CBDC as Imperfect Substitute to Bank Deposits: A Macroeconomic Perspective *

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May 14, 2024

Abstract

The impact of Central Bank Digital Currency (CBDC) is analyzed in a closed-economy model with monopolistic competition in banking and where CBDC is an imperfect substitute with bank deposits. The design of CBDC is characterized by its interest rate, its substitutability with bank deposits, and its relative liquidity. We examine how interest-bearing CBDC would affect the banking sector, public finance, GDP and welfare. Welfare may improve through three channels: seigniorage; a lower opportunity cost of money; and a redistribution away from bank owners. In our numerical analysis we find a maximum welfare improvement of almost 50 bps in consumption terms.

*We would like to thank two referees and seminar participants at the 2022 EEA-ESEM Congress, the 2022 SSES Annual Congress, the 2022 Asian Meeting of the Econometric Society and the 2022 SFI Research Days meeting.

1 Introduction

As our economies are becoming increasingly digital, central banks around the world are exploring the possibility of issuing central bank digital currency (CBDC). Since there are various ways to implement CBDCs, it is important to understand its implications. For example, CBDC could mainly substitute cash, which would have little impact on financial intermediation. Alternatively, it could substitute checking deposits and could lead to banking disintermediation. Although a growing literature is exploring the macroeconomic implications of CBDC, our understanding is still limited.[1] Under some conditions, CBDC leaves economic outcomes unchanged, as shown in Brunnermeier and Niepelt (2019). In contrast, other studies show that the disintermediation implied by CBDCs would reduce bank loans and possibly output (see Keister and Sanches (2023), or Chiu et al. (2023)), while Barrdear and Kumhof (2021) predict a large increase in output. Results depend in particular on how easily banks can substitute checking deposits by other types of funding and how substitutable are checking deposits with CBDC. The interest rate on CBDC and the competitive structure of the banking sector may also play significant roles.

The purpose of this paper is to shed light on these issues and give quantitative estimates on the potential benefits of CBDC in a model with monopolistic competition in banking, where CBDC and bank deposits are imperfect substitutes. We model imperfect substitutability by assuming that CBDC and bank deposits contribute to the formation of a composite liquid asset, which is useful to households as it reduces the transaction cost of acquiring goods for consumption.[2] Given the interest paid by each type of money, households' demand for each reflects the optimal trade-off between maximizing interest

[1] E.g., see Anhert et al. (2022), Auer et al. (2021), Infante et al. (2022), and Niepelt (2024) for recent surveys of the literature.

[2] This framework extends the idea present in Feenstra (1986), Rebelo and Vegh (1996) and Schmitt-Grohé and Uribe (2004) that money is demanded as it reduces a transaction or liquidity cost. Barrdear and Kumhof (2021) adopt a similar approach. Imperfect substitutability is also modeled by introducing CBDC in the utility function (e.g., Agur et al. (2021), or Ferrari et al. (2022)) or in search models, where CBDC is used for different transactions (e.g., Assenmacher et al. (2021)). However, several papers in the literature assume perfect substitutability between CBDC and bank deposits or focus on the interaction between cash and CBDC (e.g., Davoodalhosseini (2022)).

collection and minimizing the transaction cost, given the imperfect substitutability between the different monies.

CBDC design involves three dimensions in our model: the interest rate it pays; its liquidity relative to bank deposits – which, in the model, is the weight of CBDC in the formation of the composite liquid asset – and its degree of substitutability with bank deposits. In practice, liquidity may be related to technological aspects of the design, such as the rapidity of payments, or to any fee structure. Substitutability might involve the interoperability between CBDC and bank deposits (see Brunnermeier and Landau (2019) for discussions on this issue), or some characteristics that might differentiate the two monies and make one more suitable than the other in certain circumstances. For example CBDC might be in the form of token, might grant more or less privacy than bank deposits, might be more secure than bank deposits or might for example offer better conditions for international transactions.[3]

We analyze the welfare impact of CBDC in the steady state. We identify three channels through which CBDC may improve welfare. First, through CBDC the central bank may increase its seigniorage revenue, which, everything else equal, would allow the government to reduce income taxes. Second, if households can earn higher interest on their money (CBDC and/or deposit) holdings, they optimally choose to increase their money holdings and thus pay a lower transaction cost on consumption. Third, the introduction of CBDC may lead to a reallocation of banks' rents to the general population, whether in the form of tax reduction (first channel) or in the form of higher interest payment (second channel). If bank rents are collected by a wealthier fraction of the population, this shift implies that CBDC induces some degree of reduction of inequality.

Seigniorage is an important endogenous variable in the model. Its magnitude depends on all three dimensions of CBDC (interest rate, liquidity, substitutability). In particular, seigniorage is non-monotonic in the interest rate paid by CBDC, as a higher interest rate decreases seigniorage per unit CBDC issued, but increases its demand.

The optimal interest rate on CBDC is the one that reaches the best compromise between raising higher seigniorage to lower tax distortions or paying higher interest to

[3]Since there may be technical constraints in the choice of liquidity and substitutability, in our quantitative welfare analysis we will only consider the interest rate as a policy variable.

lower the opportunity cost of holding money. The optimal interest rate depends on how high are existing tax rates, as the higher the tax rate, the higher the distortion they bring to the economy. Thus, with a higher tax rate the potential benefit of the first channel – collecting seigniorage and lower taxes – is higher, hence the optimal interest rate on CBDC is lower. This is relevant since, as reported e.g. by Trabandt and Uhlig (2011), the amount of labor taxation differs enormously between different countries: it is around 25% in the United States and it averages more than 40% in the EU-14 countries.

However, the quantitative analysis shows that these two channels would bring only a modest welfare improvement: at the optimum they would bring an increase of only 9 basis points in consumption terms for countries with a labor tax rate of 25%, and of 20 basis points for countries with a tax rate of 45%.

The third channel we consider is the reallocation of banks' rents that may lead to a reduction of inequality. In one parameterization of the model, a 10% fraction of the population ("bankers") owns a 90% share of the banks and receives the corresponding profits.[4] CBDC allows non-bankers to take over part of the rents associated to deposits, whether in the form of tax reductions or in the form of interest on CBDC holdings. Taking into account this channel, together with the previous two, we find that welfare increases by 40 basis points if the labor tax rate is 25% and 47 basis points if the labor tax rate is 45%.

We also emphasize that these benefits require historically normal interest rates (our baseline rate is 4%). At interest rates close to zero, all three of our channels lose their efficacy: seigniorage is close to zero, the opportunity cost of holding any form of money is close to zero without the need of introducing CBDC, and banks collect zero rents from deposits, implying that there are no rents that CBDC can redistribute to the public.

Most of the literature on CBDC assumes perfect competition in banking or does not model banks explicitly. Exceptions are Andolfatto (2021) who assumes a one bank monopoly and Chiu et al. (2023) who assume Cournot competition with smaller number of banks. In these frameworks, the interest rate on CBDC affects the optimal deposit interest rate and can affect welfare through this channel. With a continuum of banks

[4]In the United States households in the top 10% of the wealth distribution own 90% of the stock. See for example "How America's 1% came to dominate equity ownership", https://www.ft.com/content/2501e154-4789-11ea-aeb3-955839e06441

in monopolistic competition, however, individual banks take the average deposit rate as given so that the deposit rate is unaffected by the CBDC interest rate.[5]

While our approach share some features with Barrdear and Kumhof (2021), our paper estimates a significantly lower welfare benefit of CBDC. Their estimate of a 3% GDP increase is due in large part to the following channel, absent from our model. When issuing CBDC, the central bank buys public debt from private investors. In their model this is assumed to result in a lower interest rate on government bonds, which brings savings to the government and general welfare improvements. Chiu and Davoodalhosseini (2023) consider a general equilibrium model where cash and deposits play different roles for payments. They find that an interest-bearing cash-like CBDC improves welfare because the main impact is the reduction in the opportunity cost of money holdings. A similar effect is also present in our framework.

Since we focus on the steady state, we do not examine the cyclical issues associated with CBDC. Using a DSGE model, Burlon et al. (2024) find a positive cyclical impact of CBDC as the increased seigniorage is transferred to households and increases their consumption. Piazzesi et al. (2022) consider varying the interest rate on CBDC for monetary policy objectives.

There are various potential channels through which CBDC could affect bank lending,[6] but there is uncertainty about the sign and the magnitude of this effect.[7] In the baseline model we abstract from these channels and an important feature of our model is that the two main functions of banks, deposit taking and credit provision, do not interact. This is because the financial markets provide an alternative source of financing, although at a higher interest rate.[8] This reduces bank profits on deposits, but it does not affect the

[5]Empirical evidence for monopolistic competition in the banking sector is provided e.g., by Drechsler, Savov and Schnabl (2017). Gerali et al. (2010) introduce monopolistic competitive banks in a DSGE model. Kurlat (2019) assumes a finite number of banks with entry.

[6]For example, because of reduced profits as in Burlon et al. (2024); because deposits, but not CBDC, are associated with credit lines, as in Piazzesi and Schneider (2022); or because of an increased cost of wholesale funding, as in Whited et al. (2022).

[7]Andolfatto (2021) and Chiu et al. (2023) show that CBDC might increase lending. Also, the evidence in the literature is mixed about whether or not an increase in bank competition has adverse consequences on banks' optimal lending choices (see e.g., De Nicolo and Boyd (2005) for a review of this literature).

[8]Alternatively, banks could borrow from the central bank as in Brunnermeier and Niepelt (2019): when the central bank expands its liabilities by issuing CBDC, it might acquire claims vis-à-vis the banking sector,

marginal cost of funding for banks, which is always equal to the risk-free rate, and for this reason it does not affect credit extension in the steady state of our model.[9]

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 describes the steady state equilibrium. Section 4 discusses the calibration and Section 5 outlines the numerical results, in terms of the relative demand for CBDC and bank deposits, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and the welfare implications. Section 6 concludes.

2 A Model with CBDC

We consider a closed economy with two types of agents – households and bank owners – firms, banks, and finally the government and the central bank. This economy is similar in many respects to the classical monetary economy in Gali (2015) and to the economy in Del Negro and Sims (2015). As in the continuous-time model of Del Negro and Sims (2015) there is an explicit role for money, as the latter mitigates the transaction cost of consumption. However, as described in detail in the next subsection, our model features multiple types of money, which are imperfect substitutes.

2.1 Demand for Bank Deposits and CBDC

Our model comprises two types of agents, households and bank owners, described in detail in Section 2.2. All the action in the model is on the part of households, which in particular generate money demand. Households decide how to allocate savings between the following assets: a nominal asset a (e.g. government bonds), paying the nominal risk-free rate r_t^*, bank deposits $d^b(j)$ for each bank j, paying a nominal interest $r_t^b(j)$, and CBDC d^c, paying nominal interest r_t^c. Both bank deposits and CBDC reduce trans-

thus providing substitute funding for banks. In Brunnermeier and Niepelt (2019) economic outcomes are unchanged if central bank funding is provided at the same conditions as deposit funding, and if the central bank pays the same interest on CBDC as banks do on deposits. In our model, the interest on substitute bank funding would be equal to the risk-free interest rate.

[9] In an extension of the model that we present in the online Appendix, we allow banks' marginal funding cost to depend on deposits, following the model of Wang, Whited, Wu and Xiao (2022). The results are very similar to those in the baseline model.

actions costs, but they are imperfect substitutes.

Bank deposits are issued by a continuum of banks of size 1 in monopolistic competition. The equilibrium interest rate on bank deposits is typically lower than the safe rate r^* due to the costs of managing deposits and to banks' market power, as discussed in section 2.4.

As in Schmitt-Grohé and Uribe (2004) and Del Negro and Sims (2015), we assume that households incur transactions costs $c_t s_t$ to consume c_t. These costs can be reduced by holding bank deposits and CDBC. More precisely, s_t is a function of money velocity $x_t \equiv p_t c_t / d_t$, where p_t is the price level and d_t is a composite of the deposits of all banks and CBDC. This composite captures the imperfect substitutability among deposits. We assume a CES structure:

$$d_t \quad = \quad \left(\alpha_c (d_t^c)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} + \alpha_b (d_t^b)^{\frac{\epsilon_{cb}-1}{\epsilon_{cb}}} \right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{1}$$

d_t^b is a composite of all bank deposits:

$$d_t^b \equiv \left(\int (d_t^b(j))^{1-\frac{1}{\epsilon^b}} dj \right)^{\frac{\epsilon^b}{\epsilon^b - 1}} \tag{2}$$

where ϵ^b is the elasticity of substitution between deposits at different banks, and ϵ_{cb} is the elasticity of substitution between bank deposits and CBDC.

The interest rate on CBDC r_t^c is set by the central bank. The relative liquidity and the elasticity of substitution between bank deposits and CBDC can be a design choice of the government. We assume that

$$\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} = 1, \ \mathit{with} \ \alpha_c^{\epsilon_{cb}} > 0, \ \alpha_b^{\epsilon_{cb}} > 0 \tag{3}$$

as in this case one unit of the numeraire good results at most in one unit of the composite d_t (when $\alpha_c^{\epsilon_{cb}}$ is allocated in CBDC and $\alpha_b^{\epsilon_{cb}}$ is allocated in bank deposits). $\frac{\alpha_c^{\epsilon_{cb}}}{\alpha_b^{\epsilon_{cb}}}$ can be interpreted as the relative liquidity of CDBC with respect to bank deposits. The world without CBDC is one where $\alpha_c = 0$ and $\alpha_b = 1$.[10]

While we do not introduce cash in our baseline model, the online Appendix includes an extension of this framework featuring cash alongside bank deposits and CBDC.

[10] If the introduction of CBDC implied $\alpha_c^{\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}} > 1$, it could improve the overall efficiency of the payment system since fewer resources would be needed to alleviate the transaction cost. However, we abstract from this effect to concentrate on the effect of the competition between bank deposits and CBDC.

2.2 Households and Bank Owners

Households are a measure-one set of agents who work in firms, consume, and save. In addition, they own a fraction of firms and banks. They derive utility from consumption and disutility from working. We assume separable CRRA preferences so that the household's periodic flow utility is given by

$$u(c_t, h_t) = log(c_t) - \frac{h_t^{1+\gamma}}{1+\gamma} \qquad \gamma \geq 0$$

where c_t is consumption and h_t denotes labor supply. The household's expected lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{4}$$

The household's budget constraint is

$$\begin{aligned} & (1-\tau_h)w_t h_t + (1+r_{t-1}^*)a_{t-1} + \int (1+r_{t-1}^b(j))d_{t-1}^b(j)dj \\ + \quad & (1+r_{t-1}^c)d_{t-1}^c + \zeta(1-\tau_b)\Pi_t^b = p_t c_t(1+s_t) + \int d_t^b(j)dj + d_t^c + a_t + p_t t_t \end{aligned} \tag{5}$$

where w_t is the (nominal) wage, a_t are holdings of the risk-free bond, and Π_t^b are bank dividends, t_t are lump-sum taxes. τ_h and τ_b are labor income and dividend tax rates. ζ is the fraction of banks that is owned by households.[11]

The remaining fraction $1-\zeta$ belongs to the second type of agent in the model, bank owners. This is a set of agents of size λ, who do not work and, importantly, are not subject to the transaction cost. Besides receiving bank dividends, they invest their wealth in the risk-free asset, and their budget constraint is simply

$$p_t c_t^{bo} + a_t^{bo} = \frac{1-\zeta}{\lambda}\Pi_t^b + (1+r_{t-1}^*)a_{t-1}^{bo} \tag{6}$$

where c^{bo} is consumption and a^{bo} are the holdings of the risk-free bond, per unit-size bank owner. The bankers' flow utility is given by

$$u_t^{bo}(c_t^{bo}) = log(c_t^{bo}) \tag{7}$$

Households maximize their utility subject to (5). First-order conditions are standard and are described in the Appendix. Below we will assume a specific form for the

[11]The firm sector is perfectly competitive. Hence, firm profits are zero and it is not important to specify the firm ownership.

transactions cost, similar to Schmitt-Grohé and Uribe (2004). This cost is a function of money velocity $x_t = \frac{p_t c_t}{d_t}$

$$s(x_t) = Ax_t + \frac{B}{x_t} - 2\sqrt{AB} \tag{8}$$

where A and B are constant parameters.

The demand equation for the deposits of each bank j is

$$d_t^b(j) = \left(\frac{r_t^* - r_t^b(j)}{r_t^* - r_t^b}\right)^{-\epsilon^b} d_t^b \tag{9}$$

where

$$r_t^* - r_t^b \equiv \left(\int (r_t^* - r_t^b(j))^{1-\epsilon^b} dj\right)^{\frac{1}{1-\epsilon^b}} \tag{10}$$

In equilibrium, all banks offers the same deposit rate r_t^b, as we see in more detail in Section 2.4. From the Euler equations, we obtain the relationship between bank deposits holdings and CBDC holdings:

$$d_t^b = \left(\frac{\alpha_b}{\alpha_c} \times \frac{r_t^* - r_t^c}{r_t^* - r_t^b}\right)^{\epsilon_{cb}} d_t^c \tag{11}$$

so that there is a simple relationship between holdings of bank deposits and the composite liquid asset

$$d_t = f_t d_t^b \tag{12}$$

with the proportionality factor f_t given by

$$f_t = \left(\frac{r_t^* - r_t^b}{\alpha_b}\right)^{\epsilon_{cb}} \left(\alpha_c^{\epsilon_{cb}}(r_t^* - r_t^c)^{1-\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}}(r_t^* - r_t^b)^{1-\epsilon_{cb}}\right)^{\frac{\epsilon_{cb}}{\epsilon_{cb}-1}} \tag{13}$$

(Notice that without CBDC, i.e., with $\alpha_c = 0$, $\alpha_b = 1$, we have $f_t = 1$ and $d_t = d_t^b$). Defining the "composite interest rate" r^{comp} such that

$$(r_t^* - r^{comp}) \equiv \left(\alpha_c^{\epsilon_{cb}}(r_t^* - r_t^c)^{1-\epsilon_{cb}} + \alpha_b^{\epsilon_{cb}}(r_t^* - r_t^b)^{1-\epsilon_{cb}}\right)^{\frac{1}{1-\epsilon_{cb}}} \tag{14}$$

(13) can be written as

$$f_t = \left(\frac{r_t^* - r_t^b}{\alpha_b(r_t^* - r_t^{comp})}\right)^{\epsilon_{cb}} \tag{15}$$

Comparing the Euler equation for the bond with that for bank deposits, money velocity is

$$x_t = \sqrt{\frac{r_t^* - r_t^{comp} + B(1 + r_t^*)}{(1 + r_t^*)A}} \tag{16}$$

so that the demand for bank deposits is

$$d_t^b = \frac{p_t c_t}{f_t} \sqrt{\frac{(1 + r_t^*)A}{r_t^* - r_t^{comp} + B(1 + r_t^*)}} \tag{17}$$

The demand for CBDC can be easily obtained by combining (11) and (17).

Finally, with simple algebra we obtain that the total cost (in terms of lost interest) paid by households to acquire money instruments and thus reduce the transaction cost satisfies the equilibrium relationship

$$d_t^b(r_t^* - r_t^b) + d_t^c(r_t^* - r_t^c) = d_t(r^* - r_t^{comp}) \tag{18}$$

where d_t is the composite money instrument defined in (1).

The interest semi-elasticity of money demand, defined as the percentage change in the demand for money instruments for a one percentage change in the *spread* between the interest paid by money and the risk-free rate, is essentially determined by the parameter B.

$$\iota = -\frac{1}{2} \times \frac{1}{B(1 + r^*) + (r^* - r^{comp})} \tag{19}$$

In the online Appendix we show an extension of the model adding cash as a third money instrument, paying zero interest. Specifically, we have a nested CES structure in which cash and CBDC are imperfect substitutes; the composite of cash and CBDC, in turn, is an imperfect substitute of bank deposits. We show that if the "composite interest" (defined similarly as in (14)) of cash and CBDC is equal to the value of r^c in the two-instrument model of this section, economic outcomes are unchanged: household holdings of the three instruments are such that in equilibrium households pay the same transaction cost of consumption, and the cost of holding money is also unchanged, given by (18).

2.3 Firms

There is a representative firm with Cobb-Douglas production function

$$y_t = z k_t^{\alpha} h_t^{1-\alpha} \tag{20}$$

where k_t is capital, installed in period $t - 1$. A fraction φ of capital can only be financed by banks (e.g., for the financing of working capital), so that $\varphi p_{t-1} k_t = l_{t-1}$, where l_{t-1}

are the loans that the firm obtains from the bank in period $t-1$, to be repaid at t. The remaining fraction $1-\varphi$ is financed by issuing bonds at interest rate r^*_{t-1}.

We assume monopolistic competition in the loan market, so that, similarly to deposits, loans are a bundle of loans from different banks[12]

$$l_t \equiv \left(\int (l_t(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}} \tag{21}$$

where ϵ^l is the elasticity of substitution for loans from different banks and the index i denotes a bank. The working capital constraint can be rewritten as

$$k_t = \frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{p_{t-1}\varphi} \tag{22}$$

Firms choose loans, capitals and labor to maximize profits, which, taking into account the working capital constraint, can be written as

$$\Pi_t = p_t z \left(\frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{\phi p_{t-1}} \right)^{\alpha} h_t^{1-\alpha} - w_t h_t - \int l_{t-1}(i) r^l_{t-1}(i) di - (1-\varphi)\, r^*_{t-1} \frac{\left(\int (l_{t-1}(i))^{1-\frac{1}{\epsilon^l}} di \right)^{\frac{\epsilon^l}{\epsilon^l-1}}}{\varphi} \tag{23}$$

We obtain that firms' loan demand is

$$l_t(i) = \left(\frac{r^l_t(i)}{r^l_t} \right)^{-\epsilon^l} l_t \tag{24}$$

where $r^l_t(i)$ is the loan interest rate charged by bank i and the "market loan rate" r^l_t is

$$r^l_t = \left(\int (r^l_t(i))^{1-\epsilon^l} di \right)^{\frac{1}{1-\epsilon^l}} \tag{25}$$

In equilibrium all banks choose the same rate r^l_t. The capital/labor ratio chosen by firms is

$$\frac{k_t}{h_t} = \left(\frac{z\alpha}{\hat{r}^K_{t-1}} \right)^{\frac{1}{1-\alpha}} \tag{26}$$

where $\hat{r}^K_{t-1} = \frac{p_{t-1}}{p_t}(\varphi r^l_{t-1} + (1-\varphi) r^*_{t-1})$ is the real cost of a unit of capital (we will denote real interest rates with "hatted" symbols . Finally, with competitive labor markets,

$$\frac{w_t}{p_t} = (1-\alpha) z \left(\frac{z\alpha}{\hat{r}^K_{t-1}} \right)^{\frac{\alpha}{1-\alpha}} \tag{27}$$

[12] Paravisini, Rappoport and Schnabl (2015) provide empirical evidence of specialization in bank lending, which supports the idea of monopolistic competition in the lending market.

2.4 Banks

We assume that there is a size-one continuum of banks in monopolistic competition in the deposit market and in the loan market. The aggregate bank balance sheet is

$$l_t + b_t^b + m_t = d_t^b + a_t^b + e_t^b \tag{28}$$

where on the asset side (LHS) we have bonds held by the banks b_t^b, required reserves m_t and loans l_t, and on the liability side (RHS) we have bank deposits d_t^b, other bank liabilities (such as bonds) a_t^b, and bank equity e_t^b.

Bonds on the asset and liability side, b_t^b and a_t^b, yield an interest rate r_t^*, whereas reserves yield an interest rate r_t^m determined by the central bank. Required reserves are a fraction ϕ of deposits: $m_t = \phi d_t^b$.

Loans are provided with proportional cost c^l at the nominal interest rate $r_t^l(j)$ for bank j. Deposits are provided with proportional cost c^b at the (nominal) rate $r_t^b(j)$. For now, we assume that costs c^l and c^b are constant. Profits of bank j are

$$\Pi_t^b(j) = (1+r_{t-1}^l(j)-c^l)l_{t-1}(j)+(1+r_{t-1}^*)(b_{t-1}^b(j)-a_{t-1}^b(j))+(1+r_{t-1}^m)m_{t-1}(j)-(1+r_{t-1}^b(j)+c^b)d_{t-1}^b(j) \tag{29}$$

Using the bank balance sheet and the reserve ratio, this can be rewritten as:

$$\Pi_t^b(j) = [(1-\phi)r_{t-1}^* + \phi r_{t-1}^m - (r_{t-1}^b(j) + c^b)]d_{t-1}^b(j) + [r_{t-1}^l(j) - c^l - r_{t-1}^*]l_{t-1}(j) \tag{30}$$

Banks choose the deposit rate and the loan rate to maximize profits. At $t-1$ bank j chooses $r_{t-1}^b(j)$ to maximize time-t profits from deposits in (30), subject to the demand for this bank's deposit (9). Notice that each bank, having size zero, takes the aggregate demand for bank deposits d^b and the aggregate deposit rate r^b as given. Similarly, the loan rate is chosen by maximizing the profits from loans in (30) subject to the loan demand (24). Again each bank takes the aggregate loan demand and aggregate loan rate as given. The two maximization problems are independent. The profit-maximizing deposit rate, the same for each bank, is

$$r_t^b(j) = r_t^b = r^* - (c^b + \phi(r_t^* - r_t^m))\frac{\epsilon^b}{\epsilon^b - 1} \tag{31}$$

Notice, however, that if there is a zero-lower-bound on the nominal interest rate, the above expression should be modified as

$$r_t^b(j) = r_t^b = max\left(0, r^* - (c^b + \phi(r_t^* - r_t^m))\frac{\epsilon^b}{\epsilon^b - 1}\right) \tag{32}$$

The optimal loan rate is

$$r_t^l(j) = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c_l) \tag{33}$$

The result in (31)-(32) allows us to formulate the following Lemma:

Lemma 1: With banks in monopolistic competition, the choice of the deposit rate by each bank is not affected by CBDC. The deposit rate does not change in reaction to a change in the relative liquidity between CBDC and bank deposits, or in reaction to a change in the interest rate paid by CBDC.

The intuition behind this somewhat surprising result is that each bank competes with other banks for deposits, but perceives the aggregate demand for bank deposits (and of CBDC) as fixed, not internalizing how the relative demand for the two monies depends on the interest paid in aggregate by the banking system. However, competition with CBDC implies lower overall demand for bank deposits, so that in equilibrium each bank relies less on deposits and more on other liabilities, such as bank bonds and/or equity.

The loan rate is unaffected by deposits or CBDC altogether. All banks choose therefore the same value (52) of the loan rate, with or without CBDC. The quantity of loans is not affected by CBDC because banks can replace deposits by borrowing from the market at interest rate r^*.

2.5 Government

The government needs to fund a constant exogenous real expenditure g. The government receives central bank profits (seigniorage) $\mathcal{S}$, levies taxes on labor income at rate τ_h and on bank profits at rate τ_b (firm profits are 0 due to perfect competition in the goods markets). It pays interest r_{t-1}^* on the debt contracted in the previous period b_{t-1}^g.[13] The government budget constraint is:

$$\tau_h w_t h_t + \tau_b \Pi_t^b + \mathcal{S}_t + b_t^g + t_t = g + (1 + r_{t-1}^*) b_{t-1}^g \tag{34}$$

The presence of CBDC may increase seigniorage collected by the central bank, in which case the government may be able to finance its expenditure by levying lower taxes. In

[13] Government bonds were not explicitly mentioned as an investment choice for households, since they are assumed to be perfect substitutes of other risk-free bonds.

particular, we will assume that with higher seigniorage the government decides to lower the most distortionary tax, i.e., the tax on labor.

2.6 Central Bank

The central bank issues the monetary base m_t, consisting in bank reserves, as well as CBDC d_t^c, and holds assets a_t^c bearing the risk-free interest rate. Assuming zero equity at the beginning of each period, its balance sheet is $m_t + d_t^c = a_t^c$.

The central bank sets three interest rates: the nominal risk-free rate r_t^*, which determines inflation, the interest on reserves r_t^m, and the interest on CBDC r_t^c. For the risk-free rate we assume a Taylor rule

$$r_t^* = \rho + \phi_\pi(\pi_t - \pi^*) \tag{35}$$

with $\phi_\pi > 1$. Here $\rho \equiv \beta^{-1} - 1$ and π^* is the inflation target. As in the classical monetary economy of Gali (2015), the Taylor rule (35) implies that inflation is uniquely determined as

$$\pi_t = \Sigma_{s+0}^{\infty}\phi_\pi^{-(s+1)}(\hat{r}_{t+s} - \rho) \tag{36}$$

and the real interest rate $\hat{r}_t \equiv r^* + \pi_{t+1}$ is determined by the consumption process

$$(1 + \hat{r}_t)^{-1} = \beta\frac{U'(c_{t+1})}{U'(c_t)} \tag{37}$$

which implies that in steady state $\hat{r} = \rho$ and inflation is at target.

The growth in monetary base is determined by the inflation target and money market equilibrium is simply given by $m_t = \phi d_t^b$. Central bank profits are given by seigniorage

$$\mathcal{S}_t = (r_{t-1}^* - r_{t-1}^m)m_{t-1} + (r_{t-1}^* - r_{t-1}^c - c^c)d_{t-1}^c \tag{38}$$

where c^c is the cost of managing CBDC, and are distributed each period to the government.

3 Steady State Equilibrium

Since there is no shock, the equilibrium is a steady state characterized by the following conditions. Given the wage paid by firms, the interest paid by the risk-free asset, by bank deposits and by CBDC, the tax rates chosen by the government, households make decisions about labor, consumption, savings in the risk-free asset, bank deposits and CBDC to maximize utility. Given the cost of capital and the cost of labor (wage), firms choose capital and labor to maximize profits. Given deposit demand and loan demand, banks choose the rate on deposits and on loans to maximize their profits. The wage is such that labor markets clear. All the equations determining steady state real variables are summarized in the Appendix.

Our purpose is twofold. First, we want to analyze the effect of the introduction of CBDC on the steady state equilibrium, as well as the effect of different CBDC design choices, such as the relative liquidity between CBDC and bank deposits and of the elasticity of substitution between the two monies. Second, we want to find the optimal choices of the government. This is discussed in the next subsection.

3.1 Optimal Government Choices

The objective of the government when introducing CBDC is to maximize welfare, by choosing interest rate on CBDC, and, within possible technological constraints, also its liquidity and substitutability relative to bank deposits. Welfare is defined as a weighted average of the households' and the banker's utility:

$$W = log(c) - \frac{h^{1+\gamma}}{1+\gamma} + \lambda \; log(c^{bo}) \tag{39}$$

The first channel available to the government to improve welfare is seigniorage: as stated in Section 2.5, we assume that government expenditure is exogenous, and higher seigniorage allows the government to lower the (distortionary) labor tax rate. Indeed, the steady state equations (64) and (58) in Appendix B show that the labor tax is reduced by seigniorage, and that a lower labor tax increases labor and therefore consumption.

Seigniorage depends in particular on the liquidity parameters of deposits and CBDC, α_b and α_c, and on the substitutability parameter ϵ_{cb}. Propositions 1 and 2 below (proved in the online Appendix) shed some light on the optimal choices in order to maximize the impact of this channel:

Proposition 1:

If $\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$ and the marginal cost of managing deposits c^c is negligible:

- **The interest rate r^c that maximizes seigniorage is larger than the interest rate on deposits r^b.**
- **If, in addition, $\epsilon_{cb} > 1.5$, the optimal value of r^c is decreasing in α_c.**
- **The peak value of seigniorage in the r^c dimension ($max_{r^c}\ \mathcal{S}$) is increasing in the CBDC liquidity parameter ($\alpha_c^{\epsilon_{cb}}$), and is increasing in the substitutability parameter ϵ_{cb} while $\alpha_c^{\epsilon_{cb}}$ remains fixed.**[14]

The condition $\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$ excludes the region of the parameter space in which the elasticity of substitution between bank deposits and CBDC is very small and/or the liquidity of bank deposits is much lower than the liquidity of CBDC. Intuitively, in the latter region the central bank can almost act as a monopolist and collect high seigniorage by choosing $r^c < r^b$. Technically, it may be a difficult task to design a CBDC with these properties, so that the condition $\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$ seems more realistic. It seems also reasonable to assume a negligible marginal cost of managing CBDC, given that most costs faced by banks, such as branch openings and marketing, would likely be much smaller for CBDC.

Proposition 2:

Suppose that the government can choose the elasticity of substitution ϵ_{cb}, but the relative liquidity between CBDC and bank deposits ($\alpha_c^{\epsilon_{cb}}/\alpha_b^{\epsilon_{cb}}$) is fixed. Under the conditions of Proposition 1 ($\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$ and negligible c^c) the maximum value of seigniorage is achieved in the limit $\epsilon_{cb} \to \infty$ (so that the two monies are perfect substitutes) and r^c is infinitesimally higher than r^b.

Proposition 2 tells us that, in the region of the parameter space defined by $\alpha_b^{\epsilon_{cb}}\epsilon_{cb} > 1$ (intuitively, unless the central bank is able to design a CBDC with very low substitutability with – or much more liquid than – bank deposits) it is optimal, from the point of view of maximizing seigniorage, to design CBDC as a perfect substitute of bank deposits and outcompete the latter by setting the interest on CBDC just infinitesimally higher than the interest on bank deposits.

Seigniorage is however not the only channel available to the government to improve

[14] We always maintain the condition $\alpha_b^{\epsilon_{cb}} + \alpha_c^{\epsilon_{cb}} = 1$. As we think about the implications of different values of the substitutability parameter, it is natural to keep the relative liquidity of the two monies, $\alpha_b^{\epsilon_{cb}}/\alpha_c^{\epsilon_{cb}}$, hence the individual $\alpha_b^{\epsilon_{cb}}$ and $\alpha_c^{\epsilon_{cb}}$, fixed.

welfare. By paying high interest on CBDC, the government/central bank can lower the opportunity cost of holding money. In this case households would hold a higher amount of liquid assets, with the effect of lowering the transaction cost. A lower transaction cost actually stimulates labor supply and increases consumption (see (55) and (58) in Appendix B). To maximize the impact of this channel, the government/central bank should set the interest rate on CBDC equal to the risk-free rate, so that households would stop holding deposits altogether and hold instead enough CBDC to reduce the transaction cost to zero. Setting the CBDC rate equal to the risk-free rate would also maximize the impact of the redistribution channel: with $r^c = r^*$ bank deposits would be fully wiped out in favor of CBDC, and the rents associated to deposits would flow to households.

The redistribution channel provides higher welfare benefits the higher is the initial level of inequality (i.e. the lower is the parameter ζ). We therefore expect the optimal r^c to be decreasing in ζ, the share of banks owned by households.

In contrast, the benefit of a lower CBDC rate – higher seigniorage, translating, everything else equal, in a lower labor tax rate – is higher when government spending, hence labor taxes, are high, given the distortionary effect of taxes on labor. We therefore expect the optimal r^c to be decreasing in government spending. The numerical results presented in Section 5, which cover different scenarios for the labor tax rate τ_h – closely related to government spending g – and ζ, confirm that the optimal value the CBDC rate is decreasing in both parameters.

An analytical proof that both the consumption-maximizing and the welfare-maximizing value of r^c are decreasing in g and ζ, under mild conditions, is provided in the online Appendix (**Proposition E1**).[15]

Table 1: Model Parameters	
Parameter	Description
$r^* = 4\%$	risk-free rate
$A = 0.0111$	Transaction cost parameter
$B = 0.07524$	Transaction cost parameter
$\gamma = 1$	Inverse Frisch elasticity
$\phi = 0.08$	reserve ratio
$\tau_b = 25\%$	tax rate on bank profits
$\varphi = 0.2$	working capital requirement
$r^m = 0$	interest rate on bank reserves
$c^b = 0.25\%$	managing cost of bank deposits
$c^l = 0.5\%$	managing cost of loans
c^c=0.25%	managing cost of CBDC
$\alpha = \frac{1}{3}$	Cobb-Douglas capital share
$\epsilon^b = 1.40$	Elasticity of substitution of bank deposits
$\epsilon^l = 6.67$	Elasticity of substitution of bank loans
$\pi^* = 0$	inflation target

4 Calibration

Table 1 summarizes our parameter choices. The parameters that are most important for our experiment are those affecting money demand and the banking system. In our baseline case we use the values for parameters A and B of the transaction cost estimated for the US economy by Schmitt-Grohe and Uribe (2004), which imply, according to (19), an interest semi-elasticity of money demand equal to -0.05. This is consistent with the estimation on the long-run money demand by Ball (2001), and also with the more recent

[15] One of the conditions amounts to demanding that the transaction cost of consumption has a negligible impact on labor. We need this assumption since, by lowering the CBDC rate to collect seigniorage and decrease the labor tax rate, we also increase the transaction cost of consumption (since money velocity x is decreasing in r^c), which also has a distortionary effect on labor. However, with any realistic calibration of the transaction cost, the latter effect is minimal relative to the effect of taxes.

estimates by Drechsler, Savov and Schnabl (2017), that, similarly to us, focus on the demand for deposits as a function of the deposit spread.[16]

For the banking system, the parameters ϵ^b (elasticity of substitution between deposits of different banks) is calibrated so that the deposit spread (difference between the deposit rate and the risk-free rate) is 2%, an historical average in the US and Europe alike.[17] The parameter ϵ^l (elasticity of substitution between loans of different banks) is calibrated so that the loan spread – difference between the loan rate and the risk-free rate – is 1%. This value is appropriate for the US but is low for other countries; however our results are not sensitive to this parameter, as the loan extension activity by banks is not affected by the introduction of CBDC.

Only indirect data is available to estimate the banks' cost of managing deposits and loans. According to call report data from the Federal Financial institution Examination Council,[18] total operating costs for US banks amount to around 2% of the value of bank assets, and fee income is around 1% of bank assets. If operating costs (net of fees) are equally distributed across assets and liabilities, then we could take 50 bps as an estimate of the cost of operating deposits and loans. However it is likely that operating costs, whose biggest component is given by employee salaries, are much higher on the investment side than on deposits. We therefore use 25 bps as baseline value of the cost of operating deposits (net of fees), but also consider (in Section 5.3) a scenario with the alternative value of 50 bps. We use 50 bps as the operational cost of loans.

The required reserve ratio ϕ is now zero in the United States but was 10% until 2020. It can be much higher in less advanced economies (for example, it is around 40% in Argentina). Our baseline value is 5%, and we consider alternative values in Section 5.3.

Another important parameter for our analysis is the inverse Frisch elasticity γ, which affects the extent to which labor taxation is distortionary. We use a standard value equal

[16] Drechsler, Savov and Schnabl (2017) find that a percentage point increase in the risk-free rate corresponds on average to a 60 bps increase in the deposit spread, and a 4% decrease in the demand for deposits. Hence, a 1% increase in the deposit spread corresponds to a 5% decrease in the demand for deposits.

[17] As pointed out by Drechsler, Savov and Schnabl (2017), the deposit spread in the US is increasing in the risk-free rate. However, a spread around 2% is an historical average. Data on deposit rates in several European countries from the World Bank open database confirm that this is the case also in Europe.

[18] Downloadable at *https://cdr.ffiec.gov*.

to 1 in our baseline scenario, but later consider a range of values from 0.25 to 4. Finally, the value of productivity (expressed by the variable z) is irrelevant to our experiment as it does not affect the *percentage change* in consumption, labor and welfare induced by CBDC, so it can be normalized to 1.

We set the baseline value of the inflation target to 0. As discussed in Section 5.3, our results are largely insensitive to this choice. We will consider different scenarios for the new parameters associated with CBDC, in particular the relative liquidity between CBDC and bank deposits, and their elasticity of substitution.

5 Results

Given our parameter calibration, in this section we outline our numerical results, in terms of the relative demand for bank deposits and CBDC, seigniorage collected by the government, the optimal choice of the interest rate on CBDC and welfare implications.

5.1 CBDC Demand and Seigniorage

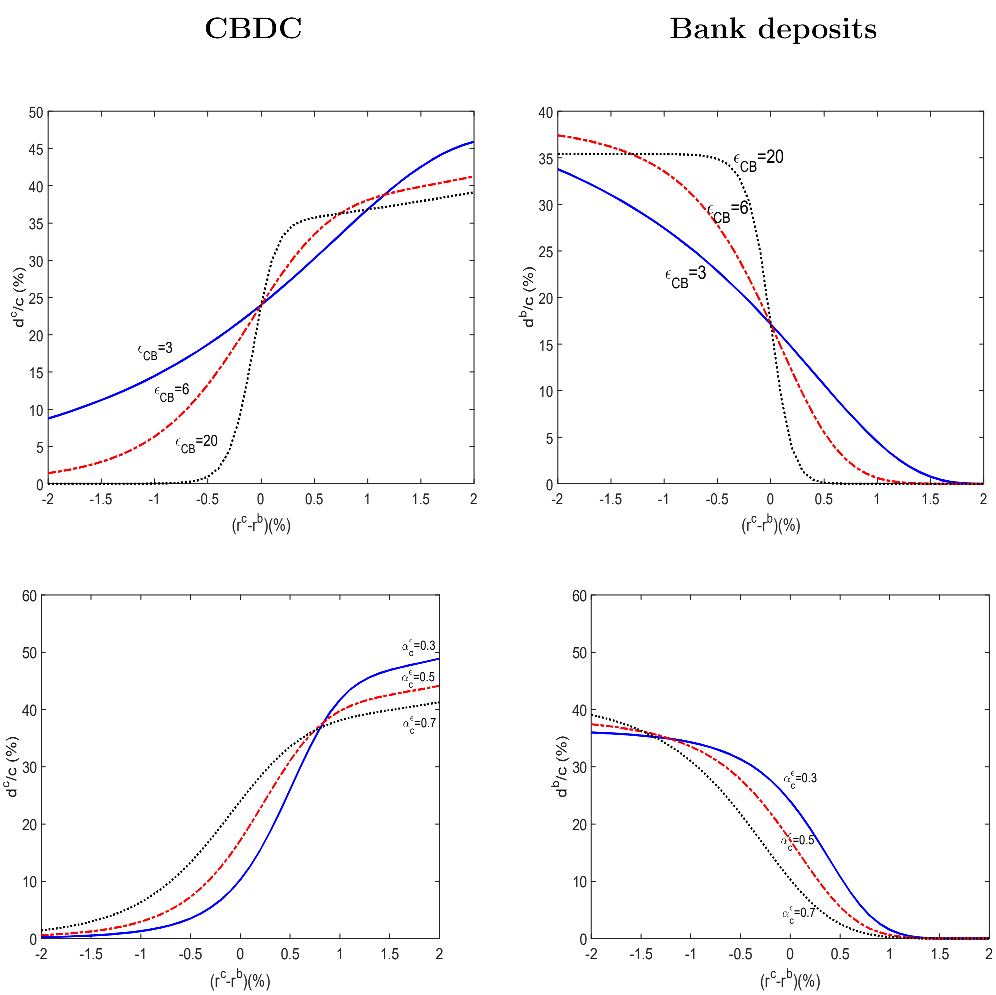
We start by examining the impact of the CDBC interest rate r_t^c on the demand for CBDC and bank deposits for different levels of substitutability and relative liquidity of CBDC. An increase in r^c tends to increase the demand for CBDC and decrease the demand for bank deposits. However, the demand for both instruments is non-monotonic in their elasticity of substitution ϵ_{cb} and in their relative liquidity.

The four panels of Figure 1 show the demand for CBDC (in the two left panels) and bank deposits (in the two right panels) when r^c is within 2 percentage points higher or lower than the interest paid by bank deposits, r^b, i.e., in a range of 4 percentage points below the risk free rate in our calibration.

In the top panels we set $\alpha_b = \alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$ (meaning that CBDC and bank deposits are equivalent from the point of view of liquidity, so that if they paid the same interest, households would allocate the same amount of resources on the two), and show demand curves for three values of ϵ_{cb}: $\epsilon_{cb} = 3$, which we take as a representative case of "low substitutability" between bank deposits and CBDC; $\epsilon_{cb} = 6$ (medium substitutability) and $\epsilon_{cb} = 20$ (high substitutability).

In the bottom panels we set $\epsilon_{cb} = 6$ (the medium substitutability case) and show the results for three different values of α_c (α_b and α_c are related by (3)). These three values are such that $\alpha_c = 0.3^{\frac{1}{\epsilon_{cb}}}$, $\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$, and $\alpha_c = 0.7^{\frac{1}{\epsilon_{cb}}}$, implying that, of the resources allocated in liquid assets (bank deposits or CBDC), households would choose to allocate 30%, 50% and 70%, respectively, in CBDC if the two paid the same interest.

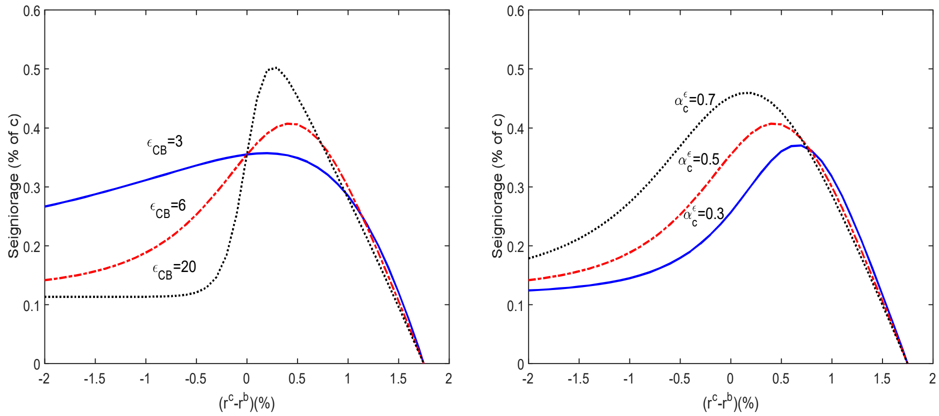
Figure 1: Demand for CBDC and bank deposits



The main takeaways from the two top panels are the following

- When the interest paid by CBDC is below the interest paid by bank deposits, demand for CBDC is decreasing in the elasticity of substitution ϵ_{cb}.

Figure 2: Seigniorage Revenues



The intuition is that the more the two instruments are substitutable, the less households are willing to hold the more costly one, i.e., CBDC.

- When the interest paid by CBDC is higher than the one paid by bank deposits, but the spread $r^c - r^b$ is not too large, the same effect persists, in the other direction: the more substitutable the two instruments, the less households are willing to hold bank deposits.

- When interest paid by CBDC $r^c - r^b$ is large enough, so that r^c is close to the risk-free rate, holdings of CBDC become *decreasing* in ϵ_{cb}. The intuition is that when the two instruments are less substitutable, it takes a higher amount of one to substitute for the other. This may be worthwhile if one instrument (CBDC in this case) is almost costless.

Next, in Figure 2 we examine how seigniorage is affected by the interest rate choice. On the left panel we set $\alpha_c = \alpha_b = 0.5^{\frac{1}{\epsilon_{cb}}}$ (equal liquidity properties for CBDC and bank deposits) and show the three curves of seigniorage as a function of $r^c - r^b$ for the three values of the elasticity of substitution previously considered: $\epsilon_{cb} = 3$, $\epsilon_{cb} = 6$ and $\epsilon_{cb} = 20$. On the right panel we fix $\epsilon_{cb} = 6$ and show the same curves for different values of α_c.

Seigniorage revenues are non-monotonic in r^c, interest paid on CBDC, as the demand for CBDC is increasing and the central bank profit per unit of CBDC is decreasing in r^c. As seen in Figure 2, the location of the interior maximum depends both on the elasticity of substitution between bank deposits and CBDC, and their relative liquidity. The main results emerging from Figure 2 are consistent with Propositions 1 and 2: in all the cases we analyze the peak of seigniorage occurs for $r^c < r^b$; however, as the liquidity of CBDC increases, the value of r^c that maximizes seigniorage gets closer and closer to r^b. Finally, the value of seigniorage at the peak is increasing in both the liquidity of CBDC and the elasticity of substitution.

5.2 Optimal Policy and Welfare gains

The main numerical results about the impact on CBDC on consumption, labor, seigniorage and welfare are summarized in Table 3, along with the optimal choice of interest rate. Specifically, the numbers in Table 3 refer to four key environments: when (pre-CBDC) labor tax rate is 25% and 45%, in two different scenarios for bank ownership that we call "case a" and "case b". The key parameters defining these two scenarios are summarized in Table 2.

Table 2: Households vs Bankers

	"case a"	"case b"
ζ	1	0.1
υ^{hh}/c	4	2
υ^{bo}/c^{bo}	/	23
λ	0	0.11

In "case a" households are effectively the only type of agent, possessing all the wealth and collecting all bank profits ($\zeta = 1$). Household wealth υ^{hh} (holdings of the risk-free asset, deposits and CBDC, in real terms) is 4 times their annual consumption, which is an historical average in the United States; the size of the banker population is $\lambda = 0$. "case b" mirrors instead, in a stylized way, the inequality in the United States, where the top 10% of households hold around 90% of shares, and consume around 4 times as much as the average member of the bottom 90% (see e.g. Piketty and Zucman (2014)). In this scenario the top 10% US households is represented by bankers, and the remaining 90% by households; hence we set the the size of the bankers population as $\lambda = 0.11$ (remember that the size of "households" is normalized to 1) and $\zeta = 0.1$ (share of bank profits collected by households); we set the household wealth v^{hh} at 2 times their consumption and bankers wealth v^{bo} to 23 times their consumption. The latter number is chosen so that bankers' consumption, entirely due to bank profits and interests in this model (see (6)), is 4 times the household consumption.

To obtain the numbers in Table 3, we set the liquidity of CBDC equal to that of bank deposits ($\alpha_c = 0.5^{\frac{1}{\epsilon_{cb}}}$), and the elasticity of substitution at the medium level, $\epsilon_{cb} = 6$.

Table 3: CBDC-induced changes in the economy

τ_h=**25%**	"case a"	"case b"	τ_h=**45%**	"case a"	"case b"
Household Consumption	+27 bps	+54 bps	Household Consumption	+41 bps	+62 bps
Bankers' Consumption	/	-119 bps	Bankers' Consumption	/	-117 bps
Labor	+22 bps	0	Labor	+26 bps	+4 bps
Labor tax rate	-0.14%	-0.12%	Labor tax rate	-0.30%	-0.27%
Optimal $(r^* - r^c)$	0.96%	0.85%	Optimal $(r^* - r^c)$	1.54%	1.42%
Seigniorage	+26 bps	+22 bps	Seigniorage	+45 bps	+ 41 bps
Welfare	+9 bps	+40 bps	Welfare	+20 bps	+47 bps

The plots in Figure 3 show the optimal CBDC rate as a function of the labor tax rate, in "case a" and "case b". We see that the optimal spread on CBDC is higher when the tax rate is higher, and when ζ is higher, i.e. in "case a". Clearly the opposite is true for the optimal r^c: the latter is decreasing in τ_l and ζ, consistently with the discussion in Section 3.1 (and with **Proposition E1** in the online Appendix),

Figure 4 shows the welfare gain when r^c is at the optimal level, for different values of the substitutability parameter. The left panel shows the welfare gain as a function of the labor tax rate, in "case a" and "case b", when $\alpha_c^{\epsilon_{cb}} = 0.5$ and $\epsilon_{cb} = 6$. The right panel shows the same, when setting $\epsilon_{cb} = 20$. We see that the welfare gain in this case is increasing in the elasticity of substitution, although very mildly. In "case a", the welfare gain ranges from a modest 7-8 bps when the labor tax rate is 20% to a more significant 18-20 bps when the labor tax rate is 45%. On the other hand, in "case b" the welfare gain ranges between 39-40 bps (when $\tau_h = 20\%$) to 47-50 bps (when $\tau_h = 45\%$).

Figure 5 shows how the welfare gain at the optimal interest level r^c changes with

Figure 3: Optimal CBDC rate

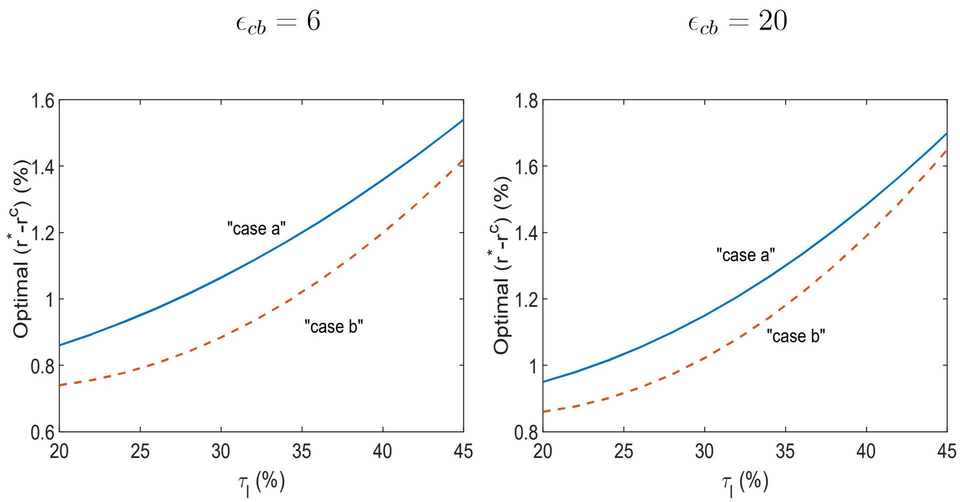
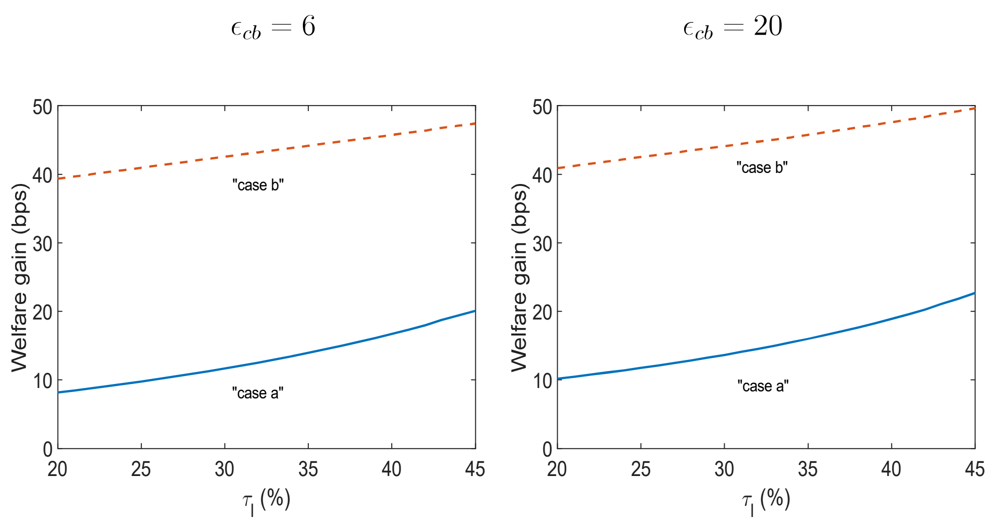


Figure 4: Welfare gain



liquidity and substitutability. We observe that, if the liquidity of CBDC is low relative to that of bank deposits, the welfare gain is quite sensitive to the elasticity of substitution between CBDC and bank deposits. Intuitively, if CBDC is significantly less liquid than bank deposits, to make CBDC attractive we need to set the interest paid by CBDC significantly higher than the interest paid by bank deposits; but if the substitutability between the two is low, demand for bank deposits continues to be high unless r^c is very close to the risk-free rate. This means that the seigniorage the central bank can collect is necessarily low, which lowers the welfare gain, especially when labor taxes are at the high end of the spectrum. The figure also shows that, everything else equal, welfare increases with CBDC liquidity and substitutability. However, if the two monies are very substitutable and CBDC is at least as liquid as bank deposits, no big gains can be achieved by further increasing the liquidity of CBDC. This may be relevant since – although disregarded in this model – it seems likely that increasing the liquidity of CBDC might involve higher costs for the central bank.

Finally, Figure 6 shows how consumption, welfare and banks' profits depend on the choice of the interest rate on CBDC.[19]

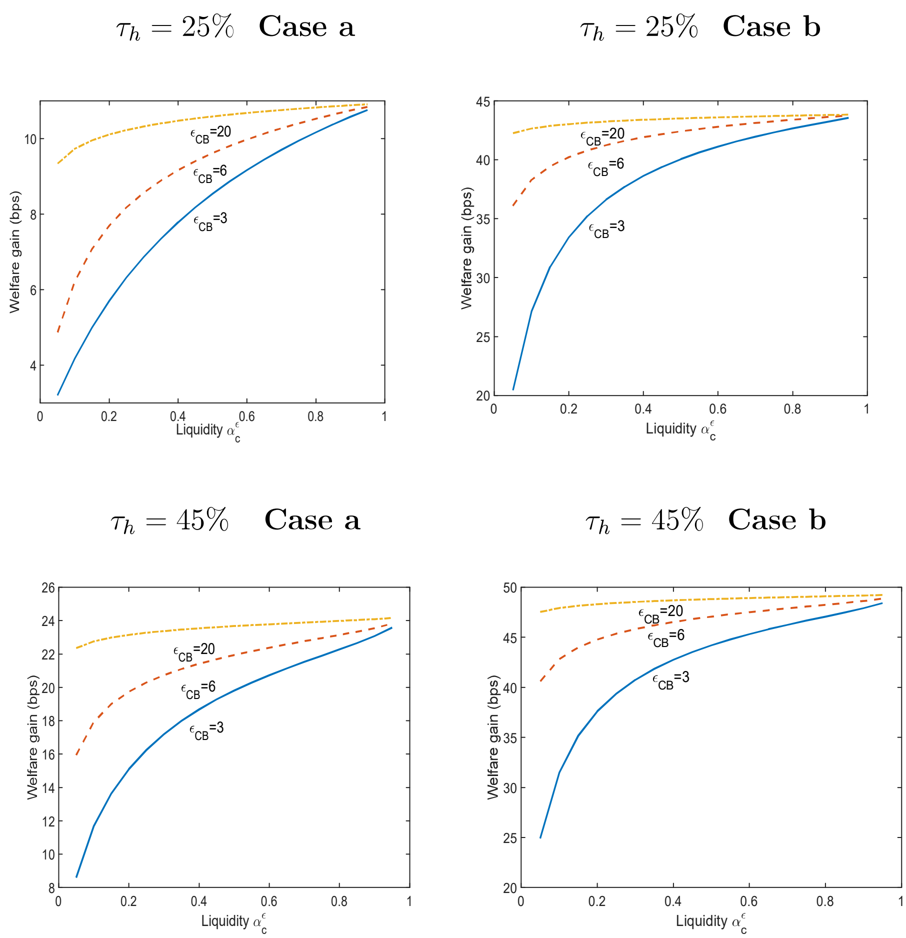
Table 4 shows the welfare improvement brought by CBDC with some alternative parameter choices. One quantity that has a significant impact on results is the Frisch elasticity of substitution, i.e., the inverse of the parameter γ, which is equal to 1 in the baseline case. We consider here two alternative values: $\gamma = 0.25$ (corresponding to Frisch elasticity equal to 4, among the highest values considered in the literature) and $\gamma = 4$ (Frisch elasticity equal to 0.25, in the low range of estimated "micro-elasticities"). As is intuitive, CBDC has the potential to bring higher welfare improvement when the elasticity is high, i.e., when taxation has a stronger distortionary effect on labor. Welfare improvements in "case a" are indeed higher when $\gamma = 0.25$ (and lower when $\gamma = 4$). However, in "case b", the welfare improvement is essentially independent of the Frisch elasticity: in this case, to maximize the redistribution from bankers to non-bankers it is optimal to set the rate on CBDC close to the risk-free rate. However, this involves small seigniorage collection, hence small tax reduction.[20]

The parameter that has the biggest impact on results is the interest semi-elasticity of money demand, governed by the parameter B of the transaction cost (see (19)). A higher semi-elasticity means that the distortion associated with the low interest on money has stronger effects on the economy, so CBDC, by paying interest close to the risk-free rate, has the potential to bring bigger welfare improvements. We consider here a value of the

[19]Our simplified model for banks identifies banks profits with net interest income (NII), abstracting from all other costs. As the figure shows, the order of magnitude for banks' profits in the model is around 1.5-2% of consumption, comparable with banks' NII in the United States, but much higher than actual banks' profits.

[20]Similarly, if we abstract from distortionary taxes and assume that all taxes are lump-sum, we obtain a lower welfare improvement in "case a", since the channel through which seigniorage can improve welfare is inactive, but essentially unchanged welfare improvement in "case b".

Figure 5: Welfare gain vs Liquidity and Elasticity of Substitution



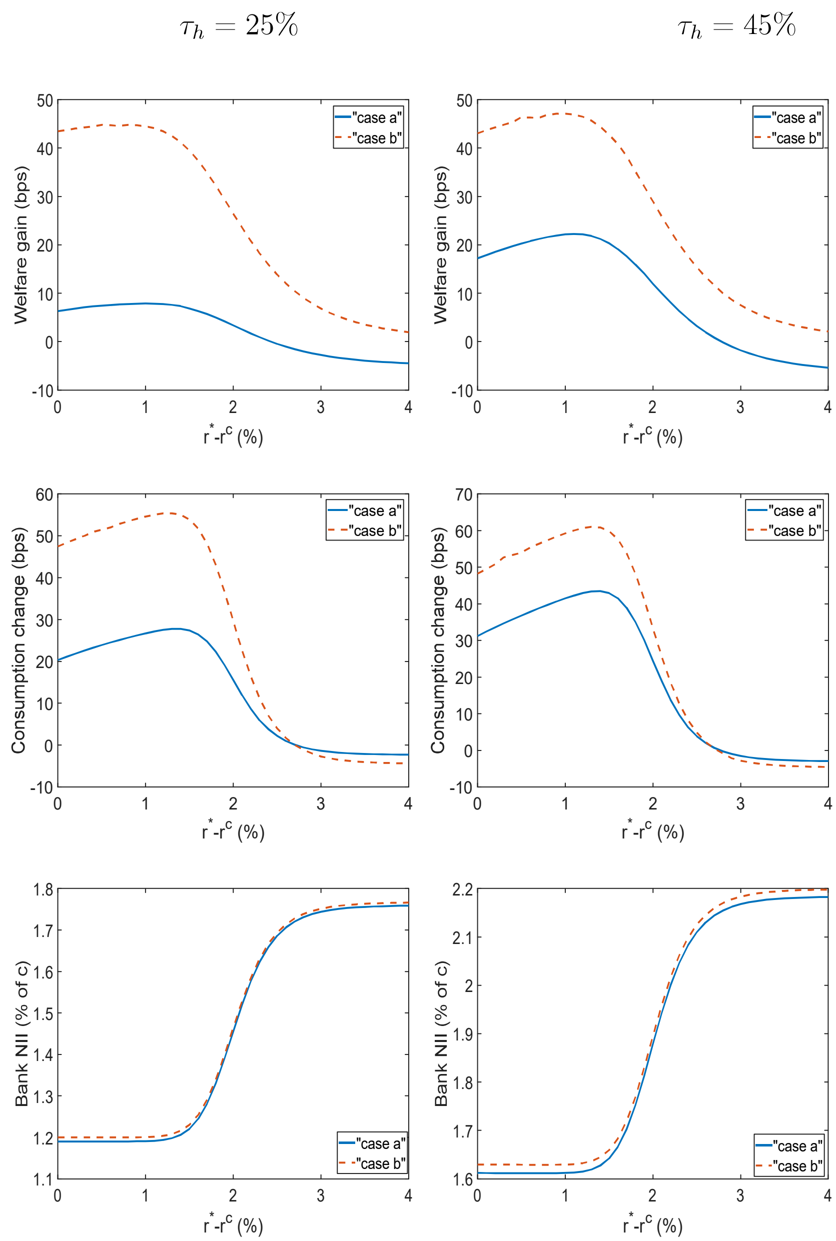
semi-elasticity equal to -0.12.[21] As labor taxes are high (45%) and at the same time the interest semi-elasticity is high, the welfare gains induced by CBDC reach 35 bps in "case a" and 68 bps in "case b".

Moreover, we show results obtained with alternative values of the cost of managing deposits and loans, reserve requirement, the corporate tax rate (used in our model as the tax rate on bank profits), banks' degree of competition in the loan market, the working capital requirement for firms –affecting the extent to which firms are dependent on bank loans– and households' wealth as a fraction of annual consumption.

We see that the impact of these parameters is not large. However, parameters af-

[21] For example, this is the value of the semi-elasticity estimated by Benati et al. (2021) for Switzerland.

Figure 6



fecting deposits have some impact on our results. In general, with parameter values implying that banks' rent collection on deposits is high (low reserve ratio, low cost of managing deposits) the introduction of CBDC has a stronger welfare impact. Similarly, if the corporate tax rate is low, implying a stronger degree of inequality between households and bankers, the introduction of CBDC has a higher potential of smoothing such inequality and improving welfare. We also report the welfare improvement in a world with no distortionary taxes. In this case the optimal CBDC rate would be equal to the risk-free rate, since there would be no scope for seigniorage to reduce taxes.

Finally, results are essentially insensitive to the inflation target, as we discuss in the next section.

5.3 Discussion

We would like to dedicate this final discussion to two topics. First, we would like to clarify why, in the context of the model, the benefits of CBDC are largely insensitive to the inflation target. Equations (51)-(64) in the Appendix show that steady-state real variables depend on the real rate $\hat{r} = \beta^{-1} - 1$; the real cost of capital $\hat{r}^K$; the spreads $r^* - r^b$, $r^* - r^c$ and $r^* - r^m$.

In our model $r^* - r^b$ is independent of inflation, as banks optimally choose r^b at a constant spread to r^* (see (51)). However, this is not strictly true for the loan rate r^l, which affects r^K. In fact, given (52) and (53), we find that the real cost of capital is given by

$$\hat{r}^K = (1-\varphi)r^* + \varphi r^l - \pi^* = \hat{r} + \varphi \frac{\epsilon^l}{\epsilon^l - 1} c^l + \varphi \left(\frac{\epsilon^l}{\epsilon^l - 1} - 1 \right) \pi^* \tag{40}$$

Hence, the real cost of capital is independent of inflation only in the perfect-competition limit $\epsilon^l \to \infty$. In our calibration the effect of inflation is however numerically small: banks' market power is such that $\frac{\epsilon^l}{\epsilon^l - 1} = 1.17$, and the fraction of firm capital financed by bank loans is $\varphi = 0.2$, meaning that one extra point of inflation affects the loan rate by 17 basis points and the cost of capital by only 3.5 basis points. Moreover, this is true before and after the introduction of CBDC, which explains why inflation is not an important factor in determining the impact of CBDC on the economy. Finally, the spreads $r^* - r^c$ and $r^* - r^m$ are policy choices. Since inflation enters the steady-state equations only through its effect on $\hat{r}^K$, if we can neglect the latter (i.e. in the limit $\epsilon^l \to \infty$), the optimal choice for $r^* - r^c$ and $r^* - r^m$ is also independent of inflation. In fact, in our alternative scenario with 2% inflation target we do not detect any significant change in the welfare-maximizing value of $r^* - r^c$. To sum up, as the inflation target moves, real variables are affected only through the change in the loan spread, but this change is numerically minor, and in particular it is negligible in assessing the effect of the introduction of CBDC in the economy.

The second topic we would like to discuss regards the robustness of our results with respect to the separation of deposit and credit decisions. In our model bank profits (30) separate into profits from deposits and from loans, since the demand for deposits and loans depends only on the corresponding rate, and the marginal funding rate for loans is equal to the policy rate.

Part of the literature has departed from this modeling choice. For example, in the model of Wang, Whited, Wu and Xiao (2022), the marginal funding cost for loans is increasing in the amount that banks borrow from funding sources other than deposits. This is motivated by the fact that non-reservable borrowing does not benefit from FDIC deposit insurance and is therefore more risky (see also Kashyap and Stein (1995)). Additionally, as deposits fluctuate, there may be switching costs as banks need to find alternative sources of funding (see also Drechsler, Savov, Schnabl (2017)). One might therefore think that the introduction of CBDC, by reducing the demand for bank deposits, would increase the marginal cost of funding for loans.[22]

As a robustness test for our model, in the online Appendix we present an extension along the lines of Wang, Whited, Wu and Xiao (2022), and find that the main results are very similar as in the baseline model. The reason is that, to avoid a big increase in the marginal funding cost, i.e. in the interest on non-reservable borrowing, banks would try to attract more deposits by raising the deposit rate (Lemma 1 does not apply in this model). In other words, they would try to actively compete with CBDC, so that disintermediation would be only partial. We find that the increase in the loan rate and the cost of capital would be modest, and that its welfare cost would be in some cases even outweighed by the welfare benefit of the increase in the deposit rate.

[22] In our view this is not necessarily the case: on the one hand switching costs would apply only in a transition phase; on the other hand, when and if banks are disintermediated, the government may decide to extend some form of insurance to bank liabilities other than deposits.

Table 4: CBDC-induced welfare changes with alternative parameter values				
	$\tau_h = 25\%$ case a	$\tau_h = 25\%$ case b	$\tau_h = 45\%$ case a	$\tau_h = 45\%$ case b
Baseline	+9 bps	+40 bps	+20 bps	+47 bps
$\gamma = 0.25$	+28 bps	+41 bps	+31 bps	+49 bps
$\gamma = 4$	+2 bps	+40 bps	+15 bps	+46 bps
$\iota = -0.12$	+21 bps	+66 bps	+35 bps	+68bps
$c_d = 0.005$	+7 bps	+34 bps	+16 bps	+36 bps
Reserve ratio = 0	+11 bps	+43 bps	+22 bps	+51 bps
Reserve ratio = 10%	+8 bps	+35 bps	+17 bps	+43 bps
$\tau_b = 35\%$	+8 bps	+34 bps	+17 bps	+42 bps
$\tau_b = 15\%$	+10 bps	+45 bps	+23 bps	+52 bps
$\epsilon^l = 4$	+9 bps	+40 bps	+20 bps	+47 bps
$\varphi = 0.3$	+9 bps	+40 bps	+20 bps	+47 bps
$\pi^* = 2\%$	+9 bps	+40 bps	+20 bps	+47 bps
	case a		case b	
Lump-sum taxes	+3bps		+35bps	

6 Conclusion

There is an intense discussion in policy circles about the potential introduction of a broad retail CBDC. While there are various microeconomic aspects related to its implementation, in this paper we consider its macroeconomic implications. Most likely, CBDC will not be a perfect substitute of cash or bank deposits. This imperfect substitutability is a key element in our analysis and we show the impact of CBDC under various degrees of substitutability. In our welfare analysis, we find that CBDC could be an instrument to mitigate two distortions in the economy: distortionary taxation and the opportunity cost of holding money, which is much higher than the cost of providing money. Clearly this benefit would be higher, the higher the extent of the distortions. In our benchmark case, we find that the benefits of CBDC in reducing distortions would be modest: even in economies with high labor taxes (around 45%), welfare would improve at most by 20 bps in consumption terms. Instead, we found higher welfare gains from the redistribution of rents associated to deposits from bankers to non-bankers. The welfare improvement to non-bankers (and to the whole population in the limit in which bankers are a negligible minority) could reach around 50 bps when taking into account this channel. The welfare gains might be higher in countries in which the Frisch elasticity and/or the interest semi-elasticity of money demand is very high. Indeed, these are the cases in which the two distortions mentioned above have stronger effect on the economy.

Appendix

A. Household FOCs

FOC with respect to consumption

$$\frac{1}{c_t} = \lambda_t(1 + s(x_t) + x_t s'(x_t)) \tag{41}$$

Specialized to the case of the transaction cost in the form (8), (A.10) becomes

$$\frac{1}{c_t} = \lambda_t(1 + 2Ax_t - 2\sqrt{AB}) \tag{42}$$

FOC with respect to hours worked

$$h_t^\gamma = \lambda_t W_t(1 - \tau_h) \tag{43}$$

FOC with respect to bank deposits d_t^b

$$\lambda_t \left(1 - (Ax_t^2 - B)\alpha_b \left(\frac{d}{d^b}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \lambda_{t+1}(1 + r_t^b) \tag{44}$$

FOC with respect to CBDC d_t^c

$$\lambda_t \left(1 - (Ax_t^2 - B)\alpha_c \left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \lambda_{t+1}(1 + r_t^c) \tag{45}$$

FOC with respect to the risk-free asset a_t

$$\lambda_t = \lambda_{t+1}(1 + r^*) \tag{46}$$

(42), (43), (44), (45) and (46) imply the three Euler equations:

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})}\left(1-(Ax_t^2-B)\alpha_b\left(\frac{d}{d_b}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \beta(1+r_t^b)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \quad (47)$$

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})}\left(1-(Ax_t^2-B)\alpha_c\left(\frac{d}{d_c}\right)^{\frac{1}{\epsilon_{cb}}}\right) = \beta(1+r_t^c)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \quad (48)$$

$$\frac{1}{c_t(1+2Ax_t-2\sqrt{AB})} = \beta(1+r^*)\frac{1}{c_{t+1}(1+2Ax_{t+1}-2\sqrt{AB})} \quad (49)$$

and the labor/leisure tradeoff condition

$$h_t^\gamma = \frac{W_t(1-\tau_h)}{c_t(1+2Ax_t-2\sqrt{AB})} \quad (50)$$

B. Steady state equations

In steady state, the real interest rate is $\hat{r} = \beta^{-1} - 1$ and the nominal risk-free rate is r^* such that $\frac{1+r^*}{1+\pi^*} = 1 + \hat{r}$; the central bank pays a constant rate r^m on bank reserves and r^c on CBDC (these can be considered as parameters of the problem); banks pay a constant rate r^b on deposits, given by

$$r^b = r^* - (c^b + \phi(r^* - r_m))\frac{\epsilon^b}{\epsilon^b - 1} \quad (51)$$

and demand a constant loan rate

$$r^l(j) = \frac{\epsilon^l}{\epsilon^l - 1}(r^* + c_l) \quad (52)$$

The unit cost of capital is

$$\hat{r}^k = \varphi r^l + (1-\varphi)r^* - \pi^* \quad (53)$$

Given these rates, households choose a constant money velocity

$$x = \sqrt{\frac{r^* - r^{comp} + B(1+r^*)}{(1+r^*)A}} \quad (54)$$

The other relevant variables of the model, consumption c, labor h, capital k, real wages $\hat{w} \equiv \frac{w}{p}$, real loans $\hat{l}$,real bank deposits $\hat{d}^b$, CBDC $\hat{d}^c$, real bank profits $\hat{\Pi}$ (we use the hatted symbols for these variables to distinguish them from their nominal counterparts) are determined by the following equations

$$
\begin{aligned}
c(1+s(x)) &= (1-\tau_h)\hat{w}h + \upsilon^{hh}\ \hat{r} - \hat{d}^b(r^* - r^b) - \hat{d}^c(r^* - r^c) + \zeta(1-\tau_b)\hat{\Pi}^b - \hat{t} \\
&= (1-\tau_h)\hat{w}h + \upsilon^{hh}\ \hat{r} - \hat{d}(r^* - r^{comp}) + \zeta(1-\tau_b)\hat{\Pi}^b - \hat{t} && (55) \\
\hat{d}^b &= \frac{c}{fx} \quad with \quad f = \left(\frac{r^* - r^b}{\alpha_b(r^* - r^{comp})}\right)^{\epsilon_{cb}} && (56) \\
\hat{d}^c &= \left(\frac{\alpha_c}{\alpha_b} \times \frac{r^* - r^b}{r^* - r^c}\right)^{\epsilon_{cb}} \hat{d}^b && (57) \\
h^\gamma &= \frac{\hat{w}(1-\tau_h)}{c(1+s(x)+xs'(x))} && (58) \\
k &= \left(\frac{z\alpha}{\hat{r}^K}\right)^{\frac{1}{1-\alpha}} h && (59) \\
\hat{w} &= (1-\alpha)z\left(\frac{z\alpha}{\hat{r}^K}\right)^{\frac{\alpha}{1-\alpha}} && (60) \\
\hat{l} &= \varphi k && (61) \\
\hat{\Pi}^b &= ((r^* - r^b - c^b) - \phi(r^* - r^m))\hat{d}^b + (r_l - r^* - c^l)\hat{l} && (62) \\
c^{bo} &= \frac{1-\zeta}{\lambda}\Pi^b + \hat{r}\upsilon^{bo} && (63)
\end{aligned}
$$

Finally, we assume that the government, in order to finance an exogenous expenditure g, chooses an exogenous tax on profits τ_b and sets the labor tax τ_h endogenously, so that

$$
\tau_h = \frac{g - \tau_b\hat{\Pi}^b - (r^* - r^m)\hat{m} - (r^* - r^c - c^c)\hat{d}^c}{\hat{w}h} \qquad (64)
$$

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