Foreign Exchange Intervention with UIP and CIP Deviations: The Case of Small Safe Haven Economies [1]

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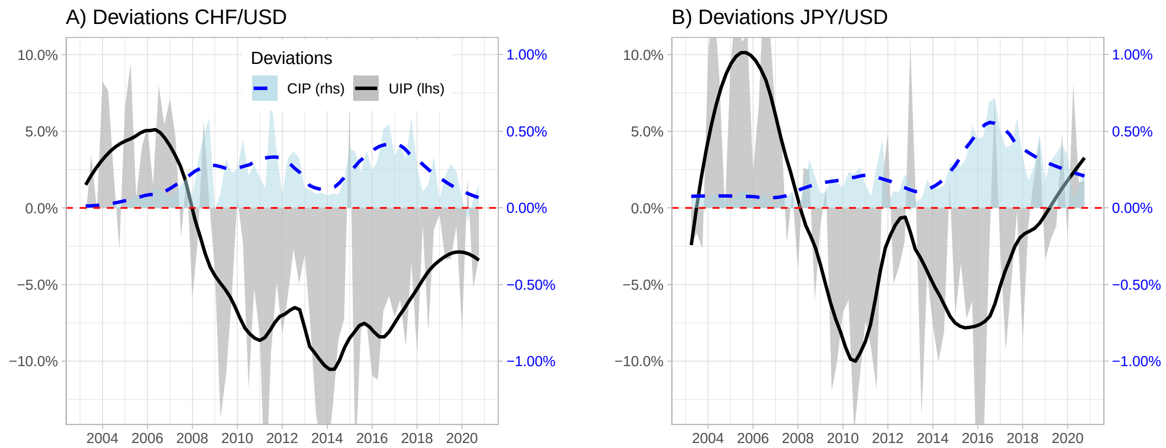
Abstract

We examine the opportunity cost of foreign exchange (FX) intervention when both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through constrained international financial intermediaries. Deviations from CIP come from limited arbitrage or through a convenience yield, while UIP deviations are also affected by risk. We show that the sign of CIP and UIP deviations may differ for safe haven countries. We examine the optimal policy of a constrained central bank planner in this context. We find that there may be a benefit, rather than a cost, of FX reserves if international intermediaries value more the safe haven properties of a currency that domestic households. We show that this has been the case for the Swiss franc and the Japanese Yen.

1 Introduction

A vast literature examines the optimal level of central bank international reserves in emerging markets (see Bianchi and Lorenzoni (2022) for a recent survey). A recurrent feature is that the accumulation of reserves bears an opportunity cost from an interest rate differential implied by an upward supply of international funding. To what extent is this opportunity cost also relevant for financially developed economies and what is the appropriate interest differential to consider? Since Foreign Exchange (FX) interventions by central banks are unhedged, it seems that the focus should be on Uncovered Interest Rate Parity (UIP). In contrast, the recent literature argues that what matters are deviations from Covered Interest Rate Parity (CIP) (e.g., Amador et al., 2020; Fanelli and Straub, 2021). This distinction between CIP and UIP appears particularly relevant for safe haven countries, since CIP and UIP deviations may be of different signs. Figure 1 shows CIP and UIP deviations for Switzerland and Japan. They are computed from the perspective of international investors and UIP deviations are estimated using survey expectation data. They show that since 2008, both countries have experienced positive CIP deviations and negative UIP deviations. The latter implies a negative excess return, which is typical of safe haven currencies.

Figure 1: UIP and CIP Deviations



Notes: This figure shows the UIP and CIP deviations in percentage points as defined in (5) and (4), taking the USD as the foreign currency and considering a 3-month horizon. Panel A) and B) considers the CHF and the JPY as the domestic currency, respectively. UIP deviations are computed using monthly data from Datastream for the 3-month Libor rates and from Consensus Economics for the exchange rate forecasts and the spot exchange rates. The CIP deviations are monthly averages of daily observations and are computed using 3-month Libor rates, spot exchange rates and forward rates with a 3-month maturity from Datastream. All returns are annualised.

To clarify these issues, we develop a model where both CIP and UIP deviations are present. We consider a small open economy that receives international capital flows through international financial intermediaries as in Gabaix and Maggiori (2015).[1] The

[1]There is a growing literature introducing financially constrained financial intermediaries. See Mag-

structure of the model is similar to those in recent papers examining the role of international reserves (see Cavallino, 2019; Amador et al., 2020; Fanelli and Straub, 2021; Basu et al., 2020; Maggiori, 2021), but financial intermediaries are risk averse. The international financial intermediaries are the marginal investors and determine both UIP and CIP deviations through their unhedged and hedged portfolio choices. These deviations typically do not coincide and may even be of different sign.

In this environment, we examine the opportunity costs of FX intervention. We model the central bank as a constrained planner. We examine the various factors influencing optimal policy decisions, focusing on various types of FX interventions. We show that the central bank incentives are similar for sterilized interventions and unsterilized interventions at the Zero Lower Bound (ZLB). In these cases, we identify the conditions under which CIP or UIP deviations matter for the opportunity cost. We find that there may be no opportunity cost, or even a benefit, of FX intervention in a safe haven country, even if it faces a negative CIP deviation.

The presence of systematic deviations from CIP in the wake of the Global Financial Crisis is a major development in international finance (see Du and Schreger (2022) for a recent survey). The theoretical literature has provided explanations for CIP deviations, but has devoted limited attention to the link between CIP and UIP deviation. Most papers analyzing interest rate differentials assume complete markets so that either there is no UIP deviations or CIP deviations are equal to UIP deviations. This is not consistent with the data.

The recent literature has followed two main approaches to explain interest rate differentials. First, there may be financial frictions that limit arbitrage, e.g., by assuming constrained financial intermediaries. The other approach is to assume differences in convenience yields. The two approaches are present in our model and determine deviations from CIP. But we do not assume complete markets, so that UIP deviations differ from CIP deviations. A basic result from this analysis is the following relationship between UIP and CIP deviations:

$$devUIP_t = devCIP_t - \frac{cov_t(M_t^*, X_{t+1}^*)}{E_t M_{t+1}^*} \tag{1}$$

where M_{t+1}^* is the stochastic discount factor (SDF) of financial intermediaries and X_{t+1}^* is the foreign currency excess return from the international intermediary perspective. If the small open economy is a safe haven country, we have $cov_t(M_t^*, X_{t+1}^*) > 0$, i.e., the safe haven currency yields a higher return in bad times. Therefore, it is possible to have a positive CIP deviation with a negative UIP deviation.

We derive equation (1) in a simple two-period small economy model with two assumptions that differ from most of the literature. First, international financial intermediaries face exchange rate risk. This risk could be hedged on the forward market, but it is not optimal to fully hedge a safe haven currency. The other assumption is that the financial

giori (2022) for a recent survey.

constraint applies to the whole foreign exchange investment of financial intermediaries, whether it is hedged or not.[2]

We analyze optimal FX intervention in this framework. Since our focus is on the opportunity cost of accumulating international reserves, we do not model its benefits, such as exchange rate stabilization or precautionary saving, which have been widely discussed in the literature. In deriving the optimal FX intervention, the central bank needs to take into account the preferences of domestic households. We show that if domestic households attribute less value to the safe haven properties of their currency than international financial intermediaries (i.e., the domestic SDF is less correlated to the excess return than for financial intermediaries), then the central bank may find it beneficial to buy foreign assets when financial markets are segmented.

For a more specific analysis, we consider a linearized version of the model where the distribution of shocks is such that the domestic currency is perceived as a safe haven by international investors. This allows us to derive precise expressions for FX intervention and CIP and UIP deviations and examine the impact of various parameters on these variables. For example, an increase in global risk leads to larger optimal FX interventions, larger positive CIP deviations and larger negative UIP deviations.

Finally, we examine this issue empirically by estimating the SDF of financial intermediaries following He et al. (2017). When considering the CHF and JPY with respect to the USD, we find that it is indeed the case that the SDF of financial intermediaries is more correlated with excess returns than the SDF of domestic households.

This paper complements the literature on the opportunity cost of FX reserves. There is a long tradition of estimating the cost and benefits of accumulating FX reserves (e.g., Jeanne and Rancière, 2011). In the case of developing or emerging economies, the opportunity cost is based on the country's borrowing cost, which implies a credit risk (e.g., Edwards, 1985).[3] In the case of safe haven countries, credit risk is negligible. Instead it is a combination of UIP and CIP deviations that matters.

By focusing on countries like Switzerland or Japan, this paper provides a different perspective on safe haven economies. A growing literature has been analyzing the special role of the US dollar as a reserve currency. Several papers have focused on the role of convenience yields in generating currency movements and expected excess returns (e.g., Jiang et al., 2021b,a; Valchev, 2020; Kekre and Lenel, 2021; Bianchi et al., 2022). We show that convenience yields are not the sole determinant for exchange rate movements and UIP deviations in safe haven economies.

The rest of the paper is organized as follows. Section 2 describes the model and the decentralized equilibrium and Section 3 analyzes the optimal policy of a constrained central bank planner. Section 4 examines a linearized model of a safe haven country. Section

[2] In contrast, in Itskhoki and Mukhin (2021), intermediation frictions generate UIP deviation without CIP deviations. This is because the intermediation frictions originate in the intermediaries' risk aversion.

[3] Yeyati and Gómez (2022) argue that when reserves are used for leaning-against-the-wind interventions, it is more appropriate to use UIP deviations.

5 analyzes empirically the behavior of the SDFs of international financial intermediaries and domestic households. Section 6 concludes.

2 The Model

Assume a small open endowment economy with domestic households, the central bank, and international financial intermediaries. There are two periods, t and $t+1$. We will call the small economy Switzerland and the rest of the world the US. International financial intermediaries buy domestic bonds and are the marginal investors determining excess returns. The foreign interest rate i_t^* is given. The price of goods in dollars is normalized to one.

2.1 International Financial Intermediaries

The international financial intermediaries are as in Gabaix and Maggiori (2015), but are risk averse. They value their expected profits at the world stochastic discount factor, denoted m_{t+1}^*. They are the marginal investors and their optimal behavior will give the deviations from CIP and UIP. They typically invest in Swiss bonds, but at the ZLB they may also hold Swiss money.[4] Denote b_t^{H*} and h_t^{H*} their net positions in Swiss bonds and money, expressed in dollars, and a_t^{H*} their total position: $a_t^{H*} = b_t^{H*} + h_t^{H*}$. When $i_t > 0$, $a_t^{H*} = b_t^{H*}$ as money is dominated by bonds. We also assume that they can use forward contracts in quantities f_t^*.

Moreover, financial intermediaries may value the liquidity of US assets. We assume that investors have operating costs that are increasing in non-US assets holdings a_t^{H*} and that it is a linear function: $\chi \cdot a_t^{H*}$, with $\chi \geq 0$. Their objective function is in dollars (and equivalently, in goods terms since the dollar price is constant):

$$V_t = E_t \left\{ m_{t+1}^* \left[a_t^{H*} \left((1+i_t)\frac{S_t}{S_{t+1}} - (1+i_t^*) \right) - f_t^* \left(\frac{1}{S_{t+1}} - \frac{1}{F_t} \right) \right] \right\} - \chi a_t^{H*}$$

a_t^{H*} represents the total funds invested in the country, covered or uncovered. $f_t^*/(1+i_t)$ is the covered amount, and $a_t^{H*} - f_t^*/(1+i_t)$ is the uncovered amount.

To capture the role of financial intermediaries, we assume, as Gabaix and Maggiori (2015), that intermediaries can divert a fraction Γa_t^{H*} of the total invested funds, after the world consumer's investment decisions are taken, but before shocks are realized. This yields a participation constraint for investors:

$$V_t \geq \Gamma (a_t^{H*})^2$$

[4] For notational convenience, we assume that financial intermediaries only potentially hold money at time t so that $h_{t-1}^{H*} = h_{t+1}^{H*} = 0$.

Consider first the FOC w/f_t^*:

$$E_t \left\{ m_{t+1}^* \left(\frac{1}{S_{t+1}} - \frac{1}{F_t} \right) \right\} = 0 \tag{2}$$

The forward market is effectively frictionless, since it does not involve a transfer of funds ex ante. This implies a relationship between the CIP deviation and the UIP deviation:

$$E_t(m_{t+1}^* Z_{t+1}^*) = E_t(m_{t+1}^* X_{t+1}^*) \tag{3}$$

where Z_{t+1}^* is the excess return hedged by a forward contract or the CIP deviation:

$$Z_{t+1}^* \equiv (1 + i_t)\frac{S_t}{F_t} - (1 + i_t^*) \tag{4}$$

and X_{t+1}^* is the domestic currency excess return, expressed in foreign currency:

$$X_{t+1}^* \equiv (1 + i_t)\frac{S_t}{S_{t+1}} - (1 + i_t^*) \tag{5}$$

Equation (3) can be rewritten as an equivalent of Equation (1):

$$E_t X_{t+1}^* = Z_{t+1}^* - \frac{cov_t(m_{t+1}^*, X_{t+1}^*)}{E_t m_{t+1}^*} \tag{6}$$

Covered and uncovered carry trades yield the same returns in expectation, up to a covariance term, because intermediaries are risk-averse.

Using Equation (2), the participation constraint can be simplified as follows:

$$E_t \left(m_{t+1}^* a_t^{H*} X_{t+1}^* \right) - \chi a_t^{H*} \geq \Gamma (a_t^{H*})^2 \tag{7}$$

If the participation constraint is binding, we have:

$$E_t \left(m_{t+1}^* X_{t+1}^* \right) = \Gamma a_t^{H*} + \chi \tag{8}$$

This, along with Equations (3) and (6), implies

$$Z_{t+1}^* = \frac{\Gamma a_t^{H*} + \chi}{E_t m_{t+1}^*} \tag{9}$$

and

$$E_t X_{t+1}^* = \frac{\Gamma a_t^{H*} + \chi}{E_t m_{t+1}^*} - \frac{cov_t(m_{t+1}^*, X_{t+1}^*)}{E_t m_{t+1}^*} \tag{10}$$

The term $\Gamma a_t^{H*} + \chi$ in Equations (9) and (10) shows the impact of limited arbitrage and of the convenience yield on CIP and UIP deviations. Indeed, the intermediation frictions, which bear on both covered and uncovered intermediated funds, affects both CIP and UIP deviations.[5]

[5] Fanelli and Straub (2021) discuss a similar setup with frictions in intermediation and frictionless forward markets in an extension of their model. They find that, in that case, intermediation frictions generate both UIP and CIP deviations.

2.2 Domestic Households

Households can hold money, H_t^H, domestic bonds B_t^H (both expressed in domestic currency), and foreign bonds b_t^F (expressed in foreign currency). Domestic bonds and money are perfect substitutes at the ZLB. We assume that households do not use the forward exchange market.

Their budget constraints are:

$$C_t = Y_t - \frac{H_t^H}{P_t} - \frac{B_t^H}{P_t} - \frac{S_t b_t^F}{P_t} + \frac{T_t}{P_t} \tag{11}$$

$$C_{t+1} = Y_{t+1} + \frac{H_t^H - H_{t+1}^H}{P_{t+1}} + (1+i_t)\frac{B_t^H}{P_{t+1}} + (1+i_t^*)\frac{S_{t+1} b_t^F}{P_{t+1}} + \frac{T_{t+1}}{P_{t+1}} \tag{12}$$

where T_t and T_{t+1} are lump-sum transfers. The real levels of domestic bonds and money holdings are $b_t^H = B_t^H/P_t$ and $h_t^H = H_t^H/P_t$.

Potentially, households could face short-selling constraints:

$$b_t^H \geq 0, \; b_t^F \geq 0 \tag{13}$$

and a cash-in-advance constraint in t and $t+1$:

$$h_t^H \geq Y_t, \; h_{t+1}^H \geq Y_{t+1} \tag{14}$$

Their utility function is:

$$U(C_t) + \beta E_t U(C_{t+1}) \tag{15}$$

Domestic households choose bonds and money holdings to maximize (15) subject to constraints (11) to (14). The first-order conditions associated with bond portfolio choices are:

$$U'(C_t) - E_t\left(\beta U'(C_{t+1})(1+i_t^*)\right) \qquad -\lambda^F = 0 \tag{16}$$

$$E_t\left(\beta U'(C_{t+1})\left[(1+i_t^*) - (1+i_t)\frac{S_t}{S_{t+1}}\right]\right) \qquad +\lambda^F - \lambda^H = 0 \tag{17}$$

where λ^H and λ^F are the multipliers associated with the short-selling constraints (13). Equation (16) shows that the borrowing constraints affect intertemporal allocations. Equation (17) shows that they prevent households from reaching their optimal portfolio allocation between domestic and foreign currency bonds.

2.3 The Government

At time t the government issues debt B_t^G (expressed in domestic currency) and transfers the funds to households:

$$B_t^G = T_t^G \tag{18}$$

At $t+1$, the government receives the central bank profits, Π_{t+1}^{CB} and repays its debt :

$$T_{t+1}^G = -(1+i_t)B_t^G + \Pi_{t+1}^{CB} \tag{19}$$

We assume that the government is passive and that the level of real debt $b_t^G = B_t^G/P_t$ is exogenous.[6]

2.4 The Central Bank

In period t, the central bank issues money H_t, and buys domestic and foreign bonds B_t^{CB} and b_t^{CBF} (expressed respectively in domestic and foreign currency). In period $t+1$, the central bank issues new money $H_{t+1} - H_t$ and distributes its profits Π_{t+1}^{CB} to the government. The central bank's budget constraint write then as follows:

$$S_t b_t^{CBF} + B_t^{CB} = H_t \tag{20}$$

$$\Pi_{t+1}^{CB} = (1+i_t^*)S_{t+1}b_t^{CBF} + (1+i_t)B_t^{CB} + H_{t+1} - H_t \tag{21}$$

In period t, the central bank has as instruments the nominal interest rate i_t, the total money supply H_t and the choice of foreign reserves b_t^{CBF} and domestic bonds B_t^{CB}. However, the interest rate cannot be negative, so the central bank loses the interest rate instrument when it hits this zero lower bound (ZLB).

In period $t+1$, we assume that the supply of money H_{t+1} is exogenous: $H_{t+1} = \bar{H}e^h$ where h is an exogenous shock. It represents variations in the net money supply to households due for instance to liquidity trading, or money velocity shocks.

From the budget constraint (20), there are two ways to change the level of reserves b_t^{CBF}. First, through a *sterilized* intervention where an increase in b_t^{CBF} is compensated by a decline in B_t^{CB}. Second, through an *unsterilized* intervention where an increase in b_t^{CBF} is associated with an expansion in H_t. Another possibility would be to allow the central bank to transfer funds to households in both periods. In that case, an increase in b_t^{CBF} could be implemented by changing transfer. We examine this fiscal foreign exchange intervention in the Online Appendix.

2.5 Equilibrium in Asset Markets

We assume that the law of one price holds (there are no frictions in the goods market), and that the foreign currency price of the good is equal to one. As a consequence, $P_t = S_t$. At $t+1$, equilibrium on the money market yields $H_{t+1}^H = S_{t+1}Y_{t+1} = He^h$, which determines S_{t+1}. Therefore, we can treat S_{t+1} as exogenous.

The supply of bonds and money are assumed to be positive, i.e., $B_t^G - B_t^{CB} > 0$ and $H_t > 0$. The amount of domestic debt held by international intermediaries is equal to the net domestic supply: $S_t b_t^{H*} = P_t B_t^G - B_t^{CB} - P_t b_t^H$. Similarly, the foreign money holdings are equal to the net domestic supply: $S_t h_t^{H*} = H_t - P_t h_t^H$. From (10), and since

[6] Alternatively, we could assume that it is the nominal debt level $= B_t^G$ that is exogenous. However, there would an incentive for the central bank to alter the real debt level by moving the exchange rate. We examine this case in the Online Appendix (to be written).

$a_t^{H*} = b_t^{H*} + h_t^{H*}$, this implies

$$\Gamma\left(b_t^G - \frac{B_t^{CB}}{S_t} - b_t^H + \frac{H_t}{S_t} - h_t^H\right) = (1+i_t)S_t E_t \frac{1}{S_{t+1}} - (1+i_t^*) + \frac{cov_t(m_t^*, X_{t+1}^*)}{E_t m_{t+1}^*} - \chi \quad (22)$$

Outside the ZLB, $h_t^{H*} = H_t/P_t - h_t^H = 0$, and households hold the minimum amount of money: $H_t/P_t = h_t^H = Y_t$. This implies that $S_t = P_t$ is determined by the equality between the demand and supply of money in the domestic economy $S_t Y_t = H_t$. Since $E_t \frac{1}{S_{t+1}}$ is exogenous, and S_t clears the money market, Equation (22) determines i_t. An increase in $cov_t(m_t^*, X_{t+1}^*)$ leads to a decline in the domestic interest rate i_t. Intuitively, the increase in covariance makes the domestic bonds more attractive to foreigners and generates an excess demand for domestic bonds. The decline in the interest rate clears this excess demand.

If the interest rate hits the ZLB, it cannot clear the domestic bond market. At the same time, the exchange rate is not determined by the money market, as now money and bonds become substitutes. We can see this by noting that h_t^{H*} can now be strictly positive, so that the net supply of money to foreigners $H_t/P_t - h_t^H$ is strictly positive in Equation (22). Since the interest rate i_t cannot adjust in the ZLB, the exchange rate adjusts. Now an increase in $cov(m_t^*, X_{t+1}^*)$ leads to a domestic currency appreciation (decrease in S_t). This reduces the excess return, which dampens the increase in demand. In the presence of sterilized interventions ($B_t^{CB} > 0$), the appreciation generates a valuation effect that decreases the net supply of bonds to the private sector. If this effect is not unrealistically large, then the appreciation should be even larger to clear the domestic bond market. However, bear in mind that B_t^{CB} is an endogenous variable chosen by the central bank and is not fixed.

3 The Central Bank as a Constrained Planner

To determine the incentives for foreign exchange interventions, we first reframe the central bank's problem as that of a constrained central planner. For that analysis, it is convenient to focus on the Home country's net and gross foreign liabilities. We then show how the resulting optimal allocation can be decentralized using foreign exchange interventions.

3.1 Gross and Net Foreign Liabilities

Gross foreign liabilities are domestic bonds and money not held domestically. They are given by

$$gfl_t = \left(b_t^G - \frac{B_t^{CB}}{S_t} - b_t^H\right) + \left(\frac{H_t}{S_t} - h_t^H\right) \quad (23)$$

The first term corresponds to the foreign holdings of domestic bonds. The second term corresponds to the foreign holdings of domestic money. In equilibrium, $gfl_t = a_t^{H*}$.

Net foreign liabilities are given by

$$nfl_t = gfl_t - (b_t^F + b_t^{CBF}) = b_t^G - b_t^H - b_t^F - h_t^H \tag{24}$$

where $b_t^F + b_t^{CBF}$ are the domestic holding of foreign assets. The second equality is obtained by replacing b_t^{CBF} with $(H_t - B_t^{CB})/S_t$, using the central bank budget constraint, and replacing gfl_t using (23).

It is useful to notice that FX intervention affects gfl_t, but not nfl_t: an increase in b_t^{CBF} will increase gfl one-for-one, through an increase in H_t (unsterilized intervention) or a decline in B^{CB} (sterilized intervention), while in nfl, the changes in b_t^{CBF} are offset either by changes in B_t^{CB} or by changes in H.

After consolidating the household's budget constraints using the equilibrium in the domestic asset markets, and substituting transfers in the household's budget constraint, we obtain the period resource constraints:

$$\begin{aligned} C_t &= Y_t + nfl_t \\ C_{t+1} &= Y_{t+1} - (1+i_t^*)nfl_t - X_{t+1}^* gfl_t + i_t \tfrac{S_t}{S_{t+1}} \left(\tfrac{H_t}{S_t} - h_t^H \right) \end{aligned} \tag{25}$$

The last term, which represents the economy's seigniorage revenue from the foreign holding of domestic money, can be neglected since we will either have $\frac{H_t}{S_t} = h_t^H$ (if $i_t > 0$) or $i_t = 0$. Here we see the channel through which the central bank balance sheet can improve households' welfare: the central bank can exploit the UIP deviation (if $X_{t+1}^* < 0$) by increasing the economy's gross foreign position by holding more foreign reserve b_t^{CBF} while issuing more domestic bonds (decreasing B_t^{CB}) or more money if the economy is at the ZLB (increasing $\frac{H_t}{S_t} - h_t^H$).[7] This channel appears clearly as a net intertemporal gain for the domestic economy if we write the intertemporal resource constraint:

$$(1+i_t^*)C_t + C_{t+1} = (1+i_t^*)Y_t + Y_{t+1} - X_{t+1}^* gfl_t. \tag{26}$$

Finally, we can also relate the country's consolidated financial liabilities to the household short-selling constraints (13). Using the central bank's budget constraint, we can see that the households' constraint on domestic bond issuance translates into a constraint on gross foreign liabilities:

$$gfl_t \leq b_t^G + b_t^{CBF} - h_t^H \tag{27}$$

However, (27) is not an effective constraint if we allow the central bank to change its holding of foreign bonds b_t^{CBF}.

Similarly, the foreign currency no-borrowing constraint implies:

$$nfl_t \leq gfl_t - b_t^{CBF} \tag{28}$$

[7] If the central bank could make transfers in t, then it could also affect the consumption profile through the economy's net position nfl_t. If the household is constrained ($b_t^H + b_t^F = 0$), then the central bank could increase the net borrowing of the economy and hence transfer consumption from $t+1$ to t. This fiscal foreign exchange intervention case is examined in the Online Appendix.

This constraint cannot be relaxed by non-fiscal FX intervention since changes in gfl_t are offset by changes in b_t^{CBF}.[8] This constraint is effective except if we allowed the central bank to perform fiscal interventions, where changes in gfl_t need not be offset by changes in b_t^{CBF}. Equations (27) and (28) are equivalent to the no-borrowing constraints (13).

3.2 The Constrained Planner's Program

Based on the previous equations, we can examine the planner's optimal choices.

Definition 1 (Constrained planner equilibrium) *A constrained planner equilibrium is an equilibrium where a planner maximizes the household's utility (15) subject to the economy's resource constraints (25); the asset pricing equation (8); the cash-in-advance constraints $h_t^H \geq Y_t$ and $\bar{H}e^h = S_{t+1}Y_{t+1}$; the non-negativity of foreign domestic money holdings $h_t^{H*} \geq 0$; the equilibrium on the market for money $H_t = S_t(h_t^H + h_t^{H*})$; the consolidated bond and money market equilibrium $a_t^{H*} = gfl_t$; the zero lower bound $i_t \geq 0$; and the foreign liability constraints (27) and (28). The planner's choice variables are $(i_t, S_t, S_{t+1}, gfl_t, nfl_t, b_t^{CBF}, H_t, h_t^H, h_t^*, a_t^*)$.*

The central bank's program is:

$$
\begin{aligned}
&\max E\Bigg\{U(C_t) + \beta U(C_{t+1}) \\
&+\eta_t \left(Y_t - C_t + nfl_t\right) \\
&+\eta_{t+1} \left[Y_{t+1} - C_{t+1} - (1+i_t^*)nfl_t + \left[(1+i_t^*) - (1+i_t)\tfrac{S_t}{S_{t+1}}\right] gfl_t + i_t \tfrac{S_t}{S_{t+1}} \left(\tfrac{H_t}{S_t} - h_t^H\right)\right] \\
&+\xi i_t \\
&+\Delta_t^H \left(h_t^H - Y_t\right) \\
&+\Delta_t^F \left(\tfrac{H_t}{S_t} - h_t^H\right) \\
&+\Lambda \left(gfl_t - b_t^{CBF} - nfl_t\right) \\
&+\tilde{\Lambda} \left(b_t^G + b_t^{CBF} - h_t^H - gfl_t\right) \\
&+\alpha_0 \left(E_t \left(m_{t+1}^* \left[(1+i_t^*) - (1+i_t)\tfrac{S_t}{S_{t+1}}\right]\right) + \Gamma gfl_t + \chi\right) \Bigg\}
\end{aligned}
$$

and we treat S_{t+1} as an exogenous variable since $S_{t+1} = He^h/Y_{t+1}$. Here, we substituted the foreign demand for domestic assets a_t^{H*} with gfl_t and h_t^{H*} with $H_t/S_t - h_t^H$.

Consider the first order conditions for assets:

$$
\begin{aligned}
&/nfl_t: && \eta_t - E_t\left(\eta_{t+1}(1+i_t^*)\right) && -\Lambda && = 0 && (29) \\
&/gfl_t: && E_t\left(\eta_{t+1}\left[(1+i_t^*) - (1+i_t)\frac{S_t}{S_{t+1}}\right]\right) && +\Lambda - \tilde{\Lambda} + \alpha_0 \Gamma && = 0 && (30) \\
&/H_t: && E_t\left(\eta_{t+1}\left[i_t \frac{S_t}{S_{t+1}}\right]\right) && +\Delta_t^F && = 0 && (31) \\
&/b_t^{CBF}: && && -\Lambda + \tilde{\Lambda} && = 0 && (32)
\end{aligned}
$$

[8] When capital controls are in place, however, Bacchetta et al. (2013) show that sterilized interventions can affect the country's intertemporal allocation.

Equation (32) implies that $\tilde{\Lambda} - \Lambda = 0$. This means that the central bank equalizes the marginal benefit of relaxing the foreign-currency and domestic-currency debt constraints by adjusting its assets and liabilities and going shorter in the asset whose shadow cost is higher and longer in the asset whose shadow cost is lower. Also note that $\eta_t = U'(C_t)$, $\eta_{t+1} = U'(C_{t+1})$, and that $m_{t+1} = \eta_{t+1}/\eta_t$ is the central bank's discount factor, which coincides with the household's (see Appendix A.1).

3.3 Optimal foreign exchange interventions

We can examine the impact of sterilized and unsterilized FX interventions by examining Equation (30) with $\Lambda - \tilde{\Lambda} = 0$.

Sterilized interventions Equation (30), with $\Lambda - \tilde{\Lambda} = 0$, can be rewritten as follows:

$$\underbrace{-E_t X^*_{t+1} - \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} + \frac{\alpha_0}{\eta_t E_t m_{t+1}}\Gamma}_{MBFX_t} = 0 \tag{33}$$

The left-hand side, $MBFX_t$, corresponds to the marginal benefit of sterilized foreign exchange interventions, that is, of expanding the central bank's balance sheet by going long in foreign bonds and short in domestic bonds. It is composed of the excess return on foreign bonds, minus the risk premium associated with this excess return, plus the marginal benefit of the resulting price distortions. If, in the absence of interventions, $MBFX_t$ is positive, then it would be optimal for the central bank to perform foreign exchange interventions. These interventions can drive the marginal benefit to zero, achieving an optimal central bank balance-sheet, as we will see in more details later.

Since $E_t X^*_{t+1} < 0$ for safe haven countries, $MBFX_t$ may be be positive. In contrast, for non-safe haven countries $MBFX_t$ is likely to be negative. In this case, FX interventions are always costly and will not be used unless there are offsetting benefits that we have not modeled.

Substituting $E_t X^*_{t+1}$ using Equation (10), we can rewrite the marginal benefit of foreign exchange interventions:

$$MBFX_t = \underbrace{-\overbrace{\frac{\Gamma g f l_t + \chi}{E_t m^*_{t+1}}}^{-devCIP} + \frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}}}_{-devUIP} - \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} + \frac{\alpha_0}{\eta_t E_t m_{t+1}}\Gamma \tag{34}$$

Equation (34) shows how CIP and UIP deviations affect the central bank incentives to buy FX reserves. This can be summarized in the following proposition:

Proposition 1 *Consider the SDF of domestic households, m_t, and of international financial intermediaries m^*_t and the excess return in foreign currency, X^*_{t+1}. The benefit (or cost) of foreign exchange intervention $MBFX_t$ depends on* *(i) CIP deviations when* $cov(m_{t+1}, X^*_{t+1}) = cov(m^*_{t+1}, X^*_{t+1})$.

(ii) UIP deviations when $cov(m_{t+1}, X^*_{t+1}) = 0$.

In fact, the intermediation friction generates two wedges that are relevant to the central bank. First, the CIP deviation, which is a riskless excess return. Second, the difference between the foreign and domestic risk premia. If the foreign and domestic agents have the same risk premium, then the central bank should seek to shut down CIP deviations. This is the case in the absence of risk, as in Amador et al. (2020), or when financial intermediaries have the same discount factor as households. In contrast, in the limit case where the domestic agents have negligible risk aversion as compared to financial intermediaries, then the sum of the two wedges is equal to the UIP deviation, and the central bank optimal policy would be to eliminate UIP deviations.[9]

However, in general, the sum of the two wedges does not coincide with either the CIP or the UIP deviations. In particular, a safe haven currency may be more desirable for foreign investors as a diversification hedge than for the domestic investors so that $cov(m^*_{t+1}, X^*_{t+1}) > cov(m_{t+1}, X^*_{t+1})$. If the difference is large enough, it can be optimal for the central bank to increase the country's long exposure to foreign currency and the country's short exposure to domestic currency.

Finally, we examine the last term in equation $MBFX_t$, which arises from a monopoly power of the central bank. The central bank has an incentive to not fully shut down its risk-adjusted foreign currency excess return in order to maximize its profit. Appendix A.2 shows that this term is equal to:

$$\frac{\alpha_0}{\eta_t E_t m_{t+1}}\Gamma = -\Gamma gfl_t \frac{E_t\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E_t m_{t+1} E_t\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)} \tag{35}$$

This term is of the same sign as $-gfl_t$, home's gross external position in domestic currency. If the country is short in domestic currency ($gfl_t > 0$), then α_0 is negative. When accumulating foreign currency assets by issuing domestic currency liabilities, the planner increases the domestic interest rate i_t and hence reduces the foreign currency excess return. The resulting opportunity cost is proportional to the economy's gross external position. This term also depends on Γ, which measures the impact of domestic currency issuance on the excess return. The higher Γ, the more difficult it is for foreign intermediaries to absorb additional foreign currency assets, the higher the impact of foreign currency issuance on the excess return.

To summarize, there could be a benefit of intervention for a safe haven currency if its hedging property is more valued by international investors.

[9]This is what Itskhoki and Mukhin (2021) implicitly assume. In their linear approximation, they take the level of risk to zero but ensure that the risk premium of the financial intermediaries remains a first order object by rescaling their risk aversion, but not that of the households. This implies that the intermediaries' risk aversion is an order of magnitude higher than that of the households. As a result, it is optimal to eliminate UIP deviations.

Unsterilized interventions Consider now unsterilized interventions. Equation (31) implies that $\Delta^H > 0$ if $i_t > 0$, meaning that $H_t/S_t = h_t^H$ constrains unsterilized interventions outside the ZLB, but not at the ZLB.

It is in principle more advantageous for the central planner to buy foreign assets by issuing money (H_t) than by selling bonds (B_t^{H*}), as long as $i_t > 0$. However, this strategy is made ineffective in equilibrium by the unwillingness of foreigners to hold money when $i_t > 0$ ($H_t/S_t = h_t^H$). As domestic households need a fixed real quantity of money, issuing more money would be purely inflationary and would not increase the capacity of the central bank to buy foreign bonds.

At the ZLB however, when $i_t = 0$, foreigners may want to hold money. In that case, sterilized and unsterilized interventions become equivalent and $MBFX_t$ is the marginal benefit of both sterilized and unsterilized interventions, so that the above analysis applies. Whether foreign bonds are acquired by increasing H_t (unsterilized intervention) or by decreasing B_t^{CB} (sterilized intervention) does not matter.

Exchange rate and interest rate determination Outside the ZLB, S_t and i_t are undetermined. The excess return, which depends on $S_t(1 + i_t)$, is set optimally by the central bank, for a given expectation of S_{t+1}, and it is compatible with different combinations of S_t and i_t. However, if the central bank fixes the supply of money to a given level $\bar{H}$, then S_t is determined by the equilibrium on the money market, and then i_t is determined by the optimal foreign exchange intervention, for a given S_t. If the optimal excess return on domestic currency becomes too low, then the interest rate hits the zero lower bound, and S_t adjusts to clear the bond market and delivers the optimal excess return.

3.4 Implementation of the Optimum in a Decentralized Equilibrium

Here we discuss how the optimum is implemented in a decentralized equilibrium by analyzing households' optimal choices. Consider the central bank's foreign exchange interventions (sterilized or unsterilized). These interventions are relevant for the economy' gross foreign liabilities. Suppose that the optimal gross foreign liability position of the economy is $\widehat{gfl}_t$.

The households' optimal portfolio allocation, characterized by Equation (17), can be compared with Equation (30). For Equation (30) to be implemented in the decentralized equilibrium, we need that

$$\lambda^H - \lambda^F = -\alpha_0\Gamma \tag{36}$$

where we used $\eta_t = U'(C_t)$, $\eta_{t+1} = U'(C_{t+1})$ and $\Lambda - \tilde{\Lambda} = 0$. In safe haven countries, where typically $\widehat{gfl}_t > 0$ and hence $\alpha_0 < 0$, optimal foreign exchange interventions are only consistent with the households being financially constrained when issuing domestic-currency bonds ($\lambda^H > 0$), since $\lambda^F \geq 0$. The central bank, as we have seen, does not fully exhaust the –private– marginal benefit of going long in foreign currency and short in domestic currency. Households can be prevented from exploiting this residual marginal benefit only if their domestic-currency borrowing constraint is binding.

In practice, the central bank desires fewer domestic liabilities than households. The optimum is then easily implementable for the central bank by supplying just the right amount of domestic liabilities to complement the existing public domestic supply and reach $\widehat{gfl}_t$, through foreign exchange interventions. More precisely, the optimal foreign exchange intervention must satisfy

$$\widehat{b}_t^{CBF} = \widehat{gfl}_t - \left(b_t^G - h_t^H\right) \tag{37}$$

The domestic currency bonds issued by foreign exchange interventions $\widehat{b}_t^{CBF}$ must close the gap between the optimal gross foreign liabilities $\widehat{gfl}_t$ and the existing real supply of domestic bonds, which is equal to the amount of government bonds that are not held by the central bank to back households' asset holding $b_t^G - h_t^H$. For that level of domestic currency bonds, households would like to issue more domestic currency bonds ($b_t^H < 0$), but they are prevented from doing so by their no-borrowing constraints. That way, the optimum is implementable.[10]

4 A Linear Version of a Safe Haven Economy

In this section, we focus on the safe haven currency case. We consider a linearized version of the model and assume lognormal shocks. We do not model the reasons behind the safe haven status and simply assume that the domestic currency is expected to appreciate when global output slows down.[11] The objective is to derive $cov(m_{t+1}^*, X_{t+1}^*)$ and $cov(m_{t+1}, X_{t+1}^*)$ to evaluate the opportunity cost of FX interventions given in (34). We assume that the SDF of international financial intermediaries is proportional to a global variable Y_t^*, that we call world output.[12] We assume that Y_{t+1}^* is log-normal with

[10]Note that if $\widehat{gfl}_t < 0$ ($\alpha_0 > 0$), then Equation (36) would imply that $\lambda^F > 0$ (since $\lambda^H \geq 0$), meaning that the household should be constrained in issuing foreign-currency bonds for the optimum to be implementable. Indeed, in that case, the central bank would typically save in domestic currency and borrow in foreign currency, but there would remain a private benefit of going short in foreign currency and long in domestic currency, which can only be consistent with a binding constraint on foreign liabilities in a decentralized economy. The optimum can be implemented by the central bank (or government) by supplying just the right amount of foreign-currency liabilities to reach $-\widehat{gfl}_t$.

[11]There is a small literature trying to provide explanations for safe haven effects, but the focus is on the US and the mechanisms do not apply to a small countries. See Maggiori (2017) or Hassan et al. (2021). Papers that model time-varying safe haven effects include Gourinchas and Rey (2022), Devereux et al. (2022), and Kekre and Lenel (2021).

[12]In the empirical section we will use the net wealth of financial intermediaries rather than global output.

$\tilde{Y}^*_{t+1} \sim N(\sigma_y^2/2, \sigma_y^2)$. This assumption normalizes the expected foreign stochastic discount factor to β under log-utility: $E_t m^*_{t+1} = E_t(\beta/Y^*_{t+1}) = \beta$.

The SDF of domestic households is proportional to domestic output Y_t. We assume that $\log(Y_{t+1}) = \alpha \log(Y^*_{t+1}) + (1-\alpha)\sigma_y^2/2$ to take into account the correlation between domestic output and the global variable, with $\alpha > 0$, while ensuring that $E(Y_{t+1}) = E(Y^*_{t+1})$. With the appropriate assumptions on h, we can assume that $S_{t+1} = He^{\rho \log(Y^*_{t+1})}$.

We make the following assumption:

Assumption 1 (Safe haven) $\rho > 0$ *and* $\alpha < 1$.

A positive ρ captures the hedging capacity of safe haven currencies: the exchange rate appreciates when global output is low, so the domestic currency is a good hedge against global output fluctuations. A low α reflects the small exposure of the domestic output to global risk. Domestic output comoves with global output, so the domestic currency is also a hedge for domestic households, but since domestic output is less volatile, domestic households will in equilibrium be willing to be short in domestic bonds.

We denote from now on the variables in log with a tilde: $\tilde{Y} = \log(Y)$ and $\tilde{Y}^* = \log(Y^*)$. We also define $\tilde{i}^*_t = \log(1 + i^*_t)$ and $\tilde{i}_t = \log(1 + i_t)$. We further assume that $Y_t = Y^*_t$. Since the optimal H_t is undetermined, we fix it to $H_t = 1$. These assumptions imply that $S_t = H_t/Y_t = 1$ outside the ZLB.

4.1 Optimal Allocations

We first solve for the equilibrium for given nfl_t and gfl_t in order to evaluate the planner's optimality conditions as a function of nfl_t and gfl_t. We will consider in turn the case where the ZLB is not binding and the case where it is. We assume log-utility and use second-order approximations.

The household's budget constraints yield:

$$\begin{aligned}\tilde{C}_t &= nfl_t - nfl_t^2 \\ \tilde{C}_{t+1} &= \tilde{Y}_{t+1}(1 + nfl_t + gfl_t) - (nfl_t - nfl_t^2)(1 + \tilde{i}^*_t) - gfl_t X^*_{t+1}\end{aligned} \tag{38}$$

See the details of the derivation in Appendix B.1. Besides, we have $\tilde{S}_{t+1} = \rho \tilde{Y}^*_{t+1}$.

The foreign and domestic interest rates must satisfy

$$E_t(e^{\tilde{m}^*_{t+1} + \tilde{i}^*_t}) = 1 \tag{39}$$

$$E_t(e^{\tilde{m}^*_{t+1} + \tilde{S}_t - \tilde{S}_{t+1} + \tilde{i}_t}) = 1 + \chi + \Gamma gfl_t \tag{40}$$

This yields:

$$\begin{aligned}1 + i^*_t &= \frac{1}{\beta} \\ (1 + i_t)S_t &= \frac{1}{\beta}(1 + \chi + \Gamma gfl_t)e^{-\frac{1}{2}(1+\rho)\rho\sigma_y^2}\end{aligned} \tag{41}$$

See Appendix B.2 for a full derivation. These equations define respectively the foreign interest rate i^*_t as a function of the subjective discount factor and $(1 + i_t)S_t$ as a function of financial frictions (Γ and χ), of the hedging properties of the domestic exchange rate and of gross foreign liabilities. This determines i_t outside the ZLB (since $S_t = 1$) and S_t at the ZLB (since $i_t = 0$).

What are the implications for foreign exchange interventions? We can write the difference in risk premia as follows

$$\frac{cov(m^*_{t+1}, X^*_{t+1})}{E_t m^*_{t+1}} - \frac{cov(m_{t+1}, X^*_{t+1})}{E_t m_{t+1}} = \frac{1}{\beta}(1 + \chi + \Gamma gfl_t)\left(1 - e^{-\Delta cov}\right) \tag{42}$$

where $\Delta cov = cov(\tilde{m}_{t+1}, \tilde{S}_{t+1}) - cov(\tilde{m}^*_{t+1}, \tilde{S}_{t+1})$ is the difference between the covariance of the log-linearized domestic currency excess return with the intermediaries SDF and the covariance with the domestic SDF. In Appendix B.3 we show that

$$\Delta cov = \rho\sigma_y^2 \left[1 - \alpha(1 + nfl_t + gfl_t) - \rho gfl_t\right] \tag{43}$$

In a safe haven economy as defined in 1, if nfl_t and gfl_t are not too high, this covariance differential is positive. This implies that the difference in covariances introduces an incentive for the central bank to perform foreign exchange interventions. The domestic economy is less exposed to global risk, so the planner is willing the go short in domestic bonds and long in foreign bonds. Notice that this covariance differential is decreasing in gfl_t. By increasing its balance-sheet exposure to exchange rate risk, the planner increases the risk exposure of the domestic household and the domestic covariance catches up to the foreign one.

However, this covariance differential is only one component of the planner's optimality condition (34). The following lemma lays down explicit expressions.

Lemma 1 *Suppose that the economy is a safe haven as in Definition 1. Denote by* $\widehat{gfl}_t$ *and* $\widehat{nfl}_t$ *the optimal gross and net foreign liabilities. We focus on solutions where* $\widehat{nfl}_t < [-\log(\beta) + \alpha^2)]/2$. *Then:*

(i) $\widehat{nfl}_t = min\{b^G_t - 1, \overline{nfl}(\widehat{gfl}_t)\}$, *where* $\overline{nfl}(gfl_t)$ *is defined in Appendix B.4;*

(i) $\widehat{gfl}_t$ *is implicitly defined by:*

$$1 - (1 + \chi + 2\Gamma gfl_t)e^{-\Delta cov} = 0$$

where Δcov *is defined by Equation (43).*

See the proof in Appendix B.4. Result (i) comes from the fact that the household may be financially constrained. In that case, $nfl_t = b^G_t - h^H_t$, where $h^H_t = 1$. Otherwise, the level of net foreign liabilities nfl equalizes the domestic and foreign discount factors in expectations. Note that in that case, the household can only issue foreign bonds, because, as we have seen, we must have $\lambda^H > 0$. Result (ii) derives from the optimality condition with respect to gfl_t, Equation (34). It reflects the fact that the planner does not fully shut down the domestic currency excess return rent (hence the extra term in Γ on the left-hand side). The planner intervenes enough to take advantage of the excess return, but takes into account the fact that interventions decrease the domestic excess return.

4.2 Comparative statics

Lemma 1 implies that, at the optimum,

$$\widehat{gfl}_t = \frac{\rho\sigma_y^2[1-\alpha(1+\widehat{nfl}_t)]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2} \tag{44}$$

This is shown formally in Appendix B.5. Since we analyze safe haven economies, we focus on the case where $\widehat{gfl}_t \geq 0$.

Note that $\widehat{gfl}_t$ depends negatively on $\widehat{nfl}_t$. Indeed, higher leverage makes the economy more vulnerable to global risk and hence reduces the incentives of the central bank to take more risk on its balance sheet. What parameters drive $\widehat{nfl}_t$? In the case where the households are unconstrained, we show in Appendix B.5 that, under some conditions, $\widehat{nfl}_t$ is positive if σ_y^2 is high and α is low, while χ and Γ are not too large, even though the path of domestic output is the same as the foreign one on average. This is due to the fact that domestic households are less exposed to global risk than the foreign investors, which lowers the domestic discount factor relative to the foreign one. This is akin to an "inverse precautionary saving motive". This low risk exposure generates a borrowing motive that can drive the domestic households to hit their no-borrowing constraint if the government debt is too low ($b_t^G - 1 < \overline{nfl}_t$).

In what follows, we suppose that the households are constrained so that $\widehat{nfl}_t = b_t^G - 1$ is given. In that case, the optimal level of intervention is given by

$$\widehat{b}_t^{CBF} = \frac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2} - (b_t^G - 1)$$

where we used equations (37) and (44).

The comparative statics for optimal FX intervention is given in the following proposition:

Proposition 2 *Consider a safe haven economy as defined in 1. Suppose that $\widehat{gfl}_t \geq 0$ and $\widehat{nfl}_t = b^G - 1$. Then optimal foreign exchange interventions, $\widehat{b}_t^{CBF}$:*

(i) are increasing in risk measures σ_y and ρ;

(ii) are decreasing in intermediaries financial frictions Γ and χ;

(iii) are decreasing in the domestic output exposure to global risk α, as long as $b_t^G > 0$;

(iv) are decreasing in the supply of government bonds b_t^G;

Points (i) to (iv) can be shown by taking the derivatives of $\widehat{b}_t^{CBF}$ with respect to σ_y, ρ,Γ, χ, α, and b_t^G.

Risk tends to increase the covariance differential Δcov, which generates an excess benefit of foreign exchange interventions, while the intermediation frictions generate a cost. The exposure of domestic output to global risk decreases the covariance differential and generates a cost. Point (iv) arises from the substitutability between government bonds and the central bank's sterilization bonds (see Equation (37)). If the government issues more bonds, then this reduces the need for the central bank to issue domestic bonds through FX interventions.

Interestingly, an increase in risk, which increases the optimal gfl_t, typically generates both a more negative UIP deviation and a more positive CIP deviation, as the financial intermediaries are obliged to absorb the excess domestic currency bonds. This is established formally in the following proposition:

Proposition 3 *Suppose Suppose that $\widehat{gfl}_t \geq 0$ and $\widehat{nfl}_t = b^G - 1$. Then:*

(i) Z_{t+1}^* is increasing in σ_y (it becomes more positive);

(ii) $E_t X_{t+1}^*$ is decreasing in σ_y (it becomes more negative) if Γ is not too large;

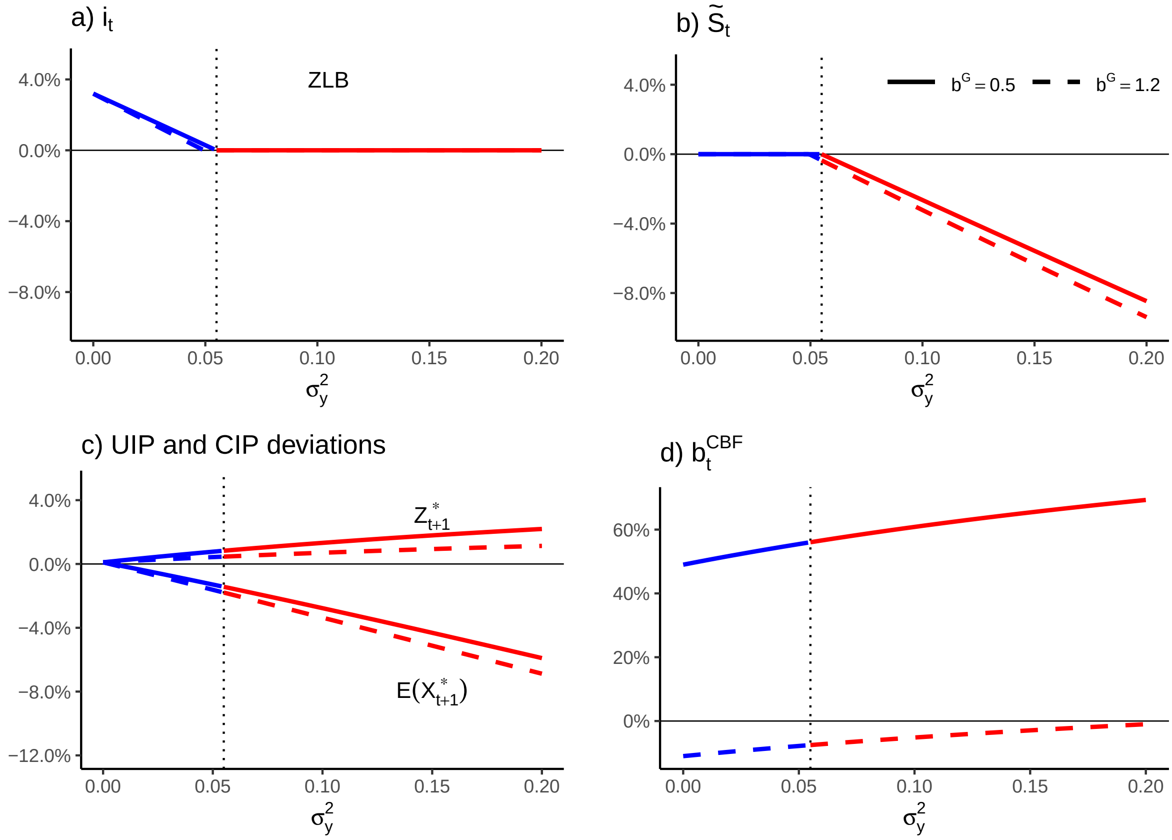
See the proof in Appendix B.6. The CIP deviation becomes more positive when risk increases, as financial intermediaries need to absorb more capita inflows (more gfl_t). As risk increases, the UIP deviation becomes more negative, because it affects positively the foreigners' risk premium. However, if the intermediation friction is large (Γ is large), the increase in the CIP deviation can offset the increase in the risk premium, and total the impact on the UIP deviation becomes ambiguous.

4.3 Numerical Illustration

These comparative statics hold both outside and inside the ZLB. The only difference is that at the ZLB, i_t is constant at zero, while S_t adjusts. To illustrate this, we show a numerical example varying the level of risk σ_y^2. Figure 2 shows the comparative statics of σ_y^2 under a baseline specification of parameters, both for ZLB and non-ZLB. We also consider two levels of b_t^G: 0.5 and 1.2. Panel a) shows the negative relationship between the domestic interest rate and risk through a standard Euler Equation argument. The ZLB is attained at $\sigma_y^2 \geq 0.055$. At the ZLB, it is the exchange rate which adjusts to accommodate higher risk through an appreciation (see Panel b)). Panel c) displays the deviations from UIP ($E_t X_{t+1}^*$) and CIP (Z_{t+1}^*). As we can see, an increase in risk leads to a more negative UIP deviation, and a more positive CIP deviation, as explained in Proposition 3. For Panels a) to c), the level of public debt b_t^G does not have a large impact.

Panel d) shows that $\widehat{b}_t^{CBF}$ increases with risk because of the positive covariance differential (see equation (43)) resulting from the assumption of safe-haven ($\alpha < 1$ and $\rho > 0$). However, the level of $\widehat{b}_t^{CBF}$ is only positive when $b_t^G = 0.5$. When b_t^G is large, the central bank prefers buying domestic bonds rather than foreign bonds. This perspective is consistent with the experience of Switzerland and Japan. Swiss public debt has been below 50% in the last 15 years, while it has been higher than 200% for Japan.

Figure 2: Comparative statics of σ_y^2



Notes: Baseline parameters : $\beta = 0.97, \chi = 0.2\,\Gamma = 0.1, \alpha = 0.6, \rho = 0.4.$

5 Empirical Analysis

In this empirical section, we argue, as other authors have done, that the CHF and the JPY are safe haven currencies. We then evaluate a key determinant of the optimality of foreign exchange interventions in these economies, namely the covariance differential $\frac{cov(m_{t+1}^*, X_{t+1}^*)}{E_t m_{t+1}^*} - \frac{cov(m_{t+1}, X_{t+1}^*)}{E_t m_{t+1}}$ in Equation (34). We show that this term is positive, which is compatible with a net benefit of foreign exchange interventions.

5.1 CHF and JPY as Safe Haven Currencies

The safe haven properties of the Swiss franc and the Japanese yen have been documented by various authors, e.g., Stavrakeva and Tang (2021), Ranaldo and Söderlind (2010), Grisse and Nitschka (2015), or Fink et al. (2022). We confirm this by relating expected excess returns to various sources of risk. We compute UIP deviations using short-term rates from Datastream and survey data from Consensus Economics.[13] In particular, we consider deviations using $i \in \{CHF, JPY\}$ as the domestic currency and the USD as the foreign one. Let us define the log excess returns of going long in the domestic currency from the international investors' perspective:

$$x_{t+1}^* = i_t - i_t^* + s_t - s_{t+1} \tag{45}$$

Table 1 shows the correlation between expected excess returns in CHF and JPY (Ex_{t+1}^*) and different measures of risk. Since 2010, this correlation is systematically positive, suggesting that agents tend to expect the CHF and JPY to yield excess returns at times of heightened uncertainty. When considering the entire sample (from 1999 to 2021), the correlation is systematically weaker or negative, which suggests that the CHF and JPY have reinforced their perceived safe-haven properties since 2010.

Table 1: Correlation between UIP deviations and (global) risk variables

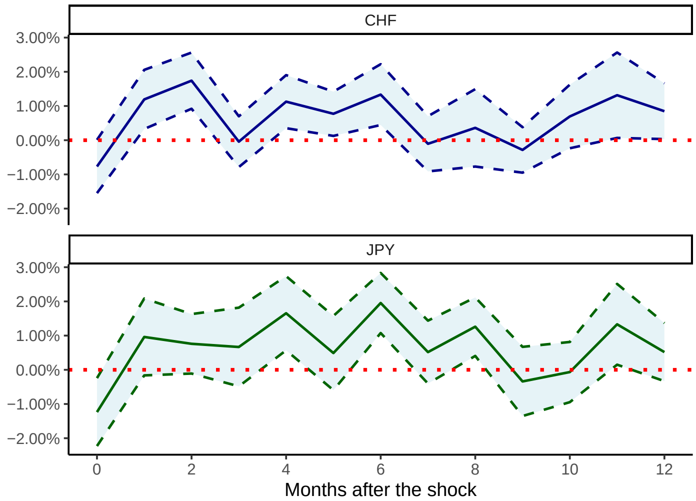
$Corr(RiskVariables, E(x_{t+1}^*))$						
	A) CHF/USD			B) JPY/USD		
Sample	USEPU	GEPU	WUI	USEPU	GEPU	WUI
1999-2021	-0.23	-0.29	-0.30	-0.11	-0.03	0.06
2010-2021	0.14	0.26	0.41	0.14	0.32	0.43

Notes: This table displays the correlation between Ex_{t+1}^* (at a 3-month horizon) and different risk variables for the whole sample and a subsample starting in 2010. Panel A) displays this correlation taking the CHF as the domestic currency and the USD as the foreign one. Similarly, Panel B) considers the JPY as the domestic currency. USEPU is the US Eonomic Policy Uncertainty index developed in Baker et al. (2016). GEPU is the Global EPU. WUI is the World Uncertainty Index developed in Ahir et al. (2022). Since WUI is only available at a quarterly frequency, we take the quarterly mean of UIP deviations when computing the correlation.

To examine the dynamic impact of uncertainty shocks, Figure 3 runs a local-projection regression (Jordà (2005)) of a Global Economic Policy Uncertainty (EPU) shock on $E(x_{t+1}^*)$. The results show that, following an unanticipated shock to the Global EPU, $E(x_{t+1}^*)$ tends to increase both for the CHF and the JPY. In other words, the CHF and the JPY are generally expected to appreciate following an uncertainty shock.

[13] See Kalemli-Özcan and Varela (2021) for a recent analysis of UIP deviations using Consensus Economics survey.

Figure 3: Local Projections to a Global EPU shock



Notes: This figure shows the results from the Local Projection of a Global EPU shock on the UIP deviations, using the CHF and the JPY as the domestic currency, respectively. Formally, we identify an uncertainty shock ($shock_t$) outside of the system by taking the residual of an AR(1) on our Global EPU variable in the spirit of Stock and Watson (2012) who uses the VIX. We then run $E(x^*_{t+h}) = \alpha^h + \beta_h shock_t + \phi^h x_t + u^h_{t+h}$ for $h = 0, ..., 12$ where x_t are control variables made of $p = 6$ lags of the dependent variable. We then report β^h at each horizon as well as the 90% confidence intervals using the Newey-West estimator.

5.2 On the Optimality of Holding Reserves

The theoretical analysis has shown that the incentives for FX interventions depend crucially on the difference between $cov(m^*_{t+1}, X^*_{t+1})$ and $cov(m_{t+1}, X^*_{t+1})$ (Equation (34)). In this subsection, we provide estimates of the covariances between x^*_{t+1} and m^*_{t+1} or m_{t+1}. For domestic households, we simply assume that $m_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$, where $1/\gamma$ is the rate of intertemporal substitution.

For international financial intermediaries, we follow the literature on intermediary asset pricing (e.g., see He and Krishnamurthy (2011) or Brunnermeier and Sannikov (2014)), and assume that their SDF is proportional to their net worth NW_t:

$$m^*_{t+1} = \beta \left(\frac{NW_{t+1}}{NW_t} \right)^{-\gamma} \tag{46}$$

As in He et al. (2017), we assume that the financial intermediaries' net worth is related to the aggregate wealth in the economy (denoted by W^W_t) and the intermediaries' capital ratio (denoted by η_t). This specification implies that the marginal utility of wealth of financial intermediaries rises when the aggregate wealth in the economy or the equity capital ratio is low (or a combination of the two). The first term captures the asset pricing effect of weaker fundamentals, while the second captures the idea that the intermediaries' risk bearing capacity is impaired when the capital ratio is low. As a result, risk aversion increases the marginal value of wealth. Using time-series and cross-sectional asset pricing tests, He et al. (2017) show that this specification captures well the marginal utility of wealth of financial intermediaries, and find supporting evidence that financial intermediaries are indeed marginal investors for a wide class of assets.

In our empirical exercise, we consider two specifications of the SDF of financial intermediaries. In the first, net worth is related to the equity capital ratio from He et al. (2017) (denoted as η_{t+1}^{HKM}) and the MSCI World Equity Index (denoted as W_{t+1}^{W}). The second uses the measure of capital ratio developed in Adrian et al. (2014), which is defined as the (inverse of) book leverage of security Brokers & Dealers and is computed using balance sheet data reported in the Flow of Funds from the Federal Reserve Board. It is computed as the ratio of total equity (total financial assets minus total financial liabilities) to total financial assets. We denote it as η_{t+1}^{AEM} and interact it with the MSCI World Equity Index to get a measure of the net worth of financial intermediaries. To convert these measures into a growth rate (as suggested by (46)), we follow the approach in He et al. (2017). In particular, we first approximate the growth rate on aggregate wealth using the excess returns on the MSCI World Equity Index, using the 3M US Libor as the risk free rate. Second, we define the intermediary capital risk factor as the residual from an AR(1) on the measures of capital ratio divided by the lagged capital ratio. $\frac{NW_{t+1}}{NW_t}$ is then defined by the interaction between the excess returns on the MSCI World portfolio and the intermediary capital risk factor.

Because our focus is on financial intermediaries, we consider a quarterly horizon to compute the excess returns, variations in net wealth and real consumption. The idea is that a quarter aligns well with the relevant horizon of financial intermediaries, especially when it comes to regulatory requirements. Du et al. (2018) notably document that CIP deviations tend to be systematically larger around quarter-ends and interpret this as evidence of the causal effect of banking regulation on asset prices. Intuitively, a good asset would be one that provides higher returns when the intermediaries' wealth is relatively low, and this may be particularly valuable at the quarter end, when regulatory constraints are more likely to bind.

Table 2 displays the covariance between quarterly excess returns and the different SDF proxies using either the CHF or the JPY as the domestic currency, keeping the USD as the foreign one. The second and third column displays the covariance between excess returns and the SDF defined using the capital ratio measure from He et al. (2017) and Adrian et al. (2014), respectively. The last column displays the covariance between the excess returns and the SDF of the Swiss and Japanese households. We assume that $\beta = 0.99$ and $\gamma = 5$. For each currency, we consider the whole sample running from 1999 to 2021 and a subsample from 2010 to 2021.

The results show that the correlations between the proxies for the SDF of financial intermediaries and excess returns are clearly positive since 2010, both using the CHF and the JPY as the domestic currency, and for both measures of the SDF. In other words, being long in CHF or JPY tend to provide higher returns when the marginal utility of wealth of financial intermediaries is high, which support that the CHF and the JPY

Table 2: $Cov(x^*_{t+1}, m^*_{t+1})$ and $Cov(x^*_{t+1}, m_{t+1})$

A) CHF domestic currency, USD foreign currency			
	Fin. Intermediaries		HH
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^W_{t+1}$	$\eta^{AEM}_{t+1} \times W^W_{t+1}$	C^{CH}_{t+1}
1999-2021	-1.83%	3.84%	-0.01%
2010-2021	1.59%	0.88%	-0.03%

B) JPY domestic currency, USD foreign currency			
$NW_{t+1} =$	$\eta^{HKM}_{t+1} \times W^W_{t+1}$	$\eta^{AEM}_{t+1} \times W^W_{t+1}$	C^{JP}_{t+1}
1999-2021	18.08%	1.91%	0.15%
2010-2021	4.71%	1.76%	0.2 %

Note:
The table shows the correlation between quarterly excess returns and different proxies of the SDF of (international) financial intermediaries and Swiss and Japanese households. Excess returns are computed according to (45). The SDF of financial intermediaries is defined as $\beta\,(NW_{t+1}/NW_t)^{-\gamma}$. For Households (HH), it is defined as $\beta\,(C_{t+1}/C_t)^{-\gamma}$. We assume $\gamma = 5$ and $\beta = 0.99$. η^{HKM}_{t+1} is retrieved from Zhiguo He's website. η^{AEM}_{t+1} is computed using balance sheet data from the Federal Reserve Flow Of Funds (Table L.130). The MSCI Equity Index is retrieved from the MSCI website and is divided by the US CPI. Real consumption growth for Switzerland and Japan ($C^{CH}_{t+1}, C^{JP}_{t+1}$) is retrieved from the FRED website. All variables are quarterly.

behave as a hedge for international intermediaries. On the other hand, the covariance between excess returns and SDF based on real consumption growth tend to be close to zero and thus systematically smaller since 2010. In the case of Switzerland, Proposition 1 implies that it is not CIP but UIP deviations that should matter for FX interventions, since $cov(m_{t+1}, x^*_{t+1}$ is about zero. For Japan, $cov(m^*_{t+1}, x^*_{t+1}) > cov(m_{t+1}, x^*_{t+1}) > 0$, so that Equation (34) should be applied.

6 Conclusion

The GFC was followed by significant changes in the international monetary system. We have been observing systematic deviations from CIP, an increased demand for safe assets, a strengthening role of the USD as a reserve currency and strong increase in central bank balance sheets. In this context, there has been a stronger demand for safe have currencies and a higher motivation of central banks in these countries for FX intervention. In the case of the Swiss National Bank, the spectacular increase in its balance sheet has occurred exclusively through the purchase of foreign assets.

The objective of this paper is to provide a simple framework to clarify some aspects of these developments. To explain the difference in UIP and CIP deviations in safe haven economies, we follow the recent literature that gives a key role to constrained international financial intermediaries. However, we assume that these intermediaries face exchange rate risk and value the hedging properties of safe haven currencies. The increased demand of these currencies may push the central bank to intervene and limit the extent of currency appreciation.

We examine the opportunity cost of FX intervention when CIP and UIP deviations are of a different sign. We show that whether CIP or UIP matters depends on how domestic residents value the hedging property of their currency compared to international investors. If they give no value to its hedging property, UIP deviations should matter. This may imply an opportunity benefit, and thus a higher incentive, for FX accumulation. We show that the incentives to accumulate FX reserves in safe haven countries increase with the level of global risk or of effective risk aversion of international intermediaries. In contrast, the incentive decreases with the level of global debt.

We also attempt to estimate the opportunity cost of intervention for Switzerland and Japan. We find that in both countries, domestic households value less the hedging properties of their currency than international investors. Overall, the incentives for intervention are stronger for Switzerland as the difference with international investors is larger and its public debt is much smaller than in Japan. While our analysis focuses on small safe haven countries, it also sheds some light on the difference between the properties of safe haven currencies and those of a reserve currency such the USD.

A Proofs - Constrained Planner Program

A.1 Other FOCs

We take the derivative with respect to h_t^H:

$$/h_t^H : -E_t\left(\eta_{t+1}\left[i_t\frac{S_t}{S_{t+1}}\right]\right) + \Delta_t^H - \Delta^F - \tilde{\Lambda} = 0 \tag{47}$$

Equations (47) and (31) then imply that $\Delta_t^H = \tilde{\Lambda} = \Lambda$. Therefore, when $\Lambda = \tilde{\Lambda} = 0$, $\Delta_t^H = 0$. This reflects the fact that, while households want to minimize their money holdings because they represent a cost (when $i_t > 0$), the amount of money held by the households is not relevant to the central bank when the economy is not constrained in its capacity to issue debt, since seigniorage is redistributed to households in period $t+1$. The cash-in advance constraint is relevant only to the extent that it also restrains the capacity of the economy to supply domestic assets to the rest of the world, just like the no-borrowing constraints.

We now take the derivatives with respect to prices:

$$/i_t : \quad -E\left[\eta_{t+1}\frac{S_t}{S_{t+1}}\left(gfl_t + \frac{H_t}{S_t} - h_t^H\right)\right] + \xi - \alpha_0 E\left(m_{t+1}^*\frac{S_t}{S_{t+1}}\right) = 0 \tag{48}$$

$$\begin{aligned} /S_t : \quad & -E\left(\eta_{t+1}\left[(1+i_t)\frac{S_t}{S_{t+1}}gfl_t - i_t\frac{S_t}{S_{t+1}}h_t^H\right]\right) - \Delta^F\frac{H_t}{S_t} \\ & -\alpha_0\left[E\left(m_{t+1}^*(1+i_t)\frac{S_t}{S_{t+1}}\right)\right] = 0 \end{aligned} \tag{49}$$

Finally, we derive with respect to consumption:

$$/C_t : \quad U'(C_t) - \eta_t = 0 \tag{50}$$

$$/C_{t+1} : \quad E\left(\beta U'(C_{t+1}) - \eta_{t+1}\right) = 0 \tag{51}$$

These equations imply that $m_{t+1}^{CB} = \eta_{t+1}/\eta_t = \beta U'(C_{t+1})/U'(C_t) = m_{t+1}$.

A.2 Monopolistic term

Here we have to distinguish two cases. Either $i_t > 0$, and in that case $\xi = 0$, $H_t/S_t = h_t^H$ and $\Delta^F > 0$. Or $i_t = 0$, and in that case $\xi > 0$ and $\bar{\Delta}^F = 0$.

In the former case, Equation (48) yields

$$\frac{\alpha_0}{\eta_t} = -gfl_t\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E\left(m_{t+1}^*\frac{S_t}{S_{t+1}}\right)} \tag{52}$$

where we have used $E(\eta_{t+1}/\eta_t) = m_{t+1}$. α_0 is of the same sign as $-gfl$, home's gross external position in domestic currency. If the country is short in domestic currency, then α_0 is negative.

Note that in that case, Equation (49) becomes redundant with (48) (we have to use Equation (31)). This means that there is some nominal indeterminacy. This nominal indeterminacy does not come from the future exchange rate, which is exogenously fixed, but from the amount of excess return adjustment that comes from i_t and S_t. In other terms, the optimal nominal money supply H_t is undetermined. For instance, if the supply of money H_t is higher, then the exchange rate S_t will be higher (more depreciated), so that the optimal interest rate i_t will be have to be lower to generate a given excess return.

In the latter case, Equation (49) yields the same equation. Note that in that case, the exchange rate is not undetermined, because $i_t = 0$.

B Proofs - Linear Case

B.1 Equation (38)

We can rewrite the resource constraints (25) as

$$\begin{aligned} C_t &= Y_t \left(1 + \tfrac{nfl_t}{Y_t}\right) \\ C_{t+1} &= Y_{t+1} \left(1 - \tfrac{nfl_t}{Y_t} \tfrac{1+i_t^*}{1+g_{t+1}} - \tfrac{gfl_t}{Y_t} \tfrac{X_{t+1}^*}{1+g_{t+1}}\right) \end{aligned}$$

with $1 + g_{t+1} = Y_{t+1}/Y_t$. We used the fact that, in equilibrium, $(H_t/S_t - h_t^H) i_t S_t/S_{t+1}$ is equal to zero (either $H_t/S_t - h_t^H = 0$ or $i_t = 0$). Taking logs and using a second-order approximation (assuming $\tilde{Y}_{t+1}$, nfl_t/Y_t, gfl_t/Y_t, X_{t+1}^* and g_{t+1} are small), we obtain

$$\begin{aligned} \tilde{C}_t &= \tilde{Y}_t + \tfrac{nfl_t}{Y_t} - \left(\tfrac{nfl_t}{Y_t}\right)^2 \\ \tilde{C}_{t+1} &= \tilde{Y}_{t+1} - \tfrac{nfl_t}{Y_t}(1 + i_t^* - g_{t+1}) + \left(\tfrac{nfl_t}{Y_t}\right)^2 (1 + i_t^*) - \tfrac{gfl_t}{Y_t}(X_{t+1}^* - g_{t+1}) \end{aligned}$$

Finally, we use the approximation $g_{t+1} = \tilde{Y}_{t+1} - \tilde{Y}_t$ along with the assumption that $Y_t = 1$ and hence $\tilde{Y}_t = 0$ to obtain Equation (38)

B.2 Equations (41)

Equation (39) yields

$$\begin{aligned} E_t(e^{\tilde{m}_{t+1}^* + \tilde{i}_t^*}) &= 1 \\ e^{E(\tilde{m}_{t+1}^*) + \frac{1}{2} V(\tilde{m}_{t+1}^*) + \tilde{i}_t^*} &= 1 \\ e^{\log(\beta) + E(\tilde{-y}_{t+1}^*) + \frac{1}{2} V(\tilde{y}_{t+1}^*) + \tilde{i}_t^*} &= 1 \\ e^{\log(\beta) + \tilde{i}_t^*} &= 1 \end{aligned}$$

Similarly, Equation (39) yields

$$
\begin{aligned}
E_t(e^{\tilde{m}^*_{t+1}-\tilde{S}_{t+1}+\tilde{i}_t+\tilde{S}_t}) &= 1+\chi+\Gamma gfl_t \\
e^{E(\tilde{m}^*_{t+1}-\tilde{S}_{t+1})+\frac{1}{2}V(\tilde{m}^*_{t+1}-\tilde{S}_{t+1})+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t \\
e^{\log(\beta)-E((1+\rho)\tilde{y}^*_{t+1})+\frac{1}{2}V((1+\rho)\tilde{y}^*_{t+1})+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t \\
e^{\log(\beta)+\frac{(1+\rho)\rho}{2}\sigma_y^2+\tilde{i}_t+\tilde{S}_t} &= 1+\chi+\Gamma gfl_t
\end{aligned}
$$

This yields (41).

B.3 Optimal foreign exchange interventions

The difference in risk premia can be written as follows

$$
\frac{cov(m^*_{t+1},X^*_{t+1})}{E_t m^*_{t+1}} - \frac{cov(m_{t+1},X^*_{t+1})}{E(m_{t+1})} = \frac{1}{\beta}(1+\chi+\Gamma gfl_t)\left(1-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\right)
$$

We used

$$
\begin{aligned}
cov(m^*_{t+1},X^*_{t+1}) &= cov\left(m^*_{t+1},(1+i_t)\tfrac{S_t}{S_{t+1}}\right) - \underbrace{cov(m^*_{t+1},(1+i^*_t))}_{=0} \\
&= E\left(m^*_{t+1}(1+i_t)\tfrac{S_t}{S_{t+1}}\right) - E\left(m^*_{t+1}\right)E\left((1+i_t)\tfrac{S_t}{S_{t+1}}\right) \\
&= E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right) - E\left(e^{\tilde{m}^*_{t+1}}\right)E\left(e^{\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right) \\
&= \underbrace{E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}_{1+\chi+\Gamma gfl_t}\left[1-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right]
\end{aligned}
$$

where we used (40), and

$$
E(m^*_{t+1}) = \beta
$$

which yields

$$
\frac{cov(m^*_{t+1},X^*_{t+1})}{E_t m^*_{t+1}} = \frac{1}{\beta}(1+\chi+\Gamma gfl_t)\left[1-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right] \tag{53}
$$

Similarly:

$$
\begin{aligned}
&\frac{cov(m_{t+1},X^*_{t+1})}{E(m_{t+1})} = \frac{cov\left(m_{t+1},(1+i_t)\frac{S_t}{S_{t+1}}\right) - \underbrace{cov(m_{t+1},(1+i^*_t))}_{=0}}{E(m_{t+1})} \\
&= \frac{E\left(m_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})} - E\left((1+i_t)\frac{S_t}{S_{t+1}}\right) \\
&= \frac{E\left(e^{\tilde{m}_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}{E\left(e^{\tilde{m}_{t+1}}\right)} - E\left(e^{\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right) \\
&= e^{-\log(\beta)+\tilde{i}_t+\tilde{S}_t-E(\tilde{S}_{t+1})+\frac{V(\tilde{S}_{t+1})}{2}-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\left[1-e^{cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}\right] \\
&= \frac{1}{\beta}e^{\tilde{i}_t+\tilde{S}_t-E(\tilde{S}_{t+1})+\frac{V(\tilde{S}_{t+1})}{2}+E(\tilde{m}^*_{t+1})+\frac{V(\tilde{m}^*_{t+1})}{2}-cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\left[e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right] \\
&= \frac{1}{\beta}\underbrace{E\left(e^{\tilde{m}^*_{t+1}+\tilde{i}_t+\tilde{S}_t-\tilde{S}_{t+1}}\right)}_{1+\chi+\Gamma gfl_t}\left[e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})-cov(\tilde{S}_{t+1},\tilde{m}_{t+1})}-e^{cov(\tilde{S}_{t+1},\tilde{m}^*_{t+1})}\right]
\end{aligned}
$$

where we used $-\log(\beta) = E(\tilde{m}^*_{t+1}) + \frac{V(\tilde{m}^*_{t+1})}{2}$. This yields Equation (42).

Now note that

$$\tilde{m}^*_{t+1} = \log(\beta) - \tilde{Y}^*_{t+1}, \tag{54}$$

and, from (38), we get

$$\tilde{m}_{t+1} = \log(\beta) - \alpha\tilde{Y}_{t+1}\left(1 + nfl_t + gfl_t\right) + (nfl_t - nfl_t^2)(2 + \tilde{i}^*_t) + gfl_t(\tilde{i}_t - \tilde{i}^*_t + \tilde{S}_t - \tilde{S}_{t+1}), \tag{55}$$

using $\tilde{Y}_{t+1} = \alpha\tilde{Y}^*_{t+1}$, $X^*_{t+1} = \tilde{i}_t - \tilde{i}^*_t + \tilde{S}_t - \tilde{S}_{t+1}$ and $\tilde{S}_{t+1} = \rho\tilde{Y}^*_{t+1}$. Therefore, we find Equation (43).

B.4 Proof of Lemma 1

Another way to write Equation (34) is:

$$\begin{aligned}
E\left(m_{t+1}(1+i^*_t)\right) - E\left(m_{t+1}(1+i_t)\frac{S_t}{S_{t+1}}\right) + \frac{\alpha_0\Gamma}{\eta_t} &= 0\\
\underbrace{(1+i^*_t)}_{\frac{1}{E(m^*_{t+1})}} E\left(m_{t+1}\right) - \underbrace{(1+i_t)}_{\frac{1+\chi+\Gamma gfl_t}{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}} E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right) + \frac{\alpha_0\Gamma}{\eta_t} &= 0\\
\frac{1}{E(m^*_{t+1})} - \frac{1+\chi+\Gamma gfl_t}{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})} + \frac{\alpha_0\Gamma}{\eta_t E(m_{t+1})} &= 0\\
1 - (1+\chi+\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} + \frac{\beta\alpha_0\Gamma E(m^*_{t+1})}{\eta_t E(m_{t+1})} &= 0
\end{aligned}$$

Equation (35) yields

$$\begin{aligned}
1 - (1+\chi+\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} - \Gamma gfl_t\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} &= 0\\
1 - (1+\chi+2\Gamma gfl_t)\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} &= 0
\end{aligned}$$

Besides,

$$\frac{\frac{E\left(m_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m_{t+1})}}{\frac{E\left(m^*_{t+1}\frac{S_t}{S_{t+1}}\right)}{E(m^*_{t+1})}} = e^{cov(\tilde{m}^*_{t+1},\tilde{S}_{t+1}) - cov(\tilde{m}_{t+1},\tilde{S}_{t+1})} = e^{-\Delta cov} \tag{56}$$

Hence result (ii) of Lemma 1.

Note that (29) implies that

$$\frac{\Lambda}{\eta_t} = 1 - E[m_{t+1}(1+i^*_t)]$$

$\Lambda = 0$ is equivalent to

$$\begin{aligned}
E[m_{t+1}(1+i^*_t)] &= 1\\
E(m_{t+1}) &= \beta\\
e^{E(\tilde{m}_{t+1}) + \frac{1}{2}V(\tilde{m}_{t+1})} &= \beta
\end{aligned}$$

where we used (41) and where $\tilde{m}_{t+1}$ is given by (55).

We have

$$E(\tilde{m}_{t+1}) = \log(\beta) - (1+nfl+gfl)\frac{\sigma_y^2}{2} + [1-\log(\beta)](nfl_t - nfl_t^2) + \left(-\frac{\rho^2\sigma_y^2}{2} - \rho\sigma_y^2 + \chi + \Gamma gfl_t\right) gfl_t$$

where we used $\log(1+\chi+\Gamma gfl_t) \simeq \chi + \Gamma gfl_t$, and

$$\frac{1}{2}V(\tilde{m}_{t+1}) = \alpha^2(1+nfl+gfl)\frac{\sigma_y^2}{2} + \frac{\rho^2\sigma_y^2}{2} gfl_t$$

Therefore, $\Lambda = 0$ is equivalent to $\tilde{m}(nfl_t, gfl_t) = 0$ with

$$\tilde{m}(nfl_t, gfl_t) = -(1-\alpha^2)(1+nfl+gfl)\frac{\sigma_y^2}{2} + [1-\log(\beta)](nfl_t - nfl_t^2) + \left(-\rho\sigma_y^2 + \chi + \Gamma gfl_t\right) gfl_t \quad (57)$$

$\tilde{m}(nfl_t, gfl_t)$ is increasing in nfl_t if $nfl_t < [-\log(\beta) + \alpha^2)]/2$. We consider only solutions that satisfy this condition. In that case, the solution is unique. Denote by $\overline{nfl}(gfl_t)$ this solution. If $\overline{nfl}(gfl_t) > b^G - h_t^H = b^G - 1$, then $nfl_t = b^G - 1$ and $\Lambda > 0$.

B.5 Solutions for $\widehat{gfl}_t$ and $\widehat{nfl}_t$

For a given nfl_t, gfl_t is implicitly defined by

$$1 - (1+\chi+2\Gamma gfl_t)e^{-\rho\sigma_y^2[1-\alpha(1+nfl_t+gfl_t)-\rho gfl_t]} = 0$$

Using $\log(1+\chi+2\Gamma gfl_t) \simeq \chi + 2\Gamma gfl_t$, this yields

$$\chi + 2\Gamma gfl_t - \rho\sigma_y^2\left[1 - \alpha(1+nfl_t+gfl_t) - \rho gfl_t\right] = 0$$

After rearranging, we obtain (44).

If $\lambda = 0$, (44) and $\tilde{m}(nfl_t, gfl_t) = 0$ jointly define nfl_t and gfl_t. If $\Lambda > 0$, then gfl_t is defined by (44) with $nfl_t = b_t^G - 1$.

Consider the case where $\Lambda = 0$. As before, consider solutions where $nfl_t < [-\log(\beta) + \alpha^2)]/2$ and denote by $\overline{nfl}(gfl_t)$ the unique solution. Suppose additionally that $1 + \overline{nfl}(gfl_t) + gfl_t > 0$. If σ_y^2 is large, and α, χ and Γ are small, then $\tilde{m}(nfl_t, gfl_t) = 0$ implies that $nfl_t - nfl_t^2 > 0$. As long as $nfl_t < 1$, this implies that $nfl_t > 0$.

Special case with $\alpha = 0$ In the special case where $\alpha = 0$, we can compute implicit solutions for nfl_t and gfl_t when $\Lambda = 0$.

First, in that case, (44) implies

$$\widehat{gfl}_t = \frac{\rho\sigma_y^2 - \chi}{2\Gamma + \rho^2\sigma_y^2}$$

and $\widehat{nfl}_t$ is the solution to the second-order polynomial equation $\tilde{m}(nfl_t, gfl_t) = 0$ that is on the increasing segment of the polynomial $\tilde{m}(nfl_t, gfl_t)$:

$$\widehat{nfl}_t = \frac{1}{2}\left[1 - \frac{\sigma_y^2}{2[1-\log(\beta)]} - \sqrt{\left[1 - \frac{\sigma_y^2}{2[1-\log(\beta)]}\right]^2 - 4\frac{\frac{\sigma_y^2}{2}(1+\widehat{gfl}_t) - (-\rho\sigma_y^2 + \chi + \Gamma\widehat{gfl}_t)\widehat{gfl}_t}{1-\log(\beta)}}\right]$$

B.6 Proof of Proposition 3

Note that the CIP deviation, as defined in (9), is increasing in gfl_t (hence (i)), since $E(m^*_{t+1}) = \beta$ is fixed, and $a_t^{H*} = gfl_t$.

Finally, note that the UIP deviation can be written as (we use (10), (53) and $E(m^*_{t+1}) = \beta$ as well):

$$\begin{aligned} E_t X^*_{t+1} &= \tfrac{1}{\beta}\left[\chi + \Gamma gfl_t - (1+\chi+\Gamma gfl_t)(1-e^{-\rho\sigma_y^2})\right] \\ &= -\tfrac{1}{\beta}\left[1-(1+\chi+\Gamma gfl_t)e^{-\rho\sigma_y^2}\right] \end{aligned}$$

where we used the results in B.3. Replacing gfl_t with $\widehat{gfl}_t$ and nfl_t with $b_t^G - 1$, we obtain

$$\begin{aligned} E_t X^*_{t+1} &= -\tfrac{1}{\beta}\left[1-\left(1+\chi+\Gamma\tfrac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2}\right)e^{-\rho\sigma_y^2}\right] \\ &\simeq -\tfrac{1}{\beta}\left[1-e^{\chi+\Gamma\frac{\rho\sigma_y^2[1-\alpha b_t^G]-\chi}{2\Gamma+\rho(\alpha+\rho)\sigma_y^2}-\rho\sigma_y^2}\right] \end{aligned}$$

The derivative of $E_t X^*_{t+1}$ with respect to σ_y^2 is of the same sign as

$$-\rho + \Gamma\frac{2\Gamma\rho(1-\alpha b_t^G)+\chi\rho(\alpha+\rho)}{[2\Gamma+\rho(\alpha+\rho)\sigma_y^2]^2}$$

Therefore, $E_t X^*_{t+1}$ is decreasing in σ_y if Γ is not too large (hence (ii)).

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