Money and Capital in a Persistent Liquidity Trap*

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Abstract

In this paper we analyze the implications of a persistent liquidity trap in a monetary model with asset scarcity. We show that a liquidity trap may lead to an increase in real cash holdings and be associated with a decline in investment and output in the medium term. This medium-term impact is a supply-side effect that may arise when firms face financial constraints and is distinct from the short-run demand-side effects arising from nominal rigidities. It occurs in particular with a persistent deleveraging shock. When the interest rate hits the zero-lower bound, cash becomes a store of value that diverts resources from investment. Policy implications differ from shorter-run analyses implied by nominal rigidities. Quantitative easing leads to a deeper liquidity trap. Exiting the trap by increasing expected inflation or applying negative interest rates does not solve the asset scarcity problem. A higher government debt helps exiting the liquidity trap and reduces asset scarcity, but may hurt investment in the medium run.
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1 Introduction

Periods of persistent liquidity traps typically coincide with substantial increases in real cash holdings, as illustrated in Figure 1 for the U.S. and Japan. These periods are also characterized by disappointing levels of investment and of output growth. Can increased real money holdings

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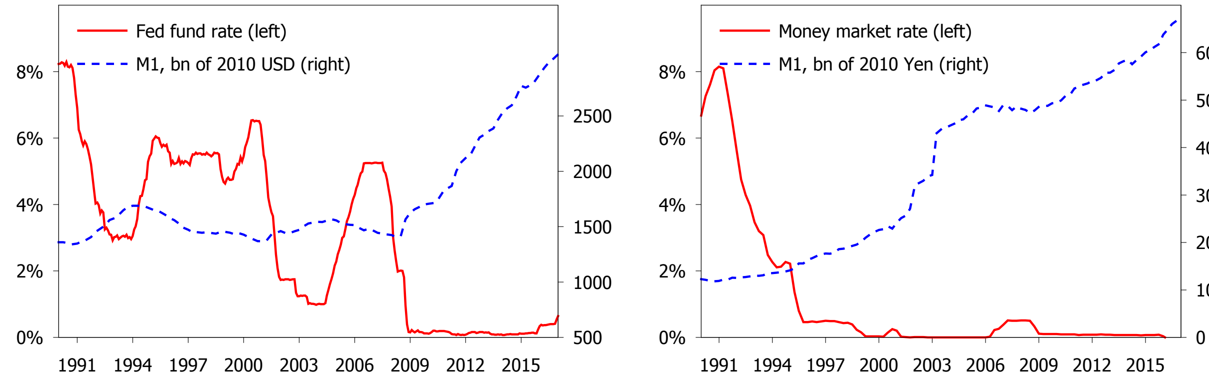
(a) United States (b) Japan



Source: International Financial Statistics and Federal Reserve. M1 is deflated by the CPI.

Figure 1: Policy rates and M1 in the US and Japan.

be associated with lower physical investment and lower growth? Most macroeconomic models would give a negative answer to this question.[1] In this paper, we argue that in a liquidity trap investment can be negatively correlated with investors' real money holdings. We consider a monetary model where prices are flexible in the medium run and where money is only held for transaction purposes in normal times. In a liquidity trap, depending on the nature of the shocks hitting the economy, investors may allocate part of their saving to money holdings. This may then hamper aggregate investment capacity and have a long-lasting impact on output.

Our paper identifies a supply-side mechanism that may contribute to a slower recovery in a liquidity trap. In the shorter run, demand-side effects dominate due to nominal rigidities. In the medium term, however, these effects die out and only the supply-side mechanism remains. This contrasts with most of the literature that only considers demand effects. The policy implications of these supply-side effects also differ from shorter-run analyses. When a liquidity trap is persistent, our analysis is therefore complementary to shorter-run demand side perspectives.

Money is introduced in a model with scarce (liquid) assets due to the lack of income pledgeability, in the spirit of Woodford (1990) and Holmström and Tirole (1998).[2] Investors find investment opportunities every other period, so that they alternate between investing and saving phases. In their investing phase, they use past liquid saving and borrow to invest, but they face credit constraints. Agents can save in two liquid assets, bonds and money. As long as the nominal interest rate is positive, money is dominated as an asset and is held only for transaction purposes. At the Zero Lower Bound (ZLB), bonds and money become substitutes

[1]In both classical monetary models and New Keynesian models, the quantity of money plays little role. E.g., see Galí (2015) for a discussion.

[2]See also Farhi and Tirole (2012) and Bacchetta and Benhima (2015) for more recent contributions.

and money is held for saving purposes as well. In this framework, we consider a persistent deleveraging shock, modeled as in Eggertsson and Krugman (2012) by a tightening of the investors' borrowing constraints.[3] This shock generates a decrease in the nominal interest rate until it hits the ZLB. This creates a gap between the effective and the shadow real interest rate that would prevail without the ZLB. In our asset-scarce model, the fall in the shadow interest rate lasts as long as the deleveraging shock.

We show that the consequences of a deleveraging shock are very different outside or at the ZLB. In both cases, a deleveraging shock reduces the supply of private bonds and leads, out of equilibrium, to excess saving. In equilibrium, the adjustment comes from restoring the supply of assets, which happens through different channels outside and at the ZLB. Outside the ZLB, the interest rate adjusts downward, which enables investors to issue more bonds, thus offsetting the tighter borrowing constraint. The shock has then no effect on capital accumulation and output (in our benchmark specification). Bonds are in fact the only store of value and fully channel funds from savers to investors. However, when the deleveraging shock brings the economy to the ZLB, the interest rate cannot fall enough to stimulate the supply of bonds by investors. Instead, savers' funds are channeled to money, as it becomes a valid store of value at the ZLB. In that case, the supply of assets is restored through an increase in the real value of money. Without money creation by the central bank, this increase happens through a temporary deflation.

Saving in money rather than bonds means that fewer funds are channelled to investment. But money can be liquidated by investors to finance investment when they get an investment opportunity. This could in principle offset the fact that they have a reduced access to external finance. However, in an asset-scarce environment, assets, money included, have a low return. A low return on saving thus implies a small amount of liquidity to finance investment. For investors in their investing phase, using money to finance investment does not compensate the fact that they have a limited ability to issue bonds. All in all, *net* resources are diverted from investment and capital remains low in the medium run. In this sense, the possibility to save in money effectively crowds out investment.

Notice the subtle role of money in this context. On the one hand, it has a special role since it is responsible for the existence of a ZLB. On the other hand, it does not have a special role as an asset at the ZLB, since bonds and money are perfect substitutes. The difference is that the real supply of money is elastic, while the real supply of bonds is limited by credit constraints. Even though money and bonds are perfect substitutes at the ZLB, the model yields a well defined

[3]With nominal rigidities, the literature has already shown that such a deleveraging shock can lead to low levels of output and employment in the short run, due to lower demand. Eggertsson and Krugman (2012), Werning (2012), Benigno et al. (2014), or Caballero and Farhi (2017) show this in New-Keynesian models. These effects are also present in our analysis.

distribution of money and bonds across agents. There is a demand for money for transaction purposes by a group of workers subject to a cash-in-advance constraint. It is still well defined at the ZLB because these workers are subject to a binding borrowing constraint. There is also a demand for both money and bonds by investors for saving purposes. Real money holdings by these agents are determined as the difference between their total desired saving and the available supply of bonds.

We start by focusing on the limiting case of a permanent liquidity trap. This case allows us to focus on steady states, which is analytically tractable and gives important insights for transitory, but persistent, deleveraging shocks. We then analyze numerically transitory shocks by assuming that the deleveraging shock ends with a constant probability in each period. On impact, the shock has more negative effects than in the medium run due to nominal rigidities—in the form of downward wage rigidity. After a few years, the economy recovers, but only partially. The economy only recovers completely when the financial constraint parameter comes back to its initial state and the economy gets out of the liquidity trap.

In our analysis, the medium-run investment slowdown is associated with a deleveraging shock that affects *investors*.[4] In contrast, a deleveraging shock affecting only workers has no medium-run effects in the liquidity trap, because it does not alter the investors' capacity to finance investment.[5] Besides, other types of shocks, such as an increase in the discount rate or a decrease in the productivity growth rate, do not have a negative long-term effect on the investment rate. Our results therefore suggest that investors' deleveraging is an important factor of growth slowdowns in persistent liquidity traps.

The policy implications of our framework differ from traditional shorter-run analyses. Typical policies advocated in a liquidity trap are fiscal policy, quantitative easing (QE), negative interest rates, or an increase in expected inflation. These policies may have their merits in the short run, but they lead to trade-offs in the medium run. QE operations, by taking public bonds away from the market, decrease the shadow real interest rate and generate a deeper and potentially longer liquidity trap. Negative nominal interest rates or an increase in expected inflation makes the liquidity trap less severe by lowering the effective real interest rate and supporting investment. However, these policies do not solve the asset scarcity problem and the lower real interest rate further deteriorates the allocation of resources across time. Instead, the increase in debt that follows from expansionary fiscal policy improves the supply of liquidity and helps exiting the liquidity trap by increasing the shadow interest rate.[6] However,

[4]Section 1 of the Online Appendix shows that the rise in cash holdings in the US comes from the less constrained firms and households, which would correspond to investors in the model.

[5]The empirical literature shows that all sectors of the private economy suffer from deleveraging in the Great Recession.

[6]Such policies are also discussed in policy circles, e.g., Kocherlakota (2015). Acharya and Dogra (2018)

while a higher supply of liquidity improves the allocation of resources across time, it can hurt investment in the medium run, leading to a lower capital stock and lower wages.

Our asset-scarce environment is characterized by a low interest rate, so it is prone to rational bubbles. When we allow for bubbles that can be held by savers, we show that they play a role similar to money, diverting resources away from investment. By sustaining a higher interest rate, the emergence of a bubble rules out money as a store of value and brings the economy out of the ZLB.

Related literature The paper is related to the recent literature on persistent ZLB equilibria. In this literature, liquidity traps usually arise when the natural rate of interest falls enough to make the nominal rate hit the ZLB (Krugman, 1998; Auerbach and Obstfeld, 2005; Eggertsson and Krugman, 2012; Werning, 2012). But even in a *persistent* liquidity trap, stagnation remains a demand-side phenomenon. Schmitt-Grohé and Uribe (2013) add permanent nominal rigidities (a non-vertical long-run Phillips curve) to Benhabib et al. (2001)'s multiple equilibrium model to get a lower output in the ZLB equilibrium. Similarly, Benigno and Fornaro (2018), in an endogenous growth model, assume permanent nominal rigidities to get a self-fulfilling ZLB steady state with low output and low growth. In the non-Ricardian models of Eggertsson and Mehrotra (2014), Caballero and Farhi (2017) and Michau (2018), long-run nominal rigidities also generate a persistently negative output gap at the ZLB.

Like us, Buera and Nicolini (2016), Guerrieri and Lorenzoni (2017) and Ragot (2016) examine the effects of a deleveraging shock at the ZLB in the absence of nominal rigidities.[7] Guerrieri and Lorenzoni (2017) focus on consumer spending in a model where households face borrowing limits, and Ragot (2016) studies optimal monetary policy in a model where money has redistributive effects due to limited participation. In both models, there is no capital accumulation. Closer to our approach, Buera and Nicolini (2016) consider a monetary model where producers need external funds to buy capital. While we focus on the negative relationship between cash holdings and capital, they study the reallocative effects of low real interest rates on total factor productivity and capital, and assume a moneyless economy in most of their paper. Like us, they discuss the trade-offs associated with the inflation policy but do not consider increases in public debt large enough to exit the ZLB. Caggese and Perez-Orive (2018) also study the reallocative effects of a low real interest rate, but when low interest rates are driven by the rise of intangible capital.

examine the role of public debt and inflation policy to exit the ZLB in an overlapping-generation model, but the trade-offs are different from our framework with constrained investors.

[7] Di Tella (2018) analyzes the role of money in a flexible price model with uncertainty shocks. Although he does not focus on the ZLB, he shows that the presence of money can reduce investment because of lower precautionary saving.

The effect of money holdings on capital of our model are reminiscent of the effects of external liquidity in other models where investors' income is not fully pledgeable, such as Woodford (1990), Holmström and Tirole (1998), and more recently Covas (2006), Angeletos and Panousi (2009), Kiyotaki and Moore (2012), Kocherlakota (2009), and Farhi and Tirole (2012). The role of money as a saving instrument is also evocative of the literature on the value of fiat money (Samuelson (1958), Townsend (1980)). In our paper, transactions are not constrained by demography or spatial separation, but by the lack of income pledgeability.

The adjustment mechanism of our model relies on a real balance effect that has been originally studied by Pigou (1943) and Patinkin (1956). More recently, Weil (1991), Ireland (2005), Bénassy (2008) and Devereux (2011) have analyzed real balance effects in OLG models.

In our model, rational bubbles arise in asset-scarce environments with a low interest rate, as in Samuelson (1958), Tirole (1985), and more recently Martin and Ventura (2012) and Farhi and Tirole (2012). Closer to our approach, Asriyan et al. (2016) introduce bubbles in a monetary environment and analyse liquidity traps.

The rest of the paper is organized as follows. Section 2 presents the basic model with infinitely-lived entrepreneurs and workers. Section 3 studies the effect of a permanent deleveraging shock in a steady state, while Section 4 analyzes the dynamic impact of persistent shocks. Section 5 examines policy options. Section 6 studies several extensions of the benchmark model: workers' deleveraging, bubbles, preference and growth shocks, financial intermediation, and inefficient saving technology. Section 7 concludes.

2 A Model with Scarce Assets and Money

We consider a heterogenous-agents, non-Ricardian monetary model where the supply of bonds and the distribution of money holdings matters. In normal times, bonds dominate money and the real interest rate adjusts to balance the supply and demand for bonds. In a liquidity trap, however, bonds and money become perfect substitutes. The supply and demand of assets are then balanced by an adjustment in real money holdings. These two adjustment mechanisms, through interest rates or money holdings, have different implications for investment and output, and therefore for policy. We show that in a liquidity trap real money holdings by investors tend to increase, which may be associated with a decline in capital and output in the medium run. This is in particular the case for a deleveraging shock, which we analyze in the next sections. We assume downward wage rigidity which may affect the short-run impact of shocks, but plays little role in the medium run. In this section, we describe the model and the equilibrium. For expositional purposes, we focus here on perfect foresight. The model will be simulated later under uncertainty.

2.1 The Setup

We model a monetary economy with heterogeneous investors, workers, and firms. There are three types of assets: bonds, money, and capital. Bonds are nominal and promise to pay one unit of currency in the next period. Denote by i_t their gross real rate of return expressed in units of currency: a bond issued in period t is traded against $1/i_t$ units of money. Under perfect foresight, the gross real return expressed in units of good is $r_t = i_t P_t / P_{t+1}$, where P_t is the price of the final good in units of currency in period t.

Money bears no interest, but it provides transaction services by relaxing a cash-in-advance constraint faced by workers. Money holdings are non-negative. In normal times, when $i > 1$, money is strictly dominated by bonds as a saving instrument. Then, only workers hold money, for transaction purposes. However, when $i = 1$, money becomes as good a saving instrument as bonds and investors start holding money as well.

Investors Following Woodford (1990), investors find investment opportunities every other period, so that they alternate between a saving period and an investment period. This simple approach is a convenient limiting case allowing to capture idiosyncratic shocks in a very tractable way.[8] Consequently, at each point in time there are two groups of identical investors, investing and saving every other period. We call investors in their saving phase S-investors, or simply savers, and denote them by S, while investors in their investment phase are called I-investors and are denoted by I. Each group is of measure 1. We assume logarithmic utility in order to get closed-form solutions. A S-investor (I-investor) maximizes $U_t^S = \log(c_t^S) + \beta E_t U_{t+1}^I$ ($U_t^I = \log(c_t^I) + \beta E_t U_{t+1}^S$), where c_t^S (c_t^I) refers to her consumption in period t and U_{t+1}^I (U_{t+1}^S) refers to her next-period's lifetime utility, subject to a sequence of budget constraints and borrowing constraints.

In period t, I-investors start with wealth $(A_t + M_t^S)/P_t$ where A_t and M_t^S are respectively nominal bond holdings and nominal money holdings inherited from their preceding saving phase. They get an investment opportunity, which consists in a match with a firm. They consume c_t^I, issue B_{t+1} nominal bonds, and invest k_{t+1} in the firm. We focus on real budget constraints, so we denote by $b_t = B_t/P_t$ and $a_t = A_t/P_t$ the real outstanding values of nominal

[8] Section 5.9 of the Online Appendix examines the more general case with idiosyncratic uncertainty on the occurrence of an investment opportunity and shows that the analysis is similar. We consider a 2-state Markov process where an investor with no investment opportunity at time $t-1$ receives an investment opportunity at time t with probability $\omega \in (0, 1]$; while an investor with an investment opportunity at time $t-1$ receives no investment opportunity at time t.

bonds issued and held by investors. We abstract from the money demand by I-investors, as it is always zero in equilibrium. Their budget constraint is

$$\frac{b_{t+1}}{r_t} + a_t + \frac{M_t^S}{P_t} = c_t^I + k_{t+1}. \tag{1}$$

In period t, S-investors start with equity k_t and outstanding nominal debt B_t inherited from their preceding investment phase. They receive a dividend $\rho_t k_t$. Then, they consume c_t^S, buy A_{t+1} nominal bonds and save M_{t+1}^S in money. Their budget constraint is

$$\rho_t k_t = c_t^S + b_t + \frac{a_{t+1}}{r_t} + \frac{M_{t+1}^S}{P_t}. \tag{2}$$

In general, the return on capital is larger than the return on bonds. Thus, I-investors choose to leverage up when they receive an investment opportunity. But they face a borrowing constraint as they can only pledge a fraction ϕ_t of dividends:

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}. \tag{3}$$

This constraint rules out default in equilibrium as it ensures that I-investors will not renegotiate their debt ex post, since creditors can always recover at least the value of the debt. Leverage ϕ_t varies exogenously over time and will be the source of macroeconomic fluctuations.

In this framework, where investment opportunities are lumpy and investors cannot fully pledge their future income, there is an asynchronicity between the investors' access to and their need for resources. This creates a demand for assets for liquidity purposes in the investors' saving phase.[9] Both bonds and money can satisfy this demand for liquidity, or demand for assets (we will use these two terms interchangeably). Capital, on the other hand, is illiquid, since it cannot be fully pledged.

Firms There is a unit measure of 2-period-lived firms, each matched with an I-investor. Firms use their investor's funds to buy capital k_t. In the following period, they hire labor h_t at real wage w_t, produce output y_t with a Cobb-Douglas function $F(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}$ and distribute profits $y_t + (1-\delta)k_t - w_t h_t$ to I-investors as dividends. As the labor market is competitive, profits are linear in k and equal to $\rho_t k_t$, with ρ_t the equilibrium return on capital.[10] For expositional clarity, we assume full depreciation, $\delta = 1$, which gives profits $\rho_t k_t = \alpha y_t$ and a

[9]We use the term liquidity in the same spirit as Woodford (1990) and Holmström and Tirole (1998).

[10]ρ is equal to $F(1, 1/\kappa(w)) + 1 - \delta - w/\kappa(w)$ where $\kappa(w)$ is the equilibrium capital-labor ratio defined by $w = F_h(\kappa(w), 1)$.

wage bill $w_t h_t = (1-\alpha)y_t$. The model will be simulated later with partial depreciation.[11]

Workers There is a unit measure of workers, endowed with one unit of labor, who maximize $U_t^w = E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}^w)$, where c_t^w refers to workers' consumption, subject to a sequence of budget constraints, borrowing constraints, cash-in-advance (CIA) constraints and $h_t \leq 1$. Their budget constraint is:

$$c_t^w + \frac{M_{t+1}^w}{P_t} + l_t^w = w_t h_t + \frac{T_t^w}{P_t} + \frac{M_t^w}{P_t} + \frac{l_{t+1}^w}{r_t}, \tag{4}$$

where $P_t l_{t+1}^w$ is the amount of nominal bonds issued in t, M^w money holdings, and T^w a monetary transfer from the government. Workers are subject to a CIA constraint: they cannot consume more than their real money holdings. Assuming the bond market opens before the market for goods, these holdings are the sum of money carried over from the previous period, monetary transfers from the government, and money borrowed on the bond market (net of debt repayment):

$$c_t^w \leq \frac{M_t^w + T_t^w}{P_t} + \frac{l_{t+1}^w}{r_t} - l_t^w. \tag{5}$$

Workers also face a borrowing constraint which limits the real value of their debt:[12]

$$l_{t+1}^w \leq \bar{l}_t^w y_{t+1}. \tag{6}$$

When $\beta r < 1$, (6) is binding in the vicinity of a steady state. Workers would prefer to dissave and always hold the minimum amount of money, so that the CIA (5) is also binding. Together with their budget constraint (4), this implies that their money holdings are simply equal to the wage bill: $M_{t+1}^w / P_t = w_t h_t$. Since the wage bill is equal to $(1-\alpha)y_t$ in equilibrium, money demand by workers is given by:

$$M_{t+1}^w = (1-\alpha) P_t y_t. \tag{7}$$

The government Denote by M_t the money supply at the beginning of period t. In period t, the government can finance transfers to agents by creating additional money $M_{t+1} - M_t$ and by issuing nominal bonds $P_t l_{t+1}^g$. For simplicity, we assume that the government only makes

[11] Analytical results are extended to the case $\delta < 1$ in Section 5.6 of the Online Appendix.

[12] We assume that the borrowing limit is linear in the wage bill and therefore proportional to output, since the equilibrium wage bill is a fraction $1-\alpha$ of output.

transfers to workers. The budget constraint of the government is:

$$\frac{M_{t+1}}{P_t} + \frac{l^g_{t+1}}{r_t} = \frac{M_t + T^w_t}{P_t} + l^g_t. \tag{8}$$

Several fiscal and monetary policies can be considered. As a benchmark case, we assume that the fiscal authority chooses a sequence of debt-to-GDP ratio $\bar{l}^g_t$ and that the monetary authority chooses the growth of money:

$$l^g_{t+1} = \bar{l}^g_t y_{t+1}, \qquad M_{t+1}/M_t = \theta_{t+1}. \tag{9}$$

Transfers to households then adjust to satisfy (8).[13] In a steady state, money growth is constant and equal to θ, which pins down steady-state inflation to θ. We make the following parametric assumption:

Assumption 1 *$\theta > \beta$.*

Assumption 1 implies that the economy can only hit the ZLB in a steady state where $\beta r < 1$, with binding borrowing constraints. Indeed, in the steady state, the nominal gross interest rate is $i = r\theta$. With Assumption 1, $i = 1$ implies $\beta r = \beta/\theta < 1$. This assumption is naturally satisfied as long as $\theta \geq 1$, that is with a non-negative steady-state inflation.

Downward wage rigidity Wages are assumed to be downwardly rigid in the spirit of Schmitt-Grohé and Uribe (2016). The nominal wage, defined by $W_t = P_t w_t$, must satisfy

$$W_t = \max\left\{\gamma W_{t-1}, W^*_t\right\}, \tag{10}$$

where $\gamma \in (0, \theta)$ is the degree of nominal rigidity and W^* is the nominal wage that would satisfy full employment ($h_t = 1$): $W^*_t = P_t(1-\alpha)k^\alpha_t$. If $W^*_t \geq \gamma W_{t-1}$, wages can adjust and there is full employment. Otherwise, there is unemployment: $h_t < 1$, where h_t is determined by $\gamma W_{t-1} = P_t(1-\alpha)\left(\frac{k_t}{h_t}\right)^\alpha$. In a steady state, the downward wage rigidity constraint is not active since prices grow at a rate θ larger than γ.

Market clearing for bonds and money Equilibrium in the two markets is given by:

$$b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1}. \tag{11}$$

$$M^S_{t+1} + M^w_{t+1} = M_{t+1}. \tag{12}$$

[13] This ensures monetary dominance.

Sequences of leverage The sequences of leverage $\{\phi_t, \bar{l}_t^w, \bar{l}_t^g\}$ are exogenous and deterministic, consistent with our assumption of perfect foresight.

2.2 Asset-scarce equilibrium with full employment

It is useful to consider the limit case with no downward wage rigidity, where $\gamma = 0$. In that case, the economy is always in full employment ($h_t = 1$).

Asset scarcity and binding borrowing constraints We focus on equilibria where borrowing constraints for I-investors and workers are binding in every period. In such "asset-scarce" equilibria, borrowing constraints prevent borrowers from supplying the saving instruments needed by savers and steady states are characterized by $\beta r < 1$.

More precisely, consider an exogenous sequence of leverage $\{\phi_t, \bar{l}_t^w\}_{t\geq 0}$, an exogenous sequence of policy parameters $\{\theta_{t+1}, \bar{l}_t^g\}_{t\geq 0}$, and initial assets $\{k_0, a_0, b_0, M_0, M_0^S, M_0^w, l_0^w, l_0^g\}$. The associated asset-scarce equilibrium is an allocation $\{y_t, c_t^I, c_t^S, c_t^w, k_{t+1}\}_{t\geq 0}$, a vector of portfolio choices $\{a_{t+1}, b_{t+1}, l_{t+1}^w, M_{t+1}^S, M_{t+1}^w\}_{t\geq 0}$, a policy $\{M_{t+1}, T_t^w, \bar{l}_t^g\}_{t\geq 0}$, and a price vector $\{r_t, \rho_{t+1}, w_t, P_t\}_{t\geq 0}$ solving the maximization problems of both groups of investors and workers with binding borrowing constraints (3) and (6), and satisfying the production function $y_t = k_t^\alpha$, the expression for equilibrium profits $\rho_t k_t = \alpha y_t$ and wages $w_t = (1-\alpha)y_t$, the government budget constraint (8) and policy rules (9), and the market-clearing conditions (11) and (12). We omit the gross nominal rate from that definition as it is simply given by $i_t = r_t P_{t+1}/P_t$. The full list of equilibrium conditions is given in Section 3.1 of the Online Appendix.

A four-equation model An asset-scarce equilibrium can be reduced to a 4-dimensional system. Real money holdings by S-investors, defined as $m_t^S = M_t^S/P_t$, turn out to play a key role in a liquidity trap. Even though money is a perfect substitute to bonds in this case, the real supply of money is elastic, while the supply of bonds is constrained. Denote $\bar{l}_t = \bar{l}_t^g + \bar{l}_t^w$ the exogenous total supply of bonds by workers and the government. In equilibrium, $(\phi_t\alpha + \bar{l}_t)y_{t+1}$ is the total supply of bonds, as $\phi_t\rho_{t+1}k_{t+1} = \phi_t\alpha y_{t+1}$ is the supply by investors in their investing phase.

The dynamics of the model can be fully described by the set of variables $\{r_t, m_{t+1}^s, k_{t+1}, P_t\}_{t\geq 0}$

which satisfies the following four equations:

$$m_{t+1}^S\left(r_t - \frac{P_t}{P_{t+1}}\right) = 0, \quad r_t \geq \frac{P_t}{P_{t+1}}, \quad m_{t+1}^S \geq 0, \tag{13}$$

$$\beta\alpha(1-\phi_{t-1})y_t = \frac{1}{r_t}\left[(\phi_t\alpha + \bar{l}_t)y_{t+1} + m_{t+1}^S\right], \tag{14}$$

$$k_{t+1} + \frac{1}{r_t}\bar{l}_t y_{t+1} + \frac{P_{t+1}}{P_t}m_{t+1}^S = \beta\left[(\alpha + \bar{l}_{t-1})y_t + m_t^S\right], \tag{15}$$

$$\frac{M_{t+1}}{P_t} = (1-\alpha)y_t + \frac{P_{t+1}}{P_t}m_{t+1}^S, \tag{16}$$

where $y_t = k_t^\alpha$. The sequence $\{\phi_t, \bar{l}_t, M_{t+1}\}$ is exogenous with $M_{t+1} = \theta_{t+1}M_t$, and there is an initial condition $\{\bar{l}_{-1}, m_0^S, k_0, M_0\}$.

Equation (13) is the complementary slackness condition (CSC) summarizing the optimal portfolio choice of S-investors. As long as $i > 1$, or equivalently $r_t > P_t/P_{t+1}$, money has a strictly lower expected return than bonds and investors do not hold any of it: $m^S = 0$. We refer to this case as "normal" periods. When $i = 1$, that is, $r_t = P_t/P_{t+1}$, investors also hold money for saving purposes, so $m^S \geq 0$. We refer to this case as "liquidity trap" periods.

Equation (14) directly derives from the Euler equation of S-investors. As they are unconstrained, their consumption satisfies the Euler condition: $1/c_t^S = \beta r_t/c_{t+1}^I$. With log-utility, consumption is a fraction $1-\beta$ of wealth for both types of investors: $c_{t+1}^I = (1-\beta)(a_{t+1}+m_{t+1}^S)$ and $c_t^S = (1-\beta)(\alpha y_t - b_t)$.[14] Substituting these expressions into the Euler equation, and using the binding borrowing constraints (3) and (6), and the market clearing condition for bonds (11), we get (14). This equation can also be interpreted as an equilibrium condition for saving instruments. The left-hand side (LHS) is the demand for saving instruments by S-investors, which depends on current income. The right-hand-side (RHS) is the supply of saving instruments. The first term is the supply of bonds by I-investors, which depends on their future pledgeable income and on their leverage ratio ϕ. The second term depends on $\bar{l}$, and represents the supply of bonds by workers and the government. Finally, the last term on the RHS corresponds to money used by S-investors as a saving instrument.

I-investors partly finance their investment by selling bonds to S-investors. Capital accumulation can then be described by Equation (15), the consolidated budget constraint of I-investors and S-investors, where I-investors' bonds have been substituted for. It obtains by aggregating (1) and (2), substituting for consumption, and using the bond market clearing condition (11). At the aggregate level, capital is equal to the part of investors' saving that is not used to buy government or workers' bonds, or to acquire money holdings (LHS). Saving is a

[14]We use a guess-and-verify method to establish this for c_t^S. We then use the Euler equation and the budget constraint for S-investors to establish it for c_{t+1}^I. The detailed proof of this property is available upon request.

fraction β of total investors' wealth, which is made up of profits, money holdings and maturing government and workers' bonds (RHS). Importantly, I-investors' financial constraint does not appear in this equation. As we will see later, how investors' funds are channeled to capital in equilibrium will depend on the ability of the rest of the economy to accommodate their excess saving (through $\bar{l}$ and m^S).

We can already get some partial-equilibrium intuitions from Equation (15) as to how money holdings by S-investors interacts with capital accumulation. First, on the LHS, an increase in m^S_{t+1}, other things equal, implies a lower capital stock, because the corresponding funds are not channeled to I-investors. Second, from the RHS, a larger m^S_t enables to increase the capital stock, because it can be liquidated to finance investment by I-investors. The investors' net bond holdings (i.e., excluding S-investors' holdings of I-investors' bonds), $\bar{l}_{t-1}y_t$, has similar effects, except that the price of money is inflation while the price of bonds is $1/r_t$.

Finally, Equation (16) is the money market equilibrium (12), where M^w has been substituted for using (7). Money supply has to be equal to the demand for money for transaction purposes plus the demand for saving purposes. Equation (16) shows how real money supply can respond to increases in money demand. With flexible prices, an increase in real money demand can be met through a price decline. With downward nominal rigidities, the adjustment would require a drop in output.

Normal and liquidity-trap steady states In the next section, we will first focus on steady state equilibria. Suppose $\phi, \bar{l}$ are constant and M_t grows at a constant gross rate θ. A steady state can be characterized by constant r, m^S, k, and a constant inflation rate $P_{t+1}/P_t = \theta$, satisfying (13) to (15). The Euler equation (14) and the aggregate budget constraint (15) become

$$\beta r\alpha(1-\phi)y = (\phi\alpha + \bar{l})y + m^S, \tag{14$'$}$$

$$k = \beta\alpha y - \left(\frac{1}{r} - \beta\right)\bar{l}y - (\theta - \beta)m^S, \tag{15$'$}$$

with $y = k^\alpha$. The CSC (13) becomes $m^S(r - \theta^{-1}) = 0$ and implies that there are two types of steady states: *normal steady states*, with $r > \theta^{-1}$ (or $i > 1$) and $m^S = 0$, and *liquidity-trap steady states*, with $r = \theta^{-1}$ (or $i = 1$) and $m^S > 0$. The path of prices P_t is determined by (16).

3 Steady-state Impact of Permanent Deleveraging

This section studies the effects of permanent deleveraging, modeled by a drop in ϕ. A permanent shock allows us to study analytically changes in steady states. This provides useful insights as to the effects of very persistent deleveraging shocks. In the next section, we simulate a persistent but non-permanent deleveraging shock in an extended version of the model.

In the following, we focus on the case $\bar{l} = 0$ where investors are in autarky: S-investors lend to I-investors. In addition to being simpler, this is also a realistic description of the US prior to the crisis: we show in Section 2.1 of the Online Appendix that the net position in financial assets of non-financial corporate businesses was indeed close to 0 in the years 2000 prior to the crisis. Afterwards, we briefly describe how the analysis would change with $\bar{l} < 0$.

3.1 Deleveraging in a Normal Steady-State

Consider first a normal steady state with $m^s = 0$. When $\bar{l} = 0$, the aggregate budget constraint ($15'$) determines the capital stock independently of leverage ϕ and the real interest rate r:

$$k = \beta\alpha y = \beta\alpha k^{\alpha}. \tag{17}$$

Leverage has no effect on the capital stock. Indeed, for a given interest rate, the shock generates a decrease in the bond supply by I-investors. Besides, as S-investors start the period with less debt, it increases their wealth and hence their demand for bonds. But, since the net supply of bonds by the rest of the economy remains unchanged at zero, adjustment to deleveraging takes place through a decrease in the interest rate, which equates the demand for bonds by S-investors with the supply by I-investors. Intuitively, saving by S-investors needs to be channelled to some asset in equilibrium, whatever the level of ϕ, and this is achieved by a decrease in interest rate.[15] This is clear from equation ($14'$), which determines r in the normal steady state as

$$r = \frac{\phi}{\beta(1-\phi)}. \tag{18}$$

Notice that a decrease in r implies a proportional decrease in $i = r\theta$ for a given steady-state inflation rate θ. Therefore, a strong contraction of credit may lead to the ZLB. This is the case when $\phi/[\beta(1-\phi)] \leq 1/\theta$. Similarly, a high enough ϕ brings the equilibrium interest rate at $1/\beta$. Beyond this, the credit constraint is not biding anymore.

[15] Note that the log-utility implies that the change in interest rate does not affect saving, as the intertemporal elasticity of substitution is equal to one. If this elasticity was larger (lower) than one, then saving would decrease (increase) and hence investment.

3.2 Deleveraging in a Liquidity Trap

If i hits the ZLB, the equilibrium becomes a liquidity trap. The effective real interest rate cannot adjust downward and remains fixed at $1/\theta$. We define the shadow real interest rate r^s as the rate that would prevail if the ZLB were not binding. It is given by the RHS of (18), i.e., $r^s = \phi/[\beta(1-\phi)]$.[16] We then define the interest rate gap as the difference between the effective and the shadow interest rates:

$$\Delta \equiv r - r^s = \frac{1}{\theta} - \frac{\phi}{\beta(1-\phi)}$$

We think of the magnitude of this gap as the depth of the liquidity trap.

In a liquidity trap steady state, the Euler equation (14′) becomes:

$$m^S = \alpha\left[(1-\phi)\frac{\beta}{\theta} - \phi\right]y. \tag{19}$$

The ratio m^S/y is decreasing in ϕ: an increase in investors' net demand for saving instruments triggered by a deleveraging shock is now accommodated by an increase in their real money holdings m^S. It is also interesting to notice that m^S is proportional to the interest rate gap Δ: $m^S = \kappa\Delta y$, where $\kappa = \alpha\beta(1-\phi)$. The magnitude of investors' real money demand is therefore also a measure of the depth of the liquidity trap.

This switch to money takes out resources from investment, as suggested by (15′), which becomes in a liquidity trap

$$k = \beta\alpha y - (\theta - \beta)m^S. \tag{20}$$

From Assumption 1, $\theta > \beta$ so that holding additional money entails a net resource cost that decreases the steady-state capital stock. On the one hand, in the steady state, S-investors partly save by holding money, with a marginal cost $P_{t+1}/P_t = \theta$. In the normal equilibrium, the corresponding amount would have indirectly financed investment through I-investors' bonds. In the liquidity trap, these resources are diverted from investment. On the other hand, investment benefits from past money holdings. The marginal propensity of I-investors to use money holdings for investment is equal to β. Since the marginal cost θ is larger than the marginal propensity β, the resource diversion is not compensated by the liquidity service of money which results in lower investment in physical capital. From the point of view of investors, money is formally equivalent to a storage technology with a low exogenous rate of return $1/\theta$. This

[16] The shadow rate goes to 0 when ϕ goes to 0. This is an extreme situation where savers, absent money, would have no instruments to trade intertemporally. Section 6 introduces an alternative inefficient saving technology, which puts a strictly positive lower bound on the shadow rate.

technology is only used in a liquidity trap.

Notice that asset scarcity is crucial for our mechanism. First, it generates a persistent drop in interest rate, making the liquidity trap persistent. Second, asset scarcity means that the return on bonds, and hence the return on money in the liquidity trap, is below $1/\beta$, so bond or money accumulation in the liquidity trap is costly.

The net resource cost for investors arises from a Pigou-Patinkin real balance effect together with an inflation tax, as can be seen by rewriting Equation (20):

$$k = \beta\alpha y - \underbrace{(\theta-1)m^S}_{\text{inflation tax}} - \underbrace{(1-\beta)m^S}_{\text{extra consumption}} .$$

Because cash is considered as net wealth by investors (a consequence of the non-Ricardian structure of the model), they consume a fraction $1-\beta$ of it. Consequently, as more financial wealth is accumulated by investors through real money balances, they consume more, and hence invest less, out of their revenues. In addition, a fraction $\theta - 1$ of cash is lost as an inflation tax, which decreases investors' revenues and further decreases investment.[17]

The upward adjustment in investors' real money holdings m^S takes place through disinflation. From (16) taken in the steady state, we have $M/P = (1-\alpha)y/\theta + m^S$. Since workers' money holdings always equal their wage bill, total real money supply M/P has to increase. For a given path of money supply, given by (9), this obtains through a downward shift in the path of prices P_t.

3.3 A Proposition

Using the above analysis, we establish the following Proposition:

Proposition 1 (Steady state with autarkic investors) *Define $\phi_T = \beta/(\theta+\beta)$ and $\phi_{max} = 1/2$. If $0 < \phi < \phi_{max}$, then there exists a locally constrained steady state with $r < 1/\beta$.*

(i) *If, additionally, $\phi \geq \phi_T$, then the steady state is normal.*

(ii) *If $\phi < \phi_T$, then the steady state is a liquidity trap.*

(iii) *In the normal steady state, the real interest rate r and the nominal interest rate i are increasing in ϕ, $m^S = 0$ and k is invariant in ϕ.*

[17] This tax is redistributed to workers through transfers. This second effect would be lower if investors also received transfers from the government.

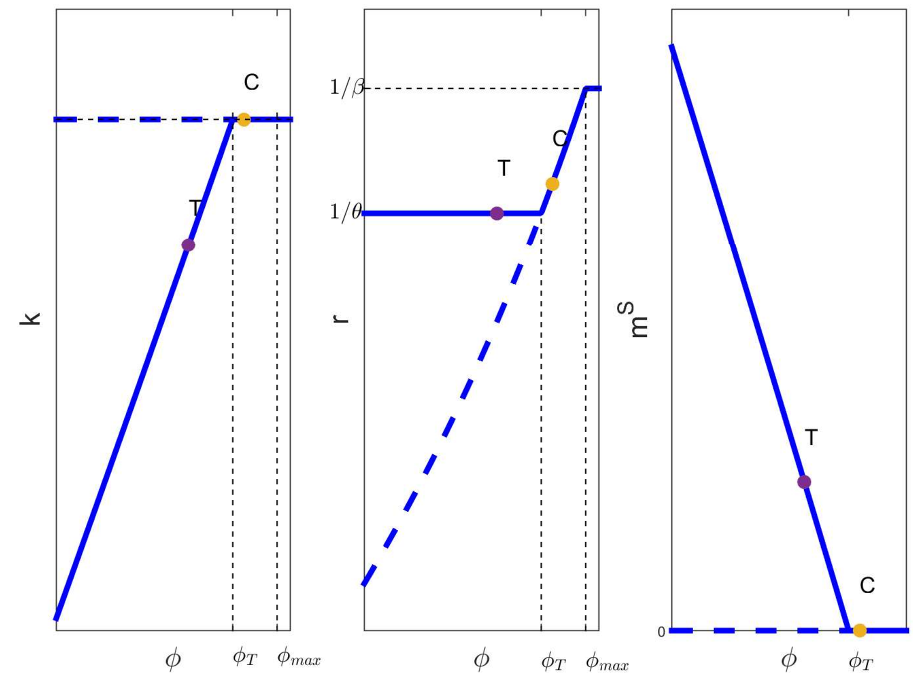


Figure 2: Steady states - Comparative statics w.r.t. ϕ, with $\bar{l} = 0$

(iv) *In the liquidity-trap steady state, the real interest rate r is invariant in ϕ, m^S/y is decreasing in ϕ and k is increasing in ϕ.*

Proof. See proof in the Online Appendix. ■

This Proposition establishes under which condition on ϕ the steady state is normal or a liquidity trap. It is illustrated in Figure 2. The solid lines show the levels of k, r, and m^S as a function of ϕ, while the broken lines show the levels of the shadow rate r^s and of k and m^S if the ZLB were not binding. For intermediate values of ϕ (between ϕ_T and ϕ_{max}), the normal real interest rate r is higher than $1/\theta$, and the steady state is normal as the nominal interest rate i is above the ZLB, as is illustrated by equilibrium C. When ϕ falls below ϕ_T, the steady state becomes a liquidity trap where the effective interest rate is $r = 1/\theta$, larger than the shadow rate r^s. It is characterized by positive real money holdings among investors, for saving purposes, as illustrated by point T on the right panel.

As long as the economy is in the normal steady state (when $\phi > \phi_T$), a permanent deleveraging shock on investors (a decrease in ϕ) has no effect on capital, but it has a negative effect on the real interest rate r. This is due to the fact that investors' bonds are still the only store of value to channel funds from S-investors to I-investors, which happens in equilibrium through a decline in r. But a deleveraging shock large enough to make the economy fall into a liquidity trap ($\phi < \phi_T$) has a negative steady-state effect on capital and output, as illustrated by point

T on the left panel. This comes from disinvestment due to the fact that money becomes an alternative store of value. This supply side effect contrasts with the recent literature, where long-run stagnation is driven by a fall in consumption demand in the presence of persistent nominal rigidities.

3.4 Discussion

The fact that higher money holdings come with lower capital and output in the steady state does not imply that investors would be better off if money did not exist. By putting a lower bound on the real rate of interest, money helps investors better smooth consumption across time. Under a mild assumption on the degree of decreasing returns to scale to capital, α, this can be shown to make both groups of investors better off in a liquidity trap steady state than they would be in the corresponding normal steady state, despite the lower capital stock (see Section 6.1 of the Online Appendix). Workers may however be hurt by lower wages.

When investors are net debtors, we have $\bar{l} < 0$. The real interest rate is then increasing in $\bar{l}$:

$$r = \frac{\phi + \bar{l}/\alpha}{\beta(1-\phi)}. \tag{21}$$

Moreover, r has a redistributive effect between investors and workers, which affects capital accumulation: when $\bar{l} < 0$, a lower interest rate reduces the cost of debt and allows investors to accumulate more capital. This implies that a deleveraging shock actually *increases* the steady-state capital stock in the normal economy.[18]

In a liquidity trap however, a deleveraging shock still has a negative effect on capital. In that case, as money and bonds are perfect substitutes, capital accumulation is affected by the total amount of net liquidity $s = m^S + \bar{l}y$, which plays the same role as cash holdings in the autarky case. Notice that we still have $m^S = \kappa\Delta y$. We therefore refer to $\bar{l}y$ as shadow liquidity, since $s = \bar{l}y$ when $\Delta = 0$. Further details of this case are found in Section 5.1 of the Online Appendix.

4 Simulated Impact of Transitory Deleveraging

Steady state comparisons are helpful to derive closed-form solutions and facilitate the analysis, but they imply a permanent liquidity trap and abstract from transition dynamics. We now

[18]The positive effects on capital accumulation of financial frictions is not an uncommon result: uninsurable risk and credit constraints in Bewley-Aiyagari models notoriously leads to an over-accumulation of capital. See Aiyagari (1994), Krusell and Smith (1997), Covas (2006), and Dávila et al. (2012).

consider a transitory deleveraging shock, using an extended version of the model described in Section 3.2 of the Online Appendix. There are two main differences with the benchmark model. First, capital only partially depreciates. Second, the deleveraging shock is persistent, but not permanent. Leverage ϕ_t is now a stochastic variable that can take two values: ϕ^H in normal times and ϕ^L for deleveraging. After a deleveraging shock hits, there is a probability λ in each period to switch back to ϕ^H and stay there. This introduces aggregate uncertainty in the model.

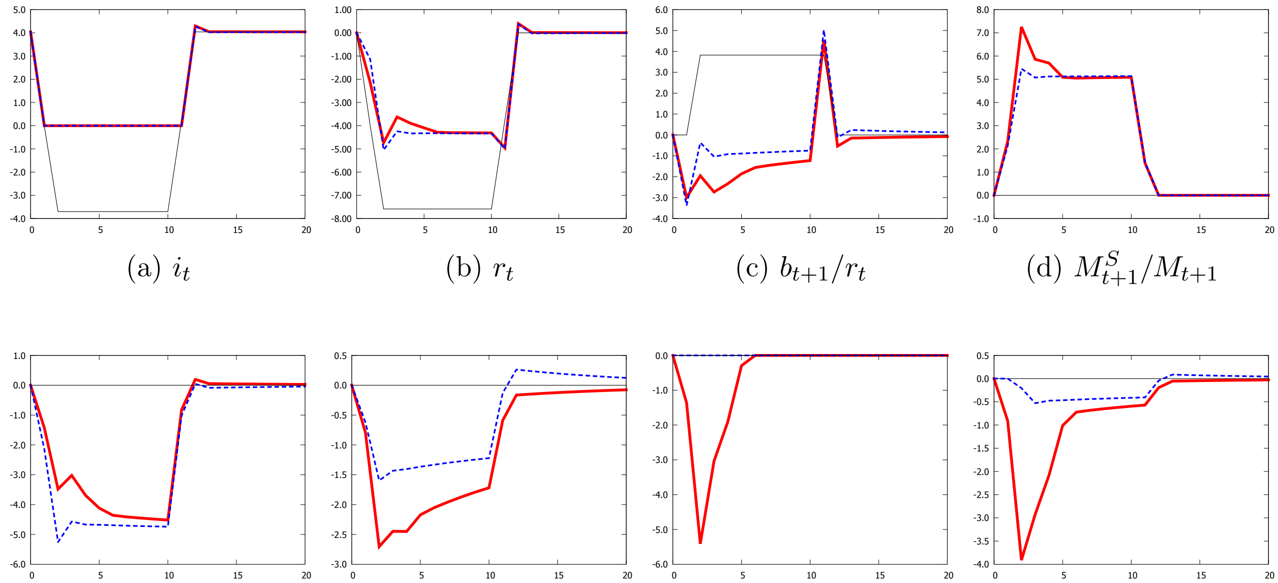
4.1 Calibration

The model is calibrated to fit the recent experience of the US at the ZLB. The time period is a year. We calibrate the balance sheet parameters $\bar{l}^g$ and $\bar{l}^w$ to match their empirical counterparts in the US in 2006. We show in Section 2.2 of the Online Appendix that the net position of the general government and the monetary authority in interest-bearing assets was about 40% of GDP. However, the net position of the rest of the world in these instruments was about -40% of US GDP. The net supply available to the domestic economy is thus approximately 0. With the assumption of autarkic entrepreneurs, this implies $\bar{l}^g = \bar{l}^w = 0$.

The discount factor β is set to 0.96 and ϕ^H to 0.495 in order to match a real interest rate of 2%, consistent with the 10-year TIPS before the crisis, and a real rate of return on capital of 4% which implies a realistic 200 bp corporate spread. We make conventional choices for the capital share $\alpha = 0.33$, the depreciation rate $\delta = 0.10$, and we set $\theta = 1.02$ to get a steady state inflation of 2%. To discipline the choice of ϕ^L, which gives the extent of deleveraging, and the degree of nominal rigidity γ, which drives the increase in unemployment, we match the response of investment and unemployment during the crisis in the US. We set $\phi^L = (1 - 0.039)\phi^H$ and $\gamma = 1.01$ to reproduce the 20% peak-to-trough variation of non-residential investment and the 5.5 pp increase in civilian unemployment of the data.[19] Finally, we set λ at 10% per year, which implies a 10-year average duration of liquidity traps.

We simulate a particular realization of the sequence of leverage. Starting from a steady state in period 0, the deleveraging shock hits in period 1 as leverage unexpectedly drops from ϕ^H to ϕ^L, and is permanently reversed in period 11 when leverage returns to its initial value ϕ^H. We construct the corresponding equilibrium by pasting a transition path corresponding to ϕ^H for $t \geq 11$ to a transition path corresponding to ϕ^L for $t = 1 \ldots 10$. In the first part of the transition, we solve for expectations of future variables taking into account the possibility that

[19] Our calibration of γ implies a 1% lower bound on wage inflation. With 2% steady-state inflation, this implies that real wages downwardly adjust by at most 1% per year.



(a) i_t (b) r_t (c) b_{t+1}/r_t (d) M^S_{t+1}/M_{t+1}

(e) P_t/θ^t (f) K_{t+1} (g) h_t (h) y_t

Figure 3: Transitory dynamics after a deleveraging shock. The shock hits in period 1 and lasts for 10 years. Thick red line: downwardly-rigid wages. Dashed blue line: flexible wages. Thin black line: no ZLB. All variables are in relative deviation from initial steady state, in percent, except interest rates and M^s/M which are in absolute deviation from initial steady state, in percent.

the shock ends in each period with probability λ. The solution method uses Dynare[20] and is described in details in Section 3.2.3 of the Online Appendix.

4.2 Results

The impact of a transitory deleveraging shock is shown in Figure 3. The dashed blue line represents the baseline case without nominal rigidities. The thin black solid line represents the outcome in the absence of ZLB. The drop in ϕ generates both a drop in the supply of and a rise in the demand for assets by investors. In the absence of ZLB, the real interest rate accommodates this excess demand for assets by dropping substantially (panels a and b). The large decrease in real rate offsets the tighter financial constraint, allowing borrowing to increase (panel c), and accommodates higher saving. With autarkic investors, capital and output are unaffected. In contrast, with the ZLB, i_t is stuck at its lower bound, which prevents the real rate from adjusting. The excess demand for saving by investors is then channeled to money: M^S increases (panel d). The increase in the demand for money is accommodated by a fall in prices (panel e). Because the real rate decrease does not offset the tighter financial constraint,

[20]We use Dynare version 4.4.3 (Adjemian et al., 2011).

borrowing decreases (panel c) and the capital stock drops on impact (panel f).

After the initial drop, the capital stock recovers somewhat, but does not go back to its initial value. As long as the economy stays in the liquidity trap, it remains persistently low. This medium-run effect corresponds to the effect of deleveraging due to the resource cost of liquidity highlighted in the steady state analysis. Output shows the same pattern as capital: an initial drop followed by a capped recovery (panel h).

Consider now the thick red line, which is drawn under the assumption of downward wage rigidity. This rigidity prevents the decrease in nominal wage needed to clear the labor market. As a result, there is unemployment, which amplifies the output drop. It also worsens the financing capacities of investors further and hence lowers the capital stock even more. On impact, labor, output and capital are more strongly hit than with flexible wages (panels f, g, and h). These demand effects are strong, but they only take place in the short run. As time goes by, real wages adjust and all variables converge toward their level under flexible wages.

The short-run impact of deleveraging is thus stronger than in the medium run, and even more so in the presence of nominal rigidities. But in the medium run, the effects caused by the scarcity of assets prevail. Contrary to the New Keynesian literature, the economy lingers at the ZLB with a lower capital stock and output level, even after wages have adjusted and the output gap has closed.

Despite its simple structure, the model matches the data for several variables relatively well. The model with nominal rigidities is calibrated to match the response of investment and employment. In addition, the structure of the model allows us to match the drop in nominal interest rate all the way down to the ZLB, since the deleveraging shock can only affect investment at the ZLB. GDP drops by 3.9% on impact in the model, which is close to the 4.4% peak-to-trough variation in the data. The price level gradually falls by about 4.5% below its trend θ^t. This is also in line with the data, since in 2017 the GDP deflator was 4.5% lower than it would have been had it grown at 2% per year since 2007. Compared to the interest rate, the real return on capital (not reported in Figure 3) is roughly stable. This implies a substantial increase in the spread between the expected real returns on capital and on bonds, by almost 5 pp. This is in line with the observation that the real rate on bonds has declined while the return on capital was roughly stable (Gomme et al., 2011, 2015).[21]

[21] From the returns on capital and Treasuries securities reported by Gomme et al. (2015), we get a similar increase in the spread by 4 to 5 pp between 2007 and the post-crisis years.

5 Policy

There is a range of macroeconomic policies that have been implemented or considered in the context of a liquidity trap. In this section, we consider the most relevant policies and study their implications in the current model. We assume the same shock as in the previous section, and study how different policies affects the response of the economy.

We start by showing that monetary policy can be very effective in the short run, effectively offsetting the contractionary effect of nominal rigidities inside a liquidity trap. We then consider various policies aimed at exiting the liquidity trap: negative interest rates, higher inflation target, QE, or an increase in government bonds. We show that these policies have different implications in the short and medium run. We also discuss their distributional effects, and the trade-offs they generate inside and outside the liquidity trap.

Exiting from a liquidity trap implies driving the interest rate gap to zero. We have:

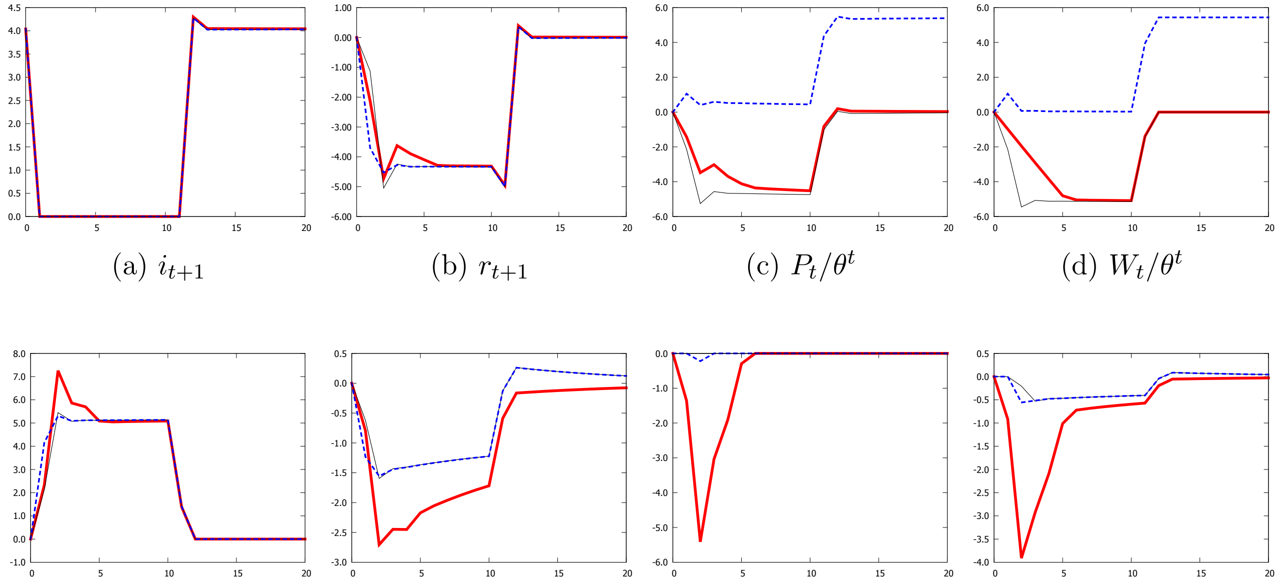
$$\Delta = \frac{i}{\theta} - \frac{\phi + \bar{l}/\alpha}{\beta(1-\phi)}$$

While a strict ZLB implies $i = 1$, we can allow $i < 1$ to analyze the impact of negative rates. The authorities can eliminate the interest rate gap either by decreasing the effective rate or by increasing the shadow rate. These two approaches have different implications as they imply a different real interest rate level. Besides the potential distributional effects on lenders and borrowers, the interest rate level affects investment and intertemporal allocations. For example, a low effective rate initially promotes investment, but distorts intertemporal consumption choices. We discuss the welfare implications and the first-best policy.

5.1 Helicopter Money

Previous studies focus on short-term effects in the presence of nominal rigidities and find that demand-side policies are paramount. Since we assume a downward wage rigidity, demand-side policies will have similar effects in our setting. However, there is one policy that is particularly efficient to undo nominal rigidities. A monetary expansion taking the form of transfers to workers ("helicopter money") can almost replicate the flexible wage equilibrium. While this policy has no effect at the ZLB in standard models, it is efficient in our non-Ricardian framework.[22] By increasing money supply, the monetary authority accommodates the demand for money by investors, making it unnecessary to decrease output or the price level.

[22] In Krugman (1998), for instance, money creation taking the form of transfers has no effect at the ZLB with pre-set prices (see footnote 11 of this work).



(a) i_{t+1} (b) r_{t+1} (c) P_t/θ^t (d) W_t/θ^t

(e) M^S_{t+1}/M_{t+1} (f) K_{t+1} (g) h_t (h) y_t

Figure 4: Helicopter money during a deleveraging shock. A deleveraging shock hits in period 1 and lasts for 10 years. Thick red line: baseline simulation with downwardly-rigid wages. Dashed blue line: downwardly-rigid wages with a monetary expansion when the shock hits. Thin black line: flexible wages. All variables are in relative deviation from initial steady state, in percent, except interest rates and M^s/M which are in absolute deviation from initial steady state, in percent.

To illustrate this, consider the same deleveraging shock as in Section 4. The thick red line in Figure 4 represents the baseline case of downwardly-rigid wages, and the thin black line is the case of flexible wages. Consider now a permanent monetary expansion taking the form of transfers to workers. In the simulation represented by the dashed blue line, the government increases M when the shock hits. The increase is calibrated so that the nominal wage converges back to its initial value as time goes by. As the figure shows, the resulting dynamics of real variables is very close to the dynamics with flexible wages. By increasing money supply, monetary policy substitutes for the fall in the price level that would obtain with flexible prices.[23]

Monetary policy is then potent to mitigate the short-run impact of deleveraging, but is unable to address its medium-run impact (unless it changes the inflation target, see below).

[23]Even a non-credible expansion can be effective, contrary to the analysis of Krugman (1998) in a Ricardian model. Suppose agents were to expect the expansion to be reversed when the shock stops. Higher current money supply would still sustain employment in the ZLB. The only difference is that the real rate would be slightly higher, and the capital stock slightly lower, due to lower expected inflation when the shock stops.

5.2 Lowering the Effective Real Interest Rate

Decreasing the effective rate can be done by increasing expected inflation through an increase in θ. This is a natural solution mentioned in the literature (e.g. Krugman, 1998). Figure 5, panel A, shows the effect of switching to a higher inflation target when the shock hits. In period 1, as the economy is hit by the deleveraging shock, the monetary authority announces that money supply will grow at a higher rate from period 2 onward.[24] The figure considers both an increase of 1 percentage point (solid red line) and of 5 percentage points (dashed red line). The latter is large enough to decrease the real rate all the way down to the shadow rate: this fully avoids the liquidity trap.

The former has both short-term and medium-term effects. In the short run, it relaxes the downward wage rigidity and reduces the drop in employment and output (panel A.4). In the medium run, the lower cost of capital sustains investment (panel A.3) and output. This increases workers' wages, but the lower real interest rate also impairs consumption smoothing. Addressing the liquidity trap by reducing the effective real interest rate drives out monetary liquidity without providing alternative liquidity and solving the underlying asset scarcity problem.

For longer lasting liquidity traps, the marginal effect of higher inflation on capital and output is actually ambiguous in the long run. The lower effective real interest rate also increases the cost of holding money for S-investors, which adversely affects capital other things equal. When money holdings by investors are large, e.g. after a large deleveraging shock, this negative effect can even dominate in the long run, as we show formally in Section 5.3 of the Online Appendix.

An alternative way to decrease the effective real rate is to implement negative nominal interest rates on cash, thereby achieving a negative effective lower bound.[25] We consider such a policy in panel B of Figure 5. The medium-term effects are the same as with a higher inflation target, but it has a smaller short-term effect as it does not relax the downward wage rigidity.

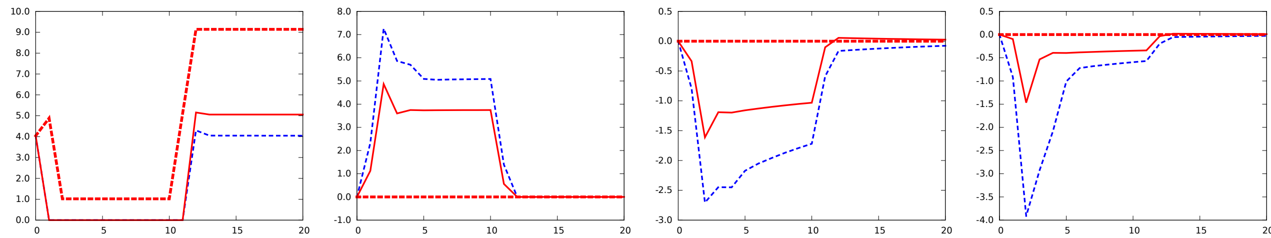
5.3 Enhancing Shadow Liquidity

The alternative to eliminate the interest rate gap is to raise the public supply of liquidity. In addition to usual short-term demand-side effects, this increases the shadow interest rate. We consider both an increase in the supply of government debt and QE policies.

[24] Notice that this policy is not the pure self-fulfilling change of target sometimes considered by the literature, but corresponds to an actual policy change: money growth increases with corresponding larger transfers to workers.

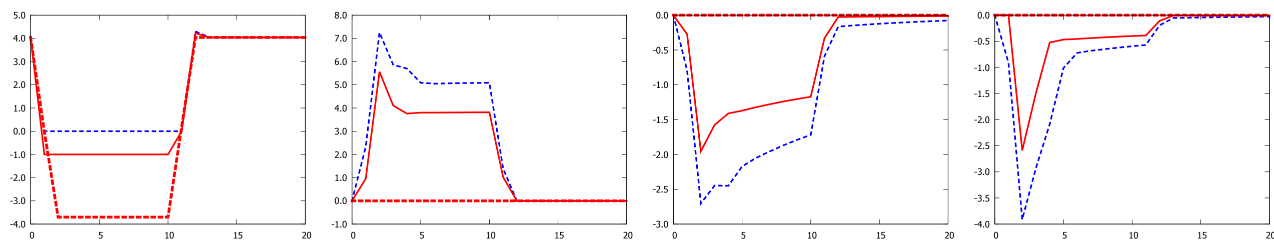
[25] Suppose that cash is replaced by Central Bank digital money, on which a negative interest rate can be charged. There would then be no ZLB on the nominal interest rate and we could have $i < 1$. A liquidity trap with a negative interest rate is a situation currently observed in several countries.

A. Higher inflation target



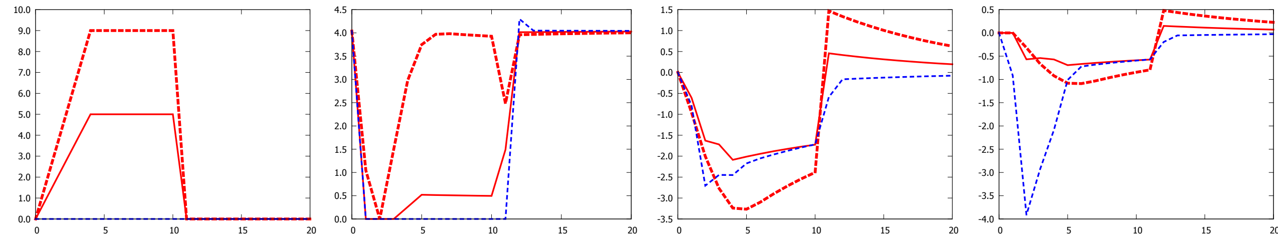
(A.1) i_t (A.2) M^S_{t+1}/M_{t+1} (A.3) K_{t+1} (A.4) y_t

B. Negative interest rate



(B.1) i_t (B.2) M^S_{t+1}/M_{t+1} (B.3) K_{t+1} (B.4) y_t

Figure 5: Higher inflation target or negative nominal interest rate during a deleveraging shock. A deleveraging shock hits in period 1 and lasts for 10 years. Dashed blue line: baseline simulation with no policy. Panel A: inflation target increases by 1% (solid red line) or 5% (dashed red line). Panel B: effective lower bound of -1% (solid red line) or -5% (dashed red line). All variables are in relative deviation from initial steady state, in percent, except interest rates and M^s/M which are in absolute deviation from initial steady state, in percent.



(a) l^g_{t+1}/Y_{t+1} (b) i_t (c) K_{t+1} (d) y_t

Figure 6: Public debt issuance during a deleveraging shock. A deleveraging shock hits in period 1 and lasts for 10 years. Solid red line: increase in public debt by 5% of GDP in 2 years. Dashed red line: increase in public debt by 9% of GDP in 2 years. Dashed blue line: baseline simulation with no policy. All variables are in relative deviation from initial steady state, in percent, except interest rates, l/Y and M^s/M which are in absolute deviation from initial steady state, in percent.

Increasing Public Debt A higher supply of government bonds, which leads to a higher $\bar{l}$, increases shadow liquidity and the shadow interest rate, and narrows the interest rate gap. A sufficient increase in public debt closes the interest rate gap and lifts the economy out of the liquidity trap. Figure 6 compares the baseline simulation with no policy (dashed blue line) with the response to a small increase in government debt—5% of GDP in two years—displayed by the solid red line, and to a larger increase—9% of GDP—displayed by the dashed red line. The counterpart of debt issuance is a transfer to workers. When the deleveraging shock stops, debt is brought back to its initial value thanks to a tax levied on workers.

These policies have different effects in the short, medium, and long run. In the short run, debt issuance has the usual demand-side effect and avoids the sharp drop in employment, output (panel d), and inflation. Indeed, similarly to helicopter money, the government supplies the market with assets, which accommodates the demand for saving instruments by S-investors.

As long as the increase in public debt is too small to exit the ZLB, it is fully equivalent to helicopter money, since bonds and money are perfect substitutes at the ZLB. By contrast, both policies considered in Figure 6 are large enough to exit the liquidity trap (panel b). This affects medium run dynamics. While the interest rate stays low with the small increase in debt, the larger increase brings the interest rate close to its value in normal times. A higher interest rate leads to better consumption smoothing and helps S-investors better transfer funds to their investment period, but it also makes borrowing more costly, which can hurt investment in the medium run, as shown in panel (c). Therefore, the medium-run effect of government debt on output is different from its short-run effect. In our baseline simulation, as can be seen from panel (d), the larger increase in debt hurts output in the medium run.

The long run effects would be different still. For a longer-lasting liquidity trap, the larger (smaller) increase in debt would increase (decrease) capital compared to the no-policy case. In general, the impact of government debt on capital, as the economy exits the liquidity trap, is ambiguous. It depends on the level of liquidity $\bar{l}$ that prevails at the exit of the liquidity trap and on the size of the deleveraging shock. In the medium to long run, it is positive for a large enough increase in debt and for a strong enough deleveraging shock. This is established formally in Section 5.2 of the Online Appendix for steady states. The Online Appendix (Section 5.10) also shows a simulation with a deleveraging shock twice as large and long as in the baseline, where a large enough increase in debt (by 18% of GDP) sustains output above the no-policy case even in the medium run.

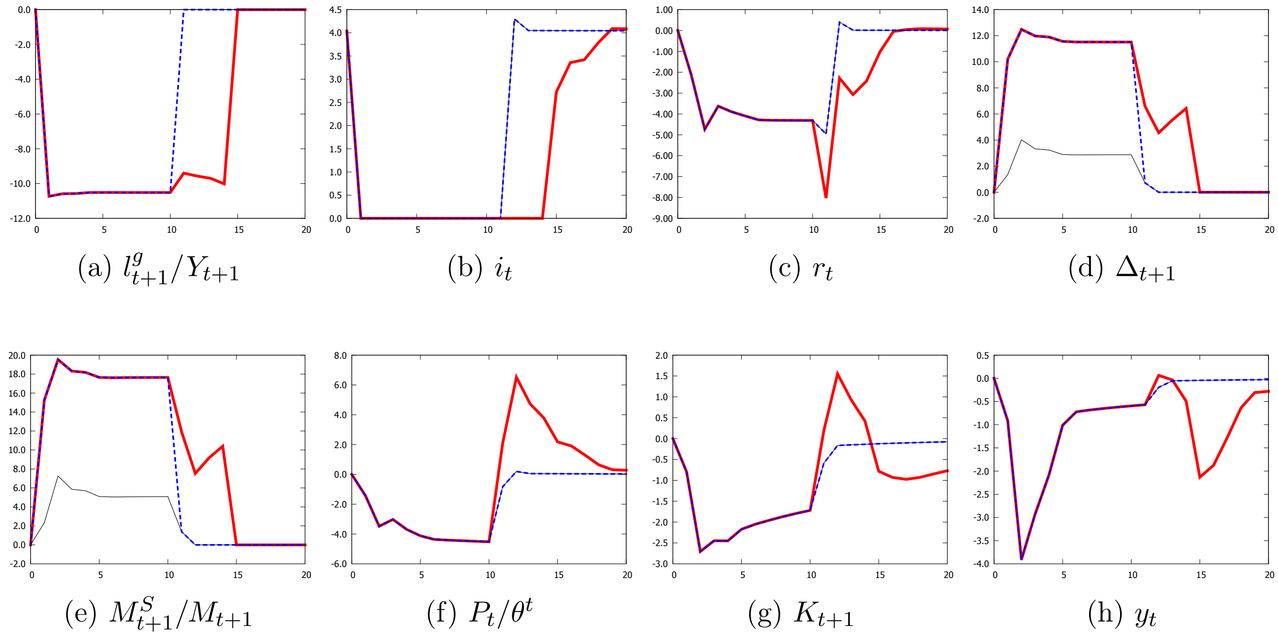
Quantitative Easing QE consists in creating money through open market operations, i.e., increasing M by decreasing Pl^g. Since money and government bonds are perfect substitutes in our setting, this has no effect per se in the liquidity trap.[26] However, QE entails a decrease in the available amount of government bonds l^g, which decreases shadow liquidity and the shadow interest rate. In that sense, QE leads to a deeper liquidity trap. This matters at the time of exit: a late exit from QE can extend the duration of the trap. Figure 7 illustrates the effect of QE with a late or early exit. We suppose that the central bank implements QE by buying bonds worth 10% of GDP when the shock hits in period 1.[27] The dashed blue line displays the case of early exit where the central bank sells the bonds in period 11 when deleveraging stops. The thick red line represents the case of late exit where the central bank announces in period 11 that it will hold the bonds for four additional years, and does so.[28] As a benchmark, the thin black line reproduces the case without QE.

As QE reduces the level of public debt available to investors (panel a), it increases the interest rate gap Δ (panel d). Inside the liquidity trap, this has no effect on real variables and only changes the composition of assets held by investors. When exit is early, QE has therefore no real effects. A late exit, by contrast, has a impact on the economy. Absent QE or with an early

[26]Note that we abstract from some potential channels of QE. In particular, the perfect substitutability of money and bonds means that there is no broad portfolio balance channel that could lower term or risk premia. It could also be that QE goes hand-in-hand with credit easing aimed at improving credit conditions for the private sector, which can alleviate the effect of deleveraging. This would consist in the government issuing new debt to lend to credit constrained investors, effectively relaxing their constraint. This does not affect government net debt but can help in getting out of the liquidity trap by increasing the shadow interest rate through a re-leveraging by investors.

[27]According to H.4.1 Federal Reserve statistical releases, the large-scale asset purchase programs of 2010–2014, usually referred to as QE2 and QE3, increased the amount of securities held at the Federal Reserve by 9 percent of GDP. The total increase since 2006 amounts to 17 percent of GDP.

[28]In the simulation presented in Figure 7, this announcement comes as a surprise. Section 5.11 of the Online Appendix presents the case where late exit is expected from the start.



(a) l^g_{t+1}/Y_{t+1} (b) i_t (c) r_t (d) Δ_{t+1}

(e) M^S_{t+1}/M_{t+1} (f) P_t/θ^t (g) K_{t+1} (h) y_t

Figure 7: Quantitative easing during a deleveraging shock. A deleveraging shock hits in period 1 and lasts for 10 years. Thick red line: quantitative easing with late exit. Dashed blue line: quantitative easing with early exit. Thin black line: baseline simulation with no quantitative easing. All variables are in relative deviation from initial steady state, in percent, except interest rates, l^g/Y and M^s/M which are in absolute deviation from initial steady state, in percent.

exit, the interest rate would increase to slow down investment as the economy relevers, bringing the capital stock close to its steady state level. Since QE deprives the economy of bonds, the interest rate stays stuck at the ZLB (panel b) and the investment boom goes unhampered (panel g). Instead, adjustment comes again from the price level, leading to stronger inflation than with an early exit (panel f). This is only temporary. As agents expect QE to be eventually reversed, the price level slowly decreases back to its steady state level, increasing the real rate and hurting capital and output. The interest rate only leaves the ZLB when QE is finally undone. Therefore, with a late exit from QE, the economy stays longer in the liquidity trap and overheats, before plummeting again, instead of quickly going back to normal.

5.4 Welfare and Pareto Efficiency

In our setting of scarce assets, an adequate supply of government debt enables the economy to reach a Pareto-*efficient* equilibrium. Indeed, by raising the real interest rate, this enables optimal consumption smoothing by all agents as well as the optimal level of capital. Proposition 9 of the Online Appendix shows this formally.[29]

But while an increase in public liquidity makes the economy *converge* to a Pareto-efficient steady state, the whole equilibrium including transition dynamics is not in general a Pareto equilibrium. As we have seen, the higher interest rate indeed initially hurts borrowers and can temporarily decrease investment even lower than its liquidity trap level. In addition, the new steady state does not always Pareto-*improve* on the initial one. When investors are initially net debtors, the efficient equilibrium may require a lower stock of capital, which lowers wages and hurts workers.

Addressing these two problems requires many additional policy instruments. Section 6.3 of the Online Appendix shows how three additional taxes/subsidies make it possible for the policy maker to implement a Pareto-efficient equilibrium path (including the transitory dynamics) that Pareto-improves on the initial liquidity trap.

6 Extensions

Workers' Deleveraging We have considered so far a deleveraging shock on investors, modeled as a decline in ϕ. Likewise, a deleveraging shock on workers can be modeled by a drop in $\bar{l}$, coming from a drop in $\bar{l}^w$.[30] Such a shock limits the economy's supply of assets and has a similar effect on the interest rate r as a deleveraging shock on investors, as can be seen from Equation (21). Workers' deleveraging can therefore also lead to the zero lower bound. Once the economy is in a liquidity trap, further decreases in $\bar{l}$ only have short-term deflationary effects. A lower supply of assets by workers leads to a higher demand for cash by S-investors. In the medium-term, after prices and wages have adjusted, higher real money holdings fully offset the effect of workers' deleveraging. Contrary to a deleveraging shock on investors, the investors' capacity to finance investment is not affected.[31]

Bubbles In our framework with scarce assets, rational bubbles can provide additional saving instruments to accommodate the demand for assets by S-investors. A bubble, when it emerges, provides enough liquidity to exit the ZLB. But it also constrains the real interest rate and

[29]The proposition shows that the efficient level of capital is given by $k = \beta\alpha y$. This level obtains when $r = 1/\beta$, which also corresponds to perfect consumption smoothing, and requires a high enough public debt. In the case where investors are net debtors out of the ZLB, capital is too high compared to a Pareto-efficient allocation and a higher public debt crowds out this inefficiently high capital stock.

[30]Note that this shock might imply a positive net position of workers ($\bar{l}^w < 0$). This is consistent with a high proportion of wealthy hand-to-mouth households, that is, households who own sizeable amounts of illiquid assets (like retirement accounts) but hold little liquid assets, as documented by Kaplan et al. (2014).

[31]Outside the liquidity trap, workers' deleveraging has a positive effect on capital as investors become net debtors. See Section 5.2 of the Online Appendix.

prevents the natural equilibrium adjustment. Section 5.4 of the Online Appendix shows that if a bubbly steady state exists, it has a zero *real* interest rate: $r = 1$. With positive steady state inflation ($\theta > 1$), this is higher then $1/\theta$, the real of return of money, so the bubble strictly dominates money as a store of value. The bubble then raises the nominal interest rate from $i = 1$ to $i = \theta$, and S-investors substitute the bubble for money in their portfolio. For a given money supply, this also reflates the economy as the price level increases to accommodate the lower money demand. This implies that a bubble dampens the short-term impact of deleveraging. However, in the medium run, the bubbly equilibrium is qualitatively similar to a liquidity trap: the bubble plays the same role as investor-held money in the liquidity trap, but with a higher real return. As with money, holding the bubble takes out resources from investment and output is lower in the bubbly equilibrium than in the normal equilibrium.

Preference and Growth Shocks In the existing literature, the shock that brings the economy to the ZLB is often assumed to be an increase in the factor of time preference. This shock, by increasing the agents' propensity to save, has a negative effect on the interest rate. A reduction in the average growth rate of productivity has also been put forward as an explanation for the secular decrease in the interest rate and for hitting the ZLB. In fact, in an infinite-horizon model, the effect of a growth slowdown is isomorphic to an increase in the factor of time preference. We therefore restrict our analysis to the latter. We find that a permanent increase in β (alternatively, a permanent fall in steady-state growth), cannot generate a fall in the investment rate in the medium run after the economy falls into a liquidity trap.

Indeed, we show in Section 5.5 of the Online Appendix that an increase in β makes the interest rate fall, and eventually hit the ZLB. In both the normal and liquidity-trap steady states, an increase in β increases the investors' propensity to save, which increases the capital stock in the medium run. As a result, whereas an increase in β can explain the emergence of a liquidity trap, it cannot explain the persistent slowdown of investment. In the presence of trend growth, the same conclusions would hold in case of a growth slowdown. In particular, with lower trend growth, less investment is required to keep the capital stock on its trend. Therefore a given amount of saving leads to an upward shift in the capital intensity of production, and hence in the investment rate.

Financial Intermediation In the benchmark model, money is modeled as *outside money* directly supplied by the government. However, in practice, cash holdings usually take the form of deposits, which are a liability of banks, and could in principle be intermediated to capital investment. We show in Section 5.7 of the Online Appendix that this is not the case. At the ZLB, banks are unable to channel deposits to credit-constrained I-investors for the same reason that savers are unable to do so in the benchmark model. Instead, banks increase their excess reserves at the central bank.

Inefficient saving technology The benchmark model assumes that bonds and money are the only available saving instruments. In Section 5.8 of the Online Appendix, we extend the model by allowing for an inefficient storage technology, with a rate of return $\sigma \in (\theta^{-1}, \beta^{-1})$ and concave installation costs. This technology starts being used by savers when the interest rate falls down to σ. Then, a moderate deleveraging shock reallocates saving to the storage technology, which crowds out "good" capital even in the normal equilibrium. This reallocative effect is similar to the one studied by Buera and Nicolini (2016). With a large enough deleveraging shock, the economy falls into the liquidity trap, the use of inefficient storage is pinned down by the real rate of interest $1/\theta$, and higher money holdings crowd out capital as in the benchmark model. One difference with the benchmark model is that the shadow rate now has a strictly positive lower bound as ϕ goes to 0, since the storage technology prevents a complete collapse of intertemporal trade, arguably a more realistic feature.

7 Conclusions

We explore the medium-term implications of a liquidity trap and find that a deleveraging shock may lead to a negative comovement between capital and investors' cash holdings. We analyze policies in a liquidity trap by examining their impact on the wedge between the effective real interest rate and the shadow rate. While most of our analysis is conducted in a stylized benchmark model, the main mechanism is robust to many extensions. Our theoretical results are derived with a permanent deleveraging shock for investors, but we show in simulations that they also obtain in the medium run for persistent shocks with nominal rigidities.

Medium-term output declines in a liquidity trap only with investors' deleveraging. Other positive shocks to saving, like workers' deleveraging or an increase in the discount rate, may also lead to a liquidity trap, but they do not depress output in the medium run. Therefore it is crucial to determine the factors that have led to a liquidity trap. Interestingly, Galí et al. (2012) suggest that financial shocks have played a key role in the slow recovery.

Our approach goes beyond Keynesian analyses that stress the role of insufficient demand in a liquidity trap. While they describe a situation of negative output gap when the adjustment of prices is hampered by nominal rigidities, we show that low investment demand leads to lower potential output even after prices have fully adjusted. Our framework then sheds light on the medium-term effects of policies used for standard demand management. In this context, we find that quantitative easing can deepen and possibly lengthen the liquidity trap. We also discuss the respective trade-offs of policies aiming at decreasing the effective real interest rate and policies aiming at increasing the shadow rate.

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