Puzzling Exchange Rate Dynamics and Delayed Portfolio Adjustment[1]

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Abstract

The objective of this paper is to show that the proposal by Froot and Thaler (1990) of delayed portfolio adjustment can account for a broad set of puzzles about the relationship between interest rates and exchange rates. The puzzles include: i) the delayed overshooting puzzle; ii) the forward discount puzzle (or Fama puzzle); iii) the predictability reversal puzzle; iv) the Engel puzzle (high interest rate currencies are stronger than implied by UIP); v) the forward guidance exchange rate puzzle; vi) the absence of a forward discount puzzle with long-term bonds. These results are derived analytically in a simple two-country model with portfolio adjustment costs. Quantitatively, this approach can match all targeted moments related to these puzzles.

1 Introduction

Richard Thaler won the 2017 Nobel Prize in Economics. Thaler has focused his research on behavior that he refers to as a deviation from "rational efficient markets." One example of this is the foreign exchange market. Focusing on the forward discount puzzle, the fact that high interest rate currencies tend to appreciate, Froot and Thaler (1990) argue that "a rational efficient markets paradigm provides no satisfactory explanation for the observed results". They suggest that gradual portfolio adjustment could solve this puzzle. Their hypothesis is that "...at least some investors are slow in responding to changes in the interest differential," arguing that "It may be that these investors need some time to think about trades before executing them, or that they simply cannot respond quickly to recent information." In Bacchetta and van Wincoop (2010) we took this proposal seriously and showed that it can indeed account for the forward discount puzzle.

The objective of this paper is to explore the role of gradual portfolio adjustment for a broader set of features in the interaction between exchange rates and interest rates. We find that gradual portfolio adjustment can account for as many as six puzzles that have been identified in the literature. The puzzles that we address are:

1. Delayed overshooting puzzle: a monetary contraction that raises the interest rate leads to a period of gradual appreciation, followed by gradual depreciation.

2. Forward discount puzzle (or Fama puzzle): high interest rate currencies have higher expected returns over the near future.

3. Predictability reversal puzzle: high interest rate currencies have lower expected returns after some period of time.

4. Engel puzzle: high interest rate currencies are stronger than implied by uncovered interest parity.

5. Forward guidance exchange rate puzzle: the exchange rate is more strongly affected by expected interest rates in the near future than the distant future.

6. LSV puzzle: current interest differentials do not predict long-term bond return differentials.

The delayed overshooting puzzle was first documented by Eichenbaum and Evans (1995) for the US and Grilli and Roubini (1996) for other countries. It should be pointed out that the subsequent studies have shown that the evidence depends on identification strategies.[1] The second puzzle, the forward discount puzzle (Fama (2006)), is the best known on this list and continues to be a well established empirical fact.[2] The predictability reversal puzzle, first documented by Bacchetta and van Wincoop (2010), is related to the forward discount puzzle. They show that while the excess return over the next quarters is positive for higher interest rate currencies (forward discount puzzle), after about 8 quarters the quarterly excess return is negative for currencies whose current interest rate is relatively high. In other words, there is a reversal in the sign of expected excess returns. Engel (2016) confirms that this is a robust puzzle.

The fourth puzzle is documented in Engel (2016). The Engel puzzle says that high interest rate currencies tend to have a stronger exchange rate than under UIP (uncovered interest rate parity). This is because the sum of all expected future excess returns is negative for high interest rate currencies. In other words, the predictability reversal will ultimately dominate and investors demand a lower sum of all future excess returns on currencies whose interest rate is currently high. Such currencies are therefore strong relative to what they would be under UIP.

The forward guidance exchange rate puzzle is developed by Galí (2020). Under UIP the exchange rate is equal to the unweighted sum of all future expected interest rate differentials. This implies that changes in expected interest rates in the near future have the same effect on the exchange rate today as changes in the expected interest differential in the more distant future. However, in the data Galí (2020) finds that expectations of interest differentials in the distant future have a much smaller effect on the current exchange rate than expectations of interest differentials in the near future.

The LSV puzzle stands for the puzzle developed by Lustig, Stathopoulos and Verdelhan (2019) (henceforth LSV). It says that the forward discount puzzle has no analogy in long-term bonds. While the international excess return on short-

[1] See for example Cushman and Zha (1997), Faust and Rogers (2003), Scholl and Uhlig (2008) or Bjørnland (2009). Cheung and Lai (2000) and Steinsson (2008) document broader evidence of a humped shape real exchange rate response to shocks.

[2] Notice, however, that the puzzle does not seem to hold when we include post-2008 data. See Bussière et al. (2018).

term bonds tends to be positive for currencies with a relatively high interest rate (forward discount puzzle), LSV find that this is not the case for long-term bonds. They show that the local excess return of long-term bonds over short-term bonds tends to be lower for high interest rate currencies and that this offsets the positive expected excess return for short-term bonds.[3] LSV find that no-arbitrage models in international finance cannot account for this.

Our objective is to show that a single friction, associated with portfolio adjustment costs, is able to account for each of these puzzles.[4] An additional objective is to do so in an analytically tractable way, which significantly facilitates the analysis and makes the results more transparent. With the exception of the LSV puzzle, the key results are summarized through a set of propositions that follow directly from the closed form analytical solution of the model. We obtain analytic tractability by assuming that agents can adjust their portfolio each period, but face a simple quadratic portfolio adjustment cost.

The first four puzzles are not entirely independent. For example, we will see that delayed overshooting can give rise to the forward discount puzzle, predictability reversal as well as the Engel puzzle. At the same time, these four puzzles are not simply different sides of the same coin. We will show that while there is considerable overlap in the regions of the parameter space that satisfy the individual puzzles, there are also big differences. For example, when the portfolio friction is very high, there is always delayed overshooting and predictability reversal, while the Engel puzzle is not satisfied and excess return predictability is weak.

The paper is part of a broader literature that has modeled portfolio frictions. There are three ways of modeling these frictions in the literature, which have a close analogy to modeling price stickyness in the goods market. The assumption that we

[3]The international excess return of long-term bonds can be written as the sum of the international excess return for short-term bonds plus the difference in local excess returns of long-term over short-term bonds.

[4]While there is a vast literature on the forward discount puzzle, which we will not review here, the other puzzles have received much less attention, and certainly not jointly. A few papers focus on delayed overshooting, e.g., Gourinchas and Tornell (2004) and Kim (2005). Recently, several papers analyze the predictability reversal puzzle. Engel (2016), Itshkoki and Mukhin (2017), and Valchev (2020) propose explanations based on liquidity shocks. Chernov and Creal (2018) and Dahlquist and Penasse (2017) focus on the role of long-term real exchange rate adjustment. The forward guidance exchange rate puzzle and LSV puzzle have only been recently documented and no solution has been proposed yet.

make of a portfolio adjustment cost is analogous to the Rotemberg (1982) cost for price changes. Other papers taking this approach include Vayanos and Woolley (2012), Gârleanu and Pedersen (2013) and Bacchetta, Tièche and van Wincoop (2020). The second approach is the most common in the literature. It assumes that there are overlapping generations of agents that change their portfolio every T periods, analogous to Taylor price setting.[5] The last approach is analogous to Calvo price setting and assumes that agents make a new portfolio decision each period with a given probability p. This approach is adopted in Bacchetta, van Wincoop, and Young (2020).

While the assumption of a simple quadratic portfolio cost is more ad hoc than the other two approaches, it has several advantages. First, as already emphasized, it is the only approach that allows for analytical tractability. The assumption that agents change their portfolio every T periods means that one needs to keep track of the wealth and portfolios of T generations of agents. The assumption that agents make a new portfolio decision with probability p makes the model even harder to solve. It significantly complicates portfolio Euler equations and substantially increases the number of state variables. Another advantage of a quadratic portfolio cost is that it leads to smooth impulse response functions in response to shocks. This is not the case when agents make new portfolio decisions every T periods. Bacchetta and van Wincoop (2010) show that this leads to a "wobbly" impulse response. This is because of the anticipation that agents changing their portfolio at the time of the shock will change their portfolio again exactly T periods later.

One limitation of the paper is that there does not exist direct evidence on the importance of gradual portfolio adjustment in the foreign exchange (FX) market. The FX market is a complex market, associated with any type of international asset trade by any type of agents. At one extreme, if a household changes the global allocation of a retirement portfolio, this would generally lead to FX trade. Such changes in retirement portfolios are done very infrequently, contributing to very gradual portfolio adjustment.[6] At the other extreme, FX hedge funds are very

[5]Recent examples include Bacchetta and van Wincoop (2010), Bogousslavky (2016), Duffie (2010), Henderschott et al. (2013) and Greenwood et al. (2018).

[6]For example, Mitchell et al. (2006) find that 80 percent of 1.2 million workers with 401(k) plans initiated no trades during a two-year period. Ameriks and Zeldes (2004) find that over a 10-year period, 44 percent of households made no changes to their TIAA-CREF portfolio allocations.

active players. However, as argued by Bacchetta and van Wincoop (2010), they manage only a very small fraction of external asset holdings. Since in addition their positions are limited by risk aversion, they alone cannot undo excess return predictability in the FX market. There exists extensive micro evidence on sluggish portfolio adjustment by households.[7] In addition, Bacchetta, Tièche and van Wincoop (2020) show that US mutual funds face significant portfolio frictions in their global equity portfolio choices.[8] But there is no clear evidence on the overall magnitude of portfolio frictions in the FX market. Therefore, while our findings show that such frictions *can* account for the puzzles, this paper takes no stand on whether the assumed frictions are quantitatively plausible.

The remainder of the paper is organized as follows. In Section 2 we discuss a two-country model with short-term bonds and gradual portfolio adjustment. In Section 3 we provide formal propositions related to the first five puzzles as well as a numerical illustration. Section 4 introduces long-term bonds in order to address the LSV puzzle. Section 5 discusses an extension of the model to infinite lives. Section 6 concludes.

2 Model with Gradual Portfolio Adjustment and Short-Term Bonds

The six puzzles can be written both in terms of real interest rates and exchange rates and in terms of nominal interest rates and exchange rates. As Engel (2016) points out, the forward discount puzzle applies equally when using real variables. Galí (2020) also uses real interest rates and exchange rates to develop the forward guidance exchange rate puzzle. An advantage of stating the puzzles in terms of real

[7] See for example Ameriks and Zeldes (2004), Bilias et al. (2010), Brunnermeier and Nagel (2008), Mitchell et al. (2006) and Giglio et al. (2019). Duffie (2010) reviews a broad range of evidence motivating models of gradual portfolio adjustment. Also related is evidence by the Investment Company Institute cited by Bacchetta, van Wincoop and Young (2020) showing that only 40 percent of US investors change their stock or mutual fund portfolio during any particular year.

[8] Related to that, Bohn and Tesar (1996) and Froot et al. (2001) find that international portfolio flows are highly persistent and strongly related to lagged returns. Bohn and Tesar (1996) conclude: "we suspect that investors may adjust their portfolios to new information gradually over time, resulting in both autocorrelated net purchases and a positive linkage with lagged returns."

variables is that the real exchange rate is stationary, while the nominal exchange rate is generally not stationary. We therefore use real variables, although we should stress that the equations can easily be written in nominal terms as well.

In this section we first describe the model and then the solution for the equilibrium real exchange rate and corresponding excess return predictability coefficients. We also discuss a calibration of the model that is used to numerically illustrate the puzzles in the next two sections. Our focus will be on portfolio choice and asset market equilibrium, but we will also discuss the goods market, for which we assume price stickiness. Full details of the general equilibrium model are left to the Online Appendix.

2.1 Model Description

There are overlapping generations of agents who live two periods. There are two countries, Home and Foreign ($h = H, F$). There are two assets, Home and Foreign nominal bonds. Agents in the Home country born at time t maximize

$$C_{H,t} + ln\left(E_t C_{H,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} - 0.5\psi\left(z_{H,t} - z_{H,t-1}\right)^2 \tag{1}$$

where $C_{H,t}$ is consumption at time t and $z_{H,t}$ is the fraction that Home agents invests in the Foreign bond. Since there are only two assets, there is only one portfolio share.

The utility of consumption is specified in an analytically convenient way as linear in time t consumption and logarithmic in the certainty equivalent of time $t+1$ consumption, with a rate of risk aversion of γ. As we will see, this specification implies that saving at time t, which will be the wealth that is invested in the assets, is always 1. This is a simple device to ignore changes in asset demand associated with changes in wealth.

The last term of (1) is a quadratic portfolio adjustment cost. It captures a utility cost of choosing a different portfolio share $z_{H,t}$ invested in Foreign bonds from that of "parents" one period ago. We do not take a stand on the micro underpinnings of this cost. It may for example be related to portfolio decision making costs or trading costs. While in the benchmark model we assume two-period lived agents, in Section 5 we consider an extension to infinite lives, where agents also need to take into account the cost of future portfolio share changes.

The budget constraint is

$$C_{H,t+1} = R^{p,H}_{t+1}(Y_{H,t} - C_{H,t}) \tag{2}$$

where $Y_{H,t}$ is real income at time t and $R^{p,H}_{t+1}$ is the real portfolio return from t to $t+1$. The income $Y_{H,t}$ is discussed in the Online Appendix, but is irrelevant for what follows. The portfolio return is

$$R^{p,H}_{t+1} = \left[z_{H,t} \frac{S_{t+1}}{S_t} e^{i^*_t} e^{-\tau_{H,t}} + (1 - z_{H,t}) e^{i_t} \right] \frac{P_t}{P_{t+1}} + T_{t+1} \tag{3}$$

Here i_t and i^*_t are the nominal interest rate on Home and Foreign bonds, S_t is the nominal exchange rate, measured in terms of the Home currency per unit of the Foreign currency, and P_t is the overall consumer price index of Home agents.

We introduce a tax $\tau_{H,t}$ of investment in the Foreign bond by Home agents, imposed by a broker or the government. There will be an analogous tax τ_{Ft} of investment in the Home bond by Foreign agents. The aggregate of this tax revenue across all Home agents is reimbursed through the term T_{t+1} in (3), which the agents take as given. This assures that the tax $\tau_{H,t}$ will only affect portfolio choice, not the overall return.

These taxes play two roles. First, their mean level τ can be set to generate realistic home bias. Second, changes in these costs generate exogenous portfolio shifts, which we refer to as financial shocks. Itskhoki and Muhkin (2019) have argued that exchange rates are disconnected from observed fundamentals primarily through unobserved financial shocks. Gabaix and Maggiori (2015) also argue that such shocks are critical to exchange rates, while Bacchetta, van Wincoop and Young (2020) show that they play a key role in the global equity market as well. While we model financial shocks as resulting from time-varying taxes, all that matters is that they generate exogenous portfolio shifts. Alternative ways that these have been modeled in the literature include noise trade, liquidity trade, hedge trade, time-varying risk-bearing capacity or time-varing investment opportunities.[9] The exact origin of such shocks plays no role in our analysis.

Substituting the budget constraint (2) into (1), we can maximize with respect to consumption $C_{H,t}$ and the portfolio $z_{H,t}$. The first-order condition with respect

[9] For different ways of modeling the portfolio shocks, see Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Gabaix and Maggiori (2015), He and Wang (1995), Spiegel and Subrahmanyam (1992) and Wang (1994).

to consumption gives $Y_{H,t} - C_{H,t} = 1$. Saving is therefore always 1 and agents invest a wealth of 1 in the two assets. The first-order condition with respect to the portfolio $z_{H,t}$ is

$$\frac{E_t e^{-\gamma r^{p,H}_{t+1} + s_{t+1} - s_t + i^*_t - \tau_{H,t} - \pi_{t+1}} - E_t e^{-\gamma r^{p}_{t+1} + i_t - \pi_{H,t+1}}}{E_t e^{(1-\gamma) r^{p,H}_{t+1}}} - \psi(z_{H,t} - z_{H,t-1}) = 0 \quad (4)$$

where inflation from t to $t+1$ is denoted as $\pi_{t+1} = p_{t+1} - p_t$, with p_t the log price level. Expectations of exponentials are computed by assuming log normality, after which we linearize around zero values of exponents. This gives

$$E_t s_{t+1} - s_t + i^*_t - i_t - \tau_{H,t} + 0.5 var_t(s_{t+1}) - cov_t(\gamma r^{p,H}_{t+1} + p_{t+1}, s_{t+1}) - \psi(z_{H,t} - z_{H,t-1}) = 0 \quad (5)$$

A first-order approximation of the log-portfolio return is

$$r^{p,H}_{t+1} = z_{H,t}(er_{t+1} - \tau_{H,t}) + i_t - \pi_{t+1} \quad (6)$$

where the excess return is $er_{t+1} = s_{t+1} - s_t + i^*_t - i_t$. Substituting (6) into (5), we can solve for the optimal portfolio as

$$z_{H,t} - \bar{z}_H = \frac{\psi}{\psi + \gamma\sigma^2}(z_{H,t-1} - \bar{z}_H) + \frac{1}{\psi + \gamma\sigma^2}(E_t er_{t+1} - \hat{\tau}_{H,t}) \quad (7)$$

Here $\hat{\tau}_{H,t} = \tau_{H,t} - \tau$, $\sigma^2 = var_t(s_{t+1})$ and

$$\bar{z}_H = \frac{0.5}{\gamma} - \frac{\tau}{\gamma\sigma^2} + \frac{\gamma - 1}{\gamma}\frac{\sigma_{s,p}}{\sigma^2} \quad (8)$$

where $\sigma_{s,p} = cov_t(s_{t+1}, p_{t+1})$. These moments are time-invariant once s_t and p_t are solved as a function of model shocks. $\bar{z}_H$ is the steady state fraction invested in the Foreign bond by Home agents.

The role of the parameters ψ and γ in the portfolio can be understood intuitively. When choosing portfolios, investors care about expected returns, risk and the cost of portfolio adjustment. A rise in ψ implies a higher weight on the cost of portfolio adjustment, leading the optimal portfolio to depend more on the lagged portfolio and less on the expected excess return. A rise in γ implies that investors are more concerned with risk, leading to a lower weight on both the lagged portfolio and the expected excess return. We will refer to the dependence of the portfolio on the lagged portfolio and expected excess return as respectively *portfolio persistence* and *return sensitivity*. A higher γ leads to both less portfolio persistence and weaker return sensitivity, while a higher ψ raises portfolio persistence, but weakens return sensitivity.[10]

The Foreign country faces an analogous problem. Its real wealth, in terms of Foreign purchasing power, is then also 1, while its optimal fraction invested in Foreign bonds is

$$z_{F,t} - \bar{z}_F = \frac{\psi}{\psi + \gamma\sigma^2}\left(z_{F,t-1} - \bar{z}_F\right) + \frac{1}{\psi + \gamma\sigma^2}(E_t er_{t+1} + \hat{\tau}_{F,t}) \tag{9}$$

where $\hat{\tau}_{F,t} = \tau_{F,t} - \tau$. As a result of symmetry we have $\bar{z}_F = 1 - \bar{z}_H$.

The real supply of bonds is assumed fixed at 1 in terms of the purchasing power of the respective countries. Let $Q_t = S_t P_t^*/P_t$ be the real exchange rate, where P_t^* is the consumer price index of the Foreign country in the Foreign currency. As a result of Walras' Law, it is sufficient to focus on the Foreign bond market equilibrium. In terms of Home country purchasing power, the value of the Foreign bond supply is Q_t, while the wealth of Home and Foreign agents is 1 and Q_t. Foreign bond market equilibrium is then

$$z_{H,t} + z_{F,t}Q_t = Q_t \tag{10}$$

Linearizing around the log real exchange rate, q_t, equal to zero, this becomes

$$z_t^A = 0.5 + bq_t \tag{11}$$

where $z_t^A = 0.5(z_{H,t} + z_{F,t})$ is the average that the two countries invest in the Foreign bond and $b = 0.5\bar{z}_H$.

It is useful to define the nominal interest rate differential as the Foreign minus the Home interest rate, $i_t^D = i_t^* - i_t$. Real interest rates are defined as the nominal interest rate minus expected inflation, so $r_t = i_t - E_t\pi_{t+1}$ and $r_t^* = i_t^* - E_t\pi_{t+1}^*$. The real interest rate differential is then $r_t^D = r_t^* - r_t = i_t^D - E_t(\pi_{t+1}^* - \pi_{t+1})$. It follows that the expected excess return can also be written in terms of real variables as $E_t er_{t+1} = E_t q_{t+1} - q_t + r_t^D$. Using this, substituting the optimal portfolios (7) and (9) into the Foreign bond market equilibrium condition (11) gives

$$E_t q_{t+1} - \theta q_t + b\psi q_{t-1} + r_t^D + 0.5\tau_t^D = 0 \tag{12}$$

where $\theta = 1 + \psi b + \gamma\sigma^2 b$ and $\tau_t^D = \tau_{Ft} - \tau_{Ht}$. Note that a rise in τ_t^D implies an exogenous portfolio shift towards the Foreign bond, away from the Home bond.

[10] These effects are similar to Gârleanu and Pedersen (2013).

2.2 Real Exchange Rate Solution

Using (12), standard solution techniques for second-order stochastic difference equations give

$$q_t = \alpha q_{t-1} + E_t \sum_{i=0}^{\infty} \frac{1}{D^{i+1}} \left(r_{t+i}^D + 0.5\tau_{t+i}^D \right) \tag{13}$$

where α and D are the roots of the characteristic equation of (12):

$$\alpha = \frac{\theta - \sqrt{\theta^2 - 4\psi b}}{2} \tag{14}$$

$$D = \frac{\theta + \sqrt{\theta^2 - 4\psi b}}{2} \tag{15}$$

It is easily verified that $0 \leq \alpha < 1$ and $D > 1$. The real exchange rate therefore depends on the lagged real exchange rate and a present discounted value of expected future real interest rate differentials.

A couple of comments about the parameters α and D are in order as they are key to the solution. Appendix B derives the following Lemma:

Lemma 1. *The following properties describe the relationship between α, D and the portfolio adjustment cost parameter ψ:*

- *As ψ rises from 0 to ∞, α rises monotonically from 0 to 1.*
- *As ψ rises from 0 to ∞, D rises monotonically from $1 + \gamma\sigma^2 b$ to ∞.*

Higher portfolio adjustment costs imply that the real exchange rate depends to a greater extent on the value of the real exchange rate during the last period and future expected real interest rates are discounted more heavily.

2.3 Goods Market and Monetary Policy

So far we have abstracted from the goods market. The solution (13) for the real exchange rate depends on the real interest rate, which is an endogenous variable and generally depends on goods market equilibrium. The real interest rate may depend on both monetary policy shocks and financial shocks, dependent on the nature of monetary policy and extent of price stickiness. To illustrate this, we briefly discuss the goods market, focusing on the key equations while leaving most of the details to the Online Appendix.

Assume a continuum of differentiated Home and Foreign goods. Agents have Cobb Douglas utility over the index of Home and Foreign goods, with a fraction φ spent on domestic goods. There is local currency pricing: Home and Foreign firms set prices of respectively $P_{H,t}$ and $P_{F,t}$ in the Home country in the Home currency and $P^*_{H,t}$ and $P^*_{F,t}$ in the Foreign country in the Foreign currency. Output is linear in labor. Firms face quadratic Rotemberg costs of price changes.

The real wage rate is constant since consumption and leisure are perfect substitutes.[11] The marginal cost of production of Home firms is therefore proportional to P_t in Home currency and P_t/S_t in Foreign currency. Similarly, the marginal costs of Foreign firms is P^*_t in Foreign currency and $S_tP^*_t$ in Home currency. Given the cost of price changes, optimal prices depend on both last period's price and the marginal cost. Prices (in logs) set by Home and Foreign firms in the Home and Foreign country are

$$p_{Ht} = (1-\kappa)p_{H,t-1} + \kappa p_t \qquad p^*_{Ht} = (1-\kappa)p^*_{H,t-1} + \kappa(p_t - s_t) \quad (16)$$

$$p_{Ft} = (1-\kappa)p_{F,t-1} + \kappa(p^*_t + s_t) \qquad p^*_{Ft} = (1-\kappa)p^*_{F,t-1} + \kappa p^*_t \quad (17)$$

Here κ depends on the cost of price adjustment, with $\kappa = 1$ implying flexible prices and $\kappa = 0$ perfectly sticky prices. Consumer price indices are

$$p_t = \varphi p_{Ht} + (1-\varphi)p_{Ft} \qquad p^*_t = (1-\varphi)p^*_{Ht} + \varphi p^*_{Ft} \quad (18)$$

Combining the price relationships (16)-(18), we arrive at

$$\pi^D_t = -\frac{\omega}{1-\omega} q_t \quad (19)$$

where $\pi^D_t = \pi^*_t - \pi_t$ is Foreign minus Home inflation and $\omega = 2(1-\varphi)\kappa/[1+(1-2\varphi)\kappa]$. ω is 1 when prices are perfectly flexible and 0 when perfectly sticky. A nominal appreciation of the Foreign currency (rise in s_t) leads to a real Foreign currency appreciation (rise in q_t). This raises the marginal cost of production in the Foreign country relative to the Home country, which lowers the price of imported goods in the Foreign country relative to the Home country. This reduces Foreign relative to Home inflation, stabilizing the real exchange rate, and more so the more flexible prices are. In the extreme where prices are perfectly flexible, the real exchange rate remains constant: $q_t = 0$.

[11] We have omitted leisure from (1) as it plays no role for portfolio choice, but it is included in an expanded utility function in the Online Appendix.

Now consider the following monetary policy:

$$i_t^D = \phi \pi_t^D + \nu_t \tag{20}$$

where ν_t captures exogenous monetary policy shocks. We need to impose the following condition on ϕ to ensure a unique solution:

$$\phi > 1 - \frac{1-\omega}{\omega}\gamma\sigma^2 b$$

In the case of flexible prices, where $\omega = 1$, this implies $\phi > 1$, the familiar Taylor principle. In addition assume AR(1) processes for monetary and financial shocks:

$$\nu_t = \rho \nu_{t-1} + \epsilon_t \tag{21}$$

$$\tau_t^D = \rho_\tau \tau_{t-1}^D + \epsilon_t^\tau \tag{22}$$

where monetary policy shocks ϵ_t are assumed to be uncorrelated with financial shocks ϵ_t^τ.

Substituting (19), (20) and $r_t^D = i_t^D - E_t \pi_{t+1}^D$ into (12) we get a second-order difference equation in the relative inflation rate:

$$E_t \pi_{t+1}^D - ((1-\omega)\theta + \omega\phi)\pi_t^D + b\psi(1-\omega)\pi_{t-1}^D = \omega\nu_t + 0.5\omega\tau_t^D \tag{23}$$

We can then solve for π_t^D, which also gives us the solution for i_t^D, r_t^D and q_t. Defining δ_1 and δ_2 as the unstable and stable characteristic roots of (23), the solutions for the real exchange rate and real interest rate are

$$q_t = \delta_2 q_{t-1} + \frac{1-\omega}{\delta_1 - \rho}\nu_t + 0.5\frac{1-\omega}{\delta_1 - \rho_\tau}\tau_t^D \tag{24}$$

$$r_t^D = (\phi - \delta_2)\delta_2 \pi_{t-1}^D + \frac{\delta_1 - \rho + (\rho + \delta_2 - \phi)\omega}{\delta_1 - \rho}\nu_t + 0.5\frac{\rho_\tau + \delta_2 - \phi}{\delta_1 - \rho_\tau}\omega\tau_t^D \tag{25}$$

Clearly, the real exchange rate and real interest rate are in general both affected by monetary policy shocks and financial shocks.

When prices are perfectly flexible ($\omega = 1$), we have $q_t = 0$ and $r_t^D = -0.5\tau_t^D$. When prices are perfectly sticky ($\omega = 0$), $\delta_1 = D$ and $\delta_2 = \alpha$ and we have

$$q_t = \alpha q_{t-1} + \frac{1}{D - \rho}\nu_t + 0.5\frac{1}{D - \rho_\tau}\tau_t^D \tag{26}$$

$$r_t^D = i_t^D \tag{27}$$

In this case the real interest differential is equal to the nominal interest differential, which is exogenously controlled through monetary policy, so that

$$r_t^D = \rho r_{t-1}^D + \epsilon_t \tag{28}$$

In what follows we assume complete price stickiness, so that $\omega = 0$ and (26) is the solution for the real exchange rate. This means that the real interest rate is controlled entirely through monetary policy. This conforms with the reality that real and nominal exchange rates are virtually indistinguishable at short to medium horizons.[12] The same results apply when we assume partial price stickiness ($0 < \omega < 1$) if we assume that central banks target the real interest differential in (28) instead of following monetary policy (20). In this case the inflation differential π_t^D is equal to $-\omega q_t/(1-\omega)$ from (19). For given real exchange rate fluctuations, relative inflation is smaller the stickier prices are (lower ω).

The financial shocks τ_t^D play a limited role in our analysis. Our assumption (28) implies that the real interest differential is not affected by financial shocks. They therefore do not directly impact excess return predictability by interest differentials. However, they do affect uncertainty σ^2 of the nominal exchange rate, which affects portfolio choice. This affects the second-order difference equation of q_t through θ. We will assume that financial shocks are large enough to match the observed exchange rate volatility in the data.

2.4 Excess Return Predictability Coefficients

Consider the following regression:

$$er_{t+k} = \alpha + \beta_k r_t^D + \varepsilon_{t+k}^{er} \tag{29}$$

Several of the puzzles are related to the excess return predictability coefficients β_k. The coefficient β_k tells us the effect of the current real interest differential on the expected excess return k periods from now. The forward discount puzzle focuses on $k = 1$, with one period usually being a month or a quarter. For the predictability reversal puzzle and the Engel puzzle we are also interested in β_k for

[12] Another drawback with assuming $\omega > 0$ is that a financial shock that leads to a higher nominal interest rate differential (a drop in τ_t^D) will lead to a Foreign currency nominal depreciation (drop in s_t). In the data higher interest rates are usually associated with a currency appreciation.

$k > 1$, which relates to the effect of the current interest differential on the excess return further into the future.

In the model, the value of β_k is equal to

$$\beta_k = \frac{cov(er_{t+k}, r_t^D)}{var(r_t^D)} \tag{30}$$

Using the solution for the real exchange rate under the assumed AR(1) process for the real interest differential, Appendix C shows that this can be written as[13]

$$\beta_k = \begin{cases} \lambda_1 \rho^{k-1} + \lambda_2 \alpha^{k-1} & \text{if } \alpha \neq \rho \\ \frac{\rho^{k-1}}{D-\rho}\left(D - \frac{1}{1+\rho} - (1-\rho)(k-1)\right) & \text{if } \alpha = \rho \end{cases} \tag{31}$$

where

$$\lambda_1 = \frac{1}{D-\rho}\left(D - \rho\frac{\alpha-1}{\alpha-\rho}\right) \tag{32}$$

$$\lambda_2 = \frac{\alpha-1}{D-\rho}\left[\frac{\rho}{\alpha-\rho} + \frac{1}{1-\alpha\rho}\right] \tag{33}$$

Lemma 2 in Appendix F characterizes the signs of λ_1 and λ_2. Both are positive for low values of ψ, but turn negative as ψ increases.

2.5 Numerical Illustrations

We provide numerical illustrations for each of the puzzles. We calibrate the parameters as follows. Parameters other than γ and ψ are calibrated to interest rates and exchange rates of the remaining G-7 countries relative to the United States (as in Engel, 2016). The real interest rate is computed as the monthly nominal interest rate minus the expected monthly inflation rate (estimated from annual inflation). We find $\rho = 0.9415$.[14] The standard deviation σ of the monthly excess return is computed as the average standard deviation of the monthly change in

[13] β_k is a continuous function of α (and therefore of ψ), but λ_1 and λ_2 are not defined at $\alpha = \rho$, which is why the expression for β_k at $\alpha = \rho$ is reported separately.

[14] The average standard deviation of the relative real interest rate innovation is 0.000342. This is only used in the impulse response of the real exchange rate to a one standard deviation interest rate shock in Figure 1. It does not affect any of the other results.

the real exchange rate, which is 0.0271.[15] We set $b = 0.085$ based on the average home bias for G-7 countries during Q2, 2017.[16] Details regarding the data for this calibration can be found in Appendix A.

We consider a wide range of values for ψ and γ, with ψ ranging from 0 to 20 and γ from 10 to 100. Somewhat arbitrarily, we set ψ and γ at respectively 15 and 50 in the benchmark. We should point out $\psi = 15$ does not imply a high welfare cost. With a two standard deviation relative interest rate shock, the welfare loss from the portfolio friction is equivalent to only a 0.005 percent drop in consumption.

3 Explaining Five Puzzles

We now use the simple model introduced above to address the first five puzzles. We do so by discussing a series of propositions and provide numerical illustrations. When describing the intuition behind the results, we will always consider an increase in the relative *Foreign* interest rate (rise in r_t^D), which leads to an appreciation of the *Foreign* currency (rise in q_t). We will always refer to the Foreign currency, so a depreciation refers to a Foreign depreciation or drop in q_t.

3.1 Delayed Overshooting Puzzle

First define

$$\bar{t} = \begin{cases} \dfrac{ln(1-\rho) - ln(1-\alpha)}{ln(\alpha) - ln(\rho)} & \text{if } \alpha \neq \rho \\ \dfrac{\rho}{1-\rho} & \text{if } \alpha = \rho \end{cases} \tag{34}$$

[15]As discussed, we can set the standard deviation of the financial shocks to match the observed standard deviation of the change in the real exchange rate. When we do this, we find that under the benchmark parameterization the interest rate shocks account for only 1 percent of the variance of real exchange rate changes. This is consistent with the well known disconnect between exchange rates and macro fundamentals. It also connects closely to Itskhoki and Mukhin (2017), who also explain exchange rate disconnect through financial shocks that are the main driver of exchange rate volatility.

[16]We combine BIS data on debt securities outstanding with external assets and liabilities for debt securities from the IMF International Investment Position Statistics. For each country we compute home bias as 1 minus the fraction invested abroad divided by the fraction of the rest of the world in the world supply of debt securities. The average home bias is 0.66. In a two-country model this home bias is $1 - 2\bar{z}_H$, implying a steady state fraction invested abroad of $\bar{z}_H$ of 0.17. Since $b = 0.5\bar{z}_H$, this gives $b = 0.085$.

Appendix D proves the following proposition:

Proposition 1. *Consider the impulse response of the real exchange rate to a positive shock to the relative Foreign interest rate r_t^D.*

- *if $\alpha < 1 - \rho$: the real exchange rate appreciates at the time of the shock and subsequently gradually depreciates back to the steady state.*
- *if $\alpha > 1 - \rho$: there is delayed overshooting. The real exchange rate appreciates at the time of the shock and keeps appreciating until time $\bar{t} > 1$. Then it gradually depreciates back to the steady state.*

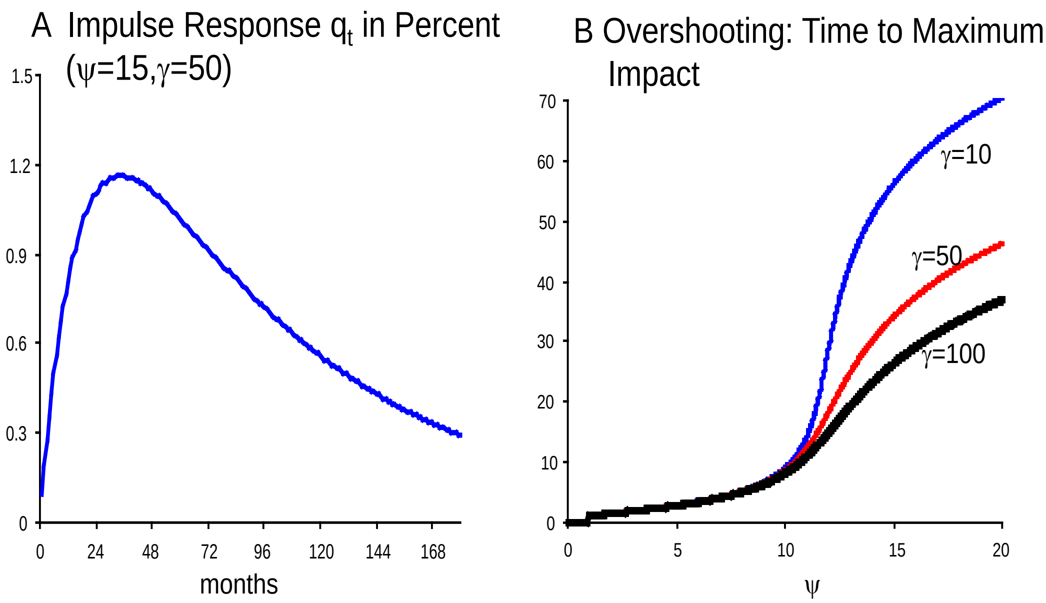
Since Lemma 1 tells us that α rises from 0 to 1 as we raise the gradual portfolio adjustment parameter ψ, Proposition 1 implies that for sufficiently large ψ, and assuming $\rho > 0$, there is delayed overshooting of the type reported by Eichenbaum and Evans (1995) and others. They show that after a monetary policy tightening, the currency continues to appreciate for another 25-39 months before it starts to depreciate. With less gradual adjustment, such that $\alpha < 1 - \rho$, there is no delayed overshooting.

To understand the intuition, consider an increase in the Foreign interest rate. There will be an immediate appreciation of the Foreign currency as investors shift to Foreign bonds. Subsequent to the shock, there are two opposing forces at work. On the one hand, the Foreign interest rate gradually declines again, which leads to a shift away from Foreign bonds and a gradual depreciation. On the other hand, to the extent that portfolios are slow to adjust, there will be a continued flow towards Foreign bonds, which leads to a continued appreciation. When ψ is sufficiently large, the second force dominates and there will be delayed overshooting.

Expression (34) indicates how long the real appreciation will last in the case of delayed overshooting. Appendix D shows that the derivative of $\bar{t}$ with respect to α is positive. A larger gradual portfolio adjustment parameter ψ, which raises α (Lemma 1), will then lead to a longer duration of the delayed overshooting. In the extreme case where α approaches 1, $\bar{t}$ approaches infinity.

Figure 1 provides a numerical illustration. The chart on the left shows the impulse response of the real exchange rate under the benchmark parameterization. The chart on the right shows the time to maximum overshooting for ψ varying from 0 to 20 and γ taking on the values 10, 50 and 100.

Figure 1: IMPULSE RESPONSE q_t AND DELAYED OVERSHOOTING



*N*otes: Panel A shows the response of q_t to a one standard deviation increase in r_t^D. Panel B shows the number of months for q_t to reach its maximum for different values of ψ and γ.

Chart A of Figure 1 shows that the real exchange rate overshoots, reaching a maximum after 35 months. This is consistent with the results in Eichenbaum and Evans (1995). Chart B shows that except for very small values of ψ, the model implies delayed overshooting. Consistent with Proposition 1, the time to maximum impact rises significantly with ψ. It is also larger the lower the rate of risk-aversion. Both of these effects are associated with *portfolio persistence.* A higher ψ and lower γ raise the persistence of the portfolio response. The more gradual portfolio response leads to a more gradual appreciation, which increases the time $\bar{t}$ to maximum overshooting.

3.2 Forward Discount Puzzle

While UIP implies that the Fama coefficient β_1 is zero, empirical evidence typically finds a positive number. Proposition 2 characterizes the sign of β_1 in the model:

Proposition 2. *The Fama predictability coefficient β_1 is positive, and larger when there is gradual portfolio adjustment ($\psi > 0$).*

The proof is given in Appendix E. Since $\beta_1 > 0$, a positive excess return is expected on the high interest rate currency, consistent with the forward discount puzzle. Moreover, Proposition 2 says that β_1 is larger when we introduce a cost of adjusting portfolios ($\psi > 0$). Even without this cost, there is some excess return predictability in the model through a risk premium channel.[17] But β_1 is always less than 1 without portfolio adjustment costs, in contrast to most empirical evidence.[18]

Proposition 1 on delayed overshooting is a useful starting point to understand the role of gradual portfolio adjustment in accounting for the forward discount puzzle. When $\alpha > 1 - \rho$, so that there is delayed overshooting, the Foreign currency is expected to continue to appreciate after the initial appreciation at the time of the shock. The Foreign currency will then have a positive expected excess return both due to the higher interest rate and the expected appreciation. Therefore the portfolio adjustment parameter ψ, which causes a gradual portfolio shift to the Foreign currency that leads to continued appreciation, increases the Fama predictability coefficient β_1.[19]

Under the benchmark parameterization the excess return predictability coefficient β_1 is equal to 3.26. Figure 2 shows how β_1 varies with ψ and γ. It rises until ψ is about 12 and then gradually declines. When ψ is low, *portfolio persistence* is weak, leading to less delayed overshooting and less excess return predictability. On the other hand, when ψ is very high, *return sensitivity* is weak. Agents then respond very little to changes in expected returns, so that the real exchange rate does not change much. This also weakens excess return predictability because the strength of the appreciation after the initial shock is weak. Therefore the predictability coefficient β_1 is largest for an intermediate value of ψ.

Figure 2 also shows that the excess return predictability coefficient β_1 is larger when risk aversion γ is smaller. A smaller γ increases both *portfolio persistence* and *return sensitivity*, both of which lead to a larger appreciation subsequent to

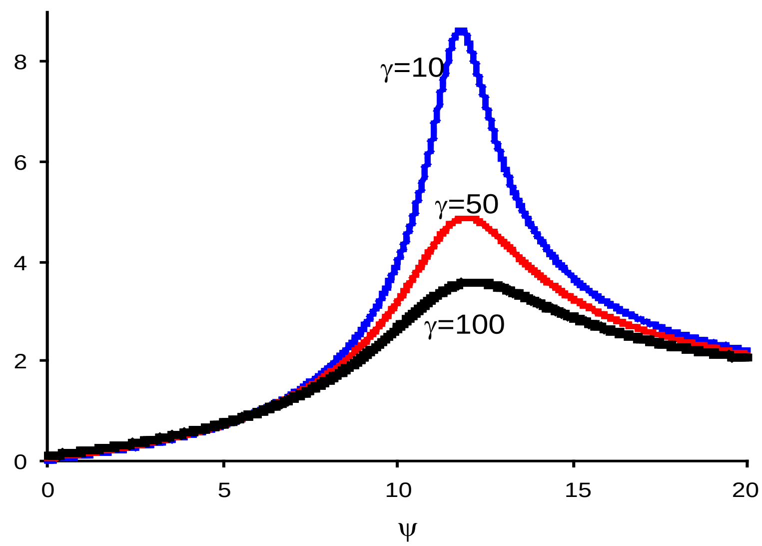
[17] Specifically, a higher Foreign real interest rate leads to a real appreciation of the Foreign currency, which increases the relative value of the Foreign bond supply. To invest a larger portfolio share in Foreign bonds, investors demand a positive expected excess return on the Foreign bond.

[18] When $\psi = 0$, we have $\beta_1 = \gamma\sigma^2 b/(\gamma\sigma^2 b + 1 - \rho)$.

[19] Even when $\alpha < 1 - \rho$, so that there is no delayed overshooting, gradual portfolio adjustment leads to a higher Fama coefficient β_1 because the rate of depreciation subsequent to the shock is smaller due to gradual portfolio adjustment. The weaker subsequent depreciation implies a higher expected excess return on the Foreign currency and therefore a larger Fama coefficient β_1.

the initial shock that enhances predictability.

Figure 2: Forward Discount Puzzle: Predictability Coefficient β_1



3.3 Predictability Reversal Puzzle

Define $\bar{\psi} = \rho\gamma\sigma^2/(1-\rho)$. Excess return predictability at longer horizons, measured by β_k, is described in the following proposition:

Proposition 3. *The following holds for* β_k*:*

- *if* $\psi \leq \bar{\psi}$*:* β_k *is positive for all* k *and drops monotonically to zero as* $k \to \infty$*.*
- *if* $\psi > \bar{\psi}$*: there is a* $\bar{k} > 1$ *such that* β_k *is positive for* $k < \bar{k}$ *and negative for* $k \geq \bar{k}$*. It converges to zero as* $k \to \infty$*.*

The proof is given in Appendix F. Proposition 3 implies that when the gradual adjustment parameter is low, the Foreign currency continues to have positive expected excess returns in all future periods, although the predictability β_k vanishes to zero over time. But when the gradual adjustment parameter is sufficiently high ($\psi > \bar{\psi}$), there will be a predictability reversal. While initially, after the increase in the Foreign interest rate, the Foreign currency is expected to have a positive expected excess return, after a certain period of time it is expected to have a negative excess return. Bacchetta and van Wincoop (2010) first documented this reversal in the sign of predictability for nominal interest rates and exchange rates. They find that a high interest rate currency has a positive expected excess return for about 5-10 quarters, after which it has a negative expected excess return. Engel (2016) reports similar findings for real interest rates and exchange rates.

The excess return on the Foreign currency is driven both by the higher Foreign interest rate and the change in the value of the Foreign currency. Under delayed overshooting the Foreign currency will at first appreciate and therefore have a positive excess return. But after time $\bar{t}$ it will start to depreciate, which contributes to a negative excess return. If $\bar{t}$ is large, by the time the Foreign currency starts to depreciate, the interest differential will be small. The excess return is then mainly driven by the Foreign currency depreciation and is therefore negative.[20]

Engel (2016) claims that models with gradual portfolio adjustment cannot account for the predictability reversal. To understand this, we first need to introduce the concept of the UIP exchange rate. From the definition of the excess return we have $q_t = E_t q_{t+1} + r_t^D - E_t er_{t+1}$. Integrating forward and assuming long-run PPP ($lim_{s\to\infty} E_t q_{t+s} = 0$), we get an expression analogous to that in Engel (2016)[21]:

$$q_t = q_t^{IP} - \sum_{i=1}^{\infty} E_t er_{t+i} \tag{35}$$

where

$$q_t^{IP} = \sum_{i=0}^{\infty} E_t r_{t+i}^D \tag{36}$$

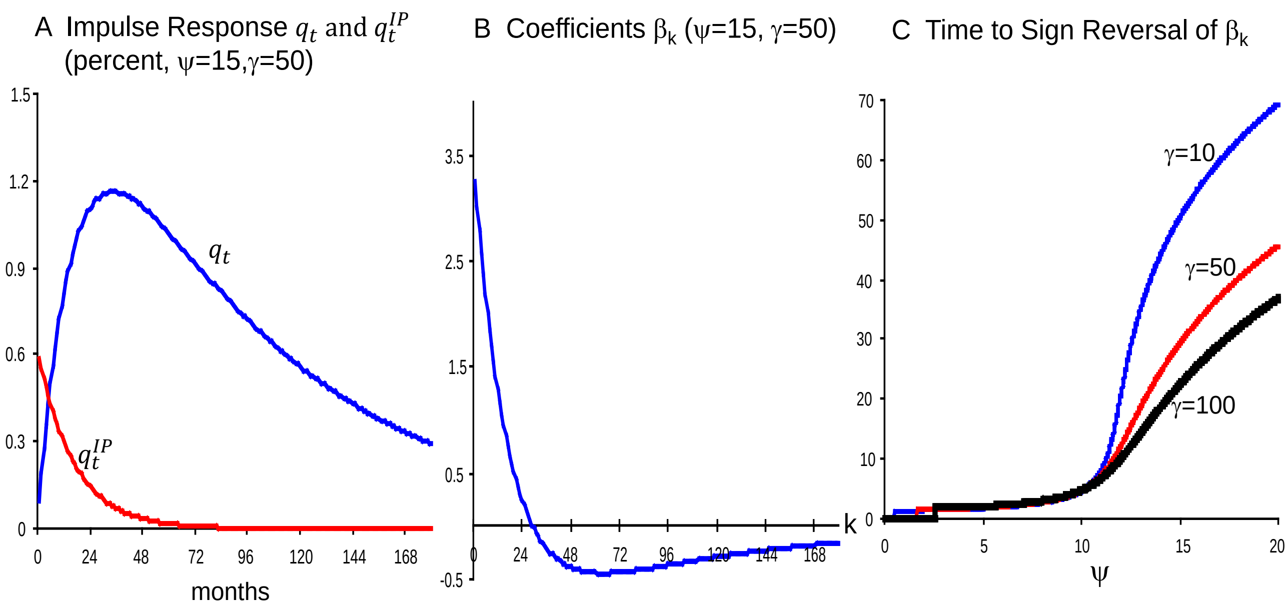
is the UIP exchange rate. It is the real exchange rate when expected future excess returns are zero. When the sum of future expected excess returns is positive, investors demand positive risk premia on the Foreign currency and we see that $q_t < q_t^{IP}$. In other words, the Foreign currency is weak.

Engel conjectures that in response to a rise in r_t^D the Foreign currency appreciates less under gradual portfolio adjustment than the UIP exchange rate and then gradually moves towards the UIP exchange rate. This implies that always $q_t < q_t^{IP}$, so that the sum of subsequent expected excess returns is always positive. In that case there can be no predictability reversal. Figure 3A shows the response of both q_t and q_t^{IP} under the benchmark parameterization. While initially $q_t < q_t^{IP}$ as a result of the weak initial portfolio adjustment, not long after that $q_t > q_t^{IP}$. The latter is consistent with predictability reversal.

[20] Delayed overshooting is not a necessary condition for predictability reversal. Dependent on parameters, predictability reversal can also happen when $\alpha + \rho < 1$.

[21] See also Dahlquist and Penasse (2017).

Figure 3: Sign Reversal of Predictability Coefficient β_k



Notes: Panel A shows the response of q_t and q_t^{IP} to a one standard deviation increase in r_t^D. Panel B shows the regression parameter β_k for different levels of k. Panel C shows the level of k for which β_k turns negative for different values of ψ and γ.

Figure 3B reports β_k for k from 1 to 180 for the benchmark case. The reversal of the predictability coefficient from positive to negative occurs after 30 months.[22] This is not too far from the reversal after 5-10 quarters reported in Bacchetta and van Wincoop (2010). It is also consistent with results reported in Engel (2016).[23]

Figure 3C considers the impact of ψ and γ on the time k where β_k reverses sign

[22]The Online Appendix discusses the implications for the national intertemporal budget constraint, which says that the net external debt is equal to the present discounted value of future trade surpluses plus gross external assets times the present value of the excess return. The net external debt is always zero in the model. A rise in r_t^D leads to a Foreign currency appreciation that raises wealth and consumption in the Home country, leading to a Home trade deficit. But this is paid for by a higher return on external assets (Foreign bonds) than external liabilities (Home bonds) until the time of predictability reversal.

[23]Engel (2016) reports results of regressions of both the ex-post and ex-ante excess return on the interest differential. The ex-ante excess return relies on a VAR to compute expected returns and delivers a somewhat shorter time to reversal of about 12 months on average.

from positive to negative. This rises with a higher ψ and lower γ. Both enhance the *portfolio persistence*, which leads to a later date $\bar{t}$ of maximum overshooting. A longer period of appreciation after the shock delays the predictability reversal.

3.4 Engel Puzzle

The Engel puzzle says that high interest rate currencies tend to be strong relative to the UIP exchange rate. More formally:

$$cov(q_t - q_t^{IP}, r_t^D) > 0 \tag{37}$$

Engel (2016) provides evidence that this condition holds in the data for 6 currencies. We will refer to it as the Engel condition. Using (35), we can also write it as

$$\sum_{k=1}^{\infty} \beta_k < 0 \tag{38}$$

which is an equivalence used by Engel (2016) as well. Predictability reversal is a necessary condition for this to hold, so that $\psi > \bar{\psi}$ is a necessary, but not a sufficient condition. Negative expected excess returns on the Foreign currency for $k \geq \bar{k}$ must more than offset the positive expected excess returns when $k < \bar{k}$.

Define (ψ_1^E, ψ_2^E) as positive values of ψ, with $\psi_1^E < \psi_2^E$, where $cov(q_t - q_t^{IP}, r_t^D) = 0$. Appendix G describes these values and proves the following proposition:

Proposition 4. *Necessary and sufficient conditions for the Engel condition to hold are*

1. $\psi_1^E < \psi < \psi_2^E$.
2. $\gamma\sigma^2 b < \frac{1-\rho}{\rho}\left(1 - \sqrt{1-\rho}\right)^2$.

Proposition 4 imposes several restrictions on parameters for the Engel condition to be satisfied. While the conditions may seem restrictive, we will see that they will hold under a broad range of parameters.

Proposition 4 tells us that the Engel condition is satisfied for intermediate values of ψ, for risk aversion γ that is not too large and for interest rate persistence ρ that is not too close to 0 or 1. The role of ψ and γ can again be related to *portfolio persistence* and *return sensitivity*. A very low value of ψ implies weak *portfolio persistence*. The lack of appreciation after the initial shock (or weak appreciation) implies that $q_t - q_t^{IP}$ will remain negative or not become very positive. On the other hand, a very high ψ implies that *return sensitivity* is weak. The portfolio will respond very little to the higher interest rate, so that $q_t - q_t^{IP}$ again remains negative. For intermediate values of ψ we see a significant appreciation after the shock, leading to a sustained positive $q_t - q_t^{IP}$ for high interest rate currencies. When γ is very large, *portfolio persistence* and *return sensitivity* are both weak, so that $q_t - q_t^{IP}$ either remains negative or does not become very positive and the Engel condition does not hold.

Finally consider the persistence ρ of the real interest rate. If the interest differential is very persistent, the Foreign currency continues to experience high interest rates for a very long time, which by itself causes positive excess returns for a long time. This is inconsistent with the Engel condition. On the other hand, when ρ is very small, the real exchange rate does not respond very much. We do not see a sustained appreciation that leads to a large positive $q_t - q_t^{IP}$ after the shock.

Figure 4: Engel Puzzle $\sum_{k=1}^{\infty} \beta_k$

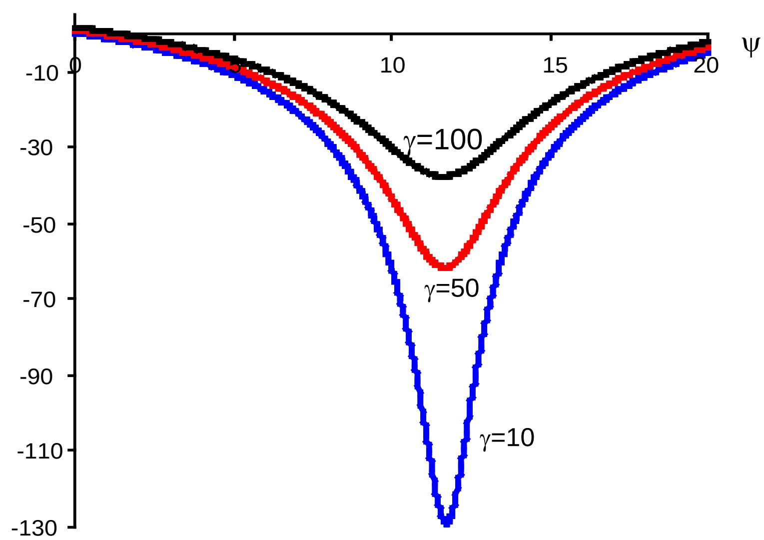


Figure 4 shows that the Engel result holds quite generally in the model as long as ψ is not too close to zero. Consistent with Proposition 4, the Engel result is stronger the lower the rate of risk-aversion γ and peaks for an intermediate value of ψ. For the benchmark parameterization the Engel coefficient $\sum_{k=1}^{\infty} \beta_k$ is equal to -25. This is similar to the estimate in Engel (2016), who finds a -31 coefficient for the G6 average exchange rate against the dollar and an average of -21 for the individual G6 currencies against the dollar.[24]

3.5 Connection Between First Four Puzzles

The puzzles discussed so far are not entirely independent. For example, we often refer back to the delayed overshooting puzzle to explain the other puzzles. Nonetheless it is not the case that they are simply different versions of a single puzzle. This is illustrated in Figure 5. In our benchmark case ($b = 0.085, \sigma = 0.0271, \gamma = 50$), Figure 5 shows the combinations of ψ and ρ that satisfy various conditions: i) delayed overshooting after at least three periods (Proposition 1); ii) $\beta_1 \geq 1$ (Proposition 2);[25] iii) predictability sign reversal after more than three periods (Proposition 3); iv) Engel condition (38) (Proposition 4).

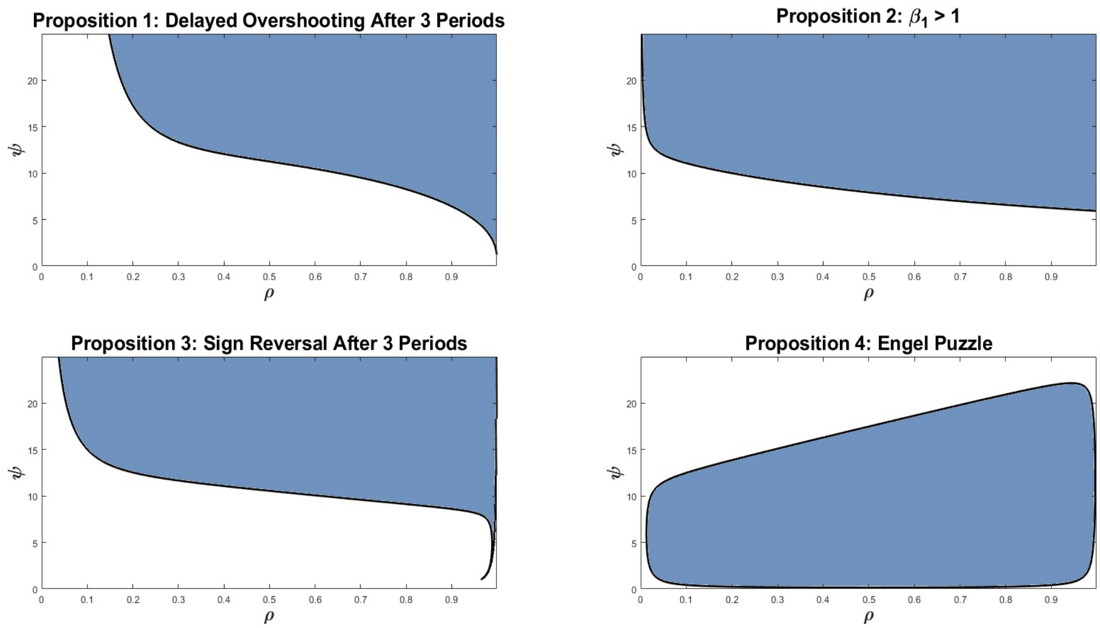
While there is significant overlap between the shaded areas in Figure 5 where the various puzzles are satisfied, there are also significant differences. This confirms that the puzzles are far from identical. With regards to ψ, we can see that it must be large enough for the first three puzzles. But the Engel puzzle is explained only for an intermediate range of ψ. Thus, when ψ is large, the Engel condition may not be satisfied, while the other puzzles are. The persistence of interest differential shocks, ρ, plays an important role for delayed overshooting and predictability reversal. In particular, as Figure 5 illustrates, ρ cannot be too small to have significant delayed overshooting and predictability reversal. But the other two puzzles can be satisfied for very small ρ. In the data, however, we found that $\rho = 0.9415$, so that the four conditions are easily satisfied simultaneously. This is in particular the case for our benchmark of $\psi = 15$.[26]

[24] Engel (2016) also reports a regression of the level of the real exchange rate q_t on r_t^D. The model implies a coefficient of 42 for the benchmark parameterization, which represents the fact that a high interest rate currency tends to be strong. Engel (2016) reports a coefficient of 43.7 when using the G6 average exchange rate against the dollar.

[25] Notice that there is predictability ($\beta_1 \geq 0$) for any value of ψ, but here we focus on a stronger level of predictability which is closer to the data ($\beta_1 \geq 1$).

[26] While our benchmark risk aversion is relatively high ($\gamma = 50$), a lower γ has a small impact on Figure 5 and, if anything, increases dark areas.

Figure 5: VALUES OF ψ AND ρ CONSISTENT WITH PUZZLES 1 TO 4



Notes: The shaded areas represents combinations of ψ and ρ that satisfy the various conditions in our benchmark case ($b = 0.085, \sigma = 0.0271, \gamma = 50$). For Proposition 1 (delayed overshooting), the condition is $\bar{t} \geq 3$. For Proposition 2 (Fama coefficient), it is $\beta_1 \geq 1$. For Proposition 3, the condition is a sign reversal after at least three periods, $k \geq 3$. For Proposition 4 it is equation (38).

3.6 Forward Guidance Exchange Rate Puzzle

The following proposition addresses the forward guidance puzzle posed by Galí (2020):

Proposition 5. *The current real exchange rate q_t depends less on expected interest differentials in the distant future than in the near future. The higher the gradual portfolio adjustment parameter ψ, the less future expected interest differentials affect q_t.*

Proposition 5 follows directly from equation (13) and Lemma 1. Future expected interest differentials are discounted at the rate $1/D$, where D is larger than 1 and rises with ψ.

Under UIP the real exchange rate is given by (36), where there is no discounting. Even when $\psi = 0$, the discount rate $1/D$ is less than 1 when we allow for exchange rate risk, which leads to a deviation from UIP. Specifically, we have $D = 1 + \gamma\sigma^2 b$. But as we shall see, the discount rate $1/D$ is very close to 1 when $\psi = 0$.

To see the role of ψ, assume that we are currently at time t and consider an expected one-period increase in the interest rate differential at $t + k$. The only reason the real exchange rate appreciates prior to $t + k$ is an expectation of subsequent appreciation. The response of q_{t+k-1} to a given higher q_{t+k} is reduced as a result of a positive ψ as portfolios are less sensitive to expected returns. For the same reason the response of q_{t+k-2} to a given expected higher q_{t+k-1} is reduced as a result of the positive ψ. When going back all the way to time t, the response of q_t can be very small when k is large. There are multiple rounds of discounting as each period the real exchange rate response to an expected higher real exchange rate next period is reduced by the positive portfolio adjustment parameter ψ.[27]

The monthly discount rate $1/D$ under our benchmark parameterization is 0.78. Future expected interest rates are therefore heavily discounted. This is consistent with results reported by Galí (2020), which imply that expected interest rates more than two years into the future have an effect on the current real exchange rate that is very small compared to the impact of expected interest rates over the next two years.[28] For comparison, when $\psi = 0$ (holding all other parameters the same), the discount rate is $1/D = 0.997$. In that case the expected interest rate two years into the future has an effect on the current exchange rate that is only 7 percent less than the effect of the current interest rate.

4 Lack of Predictability with Long-term Bonds

LSV show that while there is international excess return predictability for short-term bonds (the forward discount puzzle), this is not the case for long-term bonds. In order to address this last puzzle, we extend the model by introducing long-term

[27] While this broadly captures the intuition, the actual response of the real exchange rate is somewhat complicated by the fact that the real exchange rate not only responds to the expected real exchange rate next period, but also to the lagged real exchange rate.

[28] Galí (2020) regresses q_t on $\sum_{i=0}^{23} E_t r_{t+i}^D$ and $\sum_{i=24}^{\infty} E_t r_{t+i}^D$. We cannot do so in our model as both are proportional to r_t^D and therefore collinear. They would no longer be collinear if we adopted an AR(2) process. More generally, the precise coefficients that we would obtain for a Galí type regression depend on what we assume about the information about future expected interest differentials, which is auxiliary to the gradual portfolio adjustment aspect of the model.

bonds. We discuss the main model assumptions, leaving the solution to the Online Appendix. We instead focus on the results and intuition.

The analysis extends the OLG model of Section 2 to four assets: one-period bonds and long-term bonds in both countries. Agents in the Home country maximize

$$C_{H,t} + ln\left(E_t C_{H,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}} - \frac{1}{4}\psi \sum_{i=1}^{4}\left(z_{H,i,t} - z_{H,i,t-1}\right)^2 \tag{39}$$

For $i = 1, 2, 3, 4$, $z_{H,i,t}$ is the fraction that Home agents invest in respectively Foreign short-term bonds, Foreign long-term bonds, Home long-term bonds and Home short-term bonds.

We again assume local currency price stickiness. Optimal consumption implies again that financial wealth is always 1. Let R^L_{t+1} and $R^{L,*}_{t+1}$ be the real return on Home and Foreign long-term bonds from the perspective of respectively Home and Foreign agents. The gross real interest rates on one-period bonds are R_t and R^*_t. Consumption of Home agents at $t+1$ is equal to the portfolio return:

$$\begin{aligned} C_{H,t+1} \quad = \quad & R_t + z_{H,1,t}\left(\frac{Q_{t+1}}{Q_t}R^*_t e^{-\tau_{H,t}} - R_t\right) + z_{H,2,t}\left(\frac{Q_{t+1}}{Q_t}R^{L,*}_{t+1}e^{-\tau_{H,L,t}} - R_t\right) + \\ & z_{H,3,t}\left(R^L_{t+1} - R_t\right) + T_{t+1} \end{aligned} \tag{40}$$

There are now two time-varying costs of investing abroad: $\tau_{H,t}$ and $\tau_{H,L,t}$ for investing in respectively the Foreign short and long-term bonds. There are analogous costs of investing in Home bonds by Foreign investors. These again lead to financial shocks that are assumed to be large enough to match the observed variance matrix of the excess returns of the first three asset over Home short-term bonds. The aggregate of these costs is reimbursed through T_{t+1}.

Long-term bonds in both countries earn real coupons of κ, $(1-\delta)\kappa$, $(1-\delta)^2\kappa$, and so on. The real returns on Home and Foreign long-term bonds, from the perspective of respectively Home and Foreign agents, are then

$$R^L_{t+1} = \frac{(1-\delta)P^L_{t+1} + \kappa}{P^L_t} \tag{41}$$

$$R^{L,*}_{t+1} = \frac{(1-\delta)P^{L,*}_{t+1} + \kappa}{P^{L,*}_t} \tag{42}$$

Here P^L_t and $P^{L,*}_t$ are the prices of newly issued bonds at time t, measured in real terms from the perspective of Home and Foreign agents.

After solving for the optimal portfolios and imposing market clearing conditions, we solve for the logs of the asset prices, q_t, p_t^L and $p_t^{L,*}$ as a function of exogenous interest rate shocks. We are specifically interested in q_t and the relative long-term bond price $p_t^L - p_t^{L,*}$, which determine the excess returns of interest. They depend on shocks to the relative interest rate $r_t^D = r_t - r_t^*$, which again follows an AR(1) process.

In numerically implementing this model, our assumptions about ψ, γ, ρ and b are the same as in the benchmark parameterization discussed at the end of Section 2. In addition we set $\delta = 0.0071$, leading to a Macauley duration of the long-term bonds of 99.3 months or 8.3 years. This corresponds closely to LSV, who consider returns on 10-year coupon bonds. These have the same Macauley duration of 8.3 years when the annual interest rate is 4 percent.

There are three excess returns that are linearly related:

$$q_{t+1} - q_t + r_{t+1}^{L,*} - r_t^L = [q_{t+1} - q_t + r_t^* - r_t] + \left[(r_{t+1}^{L,*} - r_t^*) - (r_{t+1}^L - r_t)\right] \quad (43)$$

The excess return on the left is the return on Foreign minus Home long-term bonds. The first excess return on the right is the return on Foreign minus Home short-term bonds. The second excess return on the right is equal to the difference between two domestic excess returns, the excess return of long-term over short-term bonds in the Foreign country relative to the Home country.

Table 1 reports the results when regressing these three monthly excess returns on the current interest differential r_t^D, comparing the model to the moments reported by LSV based on the data. It shows the predictability numbers for the three excess returns in (43). Consistent with LSV, there is very little excess return predictability for long-term bonds in the model. The 0.65 coefficient that LSV find in the data is actually not statistically significant. The model generates excess return predictability for short-term bonds (the Fama coefficient) that is exactly the same as in the LSV data, about 2.0. The difference between the international excess return predictability of long and short-term bonds is accounted for by the last column, which again is very similar in the model and the data. Even though a higher relative interest rate r_t^D raises the expected excess return on Foreign short-term bonds, the excess return of long-term over short-term bonds significantly drops in the Foreign relative to the Home country, offsetting the positive FX excess return when investing in Foreign long-term bonds.

The intuition for the decline in the excess return of long-term over short-term

Table 1: Predictability with Long-term Bonds

Regressions on r_t^D			
	long-term Bond Excess Return $q_{t+1} - q_t + r_{t+1}^{L,*} - r_t^L$	short-term Bond Excess Return $q_{t+1} - q_t + r_t^* - r_t$	Local Bond Return Differential $(r_{t+1}^{L,*} - r_t^*) - (r_{t+1}^L - r_t)$
Benchmark model	0.35	1.99	-1.64
LSV panel estimate	0.65	1.98	-1.34

Note : The table shows the slope coefficient of a regression of the dependent variable on the interest differential r_t^D. The benchmark model is described in the text and the LSV panel estimates are from Lustig, Stathopoulos and Verdelhan (2019), Table 1.

bonds in the Foreign relative to the Home country is as follows. The higher Foreign interest rates causes especially Foreign investors to reallocate their portfolio from Foreign long-term bonds to Foreign short-term bonds (the two assets are closer substitutes for Foreign investors). This lowers the price of Foreign long-term bonds. However, the process of reallocating from Foreign long-term bonds to Foreign short-term bonds continues over time as a result of gradual portfolio adjustment, leading to a continued decline in the relative price of Foreign bonds. This implies a negative excess return of long-term bonds over short-term bonds in the Foreign country. Even though the Foreign currency is appreciating, this is approximately offset by the drop in the relative price of Foreign bonds over time. The net result is that the excess return of Foreign over Home long-term bonds is not much correlated with the interest differential.

5 Extension with Infinitely-Lived Agents

This section extends the two-period OLG structure in Section 2 to a framework where agents have infinite lives. A major implication is that portfolio demand depends on expected excess returns further into the future instead of myopic portfolio demand that only depends on the expected excess return over the next period. The real exchange rate still follows an AR(2) process. Quantitatively, however, the model with infinite lives can explain the puzzles considered in Section 3 only if we limit the information set of agents when forming expectations of future excess returns.

Assume that agents have infinite lives and maximize

$$\frac{C_{H,t}^{1-\gamma}}{1-\gamma} + E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{H,t+s+1}^{1-\gamma}}{1-\gamma} - 0.5\psi(z_{H,t+s} - z_{H,t+s-1})^2 \right) \tag{44}$$

As we will see, this specification implies that when $\beta = 0$ the optimal portfolio and equilibrium real exchange rate is the same as in the OLG setup considered so far. We can then compare to infinite lives where $\beta > 0$.

In the OLG setup, utility depends on consumption in a special way to guarantee a constant level of financial wealth of 1. This is no longer feasible in an infinite horizon setup. Wealth will generally be time varying, depending on portfolio returns and consumption decisions. This affects the equilibrium real exchange rate through portfolio demand. But for comparison to the previous results, we will abstract from this channel and focus on the impact of changes in the interest differential on portfolio demand through expected excess returns.

Home wealth evolves according to

$$W_{H,t+1} = (W_{H,t} - C_{H,t})R_{t+1}^{p,H} \tag{45}$$

with the portfolio return as in (3). Financial shocks are not explicitly introduced. As before, they only affect the variance σ^2 of the exchange rate when added.

Leaving details to Appendix H, one can write the portfolio Euler equation in terms of a second-order difference equation in the optimal portfolio that takes the form[29]

$$\psi\beta E_t(z_{H,t+1} - \bar{z}_H) - \omega(z_{H,t} - \bar{z}_H) + \psi(z_{H,t-1} - \bar{z}_H) + E_t er_{t+1} = 0 \tag{46}$$

where $\omega = \psi(1+\beta) + \gamma\sigma^2$, and the steady state portfolio is

$$\bar{z}_H = \frac{0.5}{\gamma} - \frac{\tau}{\gamma\sigma^2} + \frac{\gamma - 1}{\gamma}\frac{\sigma_{s,p}}{\sigma^2} - \frac{\sigma_{s,\lambda}}{\sigma^2}$$

Here $\sigma_{s,p} = cov(s_{t+1}, \pi_{t+1})$ and $\sigma_{s,\lambda} = cov(s_{t+1}, \lambda_{H,t+1})$, where $\lambda_{H,t} = c_{H,t} - w_{H,t}$ is the log consumption-wealth ratio.[30]

[29] Here we normalize the steady state financial wealth $W_{H,t} - C_{H,t}$ to 1.

[30] The last two terms of $\bar{z}_H$ capture respectively an inflation hedge and a hedge against time variation in expected future portfolio returns, which determine the consumption-wealth ratio.

Solving the second-order difference equation (46), we have

$$z_{H,t} - \bar{z}_H = \eta(z_{H,t-1} - \bar{z}_H) + \frac{\eta}{\psi}\sum_{s=1}^{\infty}(\beta\eta)^{s-1}E_t er_{t+s} \tag{47}$$

where

$$\eta = \frac{\omega - \sqrt{\omega^2 - 4\beta\psi^2}}{2\beta\psi}$$

The optimal portfolio (47) can be compared to the portfolio (7) in the myopic case. In both cases the optimal portfolio depends on the portfolio from the previous period and expected excess returns. But there are now two differences. First, while in the myopic case the optimal portfolio only depends on the expected excess return over the next period, it now depends on the expected present discounted value of all future excess returns, with discount rate $\beta\eta$.[31] This happens as agents wish to smooth their portfolio changes in response to these expected excess returns. Second, when $\beta > 0$ the coefficient η on the lagged portfolio is smaller than in the myopic case. Agents now wish to smooth their current portfolio relative to both past and future portfolios, leading them to give less weight to the past portfolio.

It is interesting to compare (47) with the optimal portfolio in the framework considered by Bacchetta, van Wincoop, and Young (2020). Instead of a cost of changing portfolio shares, they adopt a Calvo-type friction where agents make a new portfolio decision each period with probability p. They show that the aggregate portfolio can be approximated as in (47), where $\eta = 1 - p$ and the discount rate is $\beta(1-p)$. The latter corresponds to (47) when $\eta = 1 - p$.

An analogous solution applies to the Foreign portfolio, so that the average portfolio share becomes

$$z_t^A = 0.5 + \eta(z_{t-1}^A - 0.5) + \frac{\eta}{\psi}\sum_{s=1}^{\infty}(\beta\eta)^{s-1}E_t er_{t+s} \tag{48}$$

When solving for the equilibrium real exchange rate it is easier to go back to the first-order difference equation (46), which for the average portfolio share is

$$\psi\beta E_t\hat{z}_{t+1}^A - \omega\hat{z}_t^A + \psi\hat{z}_{t-1}^A + E_t er_{t+1} = 0 \tag{49}$$

[31] The optimal portfolio depends on expected excess returns beyond next period only if both $\beta > 0$ and $\psi > 0$. When $\psi \to 0$, then $\eta \to 0$, so that the optimal portfolio only depends on the expected excess return over the next period.

where $\hat{z}_t^A = z_t^A - 0.5$.

Define the real Home and Foreign financial wealth at time t as $A_{H,t} = W_{H,t} - C_{H,t}$ and $A_{F,t} = W_{F,t} - C_{F,t}$. In terms of Home purchasing power, the latter is $Q_t A_{F,t}$. If we again assume a real Foreign bond supply of 1, the Foreign bond market equilibrium condition is

$$A_{H,t} z_{H,t} + Q_t A_{F,t} z_{F,t} = Q_t \tag{50}$$

If the steady state real wealth of both countries is also 1, we can linearize this as

$$\hat{z}_t^A = bq_t - 0.5\left(\bar{z}_H \hat{a}_{H,t} + \bar{z}_F \hat{a}_{F,t}\right) \tag{51}$$

As discussed, for comparison to the myopic case we will ignore the second term, which captures the effect of time-varying wealth on portfolio demand.

Substituting $\hat{z}_t^A = bq_t$ into (49), we again get a second-order difference equation in the real exchange rate:

$$\mu E_t q_{t+1} - \zeta q_t + \psi b q_{t-1} + r_t^D = 0 \tag{52}$$

where $\mu = 1 + \psi b \beta$ and $\zeta = 1 + \omega b$. When $\beta = 0$, we have $\mu = 1$ and $\zeta = \theta$, so that (52) corresponds exactly to (12) in the myopic case. The general solution is analogous to before (see (13)), with

$$\alpha = \frac{\zeta - \sqrt{\zeta^2 - 4\psi b \mu}}{2\mu}$$
$$D = \frac{\zeta + \sqrt{\zeta^2 - 4\psi b \mu}}{2\mu}$$

While the solution takes the same form as in the myopic case, the coefficients α and D will be different when $\beta > 0$. The implication for the first four puzzles is illustrated in Table 2, which reports four coefficients. The first (Fama) is the Fama coefficient β_1. The second (Reverse) is the number of months until predictability reversal, so the value of k where β_k turns from positive to negative. The third (Overshoot) is the number of months after the interest rate shock that the real exchange rate reaches its highest level. The fourth (Engel) is the Engel coefficient $\sum_{k=1}^{\infty} \beta_k$. With the exception of β, the table assumes the same parameters as in the benchmark parameterization.

The first column reports the benchmark results discussed in Section 3, where $\beta = 0$. The next column changes β to 0.9966, implying an annual discount rate of 0.96. The remaining two columns will be discussed below.

In order to understand the impact of long horizons, it is useful to go back to the impulse response of the real exchange rate under the benchmark parameterization in Figure 1A. The Foreign currency appreciates at the time of the increase in r_t^D, then continues to appreciate for an additional 34 months, after which it depreciates gradually back to its starting point. If instead $\beta = 0.9966$, agents have an effective horizon of 20 months.[32] Anticipating the overshooting, followed by depreciation of the Foreign currency, investors start selling the Foreign currency much sooner. The exchange rate will then peak much sooner.

Table 2 shows that with $\beta = 0.9966$ the exchange reaches its maximum after 3.9 months, versus 34.1 months in the myopic case. While we can still account for the first three puzzles, the quantitative magnitudes change due to the more limited delayed overshooting. The Fama coefficient drops from 3.26 to 1.08. Predictability reversal now happens after 7.9 months instead of 29.3 months. We can no longer account for the Engel puzzle. The Engel coefficient turns slightly positive.

Table 2: Moments with Long Horizons

	Benchmark		RW	Carry
	$\beta = 0$	$\beta = 0.9966$	$\beta = 0.9966$	$\beta = 0.9966$
Fama	3.26	1.08	4.44	4.18
Reverse	29.3	7.9	12.2	12.4
Overshoot	34.1	3.9	18.1	18.1
Engel	-24.9	1.9	-52.0	-46.9

*N*otes: The benchmark parameterization $\psi = 15$, $\gamma = 50$, $\rho = 0.9415$, $\sigma = 0.0271$, $b = 0.085$ is assumed for all columns. Fama is the coefficient β_1. Reverse is k when β_k changes from positive to negative. Overshoot is the months after an interest rate shock that q_t reaches its maximum. Engel is equal to $\sum_{k=1}^{\infty} \beta_k$. The last two columns assume limited information when computing the expectation of future excess returns. RW assumes random walk expectations for the exchange rate, while Carry conditions expectations of all future excess returns on the current r_t^D.

The deterioration of the results with long horizons is not due to the horizon itself, but rather the strong rationality in forming expectations. The agents are aware not just of the current interest rate differential, but of the timing of all shocks that gave rise to it. Expected excess returns then depend on both the current and all past interest differentials. This does not correspond to the observed behavior of

[32]This is computed as $\sum_i (\beta\eta)^i i / \sum_i (\beta\eta)^i = 1/(1-\beta\eta)$.

FX market participants, where for example carry trade is just based on the current interest rate differential. In columns 3 and 4 of Table 2, we assume that exchange rate forecasts are based on limited information, while keeping $\beta = 0.9966$. In column 3 we assume random walk (RW) forecasts. This implies that the one-month ahead expected excess return is simply the interest differential r_t^D. In column 4 we consider a strategy similar to carry trade (Carry), where the information set is limited to the current interest differential and expectations are formed rationally conditional on this information set.

Under RW expectations, (48) implies that

$$\hat{z}_t^A = \eta \hat{z}_{t-1}^A + \frac{\eta}{\psi} \sum_{s=0}^{\infty} (\beta\eta)^s E_t r_{t+s}^D \tag{53}$$

Using the AR process for the interest differential and market equilibrium $\hat{z}_t^A = bq_t$, the real exchange rate is

$$q_t = \eta q_{t-1} + \frac{\eta}{\psi b} \frac{1}{1 - \rho\beta\eta} r_t^D$$

When only the current interest differential r_t^D is used to form expectations, the second term in the portfolio expression (48) that depends on the present discounted value of all expected future excess returns can be written as λr_t^D. In that case, using market equilibrium, the exchange rate becomes

$$q_t = \eta q_{t-1} + \frac{\lambda}{b} r_t^D$$

To make sure that expectations are rational, we compute all excess return predictability coefficients β_s and find λ such that $\lambda = (\eta/\psi) \sum_{s=1}^{\infty} (\beta\eta)^{s-1} \beta_s$.[33]

Table 2 shows that with a long horizon, we can account for all of the first four puzzles with either RW expectations or expectations conditioned on the current interest differential.[34] The Engel coefficient is again significantly negative. Compared to the second column, there is significantly longer delayed overshooting, a

[33]Specifically, we find $b/\lambda = \mu(\eta_1 - \rho) - \rho(\eta_1 - 1)/(\eta_1 - \eta) + (\eta_1 - \rho)(1 - \eta)/[(\eta_1 - \eta)(1 - \eta\rho)]$, where $\eta_1 = 1/(\beta\eta)$, $\mu = \psi b \beta$.

[34]Closely related, in a framework where agents make new portfolio decisions once in two years, Bacchetta and van Wincoop (2010) find that introducing limited information helps in accounting for the forward discount puzzle. They discuss various rationalizations for the use of such limited information. See also Bacchetta and van Wincoop (2007).

higher Fama coefficient and about one year till predictability reversal. We can also account for the Galí puzzle. This is easiest to see under RW expectations. The rate $\beta\eta$ at which future expected interest differentials are discounted in the portfolio expression (53) carries over to the real exchange rate. This discount rate is equal to 0.95. This means that an interest differential one year from now gets about half the weight as the interest differential today.

6 Conclusion

We have explored the implications of delayed portfolio adjustment for exchange rate dynamics. We have shown that when adjustment is sufficiently gradual it can solve the forward premium puzzle, as suggested by Froot and Thaler (1990). Moreover, it can explain five other puzzles related to the relationship between exchange rates and interest rates. Some of these puzzles are related, though far from identical, while others are quite separate (the Galí and LSV puzzles).

Most of the paper has assumed that investors are short lived, only caring about excess returns over the next period. We have shown that we can still account for the puzzles when agents have infinite lives, although that requires imposing an additional friction in terms of limited information processing. Assuming random walk exchange rate expectations, or expectations that are only conditioned on the current interest differential, is consistent with observed practice (e.g. the carry trade) even by sophisticated traders.

One limitation of our analysis is the absence of direct evidence on the extent of portfolio frictions in the foreign exchange market. It is therefore hard to judge whether the assumed parameters are empirically plausible. This will be an important area for additional research. In contrast to other financial markets, the complexity of the FX market is that it is connected with global trade in financial assets of any type. These trades involve both investors with significant inertia and much more active traders.

While our analysis has focused on short-term excess returns, another interesting direction for future research is to consider returns over longer horizons. This would allow us to study longer term relationships (Chinn and Meredith, 2004), as well as the link between the yield curve and exchange rates. Jointly considering equity prices will be of interest as well.

Appendix

A Data Description

To calibrate the model, we use monthly data for G7 countries over the interval December 1992 to December 2017 (the interval for which all data is available for all countries). Nominal exchange rates are end-of-period from FRED. Prices come from OECD CPI series. Nominal interest rates are end-of-period one-month Eurorates from Datastream. Long-term bond returns come from Benchmark 10Y Datastream Government Total Return Index. The monthly return is computed as $ln(TRI^i_t/TRI^i_{t-1})$ where TRI^i_t is the total return index for country i.

To compute real returns, we compute monthly inflation expectations using a regression of monthly inflation rate on lagged annual inflation. We compute short-term and long-term real return differentials and log real exchange rates for the six countries with respect to the US. We compute the moments of interest for each country pair and take the simple average of these moments.

B Proof of Lemma 1

It is immediate from the definitions of α and D that they are respectively equal to 0 and $1 + \gamma\sigma^2 b$ when $\psi = 0$. To show that they both monotonically rise with ψ, we take their derivatives:

$$\frac{\partial\alpha}{\partial\psi} = \frac{0.5b}{\sqrt{\theta^2 - 4\psi b}}\left(\sqrt{\theta^2 - 4\psi b} - (\theta - 2)\right) \tag{B.1}$$

$$\frac{\partial D}{\partial\psi} = \frac{0.5b}{\sqrt{\theta^2 - 4\psi b}}\left(\sqrt{\theta^2 - 4\psi b} + (\theta - 2)\right) \tag{B.2}$$

It is easy to see that $\sqrt{\theta^2 - 4\psi b}$ is larger than both $\theta - 2$ and $2 - \theta$. This is automatic when these are negative. When they are positive, it follows because $\theta^2 - 4\psi b > (\theta - 2)^2$. The latter can be written as $-\psi b > -\theta + 1$, which holds when substituting $\theta = 1 + \psi b + \gamma\sigma^2 b$.

Next consider the limit of $\psi \to \infty$. We can write

$$\lim_{\psi\to\infty} \alpha = 0.5 \lim_{\psi\to\infty} \frac{1 - \sqrt{1 - \frac{4\psi b}{\theta^2}}}{1/\theta} \tag{B.3}$$

Since both the numerator and denominator approach 0 when $\psi \to \infty$, we can use $L'Hopital's$ rule:

$$\lim_{\psi\to\infty} \alpha = -0.5 \lim_{\psi\to\infty} \frac{0.5(1/\theta^2)(1-\frac{4\psi b}{\theta^2})^{-0.5}(-4b+8\psi b^2/\theta)}{-b/\theta^2} = 1 \tag{B.4}$$

It is immediate that $D \to \infty$ as $\theta \to \infty$ and $D/\theta \to 1$ when $\psi \to \infty$.

C Excess Return Predictability Coefficients

We will now derive the excess return predictability coefficients

$$\beta_k = \frac{cov(er_{t+k}, r_t^D)}{var(r_t^D)} \tag{C.1}$$

From $q_t = \alpha q_{t-1} + \frac{1}{D-\rho} r_t^D$, we have

$$q_t = \frac{1}{D-\rho}\left(r_t^D + \alpha r_{t-1}^D + \alpha^2 r_{t-2}^D + ...\right) \tag{C.2}$$

Therefore

$$\begin{aligned} &er_{t+k} = q_{t+k} - q_{t+k-1} + r_{t+k-1}^D = \\ &\frac{1}{D-\rho} r_{t+k}^D + \frac{1}{D-\rho}(\alpha-1)\left(r_{t+k-1}^D + \alpha r_{t+k-2}^D + \alpha^2 r_{t+k-2}^D + ...\right) + r_{t+k-1}^D \end{aligned} \tag{C.3}$$

Then

$$\begin{aligned} cov(q_{t+k} - q_{t+k-1} + r_{t+k-1}^D, r_t^D) &= \frac{1}{D-\rho}\rho^k var(r_t^D) + \rho^{k-1} var(r_t^D) + \\ \frac{1}{D-\rho}(\alpha-1) var(r_t^D) &\left(\rho^{k-1} + \alpha\rho^{k-2} + ... + \alpha^{k-2}\rho + \frac{\alpha^{k-1}}{1-\alpha\rho}\right) \end{aligned} \tag{C.4}$$

It follows that

$$\begin{aligned} \beta_k &= \frac{1}{D-\rho}\rho^k + \rho^{k-1} + \\ &\frac{1}{D-\rho}(\alpha-1)\left(\rho^{k-1} + \alpha\rho^{k-2} + ... + \alpha^{k-2}\rho + \frac{\alpha^{k-1}}{1-\alpha\rho}\right) \end{aligned} \tag{C.5}$$

Consider the last term, but not including the ratio at the end of the large bracketed term. We can rewrite this as

$$\frac{\alpha-1}{D-\rho}\alpha^{k-1}\left(\left(\frac{\rho}{\alpha}\right)^{k-1} + ... + \left(\frac{\rho}{\alpha}\right)\right) \tag{C.6}$$

When $\alpha \neq \rho$, we can write it as

$$\frac{\alpha-1}{D-\rho}\alpha^{k-1}\frac{\left(\frac{\rho}{\alpha}\right)-\left(\frac{\rho}{\alpha}\right)^k}{1-\left(\frac{\rho}{\alpha}\right)} \tag{C.7}$$

which can be written as

$$\frac{\alpha-1}{D-\rho}\frac{\rho}{\alpha-\rho}\alpha^{k-1}-\frac{\alpha-1}{D-\rho}\frac{1}{\alpha-\rho}\rho^k \tag{C.8}$$

Adding to this the remaining terms of (C.5), we obtain the expression (31) for β_k in the text when $\alpha \neq \rho$. When $\alpha = \rho$, (C.6) is equal to

$$\frac{\alpha-1}{D-\rho}(k-1)\alpha^{k-1} \tag{C.9}$$

Adding to this the remaining terms of (C.5), we obtain the expression (31) for β_k when $\alpha = \rho$.

D Proof of Proposition 1

From $q_t = \alpha q_{t-1} + r_t^D/(D-\rho)$ we can write q_t as a function of current and past real interest rate shocks:

$$q_t = \frac{1}{D-\rho}\sum_{i=0}^{\infty}\nu_i\varepsilon_{t-i} \tag{D.1}$$

where

$$\nu_i = \begin{cases} \dfrac{\alpha^{i+1}-\rho^{i+1}}{\alpha-\rho} & \text{if } \alpha \neq \rho \\ (i+1)\rho^i & \text{if } \alpha = \rho \end{cases} \tag{D.2}$$

First assume $\alpha \neq \rho$. If the interest rate shock starts at time $t = 0$, and we normalize the shock to $D - \rho > 0$ without loss of generality, it implies that in response to this shock

$$q_t - q_{t-1} = \nu_t - \nu_{t-1} = \frac{(1-\rho)\rho^t - (1-\alpha)\alpha^t}{\alpha-\rho} \tag{D.3}$$

This implies that $q_1 - q_0 = \alpha + \rho - 1$. More generally, $q_t < q_{t-1}$ when

$$t > \bar{t} = \frac{ln(1-\rho) - ln(1-\alpha)}{ln(\alpha) - ln(\rho)} \tag{D.4}$$

while $q_t > q_{t-1}$ when $t < \bar{t}$. Below we show that $\partial\bar{t}/\partial\alpha > 0$. Since $\bar{t} = 1$ when $\alpha = 1 - \rho$, it follows that $\bar{t} < 1$ when $\alpha + \rho < 1$. The condition (D.4) is therefore satisfied for all $t \geq 1$, so that $q_t < q_{t-1}$ for all $t \geq 1$. This proves the first part of Proposition 1. When $\alpha + \rho > 1$, $\partial\bar{t}/\partial\alpha > 0$ implies that $\bar{t} > 1$. Therefore the real exchange rate continues to appreciate for at least one additional period after the shock ($t = 1$), and will start to depreciate once $t > \bar{t} > 1$. Finally, when $\alpha = \rho$, we have $q_t - q_{t-1} = \rho^{t-1}(\rho - (1-\rho)t)$ and the same results as those above apply with $\bar{t} = \rho/(1-\rho)$. In this case $\alpha + \rho < 1$ corresponds to $\rho < 0.5$, where $\bar{t} < 1$, and $\alpha + \rho > 1$ implies $\rho > 0.5$, so that $\bar{t} > 1$.

It remains to show that $\partial\bar{t}/\partial\alpha > 0$ when $\alpha \neq \rho$. We have

$$\frac{\partial\bar{t}}{\partial\alpha} = \frac{1}{\alpha(1-\alpha)}\frac{\alpha(ln\alpha - ln\rho) + (1-\alpha)(ln(1-\alpha) - ln(1-\rho))}{[ln(\alpha/\rho)]^2} \tag{D.5}$$

The sign is determined by the numerator in the large fraction. Note that it is positive for $\alpha = 0$ and $\alpha = 1$. The derivative of the numerator with respect to α is $ln(\alpha/\rho) - ln(1-\alpha)/(1-\rho)$, which is positive when $\alpha > \rho$, zero when $\alpha = \rho$ and negative when $\alpha < \rho$. The numerator of the large expression in (D.5) is therefore smallest when $\alpha = \rho$, where it is zero. It is therefore positive for all $\alpha \neq \rho$.

E Proof of Proposition 2

We have

$$\beta_1 = \lambda_1 + \lambda_2 = \frac{1}{D-\rho}\left(D - \frac{1-\alpha}{1-\alpha\rho}\right) = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(D - \alpha D\rho - 1 + \alpha\right) \tag{E.1}$$

Using that $\alpha D = \psi b$ and $\alpha + D = \theta$, we have

$$\beta_1 = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(\theta - 1 - \rho\psi b\right) = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left((1-\rho)\psi + \gamma\sigma^2\right)b > 0 \tag{E.2}$$

Next consider the second part of Proposition 2. When $\psi = 0$, we have $\alpha = 0$, $\theta = 1 + \gamma\sigma^2 b$ and $D = \theta$. The second part of Proposition 2 then holds when

$$\frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(\gamma\sigma^2 + (1-\rho)\psi\right) > \frac{1}{1+\gamma\sigma^2 b - \rho}\gamma\sigma^2 \tag{E.3}$$

This implies

$$(D-\rho)(1-\alpha\rho)\gamma\sigma^2 < (\gamma\sigma^2 + (1-\rho)\psi)(1 + \gamma\sigma^2 b - \rho) \tag{E.4}$$

Collecting terms multiplying $\gamma\sigma^2$ and using $D\alpha = \psi b$, we have

$$(D - \psi b\rho + \alpha\rho^2 - 1 - \gamma\sigma^2 b - (1-\rho)\psi b)\gamma\sigma^2 < (1-\rho)^2\psi \tag{E.5}$$

Using $D = \theta - \alpha = 1 + \psi b + \gamma\sigma^2 b - \alpha$, this becomes

$$-\alpha(1-\rho^2)\gamma\sigma^2 < (1-\rho)^2\psi \tag{E.6}$$

which clearly holds.

F Proof of Proposition 3

It first useful to characterize the signs of λ_1 and λ_2. The value of ψ where $\lambda_1 = 0$ is $\bar{\psi}$ defined in the text. Moreover, the value of ψ where $\alpha = \rho$, is $\bar{\psi} + \rho/b$. We can write the following Lemma:

Lemma 2. *There are three regions that determine the sign of λ_1 and λ_2:*

- $0 < \psi < \bar{\psi}$: $\lambda_1 > 0$ *and* $\lambda_2 > 0$
- $\bar{\psi}_1 < \psi < \bar{\psi} + \rho/b$: $\lambda_1 < 0$ *and* $\lambda_2 > 0$
- $\psi > \bar{\psi} + \rho/b$: $\lambda_1 > 0$ *and* $\lambda_2 < 0$

When $\psi = 0$, $\lambda_1 > 0$ and $\lambda_2 = 0$. When $\psi = \bar{\psi}$, $\lambda_1 = 0$ and $\lambda_2 > 0$.

Proof. First consider λ_1. Since $D - \rho > 0$, the sign is determined by

$$D - \rho\frac{\alpha - 1}{\alpha - \rho} \tag{F.1}$$

$\bar{\psi}$ is defined such that this term is equal to 0. To see this, setting (F.1) equal to zero and substituting the expressions (14) and (15) for α and D, we have

$$\frac{\theta + \sqrt{\theta^2 - 4\psi b}}{2} = \rho\frac{\theta - \sqrt{\theta^2 - 4\psi b} - 2}{\theta - \sqrt{\theta^2 - 4\psi b} - 2\rho} \tag{F.2}$$

Cross multiplying delivers

$$\psi b = \rho\theta - \rho \tag{F.3}$$

Substituting $\theta = 1 + \psi b + \gamma\sigma^2 b$ gives $\psi = (\rho/(1-\rho))\gamma\sigma^2 = \bar{\psi}$.

Now go back to (F.1). It is immediate that this term is positive when $\alpha > \rho$, so that $\lambda_1 > 0$. This happens when $\psi > \bar{\psi} + \rho/b$. So we need to consider $\psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$. Consider D and $\rho(\alpha - 1)/(\alpha - \rho)$ as functions of ψ. It follows from Lemma 1 that both rise monotonically with ψ. At $\psi = 0$, so that $\alpha = 0$, $D > \rho(\alpha - 1)/(\alpha - \rho)$. But $\rho(\alpha - 1)/(\alpha - \rho)$ rises to infinity as α approaches ρ from below, which happens when ψ approaches $\bar{\psi} + \rho/b$ from below. Therefore the schedule for $\rho(\alpha - 1)/(\alpha - \rho)$ must cross that for D between $\psi = 0$ and $\psi = \bar{\psi} + \rho/b$. This happens at $\psi = \bar{\psi}$. It follows that $\lambda_1 > 0$ when $\psi < \bar{\psi}$, $\lambda_1 = 0$ when $\psi = \bar{\psi}$ and $\lambda_1 < 0$ when $\bar{\psi} < \psi < \bar{\psi} + \rho/b$.

Next consider λ_2. It is immediate from (33) that $\lambda_2 < 0$ when $\alpha > \rho$, which happens when $\psi > \bar{\psi} + \rho/b$. So consider $\psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$. (33) then implies that $\lambda_2 > 0$ when $1/(1 - \alpha\rho) < \rho/(\rho - \alpha)$. Cross multiplying, this gives $\alpha > \alpha\rho^2$. This holds as long as $\alpha > 0$ or $\psi > 0$. When $\psi = 0$, $\alpha = 0$ and $\lambda_2 = 0$. □

The first part of Proposition 3 follows immediately from Lemma 2. When $\psi = 0$, we have $\beta_k = \lambda_1 \rho^{k-1}$, which is positive ($\lambda_1 > 0$) and monotonically declines to zero as k rises. When $0 < \psi < \bar{\psi}$, Lemma 2 says that both λ_1 and λ_2 are positive. Since $0 < \alpha < 1$, it follows that $\beta_k = \lambda_1 \rho^{k-1} + \lambda_2 \alpha^{k-1}$ is positive and monotonically declines to zero with an increase in k. Finally, when $\psi = \bar{\psi}$, Lemma 2 implies that $\beta_k = \lambda_2 \alpha^{k-1}$, with $\lambda_2 > 0$ and $0 < \alpha < 1$. It again follows that β_k is positive and declines monotonically to zero as k rises.

Next consider the second part of Proposition 3, where $\psi > \bar{\psi}$. It is immediate from (31) that $\lim_{k \to \infty} \beta_k = 0$. When $\psi \neq \bar{\psi} + \rho/b$, so that $\alpha \neq \rho$, we can write

$$\frac{\beta_k}{\alpha^{k-1}} = \lambda_1 \left(\frac{\rho}{\alpha}\right)^{k-1} + \lambda_2 \tag{F.4}$$

$$\frac{\beta_k}{\rho^{k-1}} = \lambda_2 \left(\frac{\alpha}{\rho}\right)^{k-1} + \lambda_1 \tag{F.5}$$

The sign of β_k corresponds to the sign of either of the two right hand side expressions. Assume first that $\bar{\psi} < \psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$, $\lambda_1 < 0$ and $\lambda_2 > 0$ (Lemma 2). Then (F.4) implies that $\beta_k > 0$ when $k < \bar{k}_1$ and $\beta_k < 0$ when $k > \bar{k}_1$ with

$$\bar{k}_1 = 1 + \frac{ln(-\lambda_2/\lambda_1)}{ln(\rho/\alpha)} \tag{F.6}$$

We know from Proposition 2 that $\beta_1 = \lambda_1 + \lambda_2 > 0$, so that $\lambda_2 > -\lambda_1$, which implies that $\bar{k}_1 > 1$. The $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_1$.

A similar reasoning applies to the case where $\psi > \bar{\psi} + \rho/b$, so that $\alpha > \rho$, $\lambda_1 > 0$ and $\lambda_2 < 0$ (Lemma 2). Then (F.5) implies that $\beta_k > 0$ when $k < \bar{k}_2$ and $\beta_k < 0$ when $k > \bar{k}_2$ with

$$\bar{k}_2 = 1 + \frac{ln(-\lambda_1/\lambda_2)}{ln(\alpha/\rho)} \tag{F.7}$$

From Proposition 2, $\lambda_1 > -\lambda_2$, so that $\bar{k}_2 > 1$. Again the $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_2$.

Finally consider the special case of $\psi = \bar{\psi} + \rho/b$, so that $\alpha = \rho$. In that case (31) implies that $\beta_k > 0$ when $k < \bar{k}_3$ and $\beta_k < 0$ when $k > \bar{k}_3$ with

$$\bar{k}_3 = 1 + \frac{D - (1/(1+\rho))}{1-\rho} > 1 \tag{F.8}$$

Again the $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_3$.

G Proof of Proposition 4

The Engel condition is $\sum_{k=1}^{\infty} \beta_k < 0$. We will focus here on $\alpha \neq \rho$, which is sufficient as the β_k are continuous at $\alpha = \rho$. Then

$$\sum_{k=1}^{\infty} \beta_k = \lambda_1 \frac{1}{1-\rho} + \lambda_2 \frac{1}{1-\alpha} = \frac{1}{1-\rho} - \frac{1}{(D-\rho)(1-\alpha\rho)} \tag{G.1}$$

The Engel condition can therefore be written as $(D-\rho)(1-\alpha\rho) < 1-\rho$. Using that $D\alpha = \psi b$ and $D = \theta - \alpha$, we can also write it as

$$\alpha > \frac{\psi b}{1+\rho} + \frac{\phi}{1-\rho^2} \tag{G.2}$$

where $\phi = \gamma\sigma^2 b$. Using $\theta = 1 + \psi b + \phi$ and the definition of α, this becomes

$$\sqrt{(1+\psi b+\phi)^2 - 4\psi b} < 1 - \frac{(1+\rho^2)\phi}{1-\rho^2} - \frac{1-\rho}{1+\rho} b\psi \tag{G.3}$$

We can, for convenience, refer to the left and right hand sides of (G.3) as $f(\psi)$ and $g(\psi)$. $f(\psi)$ is a convex function, which is always positive and is symmetric around the axis $\psi = (1-\phi)/b$, where it reaches a minimum. $g(\psi)$ is a line with a negative slope. Moreover $f(0) > g(0)$. These properties imply that there are only two possibilities. Either $f(\psi)$ remains above $g(\psi)$ for all ψ and therefore the Engel condition is never satisfied, or $f(\psi)$ crosses $g(\psi)$ twice and the Engel condition is satisfied for an intermediate range of ψ that we will refer to as the interval (ψ_1^E, ψ_2^E), with the boundaries of the interval equal to the solutions to $f(\psi) = g(\psi)$.

To consider the solutions of $f(\psi) = g(\psi)$, we square both sides. We need to be careful doing so. If $f^2(\psi) = g^2(\psi)$ has two solutions, it is either the case that $f(\psi) = g(\psi)$ for both solutions or $f(\psi) = -g(\psi)$ for both solutions. We know that $f(\psi)$ is convex with an axis of symmetry $\psi = (1-\phi)/b$. If it crosses the symmetric $f(\psi)$ twice, there will be two solutions that average to less that $(1-\phi)/b$ since $g(\psi)$ is a negatively sloping line.

We can write $f^2(\psi) = g^2(\psi)$ as

$$A\psi^2 + B\psi + C = 0 \tag{G.4}$$

where

$$A = \rho b^2 \tag{G.5}$$

$$B = b\rho(\phi - 1 - \rho) \tag{G.6}$$

$$C = \frac{\phi}{(1-\rho)^2}\left(1 - \rho^2 - \rho^2\phi\right) \tag{G.7}$$

In order for the Engel condition to be satisfied over some intermediate range (ψ_1^E, ψ_2^E) for ψ, two conditions need to hold. First, as discussed above, it must be the case that the average of these solutions is less than $(1-\phi)/b$, which implies $\phi < 1-\rho$. Second, it must the case that two solutions to $f^2(\psi) = g^2(\psi)$ exist, which requires $B^2 - 4AC > 0$, which can be written as

$$\rho\phi^2 - 2(2-\rho)(1-\rho)\phi + \rho(1-\rho)^2 > 0 \tag{G.8}$$

This is a quadratic that is positive when $\phi = 0$, then turns negative and then positive again. When $\phi = 1-\rho$, the quadratic is negative, so that both $\phi < 1-\rho$ and (G.8) will be satisfied when ϕ is between zero and the smaller of the two solutions to (G.8) as an equality. The latter is equal to

$$\bar{\phi} = \frac{1-\rho}{\rho}\left(1 - \sqrt{1-\rho}\right)^2 \tag{G.9}$$

To summarize, the Engel condition is satisfied if and only if $\phi < \bar{\phi}$ and $\psi_1^E < \psi < \psi_2^E)$, where $\bar{\psi}_1^E$ and ψ_2^E are the solutions to the quadratic (G.4).

H Infinite Lives

To derive the portfolio Euler equation in the infinite lives case, assume that all the additional wealth at $t+1$ due to a change in the portfolio share at t is consumed. This is fine since consumption is optimally chosen intertemporally. The portfolio Euler equation is then

$$\begin{aligned}&E_t C_{H,t+1}^{-\gamma}(W_{H,t}-C_{H,t})\left(e^{s_{t+1}-s_t+i_t^*-\tau-\pi_{t+1}}-e^{i_t-\pi_{t+1}}\right)\\&-\psi(z_{H,t}-z_{H,t-1})+\beta\psi E_t(z_{H,t+1}-z_{H,t})=0\end{aligned}\tag{H.1}$$

Define $\lambda_{H,t}$ as the log consumption-wealth ratio, where financial wealth is $W_{H,t}-C_{H,t}$. Then the first-order condition becomes

$$\begin{aligned}&(W_{H,t}-C_{H,t})^{1-\gamma}E_t\left(e^{-\gamma\lambda_{H,t+1}-\gamma r_{t+1}^{p,H}+er_{t+1}-\tau-\pi_{t+1}}-e^{-\gamma\lambda_{H,t+1}-\gamma r_{t+1}^{p,H}+i_t-\pi_{t+1}}\right)\\&-\psi(z_{H,t}-z_{H,t-1})+\beta\psi E_t(z_{H,t+1}-z_{H,t})=0\end{aligned}\tag{H.2}$$

Using the linear approximation $r_{t+1}^{p,H}=z_{H,t}(er_{t+1}-\tau)+i_t-\pi_{t+1}$ for the portfolio return, the portfolio Euler equation becomes

$$\begin{aligned}&(W_{H,t}-C_{H,t})^{1-\gamma}E_t\left(e^{-\gamma\lambda_{H,t+1}+(1-\gamma z_{H,t})(er_{t+1}-\tau)+(1-\gamma)(i_t-\pi_{t+1})}-e^{-\gamma\lambda_{H,t+1}-\gamma z_{H,t}(er_{t+1}-\tau_{H,t})+(1-\gamma)(i_t-\pi_{t+1})}\right)\\&-\psi(z_{H,t}-z_{H,t-1})+\beta\psi E_t(z_{H,t+1}-z_{H,t})=0\end{aligned}\tag{H.3}$$

Assuming that steady state financial wealth $W_{H,t}-C_{H,t}$ is 1, computing expectations of the exponentials using log normality, and approximating $e^x=1+x$, we can approximate the portfolio Euler equation as

$$E_t er_{t+1}-\gamma(z_{H,t}-\bar{z}_H)\sigma^2-\psi(z_{H,t}-z_{H,t-1})+\beta\psi E_t(z_{H,t+1}-z_{H,t})=0 \tag{H.4}$$

where

$$\bar{z}_H=\frac{0.5}{\gamma}-\frac{\tau}{\gamma\sigma^2}+\frac{\gamma-1}{\gamma}\frac{\sigma_{s,p}}{\sigma^2}-\frac{\sigma_{s,\lambda}}{\sigma^2} \tag{H.5}$$

with $\sigma^2=var(er_{t+1})=var(s_{t+1})$, $\sigma_{s,p}=cov(s_{t+1},\pi_{t+1})$ and $\sigma_{s,\lambda}=cov(s_{t+1},\lambda_{H,t+1})$. We can write this in the form of the following first difference equation in $\hat{z}_{H,t}=z_{H,t}-\bar{z}_H$:

$$\psi\beta E_t\hat{z}_{H,t+1}-\omega\hat{z}_{H,t}+\psi\hat{z}_{H,t-1}+E_t er_{t+1}=0 \tag{H.6}$$

where $\omega=\psi(1+\beta)+\gamma\sigma^2$. This corresponds to (46) in the text.

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