Online Appendix

Exchange Rates, Interest Rates, and Gradual Portfolio Adjustment

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January 2021

This Online Appendix has two sections. Section A develops a full general equilibrium model. Section B develops the algebra associated with the model with long terms bonds in Section 4 of the paper.

A General Equilibrium Model

We first describe the model, then the first-order conditions, followed by linearization and the solution.

A.1 Model

There are two countries. Each country has a continuum of agents on the interval [0,1]. There are overlapping generations, with agents living two periods. Agents make decisions about consumption, portfolio allocation and price setting. When young, agent i in the Home country produces Home good i. Output is equal to labor $L_{H,t}(i)$ supplied by the agent. The good is sold in both countries:

$$L_{H,t}(i) = x_{H,t}(i) + x^*_{H,t}(i) \tag{A.1}$$

$x_{H,t}(i)$ and $x^*_{H,t}(i)$ are the quantities sold in the Home and Foreign countries by agent i from the Home country. The revenue from selling the good, measured in the Home currency, is

$$Y_{H,t}(i) = P_{H,t}(i)x_{H,t}(i) + S_t P^*_{H,t}(i)x^*_{H,t}(i) \tag{A.2}$$

Here $P_{H,t}(i)$ is the price of Home good i in the Home market in the Home currency and $P^*_{H,t}(i)$ is the price of Home good i in the Foreign market in the Foreign currency. The nominal exchange rate S_t is measured as Home currency per unit of the Foreign currency.

The assets of agent i are equal to saving when young:

$$A_{H,t}(i) = Y_{H,t}(i) - P_t C^y_{H,t}(i) - tax_{H,t} \tag{A.3}$$

Here P_t is the consumer price index and $C^y_{H,t}(i)$ is a consumption index (defined below) when young at time t. $tax_{H,t}$ is a nominal lump sum tax. The assets are invested in Home and Foreign bonds and the returns are consumed at time $t+1$:

$$C^o_{H,t+1}(i) = R^{p,H}_{t+1}(i)\frac{A_{H,t}(i)}{P_t} \tag{A.4}$$

where the portfolio return is

$$R^{p,H}_{t+1}(i) = \left[z_{H,t}(i)\frac{S_{t+1}}{S_t}e^{i^*_t-\tau_{H,t}} + (1-z_{H,t}(i))e^{i_t}\right]\frac{P_t}{P_{t+1}} + T_{t+1} \tag{A.5}$$

Here $z_{H,t}(i)$ is the fraction invested in the Foreign bond by agent i from the Home country. The nominal interest rate is i_t for the Home bond and i^*_t for the Foreign bond. There is a cost $\tau_{H,t}$ of investment abroad that is reimbursed through a lump sum T_{t+1}. (A.5) corresponds to equation (3) in the paper.

Agent i in the Home country maximizes

$$\begin{aligned}&C^y_{H,t}(i) + ln(C^{o,CE}_{H,t+1}(i)) - \phi L_t(i)^\eta - 0.5\psi(z_{H,t}(i) - z_{H,t-1}(i))^2\\&-0.5\varphi\nu(P_{H,t}(i) - P_{H,t-1}(i))^2 - 0.5(1-\varphi)\nu(P^*_{H,t}(i) - P^*_{H,t-1}(i))^2\end{aligned} \tag{A.6}$$

Here $C^{o,CE}_{H,t+1}(i)$ is the certainty equivalent of consumption $C^o_{H,t+1}(i)$ when old, defined as

$$C^{o,CE}_{H,t+1}(i) = \left[E_t\left(C^o_{H,t+1}(i)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{A.7}$$

There is a cost of both portfolio adjustment and price adjustment. The cost of portfolio adjustment is the same as in the paper and depends on the parameter ψ. The cost of price adjustment depends on $\varphi\nu$ when sold in the Home country and $(1-\varphi)\nu$ when sold in the Foreign country. Here φ is the fraction spent on domestic goods. For given prices, agent i will supply the labor $L_{H,t}(i)$ needed to produce enough of the good to fullfil all demand by Home and Foreign agents.

Consumption is a Cobb Douglas index of Home and Foreign goods:

$$C^k_{H,t}(i) = \left(\frac{C^k_{HH,t}(i)}{\varphi}\right)^\varphi\left(\frac{C^k_{HF,t}(i)}{1-\varphi}\right)^{1-\varphi} \tag{A.8}$$

where $k = y, o$ stands for young and old consumption. $C^k_{HH,t}(i)$ and $C^k_{HF,t}(i)$ are consumption of respectively Home and Foreign goods by Home agents. They are

CES indices of individual Home and Foreign goods:

$$C^k_{HH,t}(i) = \left(\int_0^1 C^k_{HH,j,t}(i)^{\frac{\mu-1}{\mu}} dj\right)^{\frac{\mu-1}{\mu}} \tag{A.9}$$

$$C^k_{HF,t}(i) = \left(\int_0^1 C^k_{HF,j,t}(i)^{\frac{\mu-1}{\mu}} dj\right)^{\frac{\mu-1}{\mu}} \tag{A.10}$$

where $C^k_{HH,j,t}(i)$ is consumption of Home good j by Home agent i and $C^k_{HF,j,t}(i)$ is consumption of Foreign good j by Home agent i, both in group k (young or old). Also denote by P_t the overall consumer price index in the Home country, $P_{H,t}$ the price index of Home goods in Home currency and $P_{F,t}$ the price index of Foreign goods in Home currency:

$$P_t = P^{\varphi}_{H,t} P^{1-\varphi}_{F,t} \tag{A.11}$$

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\mu} dj\right)^{\frac{1}{1-\mu}} \tag{A.12}$$

$$P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\mu} dj\right)^{\frac{1}{1-\mu}} \tag{A.13}$$

The notation is again analogous for Foreign agents. Foreign agent i sells $x_{F,t}(i)$ and $x^*_{F,t}(i)$ in respectively the Home and the Foreign country and

$$L_{F,t}(i) = x_{F,t}(i) + x^*_{F,t}(i) \tag{A.14}$$

Revenue, measured in the Foreign currency, is

$$Y_{F,t}(i) = \frac{P_{F,t}(i) x_{F,t}(i)}{S_t} + P^*_{F,t}(i) x^*_{F,t}(i) \tag{A.15}$$

where $P_{F,t}(i)$ is the price of Foreign good i in the Home country in Home currency and $P^*_{F,t}(i)$ is the price of the Foreign good i in the Foreign country in the Foreign currency.

Assets of the young are

$$A_{F,t}(i) = Y_{F,t}(i) - P^*_t C^y_{F,t}(i) - tax_{F,t} \tag{A.16}$$

Then consumption at $t+1$ is

$$C^o_{F,t+1}(i) = R^{p,F}_{t+1}(i) \frac{A_{F,t}(i)}{P^*_t} \tag{A.17}$$

with portfolio return

$$R_{t+1}^{p,F}(i) = \left[z_{F,t}(i)e^{i_t^*} + (1 - z_{F,t}(i)) \frac{S_t}{S_{t+1}} e^{i_t - \tau_{F,t}} \right] \frac{P_t^*}{P_{t+1}^*} + T_{t+1}^* \tag{A.18}$$

Here $z_{F,t}(i)$ is the fraction that Foreign agent i invests in the Foreign bond. The cost $\tau_{F,t}$ of investing in the Home bond by Foreign investors is reimbursed through the lump sum T_{t+1}^*. Foreign price indices, denoted in the Foreign currency, are

$$P_t^* = \left(P_{H,t}^*\right)^{1-\varphi} \left(P_{F,t}^*\right)^{\varphi} \tag{A.19}$$

$$P_{H,t}^* = \left(\int_0^1 \left(P_{H,t}^*(j)\right)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.20}$$

$$P_{F,t}^* = \left(\int_0^1 \left(P_{F,t}^*(j)\right)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \tag{A.21}$$

The real value of the bond supply is assumed to be fixed at 1 in both countries. A constant real bond supply is accomplished through the nominal lump sum tax, $tax_{H,t}$ in the Home country in the Home currency and $tax_{F,t}$ in the Foreign country in the Foreign currency. The real bond supply is constant by assuming that the real value of the tax equals the real interest on the government debt:

$$tax_{H,t} = P_t \left(e^{i_{t-1}} \frac{P_{t-1}}{P_t} - 1 \right) \tag{A.22}$$

$$tax_{F,t} = P_t^* \left(e^{i_{t-1}^*} \frac{P_{t-1}^*}{P_t^*} - 1 \right) \tag{A.23}$$

Finally, bond market clearing conditions are

$$\int_0^1 z_{H,t}(i)A_{H,t}(i)di + \int_0^1 z_{F,t}(i)S_t A_{F,t}(i)di = P_t^* S_t \tag{A.24}$$

$$\int_0^1 (1 - z_{H,t}(i))A_{H,t}(i)di + \int_0^1 (1 - z_{F,t}(i))S_t A_{F,t}(i)di = P_t \tag{A.25}$$

Because of Walras' Law, we only need to impose the first of the two asset market clearing conditions.

A.2 First Order Conditions

Agents make decisions about consumption, portfolio allocation and prices. The first-order condition for consumption can be obtained by maximizing

$$C_{H,t}^y(i) + \frac{1}{1-\gamma} lnE_t \left(R_{t+1}^{p,H}(i)(y_{H,t}(i) - C_{H,t}^y(i) - tax_{H,t}/P_t) \right)^{1-\gamma} \tag{A.26}$$

where $y_{H,t}(i) = Y_{H,t}(i)/P_t$. This gives

$$\frac{E_t R_{t+1}^{p,H}(i)^{1-\gamma}(a_{H,t}(i))^{-\gamma}}{E_t\left(R_{t+1}^{p,H}(i)a_{H,t}(i)\right)^{1-\gamma}} = 1 \tag{A.27}$$

where $a_{H,t}(i) = A_{H,t}(i)/P_t$. The solution is

$$a_{H,t}(i) = 1 \tag{A.28}$$

Analogously, for the Foreign country we have $a_{F,t}(i) = 1$. It is then also immediate that aggregate demand for bonds is equal to the aggregate supply of bonds by adding (A.24) and (A.25) and using $A_{H,t} = P_t$, $A_{F,t} = P_t^*$.

When choosing the optimal portfolio share, Home agents maximize

$$\frac{1}{1-\gamma} lnE_t\left(R_{t+1}^{p,H}(i)\right)^{1-\gamma} - 0.5\psi(z_{H,t}(i) - z_{H,t-1}(i))^2 \tag{A.29}$$

The first-order condition is

$$\frac{E_t\left(R_{t+1}^{p,H}(i)\right)^{-\gamma}\left(e^{i_t^*+s_{t+1}-s_t-\tau_{H,t}-\pi_{t+1}} - e^{i_t-\pi_{t+1}}\right)}{E_t\left(R_{t+1}^{p,H}(i)\right)^{1-\gamma}} - \psi(z_{H,t}(i) - z_{H,t-1}(i)) = 0 \tag{A.30}$$

where $\pi_{t+1} = p_{t+1} - p_t$ is inflation and lower case letters denote logs. The analogous first-order condition for Foreign agents is

$$\frac{E_t\left(R_{t+1}^{p,F}(i)\right)^{-\gamma}\left(e^{i_t^*-\pi_{t+1}^*} - e^{i_t+s_t-s_{t+1}-\tau_{F,t}-\pi_{t+1}^*}\right)}{E_t\left(R_{t+1}^{p,F}(i)\right)^{1-\gamma}} - \psi(z_{F,t}(i) - z_{F,t-1}(i)) = 0 \tag{A.31}$$

where $\pi_{t+1}^* = p_{t+1}^* - p_t^*$ is inflation in the Foreign country in the Foreign currency.

First-order conditions for goods demand are

$$P_{H,t}C_{HH,t}^k(i) = \varphi P_t C_{H,t}^k(i) \tag{A.32}$$

$$P_{F,t}C_{HF,t}^k(i) = (1-\varphi)P_t C_{H,t}^k(i) \tag{A.33}$$

$$C_{HH,j,t}^k(i) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\mu} C_{HH,t}^k(i) \tag{A.34}$$

$$C_{HF,j,t}^k(i) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\mu} C_{HF,t}^k(i) \tag{A.35}$$

Analogously, for Foreign agents

$$P^*_{H,t} C^k_{FH,t}(j) = (1-\varphi) P^*_t C^k_{F,t}(j) \tag{A.36}$$

$$P^*_{F,t} C^k_{FF,t}(j) = \varphi P^*_t C^k_{F,t}(j) \tag{A.37}$$

$$C^{k,*}_{FH,j,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\mu} C^k_{FH,t}(j) \tag{A.38}$$

$$C^{k,*}_{FF,j,t}(j) = \left(\frac{P^*_{F,t}(j)}{P^*_{F,t}}\right)^{-\mu} C^k_{FF,t}(j) \tag{A.39}$$

Agent i in the Home country then faces the following revenue from Home good i:

$$y_{H,t}(i) = \frac{Y_{H,t}(i)}{P_t} = \varphi \left(P_{H,t}(i)\right)^{1-\mu} P^{\mu-1}_{H,t} \int_0^1 \left(C^y_{H,t}(j) + C^o_{H,t}(j)\right) dj$$
$$+(1-\varphi)\left(P^*_{H,t}(i)\right)^{1-\mu} \left(P^*_{H,t}\right)^{\mu-1} \frac{S_t P^*_t}{P_t} \int_0^1 \left(C^y_{F,t}(j) + C^o_{F,t}(j)\right) dj \tag{A.40}$$

Labor supply is equal to the quantity sold in both markets:

$$L_{H,t}(i) = \varphi \left(P_{H,t}(i)\right)^{-\mu} P^{\mu-1}_{H,t} P_t \int_0^1 \left(C^y_{H,t}(j) + C^o_{H,t}(j)\right) dj$$
$$+(1-\varphi)\left(P^*_{H,t}(i)\right)^{-\mu} \left(P^*_{H,t}\right)^{\mu-1} P^*_t \int_0^1 \left(C^y_{F,t}(j) + C^o_{F,t}(j)\right) dj \tag{A.41}$$

The Home agent i sets prices to maximize

$$y_{H,t}(i) - \phi L_{H,t}(i)^\eta - 0.5\varphi\nu(P_{H,t}(i) - P_{H,t-1}(i))^2 - 0.5(1-\varphi)\nu(P^*_{H,t}(i) - P^*_{H,t-1}(i))^2 \tag{A.42}$$

Using the expressions for $y_{H,t}(i)$ and $L_{H,t}(i)$, the first-order conditions for price setting by Home agent i are (after dividing by respectively φ and $1-\varphi$):

$$(1-\mu)\left(P_{H,t}(i)\right)^{-\mu} P^{\mu-1}_{H,t} \int_0^1 \left(C^y_{H,t}(j) + C^o_{H,t}(j)\right) dj$$
$$+\phi\mu\eta L_{H,t}(i)^{\eta-1} \left(P_{H,t}(i)\right)^{-\mu-1} P^{\mu-1}_{H,t} P_t \int_0^1 \left(C^y_{H,t}(j) + C^o_{H,t}(j)\right) dj$$
$$-\nu(P_{H,t}(i) - P_{H,t-1}(i)) = 0 \tag{A.43}$$

and

$$(1-\mu)\left(P^*_{H,t}(i)\right)^{-\mu} \left(P^*_{H,t}\right)^{\mu-1} \frac{S_t P^*_t}{P_t} \int_0^1 \left(C^y_{F,t}(j) + C^o_{F,t}(j)\right) dj$$
$$+\phi\mu\eta L_{F,t}(i)^{\eta-1} \left(P^*_{H,t}(i)\right)^{-\mu-1} \left(P^*_{H,t}\right)^{\mu-1} P^*_t \int_0^1 \left(C^y_{F,t}(j) + C^o_{F,t}(j)\right) dj$$
$$-\nu(P^*_{H,t}(i) - P^*_{H,t-1}(i)) = 0 \tag{A.44}$$

Analogous first-order conditions can be derived for Foreign agents. Agent i in the Foreign country faces the following demand $y_{F,t}(i) = Y_{F,t}(i)/P_t^*$:

$$y_{F,t}(i) = (1-\varphi)\left(P_{F,t}(i)\right)^{1-\mu} P_{F,t}^{\mu-1} \frac{P_t}{S_t P_t^*} \int_0^1 \left(C_{H,t}^y(j) + C_{H,t}^o(j)\right) dj$$
$$+\varphi \left(P_{F,t}^*(i)\right)^{1-\mu} \left(P_{F,t}^*\right)^{\mu-1} \int_0^1 \left(C_{F,t}^y(j) + C_{F,t}^o(j)\right) dj \tag{A.45}$$

Labor supply is

$$L_{F,t}(i) = (1-\varphi)\left(P_{F,t}(i)\right)^{-\mu} P_{F,t}^{\mu-1} P_t \int_0^1 \left(C_{H,t}^y(j) + C_{H,t}^o(j)\right) dj$$
$$+\varphi \left(P_{F,t}^*(i)\right)^{-\mu} \left(P_{F,t}^*\right)^{\mu-1} P_t^* \int_0^1 \left(C_{F,t}^y(j) + C_{F,t}^o(j)\right) dj \tag{A.46}$$

Foreign agent i maximizes

$$y_{F,t}(i) - \phi L_{F,t}(i)^\eta - 0.5(1-\varphi)\nu(P_{F,t}(i) - P_{F,t-1}(i))^2 - 0.5\varphi\nu(P_{F,t}^*(i) - P_{F,t-1}^*(i))^2 \tag{A.47}$$

The first order conditions are then

$$(1-\mu)\left(P_{F,t}(i)\right)^{-\mu} P_{F,t}^{\mu-1} \frac{P_t}{S_t P_t^*} \int_0^1 \left(C_{H,t}^y(j) + C_{H,t}^o(j)\right) dj$$
$$+\phi\mu\eta(L_{F,t}(i))^{\eta-1} \left(P_{F,t}(i)\right)^{-\mu-1} P_{F,t}^{\mu-1} P_t \int_0^1 \left(C_{H,t}^y(j) + C_{H,t}^o(j)\right) dj$$
$$-\nu(P_{F,t}(i) - P_{F,t-1}(i)) = 0 \tag{A.48}$$

and

$$(1-\mu)\left(P_{F,t}^*(i)\right)^{-\mu} \left(P_{F,t}^*\right)^{\mu-1} \int_0^1 \left(C_{F,t}^y(j) + C_{F,t}^o(j)\right) dj$$
$$+\phi\mu\eta(L_{F,t}(i))^{\eta-1} \left(P_{F,t}^*(i)\right)^{-\mu-1} \left(P_{F,t}^*\right)^{\mu-1} P_t^* \int_0^1 \left(C_{F,t}^y(j) + C_{F,t}^o(j)\right) dj$$
$$-\nu(P_{F,t}^*(i) - P_{F,t-1}^*(i)) = 0 \tag{A.49}$$

A.3 Linearization

The linearized price indices are

$$p_t = \varphi p_{H,t} + (1-\varphi) p_{F,t} \tag{A.50}$$
$$p_t^* = (1-\varphi) p_{H,t}^* + \varphi p_{F,t}^* \tag{A.51}$$

Define $p_{1,t} = p_{Ht} - p^*_{Ft}$ and $p_{2,t} = p^*_{Ht} - p_{Ft}$. Then

$$\tilde{p}_t = p_t - p^*_t = \varphi p_{1,t} - (1-\varphi)p_{2,t} \tag{A.52}$$

and the log real exchange rate is

$$q_t = s_t - \tilde{p}_t \tag{A.53}$$

We will use that all agents in the same country (of the same age) will make the same decisions. Therefore $z_{H,t}(i) = z_{H,t}$, etc. Using that $A_{H,t} = P_t$ and $A_{F,t} = P^*_t$, the Foreign bond market clearing condition is

$$z_{H,t}P_t + z_{F,t}S_tP^*_t = S_tP^*_t \tag{A.54}$$

Dividing by P_t, we have

$$z_{H,t} + z_{F,t}Q_t = Q_t \tag{A.55}$$

where $Q_t = S_tP^*_t/P_t$ is the real exchange rate. This corresponds to equation (10) in the paper. We linearize around a real exchange rate of 1 and $z_{F,t} = \bar{z}_F = 1 - \bar{z}_H$, where $\bar{z}_H$ is the steady state portfolio share invested in the Foreign asset by Home agents (defined below). This gives

$$z^A_t = 0.5(z_{H,t} + z_{F,t}) = 0.5 + 0.5\bar{z}_H q_t \tag{A.56}$$

This is the same as equation (11) in the paper.

In order to linearize the first-order conditions for price setting, we need to discuss the steady state around which to linearize. As in many monetary models, the steady state price level is not determined. We will linearize around price levels of 1. From the first-order condition for price setting, the steady state labor supply in both countries is

$$\bar{L} = \left(\frac{(\mu-1)}{\mu\phi\eta}\right)^{1/(\eta-1)} \tag{A.57}$$

Normalize $\phi = (\mu-1)/(\mu\eta)$ such that $\bar{L} = 1$. Then the steady state income is $\bar{Y} = 1$. From goods market clearing it then follows that $\bar{C}^y + \bar{C}^o = \bar{Y} = 1$. Using this, the first-order conditions for price setting are

$$(p_t - p_{H,t}) + (\eta-1)l_t - \bar{\nu}(p_{H,t} - p_{H,t-1}) = 0 \tag{A.58}$$

$$(p_t - p^*_{H,t} - s_t) + (\eta-1)l_t - \bar{\nu}(p^*_{H,t} - p^*_{H,t-1}) = 0 \tag{A.59}$$

where

$$\bar{\nu} = \frac{\nu}{(\mu - 1)} \tag{A.60}$$

We can also write these as

$$p_{H,t} = (1-\kappa)p_{H,t-1} + \kappa(p_t + (\eta - 1)l_t) \tag{A.61}$$

$$p^*_{H,t} = (1-\kappa)p^*_{H,t-1} + \kappa(p_t - s_t + (\eta - 1)l_t) \tag{A.62}$$

where

$$\kappa = \frac{1}{1+\bar{\nu}} \tag{A.63}$$

Analogous first-order conditions for Foreign country price setting are

$$p_{F,t} = (1-\kappa)p_{F,t-1} + \kappa(p^*_t + s_t + (\eta - 1)l^*_t) \tag{A.64}$$

$$p^*_{F,t} = (1-\kappa)p^*_{F,t-1} + \kappa(p^*_t + (\eta - 1)l^*_t) \tag{A.65}$$

These first-order conditions imply

$$p_{1,t} = (1-\kappa)p_{1,t-1} + \kappa\left(\tilde{p}_t + (\eta - 1)(l_t - l^*_t)\right) \tag{A.66}$$

$$p_{2,t} = (1-\kappa)p_{2,t-1} + \kappa\left(\tilde{p}_t - 2s_t + (\eta - 1)(l_t - l^*_t)\right) \tag{A.67}$$

From hereon we will let $\eta \to 1$. This simplifies as the last term, which depends on $l_t - l^*_t$, drops out. Then

$$\tilde{p}_t = (1-\kappa)\tilde{p}_{t-1} + (2\varphi - 1)\kappa\tilde{p}_t + 2(1-\varphi)\kappa s_t \tag{A.68}$$

or

$$\tilde{p}_t = \frac{1-\kappa}{1+(1-2\varphi)\kappa}\tilde{p}_{t-1} + \frac{2(1-\varphi)\kappa}{1+(1-2\varphi)\kappa}s_t \tag{A.69}$$

Define

$$\omega = \frac{2(1-\varphi)\kappa}{1+(1-2\varphi)\kappa} \tag{A.70}$$

Then

$$\tilde{p}_t = (1-\omega)\tilde{p}_{t-1} + \omega s_t \tag{A.71}$$

Finally, the Home first-order condition for portfolio choice implies

$$\frac{E_t e^{i^*_t + s_{t+1} - s_t - \tau - \pi_{t+1} - \gamma r^{p,H}_{t+1}} - E_t e^{i_t - \pi_{t+1} - \gamma r^p_{t+1}}}{E_t e^{(1-\gamma) r^{p,H}_{t+1}}} - \psi(z_{H,t} - z_{H,t-1}) = 0 \tag{A.72}$$

Take the expectation, using log normality. The first term then takes the form $(e^{a_1} - e^{a_2})/e^{a_3}$. Linearizing this around all $a_i = 0$, this becomes $a_1 - a_2$. This procedure gives

$$E_t\left(s_{t+1} - s_t + i_t^* - i_t - \tau_{H,t}\right) + 0.5var(s_{t+1}) - cov(s_{t+1}, \pi_{t+1} + \gamma r_{t+1}^{p,H}) - \psi(z_{H,t} - z_{H,t-1}) = 0 \tag{A.73}$$

The linearized portfolio return is

$$r_{t+1}^{p,H} = z_{H,t}(s_{t+1} - s_t + i_t^*) + (1 - z_{H,t})i_t - \pi_{t+1} \tag{A.74}$$

Substitution into the first-order condition gives

$$E_t\left(s_{t+1} - s_t + i_t^* - i_t - \tau_{H,t}\right) + (0.5 - \gamma z_{H,t})var(s_{t+1}) - (1 - \gamma)cov(s_{t+1}, p_{t+1}) - \psi(z_{H,t} - z_{H,t-1}) = 0 \tag{A.75}$$

The steady state portfolio is then

$$\bar{z}_H = \frac{0.5}{\gamma} - \frac{\tau}{\gamma var(s_{t+1})} + \frac{\gamma - 1}{\gamma}\frac{cov(s_{t+1}, p_{t+1})}{var(s_{t+1})} \tag{A.76}$$

This corresponds to equation (8) in the paper.

In deviation from steady state we have

$$z_{H,t} - \bar{z}_H = \frac{E_t(er_{t+1} - \hat{\tau}_{H,t})}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{H,t-1} - \bar{z}_H) \tag{A.77}$$

where $\sigma^2 = var(s_{t+1})$, $er_{t+1} = s_{t+1} - s_t + i_t^* - i_t$ and $\hat{\tau}_{H,t} = \tau_{H,t} - \tau$. This corresponds to equation (7) in the paper. The analogous equation for the Foreign country is

$$z_{F,t} - \bar{z}_F = \frac{E_t(er_{t+1} + \hat{\tau}_{F,t})}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{F,t-1} - \bar{z}_F) \tag{A.78}$$

where $\bar{z}_F = 1 - \bar{z}_H$ by symmetry and $\hat{\tau}_{F,t} = \tau_{F,t} - \tau$. This is equation (9) in the paper. It follows that

$$z_t^A - 0.5 = \frac{E_t(s_{t+1} - s_t + i_t^D + 0.5\tau_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^A - 0.5) \tag{A.79}$$

where $i_t^D = i_t^* - i_t$ and $\tau_t^D = \tau_{F,t} - \tau_{H,t}$.

A.4 System of Equations

Based on the results above, we end up with the following system in s_t, $\tilde{p}_t$ and z_t^A:

$$z_t^A = 0.5 + bq_t \tag{A.80}$$

$$\tilde{p}_t = (1-\omega)\tilde{p}_{t-1} + \omega s_t \tag{A.81}$$

$$z_t^A - 0.5 = \frac{E_t(s_{t+1} - s_t + i_t^D + 0.5\tau_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi}{\gamma\sigma^2 + \psi}(z_{t-1}^A - 0.5) \tag{A.82}$$

where $b = 0.5\bar{z}_H$.

Substituting (A.80) into (A.82), we have

$$bq_t = \frac{E_t(s_{t+1} - s_t + i_t^D + 0.5\tau_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi b}{\gamma\sigma^2 + \psi}q_{t-1} \tag{A.83}$$

Define the real interest differential as

$$r_t^D = i_t^D - E_t(\pi_{t+1}^* - \pi_{t+1}) \tag{A.84}$$

Then (A.83) becomes

$$bq_t = \frac{E_t(q_{t+1} - q_t + r_t^D + 0.5\tau_t^D)}{\gamma\sigma^2 + \psi} + \frac{\psi b}{\gamma\sigma^2 + \psi}q_{t-1} \tag{A.85}$$

We can write this as

$$E_t q_{t+1} - \theta q_t + \psi b q_{t-1} + r_t^D + 0.5\tau_t^D = 0 \tag{A.86}$$

where $\theta = 1 + \psi b + \gamma\sigma^2 b$. This corresponds exactly to equation (12) in the paper.

Finally, from (A.81) we have

$$\tilde{p}_t - \tilde{p}_{t-1} = \omega(s_t - \tilde{p}_t) + \omega(\tilde{p}_t - \tilde{p}_{t-1}) \tag{A.87}$$

Now use that $\tilde{p}_t - \tilde{p}_{t-1} = -\pi_t^D$, where $\pi_t^D = \pi_t^* - \pi_t$. Also use that $s_t - \tilde{p}_t = q_t$. It follows that

$$\pi_t^D = -\frac{\omega}{1-\omega}q_t \tag{A.88}$$

In the special case where $\omega = 1$ (perfectly flexible prices), we have $q_t = 0$.

A.5 National Intertemporal Budget Constraint

In a symmetric two-country model (as we assume here as well), Tille and van Wincoop (International Capital Flows, Journal of International Economics, 80(2), 157-175, 2010) show that the net external debt of a country is equal to the present discounted value of trade surpluses, plus a term that is equal to the steady value of gross assets=gross liabilities times the present discounted value of the excess return on external assets over liabilities:

$$-NFA_{H,t} = PDV(TA_{H,t}) + \bar{z}_H PDV(er_t) \tag{A.89}$$

where $NFA_{H,t}$ is the net foreign asset position of the Home country (so that the left hand side is the net external debt), $TA_{H,t}$ is the Home country trade account and $\bar{z}_H$ is the steady state share that Home investors hold in Foreign bonds, which is also the steady state gross asset (and gross liability) position. Intuitively, a country with a net external debt either needs to pay it back by running future trade surpluses or by earning higher returns on external assets than paying on external liabilities. Here we will discuss how the adjustment to an increase in r_t^D occurs within the context of this intertemporal budget constraint.

In what follows we use that in equilibrium all agents within a country are identical and therefore omit the notation for individual agents. First consider the net foreign asset position at time t, NFA_t. We will express all variables in the Home currency and divide by the Home price index P_t. So $nfa_t = NFA_t/P_t$. We have

$$nfa_t = \frac{1}{P_t}\left(z_{H,t}A_{H,t} - (1 - z_{F,t})S_t A_{F,t}\right) \tag{A.90}$$

We found $A_{H,t} = P_t$ and $A_{F,t} = P_t^*$. Therefore

$$nfa_t = z_{H,t} - (1 - z_{F,t})\frac{S_t P_t^*}{P_t} = z_{H,t} - (1 - z_{F,t})Q_t \tag{A.91}$$

This is equal to zero by the bond market clearing condition (10) in the paper. Therefore the left hand side of (A.89) is zero. Intuitively, one can also think of the net foreign asset position as half of Home minus Foreign wealth minus half of Home minus Foreign asset supply. In the model both relative Home wealth and relative Home asset supply are $P_t - S_t P_t^*$. The wealth of both countries is equal to the value of their bond supply.

Before we turn to the trade account, it is useful to do some further balance of payments accounting as a check on the equations. One useful identity is $NFA_{H,t} =$

$CA_{H,t} + VAL_{H,t}$, where $CA_{H,t}$ is the Home current account and $VAL_{H,t}$ stands for the Home country change in the net foreign asset position due to valuation effects (on external assets minus liabilities). First consider the current account, which equals saving. Start with government saving, which is equal to the tax revenue minus the interest payments. The real value of the bond supply is 1, so that the nominal bond supply in the Home country at $t-1$ is P_{t-1}. The government then needs to pay interest at time t of

$$P_{t-1}\left(e^{i_{t-1}} - 1\right)$$

Tax revenue is in equation (A.22). Government saving is then $P_{t-1} - P_t$. So real government saving is

$$\frac{P_{t-1} - P_t}{P_t}$$

Next consider saving by the agents. Saving by the young is equal to 1 in real terms ($a_{H,t} = 1$). Saving by the old is equal to their interest income minus their consumption. First consider interest income. In terms of the Home currency, the current old accumulate $A_{H,t-1} = P_{t-1}$ in assets at $t-1$. They receive interest on this that is equal to

$$P_{t-1}(1 - z_{H,t-1})(e^{i_{t-1}} - 1) + z_{H,t-1}\frac{P_{t-1}S_t}{S_{t-1}}(e^{i^*_{t-1}} - 1) \tag{A.92}$$

The first term is interest on the Home bond, while the second term is interest on the Foreign bond, both expressed in the Home currency. In addition they receive the principal

$$P_{t-1}(1 - z_{H,t-1}) + z_{H,t-1}\frac{P_{t-1}S_t}{S_{t-1}} \tag{A.93}$$

They consume both the interest and the principal. Their saving is therefore minus the principal. Dividing by P_t, and also adding real government saving and real saving by the young, total real Home saving is

$$saving_{H,t} = \frac{P_{t-1} - P_t}{P_t} + 1 - \frac{P_{t-1}}{P_t}\left(1 - z_{H,t-1} + z_{H,t-1}\frac{S_t}{S_{t-1}}\right) \tag{A.94}$$

This is equal to

$$saving_{H,t} = -\frac{P_{t-1}}{P_t} z_{H,t-1}\frac{S_t - S_{t-1}}{S_{t-1}} \tag{A.95}$$

Next consider valuation effects. These only apply to external assets, which are denominated in the foreign currency. Home agents start with $z_{H,t-1}P_{t-1}$ of these assets at $t-1$. At time t they are worth

$$z_{H,t-1}\frac{S_t}{S_{t-1}}P_{t-1}$$

The change in their value, divided by P_t, gives the real valuation effects:

$$val_t = z_{H,t-1}\frac{S_t - S_{t-1}}{S_{t-1}}\frac{P_{t-1}}{P_t}$$

Clearly

$$saving_{H,t} + val_t = 0 \tag{A.96}$$

Since the change in the net foreign asset position is zero, and the change in the NFA is equal to the current account (saving) plus valuation effects, this sum must indeed be zero.

For what follows it is also useful to have an expression for linearized saving:

$$saving_{H,t} = -\bar{z}(s_t - s_{t-1}) \tag{A.97}$$

where $\bar{z} = \bar{z}_H$. We also have $\bar{z}_F = 1 - \bar{z}$.

Next consider the trade account. It can be computed in two ways. One is to simply substract net investment income from the current account. The other is as exports minus imports. We will do it both ways as a check on the equations. Start with net investment income. As already discussed, the Home country earns interest in the Foreign bond equal to

$$z_{H,t-1}\frac{P_{t-1}S_t}{S_{t-1}}(e^{i^*_{t-1}} - 1)$$

Consider the interest that Foreign agents earn on Home bonds. They start with wealth of $A_{F,t-1} = P^*_{t-1}$ in Foreign currency, which is $S_{t-1}P^*_{t-1}$ in the Home currency, of which they invest $(1 - z_{F,t-1})S_{t-1}P^*_{t-1}$ in the Home bond, which delivers interest income

$$(1 - z_{F,t-1})S_{t-1}P^*_{t-1}(e^{i_{t-1}} - 1)$$

Net interest income, divided by P_t, is then

$$ni_{H,t} = \frac{P_{t-1}}{P_t}\left(z_{H,t-1}\frac{S_t}{S_{t-1}}(e^{i^*_{t-1}} - 1) - (1 - z_{F,t-1})Q_{t-1}(e^{i_{t-1}} - 1)\right) \tag{A.98}$$

Linearizing, we have

$$ni_{H,t} = \bar{z}\left(i^*_{t-1} - i_{t-1}\right) \tag{A.99}$$

Then

$$ta_{H,t} = saving_{H,t} - ni_{H,t} = -\bar{z}(s_t - s_{t-1}) + \bar{z}\left(i^*_{t-1} - i_{t-1}\right) = \bar{z}er_t \qquad \text{(A.100)}$$

Alternatively, we can compute the trade account as exports minus imports, which is

$$TA_{H,t} = S_t P^*_{H,t} x^*_{H,t} - P_{F,t} x_{F,t} \qquad \text{(A.101)}$$

Here $x^*_{H,t}$ and $x_{F,t}$ are the quantities that the Home country sells abroad and the Foreign country sells in Home. Using expressions for these quantities in Section A2, we have

$$ta_{H,t} = \frac{TA_{H,t}}{P_t} = (1-\varphi)Q_t\left(C^y_{F,t} + C^o_{F,t}\right) - (1-\varphi)\left(C^y_{H,t} + C^o_{H,t}\right) \qquad \text{(A.102)}$$

Using the fact that steady state consumption by the young plus the old is 1, this linearizes to

$$ta_{H,t} = (1-\varphi)q_t + (1-\varphi)\left(C^y_{F,t} + C^o_{F,t} - C^y_{H,t} - C^o_{H,t}\right) \qquad \text{(A.103)}$$

Obtaining expressions for consumption is not easy. Using that $A_{H,t} = P_t$ and $A_{F,t} = P^*_t$, we have

$$C^y_{H,t} = y_{H,t} - \frac{tax_{H,t}}{P_t} - 1$$
$$C^y_{F,t} = y_{F,t} - \frac{tax_{F,t}}{P^*_t} - 1$$

Use that

$$y_{H,t} = \varphi(C^y_{H,t} + C^o_{H,t}) + (1-\varphi)Q_t(C^y_{F,t} + C^o_{F,t})$$
$$y_{F,t} = (1-\varphi)\frac{1}{Q_t}(C^y_{H,t} + C^o_{H,t}) + \varphi(C^y_{F,t} + C^o_{F,t})$$

Therefore

$$C^y_{F,t} - C^y_{H,t} = (\varphi - (1-\varphi)Q_t)(C^y_{F,t} + C^o_{F,t}) + ((1-\varphi)\frac{1}{Q_t} - \varphi)(C^y_{H,t} + C^o_{H,t}) - e^{i^*_{t-1}}\frac{P^*_{t-1}}{P^*_t} + e^{i_{t-1}}\frac{P_{t-1}}{P_t}$$

Linearizing, this becomes

$$C^y_{F,t} - C^y_{H,t} = -2(1-\varphi)q_t + (2\varphi - 1)(C^y_{F,t} + C^o_{F,t} - C^y_{H,t} - C^o_{H,t}) - i^D_{t-1} - p^*_{t-1} + p^*_t + p_{t-1} - p_t$$

where $i_t^D = i_t^* - i_t$. It follows that

$$C_{F,t}^y - C_{H,t}^y = -q_t + \frac{2\varphi - 1}{2(1-\varphi)}(C_{F,t}^o - C_{H,t}^o) + \frac{1}{2(1-\varphi)}\left(-i_{t-1}^D - p_{t-1}^* + p_t^* + p_{t-1} - p_t\right) \tag{A.104}$$

Therefore

$$C_{F,t}^y - C_{H,t}^y + C_{F,t}^o - C_{H,t}^o = -q_t + \frac{1}{2(1-\varphi)}(C_{F,t}^o - C_{H,t}^o) + \frac{1}{2(1-\varphi)}\left(-i_{t-1}^D - p_{t-1}^* + p_t^* + p_{t-1} - p_t\right) \tag{A.105}$$

We have

$$\begin{aligned} C_{H,t}^o &= \left(z_{H,t-1}\frac{S_t}{S_{t-1}}e^{i_{t-1}^*} + (1 - z_{H,t-1})e^{i_{t-1}}\right)\frac{P_{t-1}}{P_t} \\ C_{F,t}^o &= \left(z_{F,t-1}e^{i_{t-1}^*} + (1 - z_{F,t-1})\frac{S_{t-1}}{S_t}e^{i_{t-1}}\right)\frac{P_{t-1}^*}{P_t^*} \end{aligned}$$

Linearizing, this becomes

$$\begin{aligned} C_{H,t}^o &= p_{t-1} - p_t + \bar{z}\left(s_t - s_{t-1} + i_{t-1}^*\right) + (1 - \bar{z})i_{t-1} \\ C_{F,t}^o &= p_{t-1}^* - p_t^* + \bar{z}\left(i_{t-1} + s_{t-1} - s_t\right) + (1 - \bar{z})i_{t-1}^* \end{aligned}$$

Then

$$C_{F,t}^o - C_{H,t}^o = p_{t-1}^* - p_t^* - p_{t-1} + p_t + (1 - 2\bar{z})i_{t-1}^D - 2\bar{z}(s_t - s_{t-1})$$

Substituting this into (A.105), we have

$$C_{F,t}^y - C_{H,t}^y + C_{F,t}^o - C_{H,t}^o = -q_t - \frac{1}{1-\varphi}\bar{z}(i_t^D + s_t - s_{t-1}) \tag{A.106}$$

Substituting this into (A.103), we have

$$ta_{H,t} = -\bar{z}er_t \tag{A.107}$$

This is indeed exactly the same as (A.100).

We can now turn to the intertemporal budget constraint (A.89). Notice first that this intertemporal budget constraint is indeed satisfied. The left hand side is zero, while the right hand side is the present discounted value of the trade account plus $\bar{z}$ times the excess return. Since in each period the trade account is equal to minus $\bar{z}$ times the excess return, this clearly holds. Now consider a rise in r_t^D in the model. Our analysis has shown that this leads to a period of positive excess returns on the Foreign bond, followed by negative excess returns on the Foreign bond. Positive excess returns on the Foreign bond imply from our trade account solution that the trade account is at first negative. Later on, when excess returns on the Foreign bond turns negative, the trade account becomes positive.

What accounts for the initially negative trade account? Note that there is no expenditure switching in the model as there is pricing to market. The initial depreciation of the Home currency in response to a rise in r_t^D does have the effect of raising the export price when converted to the Home currency. This by itself would raise the trade account, which of course cannot account for the actual drop in the trade account. The latter is instead caused by a rise in Home consumption relative to Foreign consumption, raising Home imports relative to exports. The old generation experiences relatively high returns in the Home country and low returns in the Foreign country when the Home currency depreciatiates. This raises relative Home consumption by the old, as seen from (A.5). The relative high consumption by old Home agents also raises the relative income of young agents in the Home country. As a result of home bias in the goods market, the old agents in the Home country raise their demand for Home goods more than for Foreign goods. We can see from (A.104) that relative Home consumption by the old indeed depends positively on relative Home consumption by the young. So to summarize, the balance of payments adjustment operates mainly through the effect of the exchange rate on asset income by the old, which they consume, and through excess returns on Foreign bonds, which corresponds to an an excess return on external assets minus liabilities for the Home country. While the Home country is at first running trade deficits in response to a rise in r_t^D, they pay for this through the positive excess return on the Foreign bond.

B Algebra for Section 4 of the Paper

B.1 Optimal Portfolios

Home agents maximize

$$C_{H,t} + ln\left(E_t C_{H,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} - \frac{1}{4}\psi \sum_{i=1}^{4} \left(z_{H,i,t} - z_{H,i,t-1}\right)^2 \tag{B.1}$$

subject to

$$C_{H,t+1} = R_t + z_{H,1,t}\left(\frac{Q_{t+1}}{Q_t}R_t^* e^{-\tau_{H,t}} - R_t\right) + z_{H,2,t}\left(\frac{Q_{t+1}}{Q_t}R_{t+1}^{L,*} e^{-\tau_{H,L,t}} - R_t\right) + z_{H,3,t}\left(R_{t+1}^L - R_t\right) + T_{t+1} \tag{B.2}$$

The aggregate of the cost of investing abroad is reimbursed through T_{t+1}, so that in the aggregate

$$C_{H,t+1} = R_t + z_{H,1,t}\left(\frac{Q_{t+1}}{Q_t}R_t^* - R_t\right) + z_{H,2,t}\left(\frac{Q_{t+1}}{Q_t}R_{t+1}^{L,*} - R_t\right) + z_{H,3,t}\left(R_{t+1}^L - R_t\right) \tag{B.3}$$

Define $\hat{z}$ as a deviation from $z_{H,4,t}$, so for example $\hat{z}_{H,1,t} = z_{H,1,t} - z_{H,4,t}$. First-order conditions for optimal portfolio choice are then

$$E_t C_{H,t+1}^{-\gamma}\left(\frac{Q_{t+1}}{Q_t}R_t^* e^{-\tau_{H,t}} - R_t\right) = 0.5\psi(\hat{z}_{H,1,t} - \hat{z}_{H,1,t-1}) \tag{B.4}$$

$$E_t C_{H,t+1}^{-\gamma}\left(\frac{Q_{t+1}}{Q_t}R_{t+1}^{L,*} e^{-\tau_{H,L,t}} - R_t\right) = 0.5\psi(\hat{z}_{H,2,t} - \hat{z}_{H,2,t-1}) \tag{B.5}$$

$$E_t C_{H,t+1}^{-\gamma}\left(R_{t+1}^L - R_t\right) = 0.5\psi(\hat{z}_{H,3,t} - \hat{z}_{H,3,t-1}) \tag{B.6}$$

Denoting logs with lower case letters, define the three excess returns over Home short term bonds as

$$er_{1,t+1} = q_{t+1} - q_t + r_t^* - r_t \tag{B.7}$$

$$er_{2,t+1} = q_{t+1} - q_t + r_{t+1}^{L,*} - r_t \tag{B.8}$$

$$er_{3,t+1} = r_{t+1}^L - r_t \tag{B.9}$$

We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c_{H,t+1} + er_{1,t+1} - \tau_{H,t}} - E_t e^{-\gamma c_{H,t+1}} = 0.5\psi(\hat{z}_{H,1,t} - \hat{z}_{H,1,t-1})e^{-r_t} \tag{B.10}$$

$$E_t e^{-\gamma c_{H,t+1} + er_{2,t+1} - \tau_{H,L,t}} - E_t e^{-\gamma c_{H,t+1}} = 0.5\psi(\hat{z}_{H,2,t} - \hat{z}_{H,2,t-1})e^{-r_t} \tag{B.11}$$

$$E_t e^{-\gamma c_{H,t+1} + er_{3,t+1}} - E_t e^{-\gamma c_{H,t+1}} = 0.5\psi(\hat{z}_{H,3,t} - \hat{z}_{H,3,t-1})e^{-r_t} \tag{B.12}$$

Using log normality of consumption and returns to compute expectations of exponentials and then linearizing (around zero values of exponents), we can write this as

$$E_t er_{1,t+1} - \tau_{H,t} + 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c_{H,t+1}) = 0.5\psi(\hat{z}_{H,1,t} - \hat{z}_{H,1,t-1}) \tag{B.13}$$

$$E_t er_{2,t+1} - \tau_{H,L,t} + 0.5\sigma_2^2 - \gamma cov(er_{2,t+1}, c_{H,t+1}) = 0.5\psi(\hat{z}_{H,2,t} - \hat{z}_{H,2,t-1}) \tag{B.14}$$

$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma cov(er_{3,t+1}, c_{H,t+1}) = 0.5\psi(\hat{z}_{H,3,t} - \hat{z}_{H,3,t-1}) \tag{B.15}$$

where $\sigma_i^2 = var(er_{i,t+1})$.

Log-linearizing (B.3), we have

$$c_{H,t+1} = r_t + z_{H,1,t}er_{1,t+1} + z_{H,2,t}er_{2,t+1} + z_{H,3,t}er_{3,t+1} \tag{B.16}$$

The first-order conditions then become

$$E_t er_{1,t+1} - \tau_{H,t} + 0.5\sigma_1^2 - \gamma z_{H,1,t}\sigma_1^2 - \gamma z_{H,2,t}\sigma_{12} - \gamma z_{H,3,t}\sigma_{13} =$$
$$0.5\psi(\hat{z}_{H,1,t} - \hat{z}_{H,1,t-1}) \tag{B.17}$$
$$E_t er_{2,t+1} - \tau_{H,L,t} + 0.5\sigma_2^2 - \gamma z_{H,1,t}\sigma_{12} + \gamma z_{H,2,t}\sigma_2^2 - \gamma z_{H,3,t}\sigma_{23} =$$
$$0.5\psi(\hat{z}_{H,2,t} - \hat{z}_{H,2,t-1}) \tag{B.18}$$
$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma z_{H,1,t}\sigma_{13} - \gamma z_{H,2,t}\sigma_{23} - \gamma z_{H,3,t}\sigma_3^2 =$$
$$0.5\psi(\hat{z}_{H,3,t} - \hat{z}_{H,3,t-1}) \tag{B.19}$$

Here σ_{ij} is the covariance between $er_{i,t+1}$ and $er_{j,t+1}$.

Define $\mathbf{er}_{t+1} = (er_{1,t+1}, er_{2,t+1}, er_{3,t+1})'$ and $\mathbf{z}_{H,t} = (z_{H,1,t}, z_{H,2,t}, z_{H,3,t})'$. $\hat{\mathbf{z}}_{H,t}$ subtracts $z_{H,4,t}$ from each element of $\mathbf{z}_{H,t}$. Then we can write the three first-order conditions for Home agents compactly as

$$E_t\mathbf{er}_{t+1} - \begin{pmatrix} \tau_{H,t} \\ \tau_{H,L,t} \\ 0 \end{pmatrix} + 0.5diag(\Sigma) - \gamma\Sigma\mathbf{z}_{H,t} = 0.5\psi(\hat{\mathbf{z}}_{H,t} - \hat{\mathbf{z}}_{H,t-1}) \tag{B.20}$$

where Σ is the variance of $\mathbf{er}_{t+1}$.

Next consider the Foreign country. We have

$$C_{F,t+1} = R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}} + z_{F,1,t}\left(R_t^* - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) + z_{F,2,t}\left(R_{t+1}^{L,*} - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) +$$
$$z_{F,3,t}\left(R_{t+1}^L e^{-\tau_{F,L,t}}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) + T_{t+1}^* \tag{B.21}$$

The cost of investment abroad is reimbursed through T_{t+1}^*, so that aggregate Foreign consumption is

$$C_{F,t+1} = R_t\frac{Q_t}{Q_{t+1}} + z_{F,1,t}\left(R_t^* - R_t\frac{Q_t}{Q_{t+1}}\right) + z_{F,2,t}\left(R_{t+1}^{L,*} - R_t\frac{Q_t}{Q_{t+1}}\right) +$$
$$z_{F,3,t}\left(R_{t+1}^L\frac{Q_t}{Q_{t+1}} - R_t\frac{Q_t}{Q_{t+1}}\right) \tag{B.22}$$

First-order conditions for optimal portfolio choice are

$$E_t(C_{F,t+1})^{-\gamma}\left(R_t^* - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}_{F,1,t} - \hat{z}_{F,1,t-1}) \tag{B.23}$$

$$E_t(C_{F,t+1})^{-\gamma}\left(R_{t+1}^{L,*} - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}_{F,2,t} - \hat{z}_{F,2,t-1}) \tag{B.24}$$

$$E_t(C_{F,t+1})^{-\gamma}\left(R_{t+1}^{L} e^{-\tau_{F,L,t}}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau_{F,t}}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}_{F,3,t} - \hat{z}_{F,3,t-1}) \tag{B.25}$$

where $\hat{z}_{F,i,t} = z_{F,i,t} - z_{F,4,t}$. We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c_{F,t+1}} - E_t e^{-\gamma c_{F,t+1} - er_{1,t+1} - \tau_{F,t}} =$$
$$0.5\psi(\hat{z}_{F,1,t} - \hat{z}_{F,1,t-1})e^{-r_t^*} \tag{B.26}$$
$$E_t e^{-\gamma c_{F,t+1} + er_{2,t+1} - er_{1,t+1}} - E_t e^{-\gamma c_{F,t+1} - er_{1,t+1} - \tau_{F,t}} =$$
$$0.5\psi(\hat{z}_{F,2,t} - \hat{z}_{F,2,t-1})e^{-r_t^*} \tag{B.27}$$
$$E_t e^{-\gamma c_{F,t+1} + er_{3,t+1} - er_{1,t+1} - \tau_{F,L,t}} - E_t e^{-\gamma c_{F,t+1} - er_{1,t+1} - \tau_{F,t}} =$$
$$0.5\psi(\hat{z}_{F,3,t} - \hat{z}_{F,3,t-1})e^{-r_t^*} \tag{B.28}$$

Assuming again that consumption and returns are log-linear, taking expectations and then linearizing (around zero values of exponents), we have

$$E_t er_{1,t+1} + \tau_{F,t} - 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c_{F,t+1}) =$$
$$0.5\psi(\hat{z}_{F,1,t} - \hat{z}_{F,1,t-1}) \tag{B.29}$$
$$E_t er_{2,t+1} + \tau_{F,t} + 0.5\sigma_2^2 - \sigma_{12} - \gamma cov(er_{2,t+1}, c_{F,t+1}) =$$
$$0.5\psi(\hat{z}_{F,2,t} - \hat{z}_{F,2,t-1}) \tag{B.30}$$
$$E_t er_{3,t+1} + \tau_{F,t} - \tau_{F,L,t} + 0.5\sigma_3^2 - \sigma_{13} - \gamma cov(er_{3,t+1}, c_{F,t+1}) =$$
$$0.5\psi(\hat{z}_{F,3,t} - \hat{z}_{F,3,t-1}) \tag{B.31}$$

Log-linearizing (B.22), we have

$$c_{F,t+1} = r_t^* - er_{1,t+1} + z_{F,1,t}er_{1,t+1} + z_{F,2,t}er_{2,t+1} + z_{F,3,t}er_{3,t+1} \tag{B.32}$$

The first-order conditions then become

$$E_t er_{1,t+1} + \tau_{F,t} + 0.5\sigma_1^2 - (1-\gamma)\sigma_1^2 - \gamma z_{F,1,t}\sigma_1^2 - \gamma z_{F,2,t}\sigma_{12} - \gamma z_{F,3,t}\sigma_{13} =$$
$$0.5\psi(\hat{z}_{F,1,t} - \hat{z}_{F,1,t-1})$$
$$E_t er_{2,t+1} + \tau_{F,t} + 0.5\sigma_2^2 - (1-\gamma)\sigma_{12} - \gamma z_{F,1,t}\sigma_{12} - \gamma z_{F,2,t}\sigma_2^2 - \gamma z_{F,3,t}\sigma_{23} =$$
$$0.5\psi(\hat{z}_{F,2,t} - \hat{z}_{F,2,t-1})$$
$$E_t er_{3,t+1} + \tau_{F,t} - \tau_{F,L,t} + 0.5\sigma_3^2 - (1-\gamma)\sigma_{13} - \gamma z_{F,1,t}\sigma_{13} - \gamma z_{F,2,t}\sigma_{23} - \gamma z_{F,3,t}\sigma_3^2 =$$
$$0.5\psi(\hat{z}_{F,3,t} - \hat{z}_{F,3,t-1})$$

We can write these first-order conditions compactly as

$$E_t \mathbf{er}_{t+1} + \begin{pmatrix} \tau_{F,t} \\ \tau_{F,t} \\ \tau_{F,t} - \tau_{F,L,t} \end{pmatrix} + 0.5diag(\Sigma) - (1-\gamma)\Sigma_1 - \gamma\Sigma\mathbf{z}_{F,t} = 0.5\psi(\hat{\mathbf{z}}_{F,t} - \hat{\mathbf{z}}_{F,t-1}) \tag{B.33}$$

where $\mathbf{z}_{F,t} = (z_{F,1,t}, z_{F,2,t}, z_{F,3,t})'$ is the vector of portfolio shares of Foreign agents and $\hat{\mathbf{z}}_{F,t}$ subtracts $z_{4,t}$ from each element of $\mathbf{z}_{F,t}$. Σ_1 is the first column of Σ.

Taking the average of (B.20) and (B.33), we have

$$E_t \mathbf{er}_{t+1} + \frac{1}{2}\begin{pmatrix} \tau_{1,t} \\ \tau_{2,t} \\ \tau_{3,t} \end{pmatrix} + \frac{1}{2}diag(\Sigma) - \frac{1}{2}(1-\gamma)\Sigma_1 - \gamma\Sigma\mathbf{z}_t^A = 0.5\psi(\hat{\mathbf{z}}_t^A - \hat{\mathbf{z}}_{t-1}^A) \tag{B.34}$$

where $\mathbf{z}_t^A = 0.5(\mathbf{z}_{H,t} + \mathbf{z}_{F,t})$ and $\hat{\mathbf{z}}_t^A = 0.5(\hat{\mathbf{z}}_{H,t} + \hat{\mathbf{z}}_{F,t})$. The relative taxes are defined as $\tau_{1,t} = \tau_{F,t} - \tau_{H,t}$, $\tau_{2,t} = \tau_{F,t} - \tau_{H,L,t}$ and $\tau_{3,t} = \tau_{F,t} - \tau_{F,L,t}$.

B.2 Market Equilibrium

Next impose asset market equilibrium:

$$z_{H,1,t} + Q_t z_{F,1,t} = Q_t b^S \tag{B.35}$$
$$z_{H,2,t} + Q_t z_{F,2,t} = Q_t P_t^{L,*} b_t \tag{B.36}$$
$$z_{H,3,t} + Q_t z_{F,3,t} = P_t^L b_t \tag{B.37}$$
$$z_{H,4,t} + Q_t z_{F,4,t} = b^S \tag{B.38}$$

Here b^S is the real supply of short-term bonds in terms of the purchasing power of both countries, while b_t is the quantity of long-term bonds in both countries.

Adding up these market clearing conditions, we have

$$(1+Q_t)(1-b^S) = b_t\left(Q_t P_t^{L,*} + P_t^L\right) \tag{B.39}$$

The steady state value of b_t must then be $\bar{b} = (1-b^S)/\bar{P}^L$, where $\bar{P}^L = \kappa/(R-1+\delta)$ is the steady state long term bond price. It follows that $\bar{b}\bar{P}^L = 1-b^S$. We refer to $\bar{b}\bar{P}^L$ as b^L, the value (in terms of purchasing power) of long term bonds in both countries. Therefore $b^S + b^L = 1$. Furthermore, linearizing (B.39) gives

$$b_t - \bar{b} = -\bar{b}p_t^{L,A} \tag{B.40}$$

where $p_t^{L,A} = 0.5(p_t^L + p_t^{L,*})$ is the average log bond price in deviation from its steady state. (B.40) is not important in what follows as excess returns depend on relative log bond prices, not average log bond prices. Since there is no investment in the model, world saving (private plus government) must be zero in equilibrium. (B.40) makes sure that this is the case. There is no endogenous mechanism in the model to equate world saving to zero.

In log-linear form the first three market clearing conditions are then

$$\mathbf{z}_t^A = 0.5\begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + 0.5\begin{pmatrix} b^S \\ b^L \\ 0 \end{pmatrix} q_t - 0.5\bar{\mathbf{z}}_F q_t + 0.25b^L\begin{pmatrix} 0 \\ -p_t^{L,D} \\ p_t^{L,D} \end{pmatrix} \tag{B.41}$$

where $\bar{\mathbf{z}}_F$ is the steady state of $\mathbf{z}_{F,t}$ and $p_t^{L,D} = p_t^L - p_t^{L,*}$ is the relative log long term bond price.

Since the steady state portfolio shares $\bar{\mathbf{z}}_F$ enter in (B.41), we need to say something about them. We will relate them to portfolio home bias. Let $\bar{z}_{H,i}$ and $\bar{z}_{F,i}$ be the steady state portfolio shares of Home and Foreign agents. By symmetry

$$\bar{z}_{H,1} + \bar{z}_{H,4} = \bar{z}_{F,1} + \bar{z}_{F,4} = b^S \tag{B.42}$$

$$\bar{z}_{H,2} + \bar{z}_{H,3} = \bar{z}_{F,2} + \bar{z}_{F,3} = b^L \tag{B.43}$$

So both Home and Foreign investors invest a fraction b^S in short term bonds and a fraction b^L in long term bonds. Within short-term bonds and within long-term bonds, the extent of average home bias is determined by the mean values of the taxes on short and long term bonds of the other country, τ and τ_L. We can vary these to to set home bias at any value for both short and long term bond. Denoting home bias as h for both short-term and long-term bonds, we have

$$h = 1 - \frac{\bar{z}_{H,1}/b^S}{0.5} = 1 - \frac{\bar{z}_{F,4}/b^S}{0.5} \tag{B.44}$$

$$h = 1 - \frac{\bar{z}_{H,2}/b^L}{0.5} = 1 - \frac{\bar{z}_{F,3}/b^L}{0.5} \tag{B.45}$$

Define "foreign country" as the country other than where the investors are located. There are four ratios in these expressions. The ratios in the first equation have as numerator the fraction of the short term bond portfolio invested in foreign short term bonds. The denominator is the short term bond supply of the foreign country as a fraction of the world short term bond supply. The ratios in the second equation are the same, but for long term bonds.

Therefore

$$\bar{z}_{H,1} = \bar{z}_{F,4} = 0.5(1-h)b^S \tag{B.46}$$

$$\bar{z}_{H,2} = \bar{z}_{F,3} = 0.5(1-h)b^L \tag{B.47}$$

These equations, together with (B.42) and (B.43) map the home bias parameter h into all steady state portfolio shares in both countries. We have

$$\bar{z}_{H,4} = \bar{z}_{F,1} = 0.5(1+h)b^S \tag{B.48}$$

$$\bar{z}_{H,3} = \bar{z}_{F,2} = 0.5(1+h)b^L \tag{B.49}$$

Define

$$\mathbf{v} = 0.25(1-h)\begin{pmatrix} b^S \\ b^L \\ -b^L \end{pmatrix} \tag{B.50}$$

Then (B.41) becomes

$$\mathbf{z}_t^A = 0.5\begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + \mathbf{v}q_t + 0.25b^L\begin{pmatrix} 0 \\ -p_t^{L,D} \\ p_t^{L,D} \end{pmatrix} \tag{B.51}$$

Combining these market equilibrium conditions with (B.34), and focusing on the deviation from the steady state, we have

$$E_t \mathbf{er}_{t+1} + \frac{1}{2} \begin{pmatrix} \tau_{1,t} \\ \tau_{2,t} \\ \tau_{3,t} \end{pmatrix} - \gamma \Sigma \mathbf{v} q_t - 0.25 \gamma b^L p_t^{L,D} \Sigma \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \tag{B.52}$$

$$0.5\psi \mathbf{v}(q_t - q_{t-1}) + \frac{\psi b^S}{8}(1-h) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (q_t - q_{t-1}) + \frac{\psi b^L}{8}(p_t^{L,D} - p_{t-1}^{L,D}) \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

B.3 Solution

In order to solve the model, we will write (B.52) as a second-order difference equation in the variables $(q_t, p_t^{L,D}, p_t^{L,A})'$. We first need to write $\mathbf{er}_{t+1}$ in terms of these variables. Log-linearizing the long term bond returns, we have

$$r_{t+1}^L = \lambda p_{t+1}^L - p_t^L \tag{B.53}$$

$$r_{t+1}^{L,*} = \lambda p_{t+1}^{L,*} - p_t^{L,*} \tag{B.54}$$

where $\lambda = (1-\delta)/R$. We then have

$$er_{1,t+1} = q_{t+1} - q_t + r_t^D \tag{B.55}$$

$$er_{2,t+1} = q_{t+1} - q_t + \lambda p_{t+1}^{L,*} - p_t^{L,*} - r_t \tag{B.56}$$

$$er_{3,t+1} = \lambda p_{t+1}^L - p_t^L - r_t \tag{B.57}$$

We can write $p_t^L = p_t^{L,A} + 0.5 p_t^{L,D}$ and $p_t^{L,*} = p_t^{L,A} - 0.5 p_t^{L,D}$, with $p_t^{L,A} = 0.5(p_t^L + p_t^{L,*})$ and $p_t^{L,D} = p_t^L - p_t^{L,*}$. Then

$$er_{2,t+1} = q_{t+1} - q_t - 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} + 0.5 p_t^{L,D} - p_t^{L,A} + 0.5 r_t^D - r_t^A \tag{B.58}$$

$$er_{3,t+1} = 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} - 0.5 p_t^{L,D} - p_t^{L,A} + 0.5 r_t^D - r_t^A \tag{B.59}$$

where $r_t^D = r_t^* - r_t$.

Next a couple of comments on the matrix Σ. Define

$$\sigma_1^2 = var_t(q_{t+1}) \tag{B.60}$$

$$\sigma_3^2 = var_t(r_{t+1}^L) = var_t(\lambda p_{t+1}^L) \tag{B.61}$$

$$\sigma_{13} = cov_t(q_{t+1}, r_{t+1}^L) = cov_t(q_{t+1}, \lambda p_{t+1}^L) \tag{B.62}$$

$$\sigma_{23} = cov_t(r_{t+1}^L, r_{t+1}^{L,*}) = cov_t(\lambda p_{t+1}^L, \lambda p_{t+1}^{L,*}) \tag{B.63}$$

We will see that q_t depends on relative shocks, while $p_t^{L,A}$ depends on average shocks (interest rate shocks and financial shocks). Since average and relative shocks are uncorrelated, we have $cov_t(q_{t+1}, r_{t+1}^L + r_{t+1}^{L,*}) = 0$ or $cov_t(q_{t+1}, r_{t+1}^{L,*}) = -cov_t(q_{t+1}, r_{t+1}^L)$. Then we have

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1^2 - \sigma_{13} & \sigma_{13} \\ \sigma_1^2 - \sigma_{13} & \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} & \sigma_{13} + \sigma_{23} \\ \sigma_{13} & \sigma_{13} + \sigma_{23} & \sigma_3^2 \end{pmatrix} \tag{B.64}$$

As discussed below, we set the size of the financial shocks (shocks to the tax rates) in order to match the data for the four moments that enter this matrix.

Consider the system (B.52). First take the third equation, plus the second equation, minus the first equation. This gives

$$\lambda E_t p_{t+1}^{L,A} - p_t^{L,A} - r_t^A + 0.5\tau_t^A - 0.5\tau_t^{L,A} = 0 \tag{B.65}$$

where $\tau_t^A = 0.5(\tau_{H,t} + \tau_{F,t})$ and $\tau_t^{L,A} = 0.5(\tau_{H,L,t} + \tau_{F,L,t})$. Assuming that r_t^A follows an AR process with AR coefficient ρ and the taxes AR processes with AR coefficient ρ_τ, the solution is

$$p_t^{L,A} = -\frac{1}{1-\lambda\rho} r_t^A + \frac{0.5}{1-\lambda\rho_\tau}\tau_t^A - \frac{0.5}{1-\lambda\rho_\tau}\tau_t^{L,A} \tag{B.66}$$

Next consider the first equation of (B.52), together with the third minus second plus first equation. This gives

$$E_t q_{t+1} - q_t + r_t^D + 0.5\tau_t^D + a_1 q_t + 0.25\gamma(\sigma_1^2 - 2\sigma_{13}) b^L p_t^{L,D} = \frac{\psi}{4} b^S (1-h)(q_t - q_{t-1}) \tag{B.67}$$

$$\lambda E_t p_{t+1}^{L,D} - p_t^{L,D} + r_t^D + 0.5\tau_t^D - 0.5\tau_t^{L,D} + 2a_2 q_t + 0.5\gamma(\sigma_{13} + \sigma_{23} - \sigma_3^2) b^L p_t^{L,D} = \frac{\psi}{4}(1-h)(b^S - b^L)(q_t - q_{t-1}) + \frac{\psi}{4} b^L (p_t^{L,D} - p_{t-1}^{L,D}) \tag{B.68}$$

where $\tau_t^D = \tau_{F,t} - \tau_{H,t}$ and $\tau_t^{L,D} = \tau_{F,L,t} - \tau_{H,L,t}$ and

$$a_1 = -0.25\gamma(1-h)\left(\sigma_1^2 b^S + (\sigma_1^2 - 2\sigma_{13}) b^L\right) \tag{B.69}$$

$$a_2 = -0.25\gamma(1-h)\left(\sigma_{13} b^S + (\sigma_{13} + \sigma_{23} - \sigma_3^2) b^L\right) \tag{B.70}$$

This system can also be written as

$$A_1 E_t \begin{pmatrix} q_{t+1} \\ p_{t+1}^{L,D} \end{pmatrix} + A_2 \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} + A_3 \begin{pmatrix} q_{t-1} \\ p_{t-1}^{L,D} \end{pmatrix} + A_4 r_t^D + A_5 \begin{pmatrix} \tau_t^D \\ \tau_t^{L,D} \end{pmatrix} = 0 \tag{B.71}$$

The matrices are defined as follows. We have

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \tag{B.72}$$

$$A_2 = \begin{pmatrix} -1 + a_1 - \frac{\psi}{4}(1-h)b^S & 0.25\gamma(\sigma_1^2 - 2\sigma_{13})b^L \\ 2a_2 - \frac{\psi}{4}(1-h)(b^S - b^L) & -1 + 0.5\gamma(\sigma_{13} + \sigma_{23} - \sigma_3^2)b^L - \frac{\psi}{4}b^L \end{pmatrix} \tag{B.73}$$

$$A_3 = \frac{\psi}{4} \begin{pmatrix} b^S(1-h) & 0 \\ (1-h)(b^S - b^L) & b^L \end{pmatrix} \tag{B.74}$$

$$A_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{B.75}$$

$$A_5 = \begin{pmatrix} 0.5 & 0 \\ 0.5 & -0.5 \end{pmatrix} \tag{B.76}$$

The system is driven by exogenous AR processes for r_t^D, τ_t^D and $\tau_t^{L,D}$:

$$r_t^D = \rho r_{t-1}^D + \varepsilon_t \tag{B.77}$$

$$\tau_t^D = \rho_\tau \tau_{t-1}^D + \varepsilon_t^\tau \tag{B.78}$$

$$\tau_t^{L,D} = \rho_\tau \tau_{t-1}^{L,D} + \varepsilon_t^{L,\tau} \tag{B.79}$$

One can write the system as a first-order difference equation of the form $AE_t x_{t+1} + Bx_t = 0$, where $x_t = (q_t, p_t^{L,D}, q_{t-1}, p_{t-1}^{L,D}, r_t^D, \tau_t^D, \tau_t^{L,D})'$. This allows us to solve for the control variables $(q_t, p_t^{L,D})$ as a function of the state variables $(q_{t-1}, p_{t-1}^{L,D}, r_t^D, \tau_t^D, \tau_t^{L,D})'$. Define

$$v_t = \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} \tag{B.80}$$

Then the solution takes the form

$$v_t = M_1 v_{t-1} + M_2 r_t^D + M_3 \tau_t^D + M_4 \tau_t^{L,D} \tag{B.81}$$

We can also integrate this and write

$$v_t = \sum_{k=0}^{\infty} M_1^k \left(M_2 r_{t-k}^D + M_3 \tau_{t-k}^D + M_4 \tau_{t-k}^{L,D} \right) \tag{B.82}$$

with M_1^0 being the identity matrix.

B.4 Calibration

For numerical analysis, we need to make assumptions about the parameters h, γ, ψ, ρ, δ, R and the variance Σ of excess returns. As in the benchmark parameterization of Section 2 of the paper, we set $h = 0.66$, $\rho = 0.9415$, $\gamma = 50$ and $\psi = 15$. We set $R = 1.0033$ for monthly data, corresponding to a 4 percent anual interest rate. LSV consider the returns on 10-year coupon bonds. A 10-year bond with face value of 1 and coupons of $R - 1 = 0.0033$ has a Macauley duration of 99.3 months or 8.3 years. We set $\delta = 0.0071$, which yields a Macauley duration of 99.3 months.

We use data on real exchange rates and long term bond returns to compute Σ. As discussed, we only need 4 independent moments: $\sigma_1^2 = var_t(q_{t+1})$, $\sigma_3^2 = var_t(\lambda p_{t+1}^L)$, $\sigma_{23} = cov(\lambda p_{t+1}^L, \lambda p_{t+1}^{L,*})$ and $\sigma_{13} = cov_t(q_{t+1}, \lambda p_{t+1}^L)$. For a given process for r_t^D and r_t^A we can always match these moments through the financial shock processes associated with τ_t^D, $\tau_t^{L,D}$, τ_t^A and $\tau_t^{L,A}$. By symmetry, the innovations in averages and differences of variables are uncorrelated. Then we have

$$
\begin{aligned}
var_t(p_{t+1}^L) &= var_t(p_{t+1}^{L,A}) + 0.25 var_t(p_{t+1}^{L,D}) \\
cov_t(p_{t+1}^L, p_{t+1}^{L,*}) &= var_t(p_{t+1}^{L,A}) - 0.25 var_t(p_{t+1}^{L,D})
\end{aligned}
$$

We can match $var_t(p_{t+1}^{L,A})$ by choosing appropriately sized shocks for τ_t^A and $\tau_t^{L,A}$, with a degree of freedom on the relative importance of these two average tax shocks. Also note that $cov_t(q_{t+1}, p_{t+1}^L) = -0.5 cov_t(q_{t+1}, p_{t+1}^{L,D})$ since $p_{t+1}^{L,A}$ depends on average shocks, which are uncorrelated with relative shocks that affect q_t. This leaves 3 moments: $var_t(q_{t+1})$, $var_t(p_{t+1}^{L,D})$ and $cov_t(q_{t+1}, p_{t+1}^{L,D})$. These depend on q_{t+1} and $p_{t+1}^{L,D}$, which in turn depend on τ_t^D and $\tau_t^{L,D}$. We can choose the variance and covariance of the innovations in τ_t^D and $\tau_t^{L,D}$ to match these 3 moments. In summary, we can always set the variance and covariance of the financial shock innovations to match the observed matrix Σ in the data.

Knowing that we can always match moments in the data of the Σ matrix by chosing large enough financial shocks in the model, we set the 4 moments of Σ based on data for real exchange rates and long-term bond returns for the G7 countries (see Appendix A of the paper). We set σ_1 equal to the same standard deviation 0.0271 of the real exchange rate as in the benchmark parameterization of the paper. We set σ_3, the standard deviation of the long-term bond return, equal to 0.0206. We set the covariance between the real exchange rate and long-term bond returns, σ_{13}, equal to 0.0000538. Finally, we set the covariance between the

Home and Foreign long-term bond returns, σ_{23}, equal to 0.000267.

B.5 Model Moments

In order to compute the predictability moments, we assume that the relative interest rate shock is uncorrelated with the financial shocks. We can then ignore the financial shocks for the purpose of computing the predictability moments. They only affect the matrix Σ.

First consider the excess return

$$er_{4,t+1} = -\lambda p_{t+1}^{L,D} + p_t^{L,D} + q_{t+1} - q_t \tag{B.83}$$

This is equal to $er_{2,r+1} - er_{3,t+1}$, which is the excess return of the Foreign long term bond over the Home long term bond. The coefficient of a regression of $er_{4,t+1}$ on r_t^D is equal to

$$\beta_1 = \frac{cov(er_{4,t+1}, r_t^D)}{var(r_t^D)} \tag{B.84}$$

Define the vectors $e_1 = (1, -\lambda)$ and $e_2 = (-1, 1)$. Then

$$er_{4,t+1} = e_1 M_2 r_{t+1}^D + \sum_{k=0}^{\infty} \left(e_1 M_1^{k+1} + e_2 M_1^k \right) M_2 r_{t-k}^D \tag{B.85}$$

We then have

$$\beta_1 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 \tag{B.86}$$

Next consider $er_{1,t+1}$, the excess return of the Foreign short term bond over the Home short term bond. Defining $e_1 = (1, 0)$ and $e_2 = (-1, 0)$, the regression coefficient of $er_{1,t+1}$ on r_t^D is

$$\beta_2 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 + 1 \tag{B.87}$$

Finally consider the difference between the Foreign and the Home local excess returns of long term over short term bonds. This is equal to $-\lambda p_{t+1}^{L,D} + p_t^{L,D} - r_t^D$. Defining $e_1 = (0, -\lambda)$ and $e_2 = (0, 1)$, this coefficient of a regression on r_t^D is

$$\beta_3 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 - 1 \tag{B.88}$$