Infrequent Random Portfolio Decisions in an Open Economy Model[1]

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Abstract

We introduce a portfolio friction in a two-country DSGE model where investors face a constant probability to make new portfolio decisions. The friction leads to a more gradual portfolio adjustment to shocks and a weaker portfolio response to changes in expected excess returns. We apply the model to monthly data for the US and rest of the world for equity portfolios. We show that the model is consistent with a broad set of evidence related to portfolios, equity prices and excess returns for an intermediate level of the friction. The evidence includes portfolio inertia, limited sensitivity to expected excess returns, a significant impact of financial shocks, excess return predictability, and asset price momentum and reversal.

JEL classification: F30, F41, G11, G12

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1 Introduction

In the last decade, DSGE open economy models have increasingly incorporated portfolio choice.[1] However, the implications of these largely frictionless models contrast sharply with the evidence on asset prices, excess returns and portfolios. In this paper we introduce a financial friction whereby investors make infrequent portfolio decisions. Analogous to Calvo price setting, investors make new portfolio decisions each period with a probability p. We analyze the implications for optimal portfolio choice and use data on equity prices, excess returns and portfolios to show that this friction allows us to more closely fit the data.

Frictionless portfolio choice models imply that investors respond only to expected excess returns in the immediate future, to which portfolios are extremely sensitive. Moreover, past portfolio choice has no impact on current portfolio choice. This is inconsistent with micro evidence on the portfolio behavior of households and mutual funds. In addition it has implications for asset prices and excess returns that are inconsistent with the data.

With regard to households portfolio choice, the Investment Company Institute reports that 60 percent make no change to their stock or mutual fund portfolio over the course of a year.[2] This is consistent with a substantial literature that has documented portfolio inertia by households.[3] Giglio et al. (2021), using a survey of US based Vanguard investors, document a response of equity portfolio shares to expected returns that is too weak to make sense in the context of frictionless models. They further provide evidence that changes in expected returns have limited explanatory power for when investors trade, but help predict the direction and the magnitude of trading conditional on its occurrence. They suggest that

[1] Examples include Benhima and Cordonier (2020), Camanho et al. (2020), Davis and van Wincoop (2018), Devereux and Sutherland (2007, 2010), Didier and Lowenkron (2012), Dou and Verdelhan (2015), Evans and Hnatkovska (2012, 2014), Gabaix and Maggiori (2015), Hau and Rey (2006), Hnatkovska (2010), Koijen and Yogo (2020), and Tille and van Wincoop (2010a,b, 2014).

[2] In the year 2001, 60 percent made no change (see Equity Ownership of America, 2002). In 2007, 57 percent made no change (see Equity and Bond Ownership in America, 2008).

[3] See for example Ameriks and Zeldes (2004), Bilias et al. (2010), Brunnermeier and Nagel (2008) and Mitchell et al. (2006). Even stronger inertia applies to retirement portfolios. The Investment Company Institute (2021) reports that for over 30 million employer-based defined contribution plans, 90 percent of investors made no changes during a given year. Mitchell et al. (2006) reports similar evidence.

this can be captured by introducing infrequent random trading à la Calvo, which we do in this paper.

Bacchetta et al. (2021), provide evidence on the importance of portfolio frictions based on the international portfolio allocation of US equity mutual funds. They report strong evidence of a gradual portfolio response and a weak response to changes in expected returns. Camanho et al. (2020) also use data for international equity mutual funds, providing evidence of infrequent portfolio rebalancing.

Frictionless portfolio choice models have several unrealistic implications for asset prices and excess returns. The first is that asset prices are very little affected by financial shocks, which can be thought of as latent asset demand shocks or exogenous portfolio shifts.[4] Since portfolios are quite sensitive to expected excess returns, very small asset price changes are sufficient to clear markets in response to such financial shocks.[5] This contrasts with evidence of large price impact of financial shocks. Gabaix and Koijen (2021), using granual IV, show that a one percent increase in US equity demand raises the equity price by five percent. They argue that this is about a factor 100 times larger than in frictionless models. This large price impact implies that financial shocks are the dominant driver of asset prices, as illustrated by Gabaix and Koijen (2021) as well. This is also consistent with Koijen and Yogo (2019), who provide evidence that latent asset demand shocks are the main driver of equity prices. Similarly, Itskhoki and Muhkin (2021) show that exchange rates are largely driven by financial shocks, which accounts for the disconnect from macro fundamentals. Gabaix and Maggiori (2015) also emphasize the importance of financial shocks for exchange rates, for which they cite a variety of evidence.

Frictionless models also have unrealistic implications for expected excess returns. The extreme sensitivity of portfolios to expected excess returns implies that in equilibrium expected excess returns are very small. Excess returns are therefore hard to predict, in contrast with lots of evidence for both equity and currency markets. Related, there is widespread evidence in many financial markets of excess return momentum and reversal. Excess returns are positively autocorre-

[4]Examples are portfolio shifts due to changes in risk aversion or the risk-bearing capacity of financial institutions, liquidity trade, noise trade or FX intervention.

[5]Tille and van Wincoop (2014) show that first-order changes in portfolio shares are associated with third-order changes in expected returns because expected excess returns are divided by second order moments (e.g. the variance of the excess return) in optimal portfolios.

lated at short horizons (momentum) and negatively over longer horizons (reversal). This is hard to explain in frictionless portfolio choice models.

Another significant feature of equity holdings is limited participation. Chien et al. (2020) and Zhang (2021) introduce limited stock market participation in a two-country model and show that this can contribute to explain limited international risk sharing. However, in contrast to infrequent portfolio adjustment, limited participation by itself does not generate the forward and backward-looking portfolio features that can explain excess return dynamics like momentum and reversal.

We introduce the portfolio friction in a two country model where investors trade equity from both countries and a risk-free bond. There are both dividend shocks and financial shocks. The model is solved with a global solution method. But to develop intuition, we derive an approximate expression of the equity portfolio share in the Home country that uses techniques related to Campbell and Viceira (1999). The optimal portfolio depends on the lagged portfolio and the expected present discounted value of all future excess returns. It is perturbed by additive exogenous portfolio shocks (financial shocks). There is also a hedge term that captures risk associated with future returns, but it is numerically not important.

We apply the model to monthly data for the United States versus the rest of the world (an aggregate of 44 countries). We consider implications for portfolio behavior, equity prices and international equity return differentials. For different values of p, we give the model the benefit of the doubt by calibrating the parameters of the financial shock process to fit some key moments.

We find that the model fits the data best for an intermediate friction of $p = 0.1$. For a stronger portfolio friction (lower p), the excess return is too autocorrelated, there is too much excess return momentum, the expected excess return is too volatile and the portfolio response to expected excess returns is too weak. But the model performs worst without the portfolio friction. Portfolios are excessively sensitive to expected excess returns compared to the data and the price impact of financial shocks is much weaker than seen in the data. The latter implies that exogenous financial flows need to be implausibly large to be consistent with various data moments. Even then the frictionless model is inconsistent with evidence related to expected excess returns. It also does not feature asset price momentum and reversal.

The paper fits into a broader literature of portfolio frictions that lead to gradual portfolio adjustment. In analogy to price setting, there are three ways of modeling gradual portfolio adjustment: investors make new portfolio decisions every T periods (like Taylor price setting), with a given probability p (like Calvo price setting) or they face a quadratic cost of changing portfolio shares (like Rotemberg cost of price changes). This is the first paper that adopts the Calvo setup. It also distinguishes itself by considering a broad set of implications for both portfolios and asset prices.

The most common assumption in this literature is staggered portfolio decisions every T periods.[6] One drawback is that it leads to bumpy impulse response functions as agents adjusting their portfolios at the time of a shock will predictably do so again in T periods. A quadratic portfolio adjustment cost has been adopted by Bacchetta and van Wincoop (2021), Bacchetta et al. (2021), Bacchetta et al. (2022), Gârleanu and Pedersen (2013) and Vayanos and Woolley (2012). This is easier to work with, but it is less realistic as it does not reflect the fact that investors change their portfolios at different times, as reported for example by Giglio et al. (2021).[7]

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 discusses an approximation of the optimal portfolio in order to develop intuition. Section 4 discusses data and calibration and Section 5 presents the empirical results. Section 6 concludes.

2 Model

There are two countries, Home (H) and Foreign (F). There is a single good. In both countries there is a continuum of agents on the interval $[0, 1]$ who have infinite lives and make decisions about consumption and portfolio allocation. Agents of

[6]For recent contributions, see Abel et al. (2007), Bacchetta and van Wincoop (2010), Bogousslavsky (2016), Chien et al. (2012), Duffie (2010), Greenwood et al. (2018) and Hendershott et al. (2013). Earlier papers examine the impact of infrequent portfolio adjustments taking the process of asset returns as exogenous, e.g. see Lynch (1996) or Gabaix and Laibson (2002).

[7]The Calvo setup implies heterogeneity in portfolio shares across investors, which is an important difference from the portfolio adjustment cost approach, but one that we will not explore here. Another difference relates to the aggregate portfolio expression. While in both cases the aggregate portfolio depends on the lagged portfolio and a present discounted value of future expected excess returns, the coefficient on the latter is very different. In the Calvo setup it depends on long-term risk about future excess returns, while in the adjustment cost setup it is largely driven by the exogenous adjustment cost parameter.

both countries can hold three assets: Home and Foreign equity and a risk-free bond.

2.1 Infrequent Decision Making

The key aspect of the model is infrequent decision making about consumption and portfolios. Analogous to Calvo price setting, we assume that agents make new decisions with a probability p. However, infrequent decision making only affects portfolio choice: we assume an intertemporal elasticity of substitution of 1, which implies that optimal consumption is a constant fraction of wealth. Agents therefore do not need to rethink their consumption choice. For portfolio choice we assume that the fraction $1-p$ of agents that does not make new portfolio decisions will hold their portfolio shares constant until the time comes that they make a new portfolio decision.[8]

2.2 Assets

Agents can invest in Home equity, Foreign equity and a one-period risk-free bond. The number of equity shares is normalized to 1 in both countries, while bonds are in zero net supply. The gross interest rate on the bond is denoted R_t. The returns on Home and Foreign equity from t to $t+1$ are

$$R_{H,t+1} = \frac{Q_{H,t+1} + D_{H,t+1}}{Q_{H,t}} \tag{1}$$

$$R_{F,t+1} = \frac{Q_{F,t+1} + D_{F,t+1}}{Q_{F,t}}, \tag{2}$$

where $Q_{H,t}$ and $Q_{F,t}$ are the prices of Home and Foreign equity shares and $D_{H,t}$, $D_{F,t}$ are dividends.

Define the relative and average log dividends as $d_t^D = d_{H,t} - d_{F,t}$ and $d_t^A = 0.5(d_{H,t} + d_{F,t})$. They are assumed to follow an AR process:

$$d_t^D = \rho_d d_{t-1}^D + \epsilon_t^{d,D} \tag{3}$$

$$d_t^A = (1-\rho_d)d + \rho_d d_{t-1}^A + \epsilon_t^{d,A}. \tag{4}$$

[8] An alternative, not explored here, is that agents hold the quantity of asset holdings constant. This is analogous to a buy-and-hold portfolio, in which case there is no rebalancing. In our specification, even the agents that do not make new portfolio decisions still trade to rebalance their portfolio. This can for example be achieved by investing in a mutual fund. While in reality a combination of both is realistic, this would significantly complicate our analysis.

The shocks to relative and average log dividends are assumed to be uncorrelated. The standard deviations of the shocks are σ_{d^D} and σ_{d^A}.

2.3 Budget Constraints

We focus mostly on describing Home agents. For Foreign agents we simply need to replace the H with an F. Consider agent i in the Home country who makes a new portfolio decision at time t. First some notation is in order. Let $W^i_{H,t}$ be the wealth of the agent at the start of period t and $cw^i_{H,t}$ the fraction of wealth that is consumed. The remainder is then invested in the three assets. A fraction $z^i_{HH,t}$ is invested in Home equity and $z^i_{HF,t}$ in Foreign equity. The remainder is invested in the bond. We also denote $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$ as the average portfolio shares of all Home agents that make new portfolio decisions at time t. In equilibrium $z^i_{HH,t} = \tilde{z}_{HH,t}$ and $z^i_{HF,t} = \tilde{z}_{HF,t}$ for investors making new portfolio decisions at time t. But we will make this substitution only after deriving the first-order conditions for agent i. For Foreign agents we denote the fractions allocated to the Home and Foreign equity as $z^i_{FH,t}$ and $z^i_{FF,t}$.

Wealth of agent i making a new consumption and portfolio decision at time t then evolves according to

$$W^i_{H,t+1} = (1 - cw^i_{H,t})W^i_{H,t}R^{p,H,i}_{t+1} \tag{5}$$

where the portfolio return is

$$\begin{aligned} R^{p,H,i}_{t+1} \quad = \quad & R_t + z^i_{HH,t}(e^{\tau_{H,t}}R_{H,t+1} - R_t) + z^i_{HF,t}(e^{-\tau_{H,t}}R_{F,t+1} - R_t) + \\ & -\tilde{z}_{HH,t}(1 - e^{\tau_{H,t}})R_{H,t+1} + \tilde{z}_{HF,t}(1 - e^{-\tau_{H,t}})R_{F,t+1}. \end{aligned} \tag{6}$$

Here $\tau_{H,t}$ is a tax on the Foreign investment return and subsidy on the Home investment return, which will be discussed further below. The aggregate of this tax/subsidy across all Home agents making a portfolio decision at time t is reimbursed through the last two terms of (6). This assures that it will only affect portfolio allocation, not overall wealth accumulation. Analogously, for Foreign agents $\tau_{F,t}$ is a tax on the Home return and subsidy on the Foreign return.

Wealth of Home agent i who does not make new consumption/portfolio decisions at time t, and last made new decisions at $t - j$, evolves according to

$$W^i_{H,t+1} = (1 - cw^i_{H,t-j})W^i_{H,t}R^{p,H,i,t-j}_{t+1} \tag{7}$$

where the portfolio return is

$$\begin{aligned} R_{t+1}^{p,H,i,t-j} &= R_t + z_{HH,t-j}^i (e^{\tau_{H,t-j}} R_{H,t+1} - R_t) + z_{HF,t-j}^i (e^{-\tau_{H,t-j}} R_{F,t+1} - R_t) + \\ & \quad -\tilde{z}_{HH,t-j}(1 - e^{\tau_{H,t-j}}) R_{H,t+1} + \tilde{z}_{HF,t-j}(1 - e^{-\tau_{H,t-j}}) R_{F,t+1}. \end{aligned} \tag{8}$$

The portfolio return has an extra $t-j$ superscript to denote when consumption/portfolio decisions were last made. The consumption-wealth ratio and portfolio shares are those chosen at $t-j$. The tax/subsidy is also at $t-j$ as it is held constant until a new portfolio decision is made.

After deriving the portfolio Euler equations, we will substitute $z_{HH,t}^i = \tilde{z}_{HH,t}$ and $z_{HF,t}^i = \tilde{z}_{HF,t}$. The same is done for portfolio shares prior to time t. The portfolio return of agents who last made a portfolio decision at time $t-j$ is then

$$R_{t+1}^{p,H,t-j} = R_t + \tilde{z}_{HH,t-j}(R_{H,t+1} - R_t) + \tilde{z}_{HF,t-j}(R_{F,t+1} - R_t) \tag{9}$$

The tax/subsidy $\tau_{H,t-j}$ no longer enters.

2.4 Financial Shocks

The tax/subsidy $\tau_{H,t}$ for Home agents and $\tau_{F,t}$ for Foreign agents plays two roles. First, their mean level τ can be set to generate realistic average portfolio home bias. Second, their changes over time generate exogenous portfolio shifts, which we will refer to as financial shocks.

Define their relative and average values as $\tau_t^D = \tau_{H,t} - \tau_{F,t}$ and $\tau_t^A = 0.5(\tau_{H,t} + \tau_{F,t})$. We assume that they follow AR processes:

$$\tau_t^D = \rho_\tau \tau_{t-1}^D + \epsilon_t^{\tau,D} \tag{10}$$

$$\tau_t^A = (1 - \rho_\tau)\tau + \rho_\tau \tau_{t-1}^A + \epsilon_{F,t}^{\tau,A} \tag{11}$$

The relative and average shocks are assumed to be uncorrelated. The standard deviations of the shocks are σ_{τ^D} and σ_{τ^A}.

A rise in τ_t^D generates an exogenous portfolio shift from Foreign to Home equity. A rise in τ_t^A generates an exogenous portfolio shift from foreign to domestic equity (increased home bias). We refer to these as financial shocks. These exogenous portfolio shifts, unrelated to endogenous changes in expected returns and risk, can be introduced in many other ways. In the literature they sometimes are modeled in the form of noise trade, liquidity trade, hedge trade, time-varying risk-bearing capacity or time-varying investment opportunities.[9] We do not wish to take a strong stand on what the exact origin of these portfolio shocks is.

2.5 Bellman Equations

Agents are assumed to have Rince preferences, which for any agent (from Home or Foreign) we can write as

$$\ln(V_t) = \max_{c_t, \mathbf{z}_t} \left\{ (1-\beta)\ln(c_t) + \beta \ln\left(\left[E_t V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right) \right\}, \tag{12}$$

where c_t is consumption and $\mathbf{z}_t$ the vector of portfolio shares. This implies an intertemporal elasticity of substitution (IES) of 1 and a rate of risk aversion of γ.

Let $V_t^{n,i}$ be the value function of Home agent i who makes new consumption/portfolio decisions at time t. Similarly, $V_t^{o,i,t-j}$ is the value function of Home agent i who does not make new decisions at time t and who last made a consumption/portfolio decisions at $t-j$. Here o stands for "old". For either of these agents, there is a probability p that they make a new portfolio decision at $t+1$ and a probability $1-p$ that they do not. We can then write the Bellman equations for these respective agents as

$$\begin{aligned} \ln(V_t^{n,i}) &= \max_{cw^i_{H,t}, z^i_{HH,t}, z^i_{HF,t}} \Big\{ (1-\beta)\ln(cw^i_{H,t} W^i_{H,t}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left(pE_t \left(V_{t+1}^{n,i}\right)^{1-\gamma} + (1-p)E_t\left(V_{t+1}^{o,i,t}\right)^{1-\gamma} \right) \Big\} \qquad (13) \\ ln(V_t^{o,i,t-j}) &= (1-\beta)\ln(cw^i_{H,t-j} W^i_{H,t}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left(pE_t \left(V_{t+1}^{n,i}\right)^{1-\gamma} + (1-p)E_t\left(V_{t+1}^{o,i,t-j}\right)^{1-\gamma} \right). \qquad (14) \end{aligned}$$

The value functions will be proportional to the wealth of the agent. We will therefore write

$$V_t^{n,i} = W^i_{H,t} e^{f^n(S_t)} \tag{15}$$

$$V_t^{o,i,t-j} = W^i_{H,t} e^{f^o(S_t, z^i_{HH,t-j}, z^i_{HF,t-j}, \tilde{z}_{HH,t-j}, \tilde{z}_{HF,t-j}, \tau_{H,t-j})}. \tag{16}$$

[9] For different ways of modeling the portfolio shocks, see Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Gabaix and Maggiori (2015), He and Wang (1995), Spiegel and Subrahmanyam (1992) and Wang (1994).

Here S_t is a vector of aggregate state variables, which will be defined below. Apart from the wealth of the agent, the value function of an agent making new consumption/portfolio decisions at time t only depends on the aggregate state S_t through the function f^n. The value function of an agent who last made a portfolio decision at $t-j$ also depends, through the function f^o, on the portfolio shares at time $t-j$, $z^i_{HH,t-j}$ and $z^i_{HF,t-j}$.[10] It also depends on $\tilde{z}_{HH,t-j}$, $\tilde{z}_{HF,t-j}$ and $\tau_{H,t-j}$, as they affect portfolio returns $R^{p,i,H,t-j}_{t+s}$ until new portfolio decisions are made. The functions f^n and f^o evaluated at their respective state variables at time t are also denoted $f^n_{H,t}$ and $f^{o,i,t-j}_{H,t}$ for Home agents.

Substituting (15) and (16) into (13) and (14), and using the wealth accumulation equations, we can write the Bellman equations as

$$\begin{aligned} f^n_{H,t} &= \max_{cw^i_{H,t}, z^i_{HH,t}, z^i_{HF,t}} \Bigg\{ (1-\beta) ln(cw^i_{H,t}) + \beta \ln(1 - cw^i_{H,t}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left(E_t \left(pe^{(1-\gamma) f^n_{H,t+1}} + (1-p) e^{(1-\gamma) f^{o,i,t}_{H,t+1}} \right) \left(R^{p,H,i}_{t+1} \right)^{1-\gamma} \right) \Bigg\} \qquad (17) \\ f^{o,i,t-j}_{H,t} &= (1-\beta) ln(cw^i_{H,t-j}) + \beta \ln(1 - cw^i_{H,t-j}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left(E_t \left(pe^{(1-\gamma) f^n_{H,t+1}} + (1-p) e^{(1-\gamma) f^{o,i,t-j}_{H,t+1}} \right) \left(R^{p,H,i,t-j}_{t+1} \right)^{1-\gamma} \right). \qquad (18) \end{aligned}$$

When the individual-specific portfolio shares $z^i_{HH,t-j}$ and $z^i_{HF,t-j}$ are evaluated at the equilibrium portfolio shares $\tilde{z}_{HH,t-j}$ and $\tilde{z}_{HF,t-j}$ for agents last making portfolio decisions at $t-j$, we omit the i supercript and write

$$f^{o,t-j}_{H,t+1} = f^o(S_{t+1}, \tilde{z}_{HH,t-j}, \tilde{z}_{HF,t-1}, \tilde{z}_{HH,t-j}, \tilde{z}_{HF,t-j}, \tau_{H,t-j})$$

It is also useful to define

$$\lambda^t_{HH,t+1} = \frac{\partial f^{o,i,t}_{H,t+1}}{\partial z^i_{HH,t}} \qquad (19)$$

$$\lambda^t_{HF,t+1} = \frac{\partial f^{o,i,t}_{H,t+1}}{\partial z^i_{HF,t}}. \qquad (20)$$

These derivatives are again evaluated by setting the agent i-specific portfolio shares equal to $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$.

[10] In principle the lagged consumption wealth decision $cw^i_{H,t-j}$ should enter as well, but we will see that this remains constant over time.

2.6 Portfolio Euler Equations

Maximizing the right hand side of (17) with respect to $cw^i_{H,t}$, $z^i_{HH,t}$ and $z^i_{HF,t}$, using the portfolio return (6), gives three first-order conditions. The first-order condition with respect to the consumption-wealth ratio simply gives $cw^i_{H,t} = 1-\beta$. Agents therefore always consume a fraction $1-\beta$ of their wealth, so that the infrequent decision making only matters for portfolio choice. Home agent i then invest $\beta W^i_{H,t}$ in the three assets.

For the portfolio Euler equations it is useful to define scaled stochastic discount factors:

$$m^{n,t-j}_{H,t+1} = \left[R^{p,H,t-j}_{t+1}\right]^{-\gamma} e^{(1-\gamma)f^n_{H,t+1}}$$
$$m^{o,t-j}_{H,t+1} = \left[R^{p,H,t-j}_{t+1}\right]^{-\gamma} e^{(1-\gamma)f^{o,t-j}_{H,t+1}}.$$

These are scaled stochastic discount factors for an agent who last made portfolio decisions at $t-j$, conditional on the agent respectively making a new portfolio decision at $t+1$ and not making a new portfolio decision at $t+1$. We also define an unconditional stochastic discount factor as $m^{t-j}_{H,t+1} = pm^{n,t-j}_{H,t+1} + (1-p)m^{o,t-j}_{H,t+1}$.[11]

After taking the derivatives of (17) with respect to $z^i_{HH,t}$ and $z^i_{HF,t}$, and then setting these portfolio shares equal to $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$, we obtain the following portfolio Euler equations

$$E_t m^t_{H,t+1}(e^{\tau_{H,t}}R_{H,t+1} - R_t) + (1-p)E_t m^{o,t}_{H,t+1} R^{p,H,t}_{t+1}\lambda^t_{HH,t+1} = 0 \qquad (21)$$
$$E_t m^t_{H,t+1}(e^{-\tau_{H,t}}R_{F,t+1} - R_t) + (1-p)E_t m^{o,t}_{H,t+1} R^{p,H,t}_{t+1}\lambda^t_{HF,t+1} = 0. \qquad (22)$$

The first terms in (21)-(22) are the expected excess returns discounted with the pricing kernel. When agents make new portfolio decisions each period ($p=1$), equating these first terms to zero gives the portfolio Euler equations. The second term applies when $p<1$, so it specifically relates to infrequent portfolio decisions. It captures the impact of future expected returns and risk beyond period $t+1$, which affect $\lambda^t_{HH,t+1}$ and $\lambda^t_{HF,t+1}$. Knowing that they may not get an opportunity to change their portfolio allocation again for some time, agents who make portfolio decisions at time t need to incorporate beliefs about expected returns and risk beyond time $t+1$.

[11] The SDF for Rince preferences is $[c_t/c_{t+1}][V^{1-\gamma}_{t+1}/E_t V^{1-\gamma}_{t+1}]$. After substituting the solution for consumption, wealth accumulation, (15)-(16), and multiplying by $\beta E_t R^{p,H,t-j}_{t+1} m^{t-j}_{H,t+1}$, the scaled discount factors are obtained.

We can then also write the Bellman equations as

$$e^{(1-\gamma)f^n_{H,t}/\beta} = \alpha E_t m^t_{H,t+1} R^{p,H,t}_{t+1} \tag{23}$$

$$e^{(1-\gamma)f^{o,t-1}_{H,t}/\beta} = \alpha E_t m^{t-1}_{H,t+1} R^{p,H,t-1}_{t+1} \tag{24}$$

where $\alpha = (1-\beta)^{(1-\gamma)(1-\beta)/\beta}\beta^{1-\gamma}$. These are for an agent who last made a portfolio decision at time t and $t-1$.

In the first-order conditions $\lambda^t_{HH,t+1}$ and $\lambda^t_{HF,t+1}$ play an important role. Their values one period earlier, so $\lambda^{t-1}_{HH,t}$ and $\lambda^{t-1}_{HF,t}$, will be control variables to be solved as a function of the state at time t. Expressions for them can be obtained by considering an agent who last made a portfolio decision at time $t-1$, but does not make a new portfolio decision at time t. Taking derivatives of (18) for $j=1$ with respect to $z^i_{HH,t-1}$ and $z^i_{HF,t-1}$, and then setting the agent i portfolio shares equal to $\tilde{z}_{HH,t-1}$ and $\tilde{z}_{HF,t-1}$, we have

$$E_t R^{p,H,t-1}_{t+1}\left(m^{t-1}_{H,t+1}\lambda^{t-1}_{HH,t} - \theta m^{o,t-1}_{H,t+1}\lambda^{t-1}_{HH,t+1}\right) = \beta E_t m^{t-1}_{H,t+1}(e^{\tau_{H,t-1}}R_{H,t+1} - R_t) \tag{25}$$

$$E_t R^{p,H,t-1}_{t+1}\left(m^{t-1}_{H,t+1}\lambda^{t-1}_{HF,t} - \theta m^{o,t-1}_{H,t+1}\lambda^{t-1}_{HF,t+1}\right) = \beta E_t m^{t-1}_{H,t+1}(e^{-\tau_{H,t-1}}R_{F,t+1} - R_t) \tag{26}$$

where $\theta = \beta(1-p)$. While we will not do so, one can use these to write the portfolio Euler equations (21)-(22) as equating an expected present discounted value of all future excess returns, multiplied by appropriate stochastic discount factors, equal to zero.

2.7 Market Clearing Conditions

There are three market clearing conditions: for Home equity, Foreign equity and bonds. Denote $z_{jk,t} = \int_0^1 z^i_{jk,t}di$ for $j=H,F$ and $k=H,F$. Similarly, aggregate Home and Foreign wealth is $W_{H,t} = \int_0^1 W^i_{H,t}di$ and $W_{F,t} = \int_0^1 W^i_{F,t}di$. Market clearing conditions will then be[12]

$$z_{HH,t}W_{H,t} + z_{FH,t}W_{F,t} = Q_{H,t}/\beta \tag{27}$$

$$z_{HF,t}W_{H,t} + z_{FF,t}W_{F,t} = Q_{F,t}/\beta \tag{28}$$

$$(1 - z_{HH,t} - z_{HF,t})W_{H,t} + (1 - z_{FH,t} - z_{FF,t})W_{F,t} = 0. \tag{29}$$

[12]This uses that $\int_0^1 z^i_{jk,t}W^i_{j,t}di = z_{jk,t}W_{j,t}$ for $j=H,F$ and $k=H,F$. This abstracts from a small aggregation issue discussed in Appendix B that portfolio shares and wealth may be cross-sectionally correlated. This turns out to be numerically unimportant.

2.8 Control and State Variables

The control and state variables are respectively

$$cv_t = \{q_{H,t}, q_{F,t}, r_t, \tilde{z}_{HH,t}, \tilde{z}_{HF,t}, \tilde{z}_{FF,t}, \tilde{z}_{FH,t}, f^n_{H,t}, f^n_{F,t}, cv_{H,t}, cv_{F,t}\}' \quad (30)$$

$$sv_t = \{S_t, s_{H,t}, s_{F,t}\}' \quad (31)$$

where $q_{H,t}$, $q_{F,t}$ and r_t are the log equity prices and interest rate, and[13]

$$\begin{aligned}
cv_{H,t} &= \{f^{o,t-1}_{H,t}, \lambda^{t-1}_{HH,t}, \lambda^{t-1}_{HF,t}\} \\
cv_{F,t} &= \{f^{o,t-1}_{F,t}, \lambda^{t-1}_{FH,t}, \lambda^{t-1}_{FF,t}\} \\
S_t &= \{d_{H,t}, d_{F,t}, \tau_{H,t}, \tau_{F,t}, w^D_t, w^D_{t-1}, z^A_{H,t-1}, z^D_{H,t-1}, z^D_{F,t-1}\} \\
s_{H,t} &= \{\tau_{H,t-1}, \tilde{z}_{HH,t-1}, \tilde{z}_{HF,t-1}\} \\
s_{F,t} &= \{\tau_{F,t-1}, \tilde{z}_{FH,t-1}, \tilde{z}_{FF,t-1}\}.
\end{aligned}$$

The last five state variables in S_t are relative log wealth $w^D_t = ln(W_{H,t}) - ln(W_{F,t})$, w^D_{t-1}, $z^A_{H,t-1} = \omega_{t-1} z_{HH,t-1} + (1-\omega_{t-1}) z_{FH,t-1}$, $z^D_{H,t-1} = z_{HH,t-1} - z_{FH,t-1}$ and $z^D_{F,t-1} = z_{FF,t-1} - z_{HF,t-1}$. Here $\omega_t = W_{Ht}/(W_{Ht} + W_{Ft})$ is the relative wealth of the Home country.

Regarding the evolution of the state variables S_t, the processes for d^D_t, d^A_t, τ^D_t and τ^A_t are given by (3), (4), (10), and (11). The portfolio share $z_{HH,t}$ evolves according to

$$z_{HH,t} = (1-p) z_{HH,t-1} + p\tilde{z}_{HH,t} \quad (32)$$

with similar equations for $z_{HF,t}$, $z_{FH,t}$, and $z_{FF,t}$. Using (5) and (9), we have[14]

$$\begin{aligned}
w^D_{t+1} \;=\;& w^D_t + ln\left(R_t + z_{HH,t}(R_{H,t+1} - R_t) + z_{HF,t}(R_{F,t+1} - R_t)\right) - \\
& ln\left(R_t + z_{FH,t}(R_{H,t+1} - R_t) + z_{FF,t}(R_{F,t+1} - R_t)\right). \quad (33)
\end{aligned}$$

These dynamic equations for portfolio shares and wealth also tell us how the last three state variables in S_t evolve.[15]

[13] Although the control variables $cv_{H,t}$ and $cv_{F,t}$ are not of separate interest to us, we need to keep track of them as their values one period later enter the portfolio Euler equations.

[14] This uses the same approximation that we made for the market clearing conditions, that $\int_0^1 z^i_{jk,t} W^i_{j,t} di = z_{jk,t} W_{j,t}$ for $j = H, F$ and $k = H, F$, which is numerically extremely accurate (Appendix B).

[15] Relative wealth is stationary in the model. In Appendix F we discuss the logic behind the stationarity and report the ergodic distribution of relative wealth w^D_t for a parameterization that is discussed in Section 4.

2.9 Definition of Equilibrium and Solution

Appendix A lists the Foreign country portfolio Euler equations, Bellman equations and first-order difference equations for $\lambda_{FH,t}^{t-1}$ and $\lambda_{FF,t}^{t-1}$. These are all derived analogously to those for the Home country.

Definition 1 *An equilibrium consists of* $\left\{q_{H,t}, q_{F,t}, r_t, \tilde{z}_{HH,t}, \tilde{z}_{HF,t}, \tilde{z}_{FF,t}, \tilde{z}_{FH,t}, f_{H,t}^{n}, f_{F,t}^{n}\right\}$ *as functions of* S_t, $\left\{f_{H,t}^{o,t-1}, \lambda_{HH,t}^{t-1}, \lambda_{HF,t}^{t-1}\right\}$ *as functions of* S_t *and* $s_{H,t}$, *and* $\left\{f_{F,t}^{o,t-1}, \lambda_{FH,t}^{t-1}, \lambda_{FF,t}^{t-1}\right\}$ *as functions of* S_t *and* $s_{F,t}$ *such that the following are satisfied: (i) The Home portfolio Euler equations (21)-(22), (ii) the Home Bellman equations (23)-(24), (iii) the Home* λ *difference equations (25)-(26), (iv) the Foreign country analogues of (21)-(26) shown in Appendix A, and (v) the market clearing conditions (27)-(29).*

The model is solved with a global solution method. Appendix C discusses the details. The large number of state and control variables (a total of 15 each) makes it challenging to obtain a global solution using standard projection methods. There is a dimensionality problem both when control variables are approximated as step functions on a rectangular grid of state variables or as polynomial functions that minimize average equation errors on a large number of points of the state space. We therefore instead follow the Taylor projection method developed in Levintal (2018).[16] This involves approximating the solution locally at various nodes of the state space, and then combining these local solutions to form the global solution. This involves far fewer parameters, although it needs to be repeated at many points of the state space.

3 Approximate Portfolio Expression

In this section we discuss an approximate portfolio expression in order to develop intuition about what is driving portfolio allocation in the model.

3.1 Notation

Since our data in the next section applies to equity portfolio shares, we focus on the equity portfolio. For agents who make new portfolio decisions, the share of the

[16]Den Haan et al. (2016) develop an analogous method. The method is applied in Fernandez-Villaverde and Levintal (2018) and Barro et al. (2018) to solve models with rare disasters.

equity portfolio that is invested in Home equity by respectively Home and Foreign agents is denoted

$$\tilde{z}^e_{HH,t} = \frac{\tilde{z}_{HH,t}}{\tilde{z}_{HH,t} + \tilde{z}_{HF,t}}$$
$$\tilde{z}^e_{FH,t} = \frac{\tilde{z}_{FH,t}}{\tilde{z}_{FH,t} + \tilde{z}_{FF,t}}.$$

We are particularly interested in the average portfolio share invested in Home equity, $\tilde{z}^{e,A}_t = 0.5(\tilde{z}^e_{HH,t} + \tilde{z}^e_{FH,t})$. The main effect of infrequent portfolio decisions relates to the way investors respond to changes in expected excess returns. Expected excess returns affect the average portfolio share $\tilde{z}^{e,A}_t$, but not the difference in portfolio shares, $\tilde{z}^{e,D}_t = \tilde{z}^e_{HH,t} - \tilde{z}^e_{FH,t}$, which is a measure of equity home bias.[17]

Some notation regarding asset returns is in order as well. The excess return of Home over Foreign equity is denoted er_{t+1}, which is equal to $r_{H,t+1} - r_{F,t+1}$. Here lower case letters denote log returns. We also denote $er_{t+1,t+i} = er_{t+1} + ... + er_{t+i}$ as the cumulative excess return of Home equity over Foreign equity over the next i periods. Other cumulative returns are denoted analogously.

3.2 Approximated Portfolio

To derive approximate portfolio shares, we follow a methodology similar to Campbell and Viceira (1999), although the portfolio problem is considerably more complicated here. After a significant amount of algebra described in the Online Appendix,[18] we find the following approximate expression for the average Home equity portfolio share:

$$\tilde{z}^{e,A}_t = 0.5 + \frac{1}{D}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{1}{(1-\theta)D}\tau^D_t + h_t \qquad (34)$$

[17] Moreover, at least up to the time of the Great Recession, there has been a trend decrease in home bias for reasons that have little to do with gradual portfolio adjustment.

[18] We start by deriving expressions for $\tilde{z}_{HH,t}$, $\tilde{z}_{HF,t}$, $\tilde{z}_{FH,t}$ and $\tilde{z}_{FF,t}$ using portfolio Euler equations, Bellman equations, and λ difference equations. We log-linearize portfolio returns, though we treat the new time t portfolio shares as unknown parameters that need to be solved and do not linearize around these variables. Most expectations take the form of $E_t e^x$, where x includes log asset returns and the Bellman variables. Assuming log normality, these expectations are approximated as $e^{Ex+0.5var(x)}$, where Ex and $var(x)$ are moments that vanish to zero in the deterministic steady state. We then approximate this as $1 + Ex + 0.5var(x)$.

where

$$D=\sum_{i=1}^{\infty}\theta^{i-1}\left[\gamma v\bar{a}r_t(er_{t+i})+2(\gamma-1)c\bar{o}v_t(er_{t+i},er_{t+1,t+i-1})\right] \tag{35}$$

Moments with a bar refer to the mean of these moments.

The optimal portfolio depends on three terms. The first is a present discounted value of expected future excess returns (international equity return differentials). The second is proportional to τ_t^D, capturing financial shocks. A rise in τ_t^D leads to an exogenous portfolio shift from Foreign to Home equity. The last term, h_t, is a hedge term. It depends on time-varying expectations of future risk and is discussed in Appendix D.

3.3 Comparison to Frictionless Portfolio

It is instructive to compare (34) to what it would be when $p=1$:

$$z_t^{e,A}=\frac{E_t er_{t+1}}{\gamma v\bar{a}r_t(er_{t+1})}+\frac{1}{\gamma v\bar{a}r_t(er_{t+1})}\tau_t^D+h_t \tag{36}$$

We will focus here on the expected excess return term. When $p<1$ the average share invested in Home equity depends on the present discounted value of all expected future excess returns, as opposed to just the expected excess return over the next period as in (36). When investors make a new portfolio decision, they have a longer effective horizon when $p<1$ as they do not know when they will make a new portfolio decision again. The discount rate is $\theta=\beta(1-p)$. A lower value of p therefore implies a longer effective horizon and a higher weight on expected excess returns further into the future. There is a close analogy between this optimal portfolio and the optimal price that a firm sets under Calvo price setting. The latter assumes that there is a probability p of firms setting a new price each period. When a firm sets a new price, the expression for the optimal price (e.g. page 45 of Gali, 2008) depends on a weighted average of expected future marginal costs, with the weight declining at the same rate $\beta(1-p)$ as in the optimal portfolio expression (34).

A lower p implies that investors are less responsive to expected excess returns in the near future. To see this, consider the portfolio response to a change in $E_t er_{t+1}$, which has a coefficient $1/D$. When $p=1$, $D=\gamma v\bar{a}r_t(er_{t+1})$, as seen in (36). When $p<1$, the expression for D is more complicated. But to get a sense, ignore the second part of the term in brackets in (35), which depends on the autocorrelation of excess returns. If we also assume that the variance is the same for all future excess returns, we have $D = \gamma var_t(er_{t+1})/(1-\theta)$. The portfolio response to a change in $E_t er_{t+1}$ is therefore a fraction $1-\theta$ of the portfolio response when $p = 1$. The smaller the p, the weaker the response. This is simply because there is more weight on excess returns further into the future and therefore less weight on the excess return in the immediate future.

There is a second reason why investors respond less to expected excess returns when $p < 1$, which is that only a limited fraction of investors make a new portfolio decision at any time. Analogous to $\tilde{z}_t^{e,A}$, we define the overall portfolio share $z_t^{e,A}$ as the average of $z_{HH,t}/(z_{HH,t} + z_{HF,t})$ and $z_{FH,t}/(z_{FH,t} + z_{FF,t})$. Linearization implies that it evolves according to

$$z_t^{e,A} = (1-p)z_{t-1}^{e,A} + p\tilde{z}_t^{e,A} \tag{37}$$

For a given response of $\tilde{z}_t^{e,A}$ to changes in expected excess returns, this implies a weaker and more gradual response of the overall portfolio share $z_t^{e,A}$.

The weaker and more gradual portfolio response to expected excess returns is a key aspect of the model. It implies that financial shocks (associated with changes in τ_t^D) have a bigger effect on asset prices as portfolios are less responsive to expected excess returns and therefore asset prices. As we will see, the gradual portfolio response also gives rise to momentum and reversal of asset prices that is commonly seen in financial markets.

3.4 Financial Flows and Price Impact

In Appendix E we show that the global solution for $\tilde{z}_t^{e,A}$ is very close to the approximation (34) without the hedge term h_t. This suggests both that the approximation (34) is quite accurate and that the hedge term is not very important quantitatively. Ignoring the hedge term, we can use (34) and (37) to write the following expression for the portfolio share $z_t^{e,A}$ allocated to Home equity across all investors from both countries (in deviation from its mean 0.5):

$$z_t^{e,A} = (1-p)z_{t-1}^{e,A} + \frac{p}{(1-\theta)D}\sum_{i=1}^{\infty}(1-\theta)\theta^{i-1}E_t er_{t+i} + 0.5f_t \tag{38}$$

where

$$f_t = \frac{2p}{(1-\theta)D}\tau_t^D \tag{39}$$

The portfolio depends on the lagged portfolio, the present discounted value of future expected excess returns and the term $0.5 f_t$ that is proportional to τ_t^D. The coefficients $(1-\theta)\theta^{i-1}$ of expected future excess returns sum to 1. A rise in τ_t^D generates an exogenous portfolio shift that leads to a flow from Foreign to Home equity. One can write the exogenous portfolio flow from Foreign to Home, as a fraction of the Home equity market, as $\Delta z^e_{HH} + \Delta z^e_{FH} = 2\Delta z_t^{e,A} = \Delta f_t$, abstracting from general equilibrium changes in expected returns. A one standard deviation shock to τ_t^D then generates a financial flow of

$$\Delta f_t = \frac{2p}{(1-\theta)D}\sigma_{\tau^D} \tag{40}$$

Following the literature, we define the price impact of the financial flow as the change in the relative price q^D at the time of the shock relative to the change in the flow as a share of the Home market:

$$M = \frac{\Delta q_t^D}{\Delta f_t} \tag{41}$$

For example, $M = 2$ means that the relative price rises by 2 percent when there is an exogenous flow towards Home equity equal to 1 percent of the market.

4 Data and Calibration

We calibrate the model to show how infrequent portfolio adjustment brings us closer to the empirical evidence on portfolios and excess returns. In this section we describe the data and the calibration for various degrees of the portfolio friction. In Section 5, we present the quantitative implications for the different parameter values and compare them to the data.

4.1 Data

We consider monthly data from November 1995 to December 2018 for the US and the rest of the world (ROW). The latter is an aggregate of 44 countries. We use data for the excess return er_t, the relative equity price q_t^D, the average portfolio share $z_t^{e,A}$ that the two countries invest in US equity, the difference $z_t^{e,D}$ in the portfolio shares that the US and ROW invest in US equity (a measure of home bias), and log dividends.

The excess return is computed as the change in the log of the MSCI US total return index minus the change in the log of the MSCI ACWI ex US total return index. The latter is an aggregate of 44 countries (ROW), not including the US. The relative equity price is computed from the MSCI price index for the US and ROW. To compute log dividends, we use data on earnings as opposed to dividends as the latter do not include share repurchases, which have become the preferred method of shareholder payments.[19] We compute earnings using the monthly MSCI series for the price index divided by the price-earnings ratio, again for the US and the aggregate of the other 44 countries.

Portfolio data is obtained from US external equity assets and liabilities from Bertaut and Tryon (2007) and Bertaut and Judson (2014), updated through the end of 2018, together with US and ROW market capitalization data. The share invested in US equity by respectively US and ROW investors is computed as

$$z^e_{HH,t} = \frac{z_{HH,t}}{z_{HH,t} + z_{HF,t}} = \frac{US\ market\ cap - US\ ext\ liab}{US\ market\ cap - US\ ext\ liab + US\ ext\ assets}$$
$$z^e_{FH,t} = \frac{z_{FH,t}}{z_{FH,t} + z_{FF,t}} = \frac{US\ ext\ liab}{ROW\ market\ cap + US\ ext\ liab - US\ ext\ assets}$$

where *US ext liab* and *US ext assets* refer to US external equity liabilities and assets. Average and relative portfolios are $z^{e,A}_t = 0.5(z^e_{HH,t} + z^e_{FH,t})$ and $z^{e,D}_t = z^e_{HH,t} - z^e_{FH,t}$. We are mainly interested in $z^{e,A}_t$, which depends on expected excess returns. $z^{e,D}_t$ is a home bias variable that is mainly driven by exogenous changes in τ^A_t.

We also consider several excess return predictability regressions. Our sample is not long enough to obtain accurate estimates of excess return predictability with just one country pair. For these regressions we therefore use panel data for excess returns of US equity relative to that of 73 foreign countries. These excess returns are regressed on their own lag as well as the relative log dividend yield. We again use MSCI data to compute excess returns, relative equity prices and relative dividends (as discussed above).

[19] The MSCI earnings data is a 12-month trailing average. Companies do not report monthly dividends. The measure is reasonable if dividends plus repurchases keep up with the 12-month trailing average of earnings. The correlation between $d_{Ht} - d_{Ft}$ computed based on relative earnings and relative dividends is 0.81.

4.2 Calibration

The numerical solution is very time consuming. We therefore only consider four different values of p: $p = 0.04, 0.1, 0.2$ and 1 (the frictionless case). Table 1 shows the calibration of the remaining parameters for each value of p. These involve the parameters of the dividend and financial shock processes, as well as γ, β and τ.

We set $\gamma = 10$ and $\beta = 0.99668$. Risk aversion of 10 is simply adopted from Bacchetta and van Wincoop (2010), who use their model of infrequent portfolio adjustment to account for the forward discount puzzle. They provide a variety of motivations for this choice. A time discount rate of 0.99668 implies a risk-free rate that is about 4 percent annualized in the risky steady state.

The dividend processes (3)-(4) are calibrated as follows. We set $\rho_d = 0.9767$ as the autocorrelation of d_t^D. We then compute $\epsilon_t^{d,D}$ and $\epsilon_t^{d,A}$ from (3)-(4), from which we obtain their standard deviations σ_{d^D} and σ_{d^A}. We set the mean log dividend equal to $d = (1-\beta)/\beta$, which implies an annualized dividend yield of 4 percent in steady state.

The financial shock processes are (10)-(11). As with dividends, there are four financial shock parameters: τ, ρ_τ, σ_{τ^D} and σ_{τ^A}. Since financial shocks are unobservable, we set these four parameters to match four moments. This gives a different set of parameters for each value of p. While the four parameters are set jointly to target the four moments, we have a specific moment in mind for each parameter.

τ is set such that steady state home bias corresponds to the 0.74 observed in the data, which is the average of the fraction that the US invests in US equity and ROW invests in ROW equity. The standard deviation σ_{τ^D} of the innovation $\epsilon^{\tau,D}$ determines the magnitude of exogenous financial flows between US and ROW equity. We set it to match the observed standard deviation of the excess return. The standard deviation σ_{τ^A} affects the volatility of portfolio home bias, captured by $z^{e,D}$. We therefore set σ_{τ^A} to target the standard deviation of $\Delta z^{e,D}$. Finally, we set the persistence ρ_τ to match the coefficient a_1 of the regression $q_{t+1}^D - q_t^D = a_1 q_t^D$. This coefficient tells us how quickly the relative price reverts to the mean. In the data $a_1 = -0.0084$ (s.e.=0.0022).[20]

We first match the moments by applying the local linear solution at the steady state over the entire state space. Using the resulting parameterization, we compute

[20] This is based on a panel regression for 73 countries.

the global solution, for which we report the model moments below. The targeted moments are therefore not exactly equal to those in the data, but they are very close.

We consider a fifth parameterization, for which the parameters are the same as for the $p = 1$ case with the exception of σ_{τ^D} and τ. As discussed below, to match the targeted moments the size of the financial flow between the two equity markets is extremely large when $p = 1$. We therefore consider a case where σ_{τ^D} is lowered such that the size of the financial flow is equal to that under the $p = 0.1$ case. At the same time τ is adjusted to match the observed portfolio home bias, as discussed above.

5 Quantitative Implications

In this section we discuss the quantitative implications of the model for different values of p, as described in the previous section, and relate them to the data. We discuss three sets of results. The first relates to the portfolio expression (38), which tells us how responsive the portfolio is to the lagged portfolio and expected excess return. We also consider evidence related to the last term of the portfolio expression, containing exogenous financial shocks. We will discuss the size of the implied financial flows, the price impact of the flows and their contribution to the variance of the excess return. In the second set of results, we consider various moments involving excess returns and equity portfolio shares. We finally consider a variety of evidence related to excess return predictability, including excess return momentum and reversal. The results for each of the five parameterizations are presented in Tables 2, 3 and 4. They show that the results are close to the data when p=0.1.

5.1 Portfolio Terms and Price Impact

We first consider the optimal portfolio expression (38). The first two rows of Table 2 report information related to the first two elements of the optimal portfolio expression: the coefficient $1 - p$ on the lagged portfolio and the coefficient $\frac{p}{(1-\theta)D}$ on the weighted average of expected future excess returns.

While both $\epsilon_t^{\tau,D}$ and $\epsilon_t^{\tau,A}$ are financial shocks in the model, in what follows we mainly refer to financial shocks as innovations in $\epsilon_t^{\tau,D}$, which change f_t in the last term of the portfolio expression and lead to financial flows between the two equity markets. The last three rows of Table 2 report several pieces of information related to these financial shocks. It first reports the standard deviation of a financial shock innovation Δf_t in the portfolio, which is $\frac{2p}{(1-\theta)D}\sigma_{\tau^D}$. It measures the size of the exogenous financial flow between the two equity markets, as a fraction of either equity market, due to a one standard deviation financial shock. The next row reports the price impact M of financial shock innovations. The last row reports the fraction of the variance of the excess return that is due to these financial shocks.

While Table 2 does not report empirical counterparts, we can draw comparisons to the literature. The first two rows of Table 2 can be compared to results reported by Bacchetta et al. (2021), Raddatz and Schmuckler (2012) and Giglio et al. (2021). Bacchetta et al. (2021) estimate a version of (38) using monthly portfolio shares of US equity mutual funds in foreign countries.[21] The other papers report the portfolio response to lagged portfolio shares or expected excess returns, but not both. Raddatz and Schmuckler (2012) report the sensitivity of mutual fund portfolio shares to lagged portfolio shares, without including expected excess returns. Giglio et al. (2021) report evidence on the response of equity portfolio shares of Vanguard investors to expected excess returns (from survey data), without including lagged portfolios.[22]

The coefficient on the lagged portfolio in Bacchetta et al. (2021) is 0.918, while the coefficient on the weighted average of expected future excess returns is 15.4.[23] These results are in line with the model when $p = 0.1$, where Table 2 shows that the coefficient on the lagged portfolio is 0.9 and the coefficient on the weighted average of expected future excess returns is 15.1. Raddatz and Schmuckler (2012, Table 5) report a coefficient of 0.9 on the lagged portfolio share (when including a destiny-fund fixed effect), also consistent with $p = 0.1$. Finally, Giglio et al. (2021)

[21] We will refer here to the results reported in the Appendix of Bacchetta et al. (2021), where they estimate a portfolio specification identical to (38). The main body of the paper considers a generalized specification where the portfolio share depends both on the lagged portfolio share and a buy-and-hold portfolio share (lagged portfolio share adjusted for valuation effects).

[22] In their Table V, they do consider a regression of the change in the equity portfolio share on the lagged portfolio share and expected returns. The dependent variable is not the actual change in the portfolio share, but the change due to active trading, removing valuation effects, and over different time windows for different investors. Nonetheless, it does show that active trading very gradually brings the portfolio back to its long-run mean, as is the case in our model.

[23] See Table B.1, column 1, of Online Appendix.

find that the equity share of Vanguard investors depends on the one-year expected excess return on equity with a coefficient 1.16. This translates to 13.9 for monthly expected returns[24], again suggesting a value of p close to 0.1.

Giglio et al. (2021) document that the portfolio response to the expected excess return is much stronger conditional on trading by investors. Moreover, they report that the timing of the trading is unpredictable. They argue that this is consistent with a Calvo type friction as we assume here, where the timing of portfolio decisions is random for each investor.

Next consider evidence related to financial shocks. While we cannot observe the magnitude of the exogenous financial shocks, there are estimates of the price impact of these shocks and the extent to which they account for equity prices. As pointed out in the introduction, Gabaix and Koijen (2021) present evidence that the price impact of financial shocks in the equity market is about 5 for the aggregate stock market. This corresponds to a price elasticity of demand of 0.2. They also review evidence in the literature on the price elasticity of the demand for individual stocks. This micro elasticity averages to about 1. In a two-country model, Home and Foreign equity are likely to be closer substitutes than stocks and bonds, but not as close as two individual US equity. Therefore, one can reasonably expect the elasticity in our model to be somewhere in between 0.2 and 1, which implies a price impact somewhere in the range of 1 to 5. The price impact $M = 2.15$ when $p = 0.1$ is consistent with this.

As pointed out by Gabaix and Koijen (2021), estimates of the price impact of financial shocks are much larger than implied by frictionless models. This large price impact of financial shocks implies that they are important drivers of asset prices. When $p = 0.1$, Table 2 reports that 96 percent of the variance of Δq_t^D is due to financial shocks. Consistent with this, Koijen and Yogo (2019) find that latent asset demand shocks are the main driver of stock prices. Similarly, Itskhoki and Muhkin (2021) find that financial shocks are the main driver of exchange rates.

The evidence in Table 2 is inconsistent with the frictionless case $p = 1$. Table 2 shows that the weight on the expected excess return is then 147, about 10 times bigger than the evidence discussed above suggests. Portfolios also do not depend on

[24]This uses column 3 of Table 3 of Giglio et al. (2021), where some outliers are removed. Without removing the outliers, the number is 8.3. Here it is implicitly assumed that the portfolio depends only on expected returns over the next 12 months, with equal weights on each month. When $p = 0.1$, our model implies that two thirds of the weight is on the first 12 months.

lagged portfolios. This is inconsistent not just with the papers discussed above, but with a large micro literature mentioned in the introduction that has documented portfolio inertia.

The price impact of financial shocks is only 0.25 when $p = 1$, well below estimates in the literature. This weak price impact means that either excessively large financial shocks are needed to account for the variance of the excess return, or the model is inconsistent with evidence that financial shocks are the main driver of equity prices. This is illustrated in the last two columns of Table 2. When the financial process parameters are set to match the variance of the excess return, a one standard deviation financial shock implies a monthly flow from the ROW to the US stock market that is 10 percent of the entire US stock market. While no direct observations of these exogenous flows exist, this is implausibly large. If instead the size of a one standard deviation financial shock innovation is the same as under $p = 0.1$ (last column), none of the variance of the excess return is driven by financial shocks.

At the same time, $p = 0.04$ appears too low. It implies a coefficient on the weighted average of expected future excess returns of 5.6. This is lower than the estimates reported above, which are closer to 15. The coefficient on the lagged portfolio is 0.96, which is on the high end based on the estimates reported above.[25]

5.2 Portfolio and Excess Return Moments

Table 3 reports a set of model moments. These include the standard deviations of the excess return and several portfolio variables ($z_t^{e,A}$, $\Delta z_t^{e,A}$ and $\Delta z_t^{e,D}$), autocorrelations of the excess return, $z_t^{e,A}$, $\Delta z_t^{e,A}$ and the relative equity price q_t^D, as well as correlations between the relative dividend change Δd_t^D and both the excess return and $\Delta z_t^{e,A}$.[26] The moments in the model are averages over 1000 simulations, with standard errors in brackets. The only targeted moments in Table 3 are the standard deviations of the excess return and $\Delta z_t^{e,D}$, which are indicated in italics.[27]

[25]On the other hand, it is consistent with micro evidence for households. The Investment Company Institute reports that 60 percent of households do not change their stock or mutual fund portfolio during a given year, which corresponds to $p = 0.04$ for monthly data.

[26]We only consider the standard deviation of the monthly change in portfolio home bias $\Delta z_t^{e,D}$, rather than $z_t^{e,D}$, as home bias trends upward in the data.

[27]The other targeted moments are the steady state home bias and the coefficient a_1 from the regression $q_{t+1}^D - q_t^D = a_1 q_t^D$, which are both exactly matched.

The moments are again consistent with the data when $p = 0.1$. Perhaps surprisingly, this is the case for most of the parameterizations. It may be most surprising that the model performs well in Table 3 even for the frictionless case $p = 1$. The reason that the model performs well is that the process of the financial shock is chosen to make the model match several key moments, particularly the standard deviation of the excess return. As already discussed, in the frictionless case an excessively large financial shock is needed to overcome the unrealistically small price impact of financial shocks.

The last column provides further perspective on this. When the size of the financial shock in the frictionless case is set equal to that under $p = 0.1$ (last column), the standard deviation of both the excess return and average equity portfolio share are less than one fourth of what they are in the data. With this more reasonably sized financial shock, the excess return and average equity portfolio share are almost entirely driven by the dividend shocks (see last two rows of Table 3, last column). In the data the correlations of both er_t and $\Delta z_t^{e,A}$ with the change in the relative dividend Δd_t^D are close to 0.2.

Table 3 does show that the autocorrelation of the excess return becomes too high when $p = 0.04$. It is more than 5 standard deviations above the data. The friction is clearly too strong when $p = 0.04$. The gradual portfolio adjustment leads to a gradual response of the relative price, which gives rise to an excess return that is too autocorrelated when $p = 0.04$.

This is further illustrated in Figure 1, which shows impulse responses of q_t^D and $z_t^{e,A}$ for a one standard deviation relative dividend shock $\epsilon_t^{d,D}$ and financial shock $\epsilon_t^{\tau,D}$. When $p = 0.04$, the gradual portfolio adjustment leads to a hump-shaped response of both the relative asset price and portfolio to both shocks. The same is also the case when $p = 0.1$, but there is less delayed overshooting in that case, leading to a smaller and more realistic autocorrelation of the excess return.

The hump-shaped response of the relative price is consistent with extensive evidence in financial markets of excess return momentum and reversal in commodity, equity, currency and bond markets (e.g. Moskowitz et al., 2012). After an initial increase in the relative price, it continues to rise for some time (momentum) and then declines (reversal). Closely related is the evidence of post-earnings announcement drift, where the equity price continues to rise for some time after a positive earnings announcement. This can be seen in Figure 1 for the relative dividend shock when $p = 0.04$ or $p = 0.1$.

Excess return momentum and reversal is further illustrated in Figure 2, which shows the impulse response of the expected excess return in response to a one standard deviation financial shock. A financial shock that raises the relative Home equity price implies a positive Home excess return. But as Figure 2 illustrates, it also implies a positive expected excess return the next month when $p = 0.04$ or $p = 0.1$. This continued positive excess return implies a positively autocorrelated excess return (momentum). But eventually the relative Home price starts to fall (Figure 1, panel C), leading to a negative excess return (reversal). This can be seen in Figure 2 for both $p = 0.04$ and $p = 0.1$.

Another problem with the frictionless case is that there is no such excess return momentum and reversal. Figure 1 shows that when $p = 1$ there is no delayed overshooting. The relative price starts to gradually fall after the initial increase, for both dividend and financial shocks. The expected excess return therefore remains negative in response to financial shocks, as shown in Figure 2.

5.3 Excess Return Predictability

Table 4 reports various moments related to excess return predictability. It first reports regressions of the excess return over the next 1, 3 and 12 months on the current relative log dividend yield. In the data the regression coefficient is estimated using panel data for 73 countries. Standard errors for the data are in parentheses. The model reports population moments, computed by simulating the model over 200,000 years. Table 4 next reports evidence on momentum and reversal by regressing the change in the relative price over the next 3 months on both the change in the relative price over the past month and the current relative price.

Finally, the last four rows of Table 4 report the standard deviation and autocorrelation of the expected excess return $E_t er_{t+1}$. In the data we can only observe an approximation of the expected excess return based on a regression. Table 4 shows that for all parameterizations of the model the standard deviation and autocorrelation of the theoretical and regression-based expected excess return are quite close. This gives us confidence that the regression-based expected excess return is a good proxy for its unobservable theoretical counterpart. The regression used to compute the expected excess return is $er_{t+1} = a_1 er_t + a_2(d_t^D - q_t^D)$, which contains both momentum and the relative dividend yield. The regression in the data is again a panel regression for 73 countries.[28]

Table 4 shows that the model again performs well when $p = 0.1$. Regressing the excess returns on the dividend yield gives coefficients that are quite close to the data for all of the three horizons. The momentum/reversal regression is also very close to the data. The positive coefficient on the lagged change in the relative price reflects momentum, while the negative coeffient on the current relative price reflects reversal. Both are close to the data. This is again associated with the humped-shaped impulse response of the relative price documented in Figure 1. Finally, the standard deviation and autocorrelation of the regression-based expected excess return are also close to those in the data when $p = 0.1$.

The frictionless case is again inconsistent with the data. The momentum coefficient is close to zero and more than 3 standard deviations below that reported in the data. Moreover, the expected excess return has an autocorrelation that is close to 1, which can be rejected by the data, where it is 0.48. The high autocorrelation of the expected excess return in the frictionless case can also be seen in Figure 2, where a financial shock leads to a very persistent drop in the expected excess return. By contrast, when $p = 0.04$ or $p = 0.1$, it is first positive and then negative, due to momentum and then reversal. This leads to a much lower autocorrelation.

The fact that there is excess return predictability at all in the frictionless case, with regression coefficients on the relative log dividend yield close to the data, is only because the size of the financial shock is set so high. The last column shows that when lowering the size of the financial shock to that under $p = 0.1$, there is virtually no excess return predictability.

Finally, Table 4 again implies that $p = 0.04$ is too low. The momentum coefficient is much too high and the expected excess return is too volatile relative to the data.

6 Conclusion

We have introduced a Calvo type portfolio friction in a two-country DSGE model for the global equity market. The optimal portfolio depends on the lagged portfolio and the present discounted value of expected excess returns across the two equity

[28]The standard errors of the standard deviation and autocorrelation of the regression-based expected excess return are obtained by computing the standard deviation and autocorrelation for 1000 draws from the estimated distribution of (a_1, a_2).

markets. The friction implies portfolio inertia (gradual adjustment of portfolios) and a weaker response to expected excess returns.

We find that the model with an intermediate level of the friction is consistent with a broad set of empirical evidence. This includes portfolio behavior by households and mutual funds, as well as evidence related to the price impact of financial shocks and the behavior of excess returns. In a frictionless model portfolios do not exhibit observed portfolio inertia and are excessively sensitive to expected excess returns. This has several counterfactual implications for asset prices, such as a weak price impact of financial shocks, either excessively large financial shocks or no expected excess return predictability, and inability to account for momentum and reversal of excess returns. On the other hand, when the friction is too strong, portfolio inertia is too strong. This leads to excess returns that are too autocorrelated, too much excess return momentum and too much volatility in expected excess returns.

Appendix

A Foreign Country Equations

First, define:

$$m_{F,t+1}^{n,t-j} = \left[R_{t+1}^{p,F,t-j}\right]^{-\gamma} e^{(1-\gamma)f_{F,t+1}^{n}}$$
$$m_{F,t+1}^{o,t-j} = \left[R_{t+1}^{p,F,t-j}\right]^{-\gamma} e^{(1-\gamma)f_{F,t+1}^{o,t-j}}$$

where the portfolio return is defined as

$$R_{t+1}^{p,F,t-j} = R_t + \tilde{z}_{FH,t-j}(R_{H,t+1} - R_t) + \tilde{z}_{FF,t-j}(R_{F,t+1} - R_t) \tag{A.1}$$

Also define $m_{F,t+1}^{t-j} = pm_{F,t+1}^{n,t-j} + (1-p)m_{F,t+1}^{o,t-j}$.

The Foreign country portfolio Euler equations are

$$E_t m_{F,t+1}^{t}(R_{F,t+1} - R_t) + (1-p)E_t m_{F,t+1}^{o,t} R_{t+1}^{p,F,t} \lambda_{FF,t+1}^{t} = 0 \tag{A.2}$$
$$E_t m_{F,t+1}^{t}(e^{-\tau_{F,t}} R_{H,t+1} - R_t) + (1-p)E_t m_{F,t+1}^{o,t} R_{t+1}^{p,F,t} \lambda_{FH,t+1}^{t} = 0 \tag{A.3}$$

The Foreign country Bellman equations are

$$e^{(1-\gamma)f_{F,t}^{n}/\beta} = \alpha E_t m_{F,t+1}^{t} R_{t+1}^{p,F,t} \tag{A.4}$$
$$e^{(1-\gamma)f_{F,t}^{o,t-1}/\beta} = \alpha E_t m_{F,t+1}^{t-1} R_{t+1}^{p,F,t-1} \tag{A.5}$$

The Foreign country λ difference equations are

$$E_t R_{t+1}^{p,F,t-1}\left(m_{F,t+1}^{t-1}\lambda_{FH,t}^{t-1} - \theta m_{F,t+1}^{o,t-1}\lambda_{FH,t+1}^{t-1}\right) = \beta E_t m_{F,t+1}^{t-1}(e^{-\tau_{F,t-1}} R_{H,t+1} - R_t) \tag{A.6}$$
$$E_t R_{t+1}^{p,F,t-1}\left(m_{F,t+1}^{t-1}\lambda_{FF,t}^{t-1} - \theta m_{F,t+1}^{o,t-1}\lambda_{FF,t+1}^{t-1}\right) = \beta E_t m_{F,t+1}^{t-1}(R_{F,t+1} - R_t) \tag{A.7}$$

B Aggregation

There is a small aggregation issue in the market clearing conditions in that asset demand involves the product of wealth and portfolio shares. Specifically, $\beta \int_0^1 z_{jk,t}^i W_{j,t}^i di$ is the total demand for country k equity by agents from country j. We have $\int_0^1 z_{jk,t}^i W_{j,t}^i di = z_{jk,t} W_{j,t} + cov(z_{jk,t}^i, W_{j,t}^i)$, where the latter is a cross-sectional covariance term. In theory the covariance term may not be exactly zero. $z^i_{HH,t}$ and $W^i_{H,t}$ could be cross sectionally correlated as a result of wealth accumulation after the most recent portfolio decision. For example, agents with a large portfolio share in the Home country will have seen their wealth rise a lot if Home equity returns have recently been relatively high. The market clearing conditions (27)-(29) ignore the covariance term. This turns out to be numerically very accurate, with the correlation between $\int_0^1 z^i_{jk,t} W^i_{j,t} di$ and $z_{jk,t} W_{j,t}$ above 0.9997 for all j, k based on the solution of the model for $p = 0.1$ discussed in Section 4. The accuracy was checked by simulating the solution over 100,000 months, keeping track of the wealth and portfolio shares of 100 million agents as an approximation of the continuum of agents in the model.

C Solution Method

We first discuss an overview of the solution method and then provide some further details.

C.1 Overview

The aim is to find a solution

$$cv_t = g(sv_t) \tag{C.8}$$

Given a particular node in the state space, Taylor projection locally approximates $g(sv_t)$ as a polynomial, which in our case will be linear. For a particular node sv^i in the state space, this takes the form

$$cv_t = cv^i + M^i(sv_t - sv^i) \tag{C.9}$$

where M^i is a matrix with a non-zero value in element (j, k) if state variable k affects control variable j. Not all control variables depend on all state variables. There are a total of 153 non-zero coefficients in M^i, plus 15 constants in the vector cv^i, for a total of 168 coefficients.

The model can be written in the form

$$E_t F(cv_t, cv_{t+1}, sv_t, sv_{t+1}) = 0 \tag{C.10}$$

$$sv_{t+1} = G(sv_t, cv_t, \epsilon_{t+1}) \tag{C.11}$$

where $\epsilon_{t+1} = (\epsilon^d_{H,t}, \epsilon^d_{F,t}, \epsilon^\tau_{H,t}, \epsilon^\tau_{F,t})'$ is the vector of shocks. F consists of the 15 equations listed in Definition 1. Equation (C.11) describes the evolution of state variables, which is further discussed in the next subsection. Using (C.9) at both t and $t+1$, together with (C.11), we can write (C.10) in the form

$$\mathcal{H}(sv_t) = E_t H(sv_t, \epsilon_{t+1}) = 0. \tag{C.12}$$

We compute expectations using an order-5 monomial method (Judd, 1998) with 33 integration nodes for the shocks. $\mathcal{H}(sv_t)$ represents the errors of the equations. At the node sv^i, the "Taylor" part of "Taylor projection" involves setting both the level of $\mathcal{H}$ and its derivatives with respect to the state variables equal to zero: $\mathcal{H}(sv^i) = 0$ and $\partial\mathcal{H}/\partial sv_t(sv^i) = 0$. These give respectively 15 and 153 constraints on the 168 parameters $\{cv^i, M^i\}$.[29] We compute numerical derivatives using two-sided finite-differences (using two or five-point stencils makes no difference). We then solve the 168 parameters $\{cv^i, M^i\}$ from the 168 equations.

We first obtain the local solution at the deterministic steady state.[30] The other nodes sv^i are obtained as follows. Since a rectangular grid is unfeasible in such a high-dimensional problem, we use the approach from Maliar and Maliar (2015). We generate a long simulation using the linear solution at the symmetric state (10 million periods). We sample every 1000 points to eliminate autocorrelation. From this sample we construct a set of 150 points using Ward's clustering algorithm. We then use symmetry to obtain the solution at another 150 points. So we have a solution at 301 points. We find that these points cover the ergodic set sufficiently well.[31]

To construct the global solution, we use the modified Shepard's inverse-weighting interpolation. Define the weights

$$w_i(sv^i, sv_t) = \frac{\tilde{w}_i(sv^i, sv_t)}{\sum_{j=1}^{241} \tilde{w}_j(sv^j, sv_t)} \tag{C.13}$$

[29] Specifically, all 15 equations depend on S_t (9 state variables), which gives 135 derivatives. In addition, the Bellman equation for $f_{Ht}^{o,t-1}$ and the difference equations for $\lambda_{HH,t}^{t-1}$ and $\lambda_{HF,t}^{t-1}$ also depend on $sv_{H,t}$ (3 state variables). This gives an additional 9 derivatives and an analogous 9 derivatives for the Foreign country. This gives a total of 153 derivatives.

[30] Variables other than portfolio shares are equal to their deterministic steady states. The portfolio shares are set at $z_{HH} = z_{FF} = \bar{z}$, $z_{HF} = z_{FH} = 1 - \bar{z}$, where $\bar{z}$ is set at an empirically realistic value (see Section 4). The value for τ is set to make sure that also $\tilde{z}_{HH} = \tilde{z}_{FF} = \bar{z}$, $\tilde{z}_{HF} = \tilde{z}_{FH} = 1 - \bar{z}$ at this symmetric node of the state space.

[31] If we create a new set of points by simulating the resulting global solution, the new set of points is very similar. For $p = 0.04$, we could only find a solution at 143 points, which with symmetry becomes 285.

where

$$\tilde{w}_i(sv^i, sv_t) = \left(\frac{max\{0, k - \|sv_i - sv_t\|\}}{k\|sv_i - sv_t\|}\right)^2.$$

$\|sv_i - sv_t\|$ is the Euclidean distance and k is set to 4.[32] Then

$$cv(sv_t) = \sum_{i=1}^{241} w_i(sv^i, sv_t)\left(cv^i + M^i(sv_t - sv^i)\right) \tag{C.14}$$

We also solve the model in the frictionless case where $p = 1$. The same solution method is followed, but the solution is significantly faster as there are far fewer state and control variables. The set of state variables consists of the exogenous state variables $d_{H,t}$, $d_{F,t}$, $\tau_{H,t}$, $\tau_{F,t}$ and relative wealth w_t^D. The other four state variables in S_t, related to lagged relative wealth and portfolio shares, as well as $s_{H,t}$ and $s_{F,t}$, are no longer state variables. The additional control variables $cv_{H,t}$ and $cv_{F,t}$ also disappear. Overall, the number of state variables is reduced from 15 to 5 and the number of control variables is reduced from 15 to 9.

C.2 Further Details

A couple of points are in order regarding the set of state variables in S_t. Adding up the market clearing conditions, the sum of wealth of both countries is proportional to the sum of their asset prices. Aggregate wealth is therefore not a state variable. The lagged bond market equilibrium condition implies $\omega_{t-1}(z_{HH,t-1} + z_{HF,t-1}) + (1 - \omega_{t-1})(z_{FH,t-1} + z_{FF,t-1}) = 1$. We therefore cannot use lagged relative wealth and all four lagged portfolio shares as state variables. We also do not use the four lagged portfolio shares as state variables. In a symmetric state, where $\omega_{t-1} = 0.5$, the four lagged portfolio shares are locally in a linear relationship (adding to 2). Also note that $z^A_{F,t-1} = \omega_{t-1}z_{HF,t-1} + (1 - \omega_{t-1})z_{FF,t-1} = 1 - z^A_{H,t-1}$ is redundant from the time $t - 1$ bond market clearing condition.

There are two reasons why only one-period lagged portfolios are in the state space. First, (32) implies that the aggregate portfolio share $z_{HH,t}$ depends on the one-period lagged portfolio share $z_{HH,t-1}$ and the new portfolio share chosen at time t. The lagged portfolio share $z_{HH,t-1}$ aggregates all portfolio shares chosen at $t - 1$ and earlier. Second, the new portfolio share $\tilde{z}_{HH,t}$ chosen at time t depends

[32] Setting k lower than 4 raises Euler equation errors, while setting it higher makes little difference to the weights.

through the portfolio Euler equation on beliefs about $\lambda^t_{HH,t+1}$. The same variable one period earlier, $\lambda^{t-1}_{HH,t}$, is one of the control variables. It depends on portfolio choice at $t-1$, but not earlier. We do not need to solve for $\lambda^{t-j}_{HH,t}$ for $j>1$, which depends on portfolio shares prior to $t-1$.

Next we discuss the evolution of the state variables. (C.11) writes the evolution of the state variables as $sv_{t+1} = G(sv_t, cv_t, \epsilon_{t+1})$. To see this, first consider the last 6 state variables at $t+1$: $sv_{H,t+1} = (\tau_{Ht}, \tilde{z}_{HH,t}, \tilde{z}_{HF,t})'$ and $sv_{F,t+1} = (\tau_{Ft}, \tilde{z}_{FH,t}, \tilde{z}_{FF,t})'$. Clearly, these are elements of sv_t and cv_t. Next consider the first 9 state variables: $S_{t+1} = (d_{H,t+1}, d_{F,t+1}, \tau_{H,t+1}, \tau_{F,t+1}, w^D_{t+1}, w^D_t, z^A_{Ht}, z^D_{Ht}, z^D_{Ft})'$. The Home and Foreign dividends and taxes/subsidies at $t+1$ depend on their values at time t (part of sv_t) and the shocks ϵ_{t+1}. Skip over w^D_{t+1} for a moment. w^D_t is part of sv_t. z^A_{Ht}, z^D_{Ht} and z^D_{Ft} depend on ω_t, $z_{HH,t}$, $z_{HF,t}$, $z_{FH,t}$ and $z_{FF,t}$. ω_t depends on w^D_t, which is part of sv_t. We can write $z_{HH,t} = (1-p)z_{HH,t-1} + p\tilde{z}_{HH,t} = (1-p)z^A_{H,t-1} + (1-p)(1-\omega_{t-1})z^D_{H,t-1} + p\tilde{z}_{HH,t}$, where ω_{t-1} depends on w^D_{t-1}. So $z_{HH,t}$ can be written as a function of state variables at time t and control variables at time t. The same is the case for the other portfolio shares.

Some more discussion is warranted regarding w^D_{t+1}. Denote all state variables at $t+1$ other than w^D_{t+1} as $\tilde{sv}_{t+1}$. It follows from the discussion above that $\tilde{sv}_{t+1} = G_s(sv_t, cv_t, \epsilon_{t+1})$ for a known function G_s. From (33), and the return expressions (1) and (2), as well as the discussion above, it follows that we can write $w^D_{t+1} = G_w(sv_t, cv_t, \epsilon_{t+1}, q_{H,t+1}, q_{F,t+1})$ for a known function G_w. At this point we substitute the linear projection (C.9) at a particular node sv^i, applied to $t+1$: $cv_{t+1} = cv^i + M^i(sv_{t+1} - sv^i)$. For a given cv^i and M^i (first and second row), this gives $q_{H,t+1}$ and $q_{F,t+1}$ as linear functions of sv_{t+1}, which in turn implies a linear function in $\tilde{sv}_{t+1}$ and w^D_{t+1}. Write these as $q_{i,t+1} = G_i(\tilde{sv}_{t+1}, w^D_{t+1})$. Then we have

$$w^D_{t+1} = G_w(sv_t, cv_t, \epsilon_{t+1}, G_H(G_s(sv_t, cv_t, \epsilon_{t+1}), w^D_{t+1}), G_F(G_s(sv_t, cv_t, \epsilon_{t+1}), w^D_{t+1})) \tag{C.15}$$

We linearize the right hand side around $w^D_{t+1} = w^D_t$, where w^D_t is the fifth element of the node sv^i, to solve for w^D_{t+1} as a function of sv_t, cv_t and ϵ_{t+1}.

For the simulation of the global solution, we use the evolution equation for w^D_{t+1} directly, which is a nonlinear equation (since returns tomorrow depend on prices tomorrow, which in turn depend on w^D_{t+1} itself. We solve this equation using Brent's method at each step in the simulation; this approach delivers the same answer as the linearized solution if we use only the local solution at the steady state.

We finally make a couple more comments on the solution of the 168 parameters. We start the solution of the 168 parameters either at the deterministic steady state or at the nearest node in the state space for which we have solved the local solution. We go in steps of 0.001 times the distance towards the new node in the state space, each time resolving the parameters, until we have reached the new node. We normalize the variables $f^n_{H,t+1}$, $f^{o,t}_{H,t+1}$, $f^n_{F,t+1}$, $f^{o,t}_{F,t+1}$ by $f^{o,t-1}_{Ht}$ and $f^{o,t-1}_{Ft}$ to avoid overflows, given the large steady state values of the f variables. We use a dampened quasi-Newton method to solve the parameters before switching to hybrid-Powell once the largest absolute value of the elements of $\mathcal{H}(sv_t)$ is less than 10^{-4}.

All codes are written in Fortran95 and compiled with the Intel Compiler, except for Ward's clustering algorithm, which is written in Matlab. All codes are available on request. No proprietary software is needed.

D Hedge Terms

The last term in (34) is

$$
\begin{aligned}
&h_t = \frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{t+i}, r^A_{t+1,t+i}) \\
&+\frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}\left(cov_t(er_{H,t+i},(1-\mu)f^n_{H,t+i}+\mu f^n_{F,t+i}) - cov_t(er_{F,t+i},\mu f^n_{H,t+i}+(1-\mu)f^n_{F,t+i})\right) \\
&+\frac{(1-2\bar{z})^2}{d}\sum_{i=1}^{\infty}\theta^{i-1}cov_t(r^A_{t+i}-r_{t+i-1},(\gamma-1)er_{t+1,t+i-1}+\gamma er_{t+i}).
\end{aligned}
$$

where $r^A_{t+1} = 0.5(r_{H,t+1}+r_{F,t+1})$, $er_{H,t+1} = r_{H,t+1}-r_t$,$er_{F,t+1} = r_{F,t+1}-r_t$, $\mu = 0.5+(\bar{z}-0.5)(D/d)$ and

$$
d = \sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma \bar{var}_t(er^{sum}_{t+i}) + 2(\gamma-1)\bar{cov}_t(er^{sum}_{t+i}, er^{sum}_{t+1,t+i-1})\right)
$$

Here $er^{sum}_{t+i} = er_{H,t+i}+er_{F,t+i}$ and $er^{sum}_{t+1,t+i-1} = er^{sum}_{t+1}+...+er^{sum}_{t+i-1}$.

The terms in h_t are hedge terms associated with time-varying risk. These involve the variance and covariance of asset return variables and the Bellman variables $f^n_{H,t+s}$, $f^n_{F,t+s}$. Analogous to expected asset returns, it is not just uncertainty about asset returns and Bellman variables over the next period that affects portfolios, but rather perceived risk at all future dates, with discount rate θ. In what follows, assume that $\gamma > 1$.

The first term in h_t implies that the average share invested in Home equity is higher when Home equity has a relatively high payoff (compared to Foreign equity) in bad future states where the world equity return has been low. Home equity is then an attractive hedge against such bad states. The second term of h_t captures a hedge against future changes in expected portfolio returns. The approximated solution for $f^n_{H,t}$ is

$$f^n_{H,t} = E_t \sum_{i=1}^{\infty} \beta^i \bar{r}^{p,H}_{t+i} \tag{D.16}$$

where $\bar{r}^{p,H}_{t+i} = r_{t+i-1} + \bar{z}er_{H,t+i} + (1-\bar{z})er_{F,t+i}$ is the Home portfolio return evaluated at the mean of portfolio shares. An analogous solution applies to $f^n_{F,t}$. The second term of (D.16) then says that the Home portfolio share is high when Home equity returns are relatively high in bad future states where subsequent future expected portfolio returns are low. The last hedge term in (D.16) is less intuitive.

E Accuracy of Approximated Portfolio in Section 3

Equation (34) gives a linear approximation of the solution of $\tilde{z}^{e,A}_t$ as the sum of three terms. We will show that the global solution is very close to just the sum of the first two terms, so ignoring the hedge term. This approximated solution is

$$\tilde{z}^{e,A,approximate}_t = 0.5 + \frac{1}{D} \sum_{i=1}^{\infty} \theta^{i-1} E_t er_{t+i} + \frac{1}{(1-\theta)D} \tau^D_t . \tag{E.17}$$

To show that this is close to $\tilde{z}^{e,A}_t$ from the global solution, we simulate the model over 230 months, the sample length used for calibration and to compute data moments in Section 5. During each month the present discounted value of expected excess returns is computed by generating 100,000 different futures of 150 months.[33] The parameter D is computed using the mean over the 230 months of the present

[33]Truncating after 150 months is sufficient as θ^{150} is equal to 0.0013, so that expected excess returns further into the future get virtually no weight.

discounted value of the moments in the expression for D, again using 100,000 futures of 150 months to compute the present discounted value of the moments.

For $p = 0.1$, Figure A1 shows both the global solution for $\tilde{z}_t^{e,A}$ and the approximated solution (E.17). The two lines are extremely close, with a correlation of 0.964. Sometimes they are indistinguishable and overlap. Any deviation that is left is caused either by the approximation itself used to derive (34) or the hedge term h_t. We have not been able to numerically approximate the time varying hedge term accurately enough as it would require an even much larger number of futures, but clearly it does not play a significant role.

F Ergodic Distribution Relative Wealth

We compute the ergodic distribution of relative wealth for $p = 0.1$ by simulating the model over one million months. The result is shown in Figure A2. Ninety five percent of the distribution is between plus and minus 0.52. The logic behind the stationarity of relative wealth is as follows. Assume that a shock leads to an increase in the relative wealth of the Home country. As a result of home bias (which is matched in the parameterization), this leads to an increase in the relative demand for Home equity. This raises the relative Home equity price and therefore lowers the expected return on Home equity relative to Foreign equity. This lowers the expected portfolio return of Home agents relative to Foreign agents, which in turn reduces the relative wealth of the Home country.

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The data underlying this article are available in the Zenodo repository, at https://zenodo.org/record/5898338.

Table 1 Calibrated Parameters

Parameter	$p = 0.04$	$p = 0.1$	$p = 0.2$	$p = 1$	$p = 1$ smaller financial shock
γ	10	10	10	10	10
β	0.99668	0.99668	0.99668	0.99668	0.99668
ρ_d	0.9767	0.9767	0.9767	0.9767	0.9767
σ_{d^D}	0.0447	0.0447	0.0447	0.0447	0.0447
σ_{d^A}	0.0325	0.0325	0.0325	0.0325	0.0325
ρ_τ	0.98084	0.99003	0.99151	0.99177	0.99177
τ	0.0000941	0.0001050	0.0001011	0.0001078	0.0000410
σ_{τ^D}	0.0006514	0.0003851	0.0003387	0.0003442	0.0000020
σ_{τ^A}	0.0000624	0.0000359	0.0000243	0.0000084	0.0000084

Notes: The table reports parameters under 5 parameterizations. The first four parameterizations set the parameters of the financial shock process (the last 4 parameters) to target 4 moments. The other parameters remain the same. The last column of parameters is the same as the previous column, except that σ_{τ^D} is lowered such that the standard deviation of a financial shock innovation $\Delta f_t = \frac{2p}{(1-\theta)D}\epsilon_t^{\tau,D}$ is the same as under the $p = 0.1$ parameterization and τ is adjusted to match the observed portfolio home bias.

Table 2 Portfolio Terms and Price Impact

	$p = 0.04$	$p = 0.1$	$p = 0.2$	$p = 1$	$p = 1$ smaller financial shock
Features Portfolio Expression					
coefficient lagged portfolio	0.96	0.9	0.8	0	0
coefficient expected excess return	5.6	15.1	30.7	147.1	2943
Financial Shock					
s.d. financial shock innovation Δf	0.0073	0.0117	0.0208	0.1012	0.0117
price impact M	3.44	2.15	1.20	0.25	0.0146
fraction $var(er_{t+1})$ due to financial shock	0.977	0.956	0.944	0.924	0.00

Notes: The table reports information about the three elements of the optimal portfolio expression $z_t^{e,A} = (1-p)z_{t-1}^{e,A} + \frac{p}{(1-\theta)D}\sum_{i=1}^{\infty}(1-\theta)\theta^{i-1}E_t er_{t+i} + 0.5f_t$. The coefficient on the lagged portfolio is $1-p$. The coefficient on the weighted average of expected future excess returns (with weights adding to 1) is $\frac{p}{(1-\theta)D}$. For the last portfolio term the table reports the standard deviation of the financial shock innovation $\Delta f_t = \frac{2p}{(1-\theta)D}\epsilon_t^{\tau,D}$, which is the size of the exogenous financial flow between the two equity as a fraction of either equity market. It also reports the price impact M, which is the ratio of the relative price change to the change in Δf_t in response to a financial shock innovation $\epsilon_t^{\tau,D}$. The last row reports the fraction of the variance of the excess return that is associated with financial shocks. The columns correspond to the parameterizations reported in Table 1.

Table 3 Data and Model Moments

	DATA	MODEL				
		$p = 0.04$	$p = 0.1$	$p = 0.2$	$p = 1$	$p = 1$ smaller financial shock
STANDARD DEVIATIONS						
er_t	0.0262	*0.0257* (0.0014)	*0.0258* (0.0010)	*0.0260* (0.0010)	*0.0262* (0.0011)	0.0058 (0.0003)
$z_t^{e,A}$	0.0297	0.0322 (0.0090)	0.0266 (0.0080)	0.0247 (0.0080)	0.0253 (0.0074)	0.0052 (0.0019)
$\Delta z_t^{e,A}$	0.0046	0.0054 (0.0004)	0.0052 (0.0004)	0.0050 (0.0003)	0.0051 (0.0003)	0.0015 (0.0003)
$\Delta z_t^{e,D}$	0.0048	*0.0047* (0.0008)	*0.0046* (0.0006)	*0.0046* (0.0004)	*0.0046* (0.0002)	0.0974 (0.0041)
AUTOCORRELATIONS						
er_t	0.0686	0.3765 (0.0559)	0.1022 (0.0605)	0.0294 (0.0590)	0.0181 (0.0601)	-0.0039 (0.0592)
$z_t^{e,A}$	0.9807	0.9828 (0.0107)	0.9741 (0.0162)	0.9710 (0.0190)	0.9721 (0.0182)	0.9665 (0.0224)
$\Delta z_t^{e,A}$	0.0971	0.3784 (0.0604)	0.0968 (0.0729)	0.0170 (0.0690)	0.0199 (0.0608)	-0.0072 (0.0692)
q_t^D	0.9803	0.9826 (0.0103)	0.9751 (0.0137)	0.9720 (0.0170)	0.9729 (0.0175)	0.9593 (0.0199)
CONTEMPORANEOUS CORRELATIONS						
$corr(\Delta d_t^D, er_t)$	0.2025	0.1155 (0.0602)	0.2044 (0.0572)	0.2304 (0.0577)	0.2351 (0.06)	0.9963 (0.0017)
$corr(\Delta d_t^D, \Delta z_t^{e,A})$	0.2709	0.1361 (0.0586)	0.2001 (0.0572)	0.2286 (0.0578)	0.2361 (0.0602)	0.9375 (0.0854)

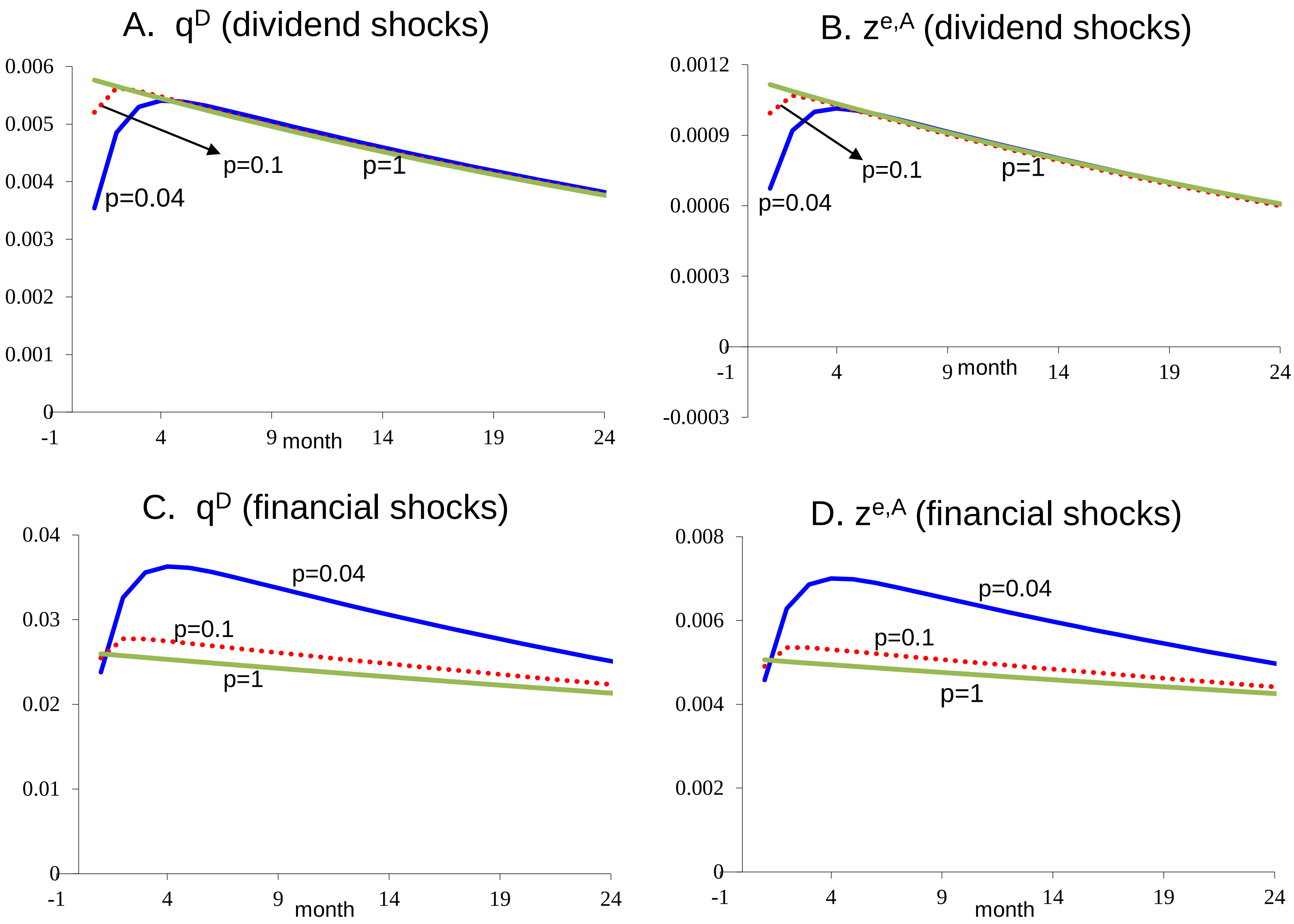
Notes: Model moments and associated standard errors (in parentheses) are based on 1000 simulations of a 278 month period. Results are shown for the parameterizations reported in Table 1. In the last column the size of a one standard deviation of the financial shock innovation Δf_t is the same as under $p = 0.1$. Of the four targeted moments, the table only reports the standard deviations of the excess return and $\Delta z_t^{e,D}$, which are indicated in italics.

Table 4 Excess Return Predictability

	DATA	MODEL				
		$p=0.04$	$p=0.1$	$p=0.2$	$p=1$	$p=1$ smaller financial shock
Predictability by Relative Dividend Yield						
$er_{t+1} = a_1(d_t^D - q_t^D)$	0.0066 (0.0020)	0.0078	0.0068	0.0066	0.0068	0.0000
$er_{t+1,t+3} = a_1(d_t^D - q_t^D)$	0.0185 (0.0038)	0.0297	0.0216	0.0201	0.0205	0.0000
$er_{t+1,t+12} = a_1(d_t^D - q_t^D)$	0.0779 (0.0090)	0.1283	0.0849	0.0775	0.0764	0.0000
Momentum and Reversal						
$q_{t+3}^D - q_t^D = a_1(q_t^D - q_{t-1}^D) + a_2 q_t^D$						
a_1	0.1020 (0.0296)	0.5740	0.1158	0.0350	0.0046	-0.0034
a_2	-0.0290 (0.0045)	-0.0520	-0.0287	-0.0260	-0.0260	-0.0513
Expected Excess Return						
s.d. theoretical $E(er)$		0.0106	0.0035	0.0024	0.0024	0.0000
a.c. theoretical $E(er)$		0.3980	0.5311	0.9527	0.9917	0.9985
s.d. regression-based $E(er)$	0.0042 (0.0011)	0.0105	0.0035	0.0024	0.0026	0.0000
a.c. regression-based $E(er)$	0.4814 (0.2020)	0.4335	0.5308	0.9470	0.9824	0.9223

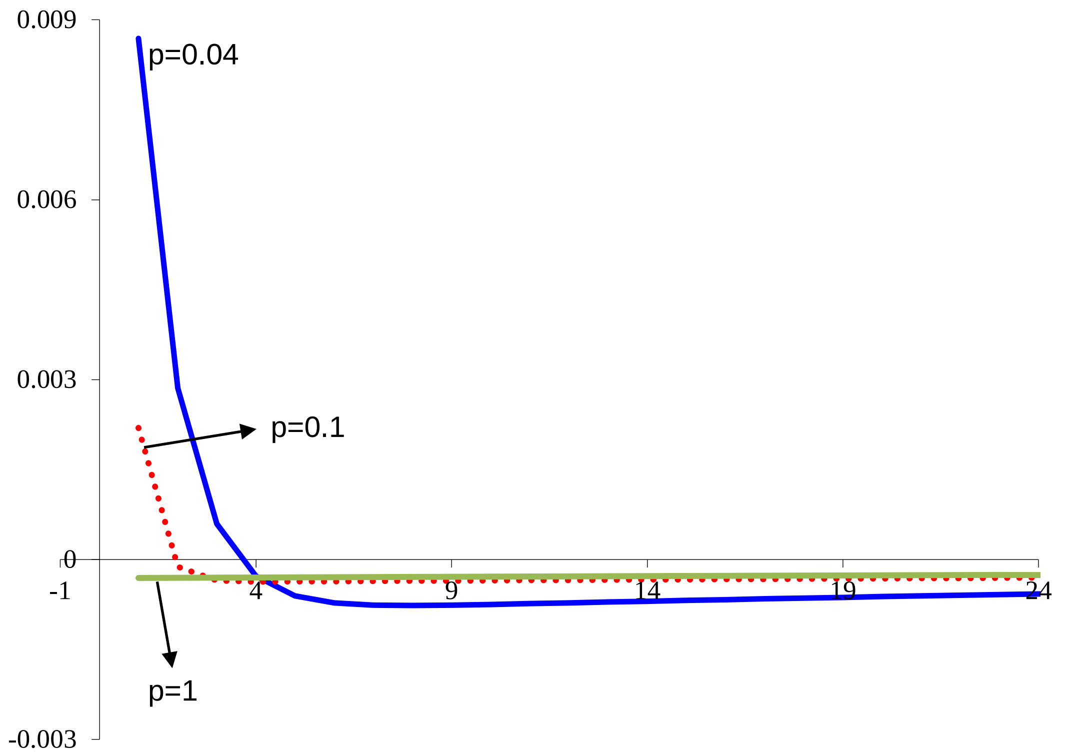
Notes: Data moments are based on panel regressions for 73 countries, with standard errors in parenthesis. Model moments are population moments computed by simulating the model over 200,000 years. Results are shown for the parameterizations reported in Table 1. $er_{t+1,t+i}$ stands for the excess return over the next i months. The theoretical $E(er)$ stands for the expected excess return $E_t er_{t+1}$ that can only be computed within the model. The regression-based $E(er)$ is the expected excess return based on the regression $er_{t+1} = a_1 er_t + a_2(d_t - q_t^D) + \epsilon_{t+1}$. The standard error of the standard deviation and autocorrelation of the regression-based $E(er)$ in the data is computed by sampling from the estimated joint distribution of (a_1, a_2), computing the standard deviation and autocorrelation for each pair of parameters.

Figure 1 Impulse Response Functions



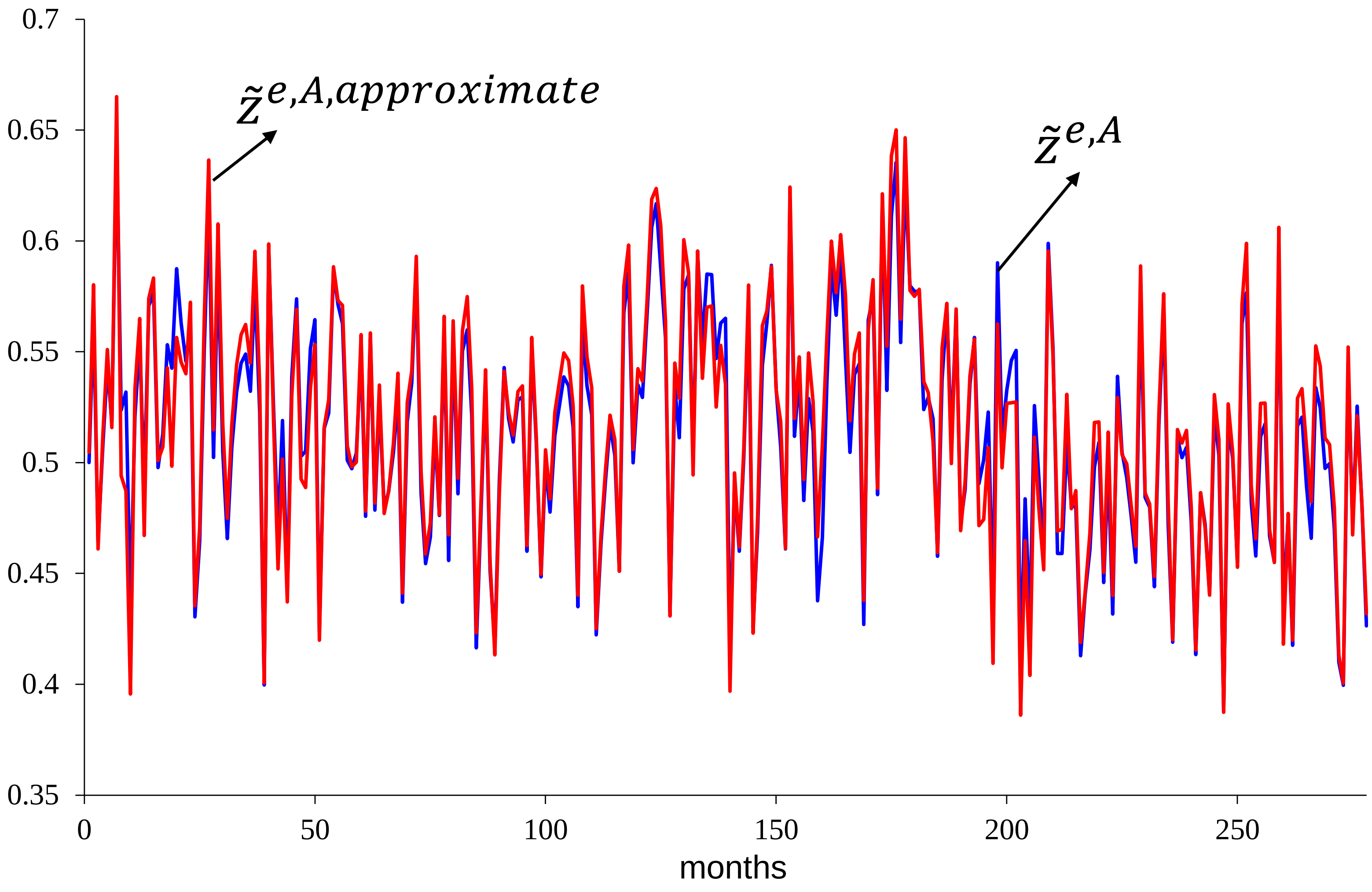
Notes: The charts show the impulse response functions of q^D and $z^{e,A}$ in response to a one standard deviation increase in $\varepsilon^{d,D}$ (dividend shock) and $\varepsilon^{\tau,D}$ (financial shock), starting from the symmetric steady state.

Figure 2 Impulse Response Expected Excess Return (Financial Shocks)



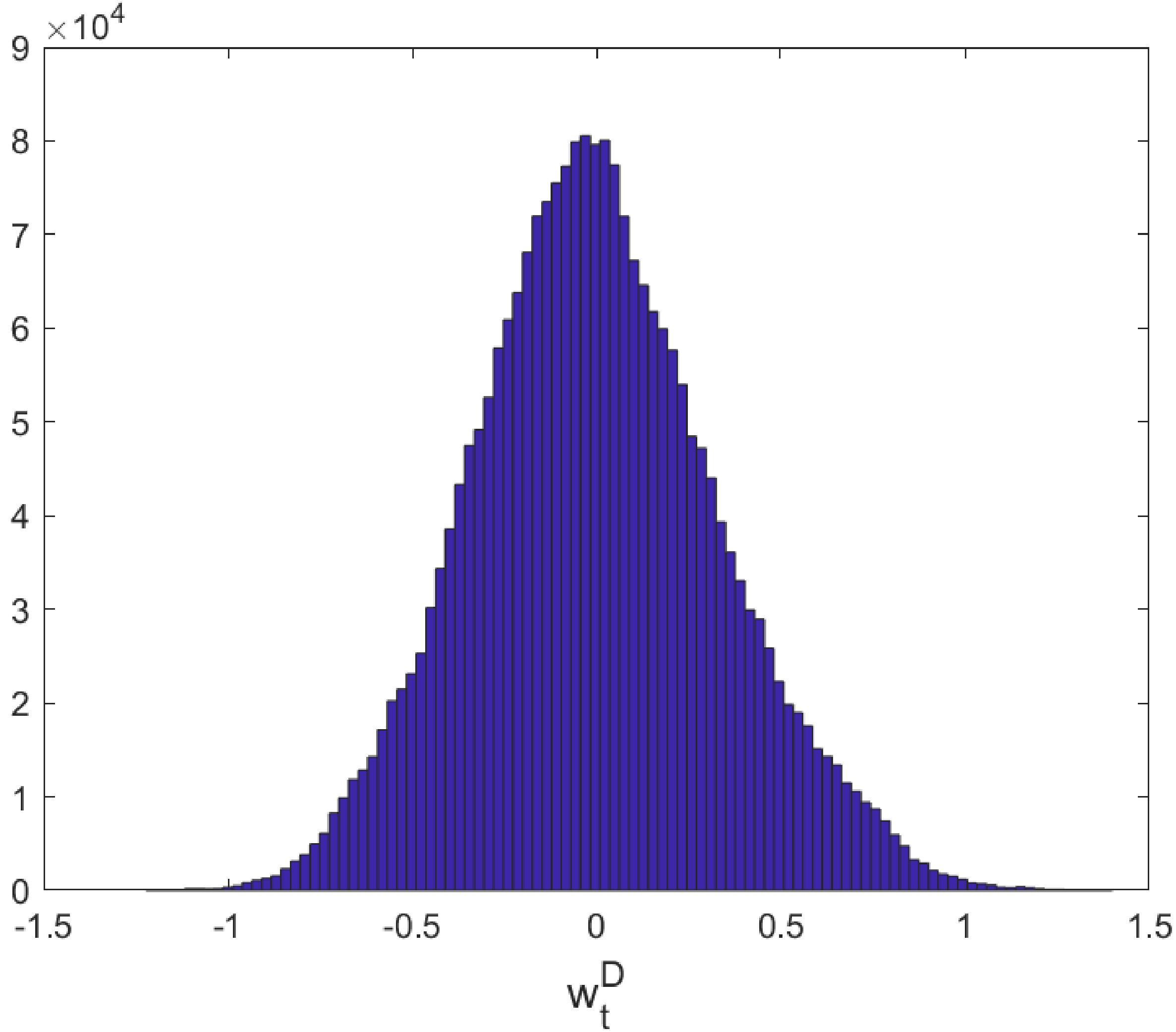
Notes: Expected excess returns are computed from impulse response functions. Excess returns in the periods subsequent to the shock are expected excess returns. The chart shows the expected excess return over time in response to a one standard deviation financial shock innovation $\varepsilon^{\tau,D}$.

Figure A1 Approximation Optimal Portfolio*



*The chart shows the global solution of the optimal portfolio based on a simulation over 278 months, as well as the approximation (34), without the hedge term h_t. The correlation between the global solution and the approximation is 0.976.

Figure A2 Ergodic Distribution Relative Wealth



Notes: The ergodic distribution is obtained by simulating the model over 2.4 million months for the parameterization where p=0.1.