International Portfolio Choice with Frictions: Evidence from Mutual Funds\(^1\)

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Abstract

Using data on international equity portfolio allocations by US mutual funds, we estimate a simple portfolio expression derived from a standard Markowitz mean-variance portfolio model extended with portfolio frictions. The optimal portfolio depends on two benchmark portfolios, the previous month and the buy-and-hold portfolio shares, and a present discounted value of expected future excess returns. We show that equity return differentials are predictable and use the expected return differentials in the mutual fund portfolio regressions. The estimated reduced form parameters are related to the structural model parameters. The estimates imply significant portfolio frictions and a modest rate of risk aversion. While mutual fund portfolios respond significantly to expected returns, portfolio frictions lead to a weaker and more gradual portfolio response to changes in expected returns.
1 Introduction

An extensive literature has introduced frictions into models of portfolio choice that lead to deviations from the standard Markowitz mean-variance portfolio.\textsuperscript{1} This is supported by micro evidence of sluggish portfolio decisions by households and helps explain various asset pricing facts. In this paper we focus on international portfolio decisions. The objective is to provide evidence on how US mutual funds allocate their equity portfolios across countries, and specifically to what extent this is affected by portfolio frictions that lead to a weaker and more gradual response to changes in expected returns. It has frequently been suggested that global investors are slow to adjust their portfolios in response to new information. In the context of US external equity investments, Bohn and Tesar (1996) comment that “we suspect that investors may adjust their portfolios to new information gradually over time, resulting in both autocorrelated net purchases and a positive linkage with lagged returns.” Froot et al. (2001) provide similar evidence. Froot and Thaler (1990), in attempting to explain the forward discount puzzle of excess return predictability in the foreign exchange market, hypothesize that “...at least some investors are slow in responding to changes in the interest differential.” More formally, Bacchetta and van Wincoop (2010, 2021), Bacchetta, Davenport and van Wincoop (2022), and Bacchetta, van Wincoop and Young (2022) show that open economy models with portfolio frictions can explain a variety of evidence related to excess return predictability in foreign exchange and equity markets as well as various data moments involving capital flows, saving, investment and aggregate US equity portfolios.\textsuperscript{2} Nonetheless none of the existing literature has provided direct evidence of portfolio frictions in international portfolio allocation data. This paper aims to fill that gap.

Our evidence is based on 15 years of monthly equity portfolio allocation data

\textsuperscript{1}Some recent contributions include Abel et al. (2007), Bogoousslavsky (2016), Chien et al. (2012), Duffie (2010), Greenwood et al. (2018), Hendershott et al. (2022) and Vayanos and Woolley (2012).

\textsuperscript{2}While there are many models of international capital flows driven by portfolio choice, these tend to abstract from portfolio frictions considered here. Examples of recent DSGE models of capital flows based on portfolio choice include Benhima and Cordonier (2022), Davis and van Wincoop (2018), Devereux and Sutherland (2007, 2010), Didier and Lowenkron (2012), Evans and Hnatkovska (2012, 2014), Gabaix and Maggiori (2015), Hnatkovska (2010) and Tille and van Wincoop (2010a, 2010b, 2014).
across 36 countries for 316 US mutual funds that report to EPFR (Emerging Portfolio Fund Research). Mutual funds are the most important players in external US equity holdings, accounting for 50 percent of all US foreign equity holdings at the end of 2019. To structure the analysis, we present a portfolio choice model that enables us to derive a simple and testable portfolio equation. While the standard Markowitz mean-variance portfolio is embedded as a special case, the model allows for deviations from the Markowitz portfolio as a result of portfolio frictions that involve costs of deviating from two benchmark portfolios. The optimal portfolio share then depends on both of these benchmark portfolios and a present discounted value of expected future excess returns. We first document that international differences in stock returns are predictable and that predictability improves over longer horizons. We then use estimates of these expected excess returns in our portfolio regressions. We find that portfolios respond to expected return differentials, but deviate gradually from benchmark portfolios. The results from the portfolio regressions are used to obtain estimates of the structural parameters of the model, such as the two portfolio frictions and risk aversion.

The simple theoretical portfolio choice model that structures the empirical analysis is analogous to Gârleanu and Pedersen (2013). It assumes that funds (investors) maximize the present discounted value of risk-adjusted portfolio returns minus quadratic costs of deviating from two benchmark portfolios. The first portfolio friction is a cost of deviating from the portfolio share during the previous month, which is the portfolio under complete rebalancing. The second is a cost of deviating from a buy-and-hold portfolio. The more important these portfolio frictions are, the more the optimal portfolio share depends on the two benchmark portfolios and the less it depends on expected excess returns. In addition, the portfolio frictions imply that the optimal portfolio depends not just on expected excess returns over the next period (as in the Markowitz portfolio), but on a present discounted value of future excess returns. The frictions lead to a more gradual response of portfolio shares to changes in expected returns.  

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3See Exhibit 19 in “Portfolio Holdings of Foreign Securities,” October 2020, Department of the Treasury.

4These two benchmark portfolios depend on the past behavior of funds themselves, reflecting sluggish portfolio adjustment. There is also a substantial literature that has focused on benchmarks that are external to the fund, such as global or regional stock or bond indices. We will not consider such benchmarks here.
One difficulty with estimating portfolio expressions is endogeneity. The error term of the portfolio expression, which for example captures latent time-varying risk, can lead to portfolio shifts that affect equity prices. While individual mutual funds are too small to impact equity prices, there may be common components across investors of such portfolio shifts. This leads to endogeneity of our explanatory variables (the lagged portfolio share, buy-and-hold portfolio share and measure of expected excess returns). We address this issue by using Two-Stage Least Squares (2SLS), using instruments that have explanatory power for the endogenous regressors but are unrelated to time-varying risk that enters the error term. The portfolio theory provides guidance in computing a fund-specific measure of portfolio risk that affects the portfolio of a fund.

We find that the funds respond to the discounted expected excess return with strong statistical significance. We also find that both benchmark portfolios are very important, so that portfolio frictions matter. Our estimates imply a humped shaped portfolio response to an expected excess return innovation. The initial portfolio response is weaker than in the absence of portfolio frictions, while the portfolio response builds gradually as a result of the frictions. The regression estimates imply a plausible rate of risk aversion of 3.2. We also find that the lagged portfolio share is at least as important as the the buy-and-hold portfolio, which is consistent with extensive portfolio rebalancing by the mutual funds.

The paper is related to various strands of literature. The first is the literature on excess return predictability. While the evidence we report on the predictability of international stock return differentials is new, the evidence on the predictability of international short term bond return differentials (UIP deviations) has been known since Fama (1984). Predictability has also been widely documented in the context of country or individual stock returns or the excess of stock returns over bond returns (for a textbook discussion, see Campbell, Lo and MacKinlay, 1997). While the latter literature focuses mainly on the US, some papers document stock return predictability in other countries or show, by pooling the data, that there is global predictability (Hjalmarsson, 2010). Cenedese et al. (2016) consider the profitability of trading strategies that exploit international equity return differentials. They sort countries into various “bins” based on the realization of variables like the dividend yield that are likely to predict future equity returns. They do not estimate portfolio expressions or excess returns, but find substantial Sharpe ratios from trading strategies that exploit in which bin countries are located.
In terms of estimation of portfolio regressions, the open economy literature is very limited, which is the motivation behind this project. Frankel and Engel (1984) invert the portfolio expression obtained from a simple frictionless mean variance portfolio model, relating expected returns on various currencies to asset supplies. They strongly reject the model. Also relevant is recent work by Koijen and Yogo (2020). As we do, they adopt a two-step approach, first estimating expected excess returns and then estimating portfolio expressions. They differ in that they do not allow for portfolio frictions and use aggregate bilateral portfolio shares in three asset classes. They also handle the endogeneity issue differently as they can use their global demand system to instrument asset prices with macroeconomic variables. Some papers have investigated the link between international capital flows (as opposed to portfolio shares) and past returns as well as expected future returns (e.g. Bohn and Tesar (1996), Froot et al. (2001), Didier and Lowenkron (2012)).

There is also a literature that has investigated international portfolio allocation using fund-level data. Raddatz and Schmukler (2012) use EPFR data to regress portfolio allocation of mutual funds across countries on either the lagged portfolio share or the buy-and-hold portfolio share, as well as the most recent return differential (of a destination country relative to the overall fund return). But the optimal portfolio does not depend on expected excess returns. Portfolios therefore only have backward looking elements (past portfolios and past returns) and no forward looking element. Raddatz, Schmukler and Williams (2017) also estimate portfolio regressions for EPFR funds. Portfolio shares are related to benchmark portfolios in the form of indices, such as global or regional stock or bond indices, as well as relative returns. There are again no forward looking elements. Disyatat and Gelos (2001) compare EPFR portfolio weights to the predictions of a simple mean-variance portfolio model, where the variance of the portfolio return is replaced by the variance of a tracking error relative to a benchmark index and expected returns are based on historical returns.

Camanho et al. (2022) use data for equity funds in various countries to consider the extent of portfolio rebalancing by different funds. They use the FactSet/LionShares database, also used by Ferreira and Matos (2008). These data cover more funds than EPFR data, as they include a broad set of institutional in-

\footnote{Curcuru et al. (2014) stress the role of future returns, but use \textit{ex post} realized returns in their regressions.}
vestors rather than just mutual funds (including for example pension funds, bank trusts and insurance companies). In addition, only a limited set of mutual funds that report to EPFR indicate their country allocation. Nonetheless, there are several advantages of EPFR data over FactSet data that make it attractive for our purpose. Since we are interested in the speed of portfolio adjustment, it is attractive that the EPFR data are available at the monthly frequency. The reporting frequency of the FactSet data is quarterly, semi-annual or annual, dependent on the country. In addition, funds reporting to EPFR report their portfolio allocation at the end of each month. Reporting dates vary significantly across funds in the FactSet data. It is important that the time of reporting matches up to the date at which we compute expected excess returns. Finally, just using data for mutual funds has the advantage that they are more homogenous than the entire set of institutional investors. They are also the most internationally oriented.

Outside the open economy literature, there is a literature on individual portfolio choice that has documented significant portfolio inertia. This literature (e.g. Ameriks and Zeldes (2004), Bilias et al. (2010), Brunnermeier and Nagel (2008), Mitchell et al. (2006)) uses data on portfolio allocation by individual households. It is consistent with gradual portfolio adjustment, although (like the other literatures discussed above) it does not relate portfolio allocation to expected excess returns as in standard portfolio theory. An exception is the recent paper by Giglio et al. (2021), which relates equity portfolio shares to expected returns based on survey data of US-based Vanguard investors. They find that portfolio shares depend positively on reported equity return expectations, but that responsiveness to expected equity returns is too weak to make sense in the context of the frictionless mean-variance portfolio choice model (implied risk aversion is excessive). They further provide evidence that changes in expected returns have limited explanatory power for when investors trade, but help predict the direction and the magnitude of trading conditional on its occurrence. They argue that the evidence is consistent with infrequent trading.

The remainder of the paper is organized as follows. Section ?? presents the model. Section ?? analyzes the predictability of international equity return differentials. Section ?? presents results from estimating the fund-level portfolio regressions. Section ?? concludes.
2 A Model of Portfolio Allocation with Financial Frictions

2.1 Portfolio Objective

Consider a fund $i$ that allocates its equity portfolio to $N$ countries. We will focus the analysis here on a specific fund $i$, although one should keep in mind that the investment universe varies across funds. We take the set of countries that a fund invests in as given and consider the portfolio allocation across these countries. One should therefore keep in mind that the number of countries $N$ in the investment universe will vary across funds.

The vectors of portfolio shares and country equity returns are

$$
\begin{align*}
\mathbf{z}_{i,t} & = \begin{pmatrix} z_{i,1,t} \\ \vdots \\ z_{i,N,t} \end{pmatrix} \\
\mathbf{R}_{t+1} & = \begin{pmatrix} R_{1,t+1} \\ \vdots \\ R_{N,t+1} \end{pmatrix}
\end{align*}
$$

(1)

where $z_{i,n,t}$ is the share that fund $i$ allocates to country $n$ at time $t$ and $R_{n,t+1}$ is the country $n$ equity return from $t$ to $t+1$.

Define the buy-and-hold portfolio as

$$
\mathbf{z}_{bh,i,n,t} = \frac{z_{i,n,t-1} + 1 + R_{n,t}}{1 + z'_{i,t-1} \mathbf{R}_t}
$$

(2)

This is the portfolio held at time $t$ in the absence of asset trade at time $t$. The buy-and-hold portfolio share only differs from the lagged portfolio share $z_{i,n,t-1}$ due to valuation effects associated with equity returns.

We consider a structure similar to Gărlăneu and Pedersen (2013), where funds maximize the present discounted value of risk-adjusted portfolio returns, but face costs of deviating from benchmark portfolios. The objective of the fund is to maximize

$$
\begin{align*}
\sum_{s=0}^{\infty} \beta^s E_{i,t} \mathbf{z}'_{i,t+s} \mathbf{R}_{t+s+1} & - 0.5 \gamma_i \sum_{s=0}^{\infty} \beta^s \mathbf{z}'_{i,t+s} \mathbf{\Omega}_{i,t} \mathbf{z}_{i,t+s} \\
-0.5 \mu_1 \sum_{s=0}^{\infty} \beta^s E_{i,t} (\mathbf{z}_{i,t+s} - \mathbf{z}_{i,t+s-1})' \mathbf{A}_{i,t} (\mathbf{z}_{i,t+s} - \mathbf{z}_{i,t+s-1}) \\
-0.5 \mu_2 \sum_{s=0}^{\infty} \beta^s E_{i,t} (\mathbf{z}_{i,t+s} - \mathbf{z}_{bh,i,t+s}') \mathbf{A}_{i,t} (\mathbf{z}_{i,t+s} - \mathbf{z}_{bh,i,t+s})
\end{align*}
$$

(3)
where $E_{i,t}$ is the expectation of fund $i$ at time $t$. The variance at time $t$ of $R_{t+s+1}$ is denoted $\Omega_{i,t}$.

The first line of (10) is a present-value version of a standard mean-variance objective. The discount rate is $\beta$ and the rate of risk aversion is $\gamma_i$. The last two lines capture the cost of deviating from the benchmark portfolios, respectively the lagged portfolios and the buy-and-hold portfolios. The parameters $\mu_{1,i}$ and $\mu_{2,i}$ determine the cost of deviating from respectively the lagged portfolios and the buy-and-hold portfolios. We follow Gârleanu and Pedersen (2013) by assuming $A_{i,t} = \Omega_{i,t}$, for which they provide micro foundations.

There can be multiple underlying frictions that generate the gradual portfolio adjustment implied by (10). Gârleanu and Pedersen (2013) think of it as transaction costs. This is particularly relevant when deviating from the buy-and-hold portfolio, which involves asset trade. There may also be costs to acquiring information or costs to portfolio reoptimization that make fund managers more conservative and stick closer to prior benchmarks. In addition, it is possible that fund managers are penalized more for bad performance if this happens after they make significant portfolio changes relative to benchmark portfolios. This can take the form of fund outflows that affect manager compensation or lead to replacement of managers.

2.2 Optimal Portfolio

For a given optimal portfolio share $z_{i,n,t}$, consider the allocation of the remaining portfolio share $1 - z_{i,n,t}$ among countries other than $n$. Specifically, define

$$z_{i,m,-n,t} = \frac{z_{i,m,t}}{\sum_{k \neq n} z_{i,k,t}} = \frac{z_{i,m,t}}{1 - z_{i,n,t}} \quad (4)$$

This is the share allocated to country $m$ of the equity portfolio outside country $n$. We can define a vector $\mathbf{z}_{i,-n,t}$, where element $m$ is equal to $z_{i,m,-n,t}$ if $m \neq n$ and element $n$ is zero. Choosing the optimal $\mathbf{z}_{i,t}$ is equivalent to choosing $z_{i,n,t}$ and $\mathbf{z}_{i,-n,t}$. The first-order condition with respect to $z_{i,n,t}$ therefore takes the optimal $\mathbf{z}_{i,-n,t}$ as given.

We define a reference portfolio return for fund $i$ and country $n$ as the return on the portfolio of countries other than country $n$, with portfolio weights $z_{i,m,-n,t}$:

$$R_{\text{ref}(i,n),t+1} = \sum_{m \neq n} z_{i,m,-n,t} R_{m,t+1} = \mathbf{z}_{i,-n,t}' \mathbf{R}_{t+1} \quad (5)$$
The excess return of country \( n \) relative to the reference portfolio is

\[
er_{i,n,t+1} = R_{n,t+1} - R_{ref(i,n),t+1} = (e_n - z_{i,-n,t})' R_{t+1}
\]

where \( e_n \) is a vector of size \( N \) with element \( n \) equal to 1 and zeros otherwise.

We maximize (??) with respect to \( z_{i,n,t} \) after substituting the identity

\[
z_{i,t} = z_{i,-n,t} + (e_n - z_{i,-n,t}) z_{i,n,t}
\]

Appendix A derives the first-order condition with respect to \( z_{i,n,t} \). After some rewriting, it becomes:

\[
E_{i,t} er_{i,n,t+1} + u_{i,n,t} - \gamma_i \sigma_{i,n}^2 \hat{z}_{i,n,t} + (\mu_{1,i} + \mu_{2,i}) \sigma_{i,n}^2 (\beta E_{i,t} \hat{z}_{i,n,t+1} - (1 + \beta) \hat{z}_{i,n,t} + \hat{z}_{i,n,t-1}) + \mu_{2,i} \sigma_{i,n}^2 \hat{z}_{i,n,t} (1 - \bar{z}_{i,n}) (er_{i,n,t} - \beta E_{i,t} er_{i,n,t+1}) = 0
\]

Here \( \hat{z}_{i,n,t} = z_{i,n,t} - \bar{z}_{i,n} \), where

\[
\bar{z}_{i,n} = -\frac{\sigma_{n,ref(i,n)}}{\sigma_{i,n}^2}
\]

is the mean (over time) of the portfolio share allocated to country \( n \) by fund \( i \), where \( \sigma_{n,ref(i,n)} \) is the mean value of \( \text{cov}_{i,t}(er_{i,n,t+1}, R_{ref(i,n),t+1}) \) and \( \sigma_{i,n}^2 \) is the mean value of \( \text{var}_{i,t}(er_{i,n,t+1}) \). While \( \bar{z}_{i,n} \) depends on the mean level of risk, the term \( u_{i,n,t} \) in (??) captures time-varying risk in deviation from its mean:

\[
u_{i,n,t} = -\gamma_i \left( \text{cov}_{i,t}(er_{i,n,t+1}, R_{ref(i,n),t+1}) - \sigma_{n,ref(i,n)} \right) - \gamma_i \bar{z}_{i,n} \left( \text{var}_{i,t}(er_{i,n,t+1}) - \sigma_{i,n}^2 \right)
\]

Appendix B derives the following solution to the second-order difference equation (??) in \( \hat{z}_{i,n,t} \):

\[
\hat{z}_{i,n,t} = \omega_i \left( \frac{\mu_{1,i}}{\mu_{1,i} + \mu_{2,i}} \hat{z}_{i,n,t-1} + \frac{\mu_{2,i}}{\mu_{1,i} + \mu_{2,i}} \bar{z}_{i,n,t} \right) + \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} E_{t} er_{i,n,t+s} + \epsilon_{i,n,t}
\]

\(^6\)It is worth noting that this second-order difference equation in the portfolio share is the same as in Bacchetta, Davenport and van Wincoop (2021) when there is only a cost of deviating from the lagged portfolio and \( \mu_{1,i} \sigma_{i,n}^2 \) is equal to the portfolio cost parameter \( \psi \) in that paper. Rather than assuming the objective (??), Bacchetta, Davenport and van Wincoop (2021) consider a setup with two countries with Rince preferences. The rate of relative risk aversion \( \gamma \) in the Rince preferences enters the second-order difference equation of the portfolio share the same as here. We therefore interpret \( \gamma \) as a rate of relative risk aversion. The portfolio cost in that paper is a quadratic cost of deviating from the past portfolio share that enters the utility function.
where $\theta_i = \mu_{1,i} + \mu_{2,i}$,
\[
\epsilon_{i,n,t} = \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} E_{i,t} u_{i,n,t+s-1} + \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} (E_{i,t} \epsilon_{i,n,t+s} - E_t \epsilon_{i,n,t+s})
\]
and
\[
\omega_i = \frac{2\theta_i}{\gamma_i + (1 + \beta) \theta_i + \sqrt{\gamma_i^2 + (1 - \beta)^2 \theta_i^2 + 2(1 + \beta) \gamma_i \theta_i}}
\]
In (12) the term $E_{i,t} \epsilon_{i,n,t+s} - E_t \epsilon_{i,n,t+s}$ captures the expected excess return by fund $i$ minus the expected excess return by the econometrician. The expectation operator for the latter is denoted $E_t$.

In general there is heterogeneity across funds in $\gamma_i$, $\mu_{1,i}$ and $\mu_{2,i}$, as well as heterogeneity across $(i, n)$ pairs with respect to $\sigma_{i,n}^2$ and risk that determines $\bar{z}_{i,n}$. The same parameters without the $i$ and $n$ subscripts will refer to their mean across funds and countries. Since we will not be able to precisely characterize the heterogeneity across funds in the data, we focus on the mean of these parameters. To do so, we linearize the optimal portfolio expression (12) with respect to these parameters equal to their mean and all $\hat{z}$ variables and excess returns equal to zero.\footnote{We therefore omit second order terms such as $(\omega_i - \omega) \hat{z}_{i,n,t-1}$, where $\omega$ is the mean of $\omega_i$. This involves the product of two variables that both have a mean of zero. In Section 4.1 we will consider heterogeneity associated with $\sigma_{i,n}^2$ that leads to heterogeneity in the coefficient on the present discounted value of the expected excess return.}

Defining $\delta = \beta \omega$, this gives
\[
z_{i,n,t} = b_{i,n} + b_1 \tilde{z}_{i,n,t-1} + b_2 \hat{z}_{i,n,t} + b_3 E_{i,n,t} + \epsilon_{i,n,t}
\]
where
\[
b_{i,n} = (1 - \omega) \bar{z}_{i,n}; \quad b_1 = \omega \frac{\mu_1}{\mu_1 + \mu_2}; \quad b_2 = \omega \frac{\mu_2}{\mu_1 + \mu_2}; \quad b_3 = \frac{\omega}{\theta \sigma^2 (1 - \delta)}
\]
\[
\omega = \frac{2\theta}{\gamma + (1 + \beta) \theta + \sqrt{\gamma^2 + (1 - \beta)^2 \theta^2 + 2(1 + \beta) \gamma \theta}}
\]
Here $\omega$, $\theta$, $\gamma$, $\mu_1$ and $\mu_2$ refer to the mean across funds of $\omega_i$, $\theta_i$, $\gamma_i$, $\mu_{1,i}$ and $\mu_{2,i}$, while $\sigma^2$ is the mean across all $(i, n)$ of $\sigma_{i,n}^2$. The present discounted value of future expected excess returns is defined as
\[
E_{i,n,t} = \sum_{s=1}^{\infty} (1 - \delta)^{s-1} E_{i,t} \epsilon_{i,n,t+s}
\]
Note that it is defined such that the weights on all future expected excess returns sum to 1.

2.3 Intuition

Equation (2.3) writes the optimal portfolio as a linear function of the two benchmark portfolios, $z_{i,n,t-1}$ and $z_{i,n,t}^{bh}$, the expected present discounted value of the excess return on country $n$ equity relative to the reference portfolio, and a time-varying error term.

First consider the role of risk aversion. In general, investors face a trade-off between risk, expected returns and the cost of deviating from the benchmark portfolios. A rise in risk aversion implies that investors are more concerned with risk, and therefore relatively less concerned with deviating from the benchmark portfolios and expected excess returns. This therefore reduces $b_1$, $b_2$ and $b_3$ (through $\omega$).

Next consider the role of the portfolio frictions. A higher relative cost of deviating from the lagged portfolio compared to the buy-and-hold portfolio leads to a higher relative weight on the lagged portfolio in the portfolio expression: $b_1/b_2 = \mu_1/\mu_2$. An increase in the aggregate portfolio friction $\theta$ implies that investors are relatively more concerned with deviating from the benchmark portfolios and therefore relatively less concerned with risk and expected returns. It therefore raises the weight on both benchmark portfolios ($b_1$ and $b_2$), while it lowers the weight on the present discounted value of expected future excess returns ($b_3$).

Portfolio frictions also have the implication that the optimal portfolio gives relatively more weight to expected excess returns further into the future. The weight on the expected excess return $s$ periods into the future is $(1 - \delta)\delta^{s-1}$ with $\delta = \beta \omega$. A higher average portfolio friction $\theta$ raises $\delta$, which implies more weight

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8In models where investors aim to minimize the tracking error relative to a benchmark, they care both about the expected portfolio return and the variance of a relative portfolio (the chosen portfolio minus a benchmark portfolio). In that case higher risk aversion still implies a weaker response to expected returns, but the weight on the benchmark portfolio is unaffected by risk aversion. See, for example, Pavlova and Sikorskaya (2022) and Disyatat and Gelos (2001). The same would happen in our model if we assume that the frictions $\mu_{1,i}$ and $\mu_{2,i}$ are proportional to risk aversion. In that case $\omega$ is unaffected by risk aversion, so that the weights $b_1$ and $b_2$ on the benchmark portfolios are unaffected by risk aversion.
on expected returns further into the future. Without portfolio frictions, all the weight is on the expected excess return in the immediate future:

\[ z_{i,n,t} = b_{i,n} + \frac{E_t r_{i,n,t+1}}{\gamma \sigma_{i,n}^2} + \epsilon_{i,n,t} \]  

(16)

Gărlăeanu and Pedersen (2013) explain the impact of the portfolio friction in terms of an “aim portfolio.” In their setup there is only a cost of deviating from the lagged portfolio. They show that the optimal portfolio is a weighted average of the lagged portfolio share and that of the aim portfolio. The aim portfolio is forward looking. It is a portfolio towards which investors trade each period. The aim portfolio is a weighted averaged of the Morkowitz portfolio and the expected aim portfolio during the next period. One can write it as the present discounted value of the current and all future Markowitz portfolios. Since the Markowitz portfolio depends on the one-period expected excess return, the aim portfolio depends on the expected present discounted value of all future expected excess returns. As a result, the optimal portfolio has both a backward-looking part (the lagged portfolio) and a forward-looking part that depends on all future Markowitz portfolios and therefore all future expected excess returns. As a result of the portfolio friction, investors wish to smooth the transition between the past portfolio and future Markowitz portfolios.

Some comments are in order regarding the error term \( \epsilon_{i,n,t} \) defined in (??). The first term is the expected present discounted value of risk in deviation from its mean. Risk is defined by \( u_{i,n,t} \) in (??) and depends on both the variance of the excess return and the covariance of the excess return with the reference portfolio. The second term captures differences between expected excess returns by fund \( i \) and that by the econometrician (denoted with the expectation operator \( E_t \)). This may include deviations from rational expectations of excess returns that lead to noise trade. It may also capture changes in expected excess returns associated with information that is not easily available to an econometrician. Waves of optimism or pessimism about a country that we cannot easily measure therefore go into the

\[ \text{We can also see that investors give more weight to expected excess returns further into the future (higher } \delta \text{) when the time discount rate } \beta \text{ is higher and the risk aversion } \gamma \text{ is lower.} \]

\[ \text{Bacchetta, van Wincoop, and Young (2022) obtain a similar dependence of the portfolio on the past portfolio and the present value of expected future excess returns in a framework where there is a given probability } p \text{ of changing the portfolio each period, analogous to Calvo price setting.} \]
error term as well.

It should finally be pointed out that there is an alternative way of writing the optimal portfolio expression. Define a valuation effect variable as the difference between the buy-and-hold portfolio and the lagged portfolio:

$$val_{i,n,t} = z_{i,n,t}^{bh} - z_{i,n,t-1}$$

(17)

Linearizing, we have $val_{i,n,t} = \bar{z}_{i,n}(1 - \bar{z}_{i,n})\epsilon_{i,n,t}$. It tells us how much the portfolio share increases due to an increase in the excess return in the absence of any asset trade.

The portfolio can then be written as

$$z_{i,n,t} = b_{i,n} + a_1 z_{i,n,t-1} + a_2 val_{i,n,t} + a_3 ER_{i,n,t} + \epsilon_{i,n,t}$$

(18)

where $a_1 = b_1 + b_2$, $a_2 = b_2$, $a_3 = b_3$. The coefficient on the valuation effect therefore corresponds to the coefficient on the buy-and-hold portfolio in (??), while the coefficient on the lagged portfolio is now the sum of the coefficients on the lagged and buy-and-hold portfolios in (??). Raddatz and Schmukler (2012) estimate a portfolio expression for mutual funds that includes the lagged portfolio and valuation effect, but not the expected excess returns.

Given estimates of the reduced form parameters $a_1$, $a_2$ and $a_3$ from the portfolio regression, we can then extract the structural parameters. We will used scaled structural parameters for the portfolio frictions, defined as $\lambda_1 = \mu_1 \sigma^2$ and $\lambda_2 = \mu_2 \sigma^2$. Then

$$\lambda_1 = \frac{a_1 - a_2}{a_3 (1 - \delta)}$$

(19)

$$\lambda_2 = \frac{a_2}{a_3 (1 - \delta)}$$

(20)

$$\gamma = \frac{1 - a_1}{a_3 \sigma^2}$$

(21)

We also have $\delta = \beta a_1$.

### 3 Predicting Cross-Country Equity Return Differentials

A key variable in the portfolio expression derived above is the expected excess return variable $ER_{i,n,t}$. This section describes how we construct estimates for expected excess return differentials. After reviewing the empirical strategy, we show
that return differentials can be predicted by standard variables: dividend-price, earnings-price and momentum. We present results for pooled linear regressions.

3.1 Outline

The excess return in a specific country is fund-specific as it depends on equity returns in the reference countries and the portfolio weights of the fund in these countries. In this section, rather than considering expected excess returns for individual funds, we consider return differentials relative to the US. Specifically, the excess return for country \( n \) at \( t + s \) is \( er_{n,t+s} = R_{n,t+s} - R_{US,t+s} \), where \( R_{n,t+s} \) and \( R_{US,t+s} \) are the equity returns of country \( n \) and the US at \( t + s \). As discussed further in the next section, the expected excess return for a specific fund can easily be computed once we know the expected excess returns relative to the US for individual countries. For a fund \( i \) it is simply the expected excess return \( E_i er_{n,t+s} \) for country \( n \) minus the weighted average of expected excess returns \( E_i er_{m,t+s} \) of the reference countries, using the portfolio shares for the reference portfolio of fund \( i \) as weights.

In the theory, portfolio shares depend on a present discounted value of expected excess returns at all future dates. We will indeed use such present values when applying the theory to US mutual fund portfolio shares in the next section. But in this section we consider either the predictability of excess returns \( er_{n,t+1} \) over the next month or cumulative excess returns \( er_{n,t,t+k} = er_{n,t+1} + ... + er_{n,t+k} \) over the next \( k \) months. We use panel regressions to report predictability at different horizons.

3.2 Panel Regressions

We use pooled regressions over 73 countries with monthly data from January 1970 to March 2019. All data in the baseline regressions come from MSCI, using the last trading day of the month. Since data availability starts later for many countries, this gives us an unbalanced panel with more than 22,000 observations.\textsuperscript{11} Returns are computed from the MSCI total return index. We consider the following benchmark regression:

\[
er_{n,t,t+k} = \alpha_n + \beta'X_{n,t} + \varepsilon_{n,t}
\]

\textsuperscript{11}There are 18 countries in the sample in 1970, increasing to 35 in 1988, 44 in 1993, etc.
where $X_{n,t}$ is a set of explanatory variables known at time $t$. Following Petersen (2009), we include a country fixed effect and cluster standard errors by month.\footnote{Results do not change much if we include time fixed effects. We notice, however, that since we consider return differentials, global stock market shocks should not matter much.} Using pooled data and assuming a common coefficient $\beta$ allows us to get more precise estimates.\footnote{Hjalmarsson (2010) shows that pooling across countries gives superior predictability.}

The explanatory variables in the benchmark specification are standard in the literature on stock return predictability,\footnote{See for example Koijen and Van Nieuwerburgh (2011) or Rapach and Zhou (2013) for surveys.} but here we consider the differential with the US. These variables are the differential in the log earning-price ratio $dep_{n,t} = \ln(E/P)_{n,t} - \ln(E/P)_{US,t}$; the differential in the log dividend-price ratio $ddp_{n,t} = \ln(D/P)_{n,t} - \ln(D/P)_{US,t}$; and momentum, measured by the current return differential $er_{n,t-1,t}$. Since we take the log of the earning-price ratio, we omit the periods where it takes a negative value.\footnote{Negative values are observed during the Asian and the Scandinavian financial crises.}

Table 1 shows the results of regression (??) for one-period ahead returns $er_{n,t,t+1}$. We see that the three variables are strongly significant and have the expected sign. From the first column, it is interesting to notice that the small coefficient of 0.0426 on momentum implies that excess returns are not very persistent. In line with the literature on return predictability, the $R^2$ is extremely low for short-horizon predictions.

The fit of equation (??) significantly improves when the horizon increases. Table 2 shows the results for one month (as in Table 1), 12 months, 24 months, and 36 months excess returns, using the three variables in the regression. We see that coefficient values increase with the horizon. Moreover, the $R^2$ increases significantly, reaching 13.7% at the 36-month horizon.

The results in Tables 1 and 2 show that there is indeed predictability of stock market return differentials and that it is particularly strong at longer horizons.\footnote{The Online Appendix shows the Clark-McCracken (2001) tests confirming out-of-sample predictability.} In Appendix C we show that the predictability is also economically significant, following an approach similar to Cenedese et al. (2016). When sorting countries each month into quintiles based on their values of momentum, dividend-price differential, or earning-price differential, we show that returns are substantially higher for higher quintiles, i.e., higher values of momentum, dividend-price, or earning-price differentials.
Table 1: Regressions One-Month Return Differential $e_{n,t,t+1}$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>0.0426**</td>
<td>0.0426**</td>
<td>0.0439**</td>
<td>0.0441**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0167)</td>
<td>(0.0177)</td>
<td>(0.0173)</td>
<td>(0.0178)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dividend-Price</td>
<td></td>
<td>0.00695***</td>
<td>0.00757***</td>
<td>0.00595***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00210)</td>
<td>(0.00208)</td>
<td>(0.00204)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earning-Price</td>
<td>0.00660***</td>
<td>0.00716***</td>
<td>0.00459**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00197)</td>
<td>(0.00196)</td>
<td>(0.00196)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.000846</td>
<td>-0.00198</td>
<td>-0.00177</td>
<td>-0.00189</td>
<td>-0.00218</td>
<td>-0.00298*</td>
</tr>
<tr>
<td></td>
<td>(0.00146)</td>
<td>(0.00153)</td>
<td>(0.00149)</td>
<td>(0.00148)</td>
<td>(0.00152)</td>
<td>(0.00153)</td>
</tr>
<tr>
<td>Observations</td>
<td>24675</td>
<td>22873</td>
<td>22033</td>
<td>22021</td>
<td>22856</td>
<td>21908</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Standard errors clustered by month in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Notes: Regressions with 73 countries over the interval 1970:01-2019:02. All regressions include a country fixed effect.

Table 2: Regressions Return Differential - Different Horizons

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_{n,t,t+1}$</td>
<td>$e_{n,t,t+12}$</td>
<td>$e_{n,t,t+24}$</td>
<td>$e_{n,t,t+36}$</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.0441**</td>
<td>0.3318***</td>
<td>0.4930***</td>
<td>0.8175***</td>
</tr>
<tr>
<td></td>
<td>(0.0178)</td>
<td>(0.0683)</td>
<td>(0.1130)</td>
<td>(0.1621)</td>
</tr>
<tr>
<td>Dividend-Price</td>
<td>0.0060***</td>
<td>0.0994***</td>
<td>0.2289***</td>
<td>0.3866***</td>
</tr>
<tr>
<td></td>
<td>(0.0020)</td>
<td>(0.0097)</td>
<td>(0.0198)</td>
<td>(0.0336)</td>
</tr>
<tr>
<td>Earning-Price</td>
<td>0.0046**</td>
<td>0.0372***</td>
<td>0.0935***</td>
<td>0.1537***</td>
</tr>
<tr>
<td></td>
<td>(0.0020)</td>
<td>(0.0090)</td>
<td>(0.0161)</td>
<td>(0.0255)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0030*</td>
<td>-0.0265***</td>
<td>-0.0563***</td>
<td>-0.0969***</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0076)</td>
<td>(0.0143)</td>
<td>(0.0212)</td>
</tr>
<tr>
<td>Observations</td>
<td>21908</td>
<td>21116</td>
<td>20254</td>
<td>19392</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.009</td>
<td>0.064</td>
<td>0.104</td>
<td>0.137</td>
</tr>
</tbody>
</table>

Standard errors clustered by month in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Notes: Regressions with 73 countries over the interval 1970:01-2019:02. All regressions include a country fixed effect.
are associated with higher returns.

4 Results for Mutual Fund Portfolios

In this section we use panel data of US-based equity funds that report country portfolio allocations to EPFR. These data are used to estimate portfolio equations implied by the model developed in Section ???. The sample runs from January 2002 to July 2016.

4.1 Sample Selection and Portfolio Shares

The US mutual funds that report their country allocation to EPFR are mostly globally or regionally oriented, with a relatively small average share allocated to US equity. The funds report their equity holdings in 135 countries (including the US) and cash holdings. Cash holdings are relatively small, on average 2.8 percent of total AUM (assets under management). In what follows we focus on the non-cash component of AUM, the equity holdings in the 135 countries. Aggregating across all funds, during an average month 7.5 percent of equity holdings are allocated to the United States. This shows the strong global bias of our funds. As discussed further below, the far majority of the funds have no US equity holdings at all.

It is useful to put the foreign equity holdings of these funds into broader perspective. At the end of the sample, July 2016, total US foreign equity holdings was $7,045 billion.\textsuperscript{17} Of that, $3,394 billion (47 percent) was held by US mutual funds.\textsuperscript{18} US equity mutual funds that report their country allocation to EPFR report a $436 billion allocation to foreign equity, which is 13 percent of all foreign equity held by US mutual funds. The remaining 87 percent is held by US funds that do not report to EPFR and funds that do report to EPFR, but do not report their country allocation.

We clean the original sample. We focus on a subset of 316 funds investing in 36 countries and remove very small portfolio shares.\textsuperscript{19} The sample selection

\textsuperscript{17}Monthly US foreign equity holdings are reported by Bertaut and Tryon (2007), later extended by Bertaut and Judson (2014), who have since further updated it through December 2018.

\textsuperscript{18}See Exhibit 18A of the 2016 report “U.S. Portfolio Holdings of Foreign Securities” from the US department of Treasury.

\textsuperscript{19}The countries are: Australia, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Finland, France, Germany, Hong-Kong, India, Indonesia, Ireland, Israel, Italy, Japan, Malaysia,
is described below. In July 2016, this cleaned sample has a $395 billion foreign equity allocation. This is 91 percent of the full sample AUM, so not much AUM is lost by cleaning the sample.

In the Online Appendix, we report some evidence of how representative this sample is in terms of the allocation across foreign countries. For July 2016, we report the portfolio shares allocated to the 35 foreign countries. We do this both for our sample of 316 mutual funds and for total US foreign equity holdings. The correlation is 0.88. Our mutual fund sample invests a higher share in emerging markets, particularly in Asian and Latin American countries. We also report time series of portfolio shares allocated to 3 regions (Europe, Asia and Latin America) from January 2002 to July 2016. These portfolio shares look quite similar to those based on total foreign equity holdings, with correlations of respectively 0.67, 0.56 and 0.79 for the three regions.\(^\text{20}\)

Regarding the selection of funds, we only include US equity funds with more than $5 million in AUM at the end of the sample. In addition, we impose that the fund must report its global equity allocations for at least 12 consecutive months during the sample. This leaves us with a total of 316 funds. We then drop very small portfolio shares and countries in which very few funds invest or for which we have insufficient MSCI data. There are two problems with small portfolio shares. First, valuation effects are very close to zero. As discussed, after linearizing we can write the valuation effect as \(\text{val}_{i,n,t} = \bar{z}_{i,n}(1 - \bar{z}_{i,n})e_{i,n,t}.\) When \(\bar{z}_{i,n}\) is very close to zero, the valuation effect is essentially zero. This makes it difficult to determine the coefficient on the valuation effect, which is needed to determine the relative importance of the two portfolio costs (deviation from the lagged portfolio and the buy-and-hold portfolio). Second, as discussed below, we assume that the return of a fund in country \(n\) is the MSCI return of country \(n\). If a fund invests very little in country \(n\), its country \(n\) portfolio is much less likely to be well represented by the MSCI return in that country.\(^\text{21}\)

To be more precise with our selection, consider the allocation of each fund to Mexico, Netherlands, Norway, Peru, Philippines, Poland, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom and United States. 

\(^{20}\)Related, Miao and Pant (2012) find that EPFR capital flows and balance of payments capital flows behave similarly for different regions of the world for both equity and bonds.

\(^{21}\)A final problem is that when the average portfolio share in a country by a fund is very small, the number of months for which the fund reports portfolio data in that country tends to be small.
the aggregate of the 135 countries. Write the corresponding portfolio share as \( k_{i,n,t} \), which is the investment by fund \( i \) in country \( n \) in month \( t \) as a share of the total allocation by fund \( i \) to the 135 countries in month \( t \). Let \( \bar{k}_{i,n} \) be the sample average, with the average taken over the months for which the fund reports the country allocation. Here we use the letter \( k \) instead of \( z \) to be clear that these are not the shares that will be used in the regression analysis.

For portfolio shares, we only consider \((i,n)\) pairs for which \( \bar{k}_{i,n} \) is at least 2 percent. Regarding countries, only 74 of the 135 have complete MSCI equity return data that are needed to compute excess returns and valuation effects. Many of these countries have very few funds that invest in them. We only include countries \( n \) in which at least 10 of the 316 funds invest during some month of the sample, not including the \((i,n)\) pairs we removed. This reduces the sample to 36 countries. The Online Appendix extends our main analysis by including smaller portfolio shares.

Funds enter and exit the sample. The 316 funds are never all reporting simultaneously. The number of funds reporting rises over time. During the first 12 months of the sample an average of 46 funds report each month, while during the last 12 months of the sample an average of 222 funds report each month. For an average country and an average month, 29 percent of reporting funds report a portfolio allocation to that country. For an average month, only 21 percent of reporting funds report a portfolio allocation to the United States. Therefore 79 percent of the funds only invest in foreign equity.

For our sample of 316 funds and 36 countries we compute \( z_{i,n,t} \) as the equity that fund \( i \) holds in country \( n \) during month \( t \), divided by the total equity allocation of fund \( i \) to the 36 countries in the sample during month \( t \). In addition, as necessary for the regressions, we only use observations \( z_{i,n,t} \) when data are available for both \( z_{i,n,t} \) and \( z_{i,n,t-1} \). This results in a total of 154,407 observations.

### 4.2 Equity Returns

We need data on equity returns both to compute an estimate of expected excess returns and to compute the buy-and-hold portfolio. We would preferably use equity returns of individual funds in each country. Unfortunately EPFR does not provide the return of funds in individual countries. We therefore follow Raddatz and Schmukler (2012) and use MSCI returns in each country. They argue that
this provides a reasonable approximation. In what follows it is therefore assumed that \( R_{n,t} \) is the equity return of country \( n \) for all funds.

We construct the expected excess return variable \( ER_{i,n,t} \) as follows. We use the average sample weights \( \bar{z}_{i,m,-n} \) to compute the reference portfolio. In the Online Appendix we show that results are similar when we use the contemporaneous weights \( z_{i,m,-n,t} \). The excess return relative to the reference portfolio can then be written as

\[
er_{i,n,t+s} = R_{n,t+s} - \sum_{m \neq n} \bar{z}_{i,m,-n} R_{m,t+s} = \sum_{m \neq n} \bar{z}_{i,m,-n} e_r_{m,t+s}
\]

where \( e_r_{n,t+s} \) is the excess return at \( t+s \) of country \( n \) relative to the United States. It then follows that

\[
ER_{i,n,t} = (1 - \delta) \sum_{s=1}^{k} \delta^{s-1} E_t \left( e_r_{n,t+s} - \sum_{m \neq n} \bar{z}_{i,m,-n} e_r_{m,t+s} \right)
\]

(23)

First some comments are in order regarding \( k \) and \( \delta \). While in the theory \( k = \infty \), in the empirical applications \( k \) is necessarily finite. We assume \( k = 24 \). We consider lower and higher values in the Online Appendix. The estimation of \( \delta \) is discussed in Section 4.5. It is consistent with its theoretical value of \( \beta a_1 \), where \( a_1 \) is the coefficient on the lagged portfolio share in (??).

We compute the expectation in (??) by estimating a panel regression of

\[
(1 - \delta) \sum_{s=1}^{k} \delta^{s-1} e_r_{n,t+s}
\]

on the same variables that we regressed \( e_r_{n,t,t+k} \) on in Section 3. However, in contrast to the previous section, here we create true forecasts using recursive regressions up to the time of the forecast rather than using the entire sample. The sample starts in January 1970. As shown in the Online Appendix, we still find predictability when restricting the sample to the 35 foreign countries and using discounted returns. The three variables, momentum, dividend-price and earning-price, are all significant.

The buy-and-hold portfolio is computed as follows. We have \( z_{i,n,t}^{bh} = z_{i,n,t-1} \frac{1+R_{n,t}}{1+R_{i,p,t}} \). For \( R_{i,p,t} \) we use the portfolio return of fund \( i \) obtained from EPFR. Since EPFR does not provide the return of funds in individual countries, we again use the country \( n \) equity return from MSCI for \( R_{n,t} \). We compute the valuation effect as \( val_{i,n,t} = z_{i,n,t}^{bh} - z_{i,n,t-1} \).
4.3 Endogeneity

An endogeneity problem arises when the error term of the portfolio regression (1) is correlated with 
\( z_{i,n,t-1}, \) \( val_{i,n,t} \) or \( ER_{i,n,t}. \) It is not hard to see how this could happen. Portfolio shifts that enter through the error term affect asset demand, which leads to changes in equilibrium equity prices.\(^{22}\) This is particularly the case when such portfolio shifts are common not just across the US mutual funds in our sample, but the broader class of investors in these equity markets.\(^{23}\) The three explanatory variables \( z_{i,n,t-1}, \) \( val_{i,n,t} \) or \( ER_{i,n,t} \) depend on the level of equity prices, changes in equity prices or both. All three may therefore be correlated with the error term.

We therefore seek instruments that are plausibly correlated with either the level or changes of equity prices, and therefore with our three explanatory variables, while at the same time being uncorrelated with the error term. The theory in Section 2 provides guidance about the nature of the error term. Equation (1) provides an expression of the error term, which is related to both risk and expectations of the excess returns that are not captured by the econometrician.

Risk depends on the present discounted value of \( u_{i,n,t}, \) which from (1) is a linear function of

\[
\text{cov}_{i,t}(er_{i,n,t}, R_{\text{ref}(i,n),t+1} + \bar{z}_{i,n}er_{i,n,t+1})
\]

Using that \( er_{i,n,t+1} = R_{n,t+1} - R_{\text{ref}(i,n),t+1}, \) we can also write this as

\[
\text{cov}_{i,t}(er_{i,n,t+1}, \bar{z}_{i,n}R_{n,t+1} + (1 - \bar{z}_{i,n})R_{\text{ref}(i,n),t+1})
\]

Country \( n \) equity is therefore riskier from the perspective of fund \( i \) when its excess return over the reference portfolio of fund \( i \) is more correlated with the overall portfolio return of the fund. The fund’s overall portfolio return is \( \bar{z}_{i,n}R_{n,t+1} + (1 - \bar{z}_{i,n})R_{\text{ref}(i,n),t+1}. \)

---

\(^{22}\)Bacchetta, van Wincoop and Young (2022) and Bacchetta, Davenport and van Wincoop (2022) discuss the impact of portfolio shocks on equilibrium asset prices and expected excess returns in the context of general equilibrium open economy models with gradual portfolio adjustment.

\(^{23}\)Portfolio shifts that are not common across investors are unlikely to generate endogeneity problems. Individual US mutual funds are much too small to have a significant effect on equity prices of other countries. On average individual funds represent 0.02 percent of US equity investment in a country, and US equity investment in a country is only a small fraction of stock market capitalization of the country.
We obtain a proxy for this risk by computing for each country $n$, fund $i$ and month $t$ the following risk measure:

$$risk_{i,n,t+1} = 10000 \text{cov} \left( er_{i,n,d \in t+1}, z_{i,n}R_{n,d \in t+1} + (1 - z_{i,n})R_{\text{ref}(i,n),d \in t+1} \right)$$

(26)

Here $d \in t + 1$ refers to days $d$ during month $t + 1$. We therefore compute the covariance using daily values of the excess return and overall portfolio return of the fund for the days during month $t + 1$. This uses MSCI data on daily equity returns for all countries in the sample.

The other component of the portfolio error term in (??) relates to the difference between expectations of the funds and our expectations as econometricians. We compute $ER_{i,n,t}$ by regressing future excess returns on momentum, the dividend-price ratio and the earnings-price ratio. Expectations of future excess returns by funds that are orthogonal to these variables end up in the error term. But this implies that this part of the error term is by construction uncorrelated with $ER_{i,n,t}$. In what follows we will therefore not be concerned with this component of the error term.

It should be said from the outset that the endogeneity problem is not easy to tackle. We are looking for instruments that have independent explanatory power for all three of our endogenous explanatory variables, while at the same time none of them can be correlated with risk. We identify a set of instruments in several steps. We start with a set of 13 variables that are plausibly correlated with equity prices or changes in equity prices, and therefore with our endogenous regressors. Next we reduce this to a set of 8 instruments that satisfy two criteria: (i) each instrument must have statistically significant explanatory power for at least one of the three endogenous regressors, (ii) none of the instruments are significant when regressing $risk_{i,n,t+1}$ on the set of instruments. We also check that the F-tests of the first-stage regressions of the regressors on the instruments are acceptable. Only when all of this is satisfied do we estimate the portfolio regression with Two-Stage Least Squares (2SLS).

We first discuss the set of 13 variables that we start with. Details regarding their computation can be found in the Appendix. Since the endogenous regressors depend on both the level and first difference of equity prices, we consider both the level and first difference for most of the variables. The first difference for a variable $x_{i,n,t}$ is defined as $\Delta x_{i,n,t} = x_{i,n,t} - x_{i,n,t-1}$. We focus the description here on the levels of the variables.
First, \( e_{i,n,t} \) is log earnings for country \( n \) equity, minus the portfolio-weighted average of log earnings for the reference countries with portfolio weights \( z_{i,m,-n} \). Earnings is obtained by multiplying the earnings-price ratio with the price index. \( d_{i,n,t} \) is defined analogously for dividends. \( i_{i,n,t} \) is the 3-month Euro Libor interest rate for country \( n \) minus the portfolio weighted average interest rates of the reference countries. \( y_{i,n,t} \) is the monthly log industrial production index for country \( n \), minus the portfolio weighted average for reference countries. \( b_{i,n,t} \) is the log book value for country \( n \) equity (market value divided by the price) minus the portfolio-weighted average for the reference countries. \( h_{i,n,t} \) is the log bond price index (mostly for 10-year government bonds) minus the weighted average for reference countries.

Together with their first differences, this gives 12 variables, all of which are naturally correlated with equity prices and equity returns. While our mutual funds invest in equity of different countries, other investors arbitrage equity returns relative to short and long term bond returns. Such arbitrage leads to a relationship between equity prices, interest rates and long-term bond prices. The industrial production variable may be correlated with equity prices in various ways. It may be related to the wealth of domestic investors, which affects demand for domestic equity and therefore the price. Industrial production growth may also affect equity prices to the extent that it helps predict future dividends. A change in the book value affects equity prices through a change in the equity supply. Finally, earnings and dividends naturally affect equity prices through the income component of equity returns.

The last variable we consider is the one-month lagged valuation effect, \( val_{i,n,t-1} \). It is associated with relative equity price changes from \( t-2 \) to \( t-1 \). It will be correlated with \( val_{i,n,t} \), one of our endogenous regressors, to the extent that there is any autocorrelation of equity price changes. Gabaix and Koijen (2022) find that financial shocks, which capture a common component of portfolio shifts across investors, have a very persistent effect on equity prices. While the error term of the portfolio expression can then be expected to be correlated with the level of the equity price, as well as the contemporaneous equity price change, it will not be much correlated with lagged price changes.

All of these variables could potentially be correlated with portfolio risk that enters the error term. For example, monetary policy may respond to a change in risk, which could lead interest rates and bond prices to be correlated with risk.
Similarly, periods of low corporate earnings may coincide with periods of increased risk. The relationship between the instruments and portfolio risk is ultimately an empirical matter that we need to check with our measure of risk.

In theory it may also be possible for the 13 variables discussed above to be correlated with a part of the error term that is unrelated to both risk and expected returns. But, as discussed, variables such as interest rates and bond prices are related to the portfolio behavior of other investors than our mutual funds. The same can be said about industrial production if it affects the wealth of domestic investors. The book value is a measure of portfolio supply rather than demand. Earnings and dividends naturally affect the portfolios of our mutual funds, but through the expected excess return.

The first step that we take to determine the validity of our variables as instruments is to conduct first-stage regressions of the three endogenous regressors on each of the 13 variables. We run regressions of \( z_{i,n,t-1}, \text{val}_{i,n,t} \) and \( ER_{i,n,t} \) on one variable at a time and remove variables that are not statistically significant at the 5 percent level for all of the endogenous regressors. This is the case for \( \Delta i_{i,n,t}, \Delta b_{i,n,t} \) and \( \Delta y_{i,n,t} \). This leaves us with 10 variables. Since we prefer to have as many instruments as possible, within the remaining set of 10 variables we identify the largest subset such that none of the variables are significant in a regression of the \( \text{risk}_{i,n,t+1} \) on these variables. For this we conservatively use 10 percent significance as a cutoff, which leads us to more easily reject variables as valid instruments than a 5 percent significance cutoff. The largest set includes 8 instruments:

\[
I_{i,n,t} = \{\text{val}_{i,n,t-1}, e_{i,n,t}, \Delta e_{i,n,t}, \Delta d_{i,n,t}, y_{i,n,t}, b_{i,n,t}, h_{i,n,t}, \Delta h_{i,n,t}\}
\]  

(27)

In going from 10 to 8 variables, we remove \( i_{i,n,t} \) and \( d_{i,n,t} \).

Table 3 reports regressions showing that these are valid instruments. The first three columns report first stage regressions of the three endogenous regressors on the 8 instruments. Each of the instruments in the set \( I_{i,n,t} \) is statistically significant for at least one of the endogenous regressors, and in most cases for two or all three of the endogenous regressors. For the regressors \( z_{i,n,t-1}, \text{val}_{i,n,t} \) and \( ER_{i,n,t} \) there are respectively 4, 4, and 8 instruments that are statistically significant at the 1 percent level. The first stage Sanderson-Windmeijer F statistics are all well above 10, again suggesting strong instruments. The last column of Table 3 reports the regression of \( \text{risk}_{i,n,t+1} \) on this set of instruments \( I_{i,n,t} \) as well as the lagged risk.
risk\(_{i,n,t}\). While lagged risk is significant, none of the instruments are significant. We therefore conclude that this set of instruments satisfies our criteria.

### 4.4 Benchmark Results

Columns (1)-(2) of Table 4 report the benchmark estimation of equation (??), both with OLS and IV. The regression includes a country-fund dummy \(b_{i,n}\). As we have seen in the theory, this is related to differences in mean portfolio shares \(\bar{z}_{i,n}\). We get the same results when we simply subtract the mean of all variables for each country-fund pair.\(^{24}\) Column (3) reports the regression (??) with IV, which replaces \(val_{i,n,t}\) with \(z_{i,n,t}^{bh}\). As discussed, these are identical regressions, just written slightly differently.

The role of endogeneity can be seen by comparing columns (1) and (2). While the differences are not large, we see that the coefficient on the valuation effect is a bit higher under OLS, while the coefficient on the expected excess return variable is a bit lower. This is intuitive. The error term is positively correlated with the valuation effect, as an exogenous financial flow towards country \(n\) raises the country \(n\) equity price. This leads to upward bias of the coefficient on \(val_{i,n,t}\). At the same time the higher price lowers the dividend-price and earnings-price ratios, which lowers the expected excess return. The negative correlation between the expected excess return and the error term therefore leads to a downward bias of the coefficient on the expected excess return variable under OLS.

Nonetheless, both the OLS and IV results yield a similar message. The weight on the lagged portfolio in (??) is highly significant and large, respectively 0.93 under OLS and 0.948 under IV. The coefficient on the valuation effect is positive and also highly significant. For the regression (??) in column (3) this means a substantial weight on both the lagged portfolio and the buy-and-hold portfolio. It appears therefore that both portfolio frictions are important, though the weight on the lagged portfolio is more than twice as big as on the buy-and-hold portfolio.

The coefficient on the expected excess return is also highly significant, 8.2 under OLS and 9.4 under IV. The standard error is substantially larger under IV, but the t-value is still a respectable 3.4 (versus 10 under OLS). The IV coefficient on

---

\(^{24}\)Not including the country-fund dummy is highly problematic. Since portfolio shares differ significantly across funds, it will bias the coefficient on the lagged portfolio share to be very close to 1. The same will happen when including imperfect controls related to \(\bar{z}_{i,n}\).
Table 3: Portfolio Regressions, First Stage, Benchmark

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>First Stage</td>
<td>Risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_{i,n,t-1}$</td>
<td>-0.005</td>
<td>15.830***</td>
<td>0.306***</td>
<td>-0.049</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(1.638)</td>
<td>(0.023)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>$e_{i,n,t}$</td>
<td>0.023***</td>
<td>-0.431</td>
<td>0.151***</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.308)</td>
<td>(0.004)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$val_{i,n,t-1}$</td>
<td>0.296***</td>
<td>-0.223***</td>
<td>-0.805***</td>
<td>-0.214</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.031)</td>
<td>(0.142)</td>
<td>(0.224)</td>
</tr>
<tr>
<td>$\Delta e_{i,n,t}$</td>
<td>-0.009**</td>
<td>2.377***</td>
<td>-0.036***</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.772)</td>
<td>(0.011)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$y_{i,n,t}$</td>
<td>-0.028***</td>
<td>1.076*</td>
<td>-0.214***</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.600)</td>
<td>(0.008)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$b_{i,n,t}$</td>
<td>0.024***</td>
<td>-0.207</td>
<td>0.131***</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.335)</td>
<td>(0.004)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$h_{i,n,t}$</td>
<td>0.000</td>
<td>-0.366</td>
<td>-0.028***</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.335)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\Delta h_{i,n,t}$</td>
<td>-0.004</td>
<td>17.971***</td>
<td>0.100***</td>
<td>-0.039</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(2.250)</td>
<td>(0.033)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>$risk_{i,n,t}$</td>
<td></td>
<td></td>
<td></td>
<td>0.450***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.110)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.000</td>
<td>-0.004</td>
<td>0.000</td>
<td>-0.004***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.137)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Observations</td>
<td>142,758</td>
<td>142,758</td>
<td>142,758</td>
<td>142,315</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.08</td>
<td>0.08</td>
<td>0.35</td>
<td>0.37</td>
</tr>
<tr>
<td>SW F-Test</td>
<td>44.38</td>
<td>29.92</td>
<td>47.15</td>
<td></td>
</tr>
</tbody>
</table>

Clustered standard errors by months in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Regressions for 36 countries over the interval 2002:01-2016:07. The variables used in columns (1)-(3) were regressed on a fund-country fixed effect to partial them out. SW F-Test stands for the Sanderson-Windmeijer F-test of excluded instruments. In columns (2), (3) and (4), we multiply the regressions coefficients by 1000 except for the respective lagged variables.
Table 4: Portfolio Regressions, Benchmark

<table>
<thead>
<tr>
<th></th>
<th>(1) OLS</th>
<th>(2) IV</th>
<th>(3) IV</th>
<th>(4) IV</th>
<th>(5) IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{i,n,t-1}$</td>
<td>0.930***</td>
<td>0.948***</td>
<td>0.655***</td>
<td>0.950***</td>
<td>0.951***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.013)</td>
<td>(0.062)</td>
<td>(0.014)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$val_{i,n,t}$</td>
<td>0.423***</td>
<td>0.293***</td>
<td>0.296***</td>
<td>0.298***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.066)</td>
<td>(0.067)</td>
<td>(0.066)</td>
<td></td>
</tr>
<tr>
<td>$ER_{i,n,t}$</td>
<td>8.243***</td>
<td>9.403***</td>
<td>9.403***</td>
<td>9.388***</td>
<td>8.970***</td>
</tr>
<tr>
<td></td>
<td>(0.817)</td>
<td>(2.742)</td>
<td>(2.742)</td>
<td>(3.245)</td>
<td>(2.833)</td>
</tr>
<tr>
<td>$z_{bh}^{inh}$</td>
<td>0.293***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$risk_{i,n,t+1}$</td>
<td></td>
<td>-1.89*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.100)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$risk_{i,n,t+1}$</td>
<td></td>
<td></td>
<td>-1.738***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.655)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 154,407 142,758 142,758 141,478 142,315

$R^2$: 0.99 0.87 0.87 0.87 0.87

Clustered standard errors by months in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Regressions for 36 countries over the interval 2002:01-2016:07. The regressions include a fund-country fixed effect. In columns (2)-(5) the instruments are $val_{i,n,t-1}$, $e_{i,n,t}$, $\Delta e_{i,n,t}$, $\Delta d_{i,n,t}$, $y_{i,n,t}$, $b_{i,n,t}$, $h_{i,n,t}$, $\Delta h_{i,n,t}$.
the expected excess return variable is similar to findings by Giglio et al. (2021), even though their data are completely different. They regress the equity share of Vanguard investors on one-year expected excess returns on equity. The cross-sectional variation of expected excess returns, obtained from survey data, is key to their results. They implicitly assume that the portfolio depends on expected excess returns over the next 12 months, with equal weight on each month. In our expected excess return variable, most of the weight is also on the first 12 months (71 percent under OLS and 63 percent under IV).

For comparability, we multiply the coefficient on the expected excess return in Giglio et al. (2021) by 12 to translate to monthly expected excess returns. Their coefficient is then 8.3. It is 13.9 when they remove some outliers from their data. These numbers are broadly consistent with our estimates. They emphasize that this weight on the expected excess return is substantially lower than what one might expect in a frictionless model.

In the last 2 columns of Table 4 we control for risk. The covariance (??) is conditional on information by the fund at time \( t \). We do not know this information set. We will use two measures of \( \text{risk}_{i,n,t+1} \). The first is the unconditional covariance based on daily returns during month \( t + 1 \). This is appropriate when funds have so much information that at time \( t \) they know the ex-post covariance measured at \( t + 1 \). The second measure regresses \( \text{risk}_{i,n,t+1} \) on \( \text{risk}_{i,n,t} \) as well as the two variables we removed because of their significance in the risk regression, \( i_{i,n,t} \) and \( d_{i,n,t} \). These time \( t \) variables are in the information set of funds. We denote the risk conditional on these three variables as \( \hat{\text{risk}}_{i,n,t+1} \). Table 4 shows that both \( \hat{\text{risk}}_{i,n,t+1} \) and \( \text{risk}_{i,n,t+1} \) are significant, with the expected negative sign. Controlling for risk has virtually no effect on the other estimated coefficients. This is not surprising as the instruments that we use have no predictive power for risk.

4.5 Retrieving the Structural Parameters

In connecting the estimates in Table 4 to the structural parameters from the theory, we will assume a time discount rate of \( \beta = 0.97 \). The time discount rate is not identified by the reduced form parameter estimates. While \( \beta = 0.97 \) may seem low with monthly data, it is important to keep in mind that the average turnover of portfolio managers is 2 percent per month (see Kostovetsky and Warner, 2015). An even lower \( \beta \) may need to be assumed if we take into account that many funds
have short lives. In the Online Appendix we consider alternative values for $\beta$.

In the theory $\delta$ is equal to $\beta a_1$, where $a_1$ is the coefficient on $z_{i,n,t-1}$ in (??). It turns out that the estimate of $a_1$ is virtually unaffected by the assumed $\delta$. In the regression, we first set $\delta = 0.9$, then estimate (??) to obtain an estimate of $a_1$ and therefore $\delta$. We then estimate (??) again when $ER_{i,n,t}$ is computed with this estimate of $\delta$.

Next we use equations (??), (??) and (??) to obtain point estimates and confidence intervals for the structural parameters $\lambda_1$, $\lambda_2$ and $\gamma$ from the point estimates and variance matrix of $a_1$, $a_2$ and $a_3$. We set $\sigma^2 = 0.00172$, which is the mean variance of the excess return across $(i,n)$. Table 5 reports results based on the OLS and IV estimates of Table 4 (columns (1) and (2)). It reports both point estimates of the structural parameters and 95 percent confidence intervals.

<table>
<thead>
<tr>
<th></th>
<th>OLS Estimate</th>
<th>95% confidence interval</th>
<th>IV Estimate (s.e.)</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.628</td>
<td>[0.520, 0.782]</td>
<td>0.866</td>
<td>[0.551, 1.944]</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.524</td>
<td>[0.409, 0.687]</td>
<td>0.387</td>
<td>[0.163, 1.093]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4.94</td>
<td>[4.120, 6.070]</td>
<td>3.215</td>
<td>[2.380, 4.965]</td>
</tr>
<tr>
<td>$\lambda_1 - \lambda_2$</td>
<td>0.104</td>
<td>[-0.028, 0.233]</td>
<td>0.479</td>
<td>[0.166, 1.070]</td>
</tr>
</tbody>
</table>

Notes: The table reports point estimates and 95 percent confidence intervals of the structural parameters implied by the regression results reported in Table 3, columns 1 and 2. It is based on 100,000 draws from the distribution of the reduced form parameters $[a_1, a_2, a_3]$.

A first point to note is that the confidence intervals for $\lambda_1$, $\lambda_2$, as well as $\lambda_1 - \lambda_2$, are much tighter based on the OLS than IV estimates. This is because in (??)-(??) the parameters $\lambda_1$ and $\lambda_2$ depend inversely on $a_3$, the coefficient on the expected excess return. We can see from Table 4 that while the magnitudes of the expected excess return coefficients for OLS and IV do not differ a lot, the standard error is much smaller for OLS, leading to substantially tighter estimates of the structural parameters.

Several points are immediate from Table 5. First, for both OLS and IV we see that $\lambda_1$ and $\lambda_2$ are positive and significant. There is therefore strong evidence that both portfolio frictions are important. Second, $\lambda_1 - \lambda_2$ is positive. It is significant under IV, which means that there is a larger cost of deviating from the lagged
portfolio than the buy-and-hold portfolio. This relates to the substantially higher coefficient on the lagged portfolio than the buy-and-hold portfolio in column (3) of Table 4.

Finally, the point estimate of $\gamma$ is 3.2 under IV, with a 95 percent confidence interval of $[2.4, 5.0]$. This is quite reasonable. By contrast, if we just regress on the one-month expected excess return (plus the fund-country fixed effect), as would be appropriate in the absence of portfolio frictions, we obtain a coefficient of 4.2 (s.e. = 0.25) with IV. Since the coefficient on the one-month expected excess return in the frictionless model is $1/(\gamma \sigma^2)$, it would imply $\gamma = 138$, which is clearly excessive. For a more reasonable level of risk aversion, the coefficient on the expected excess return would be far higher in the frictionless model. Therefore the estimates of $\lambda_1$, $\lambda_2$ and $\gamma$ all provide evidence of the importance of portfolio frictions.

4.6 Portfolio Dynamics

It is useful to consider the implication of the results above for the dynamic response of portfolios to an expected excess return innovation and compare the case with the estimated portfolio frictions to the frictionless case. For the case with frictions, the expected excess return variable is $ER_{i,n,t}$. Using the pooled data, we estimate an AR(1) process:

$$ER_{i,n,t} = a_{i,n} + \rho ER_{i,n,t-1} + \nu_{i,n,t},$$

where $a_{i,n}$ is a fund-country dummy. We obtain an AR coefficient $\rho = 0.87$ and an average standard deviation of $\nu_{i,n,t}$ across $(i, n)$ pairs of 0.00076. In the frictionless case the expected excess return is $E_{t}er_{i,n,t+1}$, for which we analogously obtain an AR coefficient of 0.51 and average standard deviation 0.0017 of the expected excess return innovation.

We make two additional assumptions. First, for the purpose of this exercise we only include the lagged portfolio share in the regression in order to abstract from valuation effects in the buy-and-hold portfolio. The coefficient on the lagged portfolio share is then 0.88 and the coefficient on the expected excess return variable is 19.6. Second, we need to make an assumption about the portfolio response in the frictionless case. We cannot use the estimated response when regressing $z_{i,n,t}$ on $E_{t}er_{i,n,t+1}$ as that is based on data that provide strong evidence of portfolio frictions.

---

25Giglio et al. (2021) also make the point that excessive risk aversion is needed to account for the response of portfolios to expected returns in a frictionless model.
As shown in (??), in the frictionless case the coefficient on the expected excess return is equal to $1/(\gamma \sigma^2)$. We again set $\sigma^2 = 0.00172$ and assume a rate of risk aversion of $\gamma = 10$. We can scale the portfolio response in the frictionless case up or down by respectively lowering or raising the rate of relative risk aversion.

Figure 1 shows the results. The initial portfolio response to a one standard deviation expected excess return innovation is much larger in the frictionless case. If we set the risk aversion equal to the $\gamma = 3.2$ implied by estimates for the model with frictions, the initial response in the frictionless case would be even much higher by a factor $3$. Apart from the initial portfolio inertia with the estimated frictions, we also see significant portfolio persistence. The portfolio response peaks after 9 months, while in the frictionless case it peaks at the time of the shock and dies out quickly.

Figure 1: Impulse Response Portfolio Share to Expected Return Shock

Bacchetta and van Wincoop (2021) refer to the initial portfolio response as return sensitivity and the gradual portfolio response as portfolio persistence. They show in a model for the foreign exchange market with portfolio frictions that both diminished return sensitivity and increased portfolio persistence are key to
accounting for a variety of currency excess return predictability puzzles.

4.7 Robustness Analysis

We now discuss extensions and robustness analysis. We first consider results when using subsets of the 8 instruments. After that we consider heterogeneity of the portfolio response to the expected excess return variable $ER_{i,n,t}$ associated with the size of the average portfolio share $\bar{z}_{i,n}$. Then we discuss alternative regression specifications as well as alternative subsamples.

Since there are three endogenous regressors, we need at least 3 instruments for the portfolio regressions. Within our set of 8 instruments, we identify 4 instruments that account for most of the explanatory power of the endogenous regressors. We will refer to these as the 4 strong instruments. We refer to the other 4 instruments as weak instruments:

$$I_{i,n,t}^{\text{strong}} = \{\Delta d_{i,n,t}, e_{i,n,t}, val_{i,n,t-1}, y_{i,n,t}\}$$

$$I_{i,n,t}^{\text{weak}} = \{\Delta e_{i,n,t}, b_{i,n,t}, h_{i,n,t}, \Delta h_{i,n,t}\}$$

While the weak instruments have statistically significant explanatory power for the endogenous regressors, their joint explanatory power is significantly weaker than for the 4 strong instruments. This can be seen in Table 6. When regressing $val_{i,n,t}$ and $ER_{i,n,t}$ on the 4 strong instruments, the adjusted $R^2$ is respectively 0.07 and 0.27. By contrast, when regressing on the 4 weak instruments the adjusted $R^2$ is respectively 0.01 and 0.06.

This significantly weaker explanatory power affects the portfolio regression. Table 7 reports the portfolio regression with all 8 instruments (benchmark), the 4 strong instruments and the 4 weak instruments. The results with the 4 strong instruments are not significantly different from those with all 8 instruments. But when we use the 4 weak instruments, the coefficient on $ER_{i,n,t}$ is close to zero and insignificant. The coefficient on $val_{i,n,t}$ more than triples. We find that the 4 strong instruments are the minimum set of instruments that we need. Adding further instruments from the set of 4 weak instruments has little effect on the portfolio regressions. Reducing the set of instruments to 3, the minimum needed to conduct the portfolio regressions, gives insufficient independent predictive power for the 3 endogenous regressors, making the portfolio regressions unreliable.

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Table 6: Portfolio Regressions, First Stage, Robustness

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>$z_{i,n,t-1}$</td>
<td>$val_{i,n,t}$</td>
<td>$ER_{i,n,t}$</td>
<td>$z_{i,n,t-1}$</td>
<td>$val_{i,n,t}$</td>
<td>$ER_{i,n,t}$</td>
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<td></td>
<td>(0.006)</td>
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<td>(0.021)</td>
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<tr>
<td>$e_{i,n,t}$</td>
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<td>0.148***</td>
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<td></td>
<td>(0.002)</td>
<td>(0.282)</td>
<td>(0.004)</td>
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<tr>
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<td>(0.001)</td>
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<td></td>
<td>(0.000)</td>
<td>(0.142)</td>
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<td>151,050</td>
<td>151,016</td>
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<td>145,848</td>
<td>145,848</td>
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<tr>
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<td>0.04</td>
<td>0.01</td>
<td>0.06</td>
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Clustered standard errors by months in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Regressions for 36 countries over the interval 2002:01-2016:07. The variables were regressed on a fund-country fixed effect. In columns (2), (3), (5) and (6), we multiply the regressions coefficients by 1000 except for $val_{i,n,t-1}$. 

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<table>
<thead>
<tr>
<th></th>
<th>(1) Benchmark</th>
<th>(2) 4 strong</th>
<th>(3) 4 weak</th>
<th>(4) Interaction</th>
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<td>0.949***</td>
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<td>(0.072)</td>
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<td>10.125***</td>
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<td>$\bar{z}<em>{i,n} \times ER</em>{i,n,t}$</td>
<td></td>
<td></td>
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</table>

Clustered standard errors by months in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Notes: Regressions for 36 countries over the interval 2002:01-2016:07. The regressions include a fund-country fixed effect. In column (1) the instruments are $val_{i,n,t-1}$, $e_{i,n,t}$, $\Delta e_{i,n,t}$, $\Delta d_{i,n,t}$, $y_{i,n,t}$, $b_{i,n,t}$, $h_{i,n,t}$, $\Delta h_{i,n,t}$. In columns (2) the instruments are $val_{i,n,t-1}$, $e_{i,n,t}$, $\Delta d_{i,n,t}$, $y_{i,n,t}$. In columns (3) the instruments are $\Delta e_{i,n,t}$, $b_{i,n,t}$, $h_{i,n,t}$, $\Delta h_{i,n,t}$. Column (4) uses the instruments of column (2) and the instruments of column (2) interacted with $\bar{z}_{i,n}$. 

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We further investigate the portfolio response to the expected excess return variable by considering heterogeneity. So far we have focused on the average coefficients $a_1, a_2$ and $a_3$ that describe the relationship between $z_{i,n,t}$ and respectively $z_{i,n,t-1}$, $val_{i,n,t}$ and $ER_{i,n,t}$. But these coefficients will in general vary across funds and countries. We consider here one dimension of this heterogeneity, associated with the average portfolio share $\bar{z}_{i,n}$ of a fund in a country. These mean portfolio shares vary substantially, as shown in the Online Appendix. The 10th, 50th and 90th percentiles are 2.7%, 6% and 20.4%.

The theory in Section 2 implies that both $\bar{z}_{i,n}$ and the coefficient on $ER_{i,n,t}$ will be higher when $\sigma_{i,n}^2$ is lower. This is the perceived risk of the excess return of country $n$ by fund $i$. The coefficients on the lagged portfolio and the buy-and-hold portfolio (or the lagged portfolio and the valuation effect) are unaffected by $\sigma_{i,n}^2$. This suggests that when the portfolio share $\bar{z}_{i,n}$ is larger, due to lower perceived risk, the portfolio response to the expected excess return $ER_{i,n,t}$ will be larger as well.

This is tested in column (4) of Table 7. Apart from the 3 standard regressors $z_{i,n,t-1}$, $val_{i,n,t}$ and $ER_{i,n,t}$, we add an interaction $\bar{z}_{i,n}ER_{i,n,t}$. We then need to double the number of instruments as we need to add interactions of the instruments with $\bar{z}_{i,n}$. To avoid using an excessively large number of instruments (16 when we use the benchmark instruments), we only use the 4 strong instruments discussed above. This gives us 8 instruments after including the interactions with $\bar{z}_{i,n}$. Table 7 shows that the coefficient of the interaction term is 95.6 and is significant. This confirms that the portfolio response to $ER_{i,n,t}$ is larger when the portfolio share $\bar{z}_{i,n}$ is larger. Specifically, the coefficient on $ER_{i,n,t}$ varies from 6.4 for small $\bar{z}_{i,n}$ (10th percentile) to 9.5 for the median $\bar{z}_{i,n}$ and 23.3 for large $\bar{z}_{i,n}$ (90th percentile). The benchmark regression result in column (1) of Table 7 is consistent with the median fund.

The Online Appendix describes additional robustness analyses for regression (??), comparable to column (2) of Table 4, both for alternative regression specifications and alternative samples. We consider 7 alternative regression specifications. The first uses country-time fixed effects instead of the instruments to deal with endogeneity. This is intended to sweep up common portfolio shifts in and out of each country in the error term. The next two consider respectively $k = 12$ and $k = 36$ to compute $ER_{i,n,t}$ in (??). Then we assume alternative values of $\beta$ of respectively 0.96 and 0.98. Next we consider weights $z_{i,m,-n,t-1}$ to compute the
reference portfolio instead of sample average weights $\bar{z}_{i,m,-n}$. We finally report a non-recursive regression, which uses data over the entire sample to compute expected excess returns as opposed to true forecasts using recursive regressions up to the time of the forecast.

We also consider 3 alternative samples. The first has a start date of January 2012 as fewer funds report country allocations at the beginning of our sample. We next restricts the sample to funds that report their global equity allocation for at least 24 consecutive months (as opposed to 12). Finally, we consider a sample where $\bar{k}_{t,n}$ is at least 1 percent, as opposed to 2 percent, which leads to the inclusion of smaller portfolio shares.

In almost all of these regressions the results remain quite similar to the benchmark results. The largest changes occur when we use country-time fixed effects instead of the instruments and when we start the sample in January, 2012. In both cases the coefficient on the expected excess return $ER_{i,n,t}$ doubles. When starting the sample in 2012, the coefficient on $val_{i,n,t}$ drops to 0.18 and is only significant at the 10 percent level. In all other regressions the coefficients remain significant at the 1 percent level.

The Online Appendix describes two further types of robustness analysis. One includes additional instruments, either the consumer confidence index, the business confidence index or leading indicators. These data are not available for all countries and reduce our sample by about twenty percent. When adding these to the benchmark 8 instruments, one at a time, the results remain virtually unchanged. We finally report the results for a log-portfolio regression, where we regress $\ln(z_{i,n,t})$ on $\ln(z_{i,n,t-1})$, $val_{i,n,t}$ and $ER_{i,n,t}$. While the theory implies that we should estimate the portfolio expression in levels, in practice portfolio shares are closer to being log-normal. The coefficients on all three regressors remain highly significant.\footnote{When we linearize the log-specification to return to levels of portfolio shares, the coefficients on $val_{i,n,t}$ and $ER_{i,n,t}$ from the log-specification need to be multiplied by $\bar{z}_{i,n}$. Using the average or median value of $\bar{z}_{i,n}$ gives coefficients on $val_{i,n,t}$ and $ER_{i,n,t}$ close to the benchmark regression (column (2) of Table 4).}

\section{Conclusion}

The objective of the paper was to provide empirical evidence on international portfolio choice and specifically the role of portfolio frictions. We developed a
simple optimal portfolio expression that relates portfolio choice to the present discounted value of expected excess returns and two benchmark portfolios, the lagged portfolio share and the buy-and-hold portfolio. We estimated the reduced form parameters of the portfolio expression with international equity portfolio data from US mutual funds, using instrumental variables to address endogeneity. We find that portfolio shares of US mutual funds depend significantly on both benchmark portfolios, with coefficients that are quite precise.

We also find that international equity return differentials are predictable and that mutual fund portfolios respond to expected excess returns. The results are consistent with a reasonable rate of risk aversion of 3.2. While the responsiveness to the present value of expected excess returns is strongly statistically significant, we also find that quantitatively the portfolio response to expected returns is much smaller than it would be in a frictionless portfolio model. Portfolio frictions make the response to changes in expected returns smaller initially and more gradual.

There is a clear need to introduce these portfolio frictions into open economy models, as recently done by Bacchetta and van Wincoop (2021), Davenport and van Wincoop (2022), and Bacchetta, van Wincoop and Young (2022) for respectively the foreign currency market, global financial markets broadly and the global equity market. It has significant implications for the response of asset prices, capital flows, saving and investment to shocks. A weaker and more gradual portfolio response to expected returns implies more excess return predictability in both the foreign exchange market and global equity markets. It also implies a much larger impact of exogenous portfolio shocks, including also central bank asset purchases, on asset prices and capital flows. The importance of such financial shocks for exchange rates and capital flows has recently been emphasized by Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) and, as these papers emphasize, is consistent with a variety of evidence.
Appendix A: First-Order Condition Optimal Portfolio

After substituting (??) into (??) we maximize with respect to $z_{i,n,t}$. To do so, use that for a function $f(z_{i,t})$ we have

$$\frac{\partial f(z_{i,t})}{\partial z_{i,n,t}} = (e_n - z_{i,-n,t})' \frac{\partial f(z_{i,t})}{\partial z_{i,t}} \quad (A.1)$$

The first-order condition with respect to $z_{i,n,t}$ is then

$$\begin{align*}
(e_n - z_{i,-n,t})' E_{i,t} R_{t+1} - \gamma_i (e_n - z_{i,-n,t})' \Omega_{i,t} - & \mu_{1,i} (e_n - z_{i,-n,t})' \Omega_{i,t} E_{i,t} (z_{i,t+1} - z_{i,t}) \\
- & \mu_{2,i} (e_n - z_{i,-n,t})' \Omega_{i,t} (z_{i,t} - z_{i,t}) + \mu_{2,i} (e_n - z_{i,-n,t})' \Omega_{i,t} E_{i,t} (z_{i,t+1} - z_{bhi,t}) = 0
\end{align*} \quad (A.2)$$

The last line uses that $z_{bhi,t+1}$ can be written as $z_{i,n,t}$ plus a time $t + 1$ valuation effect (see (??)).

Using that $z_{i,t} = z_{i,-n,t} + (e_n - z_{i,-n,t}) z_{i,n,t}$, the first line can be written as

$$E_{i,t} e_{i,n,t+1} - \gamma_i \text{cov}_{i,t}(e_{i,n,t+1}, R_{ref(i,n),t+1}) - \gamma_i z_{i,n,t} \text{var}_{i,t}(e_{i,n,t+1}) \quad (A.3)$$

In what follows we will think of moments involving the reference portfolio as evaluated at mean portfolios $\bar{z}_{i,m,-n}$ as in the data such covariances are virtually identical whether evaluated at portfolios $z_{i,m,-n,s}$ for $s = t - 1, t, t + 1$ or $\bar{z}_{i,m,-n}$. The same applies to moments with the excess return, which depend on the reference portfolio.

Next take the first term of the second line of (??), substituting $z_{i,t} = z_{i,-n,t} + (e_n - z_{i,-n,t}) z_{i,n,t}$ and $z_{i,t-1} = z_{i,-n,t-1} + (e_n - z_{i,-n,t-1}) z_{i,n,t-1}$. We can then write it as

$$-\mu_{1,i} \text{var}_{i,t}(e_{i,n,t+1})(z_{i,n,t} - z_{i,n,t-1}) \quad (A.4)$$

In analogy, the second term of the second line of (??) can be written as

$$\mu_{1,i} \beta \text{var}_{i,t}(e_{i,n,t+1})(E_t z_{i,n,t+1} - z_{i,n,t}) \quad (A.5)$$

The second line of (??) then becomes

$$\mu_{1,i} \text{var}_{i,t}(e_{i,n,t+1})(\beta E_t z_{i,n,t+1} - (1 + \beta) z_{i,n,t} + z_{i,n,t-1}) \quad (A.6)$$
Approximating the buy-and-hold portfolio as $z_{i,n,t}^{bh} - z_{i,n,t-1} = z_{i,n,t-1} (R_{n,t} - R_{t}^{i,p})$, with $R_{t}^{i,p} = \sum_{m=1}^{N} z_{i,m,t-1} R_{m,t}$, we can write the last line of (A.7) as

$$-\mu_{2,i} (e_{n} - z_{i,n-t})' \Omega_{i,t} (z_{i,t} - z_{i,t-1}) + \mu_{2,i} \beta (e_{n} - z_{i,n-t})' \Omega_{i,t} E_{t} (z_{i,t+1} - z_{i,t})$$

$$+ \mu_{2,i} (e_{n} - z_{i,n-t})' \Omega_{i,t} \left( \begin{array}{c}
  z_{i,t-1} (R_{1,t} - R_{t}^{i,p}) \\
  \vdots \\
  z_{i,N,t-2} (R_{N,t} - R_{t}^{i,p}) \\
  z_{i,N,t} (R_{N,t+1} - R_{t}^{i,p}) \\
\end{array} \right) = 0 \quad (A.7)$$

The first line is the same as the second line of (A.7), with $\mu_{1,i}$ replaced by $\mu_{2,i}$. It can therefore be written as

$$\mu_{2,i} \text{var}_{i,t}(er_{i,n,t+1})(\beta E_{t} z_{i,n,t+1} - (1 + \beta) z_{i,n,t} + z_{i,n,t-1}) \quad (A.8)$$

Take the second line of (A.7). This can be written as

$$\mu_{2,i} \sum_{m=1}^{N} \text{cov}_{i,t}(er_{i,n,t+1}, R_{m,t+1}) z_{i,m,t-1} (R_{m,t} - R_{t}^{i,p}) =$$

$$\mu_{2,i} \text{cov}_{i,t}(er_{i,n,t+1}, R_{n,t+1}) z_{i,n,t-1} (R_{n,t} - R_{t}^{i,p})$$

$$+ \mu_{2,i} (1 - z_{i,n,t-1}) \sum_{m \neq n} \text{cov}_{i,t}(er_{i,n,t+1}, R_{m,t+1}) z_{i,m,n,t-1} (R_{m,t} - R_{t}^{i,p}) \quad (A.9)$$

One can think of the summation in the last line as a cross-sectional covariance $E(xy)$, with $x = \text{cov}_{i,t}(er_{i,n,t+1}, R_{m,t+1})$ and $y = (R_{m,t} - R_{t}^{i,p})$ and $z_{i,m,n,t-1}$ the probability. Since there is no reason why the $x$ and $y$ would be correlated, when the number of countries is large enough, we can write this as $E(x)E(y)$. (A.10) then becomes

$$\mu_{2,i} \text{cov}_{i,t}(er_{i,n,t+1}, R_{n,t+1}) z_{i,n,t-1} (R_{n,t} - R_{t}^{i,p})$$

$$+ \mu_{2,i} (1 - z_{i,n,t-1}) \text{cov}_{i,t}(er_{i,n,t+1}, R_{ref(i,n),t+1}) (R_{ref(i,n),t} - R_{t}^{i,p}) \quad (A.10)$$

Using that $(1 - z_{i,n,t-1}) (R_{ref(i,n),t} - R_{t}^{i,p}) = z_{i,n,t-1} (R_{t}^{i,p} - R_{n,t})$, and that $\text{cov}_{i,t}(er_{i,n,t+1}, R_{n,t+1}) = \text{var}_{i,t}(er_{i,n,t+1}) + \text{cov}_{i,t}(er_{i,n,t+1}, R_{ref(i,n),t+1})$, this becomes

$$\mu_{2,i} \text{var}_{i,t}(er_{i,n,t+1}) z_{i,n,t-1} (R_{n,t} - R_{t}^{i,p}) = \mu_{2,i} \text{var}_{i,t}(er_{i,n,t+1}) (z_{i,n,t}^{bh} - z_{i,n,t-1}) \quad (A.11)$$
Analogously, the last line of (??) is

$$-\mu_{2,i}\beta \var_i,t (\epsilon_{i,n,t+1}^\text{bh} (E_i \epsilon_{i,n,t+1}^\text{bh} - \epsilon_{i,n,t}))$$  \hspace{1cm} (A.12)

We can write the difference between the buy-and-hold portfolio and lagged portfolio as

$$z_{i,n,t}^\text{bh} - z_{i,n,t-1} = z_{i,n,t-1} (R_{n,t} - R_t^i) = z_{i,n,t-1} (1 - z_{i,n,t-1}) \epsilon_{i,n,t}$$  \hspace{1cm} (A.13)

Combining (??) and (??), we then have

$$\mu_{2,i} \var_i,t (\epsilon_{i,n,t+1}^\text{bh} z_{i,n,t-1} (1 - z_{i,n,t-1}) \epsilon_{i,n,t} - \beta \mu_{2,i} \var_i,t (\epsilon_{i,n,t+1}^\text{bh} z_{i,n,t} (1 - z_{i,n,t})) = 0$$  \hspace{1cm} (A.14)

We can now combine all terms of (??), which gives

$$E_i \epsilon_{i,n,t+1} + u_{i,n,t} - \gamma_i \text{cov}_{i,t} (\epsilon_{i,n,t+1}, R_{\text{ref}(i,n),t+1}) - \gamma_i \hat{z}_{i,n,t} \var_i,t (\epsilon_{i,n,t+1}) + (\mu_1,i + \mu_2,i) \var_i,t (\epsilon_{i,n,t+1}) (\beta E_i \hat{z}_{i,n,t+1} - (1 + \beta) \hat{z}_{i,n,t} + z_{i,n,t-1}) + \mu_2,i \var_i,t (\epsilon_{i,n,t+1}) z_{i,n,t-1} (1 - z_{i,n,t-1}) \epsilon_{i,n,t} - \beta \mu_2,i \var_i,t (\epsilon_{i,n,t+1}) z_{i,n,t} (1 - z_{i,n,t}) E_i \epsilon_{i,n,t+1} = 0$$  \hspace{1cm} (A.15)

Define $\sigma_{i,n}^2$ as the mean of $\var_i,t (\epsilon_{i,n,t+1})$ and $\sigma_{n,\text{ref}(i,n)}$ as the mean value of $\text{cov}_{i,t} (\epsilon_{i,n,t+1}, R_{\text{ref}(i,n),t+1})$. The mean of the excess return is zero. From (??) the steady state portfolio is then

$$\bar{z}_{i,n} = -\sigma_{n,\text{ref}(i,n)} \sigma_{i,n}^2$$  \hspace{1cm} (A.16)

Linearizing (??) around the second moments equal to their mean, the portfolio shares equal to $\bar{z}_{i,n}$ and the excess returns equal to zero, we have

$$E_i \epsilon_{i,n,t+1} + u_{i,n,t} - \gamma_i \sigma_{i,n}^2 \hat{z}_{i,n,t} + (\mu_1,i + \mu_2,i) \sigma_{i,n}^2 (\beta E_i \hat{z}_{i,n,t+1} - (1 + \beta) \hat{z}_{i,n,t} + \hat{z}_{i,n,t-1}) + \mu_2,i \sigma_{i,n}^2 (1 - \bar{z}_{i,n}) (\epsilon_{i,n,t} - \beta E_i \epsilon_{i,n,t+1}) = 0$$  \hspace{1cm} (A.17)

where $\hat{z}_{i,n,t} = z_{i,n,t} - \bar{z}_{i,n}$ and

$$u_{i,n,t} = -\gamma_i \text{cov}_{i,t} (\epsilon_{i,n,t+1}, R_{\text{ref}(i,n),t+1}) - \sigma_{n,\text{ref}(i,n)} - \gamma_i \bar{z}_{i,n} \left(\var_i,t (\epsilon_{i,n,t+1}) - \sigma_{i,n}^2\right)$$  \hspace{1cm} (A.18)
Appendix B: Solution Optimal Portfolio

We now solve the second-order difference equation (??) in the portfolio share \( \hat{z}_{i,n,t} \). Collecting terms, we can write (??) as

\[
\sigma^2_{i,n} D_i \hat{z}_{i,n,t} = E_{i,t} \epsilon_{i,n,t+1} + \theta_i \sigma^2_{i,n} \hat{z}_{i,n,t-1} + \beta \theta_i \sigma^2_{i,n} E_{i,t} \hat{z}_{i,n,t+1} + \mu_{2,i} \sigma^2_{i,n} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) (\epsilon_{i,n,t} - \beta E_{i,t} \epsilon_{i,n,t+1}) + u_{i,n,t}
\]

(A.19)

where \( D_i = \gamma_i + \theta_i (1 + \beta) \).

This can be written as

\[
(L^{-2} - D_i \beta \theta_i L^{-1} + 1) \hat{z}_{i,n,t-1} = -\frac{1}{\beta \theta_i \sigma^2_{i,n}} E_{i,t} (\epsilon_{i,n,t+1} + u_{i,n,t})
\]

\[
- \frac{1}{\beta \theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) \epsilon_{i,n,t} + \frac{1}{\theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) E_{i,t} \epsilon_{i,n,t+1}
\]

where \( L^{-2} \hat{z}_{i,n,t-1} = E_{i,t} \hat{z}_{i,n,t+1} \) and \( L^{-1} \hat{z}_{i,n,t-1} = \hat{z}_{i,n,t} \). Factoring gives

\[
(L^{-1} - \omega_{1,i})(L^{-1} - \omega_{2,i}) \hat{z}_{i,n,t-1} = \frac{1}{\theta_i \sigma^2_{i,n}} E_{i,t} (\epsilon_{i,n,t+1} + u_{i,n,t})
\]

\[
- \frac{1}{\beta \theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) \epsilon_{i,n,t} + \frac{1}{\theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) E_{i,t} \epsilon_{i,n,t+1}
\]

where \( \omega_{1,i} \) and \( \omega_{2,i} \) are the roots of the characteristic equation

\[
\omega_i^2 - D_i \beta \theta_i \omega_i + 1 \beta = 0
\]

(A.20)

These roots are

\[
\omega_i = 0.5 \left( \frac{D_i}{\beta \theta_i} \pm \sqrt{\left( \frac{D_i}{\beta \theta_i} \right)^2 - (4/\beta)} \right)
\]

(A.21)

For convenience, we will refer to the stable root (with the minus sign) simply as \( \omega_i \) and the unstable root (with the positive sign) as \( \omega_{2,i} \).

Now write the solution as

\[
(L^{-1} - \omega_i) \hat{z}_{i,n,t-1} = \frac{1}{\theta_i \sigma^2_{i,n}} E_{i,t} (\epsilon_{i,n,t+1} + u_{i,n,t})
\]

\[
- \frac{1}{\beta \theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) \epsilon_{i,n,t} + \frac{1}{\theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) E_{i,t} \epsilon_{i,n,t+1}
\]

\[
- \frac{1}{\beta \theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) \epsilon_{i,n,t} + \frac{1}{\theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) E_{i,t} \epsilon_{i,n,t+1}
\]

\[
(L^{-1} - \omega_{2,i}) \hat{z}_{i,n,t-1} = \frac{1}{\theta_i \sigma^2_{i,n}} E_{i,t} (\epsilon_{i,n,t+1} + u_{i,n,t})
\]

\[
- \frac{1}{\beta \theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) \epsilon_{i,n,t} + \frac{1}{\theta_i} \mu_{2,i} \hat{z}_{i,n} (1 - \hat{z}_{i,n}) E_{i,t} \epsilon_{i,n,t+1}
\]
This implies
\[
\hat{z}_{i,n,t} = \omega_i \hat{z}_{i,n,t-1} + \frac{1}{\beta \theta_i \sigma_i^2 \omega_{2,i}} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} \left( er_{i,n,t+s} + u_{i,n,t+s-1} \right) \\
+ \frac{\mu_{2,i} \bar{z}_{i,n} (1 - \bar{z}_{i,n})}{\beta \theta_i \omega_{2,i}} er_{i,n,t} + \frac{\mu_{2,i} \bar{z}_{i,n} (1 - \bar{z}_{i,n}) \left( \frac{1}{\omega_{2,i}} - \beta \right)}{\beta \theta_i \omega_{2,i}} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} er_{i,n,t+s}
\]

To summarize, we have
\[
\hat{z}_{i,n,t} = a_{2,i} \hat{z}_{i,n,t-1} + a_{3,i,n} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} er_{i,n,t+s} + a_{4,i,n} er_{i,n,t} + \epsilon_{i,n,t}
\]
(A.22)
where \( E_t \) is the expectation operator of the econometrician based on public information and
\[
a_{2,i} = \omega_i, \\
a_{3,i,n} = \frac{1}{\beta \theta_i \sigma_i^2 \omega_{2,i}} + \frac{\mu_{2,i} \bar{z}_{i,n} (1 - \bar{z}_{i,n}) \left( \frac{1}{\omega_{2,i}} - \beta \right)}{\beta \theta_i \omega_{2,i}} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} er_{i,n,t+s} \\
a_{4,i,n} = \frac{\mu_{2,i} \bar{z}_{i,n} (1 - \bar{z}_{i,n})}{\beta \theta_i \omega_{2,i}}
\]
and
\[
\epsilon_{i,n,t} = \frac{1}{\beta \theta_i \sigma_i^2 \omega_{2,i}} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} u_{i,n,t+s-1} + a_{3,i,n} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} \left( E_{i,t} er_{i,n,t+s} - E_{i,t} er_{i,n,t+s} \right)
\]
(A.23)
Numerically the second term in \( a_{3,i,n} \) is very close to zero. We therefore abstract from it in what follows.

We can also write the solution for \( z_{i,n,t} \) as a function of the lagged portfolio and the buy-and-hold portfolio. For this, use that from linearizing (??) \( \bar{z}_{i,n,t} = z_{i,n,t-1} + \bar{z}_{i,n} (1 - \bar{z}_{i,n}) er_{i,n,t} \), so that
\[
er_{i,n,t} = \frac{\bar{z}_{i,n,t} - z_{i,n,t-1}}{\bar{z}_{i,n} (1 - \bar{z}_{i,n})}
\]
We then have
\[
\hat{z}_{i,n,t} = \left( a_{2,i} - \frac{a_{4,i,n}}{\bar{z}_{i,n} (1 - \bar{z}_{i,n})} \right) \hat{z}_{i,n,t-1} + \frac{a_{4,i,n}}{\bar{z}_{i,n} (1 - \bar{z}_{i,n})} \bar{z}_{i,n,t} + a_{3,i,n} \sum_{s=1}^{\infty} \omega_{2,i}^{1-s} E_{i,t} er_{i,n,t+s} + \epsilon_{i,n,t}
\]
(A.24)
where $z_{i,n,t}^{bh} = z_{i,n,t} - \bar{z}_{i,n}$. Use that $\omega_{2,i} = 1/(\beta \omega_i) = 1/(\beta a_{2,i})$. This gives

$$
\hat{z}_{i,n,t} = \omega_i \left( \frac{\mu_{1,i}}{\mu_{1,i} + \mu_{2,i}} \hat{z}_{i,n,t-1} + \frac{\mu_{2,i}}{\mu_{1,i} + \mu_{2,i}} z_{i,n,t}^{bh} \right) + \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} E_t r_{i,n,t+s} + \epsilon_{i,n,t}
$$

(A.25)

with

$$
\epsilon_{i,n,t} = \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} E_t u_{i,n,t+s-1} + \frac{\omega_i}{\theta_i \sigma_{i,n}^2} \sum_{s=1}^{\infty} (\beta \omega_i)^{s-1} (E_t r_{i,n,t+s} - E_t r_{i,n,t+s})
$$

(A.26)

Using the expression for $D_i$, we can also write the stable root (??) as

$$
\omega_i = \frac{2\theta_i}{\gamma_i + (1 + \beta)\theta_i + \sqrt{\gamma_i^2 + (1 - \beta)^2 \theta_i^2} + 2(1 + \beta)\gamma_i \theta_i}
$$

(A.27)

### Appendix C: Trading Strategies

To evaluate the prediction performance and estimate the economic significance of predictability reported in Section 3, we follow the literature in building trading strategies based on the three predictors used in the regressions. The analysis is close to Cenedese et al. (2016). For each month, we sort countries into quintiles based on their values of momentum, dividend-price differential, or earning-price differential. The one fifth of countries whose predictors have the lowest value are allocated to the first quintile Q1, the next fifth to the second quintile Q2, and so on. Thus, Q1 should contain low excess returns and Q5 high excess returns. For each pair month-quintile, we take the equally weighted average equity return differential with the US. Then, for each predictor variable we form a long-short HML portfolio, obtained by going long on Q5 and short on Q1. The sample is January 1970 to February 2019.

Table ?? reports the average annualized equity return by quintile and the portfolio return when the predictor is momentum, the dividend-price ratio, or the earning-price ratio (it is also possible to build strategies based on a combination of the three variables). The table shows that returns tend to be higher for higher quintiles, i.e., higher values of momentum, dividend-price, or earning-price are associated with higher returns. This is confirmed by the results in the last column that show large returns from HML portfolios. These results are in line with
Cenedese et al. (2016), who use a more restricted sample. These results therefore demonstrate the economic significance of equity return predictability, which justifies that time-varying expected excess returns are taken into account in actual portfolio allocations.

Table C1: EQUITY EXCESS RETURNS

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>HML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>.69</td>
<td>1.01</td>
<td>-.28</td>
<td>2.33</td>
<td>9.56</td>
<td>8.87</td>
</tr>
<tr>
<td>Dividend-Price</td>
<td>-2.02</td>
<td>2.62</td>
<td>.86</td>
<td>1.2</td>
<td>4.42</td>
<td>6.44</td>
</tr>
<tr>
<td>Earning-Price</td>
<td>.06</td>
<td>-1.1</td>
<td>.11</td>
<td>3.31</td>
<td>5.49</td>
<td>5.43</td>
</tr>
</tbody>
</table>

*Notes:* Mean annualized equity excess returns relative to the US by sorting countries-months in quintiles based on their values for momentum, dividend-price and earning-price. HML shows the return from borrowing in Q1 and investing in Q5. Sample: 73 countries over the horizon 1970:01-2019:02.

Appendix D: Data Appendix

We describe here the data used other than the portfolio data from EPFR that are described in detail in Section 4.1.

We obtain the following monthly MSCI data: monthly total return index, price index, earning-price ratio, dividend-price ratio and market value (market capitalization). The total return index includes both the capital gains and dividend component of the return. All data are denominated in dollars. From these MSCI data we also compute

- **Equity Return:** relative change of the total return index from the prior month.
- **Earnings:** earning-price ratio multiplied by the price index.
- **Dividend:** dividend-price ratio multiplied by the price index.
- **Book value:** market value divided by the price index.
- **Volatility:** for each country and each month, we compute the standard deviation of the daily returns, using the daily total return index from MSCI.
In addition to these MSCI data, we obtain the following variables from other sources:

- **Industrial Production.** The main source is the industrial production index from IFS. If not available, we use the manufacturing or the retail index from the IFS. When countries do not report the industrial production, the manufacturing nor the retail index, we use the monthly gross domestic product index obtained from the Leading Indicators of the OECD. Finally, for Hong-Kong and Thailand, we use quarterly real GDP data from OECD, interpolated to a monthly series. We transform the final series for each country into an index equal to 100 in July 2016.

- **Inflation.** Monthly consumer price index series are from the IFS compiled by the IMF. If the consumer price index is not available, we use the producer price index or the wholesale price index from the IFS. For Taiwan, we obtain the consumer price index from the Statistical Bureau of Taiwan. For Australia, monthly data are not available and we interpolate the monthly series from the quarterly series. We transform the final series for each country into an index equal to 100 in July 2016.

- **Nominal Interest Rate.** We use the 3-month Eurorates obtained from Datastream. The data are midpoint of the offer and bid rates. Original data are expressed at annual rates in percent. We transform the data into a monthly rate by dividing by 1200.

- **Bond Price Index.** We obtain the data on bond price index from J.P. Morgan and Merrill Lynch obtained through Datastream. We use the price index of a 10-year government bond provided by JPM. For emerging economies, when the price index of the 10-year government bond is not available, we use the Emerging Market Bond Index provided by JPM. For Taiwan and Thailand, we use the Government Bond Index provided by Merrill Lynch. When the bond price index is in local currency, we convert to dollars.
References


