CBDC as Imperfect Substitute for Bank Deposits: A Macroeconomic Perspective

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Abstract

The impact of Central Bank Digital Currency (CBDC) is analyzed in a small open economy model with monopolistic competition in banking and where CBDC is an imperfect substitute with bank deposits. The design of CBDC is characterized by its interest rate, its substitutability with bank deposits, and its relative liquidity. We examine how interest-bearing CBDC would affect the banking sector, public finance, GDP and welfare. Welfare may improve through three channels: seigniorage; a lower opportunity cost of money; and a redistribution away from bank owners. In our numerical analysis we find a maximum welfare improvement of 60 bps in consumption terms.

1 Introduction

As our economies are becoming increasingly digital, central banks around the world are exploring the possibility of issuing central bank digital currency (CBDC). Since there are various ways to implement CBDCs, it important to understand its implications. For example, CBDC could mainly substitute cash, which would have little impact on financial intermediation. Alternatively, it could substitute checking deposits and could lead to banking disintermediation. Although a growing literature is exploring the macroeconomic implications of CBDC, our understanding is still limited.[1] Under some conditions, CBDC leaves economic outcomes unchanged, as shown in Brunnermeier and Niepelt (2019). In contrast, other studies show that the disintermediation implied by CBDCs would reduce bank loans and possibly output (see Keister and Sanches (2021) or Chiu et al. (2021)), while Barrdear and Kumhof (2021) predict a large increase in output. Results depend in particular on how easily banks can substitute checking deposits by other types of funding and how substitutable are checking deposits with CBDC. The interest rate on CBDC and the competitive structure of the banking sector may also play significant roles.

The purpose of this paper is to shed light on these issues by analyzing the impact of CBDC in a standard open-economy model with monopolistic competition in banking, where CBDC and bank deposits are imperfect substitutes. The design of CBDC is characterized by its interest rate, its substitutability with bank deposits, and its relative liquidity. In this environment, we examine how different design choices for CBDC would affect the banking sector, public finance, GDP and welfare. While bank profits are affected by CBDC design, this does not affect bank funding and banks' ability to extend loans since banks can also borrow in the international capital market. In determining the optimal design, the central bank has to consider seigniorage and the opportunity cost of holding money. For example, a low CBDC interest rate generates higher seigniorage but also a higher opportunity cost. We evaluate these trade-offs and provide quantitative estimates assuming distortionary taxation.

We model imperfect substitutability by assuming that all the different monies contribute to the formation of a composite liquid asset, which is useful to households as it

[1] E.g., see Auer et al. (2021), and Niepelt (2021) for recent surveys of the literature.

reduces the transaction cost of acquiring goods for consumption.[2] Given the interest paid by each type of money, households' demand for each reflects the optimal trade-off between maximizing interest collection and minimizing the transaction cost, given the imperfect substitutability between the different monies.

In this setup, CBDC design involves three dimensions: the interest rate it pays; its liquidity relative to bank deposits – which, in the model, is the weight of CBDC in the formation of the composite liquid asset – and its degree of substitutability with bank deposits. In practice, liquidity may be related to technological aspects of the design, such as the rapidity of payments, or to any fee structure. Substitutability might involve the interoperability between CBDC and bank deposits (see Brunnermeier and Landau (2019) for discussions on this issue), or some characteristics that might differentiate the two monies and make one more suitable than the other in certain circumstances. For example CBDC might be in the form of token, might grant more or less privacy than bank deposits, might be more secure than bank deposits or might for example offer better conditions for international transactions.

While banks choose the interest rate on deposits to maximize their profit given deposit demand, the central bank chooses the interest rate paid by CBDC, the liquidity of CBDC relative to bank deposits and the degree of substitutability to maximize public welfare.

Most of the literature on CBDC assumes perfect competition in banking or does not model banks explicitly. Exceptions are Andolfatto (2021) who assumes a one bank monopoly and Chiu et al. (2021) who assume Cournot competition with smaller number of banks. In these frameworks, the interest rate on CBDC affects the optimal deposit interest rate and can affect welfare through this channel. With monopolistic competition, however, individual banks take the average deposit rate as given so that

[2]This framework extends the idea present in Feenstra (1986), Rebelo and Vegh (1996) and Schmitt-Grohé and Uribe (2004) that money is demanded as it reduces a transaction or liquidity cost. Barrdear and Kumhof (2021) adopt a similar approach. Imperfect substitutability is also modeled by introducing CBDC in the utility function (e.g., Agur et al. (2021) or Ferrari et al. (2020)) or in search models, where CBDC is used for different transactions (e.g., Assenmacher et al., 2021). However, several papers in the literature assume perfect substitutability between CBDC and bank deposits or focus on the interaction between cash and CBDC (e.g. Davoodalhosseini, 2021).

the deposit rate is unaffected by the CBDC interest rate.[3]

An important feature of our model is that the two main functions of banks, deposit taking and credit provision, do not interact. This is due to the small open economy assumption, where banks obtain substitute funding at the risk-free rate in the international markets. Alternatively, banks could borrow from the central bank as in Brunnermeier and Niepelt (2019): when the central bank expands its liabilities by issuing CBDC, it might acquire claims vis-à-vis the banking sector, thus providing substitute funding for banks. In Brunnermeier and Niepelt (2019) economic outcomes are unchanged if central bank funding is provided at the same conditions as deposit funding, and if the central bank pays the same interest on CBDC as banks do on deposits. In our model, the interest on substitute bank funding would be equal to the risk-free interest rate, which is is higher than the deposit rate. This reduces profits on deposits, but it does not affect credit extension.

While our approach share some features with Barrdear and Kumhof (2021), our paper estimates a significantly lower welfare benefit of CBDC. Their estimate of a 3% GDP increase is due in large part to the following channel. When issuing CBDC, the central bank buys public debt from private investors. This is assumed to decrease the interest rate demanded by investors, which brings savings to the government and general welfare improvements. In our context of a small open economy, the interest rate is given and this mechanism is not present.

We identify three channels through which CBDC may improve welfare. First, through CBDC the central bank may increase its seigniorage revenue, which, everything else equal, would allow the government to reduce income taxes. Second, if households can earn higher interest on their money (CBDC and/or deposit) holdings, they optimally choose to increase their money holdings and thus pay a lower transaction cost on consumption. Third, the introduction of CBDC may lead to a reallocation of banks' rents to the general population, whether in the form of tax reduction (first channel) or in the form of higher interest payment (second channel). If bank rents are collected by a wealthier fraction of the population, this shift implies that CBDC induces some degree of reduction of inequality.

[3]Empirical evidence for monopolistic competition in the banking sector is provided e.g. by Drechsler, Savov and Schnabl (2017). Gerali et al. (2010) introduce monopolistic competitive banks in a DSGE model.

Seigniorage is an important endogenous variable in the model. Its magnitude depends on all three dimensions of CBDC (interest rate, liquidity, substitutability). Seigniorage is non-monotonic in the interest rate paid by CBDC, as a higher interest rate decreases seigniorage per unit CBDC issued, but increases its demand. Everything else equal, CBDC demand and seigniorage are in most cases increasing in the liquidity of CBDC relative to bank deposits. Finally, substitutability between bank deposits and CBDC has an ambiguous role for seigniorage.

The optimal interest rate on CBDC is the one that reaches the best compromise between raising higher seigniorage to lower tax distortions or paying higher interest to lower the opportunity cost of holding money. The optimal interest rate depends on how high are existing tax rates, as the higher the tax rate, the higher the distortion they bring to the economy. Thus, with a higher tax rate the potential benefit of the first channel – collecting seigniorage and lower taxes – is higher, hence the optimal interest rate on CBDC is lower. This is relevant since, as reported e.g. by Trabandt and Uhlig (2011), the amount of labor taxation differs enormously between different countries: it is around 25% in the United States and it averages more than 40% in the EU-14 countries.

However, the quantitative analysis shows that these two channels would bring only a modest welfare improvement: at the optimum they would bring an increase of only 8 basis points in consumption terms for countries with a labor tax rate of 20%, and of 20 basis points for countries with a tax rate of 45%.

The third channel we consider is the reallocation of banks' rents that may lead to a reduction of inequality. In one parameterization of the model we consider the limit case in which a zero-size set of "bankers" own the banks and receive all the profits.[4] CBDC allows non-bankers to take over part of the rents associated to deposits, whether in the form of tax reductions or in the form of interest on CBDC holdings. Taking into account this channel, together with the previous two, we find that the welfare of

[4]This parameterization could represent the situation in which the government's welfare function assigns a much higher weight to a fraction of the population that receives a negligible share of the profits. In the United States, for example, households in the bottom 90% of the wealth distribution own only 10% of the stock. See for example "How America's 1% came to dominate equity ownership", https://www.ft.com/content/2501e154-4789-11ea-aeb3-955839e06441

non-bankers, which coincides with general welfare if the set of non-bankers has zero size, increases by 53 basis points in countries with 20% labor tax rate and by 60 basis points in countries with 45% labor tax rate.

We also emphasize that these benefits require historically normal interest rates (our baseline rate is 3%). At interest rates close to zero, all three of our channels lose their efficacy: seigniorage clearly is also close to zero, the opportunity cost of holding any form of money is close to zero without the need of introducing CBDC, and banks collect zero rents from deposits, implying that there are no rents that CBDC can redistribute to the public.

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 describes the steady state equilibrium. Section 4 discusses the calibration and Section 5 outlines the numerical results, in terms of the relative demand for CBDC and bank deposits, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and the welfare implications. Section 6 concludes.

2 A Model with CBDC

We consider a small open economy model with two types of agents – households and bank owners – firms, banks, and finally the government and the central bank. The world price level is constant at 1; purchasing power parity is assumed to hold, so that the price level is equal to the nominal exchange rate: $P_t = S_t$. The world real interest rate is also constant at r^* and uncovered interest rate parity holds. Thus we have $(1 + i_{t+1}) = (1 + r^*)(1 + \pi^e_{t+1})$, where i_{t+1} is the nominal interest rate on the domestic safe asset and π^e_{t+1} is the expected inflation rate.

Since the objective of our analysis is to examine the impact of CBDC in the long run, we focus on deterministic steady states, so that $\pi^e_{t+1} = \pi_{t+1}$. We assume that the central bank can set inflation at its target level $\overline{\pi} \geq 0$. Below we describe the model in real terms.

2.1 Demand for Bank Deposits and CBDC

Our model comprises two types of agents, households and bank owners, described in detail in Section 2.2. All the action in the model is on the part of households, which in particular generate money demand.

Households decide how to allocate savings between a risk-free internationally traded asset a, paying interest rate r^*, bank deposits $d^b(j)$ for each bank j, paying real interest $r_t^b(j)$, and CBDC d^c, paying interest rate r_t^c. All these interest rates are expressed in real terms. Both bank deposits and CBDC reduce transactions costs, but they are imperfect substitutes.

Bank deposits are issued by a continuum of banks of size 1 in monopolistic competition. The equilibrium interest rate on bank deposits is typically lower than the safe rate r^* due to the costs of managing deposits and to banks' market power, as discussed in section 2.4.

As in Schmitt-Grohé and Uribe (2004), we assume that households incur transactions costs $c_t s_t$ to consume c_t. These costs can be reduced by holding bank deposits and CDBC. More precisely, s_t is a function of money velocity $x_t \equiv c_t/d_t$, where d_t is a composite of the deposits of all banks and CBDC. This composite captures the imperfect substitutability among deposits. We assume a CES structure:

$$d_t = \left(\alpha_c (d_t^c)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} + \alpha_b (d_t^b)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}}\right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{1}$$

d_t^b is a composite of all bank deposits:

$$d_t^b \equiv \left(\int_0^1 (d_t^b(j))^{1-\frac{1}{\epsilon^b}} dj\right)^{\frac{\epsilon^b}{\epsilon^b-1}} \tag{2}$$

where $\epsilon^b > 1$ is the elasticity of substitution between deposits at different banks. $\frac{\alpha_c}{\alpha_b}$ can be interpreted as the relative liquidity of CDBC with respect to bank deposits, and ϵ_{cb} is the elasticity of substitution between bank deposits and CBDC.

The interest rate on CBDC r_t^c is set by the central bank. The relative liquidity and the elasticity of substitution between bank deposits and CBDC can be a design choice of the government. We assume that

$$\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} = 1 \tag{3}$$

as in this case one unit of the numeraire good results at most in one unit of the composite d_t (when $\alpha_c^{\epsilon_{cb}}$ is allocated in CBDC and $\alpha_b^{\epsilon_{cb}}$ is allocated in bank deposits). The world without CBDC is one where $\alpha_c = 0$ and $\alpha_b = 1$.[5]

[5] If the introduction of CBDC implied $\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} > 1$, it could improve the overall efficiency of the payment

Notice that we do not introduce cash in our analysis. We take the view that the role of cash, small relative to that of deposits, is to facilitate transaction of informal type, and that its role would remain largely unaffected by the introduction of CBDC. In the language of this paper, we could consider cash as a type of money with no degree of substitution with digital monies, maybe useful to reduce an independent transaction cost. Our analysis concentrates instead on the competition between CBDC and bank deposits.

2.2 Households and Bank Owners

Households are a measure-one set of agents who work in firms, consume, and save. In addition, they own a fraction of firms and banks. They derive utility from consumption and disutility from working. We assume separable CRRA preferences so that the household's periodic flow utility is given by

$$u(c_t, h_t) = log(c_t) - \frac{h_t^{1+\gamma}}{1+\gamma} \qquad \gamma \geq 1$$

where c_t is consumption and h_t denotes labor supply. The household's expected lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{4}$$

where we assume $\beta(1 + r^*) = 1$ for stationarity.

The household's budget constraint is

$$\begin{aligned} & (1-\tau_h)w_t h_t + (1+r^*)a_{t-1} + \int_0^1 (1+r^b_{t-1}(j))d^b_{t-1}(j)dj \\ + \quad & (1+r^c_{t-1})d^c_{t-1} + \zeta(1-\tau^b)\Pi^b_t = c_t(1+s_t) + \int_0^1 d^b_t(j)dj + d^c_t + a_t + t_t \end{aligned} \tag{5}$$

where w_t is the wage, a_t are holdings of the bond, and Π^b_t are bank dividends, t_t are lump-sum taxes, all in real terms. τ^h and τ^b are labor income and dividend tax rates. ζ is the fraction of banks that is owned by households.[6]

The remaining fraction $1-\zeta$ belongs to the second type of agent in the model, bank owners. This is a set of agents of size ν, who do not work and, importantly, are not

system since fewer resources would be needed to alleviate the transaction cost. However, we abstract from this effect to concentrate on the effect of the competition between bank deposits and CBDC.

[6]The firm sector is perfectly competitive. Hence, firm profits are zero and it is not important to specify the firm ownership.

subject to the transaction cost. Hence their wealth is invested in the risk-free asset, and their bugdet constraint is simply

$$c_t^{bo} + w_{t+1}^{bo} = \frac{1-\zeta}{\nu}\Pi_t^b + (1+r^*)w_t^{bo} \tag{6}$$

where c^{bo} and w^{bo} are consumption and wealth per unit-size bank owner.

While ζ can take any value between 0 and 1, in our numerical analysis we will consider the two extreme cases $\zeta = 1$ and $\zeta = 0$. In the first case, the banking sector is irrelevant and we fall into the representative-agent model, in which households own the banks and equally share all bank profits. In the second extreme, banks are not held by households and bank owners collect all the profits. We will consider the case where $\nu \to 0$ so that bank owners do not matter for welfare.

Households maximize their utility subject to (5). First-order conditions are standard and are described in the Appendix. Below we will assume a specific form for the transactions cost, similar to Schmitt-Grohé and Uribe (2004). This cost is a function of money velocity $x_t = \frac{c_t}{d_t}$

$$s(x_t) = Ax_t + \frac{B}{x_t} - 2\sqrt{AB} \tag{7}$$

where A and B are constant parameters.

The demand equation for the deposits of each bank j

$$d_t^b(j) = \left(\frac{r^* - r_t^b(j)}{r^* - r_t^b}\right)^{-\epsilon^b} d_t^b \tag{8}$$

where

$$r^* - r_t^b \equiv \left(\int_0^1 (r^* - r_t^b(j))^{1-\epsilon^b} dj\right)^{\frac{1}{1-\epsilon^b}} \tag{9}$$

In equilibrium, all banks offers the same deposit rate r_t^b, as we see in more detail in Section 2.4. From the Euler equations, we obtain the relationship between bank deposits holdings and CBDC holdings:

$$d_t^b = \left(\frac{\alpha_b}{\alpha_c} \times \frac{r^* - r_t^c}{r^* - r_t^b}\right)^{\epsilon_{cb}} d_t^c \tag{10}$$

so that there is a simple relationship between holdings of bank deposits and the composite liquid asset

$$d_t = f_t d_t^b \tag{11}$$

with the proportionality factor f_t given by

$$f_t = \left(\alpha_c \left(\frac{\alpha_c}{\alpha_b} \times \frac{r^* - r_t^b}{r^* - r_t^c} \right)^{\epsilon_{cb}-1} + \alpha_b \right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{12}$$

(Notice that without CBDC, i.e. with $\alpha_c = 0$, $\alpha_b = 1$, we have $f_t = 1$ and $d_t = d_t^b$). Comparing the Euler equation for the bond with that for bank deposits, money velocity is

$$x_t = \sqrt{\frac{\frac{r^* - r_t^b}{\alpha_b} f^{-\frac{1}{\epsilon}} + B(1+r^*)}{(1+r^*)A}} \tag{13}$$

so that the demand for bank deposits is

$$d_t^b = \frac{c}{f_t} \sqrt{\frac{(1+r^*)A}{\frac{r^* - r_t^b}{\alpha_b} f^{-\frac{1}{\epsilon}} + B(1+r^*)}} \tag{14}$$

The demand for CBDC can be easily obtained by combining (10) and (14).

One quantity that will be important in our numerical analysis is the interest semi-elasticity of money demand, defined as the percentage change in the demand for money instruments for a one percentage change in the *spread* between the interest paid by money and the risk-free rate. In our model, interest semi-elasticity is essentially determined by the parameter B [7]

$$\iota = -\frac{1}{2} \times \frac{1}{B(1+r^*) + (r^* - r^b)} \tag{15}$$

2.3 Firms

There is a representative firm with Cobb-Douglas production function

$$y_t = z k_t^{\alpha} h_t^{1-\alpha} \tag{16}$$

where k_t is capital. A fraction φ of capital can only be financed by banks (e.g., for the financing of working capital), so that $\varphi k_t = l_t$, where l_t are the real loans that the firm obtains from the bank in period t. The remaining fraction $1 - \varphi$ is financed by issuing bonds at interest rate r^*.

[7] This is obtained assuming that all money instruments pay the same interest. In particular, after the introduction of CBDC, in the assumption that $r^b = r^c$.

We assume monopolistic competition in the loan market, so that, similarly to deposits, loans are a bundle of loans from different banks[8]

$$l_t \equiv \left(\int_0^1 (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l - 1}} \tag{17}$$

where ϵ^l is the elasticity of substitution for loans from different banks and the index i denotes a bank. The working capital constraint can be rewritten as

$$k_t = \frac{\left(\int_0^1 (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l - 1}}}{\varphi} \tag{18}$$

Firms choose loans, capitals and labor to maximize profits, which, taking into account the working capital constraint, can be written as

$$\Pi_t = z \left(\frac{\left(\int_0^1 (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l - 1}}}{\varphi} \right)^{\alpha} h_t^{1-\alpha} - w_t h_t - \int_0^1 l_t(i) r_t^l(i) di - (1-\varphi)\, r^* \frac{\left(\int_0^1 (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l - 1}}}{\varphi} \tag{19}$$

We obtain that firms' loan demand is

$$l_t(i) = \left(\frac{r_t^l(i)}{r_t^l} \right)^{-\epsilon^l} l_t \tag{20}$$

where $r_t^l(i)$ is the loan interest rate charged by bank i and the "market loan rate" r_t^l is

$$r_t^l = \left(\int_0^1 (r_t^l(i))^{1-\epsilon^l} di \right)^{\frac{1}{1-\epsilon^l}} \tag{21}$$

In equilibrium all banks choose the same rate r_t^l. Given the constraint $\varphi k_t = l_t$, the real cost of a unit of capital is $r_t^K = \varphi r_t^l + (1-\varphi) r^*$. From the first order conditions of the firm we easily obtain

$$k_t = \left(\frac{z\alpha}{r_t^K} \right)^{\frac{1}{1-\alpha}} h_t \tag{22}$$

and (with competitive labor markets)

$$w_t = (1-\alpha) z \left(\frac{z\alpha}{r_t^K} \right)^{\frac{\alpha}{1-\alpha}} \tag{23}$$

[8] Paravisini, Rappoport and Schnabl (2015) provide empirical evidence of specialization in bank lending, which supports the idea of monopolistic competition in the lending market.

2.4 Banks

We assume that there is a size-one continuum of banks in monopolistic competition in the deposit market and in the loan market. The aggregate bank balance sheet is

$$l_t + b_t^b + m_t = d_t^b + a_t^b + e_t^b \tag{24}$$

where on the asset side (LHS) we have bonds held by the banks b_t^b, required reserves m_t and loans l_t, and on the liability side (RHS) we have bank deposits d_t^b, other bank liabilities (such as bonds) a_t^b, and bank equity e_t^b.

Bonds on the asset and liability side, b_t^b and a_t^b, yield an interest rate r^*, whereas reserves yield an interest rate r_t^m determined by the central bank. Required reserves are a fraction ϕ of deposits: $m_t = \phi d_t^b$.

Loans are provided with cost c^l at the real interest rate $r_t^l(j)$ for bank j. Deposits are provided with cost c^b at the real interest rate $r_t^b(j)$. For now, we assume that costs c^l and c^b are constant. Profits of bank j are

$$\Pi_t^b(j) = (1+r_{t-1}^l(j)-c^l)l_{t-1}(j)+(1+r^*)(b_{t-1}^b(j)-a_{t-1}^b(j))+(1+r_{t-1}^m)m_{t-1}(j)-(1+r_{t-1}^b(j)+c^b)d_{t-1}^b(j) \tag{25}$$

Using the bank balance sheet and the reserve ratio, this can be rewritten as:

$$\Pi_t^b(j) = [(1-\phi)r^* + \phi r_t^m - (r_{t-1}^b(j) + c^b)]d_{t-1}^b(j) + [r_{t-1}^l(j) - c^l - r^*]l_{t-1}(j) \tag{26}$$

In equilibrium all profit-maximizing banks choose the same deposit rate[9]

$$r_t^b(j) = r_t^b = r^* - (c^b + \phi(r^* - r_t^m))\frac{\epsilon^b}{\epsilon^b - 1} \tag{27}$$

and loan rate

$$r_t^l(j) = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c^l) \tag{28}$$

It is interesting to notice that with demand for deposits (8), each bank sets its deposit rate relative to the overall bank deposit rate, regardless of the CBDC rate. Thus, even in case CBDC pays a high interest rate, banks do not react, and continue to pay the same rate on deposits (27). The intuition behind this somewhat surprising result is that each bank competes with other banks for deposits, but perceives the aggregate demand for bank deposits (and of CBDC) as fixed, not internalizing how

[9] We impose however a zero-lower-bound condition on the nominal deposit rate $0 \leq i_t^b \equiv r_t^b + \pi_{t+1}^e$

the relative demand for the two monies depends on the interest paid in aggregate by the banking system. However, competition with CBDC implies lower overall demand for bank deposits, so that in equilibrium each bank relies less on deposits and more on other liabilities, such as bank bonds and/or equity.

The loan rate is unaffected by deposits or CBDC altogether. All banks choose therefore the same value (28) of the loan rate, with or without CBDC. The quantity of loans is not affected by CBDC because banks can freely borrow in the bond market at interest rate r^*.

2.5 Central bank

The central bank issues the monetary base m_t, fully consisting in bank reserves, on which it pays an interest r_t^m, as well as CBDC d_t^c, on which it pays a rate r_t^c. It holds assets a_t^c bearing interest rate r^*. Assuming zero equity at the beginning of each period, the central bank's balance sheet is $m_t + d_t^c = a_t^c$. Central bank profits are given by seigniorage

$$\mathcal{S} = (r^* - r_{t-1}^m)m_{t-1} + (r^* - r_{t-1}^c - c^c)d_{t-1}^c \tag{29}$$

where c^c is the cost of managing CBDC, and are distributed each period to the government. The growth in monetary base is determined by the inflation target and money market equilibrium is simply given by $m_t = \phi d_t^b$.

2.6 Government

The government needs to fund a constant exogenous real expenditure g. The government receives central bank profits, levies taxes on labor income at rate τ^h and on bank profits at rate τ^b (firm profits are 0 due to perfect competition in the goods markets). In addition, it may impose a lump-sum tax t. It pays interest r^* on the debt contracted in the previous period b_{t-1}^g. The government budget constraint is:

$$\tau^h w_t h_t + \tau^b \Pi_t^b + (r^* - r_{t-1}^m)m_{t-1} + (r^* - r_{t-1}^c - c^c)d_{t-1}^c + b_t^g + t_t = g + (1 + r^*)b_{t-1}^g \tag{30}$$

The presence of CBDC increases seigniorage received by the government. However, since with CBDC bank profits may be reduced, the government's tax revenues from bank profits may also be reduced. In subsequent sections we will analyze in detail how changes in r_t^c affect government revenues.

3 Steady State Equilibrium

Since there is no shock, equilibrium is a steady state characterized by the following conditions

- Given the wage paid by firms, the interest paid by the risk-free asset, by bank deposits and by CBDC, the tax rates chosen by the government, households make decisions about labor, consumption, savings in the risk-free asset, bank deposits and CBDC to maximize utility.
- Given the cost of capital (determined by the risk-free rate and the loan rate chosen by banks) and the cost of labor (wage), firms choose capital and labor to maximize profits.
- Given deposits demand (which also depends on the rate offered by CBDC) and loan demand, banks choose the rate on deposits and on loans to maximize their profits.
- The wage is such that labor markets clear.

All the equations determining steady state variables are summarized in the Appendix. Given the value of the other variables $(x,\ r^b,\ r^l,\ r^c,\ r^K, d^b,\ d^c,\ f,\ f_1, a, w)$, listed in the Appendix, steady state household consumption solves

$$c^{1+\frac{1}{\gamma}}\left(1+s(x)-\frac{r^b}{fx}-\frac{r^c}{f_1 x}-\zeta(1-\tau^b)\frac{((1-\phi)r^*-r^b-c^b)}{fx}\right)+c^{\frac{1}{\gamma}}(t-r^*a)=$$

$$=\quad \frac{w^{1+\frac{1}{\gamma}}(1-\tau^h)^{1+\frac{1}{\gamma}}+\zeta(1-\tau^b)(r_l-r^*-c^l)\varphi\left(\frac{z\alpha}{r^K}\right)^{\frac{1}{1-\alpha}}w^{\frac{1}{\gamma}}(1-\tau^h)^{\frac{1}{\gamma}}}{(1+s(x)+xs'(x))^{\frac{1}{\gamma}}} \tag{31}$$

We define welfare from the point of view of the households, ignoring the bankers: $W=log(c)-h^{1+\gamma}/(1+\gamma)$. This has two possible interpretations: either the government cares more about households than about bankers, or the share ν of bankers is very small, so that despite their high (per unit-size) consumption bankers' contribution to general welfare is negligible.

Our main purpose is to analyze the effect of the introduction of CBDC on the steady state equilibrium, as well as the effect of different CBDC design choices, such as the relative liquidity between CBDC and bank deposits (as measured by the ratio α_c/α_b) and of the elasticity of substitution between the two monies. Government/central bank policy can also affect the steady state variables through its choice of the reserve ratio ϕ, the rate r^m, and the tax rates τ^h and τ^b and the lump-sum tax t. All these policy choices affect the demand for CBDC and bank deposits, which in turns affects consumption and labor. Notice that, while the loan rate (28) is not affected by the parameters set by the government, capital (22) and loan demand $l = \varphi k$ can be affected as they proportional to labor.

4 Calibration

Table 1: Model Parameters

Parameter	Description
$r^* = 3\%$	risk-free rate
$A = 0.0111$	Transaction cost parameter
$B = 0.07524$	Transaction cost parameter
$\gamma = 1$	Inverse Frisch elasticity
$\phi = 0.08$	reserve ratio
$\tau^b = 25\%$	tax rate on bank profits
$\varphi = 0.2$	working capital requirement
$r^m = 0$	interest rate on bank reserves
$c^b = 0.25\%$	managing cost of bank deposits
$c^l = 0.5\%$	managing cost of loans
c^c=0.25%	managing cost of CBDC
$\alpha = \frac{1}{3}$	Cobb-Douglas capital share
$\epsilon^b = 1.40$	Elasticity of substitution of bank deposits
$\epsilon^l = 6.67$	Elasticity of substitution of bank loans
$wealth/c = 4$	wealth over consumption ratio

Table 1 summarizes our parameter choices. The parameters that are most important for our experiment are those affecting money demand and the banking system. In our baseline case we use the values for the parameters A and B of the transaction cost estimated for the US economy by Schmitt-Grohe and Uribe (2004), which imply, according to (15), an interest semi-elasticity of money demand equal to -0.05. This is consistent with the estimation on the long-run money demand by Ball (2001), and also with the more recent estimates by Drechsler, Savov and Schnabl (2017), that, similarly to us, focus on the demand for deposits as a function of the deposit spread.[10] We will however also show results with parameter values implying a wide range of the interest semi-elasticity.

For the banking system, the parameter ϵ^b (elasticity of substitution between deposits of different banks) is calibrated so that the deposit spread (difference between the deposit rate and the risk-free rate) is 2%, an historical average in the US and Europe alike.[11] The parameter ϵ^l (elasticity of substitution between loans of different banks) is calibrated so that the loan spread –difference between the loan rate and the risk-free rate – is 1%. This value is appropriate for the US but is low for other countries; however our results are not sensitive to this parameter, as the loan extension activity by banks is not affected by the introduction of CBDC.

Only indirect data is available to estimate the banks' cost of managing deposits and loans. According to call report data from the Federal Financial institution Examination Council,[12] total operating costs for US banks amount to around 2% of the value of bank assets, and fee income is around 1% of bank assets. If operating costs (net of fees) are equally distributed across assets and liabilities, then we could take 50 bps as an estimate of the cost of operating deposits and loans. However it is likely that operating costs, whose biggest component is given by employee salaries, are much higher on the investment side than on deposits. We therefore use 25 bps as baseline value of the cost of operating deposits (net of fees), but also consider a scenario with the alternative

[10] Drechsler, Savov and Schnabl (2017) find that a percentage point increase in the risk-free rate corresponds on average to a 60 bps increase in the deposit spread, and a 3% decrease in the demand for deposits. Hence, a 1% increase in the deposit spread corresponds to a 5% decrease in the demand for deposits.

[11] As pointed out by Drechsler, Savov and Schnabl (2017), the deposit spread in the US is increasing in the risk-free rate. However, a spread around 2% is an historical average. Data on deposit rates in several European countries from the World Bank open database confirm that this is the case also in Europe.

[12] Downloadable at *https://cdr.ffiec.gov*.

value of 50 bps. We use 50 bps as the operational cost of loans.

The required reserve ratio ϕ differs hugely across countries. It is now zero in the United States and 1% in the Euro area. However, it can be much higher in less advanced economies (for example, it is around 40% in Argentina). Our baseline value is 5%, closer to the value observed in advanced economies.

Another important parameter for our analysis is the inverse Frisch elasticity γ, which affects the extent to which labor taxation is distortionary. We use a standard value equal to 1 in our baseline scenario, but later consider a range of values from 0.25 to 4. Household wealth, given in our model by the sum of the household's investment in the risk-free asset, in bank deposits and in CBDC, is set to 4 times annual consumption, similar to the ratio in the US (see e.g. Piketty and Zucman (2014)). Finally, the value of productivity (expressed by the variable z) is irrelevant to our experiment as it does not affect the *percentage change* in consumption, labor and welfare induced by CBDC, so it can be normalized to 1.

New parameters associated with CBDC, in particular the relative liquidity between CBDC and bank deposits, and their substitutability, will be regarded as choice variables in the numerical analysis.

5 Results

Given our parameter calibration, in this section we outline our numerical results, in terms of the relative demand for bank deposits and CBDC, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and welfare implications.

5.1 The demand for deposits and CBDC

We start by examining the impact of the CDBC interest rate r_t^c on the demand for CBDC and bank deposits for different levels of substitutability and relative liquidity of CBDC. An increase in r^c tends to increase the demand for CBDC and decrease the demand for bank deposits. However, the demand for both instruments is non-monotonic in their elasticity of substitution ϵ_{cb} and in their relative liquidity, here measured by the parameter α_c.

The four panels of Figure 1 show the demand for CBDC (in the two left panels) and bank deposits (in the two right panels) when r^c is within 2 percentage points higher or lower than the interest paid by bank deposits, r^b, i.e., in a range of 4 percentage points below the risk free rate in our calibration. (Given the choice of the parameter ϵ^b – the elasticity of substitution between deposits of different banks – the interest paid by deposits is 2% below the risk-free rate, regardless of the interest paid by CBDC, and regardless of the other characteristics of CBDC).

In the top panels we set $\alpha_b = \alpha_c = 0.5^{\epsilon_{cb}}$ (meaning that CBDC and bank deposits are equivalent from the point of view of liquidity, so that if they paid the same interest, households would allocate the same amount of resources on the two), and show demand curves for three values of ϵ_{cb}: $\epsilon_{cb} = 3$, which we take as a representative case of "low substitutability" between bank deposits and CBDC; $\epsilon_{cb} = 6$ (medium substitutability) and $\epsilon_{cb} = 20$ (high substitutability).

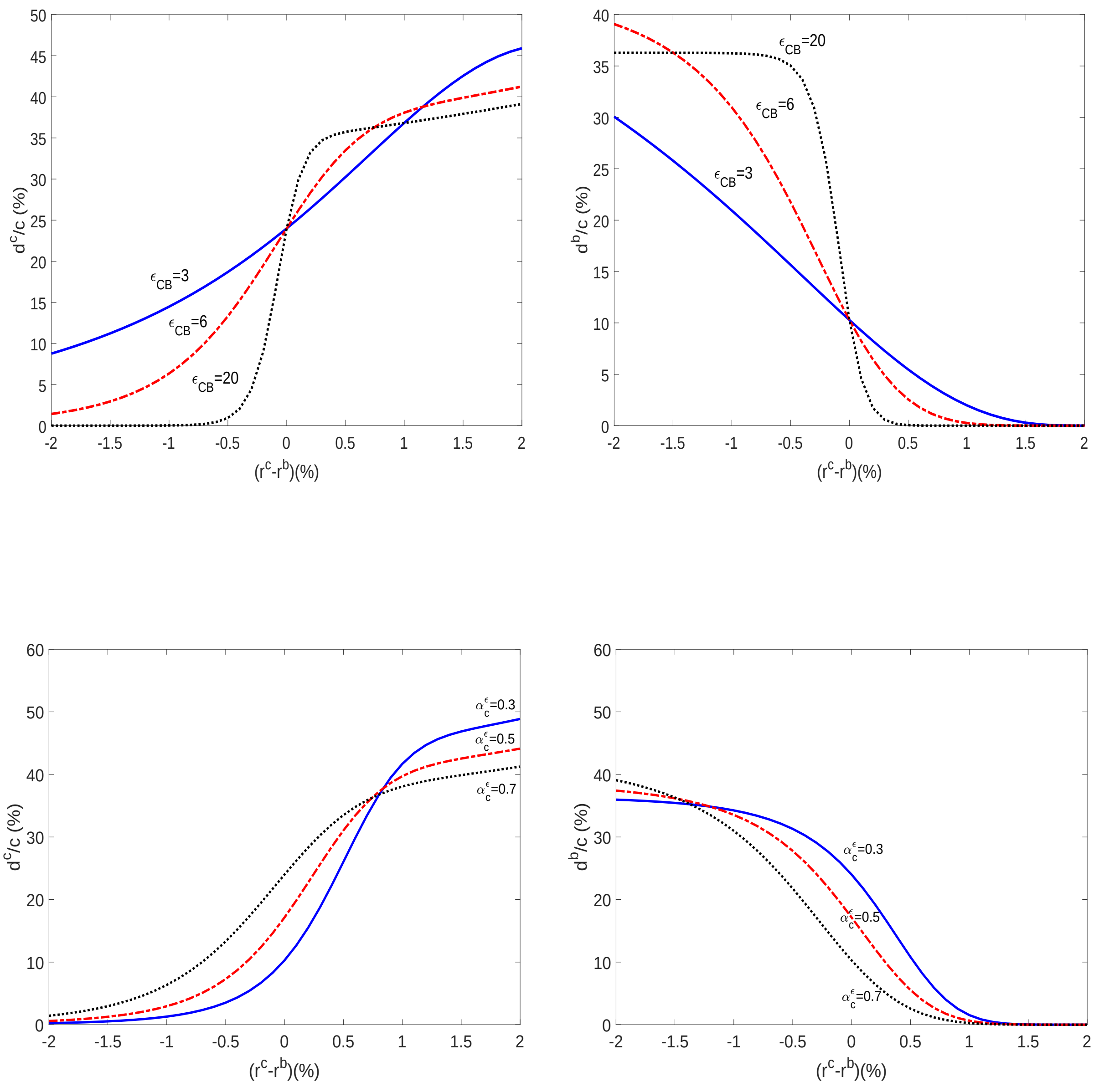
In the bottom panels we set $\epsilon_{cb} = 6$ (the medium substitutability case) and show the results for three different values of α_c (α_b and α_c are related by (3)). These three values are such that $\alpha_c = 0.3^{\frac{1}{\epsilon_{cb}}}$, $\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$, and $\alpha_c = 0.7^{\frac{1}{\epsilon_{cb}}}$, implying that, of the resources allocated in liquid assets (bank deposits or CBDC), households would choose to allocate 30%, 50% and 70%, respectively, in CBDC if the two paid the same interest.

When the interest paid by CBDC is below the interest paid by bank deposits, demand for CBDC is decreasing in the elasticity of substitution ϵ_{cb}: the more the two instruments are substitutable, the less households are willing to hold the more costly one, i.e. CBDC. When the interest paid by CBDC is higher than the one paid by bank deposits, but is not too close to the risk-free rate, the same effect persists, in the other direction: the more substitutable the two instruments, the less households are willing to hold bank deposits. However, when interest paid by CBDC is almost as high as the risk-free rate, holdings of CBDC become *decreasing* in ϵ_{cb}: holding CBDC is in this case almost costless, and highly preferable to holding bank deposits. If the two are less substitutable, rather than holding both, it is preferable to increase the holdings of liquid assets by holding *even more* of CBDC.

A similar effect occurs with respect to the relative liquidity between the two instruments. When the interest paid by CBDC is significantly below r^*, holdings of CBDC are increasing in its liquidity: given that it is costly to hold liquid asset, it is optimal to allocate more resources on the one offering better liquidity services. However, as the interest paid by CBDC approaches r^*, holding it becomes almost costless, and, if CBDC is less liquid than bank deposits, it is preferable to hold a lot of it (to compensate for the lower liquidity) rather than holding the more expensive bank deposits.

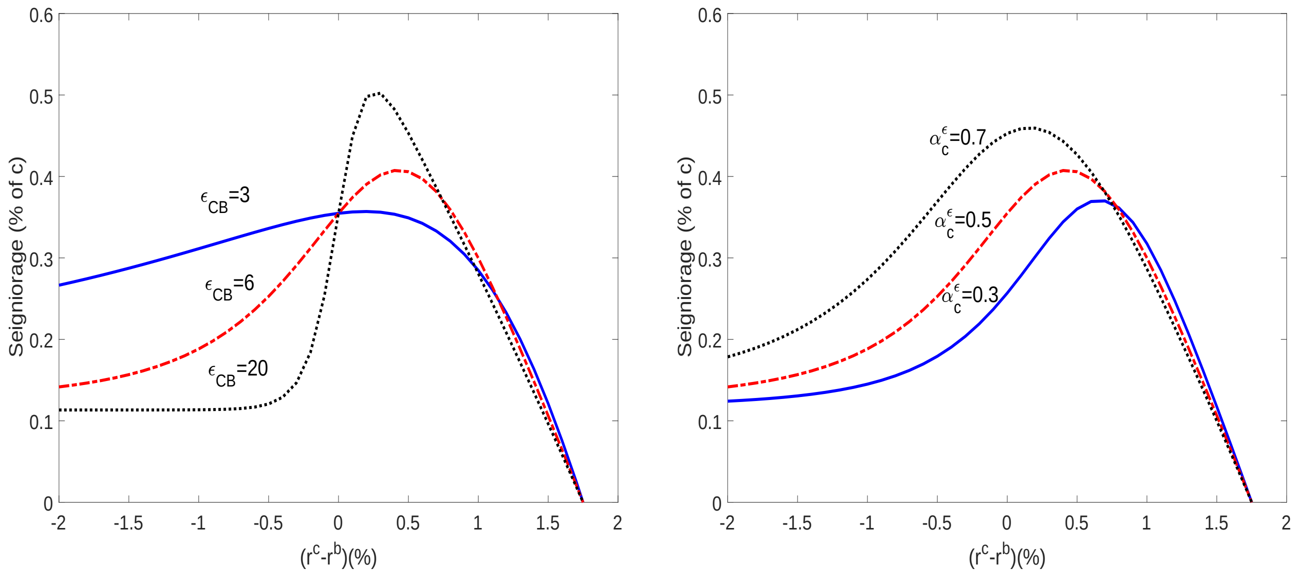
Figure 1: Demand for CBDC and bank deposits

CBDC **Bank deposits**



5.2 Seigniorage, Bank Profits, and Welfare

Figure 2: Seigniorage Revenues



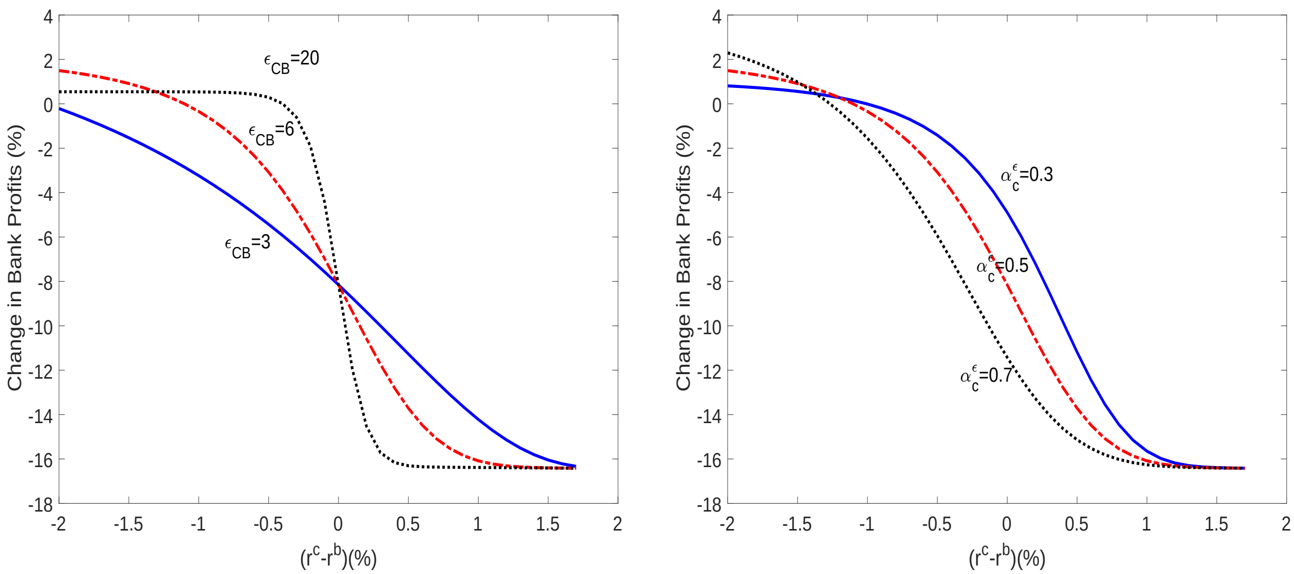
Next we examine the impact of r_t^c on seigniorage, bank profits, and welfare. The two panels of Figure 2 show the amount of seigniorage that the government can collect as a function of the interest paid on CBDC. On the left panel we set $\alpha_c = \alpha_b = 0.5^{\frac{1}{\epsilon_{cb}}}$ (equal liquidity properties for CBDC and bank deposits) and show the three curves of seigniorage as a function the interest paid by CBDC (ore precisely, as a function of the spread $r^c - r^*$) for the three values of the elasticity of substitution previously considered: $\epsilon_{cb} = 3$, $\epsilon_{cb} = 6$ and $\epsilon_{cb} = 20$.

Seigniorage is given by (29). By taking into account the constraint $m_t = \phi d_t^b$, it can be written as

$$\mathcal{S} = \phi(r^* - r_{t-1}^m)d_{t-1}^b + (r^* - r_{t-1}^c - c^c)d_{t-1}^c$$

Seigniorage revenues are non-monotonic in r^c, interest paid on CBDC, as the demand for CBDC is increasing and the central bank profit per unit of CBDC is decreasing in r^c. As seen in Figure 2, the location of the interior maximum depends both on the elasticity of substitution between bank deposits and CBDC, and their relative

Figure 3: Bank Profits



liquidity. For low elasticity of substitution, here represented by the case $\epsilon_{cb} = 3$, the peak of seigniorage revenues occurs for $r_c < r_b$; in particular for $\epsilon_{cb} = 3$ the peak occurs for r^c 30bps lower than r^b: low substitutability ensures that demand for CBDC is high even when the interest paid by it is inferior to that paid by bank deposits. For higher substitutability, the peak occurs for $r^c > r^b$. In the limit of perfect substitutability, clearly the best to maximize seigniorage is to set the interest just above that paid on bank deposits and attract the whole demand of liquid assets.

The right panel of Figure 2 shows the quite intuitive result that, for higher liquidity of CBDC relative to bank accounts, the peak of seigniorage occurs for lower r^c: if the liquidity services offered by this instruments are higher, demand decreases more slowly in the interest rate it pays, and the profit-maximizing interest rate is lower.

Figure 3 shows how bank profits are reduced (relative to the world prior to CBDC), as the spread between the interest on CBDC and on deposits increases. Bank profits can decrease up to 16%; however, as previously discussed this has no impact on banks' credit extension in our model. The reduction in bank profits reduces government tax revenues, however the increase in seigniorage is more important. For example, in the case of $\alpha_c = 0.5$ and $\epsilon_{BC=6}$, the reduction in bank profits reduces the government's revenues by 16 bps, however seigniorage increases revenues by 32 bps.

Summing up, to maximize government revenues r_t^c should be set slightly higher than r^b, unless substitutability is very low. However, if the authorities want to maximize consumers' welfare, they have an incentive to set a higher interest rate, so as to reduce the opportunity cost of holding money. By setting r^c higher than r^b, consumers face a lower opportunity cost. To minimize the opportunity cost, r^c should be set to r^*. But in this case, given that managing CBDC has a non-zero cost, the government would face negative revenues from CBDC. We quantify this tradeoff below.

5.3 Optimal Policy and Welfare gains

We now examine the broader question of what is the best choice of interest, liquidity and substitutability, that the central bank can make, in an economy with distortionary taxes. We will assume that changes in government revenues are compensated by lowering the distortionary labor tax. The tradeoff between reducing taxes or reducing the opportunity cost of holding money depends on the level of taxation: clearly in some European countries, such as Italy or France, in which the level of labor taxation is of the order or 45%, reducing taxes would be a bigger priority than in countries such as the US in which the level of labor taxation is of the order of 25%. For this reason we compute the welfare-maximizing choice as a function of the labor tax rate.

Another important consideration to determine the optimal policy is whether households share or not banks' profits. As briefly discussed in Section 2, we consider two alternative, extreme cases. In the first case, which we call "case a", households fully own banks and equally share bank profits, so that the parameter ζ in (5) is equal to 1. We call "case b" the opposite extreme, in which $\zeta = 0$ and "bankers" receive all bank profits. Assuming that their size ν is zero, they are irrelevant for welfare.

The plots in Figure 4 show the optimal interest rate choice in both cases. As we see, in "case b" the welfare-maximizing choice involves a higher interest rate on CBDC. This result can be understood the following way: when all households receive an equal share of banks profits ("case a"), welfare can only be improved via a reduction of the distortions in the economy; in contrast, when households do not receive banks' profits ("case b"), their welfare can be improved if they take over part of the resources that were previously taken by bankers. A higher interest rate reduces the demand for bank deposits and increases the demand for CBDC, and this allows households to take over a higher share of the rents associated to deposits, which were previously held by bankers.

Figure 5 shows the welfare gain in the optimal case. The left panel shows the welfare gain as a function of the labor tax rate, in "case a" and "case b", when $\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$ and $\epsilon_{cb} = 6$. The right panel shows the same, when setting $\epsilon_{cb} = 20$. We se that the welfare gain in this case is increasing in the elasticity of substitution, although very mildly. In "case a", the welfare gain ranges from a modest 7-8 bps when the labor tax rate is 20% to a more significant 18-20 bps when the labor tax rate is 45%. On the other hand, in "case b" the welfare gain would range between 52-53 bps (when $\tau_l = 20\%$) to 58-60 bps (when $\tau_l = 45\%$).

Table 2 gives more detail about the main changes in the economy (including labor, consumption, welfare) in two cases: when (pre-CBDC) labor tax rate is 25% and 45%. Here the interest rate on CBDC is set at the optimal level; the liquidity of CBDC is equal to that of bank deposits ($\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$), and the elasticity of substitution is set at the high level, $\epsilon_{cb} = 20$, that is the case that gives the slightly more positive welfare results.

Table 2: CBDC-induced changes in the economy

τ_l=**25%**	"case a"	"case b"	τ_l=**45%**	"case a"	"case b"
Consumption	+27 bps	+54 bps	Consumption	+41 bps	+62 bps
Labor	+22 bps	0	Labor	+26 bps	+4 bps
Labor tax rate	-0.12%	-0.13%	Labor tax rate	-0.30%	-0.27%
Optimal $(r^* - r^c)$	-0.96%	-0.85%	Optimal $(r^* - r^c)$	-1.53%	-1.42%
Welfare	+9 bps	+54 bps	Welfare	+20 bps	+59 bps

Finally, Figure 6 shows how the welfare gain at the optimal interest level r_c changes with liquidity and substitutability between CBDC and bank deposits. In the two top panels the labor tax rate is set at $\tau_l = 25\%$. In the bottom two panels $\tau_l = 45\%$. We see that in fact, if the liquidity of CBDC is low relative to that of bank deposits, the welfare gain is quite sensitive to the elasticity of substitution between CBDC and

Figure 4: Optimal CBDC rate

$\epsilon_{cb} = 6$ $\qquad\qquad$ $\epsilon_{cb} = 20$

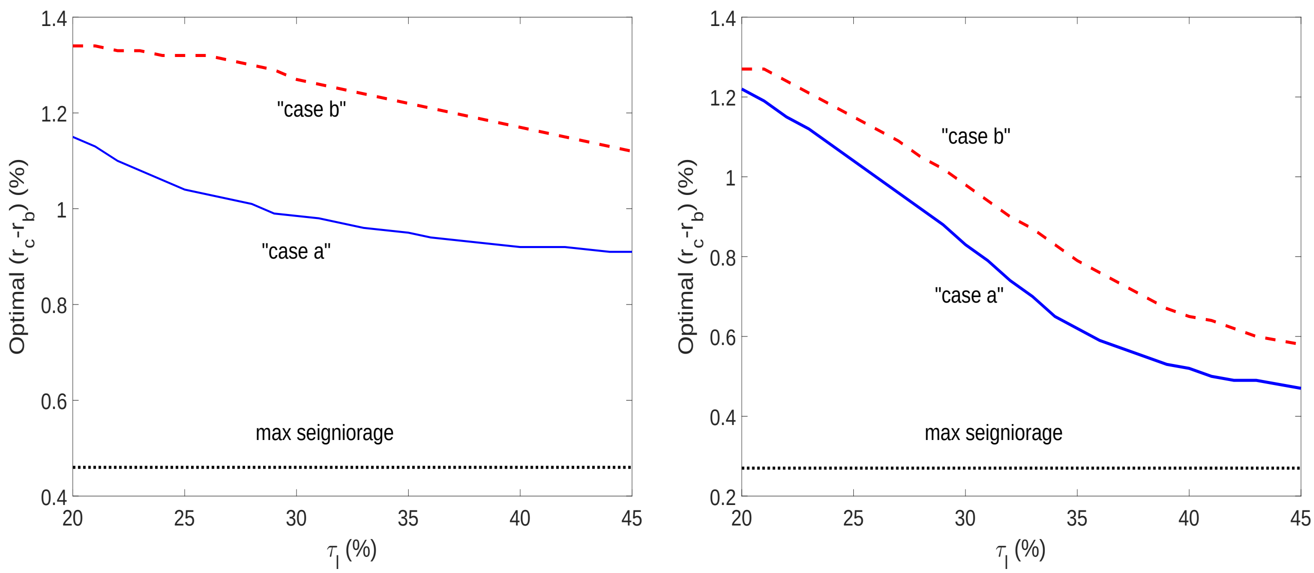
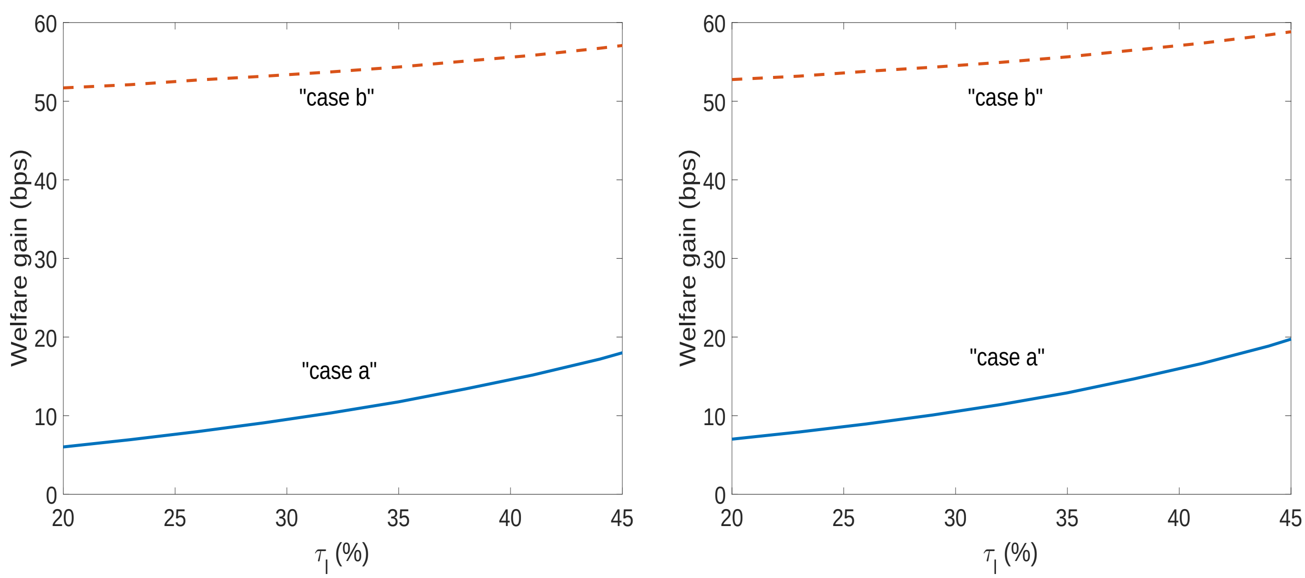


Figure 5: Welfare gain

$\epsilon_{cb} = 6$ $\qquad\qquad$ $\epsilon_{cb} = 20$



bank deposits. Intuitively, if CBDC is significantly less liquid than bank deposits, to make CBDC attractive we need to set the interest paid by CBDC, r_c, higher than the interest paid by bank deposits; and if, in addition, the substitutability between the two is low, demand for bank deposits continues to be high unless r_c is very close to the risk-free rate. This means that the seigniorage the central bank can collect is necessarily low, which lowers the welfare gain, especially when labor taxes are at the high end of the spectrum, i.e. when seigniorage used to lower tax rates would be very valuable. Figure 6 shows that, everything else equal, the more liquid is CBDC, and the more it is substitutable with bank deposits, the better it is for welfare. However the figure also shows that, if the two instruments are very substitutable and CBDC is at least as liquid as bank deposits, no big gains can be achieved by further increasing the liquidity of CBDC. This seems relevant since – although disregarded in this model – it seems likely that increasing the liquidity of CBDC might involve higher costs for the central bank.

5.4 Alternative Scenarios

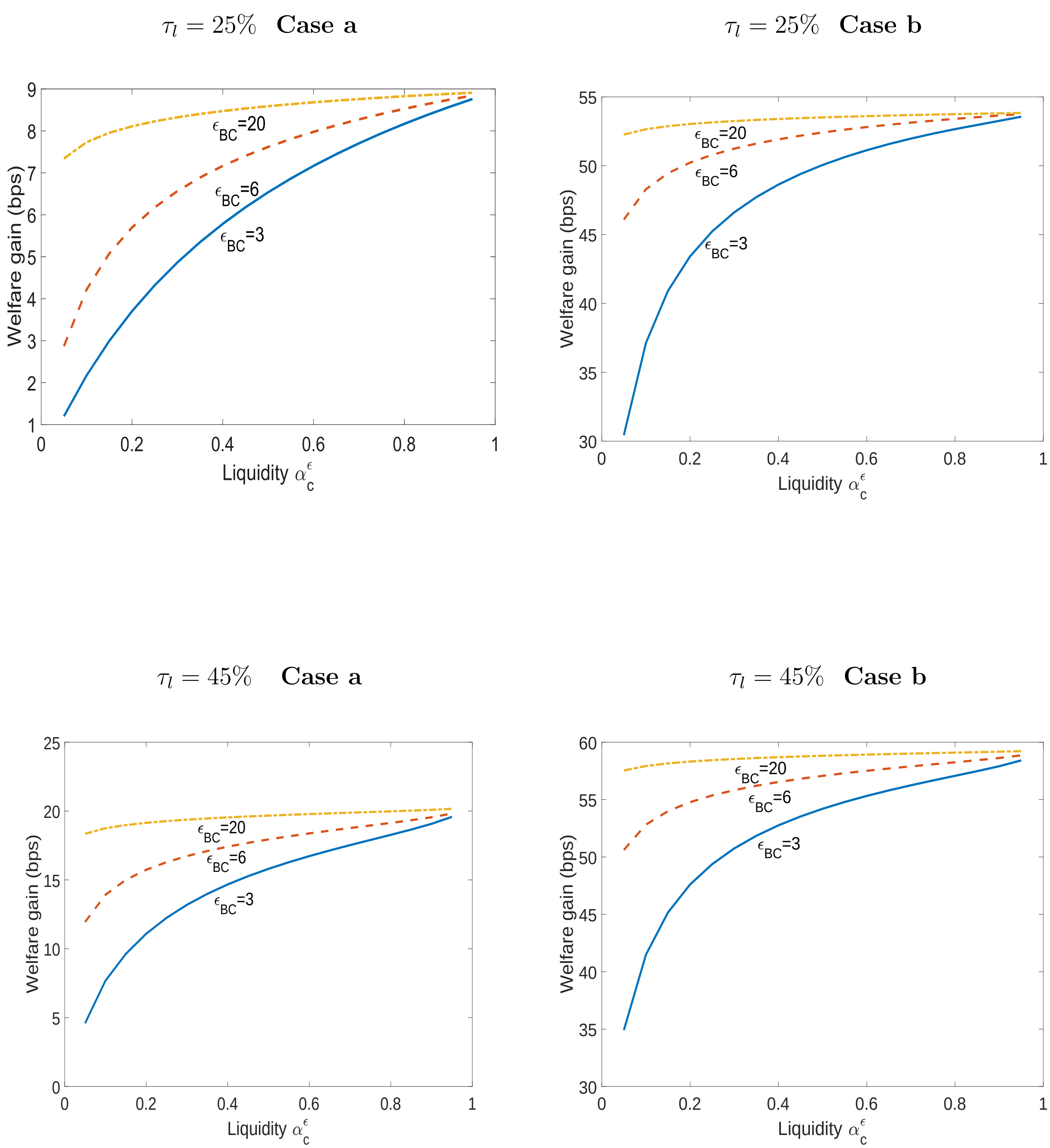
5.4.1 Lump-sum taxation

Imagine a scenario in which all taxes are lump-sum, and both the labor tax rate and the corporate tax rate are zero ($\tau^l = 0$, $\tau^b = 0$, $t > 0$, in the notation of Section 2). In this case it is inutitive that the optimal interest rate on CBDC would be equal to the risk-free rate: collecting seigniorage to decrease taxes would not be important if taxes are lump-sum; instead, choosing $r^c = r^*$ would reduce to zero the opportunity cost of holding money, and at the same time maximize the redistribution from bankers to non-bankers. Table 3 summarizes the effects of introducing CBDC in this economy.

Table 3: CBDC-induced changes in a lump-sum-tax economy

	"case a"	"case b"
Consumption	+40 bps	+54 bps
Labor	+29 bps	-2 bps
Optimal $(r_c - r^*)$	0	0
Welfare	+4 bps	+61 bps

Figure 6: Welfare gain: Liquidity and Elasticity of Substitution



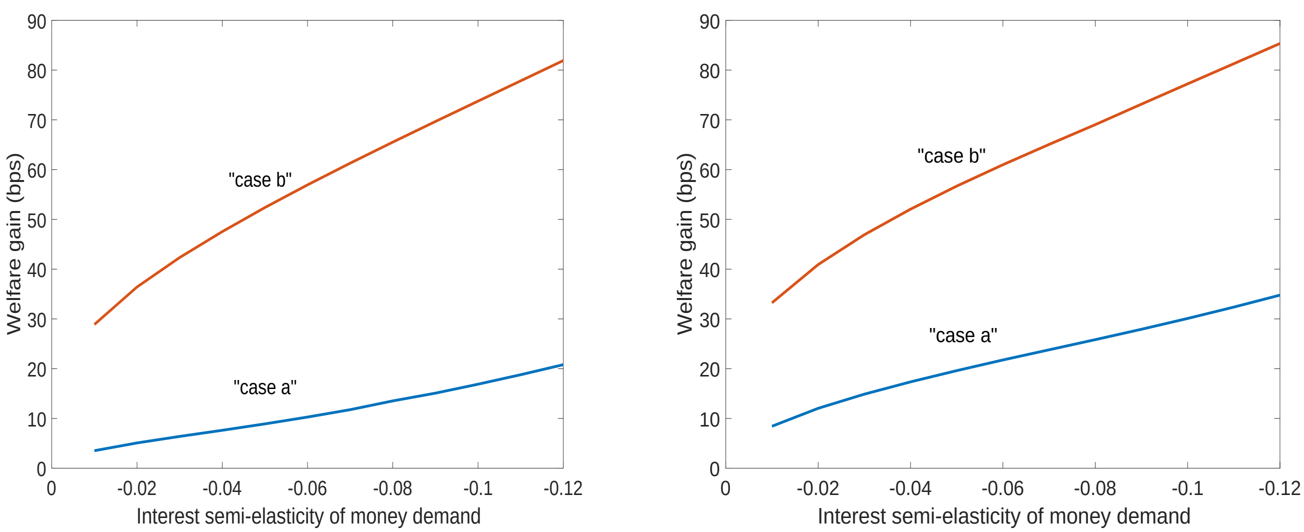
As we see from Table 3, the welfare gains in "case a" would be lower than in our baseline scenario with distortionary taxes. However in "case b" the welfare gains would be slightly larger, as, with only lump-sum taxes and in particular zero tax on bank profits, the effect of the redistribution from bankers to non-bankers would be larger.

5.4.2 Alternative money demand

We now consider the possibility that the interest semi-elasticity of money demand is different from our baseline scenario. As shown in (15), this semi-elasticity is essentially governed by the parameter B of the transaction cost. We therefore change this parameter to obtain different values of the semi-elasticity and look at the impact on the welfare gains brought by CBDC.

Figure 7: Welfare gain as a function of interest semi-elasticity of money demand

$\tau_l = 25\%$ $\tau_l = 45\%$



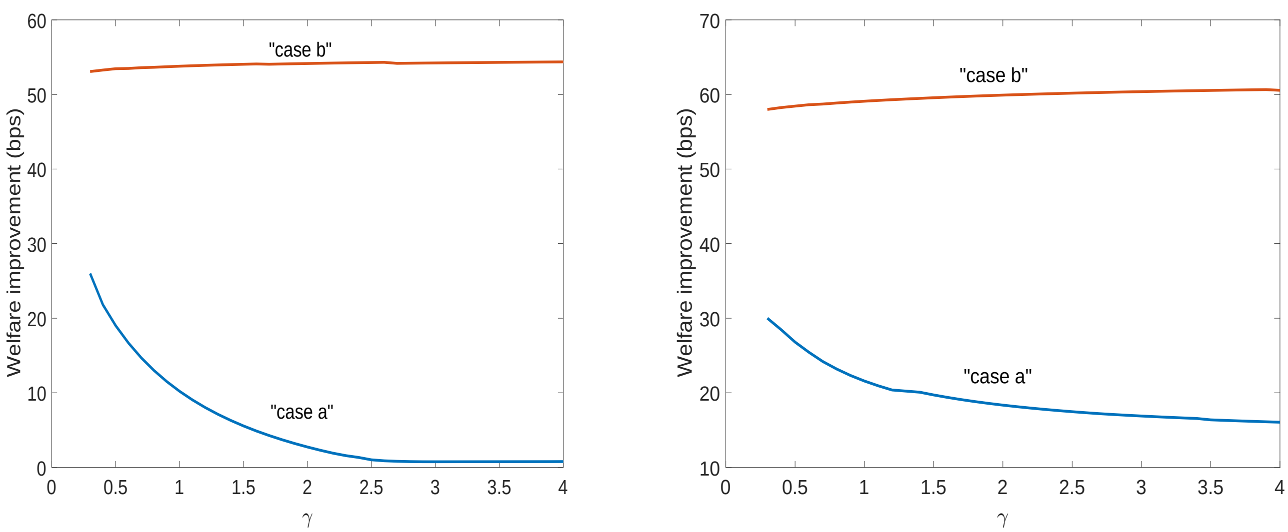
As we see from Figure 7. welfare gains increase in the interest semi-elasticity of money demand. This is intuitive as a higher semi-elasticity means that the distortion associated with the low interest on money has stronger effects on the economy, so CBDC, by paying interest close to the risk-free rate, would have the potential to bring bigger welfare improvements. As labor taxes are high (45%) and at the same time the interest semi-elasticity is high (-0.12 is the highest value we consider), the welfare gains induced by CBDC reach 35 bps in ”case a” and 85 bps in ”case b”.

5.4.3 Alternative Frisch elasticities of labor supply

The plots in Figure 8 show the welfare improvement induced by CBDC as a function of the inverse Frisch elasticity. The range for the latter goes from 0.25 (corresponding to Frisch elasticity equal to 4, one of the highest values considered in the literature) to 4 (Frisch elasticity equal to 0.25, in the low range of estimated “micro-elasticities”). As is intuitive, CBDC has the potential to bring higher welfare improvement when the elasticity is high, i.e. when taxation has a stronger distortionary effect on labor. However, Figure 8 shows that welfare improvements in “case b” are essentially independent of the Frisch elasticity: to maximize the redistribution from bankers to non-bankers it is optimal to set the rate on CBDC close to the risk-free rate. However, this involves small seigniorage collection, hence small tax reduction.

Figure 8: Welfare gain as a function of inverse Frisch elasticity

$\tau_l = 25\%$ $\tau_l = 45\%$



5.4.4 Low nominal rates

Although all plots and numbers for Section 6 have been obtained using $r = 3\%$, it is worth noting that results are essentially driven by the spreads between the risk-free rate, the rate on bank deposit and that on CBDC.

The spread $r^* - r^b$, chosen by banks, is independent of the risk-free rate, as shown by (27). Deposit and CBDC demand also depend essentially on the spreads. Seigniorage, as shown by (32), is the sum of two components, one due to reserves and one due to CBDC. The one due to CBDC is largely dominant and depends only on the spread $r^* - r^c$, rather than on the value of the risk-free rate. Hence, the value of the risk-free rate does not impact results in a significant way, and neither does inflation, affecting individual rates but not spreads.

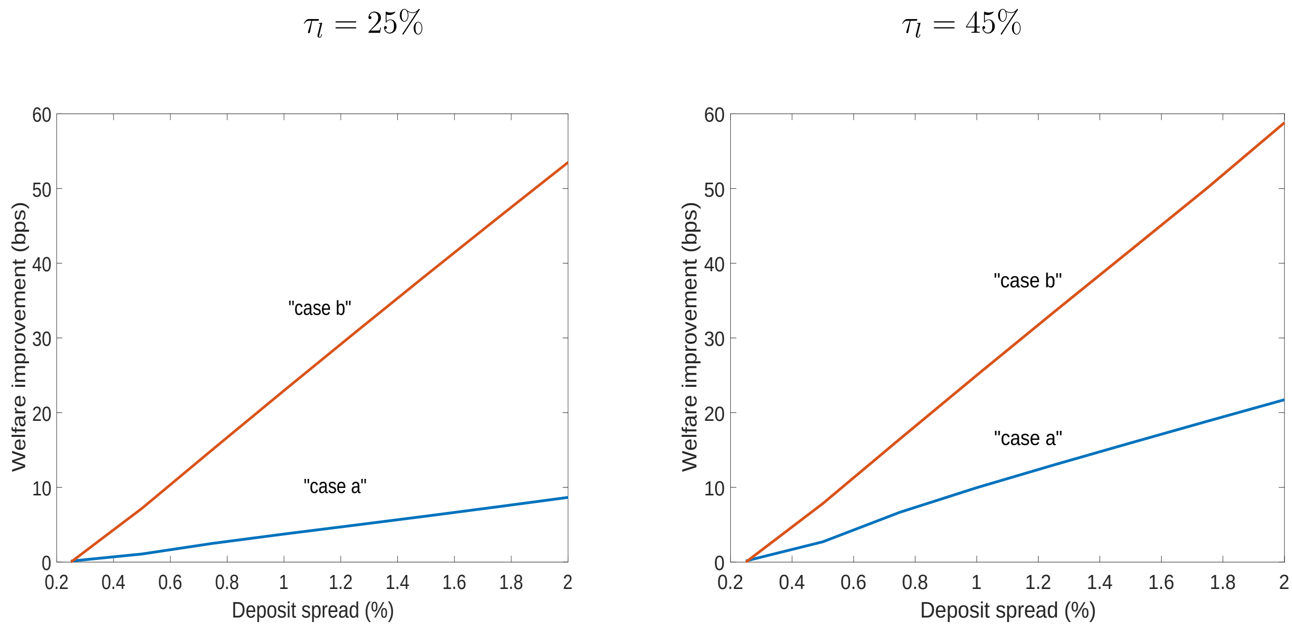
The only caveat is if there is a zero lower-bound on nominal rates. If so, assuming, as in our calibration, that the desired spread $r^* - r^b$ for banks is 2%, when the nominal risk-free rate is below 2% banks are forced to apply a spread lower than the desired value. In the limit of zero nominal rate, the spreads between all three rates are zero and all three channels analyzed in this paper lose their effectiveness. The plots in Figure 9 show that the welfare improvement brought by CBDC depends essentially linearly on the bank deposit spread $r^* - r^b$, and is zero when this spread (minus the cost of managing deposits) is zero.

5.4.5 Alternative value for other parameters

Table 4 shows the welfare improvement brought by CBDC with some alternative parameter choices. In particular, we show results obtained with alternative values of the cost of managing deposits and loans, reserve requirement, the corporate tax rate (used in our model as the tax rate on bank profits), banks' degree of competition in the loan market, the working capital requirement for firms –affecting the extent to which firms are dependent on bank loans– and households' wealth as a fraction of annual consumption.

We see that the impact of these parameters is not extremely large. However, parameters affecting deposits have some impact on our results. In general, with parameter values implying that banks' rent collection on deposits is high (low reserve ratio, low

Figure 9: Welfare gain as a function of deposit spread



cost of managing deposits) the introduction of CBDC has a stronger welfare impact. Similarly, if the corporate tax rate is low, implying a stronger degree of inequality between households and bankers, the introduction of CBDC has a higher potential of smoothing such inequality and improving welfare.

Instead, results are essentially unaffected by a change in the parameters related to loans (the loan spread, the cost of managing loans, the working capital requirement, which affects the extent to which firms need to rely on bank loans), as the loan-extension function of banks is essentially unaffected by the introduction of CBDC. Household wealth has also no impact on results.

6 Conclusion

There is an intense discussion in policy circles about the potential introduction of a broad retail CBDC. While there are various microeconomic aspects related to its implementation, in this paper we consider its macroeconomic implications. Most likely, CBDC will not be a perfect substitute of cash or bank deposits. This imperfect sub-

Table 4: CBDC-induced welfare changes with alternative parameter values

	$\tau_l = 25\%$ case a	$\tau_l = 25\%$ case b	$\tau_l = 45\%$ case a	$\tau_l = 45\%$ case b
Baseline	+9 bps	+54 bps	+20 bps	+59 bps
$c^b = 0.005$	+7 bps	+45 bps	+16 bps	+47 bps
Reserve ratio = 0	+11 bps	+58 bps	+22 bps	+63 bps
Reserve ratio = 10%	+8 bps	+ 49 bps	+17 bps	+54 bps
$\tau^b = 35\%$	+8 bps	+ 48 bps	+17 bps	+ 53 bps
$\tau^b = 15\%$	+10 bps	+ 60 bps	+23 bps	+65 bps
$\epsilon^l = 4$	+9 bps	+54 bps	+20 bps	+59 bps
$\varphi = 0.3$	+9 bps	+54 bps	+20 bps	+59 bps
$wealth/c = 2$	+9 bps	+54 bps	+20 bps	+59 bps

stitutability is a key element in our analysis and we show the impact of CBDC under various degrees of substitutability. The open-economy assumption is another crucial aspect of our model as this offers alternative financing for banks and keeps the risk-free interest rate constant, thereby limiting the real implications of CBDC.

In our welfare analysis, we find that CBDC could be an instrument to mitigate two distortions in the economy: distortionary taxation and the opportunity cost of holding money, which is much higher than the cost of providing money. Clearly this benefit would be higher, the higher the extent of the distortions. In our benchmark case, we find that the benefits of CBDC in reducing distortions would be modest: even in economies with high labor taxes (around 45%), welfare would improve at most by 20 bps in consumption terms. Instead, we found higher welfare gains from the redistribution of rents associated to deposits from bankers to non-bankers. The welfare improvement to non-bankers (and to the whole population in the limit in which bankers are a negligible minority) could reach about 60 bps when taking into account this channel. The welfare gains might be higher in countries in which the Frisch elasticity and/or the interest semi-elasticity of money demand is very high. Indeed, these are the cases in which the two distortions mentioned above have stronger effect on the economy.

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Appendix

A. Household FOCs

FOC with respect to consumption

$$\frac{1}{c_t} = \lambda_t(1 + s(x_t) + x_t s'(x_t)) \tag{32}$$

Specialized to the case of the transaction cost in the form (7), (32) becomes

$$\frac{1}{c_t} = \lambda_t(1 + 2Ax_t - 2\sqrt{AB}) \tag{33}$$

FOC with respect to hours worked

$$h_t^{\gamma} = \lambda_t W_t(1 - \tau^h) \tag{34}$$

FOC with respect to bank deposits d_t^b

$$\lambda_t \left(1 - (Ax_t^2 - B)\alpha_b \left(\frac{d}{d_b}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \lambda_{t+1}(1 + r_t^b) \tag{35}$$

FOC with respect to CBDC d_t^c

$$\lambda_t \left(1 - (Ax_t^2 - B)\alpha_c \left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \lambda_{t+1}(1 + r_t^c) \tag{36}$$

FOC with respect to the risk-free asset a_t

$$\lambda_t = \lambda_{t+1}(1 + r^*) \tag{37}$$

(33), (34), (35), (36) and (37) imply the three Euler equations

$$\begin{aligned}
\frac{1}{c_t(1 + 2Ax_t - 2\sqrt{AB})}\left(1 - (Ax_t^2 - B)\alpha_b \left(\frac{d}{d_b}\right)^{\frac{1}{\epsilon_{cb}}}\right) &= \beta(1 + r_t^b)\frac{1}{c_{t+1}(1 + 2Ax_{t+1} - 2\sqrt{AB})} && (38)\\
\frac{1}{c_t(1 + 2Ax_t - 2\sqrt{AB})}\left(1 - (Ax_t^2 - B)\alpha_c \left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right) &= \beta(1 + r_t^c)\frac{1}{c_{t+1}(1 + 2Ax_{t+1} - 2\sqrt{AB})} && (39)\\
\frac{1}{c_t(1 + 2Ax_t - 2\sqrt{AB})} &= \beta(1 + r^*)\frac{1}{c_{t+1}(1 + 2Ax_{t+1} - 2\sqrt{AB})} && (40)
\end{aligned}$$

and the labor/leisure tradeoff condition

$$h_t^{\gamma} = \frac{W_t(1 - \tau^l)}{c_t(1 + 2Ax_t - 2\sqrt{AB})} \tag{41}$$

B. Steady state equations

In steady state, the central bank pays a constant rate r^m on bank reserves and r^c on CBDC; banks pay a constant trate r^b on deposits, related to the rate on reserves and to model parameters by (27), and demand a constant loan rate r^l given by (28). The unit cost of capital is thus $r^k = \varphi r^l + (1-\varphi)r^*$. Given these rates, households choose a constant money velocity

$$x = \sqrt{\frac{\frac{r^*-r^b}{\alpha_b} f^{-\frac{1}{\epsilon_{cb}}} + B(1+r^*)}{(1+r^*)A}} \tag{42}$$

with

$$f = \left(\alpha_c \left(\frac{\alpha_c}{\alpha_b} \times \frac{r^* - r^b}{r^* - r^c}\right)^{\epsilon_{cb}-1} + \alpha_b\right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{43}$$

The other relevant variables of the model, consumption c, labor h, capital k, wages w, loans l, bank deposits d^b, CBDC d^c, bank profits Π are determined by the following equations

$$\begin{aligned}
c(1+s(x)) &= (1-\tau_h)wh + r^*a + r^b d^b + \zeta(1-\tau^b)\Pi^b - t_t && (44)\\
d^b &= \frac{c}{f}\sqrt{\frac{(1+r^*)A}{\frac{r^*-r^b}{\alpha_b} f^{-\frac{1}{\epsilon}} + B(1+r^*)}} && (45)\\
d^c &= \left(\frac{\alpha_c}{\alpha_b} \times \frac{r^* - r^b}{r^* - r^c}\right)^{\epsilon_{cb}} d^b && (46)\\
h^\gamma &= \frac{w(1-\tau^l)}{c(1+s(x)+xs'(x))} && (47)\\
k &= \left(\frac{z\alpha}{r^K}\right)^{\frac{1}{1-\alpha}} h && (48)\\
w &= (1-\alpha)z\left(\frac{z\alpha}{r^K}\right)^{\frac{\alpha}{1-\alpha}} && (49)\\
l &= \varphi k && (50)\\
\Pi &= (r^* - r^b)d^b + (r_l - r^*)l && (51)
\end{aligned}$$

Finally, given households' wealth, assets invested in the risk-free asset are

$$a = wealth - d^b - d^c \tag{52}$$