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Self-Fulfilling Risk Panics†

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Sharp surges in asset price risk are a prominent feature of financial panics, such as the turmoil in the Fall of 2008 or more recently the European debt crisis. Implied volatility, as measured by the VIX index, more than quadrupled in the wake of the Lehman Brothers failure, and tripled both in May 2010 and August 2011 in connection to the European debt crisis. Explaining such huge and sudden spikes in risk is an important theoretical challenge that the literature has yet to meet. In this paper we propose a theory for large self-fulfilling changes in beliefs about risk.

We frame our analysis in a very simple model where agents have mean-variance preferences and choose to allocate their wealth between a risk-free bond and a risky asset. The key implication is that the equilibrium asset price Q_t depends negatively on asset price risk, defined as the variance of the asset price tomorrow, $\text{var}_t(Q_{t+1})$. To see how self-fulfilling shifts in risk can arise in this context, assume that agents believe that the risk $\text{var}_t(Q_{t+1})$ depends on a variable S_t. This implies that Q_t depends on S_t as well because the asset price depends negatively on risk. Therefore Q_{t+1} depends on S_{t+1}. Now, if we assume that the distribution of S_{t+1} depends on S_t, then the risk $\text{var}_t(Q_{t+1})$ will indeed depend on S_t. This circular relationship between asset price risk and the asset price level can therefore generate self-fulfilling shifts in risk coordinated around the variable S_t. What is interesting is that S_t could be a fundamental variable like dividends, but it could also be a variable extrinsic to the model, i.e., a sunspot variable.[1]

We consider three versions of the model, which highlight the various roles of the state variable(s). We consider both an autoregressive and a Markov process for the state variable S_t. In the first version, we assume that the state variable does not affect dividends. Therefore S_t is a pure sunspot and the risky asset pays a constant dividend. We show that there is a fundamental equilibrium where the asset price is constant, and a sunspot equilibrium where the asset price, and asset price risk, fluctuate with the sunspot. Changes in the value of the sunspot can trigger both a large increase in risk and a drop in the asset price. The change in perceived risk is entirely self-fulfilling.

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[1] The key aspect of our model is that time-varying risk is self-fulfilling. This contrasts with a large and growing literature that has introduced exogenous time-varying risk in the fundamentals, such as Bansal and Yaron (2004), Bloom (2009), and many others.

In the second version of the model we assume that the state variable affects the dividend of the risky asset, and is therefore a fundamental. There is again a fundamental equilibrium, where risk is constant and changes in the asset price reflect the impact of the state variable on the dividend. There is also a so-called "sunspot-like" equilibrium in which the state variable plays a dual role. It plays both the role of a fundamental that affects dividends and the role of a sunspot that generates self-fulfilling shifts in risk. The latter role dominates.[2]

Finally, in the third version of the model there is a second state variable. In addition to S_t playing the role of a fundamental, we introduce a two-state sunspot variable that switches expectations between low and high risk states. A change in this sunspot can be seen as a shift from a tranquil to a panic state, which we call a risk panic. During such a risk panic there is a spike in asset price risk and a drop in the asset price that can be very large in magnitude. We show that the panic is larger when the fundamental is weak, and that the asset price becomes much more sensitive to the fundamental once we shift to the panic state. The role of the fundamental S_t in the panic state is the same as in the sunspot-like equilibria discussed above. In addition to its fundamental role, it becomes a focal point for self-fulfilling changes in beliefs about risk.

The paper is related to the broader literature on multiple equilibria with self-fulfilling shifts in beliefs.[3] However, the sunspot equilibria resulting from our analysis differ from the literature. First, our model has a unique non-sunspot equilibrium. Thus, in contrast to most of the literature, the role of sunspots is not to randomize over multiple fundamental (non-sunspot) equilibria.[4] A second key feature of our setting is that the self-fulfilling shift in beliefs is not about the level of a variable (the asset price) but about the level, and more generally the process, of its risk. This is critical as we wish to explain large spikes in risk.

There is also a literature focusing on self-fulfilling shifts in beliefs about risk that are due to an interaction between risk and liquidity. This occurs in limited participation models such as Pagano (1989); Allen and Gale (1994); and Jeanne and Rose (2002). When agents believe that risk is high, market participation is low. This implies low market liquidity, which leads to a large price response to asset demand shocks and therefore high risk.[5] This is quite different though from what happens in our model, where there is no concept of market liquidity. In contrast to static limited participation models, the dynamic nature of the model is critical in generating our results. In our setting, sunspot equilibria cannot occur in the absence of dynamic relation between the state variable today and its distribution tomorrow.

[2] The term "sunspot-like" equilibria was first coined by Manuelli and Peck (1992, p. 205). They write: "There are two ways that random fundamentals can influence economic outcomes. First, randomness affects resources which intrinsically affects prices and allocation. Second, the randomness can endogenously affect expectations or market psychology, thereby leading to excessive volatility." In the limiting case where fundamental uncertainty goes to zero, sunspot-like equilibria converge to pure sunspot equilibria.

[3] In terms of asset prices, there are many applications of this phenomenon for both stock prices and exchange rates. In particular, there is a large literature with self-fulfilling speculative attacks on currencies. See, e.g., Obstfeld (1986); Aghion, Bacchetta, and Banerjee (2004); or Burnside, Eichenbaum, and Rebelo (2004).

[4] There are some examples in the literature where sunspot equilibria occur even with a unique non-sunspot equilibrium. See, for example, Cass and Shell (1983) or Hens (2000). See Benhabib and Farmer (1999) or Shell (2008) for surveys on the sources of sunspots.

[5] This phenomenon is not limited to limited participation models of asset prices. For other applications see Bacchetta and van Wincoop (2006) and Walker and Whiteman (2007).

We derive our results under the assumption that agents have simple mean-variance preferences. The mean-variance portfolio model has a long history in academics and remains extensively used today. It is also widely used in the financial industry and can therefore be considered as a reasonable description of actual behavior.[6] An alternative avenue would be to introduce micro founded risk-based portfolio constraints, such as value-at-risk constraints or margin constraints, so that asset demand (and therefore the asset price itself) would depend explicitly on uncertainty about the future asset price.[7] This would, however, make the model significantly more complicated. The mean-variance portfolio assumption in this paper should then be considered as an approximation of more complex behavior.

The model is too simple to calibrate to actual data of financial panics. However, at a qualitative level it does connect to events in recent years in several ways. First, it can generate spikes in risk and a drop in asset prices that are very large, as we show through numerical illustrations. We are not aware of any other macro model that can generate the huge spikes in risk as seen during the US financial crisis in 2008 or the European debt crisis. Second, this happens without any change in fundamentals. Balance sheets of US financial institutions had started to gradually deteriorate long before the financial panic in the Fall of 2008. The same can be said for Greek debt, which did not suddenly reach its high level in May of 2010, when it first ignited a spike in risk. Finally, the last version of our model implies that a risk panic also leads to increased volatility of risk that is coordinated around news about a macro fundamental. During recent market turmoil associated with European debt, any news about Greek bailout packages has indeed had the effect of large shifts in the VIX seen around the world.

The remainder of the paper is organized as follows. In Section I we describe the model. Section II considers the model when there is one state variable that is a sunspot, following either an autoregressive or Markov process. Section III considers the case where the state variable affects the dividend of the risky asset, thus becoming a macro fundamental. This gives rise to the possibility of sunspot-like equilibria. It then extends the model by introducing a second state variable, which is a sunspot. This allows for equilibria that have the flavor of a switch between fundamental and sunspot-like equilibria. Section IV concludes.

I. A Simple Mean-Variance Portfolio Choice Model

The model is designed to keep complexity to a strict minimum. Consider an overlapping generation setup where investors are born with wealth W. They live for two periods and only consume when old. Their only problem is to allocate their wealth between a risky equity and a risk-free bond that pays a gross return $R > 1$.

[6] See Basak and Chabakauri (2010) for further motivation.

[7] A substantial literature introducing such constraints has developed in recent years. Examples are Gromb and Vayanos (2002); Brunnermeier and Pedersen (2009); and Zigrand, Shin, and Danielsson (2010). For the same reason of analytic tractability as in this paper, these constraints are often introduced in a reduced-form way rather than based on explicit micro foundations.

Equity consists of a claim on a tree with a stochastic payoff. There are K trees, each producing an exogenous stochastic output (dividend) A_t. Denoting the equity price by Q_t, the equity return from t to $t+1$ is

$$R_{K,t+1} = \frac{A_{t+1} + Q_{t+1}}{Q_t}. \tag{1}$$

In general, agents face uncertainty both about the dividend and the future equity price. The dividend is equal to

$$A_t = \bar{A} + mS_t, \tag{2}$$

where S_t is an exogenous state variable that follows a stochastic process. The dividend is constant at $\bar{A}$ when $m = 0$. In that case S_t is an extrinsic variable, or pure sunspot, with no fundamental role. When $m > 0$, S_t has a fundamental impact on the dividend. For simplicity we assume that the distribution of S_{t+1} depends at most on S_t and is time invariant. The analysis could easily be extended to processes of S_{t+1} that depend on more lags of S_t. We also assume that the unconditional distribution of S_t is such that A_t is always nonnegative.

Investors born at time t maximize a mean-variance utility over their portfolio return

$$E_t R^p_{t+1} - 0.5\gamma \operatorname{var}_t(R^p_{t+1}), \tag{3}$$

where γ measures risk aversion and the portfolio return is

$$R^p_{t+1} = \alpha_t R_{K,t+1} + (1 - \alpha_t)R.$$

α_t denotes the portfolio share invested in equity. The clearing of the equity market requires that the wealth invested in equity equates the value of existing trees:

$$\alpha_t W = Q_t K. \tag{4}$$

DEFINITION 1: *An equilibrium is a nonnegative asset price function* $Q_t = f(S_t)$ *such that* (*i*) *agents choose the portfolio share* α_t *to maximize their utility* (3), (*ii*) *the market clearing condition* (4) *is satisfied, and* (*iii*) *there are no asset price bubbles:* $\lim_{T\to\infty} E_t Q_{t+T}/R^{t+T} = 0$.

Maximization of (3) with respect to α_t gives the optimal portfolio share, which reflects the expected excess return on equity scaled by the variance of the equity return:

$$\alpha_t = \frac{E_t R_{K,t+1} - R}{\gamma \operatorname{var}_t(R_{K,t+1})}. \tag{5}$$

Using (5), the market clearing condition (4) becomes

$$E_t(A_{t+1} + Q_{t+1} - RQ_t) = \lambda \operatorname{var}_t(Q_{t+1} + A_{t+1}), \tag{6}$$

where $\lambda = \gamma K/W$. Equation (6) equates the equilibrium expected excess payoff on equity to a risk premium that depends on the variance of $Q_{t+1} + A_{t+1}$.

Iterating (6) forward, and using the no bubble condition $\lim_{T\to\infty} E_t Q_{t+T}/R^{t+T} = 0$, gives a present value expression for the equilibrium asset price:

$$Q_t = \sum_{i=1}^{\infty} \frac{1}{R^i} E_t A_{t+i} - \lambda \sum_{i=1}^{\infty} \frac{1}{R^i} E_t \operatorname{var}_{t+i-1}(Q_{t+i} + A_{t+i}). \tag{7}$$

The asset price depends on the present value of the expected future dividends and the present value of expected future risk, measured by the expected value of $\operatorname{var}_{t+i-1}(Q_{t+i} + A_{t+i})$ for $i \geq 1$.

II. Sunspot Equilibria

We first consider the case where the state variable S_t is a pure sunspot with no direct impact on the dividend. This corresponds to $m = 0$ in (2) with the dividend constant at $\bar{A}$. One solution to the asset price is immediate, which is a straightforward *fundamental equilibrium*:

$$Q_t = \bar{A}/(R - 1). \tag{8}$$

The asset price is constant and equal to the present value of the constant dividend.

However, there can be other equilibria where the asset price is affected by the sunspot variable S_t. These are *sunspot equilibria*, which are defined as follows:

DEFINITION 2: *Assume $m = 0$, so that S_t is an extrinsic variable. A pure sunspot equilibrium is an equilibrium with nonconstant asset price $Q_t = f(S_t)$.*

In the remainder of this section we focus on these equilibria. Their existence is not guaranteed and depends in particular on the process of S_t. We first derive some necessary conditions for the existence of a sunspot equilibrium, and then consider two specific examples based on, respectively, an autoregressive and Markov process for S_t.

A. *Necessary Conditions for Sunspot Equilibrium*

When $m = 0$, the present value relationship (7) becomes

$$Q_t = \frac{\bar{A}}{R - 1} - \lambda \sum_{i=1}^{\infty} \frac{1}{R^i} E_t \operatorname{var}_{t+i-1}(Q_{t+i}). \tag{9}$$

The asset price Q_t can only depend on the sunspot S_t through the asset price risk $E_t \operatorname{var}_{t+i-1}(Q_{t+i})$. If risk does not depend on the sunspot, and is therefore constant, it is immediate from (9) that the asset price is constant and thus risk is zero. The fundamental equilibrium (8) is then the only solution. This implies a simple condition for the existence of a sunspot equilibrium.

PROPOSITION 1: *A necessary condition for a sunspot equilibrium to exist is that the distribution of* S_{t+1} *depends on* S_t.

PROOF:

The proof can be given by contradiction. Consider a sunspot equilibrium $Q_t = f(S_t)$. When the distribution of S_{t+1} does not depend on S_t, then $\text{var}_t(Q_{t+1}) = \text{var}_t(f(S_{t+1}))$ does not depend on S_t. This is true for all t (current and future), so that (9) implies that Q_t does not depend on S_t.

If this condition does not hold, it breaks a key element in the circular link between the asset price and asset price risk that gives rise to the possibility of sunspot equilibria. There is another necessary condition for existence that is useful for later analysis.[8]

PROPOSITION 2: *Consider a sunspot equilibrium* $Q_t = f(S_t)$, *where* $f(S_t)$ *can be represented by an infinite order polynomial. A necessary condition for such a sunspot equilibrium to exist is that there is at least one pair* $(n_1, n_2) \in N^* \times N^*$ *such that* $\text{cov}_t(S_{t+1}^{n_1}, S_{t+1}^{n_2})$ *depends on* S_t.

PROOF:

First, notice that there can only be a sunspot equilibrium if $\text{var}_t(Q_{t+1})$ depends on S_t, since we assumed that the distribution of S_{t+1} depends at most on S_t and is not time varying. Write the solution $Q_{t+1} = f(S_{t+1})$ as an infinite order polynominal $Q_{t+1} = \sum_{n=0}^{\infty} \alpha_n S_{t+1}^n$. This implies $\text{var}_t(Q_{t+1}) = \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \alpha_{n_1}\alpha_{n_2}\text{cov}_t(S_{t+1}^{n_1}, S_{t+1}^{n_2})$. It therefore follows that for $\text{var}_t(Q_{t+1})$ to depend on S_t there must be at least one pair (n_1, n_2) with $n_1 \geq 1$ and $n_2 \geq 1$ such that $\text{cov}_t(S_{t+1}^{n_1}, S_{t+1}^{n_2})$ depends on S_t.

The condition in Proposition 2 that $f(S_t)$ can be represented by an infinite order polynomial applies to all solutions where the function $f(S)$ is continuously differentiable. It also applies when S_t can take on a finite number of values, in which case $f(S_t)$ can always be written as a finite order polynomial.

The condition in Proposition 1 is looser than in Proposition 2. To see this, consider the case where S_t follows a symmetric Markov process. It can take two values, $\overline{S}$ and $\underline{S}$, and the probability of remaining in the same state is $p > 0.5$. This implies that the distribution of S_{t+1} depends on S_t, so that the condition in Proposition 1 is satisfied. However, $\text{cov}_t(S_{t+1}^{n_1}, S_{t+1}^{n_2}) = p(1-p)(\overline{S}^{n_1} - \underline{S}^{n_1})(\overline{S}^{n_2} - \underline{S}^{n_2})$ is independent of the value of S_t. The condition of Proposition 2 is therefore not satisfied and there cannot be a sunspot equilibrium. This is because both the probability and the absolute size of a jump to another state do not depend on the current state.[9]

While the conditions in Propositions 1 and 2 are satisfied for a wide range of distributions, in the remainder of this section we focus on two examples to illustrate the existence of sunspot equilibria. We first consider an AR(1) process before turning to an asymmetric two-state Markov process.

[8] The notation N^* in the proposition stands for the set of positive natural numbers.

[9] This result only applies to a two-state Markov process. There will, in general, be sunspot equilibria for symmetric three-state Markov processes where S_t can take on the values $-\overline{s}$, 0 and $+\overline{s}$ and similarly when there are more than three states.

B. *Autoregressive Sunspot Process*

We assume that the sunspot follows an AR(1) process:

$$S_{t+1} = \rho S_t + \epsilon_{t+1} \qquad \text{with } 0 < \rho < 1. \tag{10}$$

The innovation ϵ_{t+1} has a bounded zero-mean symmetric distribution with $\epsilon_{t+1} \in [-\bar{\epsilon}, \bar{\epsilon}]$. The variance of ϵ_{t+1} is denoted by σ^2, and the variance of ϵ_{t+1}^2 is denoted ω^2. Symmetry implies that $E_t \epsilon_{t+1}^3 = 0$. Clearly the condition in Proposition 1 is satisfied when $\rho > 0$. The condition in Proposition 2 is satisfied as well because $\mathrm{cov}(S_{t+1}^2, S_{t+1}^2) = 4\rho^2\sigma^2 S_t^2 + \omega^2$ depends on S_t.

For the purpose of the next Proposition it is convenient to define a threshold value on the dividend that insures a nonnegative asset price:

$$A_1 \equiv \frac{R - \rho^2}{4\lambda \rho^2 \sigma^2}\left(\frac{R - \rho^2}{4\rho^2 \sigma^2}\omega^2 + \sigma^2 + \frac{R - 1}{(1 - \rho)^2}\bar{\epsilon}^2\right).$$

The following Proposition shows that there exists a sunspot equilibrium.

PROPOSITION 3: *Assume that the sunspot variable S_t follows the AR process* (10) *and that $\bar{A} > A_1$. Then there are two equilibria within the class of finite polynomial solutions: the fundamental equilibrium* (8) *and a sunspot equilibrium*

$$Q_t = \tilde{Q} - VS_t^2 \tag{11}$$

where

$$V = \frac{R - \rho^2}{4\lambda \rho^2 \sigma^2} > 0 \tag{12}$$

$$\tilde{Q} = \frac{1}{R - 1}\left(\bar{A} - \lambda V^2 \omega^2 - V\sigma^2\right) < \frac{\bar{A}}{R - 1}. \tag{13}$$

PROOF:

See the Appendix.

The Proposition identifies a sunspot equilibrium for a particular AR(1) process. Note that more generally within the class of AR(1) processes there is an infinite number of such equilibria as sunspot equilibria exist for any $\rho > 0$ and an infinite number of values of σ, ω, and $\bar{\epsilon}$. Also, even for a given set of parameters, Proposition 3 is limited to solutions within the class of finite order polynomials.

To understand the existence of the sunspot equilibrium in Proposition 3, it is useful to go back to the present value relation (9). In the sunspot equilibrium, risk is time-varying:

$$\mathrm{var}_t(Q_{t+1}) = 4V^2\rho^2\sigma^2 S_t^2 + V^2\omega^2.$$

There are therefore self-fulfilling shifts in perceptions of risk. This is an equilibrium because of the circular relationship between the stochastic process of the asset price and asset price risk. If agents perceive risk to depend quadratically on the sunspot, then so does Q_t from (9). Risk then depends on the variance of S_{t+1}^2, which depends on S_t^2 when $\rho > 0$. Beliefs about risk are therefore self-fulfilling and coordinated around the sunspot variable. The asset price is lower in the sunspot equilibrium than in the fundamental equilibrium and risk is higher and more volatile.

Equation (12) shows that higher risk aversion (which implies a higher λ) reduces the sensitivity of the asset price and risk to the sunspot. When investors are highly sensitive to risk, risk does not need to move much to clear the asset market. The quadratic sunspot enters in the market clearing condition (6) both through the expected excess payoff and the variance of Q_{t+1}. The parameter V affects risk more (proportional to $V^2 S_t^2$) than the expected excess payoff (proportional to VS_t^2). After changing parameters, equilibrium is therefore primarily reestablished through a change in risk. Higher risk-aversion raises the weight on risk in equation (6) (higher λ), and thus allows the market to clear with risk being less sensitive to the quadratic sunspot (lower V).

A feature of the sunspot solution $Q_t = f(S_t)$ is that it is symmetric in the sunspot: $f(S_t) = f(-S_t)$, so that the sign of the sunspot is irrelevant. This is closely related to the symmetric conditional stochastic process for S_t, which more generally is defined as a process where S_t can take on the values $\{s_j\} \equiv \{-s_j\}, j \in J$ and $\text{prob}(S_{t+1} = s_i \,|\, S_t = s_j) = \text{prob}(S_{t+1} = -s_i \,|\, S_t = -s_j) \,\forall i,\; j \in J$. It is easy to show (see Appendix (A6)) that such a symmetric process implies that when $\text{var}_t(Q_{t+1})$ is symmetric in S_t, so will Q_t, and when Q_t is symmetric in S_t, so will $\text{var}_t(Q_{t+1})$. Symmetry is therefore preserved in the self-fulfilling loop from risk to the price and back to risk. The AR(1) process, with a symmetrically distributed innovation, is an example of such a symmetric process. We next turn to a two-state Markov process that is not symmetric, resulting in a sunspot solution that is not symmetric either.

C. *Two-State Markov Sunspot Process*

We consider the example of a two-state asymmetric Markov process. We refer to the two states as the "normal" and the "bad" state, denoted by N and B, respectively. We denote the probability of being in a state $i = N, B$ next period, conditional on being in that state today, by p_i. We assume that p_B and p_N are both between 0.5 and 1 and that the normal state is more persistent: $p_N > p_B$.[10] Of course, what agents consider as "bad" is subjective as S_t plays no fundamental role when $m = 0$.

Define $p_D = p_B(1 - p_B) - p_N(1 - p_N)$ and $\kappa = 1 + R - p_N - p_B$, which are both positive under our assumptions. We also define the following threshold value for the dividend:

$$A_2 \equiv \frac{1 - p_B}{p_D}\frac{\kappa}{\lambda}\left[\kappa\frac{p_B}{p_D} - 1\right].$$

[10] If $p_N = p_B$, S_t follows a symmetric two-state Markov process. We have already seen that this does not satisfy the necessary condition for the existence of a sunspot equilibrium in Proposition 2.

The value of the asset price in state i is denoted by Q_i. The following Proposition shows that there is exactly one sunspot equilibrium in this case.

PROPOSITION 4: *Assume that the sunspot variable S_t follows a two-state Markov process with transition probabilities p_i of staying in state $i = N, B$. Assume that $0.5 < p_B < p_N < 1$ and $\overline{A} > A_2$. Then there are two equilibria. One is the fundamental equilibrium with a unique price $Q = \overline{A}/(R - 1)$ and no risk. The second is a sunspot equilibrium with $0 < Q_B < Q_N < \overline{A}/(R - 1)$,*

$$Q_D \equiv Q_N - Q_B = \frac{\kappa}{\lambda p_D} \tag{14}$$

and

$$Q_B = \frac{1}{R - 1}\left(\overline{A} - \lambda p_B (1 - p_B) Q_D^2 + (1 - p_B) Q_D\right). \tag{15}$$

PROOF:

See the Appendix.

The sunspot solution here is in many ways similar to the one for the autoregressive sunspot process considered in Section IIIB. The asset price in the sunspot equilibrium is always lower than in the fundamental equilibrium, while risk is higher than in the fundamental equilibrium. In state i the variance of Q_{t+1} is $p_i(1 - p_i)Q_D^2$. Since $p_B(1 - p_B) > p_N(1 - p_N)$ under our assumptions, risk is higher in the bad state, which results in a lower price in the bad state. Since S_t is a pure sunspot, the higher risk when we shift from state N to state B is entirely self-fulfilling.

There is one difference though in comparison to the sunspot solution for the autoregressive process. Because the sunspot process is now no longer symmetric, the price and risk are no longer a symmetric function of the sunspot. Without loss of generality we can let $S_N = 1$ and $S_B = -1$. From Proposition 4, $Q_t(S_N) > Q_t(S_B)$, which implies asymmetry. As we will see, this asymmetry is an attractive feature when we allow m to be positive in the next section.

The intuition for the self-fulfilling risk in the sunspot equilibrium again reflects the circular relationship between the stochastic process of the asset price and asset price risk. If agents believe that asset price risk is high in state B and low in state N, then indeed it will be. It leads to a low price in state B and a high price in state N. This in turn implies that risk is higher in state B, as $p_B < p_N$ means that in state B there is more uncertainty about next period's state and therefore about next period's price. Shifts in beliefs about risk across the two states are therefore self-fulfilling.

When the increase in risk from state N to state B is very large, and the drop in the price big, we can speak of a risk panic. This is a large self-fulfilling shift in perceived risk. To illustrate this, Figure 1 shows both the asset price (left panel) and asset price risk (right panel) for a particular parameterization. Asset price risk is defined as the standard deviation of next period's asset price divided by the asset price today. It is assumed that $p_N = 0.99$, so that a switch to the bad state is quite rare. The solution is shown for different values of p_B. Independent of the value of p_B we see that

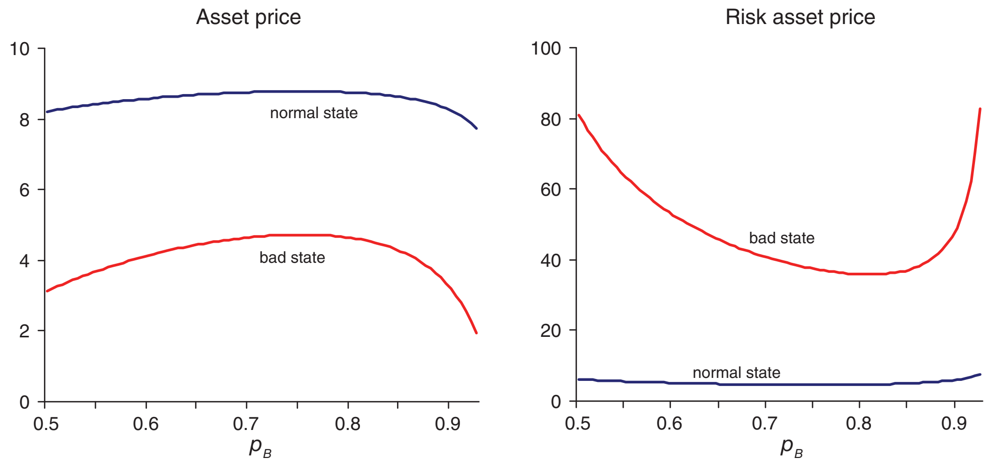

FIGURE 1. ILLUSTRATION OF RISK PANIC IN MARKOV SUNSPOT EQUILIBRIUM

Notes: The chart on the left shows the asset price in the normal state N and the bad state B as a function of the probability p_B of remaining in the bad state. The probability of remaining in the good state is held at 0.99. The chart on the right shows risk, measured as the standard deviation of the asset price next period, divided by the asset price today, in both the normal and bad states. Other than p_B, which varies along the horizontal axis, the parameterization is as follows: $\bar{A} = 1$; $R = 1.1$; $p_N = 0.99$; $\lambda = \gamma K/W = 0.5$.

a switch from state N to state B leads to an enormous spike in risk and drop in the price level.[11] For example, for $p_B = 0.7$ the risk panic involves an increase in asset price risk from 5 percent to 40 percent and a drop in the asset price by 47 percent.[12]

III. Sunspot-Like Equilibria and Risk Panics

We now turn to the case where $m > 0$, so that the state variable S_t is a fundamental that affects the dividend. We show that apart from a fundamental equilibrium there now exist so-called "sunspot-like" equilibria. In those equilibria S_t plays the dual role of a fundamental that affects the asset price through its impact on dividends and a sunspot that leads to self-fulfilling shifts in risk. We again consider the cases where S_t follows a first-order autoregressive process and an asymmetric two-state Markov process. We also consider an extension of the model where in addition to the time-varying fundamental (the dividend) there is a sunspot variable that can trigger risk panics.

[11] If p_B is too high (even if still below p_N) there is no sunspot equilibrium as the condition $\bar{A} > A_2$ is no longer satisfied.

[12] In these results, the average equity premium varies from 2 percent to 9 percent, dependent on the value of p_B. The equity premium is large here because consumption is perfectly correlated with the equity return. It obviously does depend a lot on the state, being higher in the bad state than in the good state by a factor $p_B(1 - p_B)/[p_N(1 - p_N)]$. We do not make an assumption about the rate of risk-aversion γ. Instead, we only need to choose $\lambda = \gamma K/W$, which we set at 0.5. This implies that $\gamma\alpha_t = 0.5Q_t$, which is about 4 in the good state, so that the equity portfolio share is 1 when $\gamma = 4$ or 0.4 when $\gamma = 10$.

A. *Autoregressive Dividend Process*

We assume that the process for S_t is given by (10) with the same assumptions as before. In addition we define the following threshold values for the dividend: $A_3 \equiv \max(A_{31}, A_{32}, A_{33})$, where

$$A_{31} \equiv m\overline{\epsilon}/(1-\rho)$$

$$A_{32} \equiv \lambda \frac{R^2 m^2 \sigma^2}{(R-\rho)^2} + \frac{(R-1)\, m\, \rho \overline{\epsilon}}{(R-\rho)(1-\rho)}$$

$$A_{33} \equiv \frac{R-\rho^2}{4\lambda\rho^2\sigma^2}\left(\sigma^2 + \frac{(R-1)\overline{\epsilon}^2}{(1-\rho)^2} + \frac{(R-\rho^2)\omega^2}{4\rho^2\sigma^2}\right) + \lambda\frac{\rho^2 m^2\sigma^2}{(1-\rho)^2} + \frac{(R-1)m\overline{\epsilon}}{(1-\rho)^2}.$$

PROPOSITION 5: *Assume that S_t follows the AR process* (10). *Also assume that* $\overline{A} > A_3$. *Then there are two equilibria within the class of finite polynomials. The first is a fundamental equilibrium:*

$$Q_t = \frac{1}{R-1}\left(\overline{A} - \lambda\frac{R^2 m^2\sigma^2}{(R-\rho)^2}\right) + \frac{m\rho}{R-\rho}S_t. \tag{16}$$

The second is a sunspot-like equilibrium:

$$Q_t = \tilde{Q} + vS_t - VS_t^2 \tag{17}$$

where

$$V = \frac{R-\rho^2}{4\lambda\rho^2\sigma^2} \tag{18}$$

$$v = -\frac{m}{1-\rho} \tag{19}$$

$$\tilde{Q} = \frac{1}{R-1}\big(\overline{A} - \lambda\big(V^2\omega^2 + (v+m)^2\sigma^2\big) - V\sigma^2\big). \tag{20}$$

PROOF:

See the Appendix.

In this case, Q_t is obviously affected by S_t even in the fundamental equilibrium. To see the contrast between the fundamental and the sunspot-like equilibrium, consider the present value equation (7), which in this case becomes

$$Q_t = \frac{1}{R-1}\overline{A} + \frac{m\rho}{R-\rho}S_t - \lambda\sum_{i=1}^{\infty}\frac{1}{R^i}E_t\mathrm{var}_{t+i-1}(Q_{t+i} + A_{t+i}). \tag{21}$$

In the fundamental equilibrium the asset payoff risk $\text{var}_t(Q_{t+1} + A_{t+1}) = (mR\sigma)^2/(R - \rho)^2$ is constant, so that the last term in (21) is constant. The asset price then depends positively on S_t with coefficient $m\rho/(R - \rho)$. It is more sensitive to dividend shocks when the dividend is more persistent (higher ρ). The impact of S_t on the asset price vanishes to zero as its fundamental impact becomes small $(m \rightarrow 0)$.

In the sunspot-like equilibrium, S_t plays the dual role of a fundamental and a sunspot that leads to time-varying beliefs about risk. Its fundamental role is still captured by the second term of (21) that depends positively on S_t. Its sunspot role is captured by the present value of time-varying risk through the last term of (21). We have

$$\text{var}_t(Q_{t+1} + A_{t+1}) = (v + m - 2\rho V S_t)^2 \sigma^2 + V^2 \omega^2. \tag{22}$$

This time-varying risk is self-fulfilling as it does not go away when the fundamental role of S_t vanishes with $m \rightarrow 0$. This is because V does not depend on m. The coefficient on the quadratic term is in fact the same as in the pure sunspot equilibrium. When $m \rightarrow 0$ the sunspot-like equilibrium converges to the pure sunspot equilibrium in Proposition 3. The main difference with Section III is that when $m > 0$ the self-fulfilling shifts in beliefs about risk are now coordinated around a macro fundamental rather than an external sunspot variable.

Note also that the linear coefficient on S_t is negative in the sunspot-like equilibrium (19). This is because the positive linear term in S_t in (21), associated with its fundamental role, is more than offset by the linear dependence of risk (22) on S_t that captures self-fulfilling beliefs about risk. Together with the negative quadratic term in S_t, it is therefore clear that the sunspot role dominates the fundamental role in the sunspot-like equilibrium.

Although in a very different context, not involving time-varying shifts in risk, Spear, Srivastava, and Woodford (1990, p. 281), and Manuelli and Peck (1992) also present models with sunspot-like equilibria. Spear, Srivastava, and Woodford (1990) point out that "… a sharp distinction between "sunspot equilibria" and "non sunspot equilibria" is of little interest in the case of economies subject to stochastic shocks to fundamentals." Indeed, as we raise m slightly above 0, the sunspot-like equilibrium is technically no longer a pure sunspot equilibrium, but it is effectively indistinguishable.

There is one unattractive aspect of the sunspot-like equilibrium. This relates to the symmetric process of the sunspot. As we saw in Section IIIB, this symmetric process leads to a symmetric solution for the asset price and risk when $m = 0$, which means that the sign of S_t is irrelevant. Risk is equally high when S_t is a big positive number as when it is an equally big negative number. This carries over to sunspot-like equilibria. When m is only slightly above zero, the solution is virtually identical to the sunspot solution. Symmetry of the sunspot solution then implies that risk is equally high, and the price equally low, for a large positive S_t (good fundamental) as for an equally large negative S_t (bad fundamental).

This unattractive feature continues to hold for any positive m due to the dominance of the sunspot role of S_t. In general, we have

$$\text{var}_t(Q_{t+1}) = (v - 2\rho V S_t)^2 \sigma^2 + V^2 \omega^2. \tag{23}$$

Since $v < 0$, this implies that risk is actually highest, and the asset price is lowest, when the fundamental is strongest. Related to this, a rise in dividends always lowers the asset price when $S_t > 0$. However, this unappealing result is not a general feature of sunspot-like equilibria, but is closely connected to the symmetric process for the sunspot. It does not occur when S_t follows an asymmetric two-state Markov process, which we turn to next.

B. *Two-State Markov Dividend Process*

The asymmetric two-state Markov process is analogous to that in Section IIIC, with the difference that the dividend is higher in the normal state than in the bad state: $A_N > A_B$. We define $A_D = A_N - A_B$, which converges to zero when $m \to 0$. The assumptions on the switching probabilities are as in Section IIIC, and we define the following threshold for the dividend:

$$A_4 = \frac{1 - p_B}{p_D}\Bigg(-p_B R A_D + \frac{p_B(R - p_N) + p_N(1 - p_N)}{2\lambda p_D} \times \left[\kappa + \sqrt{\kappa^2 - 4R\lambda p_D A_D}\right]\Bigg).$$

The equilibria are then given by the following proposition.

PROPOSITION 6: *Assume that the fundamental A_t follows a two-state Markov process. It takes on value A_i in state $i = N, B$, with transition probability p_i of remaining in state i. Assume that* $0.5 < p_B < p_N < 1, A_D < \frac{\kappa^2}{4R\lambda p_D}$ *and* $A_B > A_4$. *Then there are two equilibria. The values of the asset price difference $Q_D = Q_N - Q_B$ in the two equilibria are*

$$Q_D = \left[\frac{\kappa}{2\lambda p_D} - A_D\right] \pm \frac{1}{2\lambda p_D}[\kappa^2 - 4R\lambda p_D A_D]^{0.5} > 0. \tag{24}$$

Corresponding to each value of Q_D, the asset price in state B is

$$Q_B = \frac{A_B}{R - 1} + \frac{(1 - p_B)p_N(p_N + p_B - 1)A_D - (1 - p_B)[p_B(R - p_N) + p_N(1 - p_N)]Q_D}{p_D(R - 1)}. \tag{25}$$

PROOF:

See the Appendix.

Since Q_D is positive, the asset price is higher in the normal state than in the bad state. We refer to the equilibrium where Q_D takes on its lowest value (with the minus sign between the two terms in (24)) as the fundamental equilibrium, and to the other as the sunspot-like equilibrium.

In the fundamental equilibrium, the asset price differs between the normal and bad states only if the dividend differs between these two states ($Q_D \rightarrow 0$ when $A_D \rightarrow 0$). By contrast, the asset price differs across the two states in the sunspot-like equilibrium even when the dividend does not. When A_D goes to zero, Q_D converges to $\kappa(\lambda p_D)^{-1}$, which is its value in the pure sunspot equilibrium (14) in Section IIIC.

In the fundamental equilibrium asset payoff risk in state $i = N, B$ is written as

$$(26) \quad \mathrm{var}_t(Q_{t+1} + A_{t+1}) = p_i(1 - p_i)\frac{1}{(2\lambda p_D)^2}\left[\kappa - [\kappa^2 - 4R\lambda p_D A_D]^{0.5}\right]^2.$$

Equation (26) shows that risk is higher in the bad state as $p_B(1 - p_B) > p_N(1 - p_N)$, which reflects our assumption that the fundamental is riskier (i.e., more likely to change) in the bad state.[13] Risk goes to zero when the dividend becomes identical in the two states ($A_D \rightarrow 0$). The higher price in the normal state in the fundamental equilibrium follows both from the higher expected dividend in the normal state and the lower risk.

The determinants of risk are different in the sunspot-like equilibrium. Asset payoff risk in state i is now written as

$$(27) \quad \mathrm{var}_t(Q_{t+1} + A_{t+1}) = p_i(1 - p_i)\frac{1}{(2\lambda p_D)^2}\left[\kappa + [\kappa^2 - 4R\lambda p_D A_D]^{0.5}\right]^2.$$

It is still the case that risk is higher in the bad state than in the good state. This is however no longer because of the exogenously higher fundamental risk in the bad state. To the contrary, (27) shows that asset payoff risk increases in both states when fundamental risk declines ($A_D \rightarrow 0$). In the sunspot-like equilibrium risk is self-fulfilling and the main role of the fundamental S_t is as a focal point for these self-fulfilling shifts in risk.

A higher fundamental unambiguously implies a higher price and lower risk in the sunspot-like equilibrium. This is because the process for S_t is no longer symmetric as was the case for the autoregressive process. As a result the symmetry between good and bad states that we found there does not apply here.

C. *Switching Equilibria and Risk Panics*

Finally, we consider a situation that combines elements of both the sunspot and sunspot-like equilibria analyzed so far. In addition to the Markov process for the fundamental between states B and N in the previous subsection, we extend the model by introducing a two-state sunspot variable, with the states indexed as 1 and 2. The dividend is not affected by whether we are in state 1 or 2.

The presence of the two-state sunspot variable allows for a richer analysis of risk panics. It is useful to think of the sunspot in this case as a trigger variable that shifts expectations between low risk in state 1 and high risk in state 2. One can think of state 2 as a "panic state", so that the switch to that state implies a large spike in risk and drop in the asset price. We will show that while this panic is not caused by a change in

[13] The variance of A_{t+1} is $p_i(1 - p_i)A_D^2$ in state i.

the fundamental (the dividend), the magnitude of the panic depends critically on the level of the fundamental. This feature is absent from the equilibria considered thus far.

The sunspot state variable is assumed to be uncorrelated with the fundamental. In either state 1 or 2, the probability of remaining in the same state next period is $p > 0.5$. The model now has four possible states, depending on the value of the sunspot and the fundamental: $(N,1)$, $(N,2)$, $(B,1)$, and $(B,2)$. We define the asset prices in these states as, respectively, $Q_N(1)$, $Q_N(2)$, $Q_B(1)$, and $Q_B(2)$. We solve the asset prices in each state by imposing a market clearing condition for each state. We also define $Q_D(i) = Q_N(i) - Q_B(i)$ for $i = 1,2$.

In this case, the equilibria involve longer expressions that relate the asset price in each state to model parameters, which are fully described in the Appendix. Instead, the proposition below focuses on some key signs that characterize the new equilibrium that results from this setting. In the Appendix we define a cutoff A_5 for A_B and A^{max} for A_D. There is also a critical value for p:

$$\overline{p} = \frac{3R + 1 - p_N - p_B}{4R + 2 - 2p_N - 2p_B},$$

which is between 0.75 and 1.

PROPOSITION 7: *Assume that the fundamental A_t follows a two-state Markov process as in Proposition* 6. *Also assume that $A_D < A^{\max}$ and that $A_B > A_5$. Then there are four equilibria when $\overline{p} < p < 1$. The first two equilibria are the same as the fundamental and sunspot-like equilibria in Proposition* 6, *regardless of whether we are in state* 1 *or* 2. *In the third equilibrium we have*

$$Q_D(2) > Q_D(1) > 0 \tag{28}$$

$$Q_B(2) - Q_B(1) < Q_N(2) - Q_N(1) < 0. \tag{29}$$

Equilibrium 4 *is analogous to equilibrium* 3, *with the role of states* 1 *and* 2 *switched.*

PROOF:

See the Appendix.

Equilibrium 3 is the novel result in Proposition 7. A switch from state 1 to 2 involves an increase in risk and a drop in the asset price. Proposition 7 states that asset prices are lower in state 2: $Q_B(2) < Q_B(1)$ and $Q_N(2) < Q_N(1)$. This is associated with an increase in risk as there is no change in the expected level of the dividend. A numerical illustration below shows that the increase in risk and drop in the price can be very large, in which case we can speak of a risk panic.

We show in the Appendix that when p approaches 1, equilibrium 3 is such that state 1 converges to the fundamental equilibrium in Proposition 6, while state 2 converges to the sunspot-like equilibrium. A switch from state 1 to state 2 then implies a switch from the fundamental to the sunspot-like equilibrium of Proposition 6. When $p < 1$ a switch from state 1 to state 2 is not exactly a switch between the two equilibria of Proposition 6, as the very possibility of a switch increases uncertainty when we are in state 1. State 1 is then characterized by higher risk and a lower asset price than in the fundamental equilibrium in Proposition 6.

Three features characterize the role of the fundamental during and after a risk panic. First, the panic itself is not caused by a change in the fundamental, as it consists of a switch between state 1 and state 2, and not between state N and state B. Second, the panic has a larger impact when the fundamental is weak to start with. It follows from Proposition 7 that the drop in the asset price in a panic (a move from state 1 to 2) is larger when the fundamental is bad (state B) than when it is normal (state N). Finally, after a panic (once we are in state 2) the asset price becomes more volatile. As $Q_D(2) > Q_D(1)$, the asset price is more sensitive to changes in the fundamental between states N and B when we are in state 2.

Even though the panic is not caused by a change in the fundamental, the last two results show that the fundamental plays a key role as a focal point for expectations that affects both the magnitude of the panic itself, and subsequent shifts in perceived risk. When p is close to 1 we can think of the role of S_t as suddenly changing from that of a pure fundamental to that of a sunspot-like variable around which agents coordinate their perceptions of risk.

Figure 2 provides a numerical illustration of these results for a particular parameterization. The probability of staying in the normal state is $p_N = 0.99$, while the probability of staying in the bad state is $p_B = 0.9$. This means that the fundamental is 91 percent of the time in the normal state and 9 percent of the time in the bad state. The probability that the sunspot variable remains the same is $p = 0.99$. If we are currently in the low-risk state 1, the probability of switching to panic state 2 is then only 0.01.

Figure 2 considers the following experiment. We start in state $(N, 1)$ in periods 0 and 1, where the dividend is at its normal value of $A_N = 1$ and we are in the nonpanic state 1. Then at time 2 the dividend drops by 10 percent to its value $A_B = 0.9$, but we remain in the nonpanic state 1. In period 3 we switch to the panic state 2 while the fundamental remains weak at $A_B = 0.9$. In period 4 the fundamental is restored to its normal level of $A_N = 1$ but we remain in the panic state 2. Finally, starting with period 5 we return to state $(N, 1)$. Figure 2 reports both the asset price (normalized to 100 in state $(N, 1)$) and asset price risk. The latter is the standard deviation of the asset price next period, divided by the asset price today.

We see that the deterioration of the fundamental in period 2 lowers the asset price by about 13 percent. About a third of that is a result of the lower expected future dividend while the rest is the result of the exogenous increase in risk. We see though that risk spikes much more in period 3 when the economy is hit by a risk panic (a switch to state 2). This causes a much sharper additional drop in the asset price, lowering it to a level 56 percent below its starting point. What is key for this really bad outcome is that both the fundamental is weak (we are in state B) and the economy is hit by a self-fulfilling risk panic. In period 4, when the fundamental is restored to its normal level, the asset price is way up again (only 10 percent below its starting point), even though we are still in the panic state.

Figure 2 therefore illustrates that the level of the fundamental plays a key role during a risk panic. The panic is much larger when the fundamental is weak at the time of the panic. Moreover, once we reach the panic state, the asset price becomes

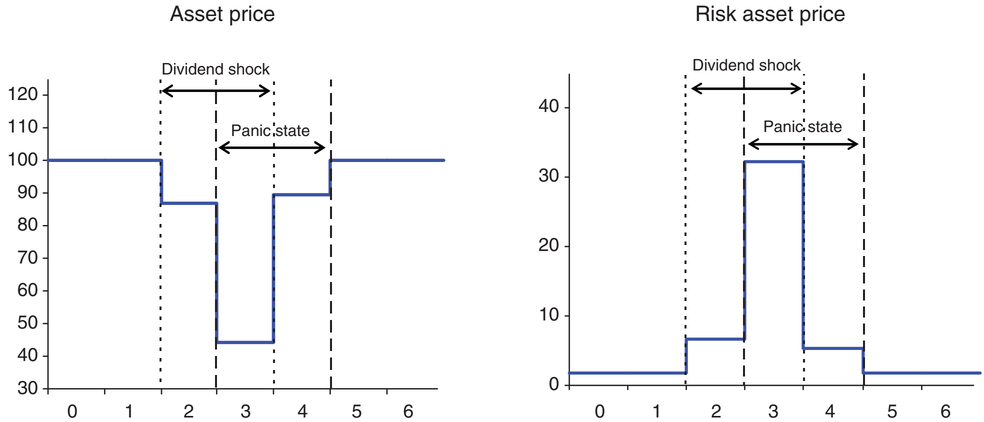

Figure 2. Illustration of Risk Panic in Switching Equilibrium

Notes: Vertical slashed lines = state 2 (panic state); vertical dotted lines = dividend shock. In periods 0 and 1 the economy is in state $(N, 1)$, where the dividend is high and there is no panic (state 1). At time 2 the dividend drops by 10 percent from 1 to 0.9. At time 3 there is a shift to state 2 (panic state). At time 4 the dividend rises back to 1, but we remain in the panic state. Starting with date 5 we return to state $(N, 1)$ where the dividend is high and there is no panic. The parameterization is as follows: $A_N = 1$; $A_B = 0.9$; $R = 1.1$; $p_N = 0.99$; $p_B = 0.9$; $p = 0.99$; $\lambda = \gamma K/W = 0.5$.

much more sensitive to changes in the fundamental. An improvement in the fundamental from state B to state N raises the asset price much more when we are in the panic state (compare period 4 to period 3) than when we are in the nonpanic state (compare period 1 to period 2). Rather than a regular fundamental, S_t becomes a gauge of fear when we switch to the panic state.

IV. Conclusion

We have developed a very simple mean-variance portfolio choice model to show that self-fulfilling shifts in risk, coordinated around either a sunspot or a macro fundamental, can occur in equilibrium. This is a result of a circular relationship between the process of asset price risk and the asset price itself. The analysis was motivated by large changes in asset price risk during recent financial crises. We have shown that the model can give rise to significant risk panics that take the form of a large sudden spike in risk and drop in the asset price. The magnitude of such panics can be particularly large when a macro fundamental is weak.

The simple model can be extended in various directions. In the working paper version (Bacchetta, Tille, and van Wincoop 2010), we consider a richer model with leveraged financial institutions and households. The richer setup enables us to examine several extensions to the basic model. We allow the interest rate to be determined endogenously and introduce another state variable, the wealth of leveraged financial institutions. We show that these extensions do not substantially change the results. In this case it is the net worth of leveraged financial institutions that becomes the focal point of fear in the market during a panic. It does not matter much for the results exactly which macro fundamental plays this role. We also consider two additional variables, leverage and market liquidity, which both collapse during a risk panic.

The analysis raises further questions that deserve to be addressed in future research. First, one can ask what happens when we consider multiple risky assets. For example, will all asset prices be equally affected by the panic? This question is particularly relevant when considering the global dimension of recent financial crises, with stock markets usually changing in lockstep around the world. In Bacchetta and van Wincoop (2012) we have started to address this question. Second, what are the policy implications? Is there anything that regulators can do to reduce the magnitude of risk panics? In Bacchetta, Tille, and van Wincoop (2011) we argue that a policy aimed at penalizing balance sheet risk exposure of financial institutions can help. Finally, it would be of interest to enrich the model to introduce other features often seen during financial panics, such as bank runs and significant contractions of the real economy.

Appendix

PROOF OF PROPOSITION 3:

First, conjecture that the solution is $Q_{t+1} = \tilde{Q} - VS_{t+1}^2$. Using (10), we have $Q_{t+1} = \tilde{Q} - V\rho^2 S_t^2 - 2V\rho S_t \epsilon_{t+1} - V\epsilon_{t+1}^2$. The expectation and variance of Q_{t+1} are therefore

$$E_t Q_{t+1} = \tilde{Q} - V\rho^2 S_t^2 - V\sigma^2$$

$$\mathrm{var}_t(Q_{t+1}) = 4V^2\rho^2\sigma^2 S_t^2 + V^2\omega^2.$$

Notice that we used the fact that $E_t \epsilon_{t+1}^3 = 0$ given the symmetry of the distribution. Substituting these into the market clearing condition (6) implies

$$\overline{A} + \tilde{Q} - V\rho^2 S_t^2 - V\sigma^2 - R\tilde{Q} + RVS_t^2 = \lambda(4V^2\rho^2\sigma^2 S_t^2 + V^2\omega^2).$$

Equating the constant terms on the left and right hand side, as well as the terms proportional to S_t^2, gives

$$\overline{A} + \tilde{Q}(1 - R) - V\sigma^2 = \lambda V^2\omega^2$$

$$V(R - \rho^2) = \lambda 4V^2\rho^2\sigma^2.$$

This has two solutions. One is the fundamental equilibrium where $V = 0$ and $\tilde{Q} = \overline{A}/(R - 1)$. The other is the sunspot equilibrium where V and $\tilde{Q}$ are as in (12) and (13). The condition $\overline{A} > A_1$ implies that Q_t is nonnegative. The lowest value of the asset price is reached when ϵ_t is constant at $\overline{\epsilon}$ or $-\overline{\epsilon}$. In that case $(S_t)^2$ reaches its maximum value of $\overline{\epsilon}^2/(1 - \rho)^2$ and Q_t its lowest value of $\tilde{Q} - V\overline{\epsilon}^2/(1 - \rho)^2$. Substituting the values for $\tilde{Q}$ and V, this is positive when $\overline{A} > A_1$. Finally, it is clear that the solution satisfies the no bubble condition $\lim_{T\to\infty} E_t(1/R)^T Q_{t+T} = 0$ as Q_t is always between 0 and $\overline{A}/(R - 1)$.

Moreover, the sunspot equilibrium (11) is the only one within the class of finite polynomial functions. Assume that the solution is

$$Q_t = \sum_{i=1}^{n} \alpha_i S_t^i \tag{A1}$$

so that $Q_{t+1} = \sum_{i=1}^{n} \alpha_i(\rho S_t + \epsilon_{t+1})^i$. Using the binomial theorem, the term in $\mathrm{var}_t(Q_{t+1})$ with the highest power of S_t is the variance of the cross-product $\alpha_n n \rho^{n-1} S_t^{n-1} \epsilon_{t+1}$, which is

$$\alpha_n^2 n^2 \rho^{2(n-1)} \sigma^2 S_t^{2(n-1)}.$$

Hence, there is a term in the market-clearing condition with the power $2(n - 1)$. This is consistent with the conjectured solution (A1) only for $n = 2$. It follows that $\alpha_n = 0$ for all $n > 2$.

PROOF OF PROPOSITION 4:

If we are in state N at time t, then

$$E_t Q_{t+1} = p_N Q_N + (1 - p_N) Q_B \tag{A2}$$

$$\mathrm{var}_t(Q_{t+1}) = p_N(1 - p_N)(Q_N - Q_B)^2. \tag{A3}$$

Similarly, if we are in state B at time t, then

$$E_t Q_{t+1} = p_B Q_B + (1 - p_B) Q_N \tag{A4}$$

$$\mathrm{var}_t(Q_{t+1}) = p_B(1 - p_B)(Q_N - Q_B)^2. \tag{A5}$$

Substituting these results into (6), the market clearing conditions in, respectively, states N and B can we written as

$$\overline{A} + p_N Q_N + (1 - p_N) Q_B - R Q_N = \lambda p_N (1 - p_N)(Q_N - Q_B)^2 \tag{A6}$$

$$\overline{A} + p_B Q_B + (1 - p_B) Q_N - R Q_B = \lambda p_B (1 - p_B)(Q_N - Q_B)^2. \tag{A7}$$

Taking the difference between these two relations, we have

$$\kappa Q_D = \lambda p_D Q_D^2. \tag{A8}$$

This has two solutions. The first is $Q_D = 0$, which gives the fundamental equilibrium $Q_N = Q_B = \overline{A}/(R - 1)$. The second solution is the sunspot equilibrium where $Q_D = \kappa/(\lambda p_D)$. Substituting this into the market clearing condition for state B, using that $Q_N = Q_B + Q_D$, we get (15). Using the expression for Q_D, Q_B is positive when $\overline{A} > A_2$. Since Q_N is larger than Q_B, it is positive as well. The no bubble condition is clearly satisfied as the asset price can take on only two finite values.

We can also show that the asset price is always higher than in the fundamental equilibrium. For this it is sufficient to show that $Q_N < \overline{A}/(R-1)$. Using (15) and $Q_N = Q_B + Q_D$, this condition is satisfied when $-\lambda p_B(1-p_B)Q_D + (R-p_B) < 0$. After substituting the expression for Q_D, the left hand side becomes $-(1-p_N) \times (p_B(1-p_B) + (R-p_B)p_N)/p_D$, which is indeed negative.

PROOF OF PROPOSITION 5:

First, conjecture the solution $Q_t = \tilde{Q} + vS_t - VS_t^2$. From the process (10), we have $Q_{t+1} + A_{t+1} = \tilde{Q} + \overline{A} + (v+m)\rho S_t - V\rho^2 S_t^2 + (v+m-2V\rho S_t)\epsilon_{t+1} - V\epsilon_{t+1}^2$. The expectation and variance of $Q_{t+1} + A_{t+1}$ are therefore

$$E_t(Q_{t+1} + A_{t+1}) = \tilde{Q} + \overline{A} + (v+m)\rho S_t - V\rho^2 S_t^2 - V\sigma^2$$

$$\mathrm{var}_t(Q_{t+1} + A_{t+1}) = (v+m-2V\rho S_t)^2\sigma^2 + V^2\omega^2.$$

Substituting these into the market clearing condition (6) implies

$$\tilde{Q} + \overline{A} + (v+m)\rho S_t - V\rho^2 S_t^2 - V\sigma^2 - R\tilde{Q} - RvS_t + RVS_t^2$$
$$= \lambda\big((v+m-2\rho VS_t)^2\sigma^2 + V^2\omega^2\big).$$

Equating the terms proportional to S_t^2, S_t and constant terms on the left and right hand side gives

$$V(R-\rho^2) = \lambda 4V^2\rho^2\sigma^2$$

$$m\rho + v(\rho - R) = -\lambda 4(v+m)V\rho\sigma^2$$

$$\overline{A} + \tilde{Q}(1-R) - V\sigma^2 = \lambda\big((v+m)^2\sigma^2 + V^2\omega^2\big).$$

The first equation implies that either $V = 0$ or $V = \frac{R-\rho^2}{4\lambda\rho^2\sigma^2}$. When $V = 0$ the other two equations imply that $v = \frac{m\rho}{R-\rho}$ and $\tilde{Q} = \frac{1}{R-1}\big(\overline{A} - \lambda\frac{R^2m^2\sigma^2}{(R-\rho)^2}\big)$. This is the fundamental equilibrium. When $V = \frac{R-\rho^2}{4\lambda\rho^2\sigma^2}$, the other two equations imply that v and $\tilde{Q}$ are as in, respectively, (19) and (20). This is the sunspot-like equilibrium.

The lowest value that the dividend can take is when S_t is at its lowest value $-\overline{\epsilon}/(1-\rho)$. The dividend is always positive when $\overline{A} > A_{31}$. In the fundamental equilibrium, the lowest value that the asset price can take is when S_t is at its lowest value $-\overline{\epsilon}/(1-\rho)$. The asset price is then positive when $\overline{A} > A_{32}$. In the sunspot-like equilibrium the lowest value that the asset price can take is when S_t is at its highest value $\overline{\epsilon}/(1-\rho)$. The asset price is then positive when $\overline{A} > A_{33}$. Therefore the condition $\overline{A} > A_3$ guarantees that the dividend and the asset prices in both equilibria are always positive.

The no bubble condition is clearly satisfied as well, since the asset price is bounded in both equilibria because S_t is bounded. Finally, the same reasoning as in

Proposition 3 can be used to show that the linear and quadratic solutions are the only ones among the class of finite polynomials.

PROOF OF PROPOSITION 6:

If we are in state N at time t, then

$$E_t(Q_{t+1} + A_{t+1} = p_N(Q_N + A_N) + (1 - p_N)(Q_B + A_B) \tag{A9}$$

$$\mathrm{var}_t(Q_{t+1} + A_{t+1}) = p_N(1 - p_N)(A_D + Q_D)^2. \tag{A10}$$

Similarly, if we are in state B at time t, then

$$E_t(Q_{t+1} + A_{t+1}) = (1 - p_B)(Q_N + A_N) + p_B(Q_B + A_B) \tag{A11}$$

$$\mathrm{var}_t\,(Q_{t+1} + A_{t+1}) = p_B(1 - p_B)(A_D + Q_D)^2. \tag{A12}$$

Substituting these results into (6), using $Q_N = Q_B + Q_D$ and $A_N = A_B + A_D$, the market clearing conditions in, respectively, states N and B can we written as

$$p_N[A_D + Q_D] - RQ_D + A_B - (R - 1)Q_B = \lambda p_N(1 - p_N)[A_D + Q_D]^2$$

$$(1 - p_B)[A_D + Q_D] + A_B - (R - 1)Q_B = \lambda p_B(1 - p_B)[A_D + Q_D]^2.$$

Taking the difference, defining $x = A_D + Q_D$, we have

$$\lambda p_D x^2 - \kappa x + RA_D = 0. \tag{A13}$$

This quadratic polynomial has two solutions when $A_D < \frac{\kappa^2}{4R\lambda p_D}$. These two solutions are

$$x = \frac{1}{2\lambda p_D}\left(\kappa \pm [\kappa^2 - 4R\lambda p_D A_D]^{0.5}\right).$$

Using $Q_D = x - A_D$, this implies (24). The corresponding values of Q_B can be found from the market clearing condition for state B. Replacing $A_D + Q_D$ with x and using $\lambda p_D x^2 = \kappa x - RA_D$, the state B market clearing condition becomes

$$Q_B = \frac{A_B}{R - 1} + \frac{1 - p_B}{(R - 1)p_D}((p_D - p_B\kappa)x + Rp_BA_D). \tag{A14}$$

This implies (25), using $x = Q_D + A_D$ and $p_D - p_B\kappa = -(p_B(R - p_N) + p_N(1 - p_N))$ from the definition of κ.

The only thing that remains to be checked is that the asset price is always positive. First note that Q_D is positive in both equilibria. This follows from the polynomial in x, which implies $x = \frac{\lambda p_D}{\kappa}x^2 + \frac{R}{\kappa}A_D$, so that $Q_D = x - A_D = \frac{\lambda p_D}{\kappa}x^2 + \frac{1}{\kappa}(R - \kappa)A_D$. This is positive because $A_D > 0$ and $R - \kappa = p_N + p_B - 1 > 0$. The asset price is then guaranteed to always be positive when it is positive at the lowest value of Q_B. Substituting the higher of the two roots of x into (A14), the resulting expression is larger than 0 when $A_B > A_4$.

PROOF OF PROPOSITION 7:

We start from the market clearing condition (6), with one condition for each of the four states $(N,1)$, $(N,2)$, $(B,1)$, and $(B,2)$. We rewrite this set of four market clearing conditions as a set of three relative market clearing conditions (taking differences between the market clearing conditions) plus the market clearing condition for the state $(N,1)$. The relative market clearing conditions only contain relative asset payoffs across the various states. Here payoff is defined as the sum of the price and the dividend. We denote the relative payoff between states $(N,1)$ and $(B,1)$ as $x = Q_D(1) + A_D$, and the relative payoff between states $(N,2)$ and $(B,2)$ as $y = Q_D(2) + A_D$. The relative payoff between states $(B,1)$ and $(B,2)$ is $Q_B(1) - Q_B(2)$. These three unknowns are solved using a system of three relative market clearing conditions. The first is the difference between the market clearing condition (6) in state $(N,1)$ and its counterpart in state $(B,1)$:

$$\text{(A15)}\quad (p[1 - p_N - p_B] + R)x + (1-p)[1 - p_N - p_B]y - RA_D$$
$$= \lambda p_D p x^2 + \lambda p_D (1-p) y^2 + \lambda p(1-p)$$
$$\times \Big[[(1-p_B)^2 - (p_N)^2](x-y)^2 + 2[1 - p_N - p_B](x-y)$$
$$\times [Q_B(1) - Q_B(2)]\Big].$$

The second is the difference between the market clearing condition in state $(N,2)$ and its counterpart in state $(B,2)$:

$$\text{(A16)}\quad (1-p)[1 - p_N - p_B]x + (p[1 - p_N - p_B] + R)y - RA_D$$
$$= \lambda p_D(1-p)x^2 + \lambda p_D p y^2 + \lambda p(1-p)$$
$$\times \Big[[(1-p_B)^2 - (p_N)^2](x-y)^2 + 2[1 - p_N - p_B](x-y)$$
$$\times [Q_B(1) - Q_B(2)]\Big].$$

The third is the difference between the market clearing condition in state $(B,1)$ and its counterpart in state $(B,2)$:

$$\text{(A17)}\quad (2p-1)(1-p_B)(x-y) - [R - (2p-1)][Q_B(1) - Q_B(2)]$$
$$= \lambda(2p-1)p_B(1-p_B)(x+y)(x-y).$$

These three equations together give the solution for all relative prices. To obtain the solution for absolute prices, we impose the market clearing condition (6) for state $(N,1)$:

$$\text{(A18)}\quad p\left[p_N(Q_N(1)+A_N)+(1-p_N)(Q_B(1)+A_B)\right]+(1-p)$$
$$\times\left[p_N(Q_N(2)+A_N)+(1-p_N)(Q_B(2)+A_B)\right]-RQ_N(1)$$
$$=\lambda p_N(1-p_N)\left(px^2+(1-p)y^2\right)+\lambda p(1-p)$$
$$\times\left[p_N(x-y)+[Q_B(1)-Q_B(2)]\right]^2.$$

Consider the solution for relative prices. We start by taking the difference between (A15) and (A16):

$$\text{(A19)}\quad \left(R+(2p-1)[1-p_N-p_B]\right)(x-y)=\lambda(2p-1)p_D(x+y)(x-y).$$

One solution of (A19) is $x=y$. Together with (A15), this implies $\lambda p_D x^2-\kappa x+RA_D=0$, which has two solutions:

$$\text{(A20)}\qquad x=y=\frac{\kappa\pm[\kappa^2-4R\lambda p_D A_D]^{0.5}}{2\lambda p_D}.$$

(A17) then implies $Q_B(1)-Q_B(2)=0$. This corresponds to the first two equilibria in Proposition 7. In each equilibrium the asset price is the same in state 1 as in state 2, and only depends on whether we are in N or B. The two equilibria correspond to the fundamental and sunspot-like equilibria of Proposition 6. Using $Q_N(1)+A_N=x+Q_B(1)+A_B$, $Q_N(2)+A_N=y+Q_B(2)+A_B$, $Q_N(1)=Q_B(1)+x-A_D$, and $\lambda p_D x^2-\kappa x+RA_D=0$, (A18) implies the expression for $Q_B(1)$ in Proposition 6.

When $x\neq y$ (A19) implies that $x+y=[R+(2p-1)(1-p_N-p_B)]\times[\lambda(2p-1)p_D]^{-1}$. (A17) then implies $Q_B(1)-Q_B(2)=\delta(y-x)$ where

$$\delta=(1-p_B)\frac{(2p-1)p_N(1-p_N-p_B)+p_BR}{p_D(1+R-2p)}>1.$$

$\delta>1$ when $p=0.5$. As δ is an increasing function of p, δ is always above 1.

The sum of (A15) and (A16) implies that

$$\text{(A21)}\qquad y-x=Q_D(2)-Q_D(1)=\pm\left[\eta\left(\kappa(x+y)-2RA_D\right.\right.$$
$$\left.\left.-0.5\lambda p_D(x+y)^2\right)\right]^{0.5},$$

where

$$\eta = \frac{1}{\frac{1}{2}\lambda p_D + 2\lambda p(1 - p)(1 - p_N - p_B)(1 - p_B + p_N - 2\delta)}.$$

An equilibrium exists only if the bracket in (A21) is positive. As $\delta > 1$ we have $1 - p_B + p_N - 2\delta < -p_B - (1 - p_N) < 0$, and η is thus positive as p_N and p_B are both above 0.5. The numerator of the bracket in (A21) is positive when

$$A_D < A^{\max} = \left[\kappa(x + y) - 0.5\lambda p_D(x + y)^2\right](2R)^{-1}.$$

A necessary (but not sufficient) condition for this is $p > \overline{p}$, where $\overline{p}$ is defined above Proposition 7. Equilibrium 3 is the value of (A21) with the positive sign on the right-hand side and equilibrium 4 is the value with the negative sign.

We now ensure that asset prices are positive in all equilibria. We focus on equilibrium 3 as equilibrium 4 is analogous. We first show that $Q_B(2)$ is the lowest value of the asset price in equilibrium 3. This equilibrium corresponds to the value of (A21) with the positive sign on the right-hand side, so we have $Q_D(2) > Q_D(1)$ and $Q_B(1) - Q_B(2) = \delta(Q_D(2) - Q_D(1)) > 0$. We can also show that $Q_D(1) > 0$ (the algebra for this is a bit lengthy and available on request). It then follows that $Q_D(2) > 0$, so that $Q_N(2) > Q_B(2)$. Also, it follows that $Q_N(1) > Q_B(1) > Q_B(2)$. It is therefore sufficient to show that $Q_B(2)$ is positive. Before we do so, note that these inequalities imply the inequalities in Proposition 7 for equilibrium 3. We have $Q_D(2) > Q_D(1) > 0$. We also have $Q_B(2) - Q_B(1) = -\delta(y - x)$ and $Q_N(2) - Q_N(1) = (y - x) + Q_B(2) - Q_B(1) = (1 - \delta)(y - x)$. It follows that $Q_B(2) - Q_B(1) < Q_N(2) - Q_N(1) < 0$.

We can solve for $Q_B(2)$ from the asset market clearing condition (A18):

$$Q_B(2) = \frac{A_B + \nu}{R - 1},$$

where

$$\text{(A22)} \qquad \nu = RA_D + (p - R)\delta(y - x) + 0.5(x + y)(p_N - R)$$
$$+ 0.5\big((2p - 1)p_N - R\big)(x - y) - \frac{\lambda p_N(1 - p_N)}{4}$$
$$\times \Big((x + y)^2 + (x - y)^2 + 2(2p - 1)(x + y)(x - y)\Big)$$
$$- \lambda p(1 - p)\big(p_N(x - y) + \delta(y - x)\big)^2.$$

$Q_B(2)$ is positive as long as $A_B > A_5 = -\nu$.

A specific case is one where the probability of switching between states 1 and 2 becomes negligible, i.e., $p \to 1$. We then have $(x + y) \to \kappa(\lambda p_D)^{-1}$ and $y - x \to \pm[\kappa^2 - 4\lambda p_D R A_D]^{0.5}(\lambda p_D)^{-1}$. This implies that

$$Q_D(i) + A_D \to \frac{\kappa \pm [\kappa^2 - 4R\lambda p_D A_D]^{0.5}}{2\lambda p_D}.$$

In equilibrium 3, state 1 is associated with the negative value in the numerator, while state 2 is associated with the positive value. From Proposition 6 it is clear that state 1 then converges to the fundamental equilibrium and state 2 to the sunspot-like equilibrium.

Symmetry

This Appendix presents the analysis underlying the discussion on symmetry in the sunspot equilibrium discussed at the end of Section IIIB. Define $Risk_t = \text{var}_t(Q_{t+1})$. We show that when the conditional distribution of the sunspot S_t is symmetric, then a symmetric solution for the price Q_t as a function of the sunspot implies a symmetric solution for $Risk_t$ as a function of the sunspot. Similarly, a symmetric solution for $Risk_t$ as a function of the sunspot implies a symmetric solution of the price as a function of the sunspot. Symmetry of the solution as a function of S_t is therefore consistent with the loop between the asset price and risk.

We define the symmetry of the distribution of the sunspot S_{t+1} conditional on the current value of the sunspot S_t as the following property:

$$\text{(A23)} \quad \text{prob}(S_{t+1} = s_j | S_t = s_i) = \text{prob}(S_{t+1} = -s_j | S_t = -s_i) \quad \forall i, j \in J,$$

where J denotes the set of possible realizations of the sunspot. (A23) shows that the distribution of S_{t+1} conditional on $S_t = s_i$ is simply the opposite of its distribution conditional on $S_t = -s_i$.

The asset price $Q_t = f(S_t)$ is a symmetric function of S_t when $f(S_t = s_i) = f(S_t = -s_i)$. Similarly risk is a symmetric function of S_t when $Risk_t(S_t = s_i) = Risk_t(S_t = -s_i)$.

We first show that when the price Q_t is a symmetric function of S_t then $Risk_t$ is also a symmetric function of S_t:

$$\begin{aligned} Risk_t(S_t = s_i) &= \text{var}_t(Q_{t+1} | S_t = s_i) \\ &= \sum_{j \in J} \text{prob}(S_{t+1} = s_j | S_t = s_i)[f(s_j)]^2 \\ &\quad - \left(\sum_{j \in J} \text{prob}(S_{t+1} = s_j | S_t = s_i) f(s_j)\right)^2 \\ &= \sum_{j \in J} \text{prob}(S_{t+1} = -s_j | S_t = -s_i)[f(-s_j)]^2 \\ &\quad - \left(\sum_{j \in J} \text{prob}(S_{t+1} = -s_j | S_t = -s_i) f(-s_j)\right)^2 \\ &= \text{var}_t(Q_{t+1} | S_t = -s_i) = Risk_t(S_t = -s_i). \end{aligned}$$

Next, we show that when $Risk_t$ is a symmetric function of S_t then the price is also a symmetric function of S_t. We know that Q_t depends on $Risk_t$ and expectations of $Risk_{t+i}$, $i > 0$. So we need to show that the expectation of $Risk_{t+i}$, $i > 0$, is symmetric in S_t when $Risk_{t+i}$ is symmetric in S_{t+i}. Since $E_t Risk_{t+i} = E_t E_{t+1} \ldots E_{t+i-1} Risk_{t+i}$ it is sufficient to show that when $Risk_{t+1}$ is a symmetric function of S_{t+1}, then its expectation at time t is symmetric in S_t. Backward induction then gets our result:

$$\begin{aligned} E_t(Risk_{t+1} | S_t = s_i) &= \sum_{j \in J} \text{prob}(S_{t+1} = s_j | S_t = s_i) Risk_{t+1}(s_j) \\ &= \sum_{j \in J} \text{prob}(S_{t+1} = -s_j | S_t = -s_i) Risk_{t+1}(-s_j) \\ &= E_t(Risk_{t+1} | S_t = -s_i). \end{aligned}$$

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