentals: R^2 of regression $s_{t+1} - s_t$ on all $f_{t+s} - f_{t+s-1}$ for $s \leq 1$ (same as Figure 2D for $k = 1$). The figures show how the explanatory power of order flow and observed fundamentals changes when respectively σ_v, σ_b, T, and ρ_b are varied, holding constant the other parameters as in the benchmark parameterization.

An increase in σ_v, implying less precise private information, reduces the link between the exchange rate and order flow and increases the link between the exchange rate and observed fundamentals. In the limit, as the noise in private signals approaches infinity, the heterogeneous-information model approaches the public-information model (with uninformative signals).

Somewhat surprisingly, an increase in the noise originating from hedge trades, by either raising the standard deviation σ_b or the persistence ρ_b, tends to strengthen the link between the exchange rate and observed fundamentals and reduce the link between the exchange rate and order flow. The effect is relatively small, however, due to offsetting factors. Order flow becomes less informative about future fundamentals with more noisy hedge trades. This reduces the impact of order flow on the exchange rate. On the other hand, the volatility of order flow increases, which contributes positively to the R^2 for order flow. The former effect slightly dominates.

It is also worthwhile pointing out that the assumed stationarity of hedge trades in the benchmark parameterization is not responsible for the much weaker relationship between the exchange rate and observed fundamentals in the short run than the long run. Even if we assume $\rho_b = 1$, so that unobserved aggregate hedge trades follow a random walk as well, this finding remains largely unaltered. The R^2 for observed fundamentals rises from 0.21 for a one-period horizon to 0.85 for a 40-period horizon.

The final panel of Figure 4 shows the impact of changing T. Initially, an increase in T leads to a closer link between order flow and the exchange rate and a weaker link between observed fundamentals and the exchange rate. The reason is that as T increases, the quality of private information improves because agents have signals about fundamentals further into the future. This implies that the impact of order flow on the exchange rate increases. Moreover, order flow itself also becomes more volatile as more private information is aggregated. Beyond a certain level of T, however, the link between the exchange rate and order flow is weakened when T is raised further. The reason is that the improved quality of information reduces the conditional variance σ^2 of next period's exchange rate. This reduces the effect of order flow on the exchange rate, as can be seen from (30).

V. Conclusion

The close relationship between order flow and exchange rates, as well as the large volume of trade in the foreign exchange market, suggest that investor heterogeneity is key to understanding exchange rate dynamics. In this paper we have explored the implications of information dispersion in a simple model of exchange rate determination. We have shown that these implications are rich and that investors' heterogeneity can be an important element in explaining the behavior of exchange rates. In particular, the model can account for some important stylized facts on the relationship between exchange rates, fundamentals, and order flow: (a) fundamentals have little explanatory power for short- to medium-run exchange rate movements, (b) over long horizons the exchange rate is closely related to observed fundamentals, (c) exchange rate changes are a weak predictor of future fundamentals, and (d) the exchange rate is closely related to order flow.

The paper should be considered only as a first step in a promising line of research. While we have mostly focused on the implications of the model for the relationship between exchange rates, fundamentals, and order flow, future work along this line should also consider the implications for other outstanding exchange rate puzzles such as the forward discount puzzle and excess volatility puzzle.[43] More broadly speaking, a natural next step is to link the theory to the data. While the extent of information dispersion and unobservable hedge trades is not known, both affect order flow. Some limited data on order flow are now available and will help tie down the key model parameters. The magnification factor may be quite large. Back-of-the-envelope calculations by Gennotte and Leland (1990) in the context of a static model for the U.S. stock market crash of October 1987 suggest that the impact of a \$6 billion unobserved supply shock was magnified by a factor of 250 due to rational confusion about the source of the stock price decline. In the context of foreign exchange markets, Carol L. Osler (2005) presents evidence that trades that are uninformative about future fundamentals have a large impact on the price.

There are several directions in which the model can be extended. The first is to explicitly model nominal rigidities as in the "new open economy macro" literature. In that literature, exchange rates are entirely driven by commonly observed macro fundamentals. Conclusions that have been drawn about optimal monetary and exchange rate policies are likely to be substantially revised when introducing investor heterogeneity. Another direction is to consider alternative information structures. For example, the information received by agents may differ in its quality or in its timing. There can also be heterogeneity about the knowledge of the underlying model. For example, in Bacchetta and van Wincoop (2004b), we show that if investors receive private signals about the persistence of shocks, the impact of observed variables on the exchange rate varies over time. The rapidly growing body of empirical work on order flow in the foreign exchange microstructure literature is likely to increase our understanding of the nature of the information structure, providing guidance to future modeling.

Appendix

A. *Derivation of Equation (7)*

It follows from (1), (2), (3), and (6) that

$$\text{(A1)} \qquad s_t = \frac{1}{1+\alpha} f_t - \frac{\alpha}{1+\alpha} \gamma \sigma_t^2 b_t + \frac{\alpha}{1+\alpha} \bar{E}_t^1 (s_{t+1}).$$

[43] See Bacchetta and van Wincoop (2003) for some discussion of the excess volatility puzzle in the context of this model. The current model yields a forward discount bias of the correct sign, but the magnitude falls short of what is found in empirical evidence. See Bacchetta and van Wincoop (2005) for an explanation of the bias based on a different type of information heterogeneity.

Therefore

$$\text{(A2)}\quad \bar{E}_t^1(s_{t+1}) = \frac{1}{1+\alpha}\bar{E}_t^1(f_{t+1}) - \frac{\alpha}{1+\alpha}\gamma\sigma_{t+1}^2\bar{E}_t^1(b_{t+1}) + \frac{\alpha}{1+\alpha}\bar{E}_t^2(s_{t+2}).$$

Substitution into (A1) yields

(A3)

$$s_t = \frac{1}{1+\alpha}\left[f_t - \gamma\sigma_t^2 b_t + \frac{\alpha}{1+\alpha}\bar{E}_t^1(f_{t+1} - \gamma\sigma_{t+1}^2 b_{t+1})\right] + \left(\frac{\alpha}{1+\alpha}\right)^2 \bar{E}_t^2(s_{t+2}).$$

Continuing to solve for s_t this way by forward induction and assuming a no-bubble solution yields (7).

B. *Solution Method with Two-Period Overlapping Investors*

The solution method is related to Townsend (1983, sect. VIII). We start with the conjectured equation (11) for s_t and check whether it is consistent with the model, in particular with equation (6). For this, we need to estimate the conditional moments of s_{t+1} and express them as a function of the model's innovations. Finally, we equate the parameters from the resulting equation to the initially conjectured equation.

B.1. *The Exchange Rate Equation.*—From (1)–(3), and the definition of f_t, it is easy to see that $i_t^* - i_t = (f_t - s_t)/\alpha$. Thus, (6) gives (for a constant σ_t^2)

(B1)

$$s_t = \frac{\alpha}{1+\alpha}\bar{E}_t(s_{t+1}) + \frac{f_t}{1+\alpha} - \frac{\alpha}{1+\alpha}\gamma b_t \sigma^2.$$

We want to express (B1) in terms of current and past innovations. First, we have $f_t = D(L)\varepsilon_t^f$. Second, using (5) we can write $b_t = C(L)\varepsilon_t^b$, where $C(L) = 1 + \rho_b L + \rho_b^2 L^2 + \cdots$. What remains to be computed are $\bar{E}(s_{t+1})$ and σ^2.

Applying (11) to s_{t+1}, writing $A(L) = a_1 + a_2 L + a_3 L^2 + \cdots$ and $B(L) = b_1 + b_2 L + b_3 L^2 + \cdots$, we have

$$\text{(B2)}\quad s_{t+1} = a_1\varepsilon_{t+T+1}^f + b_1\varepsilon_{t+1}^b + \boldsymbol{\theta}'\boldsymbol{\xi}_t + A^*(L)\varepsilon_t^f + B^*(L)\varepsilon_{t-T}^b,$$

where $\boldsymbol{\xi}_t' = (\varepsilon_{t+T}^f, \dots, \varepsilon_{t+1}^f, \varepsilon_t^b, \dots, \varepsilon_{t-T+1}^b)$ represents the vector of unobservable innovations, $\boldsymbol{\theta}' = (a_2, a_3, \dots, a_{T+1}, b_2, \dots, b_{T+1})$, and $A^*(L) = a_{T+2} + a_{T+3}L + \cdots$ (with a similar definition for $B^*(L)$). Thus, we have (since ε_j^f and ε_{j-T}^b are known for $j \le t$)

$$\text{(B3)}\quad E_t^i(s_{t+1}) = \boldsymbol{\theta}' E_t^i(\boldsymbol{\xi}_t) + A^*(L)\varepsilon_t^f + B^*(L)\varepsilon_{t-T}^b$$

and

$$\text{(B4)}\quad \sigma^2 = var_t(s_{t+1}) = a_1^2\sigma_f^2 + b_1^2\sigma_b^2 + \boldsymbol{\theta}' var_t(\boldsymbol{\xi}_t)\boldsymbol{\theta}.$$

We need to estimate the conditional expectation and variance of the unobservable $\boldsymbol{\xi}_t$ as a function of past innovations.

B.2. *Conditional Moments.*—We follow the strategy of Townsend (1983, p. 556), but use the notation of James D. Hamilton (1994, chap. 13). First, to focus on the informational content of observable variables, we subtract the known components from the observables s_t and v_t^i and define these new variables as s_t^* and v_t^{i*}. Let the vector of these observables be $\mathbf{Y}_t^i = (s_t^*, s_{t-1}^*, \dots, s_{t-T+1}^*, v_t^{i*}, \dots, v_{t-T+1}^{i*})$. This vector provides information on the vector of unobservables $\boldsymbol{\xi}_t$. From (B2) and (9), we can write

$$\text{(B5)}\quad \mathbf{Y}_t^i = \mathbf{H}'\boldsymbol{\xi}_t + \mathbf{w}_t^i,$$

where $\mathbf{w}_t^i = (0, \dots, 0, \varepsilon_t^{vi}, \dots, \varepsilon_{t-T+1}^{vi})'$ and

$$\mathbf{H}' = \begin{bmatrix} a_1 & a_2 & \cdots & a_T & b_1 & b_2 & \cdots & b_T \\ 0 & a_1 & \cdots & a_{T-1} & 0 & b_1 & \cdots & b_{T-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_1 & 0 & 0 & \cdots & b_1 \\ d_1 & d_2 & \cdots & d_T & 0 & 0 & \cdots & 0 \\ 0 & d_1 & \cdots & d_{T-1} & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & d_1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

The unconditional means of $\boldsymbol{\xi}_t$ and $\mathbf{w}_t^i$ are zero.

Define their unconditional variances as $\tilde{\mathbf{P}}$ and $\mathbf{R}$. Then we have (applying equations (17) and (18) in Townsend)

$$E_t^i(\boldsymbol{\xi}_t) = \mathbf{M}\mathbf{Y}_t^i, \tag{B6}$$

where

$$\mathbf{M} = \tilde{\mathbf{P}}\mathbf{H}[\mathbf{H}'\tilde{\mathbf{P}}\mathbf{H} + \mathbf{R}]^{-1}. \tag{B7}$$

Moreover, $\mathbf{P} \equiv var_t(\boldsymbol{\xi}_t)$ is given by

$$\mathbf{P} = \tilde{\mathbf{P}} - \mathbf{M}\mathbf{H}'\tilde{\mathbf{P}}. \tag{B8}$$

B.3. *Solution.*—First, σ^2 can easily be derived from (B4) and (B8). Second, substituting (B6) and (B5) into (B3), and averaging over investors, gives the average expectation in terms of innovations

$$\bar{E}_t(s_{t+1}) = \boldsymbol{\theta}'\mathbf{M}\mathbf{H}'\boldsymbol{\xi}_t + A^*(L)\varepsilon_t^f + B^*(L)\varepsilon_{t-T}^b. \tag{B9}$$

We can then substitute $\bar{E}_t(s_{t+1})$ and σ^2 into (B1) so that we have an expression for s_t that has the same form as (11). We then need to solve a fixed-point problem.

Although $A(L)$ and $B(L)$ are infinite lag operators, we need only solve a finitely dimensional fixed-point problem in the set of parameters $(a_1, a_2, \ldots, a_T, b_1, \ldots, b_{T+1})$. This can be seen as follows. First, it is easily verified by equating the parameters of the conjectured and equilibrium exchange rate equation for lags T and greater that $b_{T+s+1} = ((1+\alpha)/\alpha)b_{T+s} + \gamma\sigma^2\rho_b^{T+s-1}$ and $a_{T+s+1} = ((1+\alpha)/\alpha)a_{T+s} - (1/\alpha)d_s$ for $s \geq 1$. Assuming nonexplosive coefficients, the solutions to these difference equations give us the coefficients for lags $T+1$ and greater: $b_{T+1} = -\alpha\gamma\sigma^2\rho_b^T/(1+\alpha-\alpha\rho_b)$, $b_{T+s} = (\rho_b)^{s-1}b_{T+1}$ for $s \geq 2$, $a_{T+1} = (1/\alpha)\sum_{s=1}^{\infty}(\alpha/(1+\alpha))^s d_s$, and $a_{T+s+1} = ((1+\alpha)/\alpha)a_{T+s} - (1/\alpha)d_s$ for $s \geq 1$. When the fundamental follows a random walk, $d_s = 1\ \forall s$, so that $a_{T+s} = 1\ \forall s \geq 1$.

The fixed-point problem in the parameters $(a_1, a_2, \ldots, a_T, b_1, \ldots, b_{T+1})$ consists of $2T+1$ equations. One of them is the $b_{T+1} = -\alpha\gamma\sigma^2\rho_b^T/(1+\alpha-\alpha\rho_b)$. The other $2T$ equations equate the parameters of the conjectured and equilibrium exchange rate equations up to lag $T-1$. The conjectured parameters $(a_1, a_2, \ldots, a_T, b_1, \ldots, b_{T+1})$, together with the solution for a_{T+1} above, allow us to compute $\boldsymbol{\theta}$, $\mathbf{H}$, $\mathbf{M}$ and σ^2, and therefore the parameters of the equilibrium exchange rate equation. We use the Gauss NLSYS routine to solve the $2T+1$ nonlinear equations.

After having found the solution, we can also verify that the polynomial $B(L)$ is invertible, which is necessary to extract information about hedge trade innovations at $t-T$ and earlier from exchange rates at $t-T$ and earlier. Using that $b_{T+s} = (\rho_b)^{s-1}b_{T+1}$ for $s \geq 2$, we have

$$B(L) = \sum_{i=1}^{T} b_i L^{i-1} + b_{T+1}L^T \sum_{i=0}^{\infty} \rho_b^i L^i = \sum_{i=1}^{T} b_i L^{i-1} + \frac{b_{T+1}L^T}{1-\rho_b L}. \tag{B10}$$

$B(L)$ is invertible when the roots of the polynomial are outside the unit circle. Setting $B(L) = 0$, multiplying by $1-\rho_b L$, yields

$$b_1 + \sum_{i=1}^{T}(b_{i+1} - \rho_b b_i)L^i = 0. \tag{B11}$$

This amounts to solving the roots of an ordinary T-order polynomial, which is done with the routine polyroot in Gauss. The roots are indeed outside the unit circle for all parameterizations considered in the paper. For the benchmark parameterization, the roots are (rounding to the second digit after the decimal point): $(-1.43$, $-1.03+0.98i$, $-1.03-0.98i$, $-0.07+1.39i$, $-0.07-1.39i$, $0.89+0.98i$, $0.89-0.98i$, $1.28)$.

C. *Higher-Order Expectations*

We show how (32) follows from Proposition 1 in Bacchetta and van Wincoop (2004a). Bacchetta and van Wincoop (2004a) define the higher-order wedge Δ_t as the present value of deviations between higher-order and first-order expectations. In our application (assuming $\rho_b = 0$):

$$\Delta_t = \sum_{s=2}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^s[\bar{E}_t^s f_{t+s} - \bar{E}_t f_{t+s}]. \tag{C1}$$

Define $PV_t = \sum_{s=1}^{\infty} (\alpha/(1+\alpha))^s f_{t+s}$ as the present discounted value of future observed fundamentals. Let $\mathbf{V}_t^i$ be the set of private signals available at time t that are still informative about PV_{t+1} at $t+1$. In our application $\mathbf{V}_t^i = (v_{t-T+2}^i, \dots, v_t^i)'$. Let $\bar{\mathbf{V}}_t$ denote the average across investors of the vector $\mathbf{V}_t^i$. Proposition 1 of Bacchetta and van Wincoop (2004a) then says that

$$\text{(C2)} \qquad \Delta_t = \mathbf{\Pi}_t'(\bar{E}_t\bar{\mathbf{V}}_t - \bar{\mathbf{V}}_t),$$

where $\mathbf{\Pi}_t = (1/R^2)(\mathbf{I} - \mathbf{\Psi})^{-1}\boldsymbol{\theta}$, $\boldsymbol{\theta}' = \partial E_{t+1}^i PV_{t+1}/\partial \mathbf{V}_t^i$ and $\mathbf{\Psi}' = \partial E_{t+1}^i \bar{\mathbf{V}}_{t+1}/\partial \mathbf{V}_t^i$.

In our context $\bar{\mathbf{V}}_t = (f_{t+2}, \dots, f_{t+T})$. For $\rho_b = 0$ equations (7), (C1), and (C2) then lead to (32) with $\mathbf{\Pi}/(1+\alpha) = (\pi_2, \dots, \pi_T)'$.

D. *Order Flow*

In this section we describe our measure of order flow when the observable fundamental follows a random walk. Using the notation and results from Appendix B, we have

$$\text{(D1)} \qquad b_{Ft}^i = \frac{\boldsymbol{\theta}'\mathbf{M}\mathbf{Y}_t^i + f_t - nb_{t-T} - s_t + i_t^* - i_t}{\gamma\sigma_t^2} - b_t^i,$$

where $n = \alpha\gamma\sigma^2\rho_b^{T+1}/(1+\alpha-\alpha\rho b)$. Let $\boldsymbol{\mu} = (\mu_1, \dots, \mu_t)'$ be the last T elements of $\mathbf{M}'\boldsymbol{\theta}$, divided by $\gamma\sigma^2$. The component of demand that depends on private information is therefore

$$\text{(D2)} \qquad \sum_{s=1}^{T} \mu_s v_{t+1-s}^{i*} - b_t^i.$$

Using that $v_{t+1-s}^{i*} = \varepsilon_{t+1}^f + \cdots + \varepsilon_{t+1-s+T}^f + \varepsilon_{t+1-s}^{vi}$, (D2) aggregates to

$$\text{(D3)} \qquad \boldsymbol{\eta}'\boldsymbol{\xi}_t - \rho_b^T b_{t-T},$$

where $\boldsymbol{\eta}' = (\eta_1, \dots, \eta_{2T})$ with $\eta_s = \mu_1 + \cdots + \mu_s$ and $\eta_{T+s} = -\rho_b^{s-1}$ for $s = 1, \dots, T$. Order flow $x_t - x_{t-1}$ is defined as the component of (D3) that is orthogonal to public information (other than s_t). Public information that helps predict this term includes b_{t-T} and $s_{t-1}^*, \dots, s_{t-T+1}^*$. Order flow is then the error term of a regression of $\boldsymbol{\eta}'\boldsymbol{\xi}_t$ on $s_{t-1}^*, \dots, s_{t-T+1}^*$. Defining $\mathbf{H}_s$ as rows 2 to T of the matrix $\mathbf{H}$ defined in Appendix B.2, it follows from Appendix B.2 that $\bar{E}_t(\boldsymbol{\xi}_t | s_{t-1}^*, \dots, s_{t-T+1}^*) = \mathbf{M_s H_s'}\boldsymbol{\xi}_t$, where $M_s = \tilde{\mathbf{P}}\mathbf{H}_s[\mathbf{H}_s'\tilde{\mathbf{P}}\mathbf{H}_s]^{-1}$. It follows that

$$\text{(D4)} \qquad x_t - x_{t-1} = \boldsymbol{\eta}'(\mathbf{I} - \mathbf{M}_s\mathbf{H}_s')\boldsymbol{\xi}_t.$$

We can also show that there is a cointegrating relationship between the exchange rate, cumulative order flow, and $\hat{b}_t = \sum_{s=0}^{\infty} \varepsilon_{t-T-s}^b$. When f follows a random walk, the equilibrium exchange rate can be written as (see Appendix B.3)

$$\text{(D5)} \qquad s_t = f_t - \phi b_{t-T} + \boldsymbol{\tau}'\boldsymbol{\xi}_t.$$

Order flow is equal to

$$\text{(D6)} \qquad x_t - x_{t-1} = \boldsymbol{\nu}'\boldsymbol{\xi}_t,$$

where $\boldsymbol{\nu}' = \boldsymbol{\eta}'(\mathbf{I} - \mathbf{M}_s\mathbf{H}_s')$. It therefore follows that cumulative order flow is equal to

$$\text{(D7)} \qquad x_t = (\nu_1 + \cdots + \nu_T)f_t + (\nu_{T+1} + \cdots + \nu_{2T})\hat{b}_t + \boldsymbol{\psi}'\boldsymbol{\xi}_t,$$

where $\boldsymbol{\psi}$ depends on the parameters in the vector $\boldsymbol{\nu}$. It follows from (D5) and (D7) that there is a cointegrating relationship between s_t, x_t, and $\hat{b}_t$. Note that the latter follows a random walk since $\hat{b}_t - \hat{b}_{t-1} = \varepsilon_{t-T}^b$. This cointegrating relationship holds both for $\rho_b < 1$ and $\rho_b = 1$. In the latter case $b_{t-T} = \hat{b}_t$.

REFERENCES

Admati, Anat R. "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets." *Econometrica*, 1985, *53*(3), pp. 629–57.

Allen, Franklin; Morris, Stephen and Shin, Hyun Song. "Beauty Contests and Iterated Expectations in Asset Markets." *Review of Financial Studies* (forthcoming).

Bacchetta, Philippe and van Wincoop, Eric. "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" National Bureau of Economic Research, Inc., NBER Working Papers: No. 9498, 2003.

Bacchetta, Philippe and van Wincoop, Eric. "Higher Order Expectations in Asset Pricing." International Center for Financial Asset

Management and Engineering, FAME Research Paper Series: No. 100, 2004a.

Bacchetta, Philippe and van Wincoop, Eric. "A Scapegoat Model of Exchange-Rate Fluctuations." *American Economic Review*, 2004b, *94*(2), pp. 114–18.

Bacchetta, Philippe and van Wincoop, Eric. "Rational Inattention: A Solution to the Forward Discount Puzzle." Unpublished Paper, 2005.

Bjønnes, Geir H.; Rime, Dafinn and Solheim, Haakon O. A. "Liquidity Provisions in the Overnight Foreign Exchange Market." *Journal of International Money and Finance*, 2005, *24*(2), pp. 175–96.

Brennan, Michael J. and Cao, H. Henry. "International Portfolio Investment Flows." *Journal of Finance*, 1997, *52*(5), pp. 1851–80.

Brown, David P. and Jennings, Robert H. "On Technical Analysis." *Review of Financial Studies*, 1989, *2*(4), pp. 527–51.

Brunnermeier, Markus K. *Asset pricing under asymmetric information: Bubbles, crashes, technical analysis, and herding*. Oxford: Oxford University Press, 2001.

Cheung, Yin-Wong; Chinn, Menzie D. and Pascual, Antonio G. "Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive?" *Journal of International Money and Finance* (forthcoming).

Chinn, Menzie D. and Meese, Richard A. "Banking on Currency Forecasts: How Predictable Is Change in Money?" *Journal of International Economics*, 1995, *38*(1–2), pp. 161–78.

Chionis, Dionysios and MacDonald, Ronald. "Aggregate and Disaggregate Measures of the Foreign Exchange Risk Premium." *International Review of Economics and Finance*, 2002, *11*(1), pp. 57–84.

DeMarzo, Peter and Skiadas, Costis. "Aggregation, Determinacy, and Informational Efficiency for a Class of Economies with Asymmetric Information." *Journal of Economic Theory*, 1998, *80*(1), pp. 123–52.

Devereux, Michael B. and Engel, Charles. "Exchange Rate Pass-through, Exchange Rate Volatility, and Exchange Rate Disconnect." *Journal of Monetary Economics*, 2002, *49*(5), pp. 913–40.

Dow, James and Gorton, Gary. "Profitable Informed Trading in a Simple General Equilibrium Model of Asset Pricing." *Journal of Economic Theory*, 1995, *67*(2), pp. 327–69.

Elliot, Graham and Ito, Takatoshi. "Heterogeneous Expectations and Tests of Efficiency in the Yen/Dollar Forward Exchange Rate Market." *Journal of Monetary Economics*, 1999, *43*(2), pp. 435–56.

Engel, Charles and West, Kenneth D. "Exchange Rates and Fundamentals." *Journal of Political Economy*, 2005, *113*(3), pp. 485–517.

Evans, Martin D. D. and Lyons, Richard K. "Informational Integration and FX Trading." *Journal of International Money and Finance*, 2002a, *21*(6), pp. 807–31.

Evans, Martin D. D. and Lyons, Richard K. "Order Flow and Exchange Rate Dynamics." *Journal of Political Economy*, 2002b, *110*(1), pp. 170–80.

Evans, Martin D. D. and Lyons, Richard K. "A New Micro Model of Exchange Rate Dynamics." National Bureau of Economic Research, Inc., NBER Working Papers: No. 10379, 2004.

Foster, F. Douglas and Viswanathan, S. "Strategic Trading When Agents Forecast the Forecasts of Others." *Journal of Finance*, 1996, *51*(4), pp. 1437–78.

Froot, Kenneth A. and Ramadorai, Tarun. "Currency Returns, Institutional Investor Flows, and Exchange Rate Fundamentals." *Journal of Finance*, 2005, *60*(3), pp. 1535–66.

Gennotte, Gerard and Leland, Hayne. "Market Liquidity, Hedging, and Crashes." *American Economic Review*, 1990, *80*(5), pp. 999–1021.

Gourinchas, Pierre-Olivier and Rey, Hélène. "International Financial Adjustment." National Bureau of Economic Research, Inc., NBER Working Papers: No. 11155, 2005.

Gourinchas, Pierre-Olivier and Tornell, Aaron. "Exchange Rate Puzzles and Distorted Beliefs." *Journal of International Economics*, 2004, *64*(2), pp. 303–33.

Grossman, Sanford J. "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information." *Journal of Finance*, 1976, *31*(2), pp. 573–85.

Grundy, Bruce D. and McNichols, Maureen. "Trade and the Revelation of Information through Prices and Direct Disclosure." *Review of Financial Studies*, 1989, *2*(4), pp. 495–526.

Hamilton, James D. *Time series analysis*. Princeton: Princeton University Press, 1994.

Hau, Harald. "Competitive Entry and Endogenous Risk in the Foreign Exchange Market."

Review of Financial Studies, 1998, *11*(4), pp. 757–87.

Hau, Harald; Killeen, William P. and Moore, Michael. "How Has the Euro Changed the Foreign Exchange Market?" *Economic Policy*, 2002, *17*(34), pp. 149–91.

He, Hua and Wang, Jiang. "Differential Information and Dynamic Behavior of Stock Trading Volume." *Review of Financial Studies*, 1995, *8*(4), pp. 919–72.

Hussman, John P. "Market Efficiency and Inefficiency in Rational Expectations Equilibria: Dynamic Effects of Heterogeneous Information and Noise." *Journal of Economic Dynamics and Control*, 1992, *16*(3–4), pp. 655–80.

Ito, Takatoshi. "Foreign Exchange Rate Expectations: Micro Survey Data." *American Economic Review*, 1990, *80*(3), pp. 434–49.

Jeanne, Olivier and Rose, Andrew K. "Noise Trading and Exchange Rate Regimes." *Quarterly Journal of Economics*, 2002, *117*(2), pp. 537–69.

Kasa, Kenneth. "Forecasting the Forecasts of Others in the Frequency Domain." *Review of Economic Dynamics*, 2000, *3*(4), pp. 726–56.

Kasa, Kenneth; Walker, Todd B. and Whiteman, Charles H. "Asset Pricing with Heterogeneous Beliefs: A Frequency-Domain Approach." Unpublished Paper, 2004.

King, Robert G. "Monetary Policy and the Information Content of Prices." *Journal of Political Economy*, 1982, *90*(2), pp. 247–79.

Kollman, Robert. "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles." *Journal of Monetary Economics*, 2002, *49*(5), pp. 989–1015.

Lucas, Robert E., Jr. "Expectations and the Neutrality of Money." *Journal of Economic Theory*, 1972, *4*(2), pp. 103–24.

Lyons, Richard K. *The microstructure approach to exchange rates*. Cambridge, MA: MIT Press, 2001.

MacDonald, Ronald and Marsh, Ian W. "Currency Forecasters Are Heterogeneous: Confirmation and Consequences." *Journal of International Money and Finance*, 1996, *15*(5), pp. 665–85.

MacDonald, Ronald and Taylor, Mark P. "The Monetary Approach to the Exchange Rate: Rational Expectations, Long-Run Equilibrium, and Forecasting." *International Monetary Fund Staff Papers*, 1993, *40*(1), pp. 89–107.

Mankiw, N. Gregory and Reis, Ricardo. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics*, 2002, *117*(4), pp. 1295–1328.

Mark, Nelson C. "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability." *American Economic Review*, 1995, *85*(1), pp. 201–18.

Mark, Nelson C. and Sul, Donggyu. "Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post–Bretton Woods Panel." *Journal of International Economics*, 2001, *53*(1), pp. 29–52.

Mark, Nelson C. and Wu, Yangru. "Rethinking Deviations from Uncovered Interest Parity: The Role of Covariance Risk and Noise." *Economic Journal*, 1998, *108*(451), pp. 1686–1706.

McCafferty, Stephen and Driskill, Robert. "Problems of Existence and Uniqueness in Nonlinear Rational Expectations Models." *Econometrica*, 1980, *48*(5), pp. 1313–17.

Meese, Richard A. and Rogoff, Kenneth. "Empirical Exchange Rate Models of the Seventies: Do They Fit out of Sample?" *Journal of International Economics*, 1983, *14*(1–2), pp. 3–24.

Morris, Stephen and Shin, Hyun Song. "Social Value of Public Information." *American Economic Review*, 2002, *92*(5), pp. 1521–34.

Mussa, Michael. "The Exchange Rate, the Balance of Payments and Monetary and Fiscal Policy under a Regime of Controlled Floating." *Scandinavian Journal of Economics*, 1976, *78*(2), pp. 229–48.

Obstfeld, Maurice and Rogoff, Kenneth. "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" in Ben S. Bernanke and Kenneth Rogoff, eds., *NBER macroeconomics annual 2000*. Cambridge, MA: MIT Press, 2001, pp. 339–90.

Osler, Carol L. "Stop-Loss Orders and Price Cascades in Currency Markets." *Journal of International Money and Finance*, 2005, *24*(2), pp. 219–41.

Pearlman, Joseph G. and Sargent, Thomas J. "Knowing the Forecasts of Others." *Review of Economic Dynamics*, 2005, *8*(2), pp. 480–97.

Romer, David. "Rational Asset-Price Movements without News." *American Economic Review*, 1993, *83*(5), pp. 1112–30.

Sargent, Thomas J. "Equilibrium with Signal Extraction from Endogenous Variables." *Journal of Economic Dynamics and Control*, 1991, *15*(2), pp. 245–73.

Singleton, Kenneth J. "Asset Prices in a Time-Series Model with Disparately Informed, Competitive Traders," in William A. Barnett and Kenneth J. Singleton, eds., *New approaches to monetary economics: Proceedings of the Second International Symposium in Economic Theory and Econometrics*. Cambridge: Cambridge University Press, 1987, pp. 249–72.

Spiegel, Matthew and Subrahmanyam, Avanidhar. "Informed Speculation and Hedging in a Noncompetitive Securities Market." *Review of Financial Studies*, 1992, *5*(2), pp. 307–29.

Townsend, Robert M. "Forecasting the Forecasts of Others." *Journal of Political Economy*, 1983, *91*(4), pp. 546–88.

Vives, Xavier. "Short-Term Investment and the Informational Efficiency of the Market." *Review of Financial Studies*, 1995, *8*(1), pp. 125–60.

Wang, Jiang. "A Model of Intertemporal Asset Prices under Asymmetric Information." *Review of Economic Studies*, 1993, *60*(2), pp. 249–82.

Wang, Jiang. "A Model of Competitive Stock Trading Volume." *Journal of Political Economy*, 1994, *120*(1), pp. 127–68.

Woodford, Michael. "Imperfect Common Knowledge and the Effects of Monetary Policy," in Philippe Aghion, Roman Frydam, Joseph Stiglitz, and Michael Woodford, eds., *Knowledge, information and expectations in modern macroeconomics: In honor of Edmund S. Phelps*. Princeton, Princeton University Press, 2003, pp. 25–58.