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Optimal Exchange Rate Policy in a Growing Semi-open Economy

PHILIPPE BACCHETTA, KENZA BENHIMA, and YANNICK KALANTZIS*

This paper considers an alternative perspective to China's exchange rate policy. It studies a semi-open economy where the private sector has no access to international capital markets but the central bank has full access. Moreover, it assumes limited financial development generating a large demand for saving instruments by the private sector. The paper analyzes the optimal exchange rate policy by modeling the central bank as a Ramsey planner. Its main result is that in a growth acceleration episode it is optimal to have an initial real depreciation of the currency combined with an accumulation of reserves, which is consistent with the Chinese experience. This depreciation is followed by an appreciation in the long run. The paper also shows that the optimal exchange rate path is close to the one that would result in an economy with full capital mobility and no central bank intervention. [JEL E58, F31, F32, F38.]

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*Philippe Bacchetta is a Swiss Finance Institute Professor at the University of Lausanne (HEC) and a CEPR Research Fellow. Kenza Benhima is a Professor at the University of Lausanne (HEC) and a CEPR Research Affiliate. Yannick Kalantzis is a Senior Economist at the Banque de France. The authors would like to thank two referees, Menzie Chinn, Roberto Chang, Nicolas Coeurdacier, Giovanni Lombardo, Charles Engel, as well as participants at seminars at the City University of Hong Kong, the Hong Kong Monetary Authority, Università di Napoli Federico II, the London Conference on International Capital Flows and Spillovers, the Bank of Canada—ECB workshop on exchange rates, and the Summer Workshop in International Finance and Macro Finance, for useful comments. Bacchetta and Benhima gratefully acknowledge financial support from the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK). Bacchetta also acknowledges support from the ERC Advanced Grant #269573, and the Hong Kong Institute of Monetary Research.

In recent years we have seen a heated debate over Chinese exchange rate policy and the enormous accumulation of international reserves by its central bank. Although the increase in reserves has been considered as a major contributor to global imbalances, the renminbi (RMB) has typically been viewed as undervalued.[1] For example, Frankel (2010) clearly states "An appreciation would improve economic welfare." However, these views are not universally shared. For example, McKinnon (2010) gives two main arguments against more RMB flexibility. First, a flexible exchange rate is not desirable given the limited international use of the RMB. Second, an appreciation will not necessarily reduce the huge current account surplus, unless it reduces the difference between aggregate saving and aggregate investment.

This paper will focus on the second argument of McKinnon, namely the connection between the exchange rate level and net saving. We examine the optimal exchange rate policy in a dynamic intertemporal model that incorporates four basic features of the Chinese economy: (i) limited capital mobility; (ii) a net capital outflow taking the form of an accumulation of central bank international reserves; (iii) underdeveloped financial markets; and (iv) a very high growth rate. Growth is assumed to arise from exogenous increases in endowments. In such a context the central bank is modeled as a Ramsey planner who can choose the optimal path of the exchange rate and of international reserves. Our main result is that in a growth acceleration episode it is optimal to have an initial real depreciation of the currency combined with an accumulation of reserves. This depreciation is followed by an appreciation in the long run. We also show that the optimal exchange rate path is close to the one that would obtain in an economy with full capital mobility and no central bank intervention. The main reason for an optimal depreciation is financial underdevelopment implying a limited supply of financial assets. With a developed financial system, an initial appreciation would be optimal.

Studying the link between the real exchange rate and net saving naturally requires an intertemporal approach, in contrast to many analyses that examine the relationship between the exchange rate and the trade balance. The standard model analyzing this link is the representative-individual infinite-horizon model with traded and nontraded goods.[2] We deviate from this benchmark model to incorporate the four features mentioned above. First, we assume low financial development, in the form of credit constraints. This may significantly affect saving behavior, especially with high growth rates. We follow Woodford (1990) and introduce credit-constrained heterogeneous households who alternate between high and low endowments. With strong credit frictions, low-endowment households cannot borrow and therefore have an incentive to save more in high-endowment periods. With a growth acceleration episode, the desire to save is even higher as households will feel even more constrained in future low-endowment periods.

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[1] For some recent contributions on this debate, see Cheung, Chinn, and Fujii (2011), Frankel (2010), or Goldstein and Lardy (2008).

[2] See Obstfeld and Rogoff (1996, ch. 4). For example, Obstfeld and Rogoff (2000, 2007) use this standard framework to analyze U.S. net saving and the dollar exchange rate.

In an open economy, households could save more by buying foreign assets. This would allow aggregate saving to increase and would lead to a current account surplus. Moreover, this would initially depreciate the real exchange rate: a higher saving rate reduces demand and pressure on domestic prices, which implies a real depreciation. However, we consider a "semi-open" economy, which is an economy where the private sector does not have any access to the international capital market, but the central bank does. Therefore, there is a lack of financial assets available for consumers.[3,4] In this context of an "excess" demand for saving, or asset scarcity, the government or the central bank can provide domestic assets to accommodate the saving need. A natural way of changing the amount of domestic assets is for the central bank to serve as intermediary between the international capital market and domestic savers. Thus, an accumulation of international reserves at the central bank can be translated into an increase in the supply of domestic assets and an increase in private saving.[5]

This policy will also affect the real exchange rate. As in the open economy, a higher saving rate implies a real depreciation. Therefore the optimal exchange rate policy is directly tied to asset provision and reserve policy.[6] A natural question will be to compare the optimal policy in the semi-open economy to the decentralized equilibrium in the open economy.

To analyze the optimal exchange rate policy in the context of a semi-open economy, we take a dynamic optimal taxation approach, by modeling the central bank as a Ramsey planner. The central bank takes the government behavior as given and therefore has much fewer instruments than in standard optimal taxation analyses. Although this approach has not been used for exchange rate policy (and even less in the Chinese context), there is growing interest in using these tools in international macroeconomics.[7] In a growth acceleration episode, we find that it is optimal to increase the supply of domestic assets financed by international reserves. Therefore, it is also optimal to let the currency depreciate. We also find that the optimal depreciation with capital controls is close to the depreciation that would occur in an open economy. The need for reserve accumulation and currency depreciation will be stronger when the lack of saving instruments is acute, that is, with low financial development.

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[3] This implies that a capital account liberalization would lead to a net private capital outflow. Several papers in the literature predict such an outcome for China using totally different perspectives. For example, see He and others (2012a).

[4] This simple framework with credit constraints enables to capture two key features found in the recent literature on global imbalances: insufficient supply of domestic assets (see Caballero, Farhi, and Gourinchas, 2008) and precautionary saving (see Mendoza, Quadrini, and Ros-Rull, 2009).

[5] Since 2000, the Chinese central bank has increased its liabilities with the domestic banking sector at about the same rate as international reserves. These liabilities mainly take the form of central bank bonds and commercial banks reserves.

[6] In a recent paper, Jeanne (2012) considers a semi-open economy with traded and nontraded goods and shows how exogenous changes in international reserves alter intertemporal consumption choices, as well as the real exchange rate.

[7] Farhi, Gopinath, and Itskhoki (2012) show that a simple combination of taxes can replicate nominal exchange rate policy, but they do not consider a Ramsey planner.

The structure of the semi-open economy is similar to Bacchetta, Benhima, and Kalantzis (2013), but we consider traded and nontraded goods to determine real exchange rate movements. In our previous paper with a single good, the optimal policy was determined by various trade-offs caused by changes in the interest rate. Indeed, in the presence of credit constraints and growth, the role of policy is to help agents consume more in early periods, when the constraint is more binding. This can be achieved by either a high or a low interest rate, depending on the level of growth, risk, and credit constraint. The optimal semi-open economy then consists in accumulating more or less reserves than the open economy. The introduction of the real exchange rate adds an incentive to appreciate the currency in order to stimulate income and the value of collateral in early periods, which implies that the central bank would tend to accumulate less reserves. Our results suggest that this either mitigates or reinforces the policy induced by the interest rate trade-offs, but does not dramatically alter optimal policy. Exchange rate dynamics is more a consequence of reserve policy than a driver of that policy.

Our results are broadly consistent with the experience of China after it joined the World Trade Organization (WTO) in December 2001.[8] Figure 1 documents the relevant stylized facts. First, China experienced a growth acceleration: the growth rate of GDP per capita increased from 7 percent in 2001 to 14 percent in 2009. Second, the central bank started accumulating large amounts of international reserves, from 16 percent of GDP in 2001 to almost 50 percent at the end of the decade. Third, over that same period, the real effective exchange rate initially depreciated, from 2001 to 2005, before appreciating after 2005. Fourth, these dynamics coincided with an increase in aggregate net saving as represented by the current account (from 1.3 percent of GDP in 2001 to 10 percent in 2007), consistent with the mechanism we describe below.

A standard perspective for the last decade is that China experienced an exogenous increase in export demand. To prevent a nominal appreciation, the central bank intervened in the foreign exchange market and accumulated reserves. Moreover, it sterilized the increase in reserves to avoid inflation. Our alternative perspective starts from an exogenous growth acceleration that increases saving by Chinese consumers, mainly in the form of bank deposits. The implied increase in liabilities of the Chinese banking sector was translated into an increase in central bank liabilities, through required reserves and central bank bills. In this context, the optimal policy of the central bank is to purchase foreign currency assets and to let the real exchange rate depreciate.[9] Thus, the central bank served as intermediary

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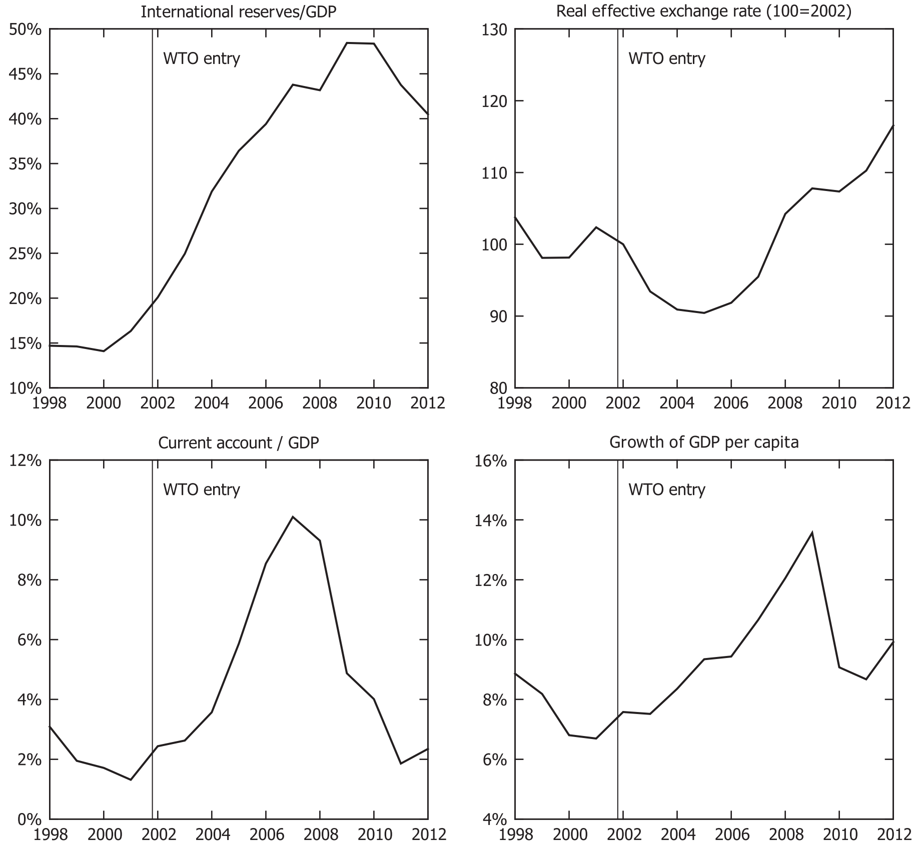
[8] We focus on the years 2000 as China was not truly a market economy until the late 1990s. For instance, a significant share of producer and retail prices were not market-determined until the second half of the 1990s. The People's Bank of China only became an autonomous central bank in the modern sense after a law was passed in March 1995. See OECD (2009) for details on the reform process.

[9] We consider a real model and do not model inflation explicitly. Introducing a nominal sector with flexible prices would allow to distinguish between nominal and real exchange rate fluctuations, but would not change our main analysis. Notice, however, that the nominal trade-weighted RMB has moved closely to its real value since 2000.

Figure 1. Stylized Facts in the Chinese Economy



Sources: International reserves, real effective exchange rate: IMF International Financial Statistics; current account: State Administration of Foreign Exchange; GDP per capita: National Bureau of Statistics. Authors' calculations.

Note: The vertical line indicates the date at which China joined the World Trade Organization (December 2001). An increase in the index of real effective exchange rate corresponds to an appreciation.

between the private sector and the international capital market, as argued in particular by Song, Storesletten, and Zilibotti (2011). We notice that both the standard and our alternative approach are consistent with the increase in international reserves and current account illustrated in Figure 1. However, the standard perspective is not consistent with the real depreciation between 2001 and 2005, as an increase in export demand should lead to a real appreciation.

As in several recent papers, one feature of our analysis is the interaction between real exchange rate movements and a credit constraint.[10] It is well known

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[10] For example, Bianchi (2011), Korinek (2011), Benigno and others (2013). Céspedes, Chang, and Velasco (2012) examine central bank intervention with such an externality in the context of capital inflows.

that this feature creates pecuniary externalities through the value of the collateral and therefore a role for policy intervention. It turns out, however, that this effect plays little role in our context. On the other hand, there is no real externality from exchange rate movements. Korinek and Servén (2011) and Benigno and Fornaro (2012) assume learning by doing in the export sector, which gives an incentive for currency depreciation and reserve accumulation. In these two papers, there is a trade-off between lower consumption today and higher productivity tomorrow. In our model, the trade-off is between lower consumption today and higher saving that allow higher consumption tomorrow. Even though there is no long-term productivity gain in our model, there is a substantial welfare gain in accumulating reserves and initially depreciating the currency.

In the following section, we lay out the model. Section II describes the model equilibrium. Section III describes the Ramsey problem and derives several analytical results about the optimal policy. Section IV presents numerical simulations and Section V concludes.

I. Model

The economy is inhabited by infinitely lived households who receive endowments in traded and nontraded goods and consume both goods. The relative price of nontraded goods in terms of traded goods, p_t, is the real exchange rate.[11] Following Woodford (1990, section I), endowments alternate between low and high levels and there are two groups of mass one of households.[12] This structure implies that in a given period half of households have a high endowment and typically would like to save, whereas the other half have a low endowment and would like to borrow.[13] Households trade one-period local assets. Without loss of generality, these assets are denominated in the traded good.[14] There is a gross interest rate r_t (measured in traded goods) on lending and borrowing.

We assume that households do not have access to international capital markets. Therefore, high-endowment households can save either by lending to low-endowment households or by holding central bank assets.[15] However, high-

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[11] In general there can be differences between the relative price of traded and nontraded goods and the commonly measured real exchange rate. We will abstract from these differences. He and others (2012b) estimate that in the case of China the relative price of traded and nontraded goods shows a stronger appreciation in recent years than standard real exchange rates measures.

[12] There are four basic differences with Woodford (1990):(i) consumers may be able to borrow; (ii) there is a Ramsey planner; (iii) there is no capital stock; (iv) there are traded and nontraded goods.

[13] This simple structure can account for three major explanations for the Chinese propensity to save that are rooted in the lack of welfare state: income risk, the need for savings in the perspective of health-related expenditures, and retirement. Other factors can explain high saving in China (for example, see Yang, Zhang, and Zhou, 2011), such as education or the gender imbalance, but adding these factors would not change the main results of our analysis.

[14] In the absence of uncertainty, the denomination of assets has no consequence on equilibrium allocations.

[15] In reality, the lending between high and low endowment households goes through the banking sector, with bank deposits and bank loans. However, modeling financial intermediaries would not affect our analysis.

endowment households may be reluctant to lend to other households because of credit market frictions and may thus be looking for other saving instruments.

In addition to households there is a Ramsey planner, that we call a central bank, who can issue local assets and hold international reserves, thereby affecting the real exchange rate. When credit constraints are tight, the opportunities to save for high-endowment households are limited. In this case the provision of local assets by the central bank may be desirable.

Households

At time t, a first group of households receives an endowment of traded and nontraded goods Y_t^T and Y_t^N. We denote the total resources of this first group in terms of traded goods by $Y_t = Y_t^T + p_t Y_t^N$. The second group receives aY_t^T and aY_t^N with $0 \leq a < 1$, so its total resources in terms of traded goods are aY_t. At t+1, the first group receives aY_{t+1}^T and aY_{t+1}^N while the second receives Y_{t+1}^T and Y_{t+1}^N, and so on. Thus, in each period, one group receives Y while the other group receives aY. We refer to the group with Y as cash-rich households, or savers, and the group with aY as cash-poor households, or borrowers. Each household alternates between a cash-rich and a cash-poor state, and each period there is an equally sized population of rich and poor. Cash-rich households will hold assets A, whereas cash-poor households borrow L. Households also receive profits from the central bank. These profits are distributed equally between the two groups so that each household receives $\pi_t/2$ in traded goods at period t. Profits can be negative, in which case households pay a lump-sum tax.

Households maximize:

$$\sum_{s=0}^{\infty} \beta^s u\left(c_s^T, c_s^N\right). \tag{1}$$

We will focus on separable iso-elastic utility functions $u(c_s^T, c_s^N) = v(c_s^T) + \kappa v(c_s^N)$ with

$$v(c) = \tfrac{c^{1-\sigma}}{1-\sigma} \qquad \text{for } \sigma > 0, \sigma \neq 1$$
$$v(c) = \ln c \qquad \text{for } \sigma = 1.$$

We denote consumption of traded (nontraded) goods during the cash-rich period as c^{AT} (c^{AN}). Consumption of traded (nontraded) goods during the cash-poor period is denoted c^{LT}(c^{LN}). Consider a household that is cash-rich at time t and cash-poor at date t+1. Its budget constraints at t and t+1 are:

$$Y_t - r_t L_t + \pi_t/2 = c_t^{AT} + p_t c_t^{AN} + A_{t+1}, \tag{2}$$

$$aY_{t+1} + r_{t+1} A_{t+1} + \pi_{t+1}/2 = c_{t+1}^{LT} + p_{t+1} c_{t+1}^{LN} - L_{t+2}. \tag{3}$$

The household income at date t, which is composed of endowment Y_t minus debt repayments $r_t L_t$ plus central bank profits, is allocated to buying assets A_{t+1}, traded goods c_t^{AT}, and nontraded goods c_t^{AN}. We will focus on sequences of endowments such that $A_{t+1}>0$. In the following period, at $t+1$, its income is composed of the return on assets, $r_{t+1}A_{t+1}$, of aY_{t+1} and of central bank profits. This has to pay for consumption of traded and nontraded goods, c_{t+1}^{LT} and c_{t+1}^{LN}. Typically the cash-poor household will borrow, so that at the optimum $L_{t+2}\geq 0$.

The cash-poor household might face a credit constraint when borrowing at date $t+1$. Owing to standard moral hazard arguments, a fraction $0\leq\phi<1$ of the total endowment is used as collateral for bond repayment:

$$r_{t+2}L_{t+2} \leq \phi Y_{t+2}. \tag{4}$$

The multiplier associated with this constraint is denoted $v'(c_{t+2}^{AT})\lambda_{t+2}$.

Cash-rich households at time t satisfy the following Euler equation:

$$v'(c_t^{AT}) = \beta r_{t+1} v'(c_{t+1}^{LT}). \tag{5}$$

Similarly, poor households at date t satisfy the following Euler equation:

$$v'(c_t^{LT}) = \beta r_{t+1} v'(c_{t+1}^{AT})(1+\lambda_{t+1}). \tag{6}$$

The intertemporal choice of a cash-poor household is distorted when the credit constraint is binding, because $\lambda_{t+1}>0$. The following slackness condition has also to be satisfied:

$$(\phi Y_{t+1} - r_{t+1}L_{t+1})\lambda_{t+1} = 0. \tag{7}$$

The Real Exchange Rate

The first-order conditions give:

$$p_t = \kappa\frac{v'(c_t^{LN})}{v'(c_t^{LT})} = \kappa\frac{v'(c_t^{AN})}{v'(c_t^{AT})}. \tag{8}$$

In equilibrium total nontraded consumption is equal to total nontraded endowment:

$$c_t^{AN} + c_t^{LN} = (1+a)Y_t^N. \tag{9}$$

In this case, the first-order conditions imply:

$$p_t = \kappa\left(\frac{c_t^{AT}+c_t^{LT}}{(1+a)Y_t^N}\right)^{\sigma}. \tag{10}$$

As this is an endowment economy, the real exchange rates simply depends on the ratio between traded consumption and nontraded output. The evolution of traded good consumption is obviously affected by the presence of credit constraints. Consider for example an increase in the growth rate of all endowments. As we shall see, the credit constraint then implies higher saving so that $c_t^{AT}+c_t^{LT}$ increases initially less than the endowment. This implies a decline in p_t and thus a depreciation.

The depreciation in a period of strong growth is thus associated with an increase in saving. How is this possible in the aggregate? In an open economy households would buy foreign assets. In a semi-open economy, this is possible if the central bank issues local assets, financed by the accumulation of reserves. Thus, as shown by Jeanne (2012), the accumulation of reserves is directly related to saving and to the exchange rate. In this paper we will determine the optimal exchange rate/reserves policy.

Central Bank Policy

The central bank issues domestic assets B_{t+1} at time t paying a gross interest rate r_{t+1}. It has access to foreign reserves B^*_{t+1} (denominated in traded goods) that yield the world interest rate r^*. We assume that $r^* = 1/\beta$. Private agents cannot buy external bonds directly, so the domestic interest rate is determined in the domestic bond market. Equilibrium in this market is:

$$B_{t+1} = A_{t+1} - L_{t+1}. \tag{11}$$

In the presence of capital controls, only the central bank has access to external assets, so it has a monopoly over the supply of bonds to domestic agents. It can therefore manipulate the domestic interest rate r_{t+1} by appropriately setting the supply of bonds B. The possibility of accumulating reserves B^* enables the central bank to change the domestic supply of bonds by simply expanding its balance sheet. The central bank can then match the desired domestic saving by accumulating reserves.

When the central bank policy creates a wedge between r_{t+1} and r^*, this generates revenues or losses. We assume that the central bank transfers directly its profits π_t to households.[16] The central bank budget constraint is:

$$B^*_{t+1} + r_t B_t + \pi_t = r^* B^*_t + B_{t+1}. \tag{12}$$

We impose the usual no-ponzi condition to the central bank net asset position:

$$\lim_{T\to\infty} \frac{B^*_T - B_T}{(r^*)^T} = 0. \tag{13}$$

In general, profits $\{\pi_t\}_{t\geq 0}$ have to satisfy the sequence of budget constraints (equation (12)) and the no-ponzi condition (equation (13)) given the policy $\{B_{t+1}, B^*_{t+1}\}_{t\geq 0}$. In the following, we focus on the realistic case where the central bank transfers its revenues or losses to households on a period-by-period basis:

$$\pi_t = (r^* - 1)B^*_t - (r_t - 1)B_t. \tag{14}$$

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[16] In practice, central banks usually transfer their profits to the government, which relaxes the government budget constraint. In Bacchetta, Benhima, and Kalantzis (2013), we explicitly introduce the government and distortionary taxes.

With this assumption, a change in international reserves has to be matched by an increase in the supply of bonds: $B^*_{t+1}-B^*_t=B_{t+1}-B_t$. Assuming that $B^*_0=B_0$, we have:

$$B^*_t=B_t. \tag{15}$$

This implies that the central bank is neither a net saver nor a net borrower.[17]

Notice that the closed economy and the open economy are special cases nested in our semi-open economy framework. The central bank can always choose to "replicate" the open economy by supplying the domestic market with bonds at the world interest rate $r_{t+1}=r^*$. It can also mimic the closed economy by not buying reserves: $B^*_{t+1}=0$. By choosing the level of reserves, the central bank also chooses both the capital account policy and the exchange rate policy, as the level of B^* both determines the level of domestic interest rate and the real exchange rate p.

As a Ramsey planner, the central bank will choose a policy $\{\pi_t, B_{t+1}, B^*_{t+1}\}_{t\geq 0}$ to maximize its social objective:

$$\sum_{s=0}^{\infty}\beta^s\left[u(c^{AT}_s,c^{AN}_s)+u(c^{LT}_s,c^{LN}_s))\right]. \tag{16}$$

We will then analyze the optimal exchange rate policy in this context. If the optimal policy replicates the open economy, then capital controls are unnecessary. But if the optimal policy differs from the open economy, it means that capital controls are welfare-improving. Notice, however, that optimal policies are not necessarily Pareto superior, as one of the groups may have a lower welfare.

II. Competitive Equilibrium

In this section, we examine the properties of a competitive equilibrium for a given policy. First, we describe how the reserve policy is equivalent to an exchange rate policy and how it affects the bond market. Then, we analyze the steady state and determine the conditions under which the economy is constrained.

We define a competitive equilibrium as follows:

Definition 1 (Competitive equilibrium) *Given endowment streams* $\{Y^T_t,Y^N_t\}_{t\geq 0}$ *and initial conditions* $(r_0, A_0, L_0, B_0, B^*_0)$ with $B_0=A_0-L_0$, *a competitive equilibrium is a sequence of prices* $\{p_t,r_{t+1}\}_{t\geq 0}$ *and Lagrange multipliers* $\{\lambda_{t+1}\}_{t\geq 0}$, *an allocation* $\{A_{t+1}, L_{t+1}, c^{AT}_t, c^{LT}_t, c^{AN}_t, c^{LN}_t\}_{t\geq 0}$, *and a policy* $\{\pi_t, B_{t+1}, B^*_{t+1}\}_{t\geq 0}$ *such that:* (i) *given the price system and the policy, the allocation and the Lagrange multipliers solve the households' problems* (*equations* (2)–(7) *are satisfied);* (ii) *given the allocation and the price system, the policy satisfies the*

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[17] This is an important assumption since this prevents the central bank from borrowing from the rest of the world and distribute resources to the households in order to overcome the borrowing constraints. It is however realistic since most central banks distribute profits on an annual basis (this is similar to the assumption made in many models that firms distribute all their profits every period).

sequence of central bank budget constraints (*equation* (12)) *and the no-ponzi condition* (*equation* (13)); (iii) *the markets for nontraded goods* (*equation* (9)) *and domestic bonds* (*equation* (11)) *clear.*

As explained earlier, we will restrict the analysis to the subset of policies defined by the profit distribution rule (equation (14)) and assume that $B_0^*=B_0$ so that the holding of reserves equals the supply of bonds by the central bank (equation (15)).

Central Bank Policy, the Real Exchange Rate, and the Real Interest Rate

With separable iso-elastic utility, intratemporal optimization by households implies that the real exchange rate depends on the aggregate consumption of traded goods as shown by Equation (10). Using the budget constraints (2), (3), and (12) together with the market-clearing conditions (9) and (11), we can derive a current account identity:

$$B_{t+1}^* - B_t^* = (1+a)Y_t^T + (r^*-1)B_t^* - (c_t^{AT} + c_t^{LT}). \tag{17}$$

Substituting Equation (17) into Equation (10), we clearly see how choosing the increase in reserves $B_{t+1}^*-B_t^*$ is equivalent to setting the real exchange rate p_t:

$$p_t = \kappa \left[\frac{(1+a)Y_t^T + (r^*-1)B_t^* - (B_{t+1}^* - B_t^*)}{(1+a)Y_t^N} \right]^{\sigma}. \tag{18}$$

By buying more reserves, and issuing the corresponding amount of domestic bonds, the central bank can depreciate the real exchange rate, as explained in Jeanne (2012): in the semi-open economy, reserve policy and exchange rate policy are equivalent.

While accumulating more reserves during the transition, that is, choosing a higher *flow* $B_{t+1}^*-B_t^*$, depreciates the real exchange rate, a larger *stock* of reserves in the steady state appreciates the real exchange rate if $r^*>1$, as it makes domestic agents richer and increase their demand for nontraded goods. In the steady state, Equation (18) can indeed be rewritten as

$$p = \kappa \left[\frac{Y^T}{Y^N} + (r^*-1)\frac{B^*}{(1+a)Y^N} \right]^{\sigma}.$$

The exchange rate policy also has an effect on the domestic bond market and the domestic interest rate. As the stock of reserves is equal to the supply of domestic bonds by the central bank, depreciating the exchange rate requires increasing the supply of bonds. This leads to a higher domestic interest rate. Such a policy might be desirable when borrowing constraints are binding.

To see this, consider the demand for assets by savers in the case of log utility ($\sigma = 1$) where we can get closed-form solutions:[18]

$$A_{t+1} = \frac{1}{1+\beta}\left(\beta(Y_t - r_t L_t + \pi_t/2) - \frac{aY_{t+1} + \pi_{t+1}/2}{r_{t+1}} - \frac{L_{t+2}}{r_{t+1}}\right). \tag{19}$$

The effect of a binding borrowing constraint is to decrease future borrowing L_{t+2}, which leads to a larger demand for saving instruments A_{t+1}. At the same time, a binding borrowing constraint also decreases current borrowing by cash-poor households L_{t+1}, as implied by Equation (7) when it holds as an equality. Absent any policy intervention, the excess demand for and the constrained supply of bonds by the private sector would lead to an abnormally low interest rate r_{t+1} to clear the market, compared with a frictionless economy. By providing more bonds to the domestic market, a policy of real exchange rate depreciation can alleviate the limited supply of bonds by cash-poor households and accommodate the need for saving by cash-rich households.

Symmetric Steady States

How central bank policy can alleviate borrowing constraints by providing domestic bonds can be analyzed precisely in deterministic symmetric steady states, defined as follows.

Definition 2 (Symmetric Steady State) *Consider a constant endowment stream* $(Y_t^T, Y_t^N) = (Y^T, Y^N)$ for $t \geq 0$. *A symmetric steady state is a constant price vector* (p, r), *Lagrange multiplier* λ, *allocation* $(A, L, c^{AT}, c^{LT}, c^{AN}, c^{LN})$, *and policy* (π, B, B^*) *that form a competitive equilibrium associated to the endowment stream* (Y^T, Y^N) *and the initial conditions* (r, A, L, B, B^*).

In a symmetric steady state, endowments and consumptions of a given individual can still fluctuate through time; but their distributions across agents are stationary. Such a steady state is symmetric in the sense that all individuals have the same state-contingent consumption and wealth.

The next step is to determine when the economy is constrained in the steady state. Define the following parameter $\overline{b}$:

$$\overline{b} = \frac{\beta(1+\kappa)\left(\frac{1-a}{1+\beta} - 2\phi\right)}{1 - \frac{\kappa(1-\beta)}{1+a}\left(\frac{1-a}{1+\beta} - 2\phi\right)}.$$

The denominator of $\overline{b}$ is strictly positive when $\kappa < (1+\beta)/(1-\beta)(1-a)/(1+a)$, a weak condition which we assume throughout.[19]

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[18] This equation follows from the Euler equation (5) and the budget constraints ((equations (2) and (3)).

[19] For example, with log-utility and $\beta = 0.95$, this condition holds as long as tradable consumption represents at least 2.5 percent of total consumption.

The following proposition shows that the steady states of the model depend on how the amount of bonds B/Y^T compares with $\overline{b}$.

Proposition 1 *Assume the profit distribution* (*equation* (14)), *with* $B_0 = B_0^*$ *and log utility. For all* $(Y^T, Y^N, B^*) \in \Re^{+*2} \times \Re^+$, *there is a unique symmetric steady state.*

If $B^* / Y^T < \overline{b}$, *the credit constraint is binding, the interest rate* $r < r^*$ *increases with* $(B^*)/(Y^T)$ *and the ratio of relative traded consumption is given by* $(c^{LT})/(c^{AT}) = r/r^* < 1$.

If on the contrary $B^* / Y^T \geq \overline{b}$, *the credit constraint does not bind and* $r = r^*$.

Proof. See section "Proof of Proposition 1" of Appendix. ☐

The proposition shows how the accumulation of reserves, or equivalently the issuance of domestic bonds, determines the extent to which households can smooth consumption despite the borrowing constraint. A higher level of reserves B^* and domestic bonds B means that cash-rich households can save more and receive a larger return on their saving, resulting in smaller fluctuations of tradable consumption through time. When the supply of bonds is large enough, cash-rich households can accumulate enough assets to completely overcome their borrowing constraint and perfectly smooth consumption.

A direct corollary of Proposition 1 is that the borrowing constraint never binds in a steady state of the open economy and that the net foreign asset position of an open economy, B^*, is necessarily larger than $\overline{b}Y^T$ in a steady state. For stringent enough borrowing constraints (that is, low enough ϕ), $\overline{b}$ is positive, and the open economy has positive net foreign assets in the steady state.

III. Optimal Exchange Rate Policy

The Ramsey Problem

To analyze optimal policy we now turn to the optimization problem of the Ramsey planner. We consider the log utility case. Without loss of generality, we assume zero initial net assets ($B_0^* - B_0 = 0$). The planner maximizes its objective (16) subject to the household budget constraints, their first-order conditions, the borrowing constraint, the complementary slackness condition, the market-clearing conditions for bonds, and the resource constraint for both nontradable goods (given by the market-clearing condition (equation (9))) and tradable goods (given by the current account identity (equation (17))).[20] Using the optimality conditions, the value of nontradable consumption in terms of tradables is suppressed from the Ramsey program, namely $p_t c_t^{AN} = \kappa c_t^{AT}$ and $p_t c_t^{LN} = \kappa c_t^{LT}$.

Maximization is then carried out with respect to $\{L_{t+1}, A_{t+1}, c_t^{AT}, c_t^{LT}, r_{t+1}, p_t, \lambda_{t+1}, \pi_t, B_t^*\}_{t \geq 0}$. The Lagrangian of the Ramsey problem in the semi-open economy

---

[20] Given the household budget constraints and the market-clearing conditions, the current account identity is equivalent to the budget constraint of the central bank.

is then defined as follows:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \big\{ & (1+\kappa)\ln(c_t^{AT}) + (1+\kappa)\ln(c_t^{LT}) - 2\kappa \ln p_t \\
& + \gamma_t^A \left[Y_t^T + p_t Y_t^N - (1+\kappa)c_t^{AT} + \pi_t/2 - A_{t+1} - r_t L_t \right] \\
& + \gamma_t^L \left[a(Y_t^T + p_t Y_t^N) + r_t A_t + L_{t+1} - (1+\kappa)c_t^{LT} + \pi_t/2 \right] \\
& + \gamma_t^G \left[r^* B_t^* - B_{t+1}^* + (1+a)Y_t^T - c_t^{AT} - c_t^{LT} \right] \\
& + \gamma_t^B \left[B_{t+1}^* + L_{t+1} - A_{t+1} \right] \\
& + \gamma_t^N \left[(1+a)p_t Y_t^N - \kappa(c_t^{AT} + c_t^{LT}) \right] \\
& + \kappa_t^A \left[v'(c_t^{AT}) - \beta r_{t+1} v'(c_{t+1}^{LT}) \right] \\
& + \kappa_t^L \left[v'(c_t^{LT}) - \beta r_{t+1} v'(c_{t+1}^{AT})(1+\lambda_{t+1}) \right] \\
& + \Gamma_t \left[\phi(Y_t^T + p_t Y_t^N) - r_t L_t \right] \\
& + \Delta_t \left[(\phi(Y_t^T + p_t Y_t^N) - r_t L_t)\lambda_t \right] \big\}.
\end{aligned}
$$

The planner takes as constraints both the borrowing constraint (which does not necessarily bind) and the complementary slackness condition, which both enter in the definition of the competitive equilibrium. It is useful to define $\Lambda_t = \Gamma_t + \lambda_t \Delta_t$. When the borrowing constraint does not bind, we have $\Lambda_t = 0$.

Although the full solution to this dynamic optimization has to be solved numerically, some interesting properties can be derived analytically. In particular, the steady state can be fully characterized. As regards transition dynamics, one can ask whether the planner wants to deviate from the closed economy regime characterized by $B^* = 0$ and a constant real exchange rate. One can also determine whether the planner wants to deviate from the open economy regime with $r = r^*$. We analyze these cases in the rest of this section and turn to full numerical solutions in Section IV.

Optimal Level of Reserves in the Steady State

To study the optimal accumulation of reserves, we focus on the first-order condition with respect to B_{t+1}^*:

$$
-(\gamma_t^G - \gamma_{t+1}^G) + \gamma_t^B = 0.
$$

Using the other FOCs of the planner's program, we can replace γ_t^B to get (see section "Derivation of Equation (20)" of Appendix for details):

$$
-\left(\gamma_t^G - \gamma_{t+1}^G\right) + \beta r_{t+1} \frac{\Lambda_{t+1}}{2} = 0. \tag{20}
$$

The first term reflects the usual motive of intertemporal smoothing. The Lagrange multiplier γ^G is the shadow cost of the resource constraint for tradable goods. When the tradable endowment is growing, this multiplier should decrease over time in the absence of policy intervention (that is, in a closed economy with $B_t^* = 0$), making the first term negative. This first effect makes the planner want to borrow abroad and appreciate the real exchange rate. The second term captures the effect of the borrowing constraint. With a binding borrowing constraint, the planner wants to accumulate reserves and depreciate the exchange rate. The optimal policy balances those two effects.

When the borrowing constraint does not bind, both terms are equal to zero and borrowing abroad allows the planner to get a constant shadow cost γ^G and achieve perfect intertemporal smoothing. A binding borrowing constraint provides a motive to borrow less than in a frictionless economy, and to potentially accumulate reserves.

This can be seen clearly in a steady state. Then, the first term disappears and Equation (20) simply becomes $\Lambda = 0$. The steady-state optimal policy consists in completely relaxing borrowing constraints. Using Proposition 1, we can then characterize the optimal level of reserves in a steady state.

Proposition 2 *A symmetric steady state with optimal central bank policy is identical to an open economy. It has positive foreign reserves when* $2\phi(1+\beta)<1-a$.

Proof. From Equation (20) taken in the steady state, we have $\Lambda = 0$. Therefore, the borrowing constraint does not bind in the steady state. From Proposition 1, this implies $r = r^*$ so that this steady state is identical to an open economy. It also implies $B^* \geq \bar{b}Y^T$. Given our assumption that $\kappa<(1+\beta)/(1-\beta)(1-a)/(1+a)$, the condition $2\phi(1+\beta)<1-a$ implies $\bar{b}>0$ and therefore $B^*>0$. ☐

Transition Dynamics

Consider now the case of transitory dynamics where endowments of both tradable and nontradable goods grow at the rate g_t: $Y^T_{t+1} = (1+g_{t+1})Y^T_t$ and $Y^N_{t+1} = (1+g_{t+1})Y^N_t$. Assume that $a(1+g_{t+1})<1$ so that endowments still decline for cash-poor households.

Comparing with the Closed Economy

To study the optimal reserve policy, we consider the closed economy and determine whether the planner wants to deviate from it. Denote by $\tilde{J}_{t+1}$ the left-hand side of Equation (20) evaluated in the closed economy with $B_t = B_{t+1} = 0$. In general, any deviation of $\tilde{J}_{t+1}$ from zero means that the central bank can improve welfare by changing the level of reserves and the real exchange rate. When $\tilde{J}_{t+1}$ is positive, social welfare can be increased by buying reserves and depreciating the real exchange rate below its value in the closed economy.

The expression for $\tilde{J}_{t+1}$ can be solved explicitly in the case of a full borrowing constraint $\phi = 0$.[21] In section "Derivation of Equation (21)" of Appendix, we show that $\tilde{J}_{t+1}$ is then given by:

$$\tilde{J}_{t+1} = \frac{1+a}{2aY^T_{t+1}}\left(1 - \frac{\tilde{r}_{t+1}}{r^*}\right). \tag{21}$$

---

[21] From Proposition 2, we already know that it is optimal to accumulate reserves in the steady state when $\phi = 0$.

where $\tilde{r}_{t+1}$ is the closed-economy interest rate. The planner finds it socially optimal to accumulate reserves and depreciate the real exchange rate during the transitory dynamics, when the closed economy interest rate is strictly lower than the world interest rate.

It easy to see that $\tilde{r}_{t+1} < r^*$ under our assumption $a(1+g_{t+1})<1$. Using the fact that $\pi_t = 0$ in the closed economy, the demand for bonds by savers (equation (19)) becomes

$$A_{t+1} = \frac{1}{1+\beta}\left(\beta Y_t - \frac{a(1+g_{t+1})Y_t}{\tilde{r}_{t+1}}\right).$$

Market clearing on the bond market implies $A_{t+1} = 0$ so that the closed-economy interest rate $\tilde{r}_{t+1}$ is given by $\beta\tilde{r}_{t+1} = a(1+g_{t+1})$. As $r^* = 1/\beta$, we have $\tilde{r}_{t+1} < r^*$, so that reserve accumulation and currency depreciation are optimal when starting from the closed economy.

Comparing with the Open Economy

So far, we have shown that it is optimal to reproduce the open economy in the steady state and to accumulate reserves if one starts from a closed economy with tight borrowing constraints. An interesting question is whether the optimal reserve policy consists in simply replicating the open economy.

To answer this question, we evaluate the left-hand side of Equation (20) at $r_{t+1} = r^*$. Let us denote this expression by J^*_{t+1}. Any deviation of J^*_{t+1} from zero means that the open economy is suboptimal and that the central bank can improve welfare by accumulating (or decumulating) reserves with respect to the open economy. When J^*_{t+1} is positive, social welfare can be increased by accumulating more reserves than the open economy. We obtain the following:

$$\begin{aligned} J^*_{t+1} = \frac{1+\beta}{\beta c^{AT}_t}&\left[\underbrace{\left(\sum_{i=1}^{\infty}\Lambda_{t+2i}\right)A_{t+1}}_{R_1} - \underbrace{\left(\sum_{i=0}^{\infty}\Lambda_{t+1+2i}\right)L_{t+1}}_{R_2} - \underbrace{\frac{1}{2}\left(\sum_{i=1}^{\infty}\Lambda_{t+1+i}\right)(A_{t+1}-L_{t+1})}_{R_3}\right] \\ &+\kappa\left[\underbrace{-\frac{\gamma^A_t + a\gamma^L_t}{1+a}}_{P_1}\underbrace{-\frac{\phi\Lambda_t}{1+a}}_{P_2} + \underbrace{\frac{2}{c^{LT}_t + c^{AT}_t}}_{P_3} - \left(\underbrace{-\frac{\gamma^A_{t+1} + a\gamma^L_{t+1}}{1+a}}_{P'_1}\underbrace{-\frac{\phi\Lambda_{t+1}}{1+a}}_{P'_2} + \underbrace{\frac{2}{c^{LT}_{t+1} + c^{AT}_{t+1}} + \frac{\Lambda_{t+1}}{2}}_{P'_3}\right)\right] \end{aligned} \quad (22)$$

with Lagrange multipliers of savers' budget constraints given by $\gamma^A_t = -\gamma^L_t = \sum_{s\geq 1}(-1)^s(\Lambda_{t+s})/(2)$ (see section "Derivation of Equation (22)" of Appendix).

In the steady state, J^*_{t+1} converges to zero as Λ goes to zero and the consumption of tradables converges to its steady-state level. This confirms that an open economy in the steady state is at the Ramsey optimum. However, in the transition, the open economy could deviate from the optimum. To interpret condition (22), it is useful to notice that a change in reserves affects welfare through two channels: movements in the real interest rate and movements in the real exchange rate. The first line of Equation (22) (terms R_1, R_2, and R_3)

corresponds to the interest rate channel. It arises whether there are nontradable goods in the economy or not (and is also present in Bacchetta, Benhima, and Kalantzis, 2013). The second line (terms P_1, P_2, P_3, P_1', P_2', P_3',) corresponds to the real exchange rate channel and disappears if $\kappa = 0$.

Consider the first line. An increase (decrease) in reserves leads to a higher (lower) interest rate than in the open economy. Changes in the interest rate then affect the utility of both cash-rich and cash-poor agents. The first term (R_1) corresponds to the net effect of the interest rate on savers and is positive, as they benefit from higher returns on saving, which alleviates their future constraints. The second term (R_2) corresponds to the net effect on borrowers. This term is negative because a high interest rate hurts the borrowing households through higher interest payments, which makes both their current and future constraints more stringent. The third term (R_3) corresponds to the effect of central bank profits. Indeed, if $A>L$ and $r>r^*$, the interest payments on domestic debt are higher than the proceeds from external reserves, so that central bank profits are negative and households need to pay a lump sum tax to balance the budget. This first line can be both negative or positive depending on whether R_1 is greater than R_2+R_3. Bacchetta, Benhima, and Kalantzis (2013) study this trade-off in detail and show under what conditions the planner wants to increase (decrease) the interest rate above (below) the world level. In particular, they show that the sum of those three terms is positive when households' saving A is high and their borrowing L is low. In that case, a higher interest rate today increases aggregate welfare by making transfers to savers, which they receive tomorrow when they become borrowers, without too much directly hurting borrowers today.

Consider now the second line, which reflects the real exchange rate consequences of changing the level of reserves: an increase in reserves depresses the current real exchange rate (terms P_1, P_2, P_3) but increases the future consumption of tradable goods and appreciates the future real exchange rate (terms P_1', P_2', P_3').

The terms P_1 and P_1' capture the effect of the real exchange rate on household income. A more appreciated real exchange rate today increases the income of both savers (γ_t^A) and borrowers ($a\gamma_t^L$), and decreases it in the following period. This is, in principle, what the central bank would like to achieve given that households are more constrained in early periods. This channel should lead to a decrease in reserves in order to appreciate the currency. The terms P_2 and P_2' represent the effect of the collateral value: by appreciating the current real exchange rate, the government makes the credit constraint less stringent, as long as creditors admit a share $\phi>0$ of nontradable goods as collateral. On the other hand, a more depreciated future real exchange rate worsens future constraints. This channel, as well, should lead to a decrease in reserves. Finally, the terms P_3 and P_3' capture the effect of the real exchange rate on consumption. A more depreciated real exchange rate today lowers the price of nontradable consumption and frees resources for tradable consumption, which is valued at the marginal utility of average consumption, $[(c_t^{LT}+c_t^{AT})/2]^{-1}$. The reverse is true for a more appreciated real exchange rate tomorrow, taking into account the average shadow price of the borrowing constraint $\Lambda_{t+1}/2$. These two terms, P_3 and P_3', are similar to a Euler equation for the planner. As for the interest rate channel, the consequences of this channel on reserve accumulation is ambiguous. It depends whether the economy is in a situation where the central bank wants to encourage borrowing or saving.

To summarize, it is in general optimal to deviate from the open economy in the transition due to several effects. The size and sign of the deviations is a quantitative question that is examined in the next section.

IV. Numerical Simulations of Optimal Policies

We examine the full solution to the Ramsey problem in two specific cases. First, to illustrate the theoretical results in the previous section, we consider a constrained closed economy and determine its optimal path to its unconstrained steady state. Second, we analyze the optimal policy in a growth acceleration episode similar to the one experienced by the Chinese economy.

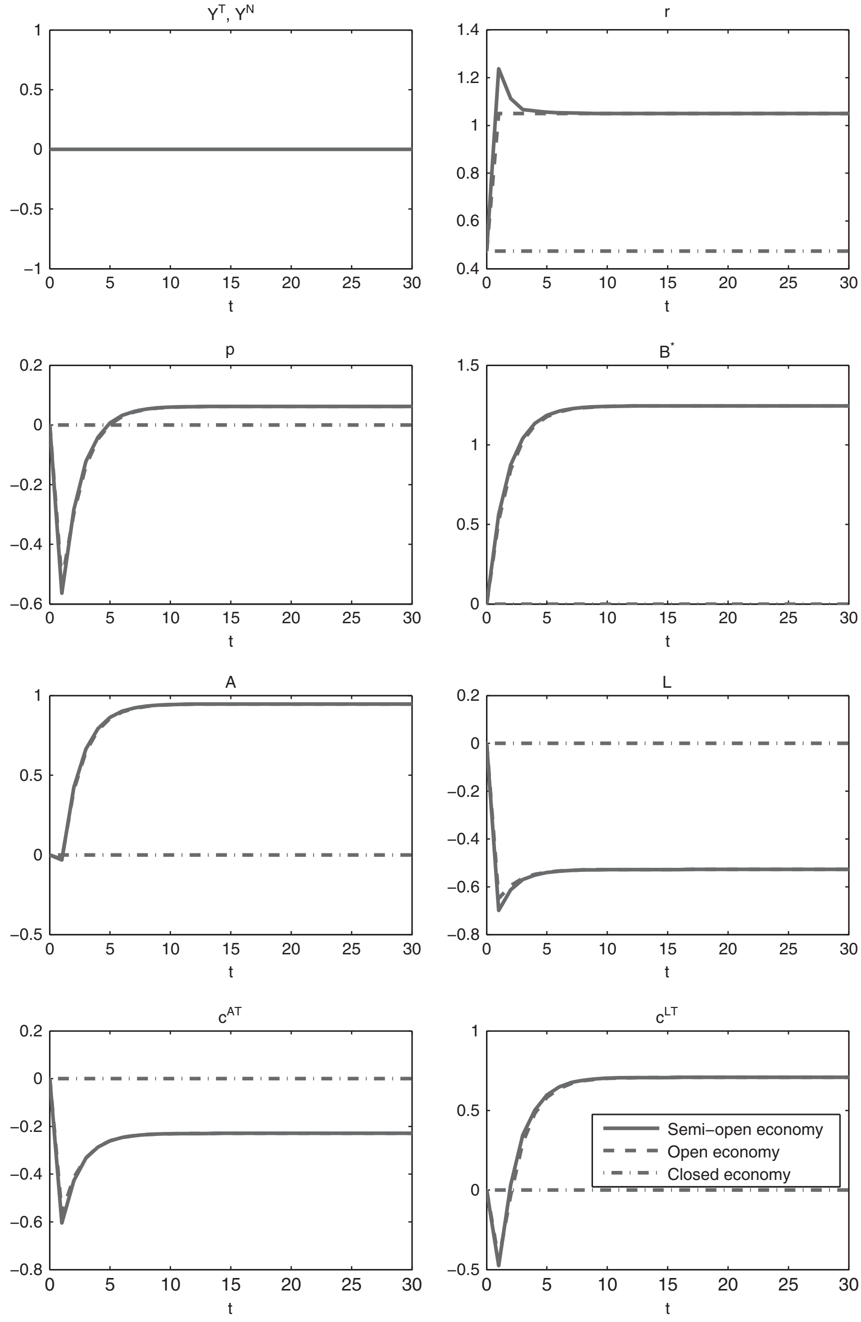
Real Exchange Rate Dynamics in Opening-up Economies

Consider a closed economy characterized by strong borrowing constraints: $2\phi(1+\beta)<1-a$. We know from Proposition 2 that in such a case, the steady-state optimal policy consists in accumulating enough reserves to completely overcome the borrowing constraints. We illustrate this result numerically and examine the whole dynamics of the optimal policy. We simulate a baseline case, with $\beta=1/1.05$ and $\kappa=3$, implying that nontradables represent 75 percent of consumption (as in Obstfeld and Rogoff, 2000). We choose low values for both ϕ and a to satisfy the aforementioned condition: $\phi=0.1$, $a=0$. This corresponds to an economy with strong borrowing constraints and a high volatility of individual incomes. We assume zero growth. For comparison purposes, we simulate the closed economy and the open economy, along with the optimal semi-open economy.

These dynamics are represented in Figure 2 in deviations from the steady state. Consider first the dynamics of the open economy, represented by the dashed line. In the long run, the economy converges to its unconstrained steady state with a higher level of foreign assets, which gives households the means to smooth their consumption of tradable goods. However, in the short run, the economy does not have enough foreign assets yet and is constrained. As a result of the sharp increase in the interest rate, cash-poor households are less able to borrow and have to decrease their consumption of tradables. Anticipating this, cash-rich households cut on their tradable consumption in order to accumulate assets. Consequently, the price of nontradable goods decreases on impact. As the economy accumulates foreign assets, the consumption of tradable goods increases and there is a real appreciation. In the long run, the real exchange rate is slightly higher than in the closed economy steady state because the consumption of tradables is higher thanks to the positive foreign asset position.

Consider now the dynamics of the optimal semi-open economy, represented by the solid line. The economy converges to a similar unconstrained steady state with positive reserves. This illustrates our result that $\tilde{J}>0$ for low ϕ. However, the initial increase in reserves is stronger than in the open economy, so that the interest rate initially jumps to a higher level than the world rate. This corresponds to the case $J^*>0$. As explained in Bacchetta, Benhima, and Kalantzis (2013), this happens in

Figure 2. Optimal Policy in a Closed Economy



Note: We assume that the economy starts at the closed economy steady state. At $t = 1$, the economy either stays in the closed economy ("closed economy"), switches to an open economy ("open economy"), or switches to an optimal semi-open economy ("semi-open economy"). All variables are in deviations from the initial steady state, except B^* and r, which are in levels. The baseline calibration simulated here is characterized by the following parameter values: $\phi = 0.1$, $a = 0$, $\kappa = 3$, and $\sigma = 1$ (log-utility).

our baseline calibration because, with stringent credit constraints, the government can achieve a transfer to cash-poor agents, who have a high marginal utility, by increasing the interest rate. A higher interest rate indeed increases the return on savings, which are part of cash-poor agents' income, without increasing interest payments too much, as L is low. This corresponds to the interest rate channel described in the first line of Equation (22). Adding a real exchange rate channel does not reverse this prediction.

Overall, the utility gain of moving from a closed to a semi-open economy is quite substantial. When switching to a semi-open economy, households gain the equivalent of 7.4 percent of their consumption under a closed economy.[22]

Real Exchange Rate Dynamics in Catching-up Economies

We now turn to the case of a growing economy. We assume that the economy experiences persistent growth but converges to a stationary steady state: $g_{t+1} = \mu g_t$, with $\mu < 1$. This corresponds to a catching-up economy. Importantly, tradable and nontradable endowments grow at the same rate.

Baseline Simulation

We consider the same baseline case as before, with $\kappa = 3$, $\phi = 0.1$ and $a = 0$, and choose $\mu = 0.9$. We start from a symmetric steady state at $t = 0$, where agents are marginally unconstrained. That is, we assume that $B_0^* = \overline{b} Y_0^T$. At $t = 1$, the economy is hit by a positive growth shock $g_1 = 10$ percent.

The optimal semi-open economy dynamics are presented in Figure 3. Before the shock hits, borrowing constraints are just at the limit of binding. When the shock hits, agents now expect persistent growth and want to borrow more from their future income. This makes their borrowing constraint strictly binding in their cash-poor periods. Anticipating this, they accumulate assets A in their cash-rich periods. This accumulation is made possible by an increase in B and thus in net foreign assets B^*. As in the previous simulation, the increase in B^* is so strong that the domestic interest rate r_t rises above r^*, as discussed above.

It is interesting to consider the real exchange rate implications of such a policy. As the consumption of tradable goods is initially depressed relatively to the consumption of nontradables due to the accumulation of foreign assets, there is an initial depreciation. However, as the accumulation of foreign assets increases the tradable revenues of the economy relative to nontradables, the real exchange rate starts appreciating after a few periods. Our model therefore features an appreciating currency in catching-up economies, similar to a Balassa-Samuelson effect. But contrary to the Balassa-Samuelson effect, this appreciation is not generated by TFP catch-up in the tradable sector (we assume the same growth rate in both sectors) but by credit constraints.

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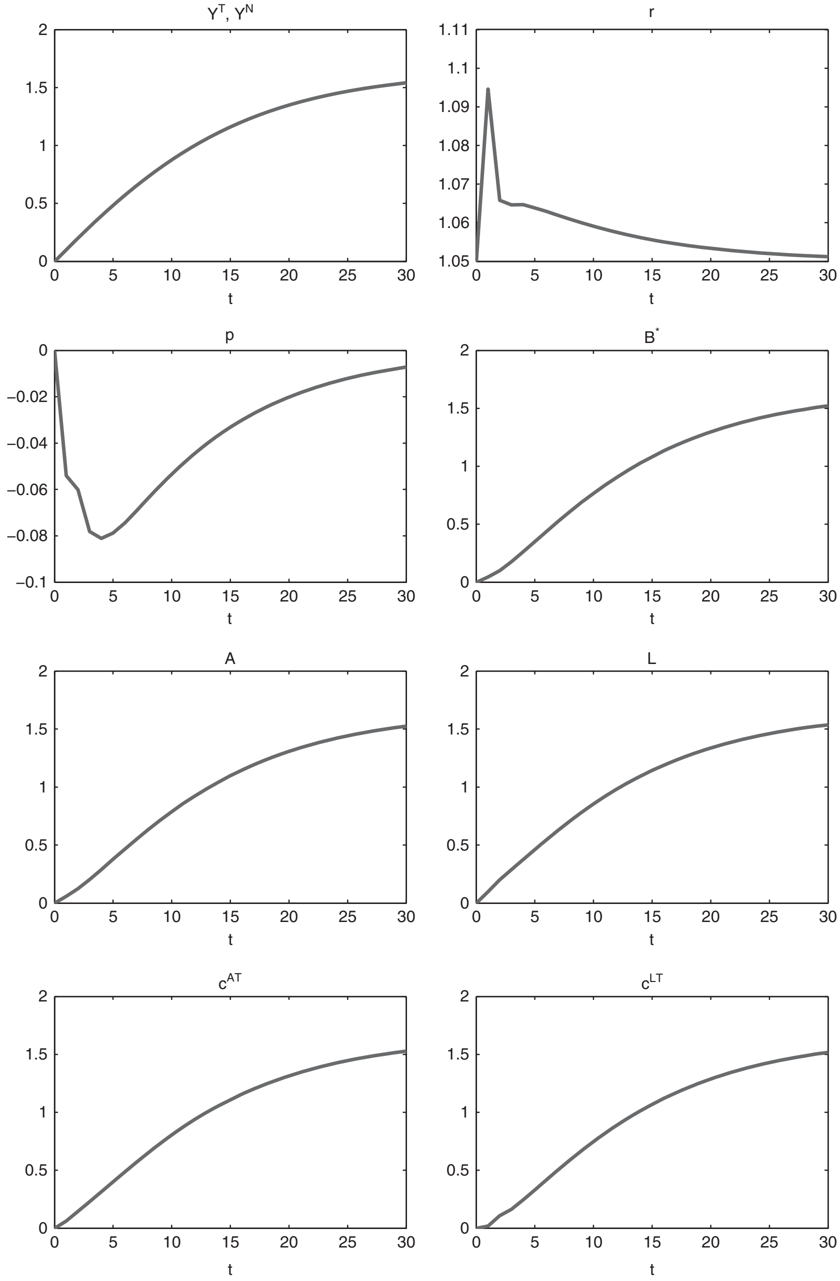
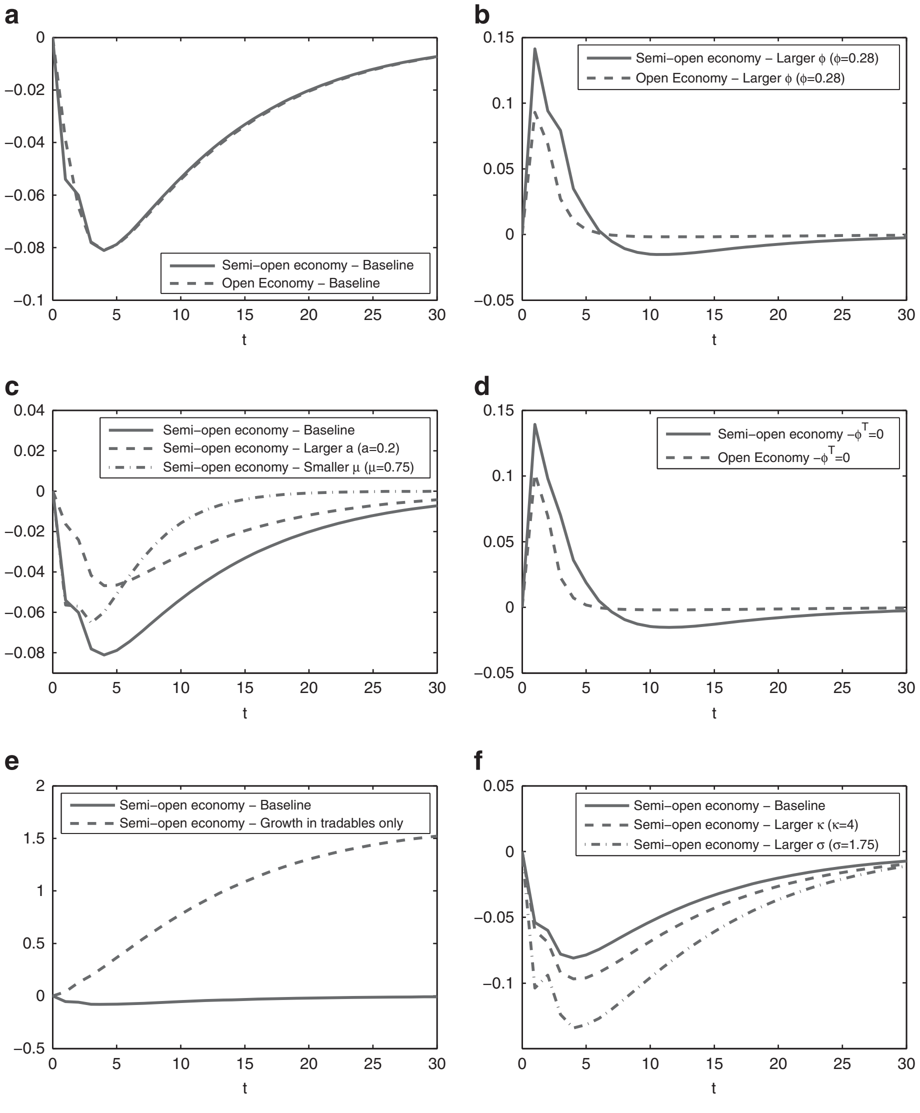
[22] This holds under the veil of ignorance, that is if the households did not know whether they would switch to a semi-open economy when they are borrowers or when they are savers. However, both borrowers and savers would agree to switch, as they respectively gain the equivalent of 7.5 percent and 7.3 percent of their consumption under the closed economy.

Figure 3. Optimal Policy in a Catching-Up Economy—Baseline



Note: We assume that $g_{t+1} = \mu g_t$. At $t = 1$, the economy is hit by a growth shock $g_1 = 10$ percent. All variables are in deviations from the initial steady state, except r, which is in level. The baseline calibration simulated here is characterized by the following parameter values: $\phi = 0.1$, $a = 0$, $\kappa = 3$, $\mu = 0.9$ and $\sigma = 1$ (log-utility).

Figure 4. Optimal Evolution of the Real Exchange Rate in a Catching-Up Economy —Sensitivity Analysis



Note: Evolution of the real exchange rate p under the optimal policy, in deviation from the initial steady state. We assume that $g_{t+1} = \mu g_t$. At $t = 1$, the economy is hit by a growth shock $g_1 = 10$ percent. The baseline calibration is characterized by the following parameter values: $\phi = 0.1$, $a = 0$, $\kappa = 3$, $\mu = 0.9$ and $\sigma = 1$ (log-utility).

In order to assess the role of policy, we compare the dynamics of the real exchange rate in the optimal semi-open economy and in the open economy, both in the baseline calibration. The results are represented in panel (a) of Figure 4. The real exchange rate has a similar behavior in the open and semi-open economy. This suggests that the initial depreciation as well as the subsequent appreciation are natural outcomes of a growth acceleration in a credit-constrained economy and would occur without policy intervention. The only difference is that, in the optimal semi-open economy, the real exchange rate is slightly less depreciated as the government is able to somewhat alleviate the credit constraints. But this is the case only after a few periods, as in the beginning the government accumulates more foreign assets than in the open economy, which depresses the consumption of tradables and depreciates the real exchange rate.

Sensitivity Analysis

To further assess the role of credit constraints, we compare these dynamics to those obtained when the agents can pledge a larger share of their income as collateral. We consider the case where $\phi = 0.28$, which is represented in panel (b) of Figure 4. Here ϕ is large enough for the economy to be in an initial negative foreign asset position, but is still small enough for the credit constraints to be binding. The dynamics of the real exchange rate are now reversed: the country experiences first an appreciation and then a depreciation. Indeed, agents are now able to better smooth their consumption of tradables, which is impossible for nontradables by definition. As a result, they initially consume relatively more tradables than nontradables, hence the initial real appreciation. In the optimal semi-open economy, the real exchange rate appreciates even more initially. This is because the optimal policy with large ϕ consists in maintaining a relatively low domestic interest rate in order to make transfers to agents and alleviate the credit constraint of borrowers. This implies that the central bank accumulates fewer reserves than in the open economy, which stimulates the consumption of tradables and appreciates further the currency.

Notice that the difference between the open economy and the semi-open economy is more substantial in the case $\phi = 0.28$ than in the case $\phi = 0.1$. This is due to the collateral and income channels described in the section "Comparing with the Open Economy." Those two channels create an incentive to appreciate the currency. This reinforces the relative appreciation observed in the optimal semi-open economy with high ϕ and mitigates the relative depreciation observed in the optimal semi-open economy with low ϕ.

As sensitivity checks we consider the cases with a less persistent growth episode, $\mu = 0.75$, and a smaller income variability with $a = 0.2$. These cases are represented in panel (c) of Figure 4. The dynamics of p are similar to the baseline in both cases, except that the initial depreciation is smaller and shorter. Indeed, with less persistent growth and with smaller income variability, the constraints are less binding, which mitigates the initial depreciation.

As mentioned in the Introduction, the literature has highlighted the role of pecuniary externalities through the collateral value in order to justify the use of capital controls or, equivalently, of real exchange rate manipulation. This motive is present in our model. It is represented by the terms P_2 and P_2' in the second line of Equation (22). It reflects the desire of the central bank to appreciate the real exchange rate in order to inflate the value of the collateral and relax the constraint. In order to assess the role of this effect, we distinguish between the share of tradable and nontradable goods that can be used as collateral (for example, as in Bianchi, 2011), that is,

$$r_{t+2}L_{t+2} \leq \phi^T Y_{t+2}^T + \phi^N p_{t+2} Y_{t+2}^N. \tag{23}$$

This pecuniary externality arises only through ϕ^N. We therefore set ϕ^T to zero and set $\phi^N = 0.28(1+\kappa)/\kappa$ so that agents face the same "average" credit constraint as in the case with larger ϕ, represented in panel (b) of Figure 4. We choose the simulation with larger ϕ as a benchmark, rather than the baseline, to give some scope for the pecuniary externality. Indeed, with ϕ close to zero, this externality vanishes. In addition, the economy in the case with larger ϕ is a net debtor, as is usual in the literature on pecuniary externalities. The results are represented in panel (d) of Figure 4. The dynamics of the real exchange rate are almost identical to the case where ϕ^T and ϕ^N are equal, which shows that the collateral value motive is dominated by the other motives for reserve accumulation.

In the baseline case, we assume that growth affects both the tradable and the nontradable sector. In panel (e), we represent the case where growth occurs only in the tradable sector, which is also the assumption made in Balassa-Samuelson. In that case, there is a clear appreciation trend in the currency. Again, this is due to the credit constraint as the consumption of tradable goods is tightly dependent on the endowment. Besides, as the consumption of tradables increases relatively to nontradables, there is no initial depreciation. However, the real exchange rate is still relatively depreciated as compared with an economy without constraint. Indeed, without constraint, the consumption of tradable goods and thus the real exchange rate would adjust immediately to their long-run level.

Empirically, both the tradable and nontradable sectors grew at a high rate in China during the years 2000. The tradable sector, defined as manufacturing and agriculture, grew at an average rate of 8.6 log-points per year in real terms between 2000 and 2010, compared with 10.2 log-points for manufacturing alone. During the same period the nontradable sector, defined as services and nonmanufacturing industry, grew at the slightly higher rate of 11 log-points.[23] Hence, our baseline case of homogeneous growth across sectors seems to be a reasonable approximation of the Chinese dynamics.

Finally, in panel (f), we represent the effect of parameters related to real exchange rate determination. Namely, we consider the case with a stronger preference for nontradables, $\kappa = 4$ and the case with a lower elasticity of substitution between tradable and nontradable goods, that is, with $\sigma = 1.75$. Qualitatively, the dynamics of the real exchange rate with a larger κ or with a larger σ is similar to the baseline case. Quantitatively, the initial depreciation is stronger. This is because both a stronger preference for nontradable goods and a lower degree of substitutability make the real exchange rate more sensitive to changes in tradable and nontradable consumption.[24]

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[23] Authors' calculation based on the World Development Indicators from the World Bank.

[24] As apparent in the graph, the real exchange rate might exhibit some mild oscillations. This is due to heterogeneity: the motive for changing the real exchange rate can fluctuate over time as the

V. Conclusions

This paper has examined the optimal exchange rate policy in an economy with strong capital controls and tight credit constraints. On the one hand, we found it optimal to reproduce an unconstrained and open economy in the long run. On the other hand, the optimal policy in transitions is more complex, in particular due to agents heterogeneity. However, in the case of growth acceleration, the difference between the evolution of the real exchange rate in the optimal policy and in the open economy was found to be small. In other words, the optimal exchange rate policy is close to reproduce the open economy. In an open economy, an increase in growth would lead to an increase in aggregate saving when credit constraints are tight. This would lead to an initial capital outflow with a currency depreciation. Over time, however, saving and capital outflow would decline and the currency would appreciate. This gradual appreciation in a growing economy is not caused by sectoral growth differentials as with the Balassa-Samuelson effect, but by declining saving rates. The optimal policy should broadly accommodate these real exchange rate dynamics.

The analysis has focused on real exchange rate adjustments in the context of sustained structural shocks, thereby taking a longer run perspective. There are several interesting aspects that we have left aside. For example, what would be the role of the exchange rate regime. On this topic, Aghion and others (2009) would suggest that a fixed exchange rate can deliver a higher productivity growth in a context of low financial development. Another interesting question would be the optimal policy in the case of domestic financial liberalization.

Finally, the paper has studied central bank policy considering fiscal developments as given. But several fiscal measures could potentially alleviate the need for saving instruments. For example, investment in public infrastructure could provide additional saving instruments to the private sector and decrease the need for reserve accumulation by the central bank. In recent years, the Chinese government has actually engaged in such a plan of large investments and international reserves at the central bank have started to decline.

APPENDIX

Proof of Proposition 1

Equations (5) and (6), taken in the steady state, imply that $(\beta r)^2(1+\lambda) = 1$. As $\lambda \geq 0$, it follows that $\beta r \leq 1$. Therefore, we look for an equilibrium interest rate $r \in (0, r^*]$.

Assume first that the borrowing constraint (equation (4)) is binding. Then, using the demand for bonds (equation (19)) and the fact that $B_t^* = B_t$, the market-clearing condition for

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agent with higher marginal utility switches from borrower to saver. This is the case in the simulation with a higher σ, where the real exchange rate initially depreciates before appreciating again. Initially, the planner accumulates reserves in order to maintain a high interest rate, which benefits the initial saver (this is captured by R_1) at the expense of the initial borrower (this is captured by R_2 and R_3). Because σ is larger, this however depreciates the currency even more than in the baseline simulation, which hurts the initial borrower further by decreasing revenues and making the constraint more stringent (these effects are summarized by P_1 and P_2 respectively). The following appreciation compensates for that by stimulating the next period's revenues of the initial borrower (P_1' term).

bonds (equation (11)), taken in the steady state, can be rewritten:

$$B^* + \frac{\phi Y}{r} = \frac{1}{1+\beta}\left(\beta[(1-\phi)Y+\pi/2] - \frac{aY+\pi/2}{r} - \frac{\varphi Y}{r^2}\right).$$

From the profit distribution (equation (14)), we have $\pi = (r^* - r)B^* = (1/\beta - r)B^*$. Then, $1/r$ is the solution of a third-degree polynomial: $P(1/r)=0$, with

$$P(X) = \phi\frac{Y}{Y^T}X^3 + \left(a+\phi(1+\beta)]\frac{Y}{Y^T} + \frac{B^*}{2\beta Y^T}\right)X^2 - \left((1-\phi)\beta\frac{Y}{Y^T} - \beta B^*\right)X + \frac{\beta B^*}{2Y^T}$$

where Y/Y^T can be derived from Equation (18):

$$Y/Y^T = 1 + pY^N/Y^T = 1 + \kappa + \frac{\kappa}{1+a}\frac{1-\beta}{\beta}\frac{B^*}{Y^T}.$$

We have $P(0) \geq 0$ for $B^* \geq 0$. In addition, $P(\beta) = P(1/r^*) < 0$ if and only if

$$\left[1 - \frac{\kappa(1-\beta)}{1+a}\left(\frac{1-a}{1+\beta} - 2\phi\right)\right]B^* < \beta(1+\kappa)Y^T\left(\frac{1-a}{1+\beta} - 2\phi\right).$$

This condition is equivalent to $B^*/Y^T < \overline{b}$ when the left-hand side is strictly positive, which we have assumed. Finally, $P(X) \to +\infty$ when $X \to +\infty$ and $P(X) \to -\infty$ when $X \to -\infty$. It follows that P has three roots: one negative root, one root on $(0, \beta)$, and one root on $(\beta, +\infty)$. Since the equilibrium interest rate has to be in $(0, r^*]$, we must have $X \geq \beta$ so that we can discard the first two roots. We conclude that there is a unique interest rate $r \in (0, r^*]$ that clears the market for bonds and that this interest rate is strictly lower than r^*. Given r, it is straightforward to derive all the other variables in the steady state.

The interest rate r is an increasing function of B^*/Y^T. To see this, compute the derivative $dP/d(B^*/Y^T)$ evaluated at the root $\overline{X}$. It has the sign of $-(B^*/2\beta Y^T\overline{X}^2 + \beta(B^*/Y^T)\overline{X} + \beta(B^*/2Y^T)) < 0$. Since P is increasing around $\overline{X}$, then $\overline{X}$ is a decreasing function of B^*/Y^T. Therefore, $r = 1/\overline{X}$ increases with B^*/Y^T.

Finally, the ratio of related traded consumption c^{LT}/c^{AT} is given by the first-order condition (equation (5)) and is equal to $\beta r = r/r^* < 1$.

Assume now that the borrowing constraint does not bind. From the first-order conditions (equations (5) and (6)) when $\lambda = 0$, we must have $\beta r = 1$ in any symmetric steady state, that is, $r = r^*$. Then, it is easy to compute all the other variables in the steady state, to check that the borrowing constraint indeed does not bind, and that $B^*/Y^T \geq \overline{b}$.

Derivation of Equation (20)

The first-order conditions with respect to A_{t+1}, L_{t+1}, and π_t are:

$$\begin{aligned}
&\mathrm{FOC}(A_{t+1}) \quad \gamma_t^A + \gamma_t^B = \beta r_{t+1}\gamma_{t+1}^L,\\
&\mathrm{FOC}(L_{t+1}) \quad \gamma_t^L + \gamma_t^B = \beta r_{t+1}(\gamma_{t+1}^A + \Lambda_{t+1}),\\
&\mathrm{FOC}(\pi_t) \quad\;\;\; \gamma_t^A + \gamma_t^L = 0.
\end{aligned}$$

The sum of the first-order conditions with respect to A_{t+1} and L_{t+1}, together with the last one, gives $\gamma_t^B = \beta r_{t+1}\Lambda_{t+1/2}$. This proves Equation (20).

The difference of the first-order conditions with respect to A_{t+1} and L_{t+1} gives

$$\gamma_t^A - \gamma_t^L = -\beta r_{t+1}(\gamma_{t+1}^A - \gamma_{t+1}^L + \Lambda_{\ t+1}). \tag{A.1}$$

Derivation of Equation (21)

From the first-order condition with respect to λ_{t+1}, we have: $\kappa_t^L \beta r_{t+1} v'(c_{t+1}^{AT}) = \Delta_{t+1}[\phi(Y_{t+1}^T + p_{t+1} Y_{t+1}^N) - r_{t+1} L_{t+1}]$ from which we can deduce that $\kappa_t^L = 0$.

Consider the first-order conditions with respect to c_t^{AT}

$$\frac{1+\kappa}{c_t^{AT}} - (1+\kappa)\gamma_t^A - \gamma_t^G - \kappa\gamma_t^N - \frac{\kappa_t^A}{(c_t^{AT})^2} = 0, \tag{A.2}$$

and with respect to c_t^{LT}

$$\frac{1+\kappa}{c_t^{LT}} - (1+\kappa)\gamma_t^L - \gamma_t^G - \kappa\gamma_t^N + \frac{\kappa_{t-1}^A r_t}{(c_t^{LT})^2} = 0. \tag{A.3}$$

To get an expression for γ_t^G, we first need to compute γ_t^A, γ_t^L, γ_t^N, and κ_t^A.

The first-order condition with respect to r_{t+1} is

$$\frac{\kappa_t^A}{c_{t+1}^{LT}} = \gamma_{t+1}^L A_{t+1} - (\gamma_{t+1}^A + \Lambda_{t+1}) L_{t+1}. \tag{A.4}$$

In the closed economy, we have $A_{t+1} = L_{+1}$. If in addition $\phi = 0$, we get $A_{t+1} = L_{t+1} = 0$, so that $\kappa_t^A = 0$.

The Lagrange multiplier γ_t^N is given by the first-order condition with respect to p_t, together with (10):

$$\gamma_t^N = \frac{2}{c_t^{AT} + c_t^{LT}} - \frac{\gamma_t^A + a\gamma_t^L}{1+a} - \frac{\phi\Lambda_t}{1+a}. \tag{A.5}$$

Finally, from the first-order conditions (equations (A.2) and (A.3)), together with the Euler Equation (6), we can show that $\Lambda_t = \lambda_t / c_{t+1}^{AT}$.

We can now evaluate γ_t^G and Λ_t in the closed economy with $\phi = 0$. In this case, we have $A_{t+1} = L_{t+1} = B_{t+1}^* = 0$ and therefore $\pi_t = 0$. From the current account identity (equation (17)), and the budget constraints (equations (2) and (3)), we get $c_t^{AT} = Y_t^T$ and $c_t^{LT} = aY_t^T$. The Euler Equation (5) implies that $\beta r_{t+1} = a(1+g_{t+1})$. From Equation (6), we get $\lambda_{t+1} = 1/(a^2) - 1$. Therefore, $\Lambda_t = (1/(a^2) - 1)(1)/(Y_t^T)$. Then, we can iterate Equation (A.1) forward to get $\gamma_t^A = -\gamma_t^L = -(1-a)/(2aY_t^T)$. From Equation (A.5), we get $\gamma_t^N = (1+a)/(2aY_t^T)$. Then, Equation (A.2) yields $\gamma_t^G = (1+a)/(2aY_t^T)$. Therefore,

$$\begin{aligned} \tilde{J}_{t+1} &= \gamma_{t+1}^G - \gamma_t^G + \beta r_{t+1} \frac{\Lambda_{t+1}}{2} \\ &= \frac{1+a}{2aY_t^T}[(1+g_{t+1})^{-1} - 1] + \frac{a(1+g_{t+1})(1-a^2)}{2a^2 Y_{t+1}^T}, \end{aligned}$$

which yields Equation (21).

Derivation of Equation (22)

By definition, J^* is the left-hand side of Equation (20), evaluated at $r_{t+1} = r^* = 1/\beta$, so we have:

$$J_{t+1}^* = -(\gamma_t^G - \gamma_{t+1}^G) + \frac{\Lambda_{t+1}}{2}. \tag{A.6}$$

Subtracting Equation (A.3) at t+1 from Equation (A.2) at t, and using the fact that $c_t^{AT} = c_{t+1}^{LT}$ in the open economy, we obtain:

$$\gamma_t^G - \gamma_{t+1}^G = (1+\kappa)(\gamma_{t+1}^L - \gamma_t^A) + \kappa(\gamma_{t+1}^N - \gamma_t^N) - \frac{1+\beta}{\beta}\frac{\kappa_t^A}{(c_t^{AT})^2}. \tag{A.7}$$

By iterating Equation (A.1) forward when $\beta r_{t+1} = 1$, and given that $\gamma_t^A = -\gamma_t^L$, we get $\gamma_t^A = -\gamma_t^L = \sum_{s\geq 1}(-1)^s(\Lambda_{t+s})/(2)$. Then, Equation (A.4) in the open economy can be rewritten:

$$\frac{\kappa_t^A}{(c_t^{AT})^2} = \frac{1}{c_t^{AT}}\left[\left(\sum_{i=1}^{\infty}\Lambda_{t+2i}\right)A_{t+1} - \left(\sum_{i=0}^{\infty}\Lambda_{t+1+2i}\right)L_{t+1} - \frac{1}{2}\left(\sum_{i=1}^{\infty}\Lambda_{t+1+i}\right)(A_{t+1} - L_{t+1})\right]. \tag{A.8}$$

Injecting Equations (A.5) and (A.8) in Equation (A.7), and replacing $\gamma_t^G - \gamma_{t+1}^G$ in Equation (A.6), we obtain Equation (22).

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