

Online Appendix

Dollar Shortages, CIP Deviations, and the Safe Haven Role of the Dollar

Philippe Bacchetta, J. Scott Davis and Eric van Wincoop
November 2023

This Online Appendix has 6 sections. Section A derives the optimal portfolios of borrowers and lenders. Section B derives the spot market equilibrium. Section C shows that the FX market equilibrium (current account=net capital outflows) follows both from the spot market equilibrium and from equating relative money demand to relative money supply. Section D derives the linearized spot and swap market equilibria when there is a portfolio shift from US lenders to Europe to US lenders to the US. Section E considers an extension with UIP Arbitrageurs. Finally, Section F discusses convenience yield shocks.

A Derivation Optimal Portfolios

A.1 Home Agents $j = 1, 2$

Period 2 consumption of Home agents $j = 1, 2$ is

$$P_2 C_{H,j,2} = Y_{H,j,2} + \Pi_{H,2} + M_{H,j,1}^{\$} + S_2 M_{H,j,1}^{\epsilon} + W_{H,j,1} + (S_2(1+i_1^{\epsilon,H}) - S_1) B_{H,j,1}^{\epsilon,H} \quad (\text{A.1})$$

Period 2 consumption is equal to period 2 income $Y_{H,j,2} + \Pi_{H,2}$ plus the period 2 value of period 1 money balances, plus period 1 financial wealth, plus the excess return on period 1 euro asset holdings. Period 2 income is

$$Y_{H,j,2} = a_j e^{\kappa_H \bar{\theta} s_2} \quad (\text{A.2})$$

Using the approximation $i_1^{\epsilon,H} = -i_1^{\$,F}$ and linearization $p_2 = 0.5s_2$, log-linearizing (A.1) around $C_{H,j,2} = \bar{C}_{H,j,2}$ and $s_2 = s_1 = 0$, we have

$$\begin{aligned} \bar{C}_{H,j,2} + \bar{C}_{H,j,2}(c_{H,j,2} - \bar{c}_{H,j,2}) &= a_j + \rho_{H,j} s_2 + \Pi_{H,2} \\ &+ M_{H,j,1}^{\$} + M_{H,j,1}^{\epsilon} + M_{H,j,1}^{\epsilon} s_2 + W_{H,j,1} - B_{H,j,1}^{\epsilon,H} \left(i_1^{\$,F} + s_1 - s_2 \right) \end{aligned}$$

where $\rho_{H,j} = \kappa_H a_j \bar{\theta} - 0.5 \bar{C}_{H,j,2}$.

Agents maximize the mean-variance objective:

$$E c_{H,j,2} - 0.5 \gamma \text{var}(c_{H,j,2}) \quad (\text{A.3})$$

This implies that agents maximize

$$\begin{aligned} & \frac{1}{\bar{C}_{H,j,2}} \left(a_j + \Pi_{H,2} + M_{H,j,1}^{\$} + M_{H,j,1}^{\epsilon} + W_{H,j,1} - B_{H,j,1}^{\epsilon,H} (i_1^{\$,F} + s_1) \right) \\ & - 0.5 \frac{\gamma}{\bar{C}_{H,j,2}^2} \left(\rho_{H,j} + M_{H,j,1}^{\epsilon} + B_{H,j,1}^{\epsilon,H} \right)^2 \text{var}(s_2) \end{aligned}$$

The first-order condition with respect to $B_{H,j,1}^{\epsilon,H}$ gives

$$B_{H,j,1}^{\epsilon,H} = -\rho_{H,j} - M_{H,j,1}^{\epsilon} - \bar{C}_{H,j,2} \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} \quad (\text{A.4})$$

We have

$$B_{H,j,1}^{\$,H} = W_{H,j,1} - S_1 B_{H,j,1}^{\epsilon,H} \quad (\text{A.5})$$

A.2 Home Agent $j = 3$

In this subsection the subscript j always refers to $j = 3$. Period 2 consumption of Home agent $j = 3$ is

$$P_2 C_{H,j,2} = Y_{H,j,2} + \Pi_{H,2} + M_{H,j,1}^{\$} + S_2 M_{H,j,1}^{\epsilon} + (1 + i_1^{\$,F}) W_{H,j,1} + (S_2 - S_1 (1 + i_1^{\$,F})) B_{H,j,1}^{\epsilon,F} \quad (\text{A.6})$$

The last term captures the excess return on euro assets, measured in dollars.

Again use that $Y_{H,j,2} = a_j e^{\kappa_H \bar{\theta} s_2}$. Using the linearization $p_2 = 0.5 s_2$, log-linearizing (A.6) around $C_{H,j,2} = \bar{C}_{H,j,2}$, $i_1^{\$,F} = 0$ and $s_2 = s_1 = 0$, we have

$$\begin{aligned} & \bar{C}_{H,j,2} + \bar{C}_{H,j,2} (c_{H,j,2} - \bar{c}_{H,j,2}) = a_j + \rho_{H,j} s_2 + \Pi_{H,2} \\ & + M_{H,j,1}^{\$} + M_{H,j,1}^{\epsilon} + M_{H,j,1}^{\epsilon} s_2 + (1 + i_1^{\$,F}) W_{H,j,1} - B_{H,j,1}^{\epsilon,F} (i_1^{\$,F} + s_1 - s_2) \end{aligned}$$

where $\rho_{H,j} = \kappa_H a_j \bar{\theta} - 0.5 \bar{C}_{H,j,2}$.

Agents maximize the mean-variance objective $E c_{H,j,2} - 0.5 \gamma \text{var}(c_{H,j,2})$. The objective is therefore

$$\begin{aligned} & \frac{1}{\bar{C}_{H,j,2}} \left(a_j + \Pi_{H,2} + M_{H,j,1}^{\$} + M_{H,j,1}^{\epsilon} + (1 + i_1^{\$,F}) W_{H,j,1} - B_{H,j,1}^{\epsilon,F} (i_1^{\$,F} + s_1) \right) \\ & - 0.5 \frac{\gamma}{\bar{C}_{H,j,2}^2} \left(\rho_{H,j} + M_{H,j,1}^{\epsilon} + B_{H,j,1}^{\epsilon,F} \right)^2 \text{var}(s_2) \end{aligned}$$

The first-order condition with respect to $B_{H,j,1}^{\epsilon,F}$ gives

$$B_{H,j,1}^{\epsilon,F} = -\rho_{H,j} - M_{H,j,1}^{\epsilon} - \bar{C}_{H,j,2} \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} \quad (\text{A.7})$$

We have

$$B_{H,j,1}^{\$,F} = W_{H,j,1} - S_1 B_{H,j,1}^{\epsilon,F} \quad (\text{A.8})$$

A.3 Foreign Agents $j = 1, 2$

From the period 2 budget constraint we have

$$P_2^* C_{F,j,2} = Y_{F,j,2} + \Pi_{F,2} + \frac{1}{S_2} M_{F,j,1}^{\$} + M_{F,j,1}^{\epsilon} + W_{F,j,1} + \left(\frac{1 + i_1^{\$,F}}{S_2} - \frac{1}{S_1} \right) B_{F,j,1}^{\$,F} \quad (\text{A.9})$$

Period 2 consumption is equal to period 2 income $Y_{F,j,2} + \Pi_{F,2}$ plus the period 2 value of period 1 money balances, plus period 1 financial wealth, plus the excess return on period 1 dollar asset holdings. Period 2 income is

$$Y_{F,j,2} = a_j e^{-\kappa_F \bar{\theta} s_2} \quad (\text{A.10})$$

Using that $p_2^* = -0.5s_2$, log-linearizing (A.9) around $C_{F,j,2} = \bar{C}_{F,j,2}$, $s_2 = s_1 = 0$ and $i_1^{\$,F} = 0$, we have

$$\bar{C}_{F,j,2} + \bar{C}_{F,j,2}(c_{F,j,2} - \bar{c}_{F,j,2}) = a_j - \rho_{F,j}s_2 + \Pi_{F,2} + M_{F,j,1}^{\epsilon} + (1 - s_2)M_{F,j,1}^{\$} + W_{F,j,1} + (i_1^{\$,F} - s_2 + s_1)B_{F,j,1}^{\$,F} \quad (\text{A.11})$$

where $\rho_{F,j} = \kappa_F a_j \bar{\theta} - 0.5\bar{C}_{F,j,2}$.

Agents maximize the following mean-variance objective:

$$E c_{F,j,2} - 0.5\gamma \text{var}(c_{F,j,2}) \quad (\text{A.12})$$

This is

$$\frac{1}{\bar{C}_{F,j,2}} \left(a_j + \Pi_{F,2} + M_{F,j,1}^{\epsilon} + M_{F,j,1}^{\$} + W_{F,j,1} + B_{F,j,1}^{\$,F} (i_1^{\$,F} + s_1) \right) - 0.5 \frac{\gamma}{\bar{C}_{F,j,2}^2} \left(\rho_{F,j} + M_{F,j,1}^{\$} + B_{F,j,1}^{\$,F} \right)^2 \text{var}(s_2)$$

The first-order condition with respect to $B_{F,j,1}^{\$,F}$ gives

$$B_{F,j,1}^{\$,F} = -\rho_{F,j} - M_{F,j,1}^{\$} + \bar{C}_{F,j,2} \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} \quad (\text{A.13})$$

We also have

$$B_{F,j,1}^{\epsilon,F} = W_{F,j,1} - \frac{1}{S_1} B_{F,j,1}^{\$,F} \quad (\text{A.14})$$

A.4 Foreign Agents $j = 3$

In this subsection the subscript j always refers to $j = 3$. From the period 2 budget constraint we have

$$P_2^* C_{F,j,2} = Y_{F,j,2} + \Pi_{F,2} + \frac{1}{S_2} M_{F,j,1}^{\$} + M_{F,j,1}^{\epsilon} + (1 + i_1^{\epsilon,H}) W_{F,j,1} + \left(\frac{1}{S_2} - \frac{1 + i_1^{\epsilon,H}}{S_1} \right) B_{F,j,1}^{\$,H} \quad (\text{A.15})$$

The last term is the excess return, in euros, of dollar holdings in the Home country.

Now again use that $Y_{F,j,2} = a_j e^{-\kappa_F \bar{\theta} s_2}$. Also use the approximation $i_1^{\epsilon,H} = -i_1^{\$,F}$ and $p_2^* = -0.5 s_2$. Log-linearizing (A.15) around $C_{F,j,2} = \bar{C}_{F,j,2}$, $s_2 = s_1 = 0$ and $i_1^{\$,F} = 0$, we have

$$\bar{C}_{F,j,2} + \bar{C}_{F,j,2} (c_{F,j,2} - \bar{c}_{F,j,2}) = a_j - \rho_{F,j} s_2 + \Pi_{F,2} + M_{F,j,1}^{\epsilon} + (1 - s_2) M_{F,j,1}^{\$} + (1 + i_1^{\epsilon,H}) W_{F,j,1} + (i_1^{\$,F} - s_2 + s_1) B_{F,j,1}^{\$,H} \quad (\text{A.16})$$

where $\rho_{F,j} = \kappa_F a_j \bar{\theta} - 0.5 \bar{C}_{F,j,2}$.

The objective $E c_{F,j,2} - 0.5 \gamma \text{var}(c_{F,j,2})$ implies that these agents maximize

$$\frac{1}{\bar{C}_{F,j,2}} \left(a_j + \Pi_{F,2} + M_{F,j,1}^{\epsilon} + M_{F,j,1}^{\$} + (1 + i_1^{\epsilon,H}) W_{F,j,1} + B_{F,j,1}^{\$,H} (i_1^{\$,F} + s_1) \right) - 0.5 \frac{\gamma}{\bar{C}_{F,j,2}^2} \left(\rho_{F,j} + M_{F,j,1}^{\$} + B_{F,j,1}^{\$,H} \right)^2 \text{var}(s_2)$$

The first-order condition with respect to $B_{F,j,1}^{\$,H}$ gives

$$B_{F,j,1}^{\$,H} = -\rho_{F,j} - M_{F,j,1}^{\$} + \bar{C}_{F,j,2} \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} \quad (\text{A.17})$$

We also have

$$B_{F,j,1}^{\epsilon,H} = W_{F,j,1} - \frac{1}{S_1} B_{F,j,1}^{\$,H} \quad (\text{A.18})$$

B Spot Market Equilibrium

In order to derive the spot market equilibrium, we need to track changes in foreign currency positions of all borrowers and lenders. After discussing each set of agents individually, we derive the spot market equilibrium.

B.1 European Borrowers ($j = 1$)

For the discussion in this subsection we set $j = 1$ as we consider European borrowers. They enter period 1 with $M_{F,j,0}^\$$ dollar balances, while at the end of period 1 they have $M_{F,j,1}^\$$ dollar balances. During period 1 these dollar balances change due to dollar invoiced income, dollar invoiced consumption, new dollar debt incurred in period 1, payments on period 0 dollar debt and spot market purchases of dollars in period 1. All dollar borrowing will be treated the same, whether direct or synthetic as it leads to the same amount of dollars going in and out.

With regard to income, all European agents receive the same dollar income from exports to the US that is invoiced in dollars. This is equal to

$$Y_{F,1}^\$ = \sum_{j=1}^3 \alpha_j C_{HF,j,1}^\$ \quad (\text{B.1})$$

Their spending in dollars on goods imported from the US that are invoiced in dollars is $C_{FH,j,1}^\$$. European agents (borrowers and lenders) receive half of the dollar profits of CIP arbitrageurs, so $0.5\Pi_1$. These add to their dollar balances. Finally, $Q_{F,j,1}^{\$,spot}$ denotes dollars that are purchased on the spot market in period 1 in exchange for euros.

We then have

$$M_{F,j,1}^\$ = M_{F,j,0}^\$ + 0.5\Pi_1 + Y_{F,1}^\$ - C_{FH,j,1}^\$ - B_{F,j,1}^{\$,F} + (1 + i_0^{\$,F})B_{F,j,0}^{\$,F} + Q_{F,j,1}^{\$,spot} \quad (\text{B.2})$$

The explanation is as follows. Foreign borrowers start with $M_{F,j,0}^\$$ dollar money balances from period 0. Then they receive the dollar profits of CIP arbitrageurs and dollar income from exports. They need to make a dollar payment of $C_{FH,j,1}^\$$ for buying US goods invoiced in dollars. They receive $-B_{F,j,1}^{\$,F}$ from dollar borrowing in period 1 and need to pay $-(1 + i_0^{\$,F})B_{F,j,0}^{\$,F}$ on period 0 dollar borrowing (principal plus interest). Finally, $Q_{F,j,1}^{\$,spot}$ are dollars are purchased in the spot market in period 1.

It follows that the spot market purchases by European borrowers are ($dX = X_1 - X_0$)

$$Q_{F,j,1}^{\$,spot} = dM_{F,j,1}^\$ - 0.5\Pi_1 - Y_{F,1}^\$ + C_{FH,j,1}^\$ + B_{F,j,1}^{\$,F} - (1 + i_0^{\$,F})B_{F,j,0}^{\$,F} \quad (\text{B.3})$$

we can also write this as

$$Q_{F,j,1}^{\$,spot} = dM_{F,j,1}^\$ - 0.5\Pi_1 - Y_{F,1}^\$ + C_{FH,j,1}^\$ + dB_{F,j,1}^{\$,F} - i_0^{\$,F} B_{F,j,0}^{\$,F} \quad (\text{B.4})$$

B.2 European Lenders

First consider European domestic lenders ($j = 2$). Following the same reasoning as for borrowers, we have

$$M_{F,j,1}^{\$} = M_{F,j,0}^{\$} + 0.5\Pi_1 + Y_{F,1}^{\$} - C_{FH,j,1}^{\$} - B_{F,j,1}^{\$,F} + (1 + i_0^{\$,F})B_{F,l,0}^{\$,F} + Q_{F,j,1}^{\$,spot} \quad (\text{B.5})$$

It follows that the purchases of dollars in the spot market by European domestic lenders are

$$Q_{F,j,1}^{\$,spot} = dM_{F,j,1}^{\$} - 0.5\Pi_1 - Y_{F,1}^{\$} + C_{FH,j,1}^{\$} + dB_{F,j,1}^{\$,F} - i_0^{\$,F}B_{F,j,0}^{\$,F} \quad (\text{B.6})$$

Similarly, for European foreign lenders ($j = 3$), who receive no interest on their dollar investments from period 0, we have

$$Q_{F,j,1}^{\$,spot} = dM_{F,j,1}^{\$} - 0.5\Pi_1 - Y_{F,1}^{\$} + C_{FH,j,1}^{\$} + dB_{F,j,1}^{\$,H} \quad (\text{B.7})$$

B.3 US Borrowers

For all US agents we need to keep track of their euro balances. They receive euro income for exports to Europe that are invoiced in euros equal to

$$Y_{H,1}^{\epsilon} = \sum_{j=1}^3 \alpha_j C_{FH,j,1}^{\epsilon} \quad (\text{B.8})$$

We denote purchases of euros in exchange for dollars by US agents as $Q_{H,j,1}^{\epsilon,spot}$.

For US borrowers ($j = 1$) we have

$$M_{H,j,1}^{\epsilon} = M_{H,j,0}^{\epsilon} + Y_{H,1}^{\epsilon} - C_{HF,j,1}^{\epsilon} - B_{H,j,1}^{\epsilon,H} + (1 + i_0^{\epsilon,H})B_{H,j,0}^{\epsilon,H} + Q_{H,j,1}^{\epsilon,spot} \quad (\text{B.9})$$

Therefore

$$Q_{H,j,1}^{\epsilon,spot} = dM_{H,j,1}^{\epsilon} - Y_{H,1}^{\epsilon} + C_{HF,j,1}^{\epsilon} + dB_{H,j,1}^{\epsilon,H} - i_0^{\epsilon,H}B_{H,j,0}^{\epsilon,H} \quad (\text{B.10})$$

B.4 US Lenders

For US domestic lenders ($j = 2$) we have

$$M_{H,j,1}^{\epsilon} = M_{H,j,0}^{\epsilon} + Y_{H,1}^{\epsilon} - C_{HF,j,1}^{\epsilon} - B_{H,j,1}^{\epsilon,H} + (1 + i_0^{\epsilon,H})B_{H,j,0}^{\epsilon,H} + Q_{H,j,1}^{\epsilon,spot} \quad (\text{B.11})$$

Therefore

$$Q_{H,j,1}^{\epsilon,spot} = dM_{H,j,1}^{\epsilon} - Y_{H,1}^{\epsilon} + C_{HF,j,1}^{\epsilon} + dB_{H,j,1}^{\epsilon,H} - i_0^{\epsilon,H} B_{H,j,0}^{\epsilon,H} \quad (\text{B.12})$$

Similarly, for US foreign lenders ($j = 3$), who receive no euro interest on euro holdings in Europe, we have

$$Q_{H,j,1}^{\epsilon,spot} = dM_{H,j,1}^{\epsilon} - Y_{H,1}^{\epsilon} + C_{HF,j,1}^{\epsilon} + dB_{H,j,1}^{\epsilon,F} \quad (\text{B.13})$$

B.5 Equilibrium

Spot market equilibrium is

$$\sum_{j=1}^3 \alpha_j Q_{F,j,1}^{\$,spot} + (1 - (F_1/S_1)) (D_{F,1}^{\$,syn} - D_{CIP,1}^{\$,H}) = S_1 \sum_{j=1}^3 \alpha_j Q_{H,j,1}^{\epsilon,spot} \quad (\text{B.14})$$

The second term on the left hand side relates to small spot market transactions associated with synthetic dollar borrowing and lending, discussed in Appendix A of the paper. From the swap market equilibrium, we have

$$D_{F,1}^{\$,syn} - D_{CIP,1}^{\$,H} = \frac{S_1}{F_1} S_1 D_{H,1}^{\epsilon,syn} \quad (\text{B.15})$$

so that

$$(1 - (F_1/S_1)) (D_{F,1}^{\$,syn} - D_{CIP,1}^{\$,H}) = \left(\frac{S_1}{F_1} - 1 \right) S_1 D_{H,1}^{\epsilon,syn} = i_1^{\epsilon,H} S_1 D_{H,1}^{\epsilon,syn} \quad (\text{B.16})$$

Substituting the expressions for $Q_{F,j,1}^{\$,spot}$ and $Q_{H,j,1}^{\epsilon,spot}$ of Home and Foreign agents into the spot market equilibrium, we have

$$\begin{aligned} & \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$} - 0.5\Pi_1 - Y_{F,1}^{\$} + \sum_{j=1}^3 \alpha_j C_{FH,j,1}^{\$} + \sum_{j=1}^2 \alpha_j dB_{F,j,1}^{\$,F} \\ & + \alpha_3 dB_{F,3,1}^{\$,H} - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + i_1^{\epsilon,H} S_1 D_{H,1}^{\epsilon,syn} = \\ & S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon} - S_1 Y_{H,1}^{\epsilon} + S_1 \sum_{j=1}^3 \alpha_j C_{HF,j,1}^{\epsilon} \\ & + S_1 \sum_{j=1}^2 \alpha_j dB_{H,j,1}^{\epsilon,H} + \alpha_3 S_1 dB_{H,3,1}^{\epsilon,F} - S_1 i_0^{\epsilon,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \end{aligned}$$

The US trade account is equal to

$$TA_{H,1}^{\$} = Y_{H,1} - \sum_{j=1}^3 \alpha_j P_1 C_{H,j,1} \quad (\text{B.17})$$

where

$$Y_{H,1} = \sum_{j=1}^3 \alpha_j (C_{HH,j,1} + C_{FH,j,1}^{\$} + S_1 C_{FH,j,1}^{\epsilon}) \quad (\text{B.18})$$

$$P_1 C_{H,j,1} = C_{HH,j,1} + C_{HF,j,1}^{\$} + S_1 C_{HF,j,1}^{\epsilon} \quad (\text{B.19})$$

Therefore

$$TA_{H,1}^{\$} = \sum_{j=1}^3 \alpha_j (C_{FH,j,1}^{\$} + S_1 C_{FH,j,1}^{\epsilon} - C_{HF,j,1}^{\$} - S_1 C_{HF,j,1}^{\epsilon}) \quad (\text{B.20})$$

It follows that

$$\begin{aligned} -Y_{F,1}^{\$} + \sum_{j=1}^3 \alpha_j C_{FH,j,1}^{\$} + S_1 Y_{H,1}^{\epsilon} - S_1 \sum_{j=1}^3 \alpha_j C_{HF,j,1}^{\epsilon} = \\ - \sum_{j=1}^3 \alpha_j C_{HF,j,1}^{\$} + \sum_{j=1}^3 \alpha_j C_{FH,j,1}^{\$} + S_1 \sum_{j=1}^3 \alpha_j C_{FH,j,1}^{\epsilon} - S_1 \sum_{j=1}^3 \alpha_j C_{HF,j,1}^{\epsilon} = TA_{H,1}^{\$} \end{aligned} \quad (\text{B.21})$$

We can then write the spot market equilibrium as

$$\begin{aligned} \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$} - S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon} + TA_{H,1}^{\$} - 0.5\Pi_1 + i_1^{\epsilon,H} S_1 D_{H,1}^{\epsilon,syn} \\ - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + S_1 i_0^{\epsilon,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \\ + \sum_{j=1}^2 \alpha_j dB_{F,j,1}^{\$,F} + \alpha_3 dB_{F,3,1}^{\$,H} - S_1 \sum_{j=1}^2 \alpha_j dB_{H,j,1}^{\epsilon,H} - \alpha_3 S_1 dB_{H,3,1}^{\epsilon,F} = 0 \end{aligned} \quad (\text{B.22})$$

C Foreign Exchange Market Equilibrium

In this section we first show that by combining the spot and swap market equilibrium we obtain the FX market equilibrium, whereby the current account is equal to net capital outflows. After that we show that the same FX market equilibrium can be derived by setting the relative money supply equal to relative money demand.

C.1 FX Market Equilibrium Derived from Spot and Swap Market Equilibrium

Consider the term

$$\frac{S_1}{F_1} \left(Q_{F,1}^{\$,swap} + Q_{H,1}^{\$,swap} + Q_{CIP,1}^{\$,swap} \right) - \frac{S_0}{F_0} \left(Q_{F,0}^{\$,swap} + Q_{H,0}^{\$,swap} + Q_{CIP,0}^{\$,swap} \right) \quad (C.1)$$

where $Q_{F,t}^{\$,swap} = \sum_{j=1}^3 \alpha_j Q_{F,j,t}^{\$,swap}$ and $Q_{H,t}^{\$,swap} = \sum_{j=1}^3 \alpha_j Q_{H,j,t}^{\$,swap}$. This term is clearly zero as swap market equilibrium in both periods implies that the terms in brackets add to zero. Using Appendix A of the paper, we can write (C.1) as

$$D_{F,1}^{\$,syn} - \frac{S_1}{F_1} S_1 D_{H,1}^{\$,syn} - D_{CIP,1}^{\$,H} - D_{F,0}^{\$,syn} + \frac{S_0}{F_0} S_0 D_{H,0}^{\$,syn} + D_{CIP,0}^{\$,H} \quad (C.2)$$

Write this as

$$\begin{aligned} & -\alpha_1 dB_{F,1,1}^{\$,F} - \alpha_2 dB_{F,2,1}^{\$,F} - \alpha_3 dB_{H,3,1}^{\$,F} - dD_{CIP,1}^{\$,H} \\ & - (1 + i_1^{\$,H}) S_1 D_{H,1}^{\$,syn} + (1 + i_0^{\$,H}) S_0 D_{H,0}^{\$,syn} \end{aligned}$$

or

$$\begin{aligned} & -\alpha_1 dB_{F,1,1}^{\$,F} - \alpha_2 dB_{F,2,1}^{\$,F} - \alpha_3 dB_{H,3,1}^{\$,F} - dD_{CIP,1}^{\$,H} \\ & - i_1^{\$,H} S_1 D_{H,1}^{\$,syn} + i_0^{\$,H} S_0 D_{H,0}^{\$,syn} \\ & + S_1 \alpha_1 B_{H,1,1}^{\$,H} + S_1 \alpha_2 B_{H,2,1}^{\$,H} + S_1 \alpha_3 B_{F,3,1}^{\$,H} \\ & - S_0 \alpha_1 B_{H,1,0}^{\$,H} - S_0 \alpha_2 B_{H,2,0}^{\$,H} - S_0 \alpha_3 B_{F,3,0}^{\$,H} \end{aligned}$$

Since this term is equal to zero, we can add it to the spot market equilibrium (B.22):

$$\begin{aligned} & \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$,} - S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\$,} + TA_{H,1}^{\$,} - 0.5\Pi_1 \\ & - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + S_1 i_0^{\$,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\$,H} \\ & + \alpha_3 dB_{F,3,1}^{\$,H} - \alpha_3 S_1 dB_{H,3,1}^{\$,F} - \alpha_3 dB_{H,3,1}^{\$,F} - dD_{CIP,1}^{\$,H} \\ & + i_0^{\$,H} S_0 D_{H,0}^{\$,syn} + S_1 \alpha_3 dB_{F,3,1}^{\$,H} \\ & + (S_1 - S_0) \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\$,H} + (S_1 - S_0) \alpha_3 B_{F,3,0}^{\$,H} = 0 \end{aligned}$$

Re-ordering the terms, we can write this as

$$\begin{aligned}
& - \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon} \\
& - \alpha_3 dB_{F,3,1}^{\$,H} + \alpha_3 S_1 dB_{H,3,1}^{\epsilon,F} + \alpha_3 dB_{H,3,1}^{\$,F} - S_1 \alpha_3 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H} = \\
& + TA_{H,1}^{\$} - 0.5\Pi_1 - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + S_1 i_0^{\epsilon,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\epsilon,H} S_0 D_{H,0}^{\epsilon,syn} \\
& + (S_1 - S_0) \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + (S_1 - S_0) \alpha_3 B_{F,3,0}^{\epsilon,H} \tag{C.3}
\end{aligned}$$

We can write the right hand side as

$$\begin{aligned}
& TA_{H,1}^{\$} - 0.5\Pi_1 + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} + S_1 i_0^{\epsilon,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \\
& + i_0^{\$,F} D_{F,0}^{\$,syn} + i_0^{\epsilon,H} S_0 D_{H,0}^{\epsilon,syn} - (S_1 - S_0) D_{H,0}^{\epsilon,syn} \tag{C.4}
\end{aligned}$$

Consider the period 0 swap market equilibrium, which can be written as

$$(1 + i_0^{\$,F}) D_{F,0}^{\$,syn} - S_0 D_{H,0}^{\epsilon,syn} = (1 + i_0^{\$,F}) D_{CIP,0}^{\$,H} \tag{C.5}$$

Multiply by $i_0^{\$,F}/(1 + i_0^{\$,F})$ and note that this is equal to $-i_0^{\epsilon,H}$. We then have

$$i_0^{\$,F} D_{F,0}^{\$,syn} + S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} = i_0^{\$,F} D_{CIP,0}^{\$,H} = \Pi_1 \tag{C.6}$$

Substituting this back into (C.4), we have

$$\begin{aligned}
& TA_{H,1}^{\$} + 0.5\Pi_1 + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} + S_1 i_0^{\epsilon,H} \sum_{j=0}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \\
& - (S_1 - S_0) D_{H,0}^{\epsilon,syn}
\end{aligned}$$

Using that $D_{H,0}^{\epsilon,syn} = - \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} - \alpha_3 B_{F,3,0}^{\epsilon,H}$, we can write the last equation as

$$\begin{aligned}
& TA_{H,1}^{\$} + 0.5\Pi_1 + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - \alpha_3 i_0^{\epsilon,H} S_1 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \\
& - (S_1 - S_0) (1 + i_0^{\epsilon,H}) D_{H,0}^{\epsilon,syn}
\end{aligned}$$

Replace the right hand side of (C.3) with this expression. This gives

$$\begin{aligned}
& -\sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon} + (S_1 - S_0)(1 + i_0^{\epsilon,H}) D_{H,0}^{\epsilon,syn} \\
& -\alpha_3 dB_{F,3,1}^{\$,H} + \alpha_3 S_1 dB_{H,3,1}^{\epsilon,F} + \alpha_3 dB_{H,3,1}^{\$,F} - S_1 \alpha_3 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H} = \\
& TA_{H,1}^{\$} + 0.5\Pi_1 + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - \alpha_3 i_0^{\epsilon,H} S_1 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.7)
\end{aligned}$$

We can finally rewrite this as

$$\begin{aligned}
& -\sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon} + (S_1 - S_0) D_{H,0}^{\epsilon,syn} \\
& -\alpha_3 dB_{F,3,1}^{\$,H} + \alpha_3 S_1 dB_{H,3,1}^{\epsilon,F} + \alpha_3 dB_{H,3,1}^{\$,F} - S_1 \alpha_3 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H} = \\
& TA_{H,1}^{\$} + 0.5\Pi_1 + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - \alpha_3 i_0^{\epsilon,H} S_1 B_{F,3,0}^{\epsilon,H} - S_1 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.8)
\end{aligned}$$

This says that US net capital outflows (left hand side) is equal to the US current account (right hand side), which is the standard FX market equilibrium condition. To see this, first ignore the terms involving $D_{H,0}^{\epsilon,syn}$ on both sides of the equation. The terms on the first line are associated with US capital outflows minus inflows involving money balances. These are purchases of euro money balances by the US agents minus purchases of dollar money balances by European agents. The terms on the second line involve US capital outflows minus inflows of the other assets. The first four terms measure purchases by US lenders of dollar and euro assets in Europe (capital outflows) minus purchases by European lenders of dollar and euro assets in the US (capital inflows). The last term on the second line measures the activity by CIP arbitrageurs, who borrow $dD_{CIP,1}^{\$,H}$ dollars in the US and lend it to Europe, also a net capital outflow.

The last line is equal to the current account, which is the trade account plus net US investment income abroad. The latter first includes the US share of the profits by CIP arbitrageurs. It also includes the interest on dollar assets in Europe held by US lenders and subtracts the interests on euro assets in the US held by European lenders.

Finally consider the two terms involving $D_{H,0}^{\epsilon,syn}$. This is net synthetic euro borrowing by the US. It requires a swap transaction, selling $S_0 D_{H,0}^{\epsilon,syn}$ dollar swaps at time 0. At time 1 this swap involves receiving $S_0 D_{H,0}^{\epsilon,syn}$ dollars in exchange for $(S_0/F_0) D_{H,0}^{\epsilon,syn}$ euros. In terms of dollars, the time 1 market value of this exchange

is

$$\left(S_0 - S_1 \frac{S_0}{F_0}\right) D_{H,0}^{\epsilon, syn} \quad (C.9)$$

We can also write this as

$$\left(S_0 - S_1(1 + i_0^{\epsilon, H})\right) D_{H,0}^{\epsilon, syn} = -S_1 i_0^{\epsilon, H} D_{H,0}^{\epsilon, syn} - (S_1 - S_0) D_{H,0}^{\epsilon, syn} \quad (C.10)$$

Following the BPM6 IMF Balance of Payments manual, the interest term $-S_1 i_0^{\epsilon, H} D_{H,0}^{\epsilon, syn}$ is included in the current account, while the capital gains term $(S_1 - S_0) D_{H,0}^{\epsilon, syn}$ is included in net capital flows. These are respectively the last term of the last line of (C.8) and the last term of the first line of (C.8).

C.2 FX Market Equilibrium Derived from Relative Money Market Equilibrium

We now show that the foreign exchange market equilibrium can also be derived from relative money market equilibrium. The two money market equilibrium equations are

$$M_1^{\$} = \sum_{j=1}^3 \alpha_j M_{H,j,1}^{\$} + \sum_{j=1}^3 \alpha_j M_{F,j,1}^{\$} \quad (C.11)$$

$$M_1^{\epsilon} = \sum_{j=1}^3 \alpha_j M_{H,j,1}^{\epsilon} + \sum_{j=1}^3 \alpha_j M_{F,j,1}^{\epsilon} \quad (C.12)$$

From the central bank balance sheets and bonds market clearing we have

$$M_1^{\$} = B_{CB,1}^{\$, H} \quad (C.13)$$

$$M_1^{\epsilon} = B_{CB,1}^{\epsilon, F} \quad (C.14)$$

We next need to impose onshore dollar and euro bond market equilibrium. Some of the demand for onshore bonds is associated with synthetic assets. For example, net synthetic borrowing of dollars of $D_{F,1}^{\$, syn}$ in Europe implies first borrowing $D_{F,1}^{\$, syn} / S_1$ euros. Similarly, net synthetic euro borrowing of $D_{H,1}^{\epsilon, syn}$ in the US implies first borrowing $S_1 D_{H,1}^{\epsilon, syn}$ dollars.

Using the expressions in the text for synthetic dollar and euro borrowing, the onshore US dollar bond market equilibrium and onshore European euro bond mar-

ket equilibrium are then

$$B_{CB,1}^{\$,H} + \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\$,H} + \alpha_3 B_{F,3,1}^{\$,H} = S_1 D_{H,1}^{\epsilon, syn} + D_{CIP,1}^{\$,H} \quad (C.15)$$

$$B_{CB,1}^{\epsilon, F} + \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\epsilon, F} + \alpha_3 B_{H,3,1}^{\epsilon, F} + \frac{1}{S_1} D_{CIP,1}^{\$,H} = \frac{1}{S_1} D_{F,1}^{\$, syn} \quad (C.16)$$

From this we can write the central bank bond holdings as a function of all the other bond holdings. The money supplies are then

$$M_1^{\$} = - \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\$,H} - \alpha_3 B_{F,3,1}^{\$,H} + S_1 D_{H,1}^{\epsilon, syn} + D_{CIP,1}^{\$,H} \quad (C.17)$$

$$M_1^{\epsilon} = - \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\epsilon, F} - \alpha_3 B_{H,3,1}^{\epsilon, F} - \frac{1}{S_1} D_{CIP,1}^{\$,H} + \frac{1}{S_1} D_{F,1}^{\$, syn} \quad (C.18)$$

Total financial wealth of both countries at the start of period 1 is (in domestic currency)

$$\begin{aligned} \tilde{W}_{H,0} &= \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\$,H} + S_1 \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon, H} + \alpha_3 B_{H,3,0}^{\$,F} + S_1 \alpha_3 B_{H,3,0}^{\epsilon, F} + \\ &+ S_1 i_0^{\epsilon, H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon, H} + i_0^{\$, F} \alpha_3 B_{H,3,0}^{\$,F} \end{aligned} \quad (C.19)$$

$$\begin{aligned} \tilde{W}_{F,0} &= \frac{1}{S_1} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\epsilon, F} + \frac{1}{S_1} \alpha_3 B_{F,3,0}^{\$,H} + \alpha_3 B_{F,3,0}^{\epsilon, H} \\ &+ \frac{1}{S_1} i_0^{\$, F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} + i_0^{\epsilon, H} \alpha_3 B_{F,3,0}^{\epsilon, H} \end{aligned} \quad (C.20)$$

Adding non-asset income to the financial wealth at the start of period 1, and

subtracting consumption and changes in money balances, we can write

$$\begin{aligned} \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\$,H} &= -S_1 \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\epsilon,H} - \alpha_3 B_{H,3,1}^{\$,F} - S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} + \tilde{W}_{H,0} + Y_{H,1} + \Pi_{H,1} \\ &- P_1 \sum_{j=1}^3 \alpha_j C_{H,j,1} + \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\$} - M_{H,j,1}^{\$}) + S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \end{aligned} \quad (C.21)$$

$$\begin{aligned} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\epsilon,F} &= -\frac{1}{S_1} \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\$,F} - \frac{1}{S_1} \alpha_3 B_{F,3,1}^{\$,H} - \alpha_3 B_{F,3,1}^{\epsilon,H} + \tilde{W}_{F,0} + \Pi_{F,1} + Y_{F,1} \\ &- P_1^* \sum_{j=1}^3 \alpha_j C_{F,j,1} + \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\epsilon} - M_{F,j,1}^{\epsilon}) + \frac{1}{S_1} \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) \end{aligned} \quad (C.22)$$

This gives

$$\begin{aligned} M_1^{\$} &= S_1 \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\epsilon,H} + \alpha_3 B_{H,3,1}^{\$,F} + S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - \tilde{W}_{H,0} - Y_{H,1} - \Pi_{H,1} \\ &+ P_1 \sum_{j=1}^3 \alpha_j C_{H,j,1} - \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\$} - M_{H,j,1}^{\$}) - S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \\ &- \alpha_3 B_{F,3,1}^{\$,H} + S_1 D_{H,1}^{\epsilon,syn} + D_{CIP,1}^{\$,H} \\ M_1^{\epsilon} &= \frac{1}{S_1} \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\$,F} + \frac{1}{S_1} \alpha_3 B_{F,3,1}^{\$,H} + \alpha_3 B_{F,3,1}^{\epsilon,H} - \tilde{W}_{F,0} - \Pi_{F,1} - Y_{F,1} \\ &+ P_1^* \sum_{j=1}^3 \alpha_j C_{F,j,1} - \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\epsilon} - M_{F,j,1}^{\epsilon}) - \frac{1}{S_1} \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) \\ &- \alpha_3 B_{H,3,1}^{\epsilon,F} - \frac{1}{S_1} D_{CIP,1}^{\$,H} + \frac{1}{S_1} D_{F,1}^{\$,syn} \end{aligned}$$

Setting these money supplies equal to money demand and using that the trade US trade account is $TA_{H,1}^{\$} = Y_{H,1} - P_1 \sum_{j=1}^3 \alpha_j C_{H,j,1}$, and the euro trade account

in Europe is $TA_{F,1}^{\epsilon} = Y_{F,1} - P_1^* \sum_{j=1}^3 \alpha_j C_{F,j,1}$, we have

$$\begin{aligned}
M_0^{\$} &= S_1 \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\epsilon,H} + \alpha_3 B_{H,3,1}^{\$,F} + S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - \tilde{W}_{H,0} - TA_{H,1}^{\$} - \Pi_{H,1} \\
&\quad + \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) - S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \\
&\quad - \alpha_3 B_{F,3,1}^{\$,H} + S_1 D_{H,1}^{\epsilon,syn} + D_{CIP,1}^{\$,H} \\
M_0^{\epsilon} &= \frac{1}{S_1} \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\$,F} + \frac{1}{S_1} \alpha_3 B_{F,3,1}^{\$,H} + \alpha_3 B_{F,3,1}^{\epsilon,H} - \tilde{W}_{F,0} - \Pi_{F,1} - TA_{F,1}^{\epsilon} \\
&\quad + \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) - \frac{1}{S_1} \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) \\
&\quad - \alpha_3 B_{H,3,1}^{\epsilon,F} - \frac{1}{S_1} D_{CIP,1}^{\$,H} + \frac{1}{S_1} D_{F,1}^{\$,syn}
\end{aligned}$$

Multiplying the second equation with S_1 and subtracting from the first, using that $S_1 TA_{F,1}^{\epsilon} = -TA_{H,1}^{\$}$ and $\Pi_{H,1} = S_1 \Pi_{F,1}$, we have

$$\begin{aligned}
M_0^{\$} - S_1 M_0^{\epsilon} &= S_1 \sum_{j=1}^2 \alpha_j B_{H,j,1}^{\epsilon,H} + \alpha_3 B_{H,3,1}^{\$,F} + S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - \tilde{W}_{H,0} - 2TA_{H,1}^{\$} \\
&\quad + 2 \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) - 2S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \\
&\quad - \alpha_3 B_{F,3,1}^{\$,H} + S_1 D_{H,1}^{\epsilon,syn} + 2D_{CIP,1}^{\$,H} \\
&\quad - \sum_{j=1}^2 \alpha_j B_{F,j,1}^{\$,F} - \alpha_3 B_{F,3,1}^{\$,H} - S_1 \alpha_3 B_{F,3,1}^{\epsilon,H} + S_1 \tilde{W}_{F,0} \\
&\quad + S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - D_{F,1}^{\$,syn}
\end{aligned}$$

Now substitute the expressions for synthetic borrowing: $D_{F,1}^{\$,syn} = -\alpha_1 B_{F,1,1}^{\$,F} - \alpha_2 B_{F,2,1}^{\$,F} - \alpha_3 B_{H,3,1}^{\$,F}$ and $D_{H,1}^{\epsilon,syn} = -\alpha_1 B_{H,1,1}^{\epsilon,H} - \alpha_2 B_{H,2,1}^{\epsilon,H} - \alpha_3 B_{F,3,1}^{\epsilon,H}$. Collecting terms, this gives

$$\begin{aligned}
&M_0^{\$} - S_1 M_0^{\epsilon} + \tilde{W}_{H,0} - S_1 \tilde{W}_{F,0} = \\
&2\alpha_3 B_{H,3,1}^{\$,F} + 2S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - 2TA_{H,1}^{\$} \\
&+ 2 \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) - 2S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \\
&- 2\alpha_3 B_{F,3,1}^{\$,H} - 2\alpha_3 S_1 B_{F,3,1}^{\epsilon,H} + 2D_{CIP,1}^{\$,H} \tag{C.23}
\end{aligned}$$

Using the expressions for $\tilde{W}_{H,0}$ and $\tilde{W}_{F,0}$, and using that $M_0^{\$} = B_{CB,0}^{\$,H}$ and $M_0^{\epsilon} = B_{CB,0}^{\epsilon,F}$, we can write out the first line of (C.23) as

$$\begin{aligned}
& B_{CB,0}^{\$,H} - S_1 B_{CB,0}^{\epsilon,F} + \tag{C.24} \\
& \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\$,H} + S_1 \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + \alpha_3 B_{H,3,0}^{\$,F} + S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} \\
& + S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} \\
& - \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\epsilon,F} - \alpha_3 B_{F,3,0}^{\$,H} - S_1 \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H}
\end{aligned}$$

From the period 0 version of the onshore bond market equilibria (C.15)-(C.16), we have

$$\sum_{j=1}^2 \alpha_j B_{H,j,0}^{\$,H} = -B_{CB,0}^{\$,H} - \alpha_3 B_{F,3,0}^{\$,H} + S_0 D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} \tag{C.25}$$

$$\sum_{j=1}^2 \alpha_j B_{F,j,0}^{\epsilon,F} = -B_{CB,0}^{\epsilon,F} - \alpha_3 B_{H,3,0}^{\epsilon,F} - \frac{1}{S_0} D_{CIP,0}^{\$,H} + \frac{1}{S_0} D_{F,0}^{\$,syn} \tag{C.26}$$

Substitute these into (C.24), which then becomes

$$\begin{aligned}
& B_{CB,0}^{\$,H} - S_1 B_{CB,0}^{\epsilon,F} \tag{C.27} \\
& - B_{CB,0}^{\$,H} - \alpha_3 B_{F,3,0}^{\$,H} + S_0 D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} + S_1 \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + \alpha_3 B_{H,3,0}^{\$,F} + S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} \\
& + S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} \\
& + S_1 B_{CB,0}^{\epsilon,F} + S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} + \frac{S_1}{S_0} D_{CIP,0}^{\$,H} - \frac{S_1}{S_0} D_{F,0}^{\$,syn} - \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - \alpha_3 B_{F,3,0}^{\$,H} - S_1 \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H}
\end{aligned}$$

Collecting terms, this is

$$\begin{aligned}
& -2\alpha_3 B_{F,3,0}^{\$,H} + S_0 D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} + S_1 \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + \alpha_3 B_{H,3,0}^{\$,F} + 2S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} \\
& + S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} \\
& + \frac{S_1}{S_0} D_{CIP,0}^{\$,H} - \frac{S_1}{S_0} D_{F,0}^{\$,syn} - \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} \tag{C.28}
\end{aligned}$$

Using again the definitions for synthetic borrowing, this can be written as

$$\begin{aligned}
& -2\alpha_3 B_{F,3,0}^{\$,H} + (S_0 - S_1) D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} + 2S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} \tag{C.29} \\
& + \frac{S_1}{S_0} D_{CIP,0}^{\$,H} + \left(1 - \frac{S_1}{S_0}\right) D_{F,0}^{\$,syn} + 2\alpha_3 B_{H,3,0}^{\$,F} - 2S_1 \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& + S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H}
\end{aligned}$$

We have now shown that the first line in (C.23) is equal to (C.29). Making this replacement, (C.23) becomes

$$\begin{aligned}
& -2\alpha_3 B_{F,3,0}^{\$,H} + (S_0 - S_1) D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} + 2S_1 \alpha_3 B_{H,3,0}^{\epsilon,F} \\
& + \frac{S_1}{S_0} D_{CIP,0}^{\$,H} + \left(1 - \frac{S_1}{S_0}\right) D_{F,0}^{\$,syn} + 2\alpha_3 B_{H,3,0}^{\$,F} - 2S_1 \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& + S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} = \\
& 2\alpha_3 B_{H,3,1}^{\$,F} + 2S_1 \alpha_3 B_{H,3,1}^{\epsilon,F} - 2TA_{H,1}^{\$} \\
& + 2 \sum_{j=1}^3 \alpha_j (M_{F,j,0}^{\$} - M_{F,j,1}^{\$}) - 2S_1 \sum_{j=1}^3 \alpha_j (M_{H,j,0}^{\epsilon} - M_{H,j,1}^{\epsilon}) \\
& - 2\alpha_3 B_{F,3,1}^{\$,H} - 2\alpha_3 S_1 B_{F,3,1}^{\epsilon,H} + 2D_{CIP,1}^{\$,H} \tag{C.30}
\end{aligned}$$

After dividing by 2, and collecting terms, we can rewrite this as

$$\begin{aligned}
& \alpha_3 dB_{H,3,1}^{\$,F} + S_1 \alpha_3 dB_{H,3,1}^{\epsilon,F} - \alpha_3 dB_{F,3,1}^{\$,H} - \alpha_3 S_1 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H} \\
& - \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$,F} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon,H} = \\
& TA_{H,1}^{\$,H} + 0.5 S_1 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} + 0.5 i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - 0.5 i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - 0.5 S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} \\
& + 0.5 (S_0 - S_1) D_{H,0}^{\epsilon,syn} + 0.5 \left(\frac{S_1}{S_0} - 1 \right) D_{CIP,0}^{\$,H} + 0.5 \left(1 - \frac{S_1}{S_0} \right) D_{F,0}^{\$,syn} \tag{C.31}
\end{aligned}$$

Using the period 0 expressions for synthetic borrowing, we can write

$$\begin{aligned}
& 0.5 i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} + 0.5 S_0 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \\
& - 0.5 i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - 0.5 S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} = \\
& i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - 0.5 S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} + 0.5 i_0^{\$,F} D_{F,0}^{\$,syn} \tag{C.32}
\end{aligned}$$

Multiplying the time 0 swap market equilibrium by $i_0^{\epsilon,H}$, we have

$$0.5 i_0^{\$,F} D_{F,0}^{\$,syn} = \Pi_{H,1} - 0.5 S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \tag{C.33}$$

so that

$$\begin{aligned}
& 0.5 i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} + 0.5 S_0 i_0^{\epsilon,H} \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} \\
& - 0.5 i_0^{\$,F} \sum_{j=1}^2 \alpha_j B_{F,j,0}^{\$,F} - 0.5 S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} = \\
& i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} + \Pi_{H,1} \tag{C.34}
\end{aligned}$$

This implies that we can write the right hand side of (C.31) as

$$\begin{aligned}
& TA_{H,1}^{\$,H} + \Pi_{H,1} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \tag{C.35} \\
& + 0.5 \left(1 - \frac{S_1}{S_0} \right) D_{F,0}^{\$,syn} - 0.5 (S_1 - S_0) D_{H,0}^{\epsilon,syn} + 0.5 \left(\frac{S_1}{S_0} - 1 \right) D_{CIP,0}^{\$,H} \\
& + 0.5 (S_1 - S_0) i_0^{\epsilon,H} \left(\sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} - \alpha_3 B_{F,3,0}^{\epsilon,H} \right)
\end{aligned}$$

We need to rewrite the last two lines. It is equal to $0.5(S_1 - S_0)$ times

$$-\frac{1}{S_0}D_{F,0}^{\$,syn} - D_{H,0}^{\epsilon,syn} + \frac{1}{S_0}D_{CIP,0}^{\$,H} + i_0^{\epsilon,H} \left(\sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} - \alpha_3 B_{F,3,0}^{\epsilon,H} \right) \quad (C.36)$$

Use that from the period 0 swap market equilibrium we have

$$D_{F,0}^{\$,syn} = (1 + i_0^{\epsilon,H})S_0 D_{H,0}^{\epsilon,syn} + D_{CIP,0}^{\$,H} \quad (C.37)$$

Then (C.36) becomes

$$-2D_{H,0}^{\epsilon,syn} + i_0^{\epsilon,H} \left(-D_{H,0}^{\epsilon,syn} + \sum_{j=1}^2 \alpha_j B_{H,j,0}^{\epsilon,H} - \alpha_3 B_{F,3,0}^{\epsilon,H} \right) \quad (C.38)$$

This is equal to

$$-2D_{H,0}^{\epsilon,syn} - 2i_0^{\epsilon,H} \left(D_{H,0}^{\epsilon,syn} + \alpha_3 B_{F,3,0}^{\epsilon,H} \right) \quad (C.39)$$

Then (C.35) becomes

$$TA_{H,1}^{\$,F} + \Pi_{H,1} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_0 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.40)$$

$$-(S_1 - S_0) D_{H,0}^{\epsilon,syn} - (S_1 - S_0) i_0^{\epsilon,H} \left(D_{H,0}^{\epsilon,syn} + \alpha_3 B_{F,3,0}^{\epsilon,H} \right)$$

This is equal to

$$TA_{H,1}^{\$,F} + \Pi_{H,1} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.41)$$

$$-(S_1 - S_0)(1 + i_0^{\epsilon,H}) D_{H,0}^{\epsilon,syn}$$

Then (C.31) becomes

$$\alpha_3 dB_{H,3,1}^{\$,F} + S_1 \alpha_3 dB_{H,3,1}^{\epsilon,F} - \alpha_3 dB_{F,3,1}^{\$,H} - \alpha_3 S_1 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H}$$

$$- \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$,F} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon,F} =$$

$$TA_{H,1}^{\$,F} + \Pi_{H,1} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_0 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.42)$$

$$-(S_1 - S_0)(1 + i_0^{\epsilon,H}) D_{H,0}^{\epsilon,syn} \quad (C.43)$$

We can finally rewrite this as

$$\alpha_3 dB_{H,3,1}^{\$,F} + S_1 \alpha_3 dB_{H,3,1}^{\epsilon,F} - \alpha_3 dB_{F,3,1}^{\$,H} - \alpha_3 S_1 dB_{F,3,1}^{\epsilon,H} + dD_{CIP,1}^{\$,H}$$

$$- \sum_{j=1}^3 \alpha_j dM_{F,j,1}^{\$,F} + S_1 \sum_{j=1}^3 \alpha_j dM_{H,j,1}^{\epsilon,F} + (S_1 - S_0) D_{H,0}^{\epsilon,syn} =$$

$$TA_{H,1}^{\$,F} + \Pi_{H,1} + i_0^{\$,F} \alpha_3 B_{H,3,0}^{\$,F} - S_1 i_0^{\epsilon,H} \alpha_3 B_{F,3,0}^{\epsilon,H} - S_1 i_0^{\epsilon,H} D_{H,0}^{\epsilon,syn} \quad (C.44)$$

This is exactly the same as the FX market equilibrium (C.8) derived from the spot and swap market equilibria.

D US Lender Portfolio Shift

Here we consider the shock in Section 4.4 of the paper, which involves an increase in λ for US investors, which we refer to as λ_H . We will derive the linearized spot and swap market equilibria, extending the derivation in Appendix D of the paper to introduce shocks to λ_H .

The swap market equilibrium is

$$\left(1 + i_1^{$,F}\right) D_{F,1}^{$,syn} - S_1 D_{H,1}^{\epsilon,syn} - \left(1 + i_1^{$,F}\right) D_{CIP,1}^{$,H} = 0 \quad (D.1)$$

This is linearized as (recall we start from zero pre-shock excess demand in offshore markets)

$$\hat{D}_{F,1}^{$,syn} - \hat{D}_{H,1}^{\epsilon,syn} - D_{CIP,1}^{$,H} = 0 \quad (D.2)$$

We have

$$D_{F,1}^{$,syn} - D_{H,1}^{\epsilon,syn} = -\alpha_1 B_{F,1,1}^{$,F} - \alpha_2 B_{F,2,1}^{$,F} - n(1 - \lambda_{US}) B_{H,3,1}^{$,F} + \alpha_1 B_{H,1,1}^{\epsilon,H} + n\lambda_{US} B_{H,2,1}^{\epsilon,H} + \alpha_3 B_{F,3,1}^{\epsilon,H} \quad (D.3)$$

We defined $\alpha_1 = 1 - n$, $\alpha_2 = n\lambda$ and $\alpha_3 = n(1 - \lambda)$. We now allow λ to be different between US and European agents. The pre-shock λ levels remain the same for the US and Europe, but we consider an increase in λ for the US. This is made explicit above.

After substituting the portfolio expressions in (D.3), Appendix D derives an expression for $\hat{D}_{F,1}^{$,syn} - \hat{D}_{H,1}^{\epsilon,syn}$. This is still correct, but we now need to add an additional term associated with a change in λ_{US} . This term is equal to

$$n \left(\bar{B}_{H,2,1}^{\epsilon,H} + \bar{B}_{H,3,1}^{$,F} \right) \hat{\lambda}_H \quad (D.4)$$

Adding this term to the swap market equilibrium in Appendix D, it becomes

$$\nu_2 s_1 - 2 \frac{i_1^{$,F} + s_1}{\gamma var(s_2)} - 2\alpha_3 \nu_1 s_1 - \frac{1}{\phi} i_1^{$,F} + n \hat{\lambda}_H \left(\bar{B}_{H,3,1}^{$,F} + \bar{B}_{H,2,1}^{\epsilon,H} \right) = 0 \quad (D.5)$$

We have

$$\bar{B}_{H,3,1}^{$,F} + \bar{B}_{H,2,1}^{\epsilon,H} = 1 - a_l - \psi = W_{l,0} \quad (D.6)$$

so that the swap market equilibrium then becomes

$$\nu_2 s_1 - 2 \frac{i_1^{$,F} + s_1}{\gamma var(s_2)} - 2\alpha_3 \nu_1 s_1 - \frac{1}{\phi} i_1^{$,F} + n W_{l,0} \hat{\lambda}_H = 0 \quad (D.7)$$

Next consider the spot market equilibrium (B.14), repeated here for convenience:

$$\sum_{j=1}^3 \alpha_j Q_{F,j,1}^{\$,spot} + (1 - (F_1/S_1)) \left(D_{F,1}^{\$,syn} - D_{CIP,1}^{\$,H} \right) = S_1 \sum_{j=1}^3 \alpha_j Q_{H,j,1}^{\epsilon,spot} \quad (\text{D.8})$$

The only thing that needs to change to the derivation of the spot market equilibrium in Appendix B is that α_2 and α_3 on the right hand side, for US lenders, will change to $n\lambda_H$ and $n(1 - \lambda_H)$. A shock to λ_{US} adds an additional term on the right hand side of the linearized spot market equilibrium of

$$n \left(\bar{Q}_{H,2,1}^{\epsilon,spot} - \bar{Q}_{H,3,1}^{\epsilon,spot} \right) \hat{\lambda}_{US} \quad (\text{D.9})$$

Using the expressions for $Q_{H,2,1}^{\epsilon,spot}$ and $Q_{H,3,1}^{\epsilon,spot}$ in Appendix D, their pre-shock levels are

$$\bar{Q}_{H,2,1}^{\epsilon,spot} = -\bar{Y}_{H,1}^{\epsilon} + \bar{C}_{HF,2,1}^{\epsilon} \quad (\text{D.10})$$

$$\bar{Q}_{H,3,1}^{\epsilon,spot} = -\bar{Y}_{H,1}^{\epsilon} + \bar{C}_{HF,3,1}^{\epsilon} \quad (\text{D.11})$$

But these are identical as $\bar{C}_{HF,j,1}^{\$} = \omega a^{\$} \bar{C}_{H,j,1}$ and $\bar{C}_{H,j,1} = 1$ for all j (see Appendix C of the paper). It then follows that the additional term in the spot market equilibrium is zero, so that the spot market equilibrium remains the same as derived in Appendix D of the paper.

E UIP Arbitrageurs

This section introduces UIP arbitrageurs to the model. They arbitrage between onshore dollar bonds and onshore euro bonds. UIP arbitrageurs enter period 1 with zero wealth. Let $B_{UIP,1}^{\$,H}$ be their onshore dollar bond position and $B_{UIP,1}^{\epsilon,F}$ their onshore euro bond position, so that $B_{UIP,1}^{\$,H} + S_1 B_{UIP,1}^{\epsilon,F} = 0$. Since onshore interest rates are zero, this portfolio yields a period 2 return of $\pi_{UIP} = B_{UIP,1}^{\$,H} \left(1 - \frac{S_2}{S_1} \right)$. Linearized, this is $\pi_{UIP} = B_{UIP,1}^{\$,H} (s_1 - s_2)$. Assume that they choose a mean-variance portfolio that maximizes $E(\pi_{UIP}) - 0.5\tilde{\gamma}var(\pi_{UIP})$, where $\tilde{\gamma}$ represents their risk aversion. Using that $E_1(s_2) = 0$, the optimal portfolio is

$$B_{UIP,1}^{\$,H} = \frac{s_1}{\tilde{\gamma}var(s_2)} \quad (\text{E.1})$$

The transaction by UIP arbitrageurs does not affect the swap market as they only hold positions in the onshore markets. If $s_1 > 0$, they borrow euros and exchange them for $B_{UIP,1}^{\$,H}$ dollars on the spot market. Adding this to the spot market schedule, the spot and swap market equilibrium schedules in equations (36)-(37) in the text (related to liquidity preference shocks) become

$$\nu_1 s_1 + 2 \frac{i_1^{\$,F} + s_1}{\gamma var(s_2)} + \frac{s_1}{\tilde{\gamma} var(s_2)} = 0 \quad (\text{E.2})$$

$$(2\alpha_3 \nu_1 - \nu_2) s_1 + 2 \frac{i_1^{\$,F} + s_1}{\gamma var(s_2)} + \frac{1}{\phi} i_1^{\$,F} - \hat{\psi} \omega [a^\$ - a^\text{€}] = 0 \quad (\text{E.3})$$

The only change is that the spot market schedule becomes a bit steeper than it already was. The shock $\hat{\psi}$ has the same qualitative effect as discussed in the text.

F Convenience Yield Shock

This section considers the effect of a rise in the US convenience yield. Assume that the onshore US asset has a convenience benefit (e.g., liquidity) that is equivalent to an increase in the return by η . The portfolios of borrowers and lenders in the model are not affected by this convenience benefit. Borrowers and lenders in the European market choose between offshore dollar assets and euro assets. Borrowers and lenders in the US market choose between onshore dollar assets and offshore euro assets. But we can think of them as only buying (or borrowing) onshore dollar assets, while swapping part of it into euros. Whether they hedge or not, in both cases they hold (or borrow) US onshore assets, so that the relative return is unaffected by whether the onshore dollar asset has a convenience yield. We will also assume that this convenience benefit does not apply to CIP arbitrageurs, who borrow and lend in similar types of assets in the wholesale market.

We introduce the UIP arbitrageurs from Section E. They arbitrage between onshore dollar assets and onshore euro assets. Different from Section E though, we now assume that the onshore dollar assets have a convenience yield. As in Section E, the linearized profit of UIP arbitrageurs is $\pi_{UIP} = B_{UIP,1}^{\$,H}(s_1 - s_2)$. But now they maximize $E(\pi_{UIP}) - 0.5\tilde{\gamma}var(\pi_{UIP}) + \eta B_{UIP,1}^{\$,H}$. The last term, not present in Section E, captures the non-pecuniary convenience benefit from holding the onshore dollar bond. The optimal portfolio is then

$$B_{UIP,1}^{\$,H} = \frac{s_1 + \eta}{\tilde{\gamma} var(s_2)} \quad (\text{F.1})$$

The convenience benefit leads to a greater demand for the onshore dollar bond.

Consider again the swap and spot market equilibrium schedules (36)-(37) in the text. We now set $\hat{\psi} = 0$. UIP arbitrageurs borrow euros, which they exchange for $B_{UIP,1}^{\$,H}$ dollars on the spot market. Adding this to the spot market schedule, the equilibrium schedules become

$$\nu_1 s_1 + 2 \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} + \frac{s_1 + \eta}{\tilde{\gamma} \text{var}(s_2)} = 0 \quad (\text{F.2})$$

$$(2\alpha_3 \nu_1 - \nu_2) s_1 + 2 \frac{i_1^{\$,F} + s_1}{\gamma \text{var}(s_2)} + \frac{i_1^{\$,F}}{\phi} = 0 \quad (\text{F.3})$$

When $\eta = 0$, the pre-shock equilibrium with $i_1^{\$,F} = s_1 = 0$ still holds. Now consider a rise in the convenience yield η . This shifts the spot market schedule downward. Chart A of Figure 5 in the paper shows the case of imperfect CIP arbitrage (post 2007), while Chart B shows the case of perfect CIP arbitrage (pre-2007) where $\phi \rightarrow 0$ and the swap market schedule is horizontal. In both cases the dollar appreciates (s_1 drops), while under imperfect CIP arbitrage the CIP deviation rises.

As discussed in the text, there are two problems with this shock as an explanation for the empirical evidence in Table 1 of the paper. First, the dollar appreciates even under perfect CIP arbitrage. Table 1 in the paper shows that increased financial stress does not lead to a dollar appreciation prior to 2007. Second, Diamond and Van Tassel (2023) show that while convenience yields rise during financial crises, the difference between the US and foreign convenience yields generally does not. Table A1 provides further evidence. It regresses the change in the relative US convenience yield on the change in the risk measures in Table 1 of the paper. The convenience yield is computed as the 3m Libor rate minus the 3m Treasury rate. The trade-weighted average convenience yield in non-US advanced countries is subtracted from the US convenience yield. Table A1 shows that a change in risk does not affect the change in the relative US convenience yield for any of the risk measures prior to 2007, while the effect is insignificant for 6 of the 8 risk measures since 2007.

Table A-1: Regression of convenience yield on measures of risk

	Dependent Variable: $\Delta CY_{US,t}$								
	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		VIX	MOVE	BEX (1)	BEX (2)	HKM (1)	HKM (2)	GZ (1)	GZ (2)
<u>1999-2006</u>									
CY_{t-1}	-0.320*** (0.099)	-0.324*** (0.096)	-0.315*** (0.105)	-0.322*** (0.099)	-0.327*** (0.102)	-0.323*** (0.102)	-0.311*** (0.101)	-0.316*** (0.100)	-0.323*** (0.098)
$\Delta Risk_t$		-0.021 (0.060)	0.039 (0.102)	-0.155 (0.109)	-0.060 (0.104)	-0.013 (0.013)	-0.003 (0.013)	0.145 (0.100)	0.011 (0.015)
$Risk_{t-1}$		0.023 (0.029)	0.000 (0.041)	-0.021 (0.050)	-0.008 (0.052)	0.006 (0.016)	-0.015 (0.014)	-0.003 (0.030)	0.004 (0.008)
\bar{R}^2	0.143	0.130	0.125	0.135	0.125	0.136	0.143	0.135	0.127
<u>2007-2021</u>									
CY_{t-1}	-0.169 (0.112)	-0.184* (0.100)	-0.206** (0.102)	-0.184* (0.096)	-0.196* (0.104)	-0.177* (0.107)	-0.176 (0.108)	-0.206** (0.100)	-0.214** (0.094)
$\Delta Risk_t$		0.177** (0.090)	0.236 (0.167)	0.121 (0.183)	0.213 (0.200)	-0.006 (0.015)	-0.007 (0.014)	0.327* (0.199)	0.111*** (0.042)
$Risk_{t-1}$		0.029 (0.042)	0.053 (0.037)	0.041 (0.098)	0.080 (0.082)	-0.014 (0.019)	-0.015 (0.019)	0.033 (0.040)	0.013 (0.016)
\bar{R}^2	0.073	0.114	0.110	0.072	0.083	0.082	0.088	0.111	0.169

Notes: $CY_{US,t}$ is the convenience yield on dollar assets relative to the convenience yield on foreign assets, computed as the difference between the US 3m Libor and the US 3m treasury minus the difference between the foreign 3m Libor and the foreign 3m government bond yield. The foreign interest rates are a trade-weighted average of rates from advanced countries, using the same trade weights as the variables in Table 1 in the text. $Risk_t$ is the level of one of eight risk measures: (1) the log of the VIX, (2) the log of the MOVE index, (3) the log of the risk aversion index from Bekaert et al. (2021), (4) the log of the uncertainty index from Bekaert et al. (2021), (5) the normalized intermediary capital risk factor from He et al. (2017), (6) the normalized intermediary value weighted investment return from He et al. (2017), (7) the log of the bond spread on senior unsecured debt of nonfinancial firms from Gilchrist and Zakrajšek (2012), (8) the normalized excess bond premium from Gilchrist and Zakrajšek (2012). The operator Δ is the month-over-month change. For scaling, all risk variables in the regression are divided by 100. All regressions include a constant and robust standard errors are written in parentheses, ***/**/* denotes significance at the 1/5/10% level.

References

- Bekaert, Geert, Eric C. Engstrom, and Nancy R. Xu. 2021. “The Time Variation in Risk Appetite and Uncertainty”. *Management Science* 68(6), 3975–4004.
- Diamond, William and Peter Van Tassel. 2023. “Risk-Free Rates and Convenience Yields Around the World”. *Federal Reserve Bank of New York Staff Reports No. 1032*.
- Gilchrist, Simon and Egon Zakrajšek. 2012. “Credit Spreads and Business Cycle Fluctuations”. *American Economic Review* 102(4), 1692–1720.
- He, Zhiguo, Bryan Kelly, and Asaf Manela. 2017. “Intermediary Asset Pricing: New Evidence from Many Asset Classes”. *Journal of Financial Economics* 126(1), 1–35.